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Shear Transfer In Concrete Having Reinforcement At An Angle To The Shear Plane

By
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Synopsis: This paper presents an experimental and analytical study of shear transfer across a plane inclined at an arbitrary angle to a parallel or orthogonal array of reinforcement. Twenty three push-off type specimens were tested, having either orthogonal or parallel arrays of reinforcement arranged at various angles to the shear plane. About two thirds of the specimens were cracked along the shear plane before being subject to shear. Hypotheses previously developed (4) for shear transfer in concrete having reinforcement at right angles to the shear plane, were extended to the cases under study. The results of the tests indicated that these hypotheses for shear transfer behavior are applicable to the case of concrete having reinforcement at an angle to the shear plane, provided the component of the bar forces parallel to the shear plane is taken into account in calculating shear resistance.

Keywords: cracking (fracturing); loads (forces); precast concrete; reinforced concrete; reinforcing steels; research; shear properties; stress transfer; structural analysis; tests.

18 shear in reinforced concrete

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INTRODUCTION

Situations can arise in design where the transfer of shear across a specific plane must be considered. Examples of such situations in precast concrete construction have been discussed by Birkeland and Birkeland (1) and by Mast (2), who also pointed out the need to consider the case of a crack existing in the shear plane before shear is applied. Such cracks can occur for a variety of reasons unrelated to shear, such as tension forces caused by restraint of shrinkage or temperature deformations, etcetera.

In the design of prestressed concrete reactor containment vessels for lateral forces, it is necessary to calculate the in-plane shear which can be transferred across cracks arbitrarily inclined to the direction of the longitudinal and circumferential reinforcement usually present near the inner and outer faces of the shell. In the general case, the shears are accompanied by in-plane tension forces. Lacking data on actual behavior, designers faced with the problem have produced conservative behavior models for design purposes. As a step toward solution of the general problem, this paper presents an experimental and analytical study of shear transfer across a plane inclined at an arbitrary angle to the reinforcement, a situation which may also occur in precast concrete connection design. Both parallel and orthogonal arrays of reinforcement were considered in the study.

EXPERIMENTAL STUDY

Test Specimens

The test specimens were designed to simulate the conditions existing in a shear plane crossing a large array of equally spaced reinforcing bars at an arbitrary angle θ as in Fig. 1. They took the form of push-off specimens reinforced with closed #3 bar stirrups arranged parallel to one another or orthogonally at various angles to the shear plane as shown in Fig. 2. When loaded axially as shown by the arrows V in Fig. 2, shear without moment is produced on the shear plane indicated. If adequate longitudinal and end reinforcement is provided, failure of this type of specimen occurs along the shear plane. The length of the shear plane varied with the angle θ between the stirrups and the shear plane and was equal to the length of the shear plane contained within the 10 inch square superposed on the reinforcement array in Fig. 1. This was done so that the area of concrete involved along with each reinforcing bar should be the same as when a large array of reinforcing bars is crossed by an arbitrarily inclined shear plane. Closed stirrups were used to ensure that the full yield strength of the reinforcement could be developed in tension where it crossed the shear plane.

Three series of specimens with orthogonal reinforcement and two with parallel reinforcement were tested. The spacing of the orthogonal reinforcement was the same in both directions and was 5 inches for Series 1 and 2 and 2-1/2 inches for Series 3. The spacing of the parallel stirrups in Series 4 and 5 was 5 inches. The details of the specimens are given in Tables 1 and 2. All the specimens were 7 inches thick and 14 inches wide. The specimens of Series 1, 3 and 4 were cracked along the shear plane by the application of a transverse line load before being tested. The specimens of Series 2 and 5 were uncracked at the commencement of the shear test. The concrete had a nominal compressive strength of 4000 psi at the time of test. It was made using Type III Portland cement, 3/4 inch maximum size glacial outwash gravel and natural sand. The mix proportions were 1.0: 2.87: 3.54 (cement: sand: gravel) with water added to produce a 3-inch slump. The stirrups were made from #3 rebar which conformed to ASTM Specification A615 and had a yield strength of 50 ksi.

Test Procedures

The specimens were loaded monotonically to failure in about 35 increments, using a hydraulic testing machine as shown in Fig. 3. The load was applied through the plates and rollers in order to prevent lateral confinement of the specimen by the testing machine structure. After each increment of load was applied, measurements were made of slip along the shear plane, separation across the shear plane, and of average strain in 10-inch gage lengths aligned with the stirrup reinforcement. The measurements were made using mechanical gages attached to reference points epoxied to the face of the specimen as may be seen in Fig. 3.

Test Results

Typical shear stress-slip curves for specimens with and without cracks in the shear plane before testing are shown in Figs. 4 and 5. There was slip from the beginning of the test in the case of the initially cracked specimens. In the initially uncracked specimens there was no relative movement between the concrete on opposite sides of the shear plane until diagonal tension cracks occurred at shear stresses of between 400 and 600 psi. These short diagonal tension cracks crossed the shear plane at intervals of a few inches and inclined at from 25 to 45 degrees. After diagonal tension cracking, relative movement (subsequently referred to as "slip) occurred as a result of rotation of the short concrete struts formed by the diagonal tension cracks. No diagonal tension cracks occurred in the initially cracked specimens.

The load at which the slip increased continuously without increase in load was taken to be the ultimate load. At ultimate, some compression spalling was observed in the region of diagonal tension cracking. The ultimate strengths attained in the tests are given in

Table 3, both as the total shear on the shear plane, V_u, and as an average ultimate shear stress, v_u, obtained by dividing V_u by the area of the shear plane.

Although the mechanical extensometers could not measure the actual local strain in the reinforcement where it crossed the shear plane, the readings obtained indicated the following trends in behavior:

(a) For initially cracked specimens - the bars inclined at both 26.5° and 45° were in substantial compression. The bars at 63.5° were only lightly strained, some in compression, some in tension. Bars inclined at 90° or more to the shear plane were subject to considerable tension strains.

(b) For initially uncracked specimens - the bars inclined at 26.5° were subject to small amounts of compression. The bars at 45° were subject to a very small tension strain. Those at 63.5° were subject to about three times as much tension as those at 45°, and bars at greater angles to the shear plane were subject to very substantial tension strains at failure.

HYPOTHESES FOR BEHAVIOR

Shear Transfer Behavior of Initially Uncracked Concrete With Orthogonal Reinforcement at Angle θ to the Shear Plane

External loads are assumed to cause a shear stress v along the shear plane and direct stresses σ_{Ny} and σ_{Nx} parallel to and normal to the shear plane, respectively. Under increasing shear, the principal tensile stress in the concrete becomes equal to the tensile strength of the concrete and several short diagonal tension cracks occur along the length of the shear plane and inclined to it at an angle α. This situation is shown in Fig. 6. The angle α will depend upon the particular combination of v, σ_{Nx} and σ_{Ny} existing at the time of cracking.

When the shear is further increased, a truss action develops as shown in Fig. 7(a). Diagonal struts of concrete are formed by the short, parallel diagonal tension cracks. When shear acts on the truss, the struts tend to rotate, causing a displacement of the concrete on one side of the shear plane relative to that on the other. This relative displacement produces stresses in the reinforcement crossing the shear plane. The resultant of the reinforcing bar forces provides the forces T and T_v at the ends of each strut normal to and parallel to the shear plane respectively. Because the diagonal struts are continuous with the concrete on both sides of the shear plane, there will be both compression and transverse shear in the struts. The applied shear is therefore resisted by the component of the reinforcing bar forces parallel to the shear plane, plus the components of the strut compression and shear forces acting parallel to the shear plane as shown in Fig. 7(b).

When reinforcement is provided normal to the shear plane, it has been found (3) to develop its yield strength at ultimate, provided a failure of the concrete does not occur first. It is therefore conservative to assume that at ultimate the strain in reinforcement normal to the shear plane is equal to its yield strain, ε_y. The following additional assumptions are also made to enable the stress in bars inclined to the shear plane to be calculated:

1. The stress in the reinforcement is proportional to the component of the relative displacement of concrete on the two sides of the shear plane in the direction of the reinforcement.

2. The relative displacement at ultimate, δ_u, is constant and equal to that necessary to produce a strain ε_y in reinforcement normal to the shear plane.

Since the relative displacement occurs as a result of rotation of the inclined concrete struts, it will occur in a direction at right angles to the struts. This is borne out by observation. The direction of the relative displacement is therefore at $(90 + \alpha)^\circ$ to the shear plane. Referring to Fig. 6(b) and using the foregoing assumptions, the strain at ultimate ε_s in reinforcement at angle θ to the shear plane is given by:

$$\varepsilon_s = C_1 \delta_u \cos(90 + \alpha - \theta)$$

where C_1 is a constant

when $\theta = 90^\circ$, $\varepsilon_s = \varepsilon_y$

$$\therefore \varepsilon_y = C_1 \delta_u \cos \alpha$$

Hence we may write

$$\varepsilon_s = \varepsilon_y \sec \alpha \cos(90 + \alpha - \theta) \quad \ldots\ldots\ldots\ldots \quad (1)$$

(+ tension, - compression)

Equating ε_s to $-\varepsilon_y$ in Equation (1), we find that the reinforcement yields in compression when θ is equal to or less than $(2\alpha - 90)^\circ$. The following expressions can now be written for the stress at ultimate f_s in reinforcement at an angle θ to the shear plane:

$$\left.\begin{array}{ll} 0 < \theta < (2\alpha - 90)^\circ, & f_s = -f_y \\ (2\alpha - 90)^\circ \le \theta \le 90^\circ, & f_s = f_y \sec\alpha \cos(90 + \alpha - \theta) \\ 90^\circ \le \theta \le 180^\circ, & f_s = f_y \end{array}\right\} \quad \ldots \quad (2)$$

Shear transfer failure will finally occur when the concrete struts fail under the combined action of the compression and shear in the struts, while the reinforcement is stressed as indicated above.

Consider an element of concrete lying in the shear plane at the middle of the thickness of a strut. With reference to coordinates x' and y', the stresses acting on the element will be as shown in Fig. 7(c). They comprise a compression stress $\sigma_{y'}$ acting parallel to the direction of the diagonal tension cracks and shear stresses $\tau_{x'y'}$ oriented as shown. Because the faces of the strut formed by the diagonal tension cracks are unloaded free surfaces, $\sigma_{x'}$ is zero. It has been shown elsewhere (4) that pairs of values of $\sigma_{y'}$ and $\tau_{x'y'}$ at failure of the concrete can be obtained from the concrete failure envelope using a simple geometric construction.

The state of stress in the element on the shear plane can also be expressed as σ_x, σ_y and τ_{xy} with respect to the axes x and y, normal and parallel to the shear plane respectively. These stresses can be stated in terms of $\sigma_{y'}$ and $\tau_{x'y'}$ as follows:

$$\sigma_x = \sigma_{y'} \sin^2\alpha - 2\tau_{x'y'} \sin\alpha . \cos\alpha \quad \ldots\ldots\ldots\ldots \quad (3)$$

$$\sigma_y = \sigma_{y'} \cos^2\alpha + 2\tau_{x'y'} \sin\alpha . \cos\alpha \quad \ldots\ldots\ldots\ldots \quad (4)$$

$$\tau_{xy} = \sigma_{y'} \sin\alpha . \cos\alpha - \tau_{x'y'}(\cos^2\alpha - \sin^2\alpha) \quad \ldots\ldots \quad (5)$$

When there is no externally applied direct stress σ_{Nx}, α is usually about 45°. In this case:

$$\sigma_x = \sigma_{y'}/2 - \tau_{x'y'} \quad \ldots\ldots\ldots\ldots \quad (3a)$$

$$\sigma_y = \sigma_{y'}/2 + \tau_{x'y'} \quad \ldots\ldots\ldots\ldots \quad (4a)$$

$$\tau_{xy} = \sigma_{y'}/2 \quad \ldots\ldots\ldots\ldots \quad (5a)$$

Now at failure, σ_x is the direct stress acting across the shear plane as a result of the stresses in the reinforcement, plus any externally applied direct stress σ_{Nx} acting across the shear plane at failure, i.e.:

$$\sigma_x = \frac{F}{lw} + \sigma_{Nx} \quad \ldots\ldots\ldots\ldots \quad (6)$$

where l and w are the length and width of the shear plane and F is the total reinforcement force acting across the shear plane.

$$F = \left(\frac{l}{s_a} \sin\theta\right) A_{sa} f_{sa} \sin\theta + \left(\frac{l}{s_b} \cos\theta\right) A_{sb} f_{sb} \cos\theta \quad \ldots \quad (7)$$

$$\therefore \frac{F}{lw} = \frac{A_{sa} f_{sa}}{s_a w} (\sin^2\theta) + \frac{A_{sb} f_{sb}}{s_b w} (\cos^2\theta) \quad \ldots\ldots\ldots\ldots \quad (8)$$

where f_{sa} and f_{sb} are the stresses in, and s_a and s_b are the spacing of the reinforcing bars crossing the shear plane at angles θ and

(90 + θ) respectively. f_{sa} is calculated using Equation (2), f_{sb} will be equal to f_y. Hence Equation (6) may be restated as:

$$\sigma_x = \frac{A_{sa}f_{sa}}{s_a w}(\sin^2\theta) + \frac{A_{sb}f_y}{s_b w}(\cos^2\theta) + \sigma_{Nx} \quad \text{...........} \quad (6a)$$

At failure τ_{xy} is the shear stress in the shear plane at the center of the strut. We may then write

$$\frac{V_u - F_v}{lw} = K\tau_{xy} \quad \text{.................................} \quad (9)$$

or

$$v_u = \frac{V_u}{lw} = \frac{F_v}{lw} + K\,\tau_{xy} \quad \text{.............................} \quad (10)$$

where F_v is the component of the bar forces parallel to the shear plane, opposing shear, and K is a coefficient.

$$F_v = -(\frac{l}{s_a}\sin\theta)A_{sa}f_{sa}\cos\theta + (\frac{l}{s_b}\cos\ )A_{sb}f_{sb}\sin\theta \quad \text{....} \quad (11)$$

$$\frac{F_v}{lw} = \sin\theta.\ \cos\theta[-\frac{A_{sa}f_{sa}}{s_a w} + \frac{A_{sb}f_{sb}}{s_b w}] \quad \text{.................} \quad (12)$$

Hence Equation (10) may be restated as

$$v_u = \frac{\sin 2\theta}{2}[\frac{A_{sb}f_y}{s_b w} - \frac{A_{sa}f_{sa}}{s_a w}] + K\,\tau_{xy} \quad \text{.................} \quad (10a)$$

The coefficient K relates the true maximum shear stress in the concrete to the nominal average shear stress in the concrete. Factors influencing the value of K ahve been discussed previously (4) and it was found that in push-off specimens with reinforcement normal to the shear plane an appropriate value of K was 0.84.

Using Equations (3) and (5) it is possible to calculate pairs of values of σ_x and τ_{xy} corresponding to shear transfer failure, and a plot can be made of the relationship between σ_x and τ_{xy} for a particular strength of concrete. The shear transfer strength corresponding to a particular arrangement of reinforcement is then obtained as follows. First, calculate the value of σ_x corresponding to the reinforcement provided using Equation (6a). From the plot relating σ_x and τ_{xy} for the concrete, read off τ_{xy}. Substitute τ_{xy} in Equation (10a) to obtain the shear transfer strength v_u.

For the isotropic orthogonal reinforcement used in the tests reported here Equations (6a) and (10a) respectively reduce to

$$\sigma_x = \frac{A_s}{sw}[f_{sa}\sin^2\theta + f_y\cos^2\theta] + \sigma_{Nx} \quad \text{.................. (6b)}$$

and $$v_u = \frac{A_s}{2sw}\sin 2\theta\,[f_y - f_{sa}] + K\,\tau_{xy} \quad \text{.................. (10b)}$$

For the case where σ_{Nx} is zero, α may be taken as 45° and f_{sa} is then given by

$$0<\theta<90^\circ \quad f_{sa} = f_y \sec 45.\cos(135-\theta)$$

$$= 1.4\, f_y \cos(135-\theta) \quad \text{.................. (13)}$$

Equations (6a) and (10a) now become

$$\sigma_x = \frac{A_s f_y}{sw}[1.4\cos(135-\theta)\sin^2\theta + \cos^2\theta] \quad \text{............... (6c)}$$

and $$v_u = \frac{A_s f_y}{2sw}\sin 2\theta[1-1.4\cos(135-\theta)] + K\,\tau_{xy} \quad \text{............. (10c)}$$

In Fig. 8 a comparison is made between the shear transfer strengths measured in the tests of Series 2 and the calculated values obtained using Equations (3), (5), (6) and (10) as described above and using a value of 0.84 for K. The intrinsic shape of the failure envelope used to obtain the values of $\sigma_{y'}$ and $\tau_{x'y'}$ substituted in Equations (3) and (5) was obtained from biaxial tests of concrete reported by Kupfer, Hilsdorf and Rüsch (6). Use was made of data reported for concrete having a ratio of tensile to compressive strength corresponding to that of the concrete used in the push-off specimens. Also plotted in Fig. 8 as an open point at θ = 0 and 90° is the average shear transfer strength for the case of reinforcement normal to the shear plane, based on several previous tests (4). It can be seen that although there is a fair amount of experimental scatter, the proposed method of calculation yields a reasonably close estimate of the ultimate shear transfer strength. The average value of v_u(test)/v_u(calc.) for the tests of Series 2 is 1.07. It can be seen that the maximum shear transfer stress is developed when θ is 20°. At this angle the contribution to shear transfer strength of the rebar force component parallel to the shear plane has already reached about 90 percent of its maximum value, and the truss action component of the shear resistance is only reduced a small amount from that corresponding to θ = 0°. As θ increases beyond 20°, the truss action resistance decreases faster than the rebar force component parallel to the shear plane increases, so that the net shear transfer strength decreases to a minimum at about θ = 68°. Beyond this value of θ, the truss action resistance increases significantly so that the net shear transfer strength increases once more.

Shear Transfer Behavior of Initially Uncracked Concrete With Reinforcement at Angle θ to the Shear Plane

The shear transfer behavior of initially uncracked concrete in which the shear plane is crossed at angle θ by a parallel array of bars will be exactly similar to the case of orthogonally reinforced concrete. Equations (6) and (10) for σ_x and v_u will apply to this case providing the appropriate values of F and F_v are substituted in the equations. For bars A_s at spacing s,

$$\frac{F}{lw} = \frac{A_s f_s}{sw} \sin^2\theta \qquad \text{........................} \qquad (14)$$

and

$$\frac{F_v}{lw} = - \sin\theta . \cos\theta \frac{A_s f_s}{sw} \qquad \text{........................} \qquad (15)$$

The stress in the bars f_s will be calculated using Equation (2). For the case where $\sigma_{Nx} = 0$ and taking α as 45°.

$0 < \theta \leq 90°$ $\quad f_s = 1.4\ f_y \cos(135-\theta)$ $\quad$ (as before)

$90° \leq \theta \leq 180°$ $\quad f_s = f_y$

Then

$$\sigma_x = \frac{A_s f_s}{sw} \sin^2\theta \qquad \text{........................} \qquad (6d)$$

and

$$v_u = - \frac{A_s f_s}{2sw} \sin 2\theta + K\ \tau_{xy} \qquad \text{...................} \qquad (10d)$$

In Fig. 9 a comparison is made between the shear transfer strengths measured in the tests of Series 5 and the calculated values obtained using Equations (6d) and (10d) as described for the case of orthogonally reinforced concrete, and using a value of 0.84 for K. Also plotted as open points at θ = 90° and 180° are measured shear transfer strengths obtained in earlier tests (4). It can be seen that the method of calculation proposed correctly predicts the trends in behavior and yields a reasonably close estimate of the ultimate shear transfer strength. In this case, the maximum shear stress is developed when θ is 113°. The average value of v_u(test)/v_u(calc.) for the tests of Series 5 is 1.0.

Shear Transfer Behavior of Initially Cracked Concrete With Orthogonal Reinforcement at Angle θ to the Shear Plane

When initially cracked concrete is subject to shear along the crack, slip will occur. The faces of the crack are rough, and hence when slip occurs, the crack faces are forced to separate. The relative displacement of the concrete on the two sides of the crack produces strains in the reinforcement crossing the crack. The resultant of the forces induced in the reinforcement will have a component parallel to the crack resisting the applied shear, and a component normal to the crack which produces a compression force across the crack. This compression force produces a frictional resistance to sliding between the faces of the crack, thus opposing the applied shear. The relative movement of the concrete on opposite sides of the crack also subjects the individual reinforcing bars to a shearing action. The resistance of the bars to this shearing action, sometimes referred to as dowel action, also contributes to the shearing resistance. A further contribution to shearing resistance is made by the resistance to the shearing off of asperities projecting from the faces of the crack.

In an under-reinforced shear plane with reinforcement normal to the crack, it has been found (3) that the amount of separation of the crack faces is sufficient to cause the reinforcement to yield. It will therefore be conservative to assume that the strain at ultimate in reinforcement normal to the crack is equal to its yield strain, ε_y. The following additional assumptions will also be made to enable the stress in bars inclined to the crack to be calculated:

1. The strain in the reinforcement is proportional to the component of the relative displacement of the faces of the crack in the direction of the reinforcement.

2. The relative displacement at ultimate, δ_u, is constant and equal to that necessary to produce a strain ε_y in reinforcement normal to the crack.

3. The relative displacement δ_u takes place in a direction at angle ϕ to the crack. ϕ is equal to $\tan^{-1}\mu$, where μ is the coefficient of friction between the crack faces. $\mu = 0.8$, hence $\phi = 38.7°$. this is based on the simplified "saw tooth" model of frictional resistance in which the slope of the saw teeth is $\tan^{-1}\mu$, and relative displacement takes place parallel to the sloping face of the teeth. The directions of the actual measured relative displacements were in reasonable agreement with this assumption.

Referring to Fig. 10(b) and using the foregoing assumptions, the strain at ultimate ε_s in reinforcement at angle θ to the crack is given by:

$$\varepsilon_s = C_2\delta_u\cos(\theta+\phi) \quad \text{........................} \quad (16)$$

where C_2 is a constant.

When $\theta = 90°$, $\varepsilon_s = \varepsilon_y$

$$\therefore \quad \varepsilon_y = C_2\delta_u\cos(90+\phi) = -C_2\delta_u\sin\phi$$

Hence we may write

$$\varepsilon_s = -\varepsilon_y \operatorname{cosec}\phi \cos(\theta+\phi) \quad \text{........................} \quad (17)$$

(+ tension, - compression)

Substituting ϕ = 38.7° in Eq. (17), we find that when θ is equal to or less than 12.7°, the reinforcement yields in compression, and when θ is equal to or greater than 90° the reinforcement yields in tension. We may now write the following expressions for the stress at ultimate f_s, in reinforcement at angle θ to the crack,

$$\left.\begin{aligned} 0 < \theta \le 12.7°, \quad & f_s = -f_y \\ 12.7° \le \theta \le 90°, \quad & f_s = -f_y \text{ cosec } 38.7 \cos(\theta+38.7) \\ & \quad = -1.6\, f_y \cos(\theta+38.7) \\ 90° \le \theta \le 180°, \quad & f_s = f_y \end{aligned}\right\} \qquad (18)$$

It has been proposed (4) that for the case of reinforcement crossing a crack at right angles the ultimate shear transfer strength may, for purposes of design, be taken as:

$$v_u = 200 + 0.8\rho f_y, \text{ but } \not> 0.3f'_c \quad \ldots\ldots\ldots\ldots \quad (19)$$

providing ρf_y is $\ge$ 200 psi.

This equation was shown to be a lower bound to all available push-off and pull-off test data for initially cracked specimens. The following modifications of Equation (19) predict mean values for shear transfer strength when the reinforcement is at 90° to the crack:

$$v_u = 400 + 0.8\rho f_y, \quad \text{but } \not> 0.3f'_c \quad \text{(psi)} \quad \ldots\ldots\ldots \quad (20)$$

or $$V_u = 400lw + 0.8F, \quad \text{but } \not> 0.3f'_c lw \quad \text{(lbs)} \quad \ldots\ldots\ldots \quad (21)$$

where l and w are the length and width of the shear plane, and F is the total reinforcement force acting across the shear plane.

It is proposed to adapt Equation (21) to the case of a crack crossing an orthogonal array of reinforcement as shown in Fig. 10(a) by adding a term F_v equal to the component of the bar forces parallel to the crack, opposing the shear:

$$V_u = 400lw + 0.8F + F_v \quad \text{(lbs)} \quad \ldots\ldots\ldots\ldots \quad (22)$$

where $$F = (\frac{l}{s}\sin\theta)\, A_s f_{sa} \sin\theta + (\frac{l}{s}\cos\theta)\, A_s f_{sb} \cos\theta$$

$$= \frac{A_s l}{s}[f_{sa} \sin^2\theta + f_{sb} \cos^2\theta] \quad \ldots\ldots\ldots\ldots \quad (23)$$

and $$F_v = -(\frac{l}{s}\sin\theta)\, A_s f_{sa} \cos\theta + (\frac{l}{s}\cos\theta)\, A_s f_{sb} \sin\theta$$

$$= \frac{A_s l}{2s}\sin^2\theta[-f_{sa} + f_{sb}] \quad \ldots\ldots\ldots\ldots \quad (24)$$

f_{sa} and f_{sb} are the stresses in the reinforcement crossing the crack at angles θ and (90+θ) respectively. f_{sa} is calculated using Eq. (18), f_{sb} will be equal to f_y.

The first term of Equations (21) and (22) primarily represents the contribution to shear transfer strength of dowel action of the reinforcement. The use of the same value in Equation (22) as in Equation (21) is to some extent arbitrary, but is also based on the following qualitative reasoning. The dowel force normal to the rebar will be small when θ is small, but it will increase as θ increases to 90°. As θ becomes larger than 90°, the displacement at right angles to the bar which causes dowel action gradually reduces to zero as the bar finally becomes aligned with the direction of relative displacement, and hence the dowel force developed decreases to zero. Shear along the crack is resisted by the component of the dowel force parallel to the crack, which will be sin θ times the dowel force normal to the bar. The exact way in which the dowel force normal to the bar varies with θ is not known but could perhaps be approximated as being proportional to sinθ. The component parallel to the crack would then be proportional to $\sin^2\theta$, say equal to $D\sin^2\theta$. Since one set of bars is at angle θ and the other at (90+θ) in any instance, the component parallel to the crack of the dowel forces in both sets of reinforcement may be written $D[\sin^2\theta + \sin^2(90+\theta)]=D$, i.e., the dowel force contribution is independent of θ according to the foregoing argument. The variation of the dowel force component parallel to the shear plane with angle θ, reported by Dulacska, (5) can be approximated reasonably well by an expression of the form $D\sin^2\theta$ for the range of values of θ(50° to 80°) included in her tests. Additional support for the use of the constant $400lw$ as the first term in Equation (22) is provided by the correlation obtained between Equation (22) and the test data reported here.

Equation (22) may also be expressed in the form of stresses:

$$v_u = V_u/lw$$

$$= 400 + 0.8\frac{A_s}{sw}[f_{sa}\sin^2\theta + f_{sb}\cos^2\theta] + \frac{A_s}{2sw}\sin 2\theta[-f_{sa} + f_{sb}] \quad \ldots\ldots\ldots\ldots \quad (25)$$

where f_{sa} and f_{sb} are calculated using Equations (18).

In Fig. 11 a comparison is made between the shear transfer strengths measured in the tests of Series I and the calculated values obtained using Equation (25). It can be seen that Equation (25) correctly predicts the way in which the ultimate shear transfer strength changes as angle θ changes, and for this series (s=5 in.) predicts the ultimate strengths very closely. When applied to Series 3 (s=2-1/2 in) the results were not quite so consistent, the ratios of v_u(test)/v_u(calc) for specimens 3.1 through 3.4 being 0.84, 1.13, 0.99 and 1.16, for an average value of v_u(test)/v_u(calc) of 1.03. The maximum and minimum values of shear stress occur at θ= 22° and 72° respectively.

It is interesting to note that the difference between the calculated shear transfer strength of initially uncracked concrete and that of initially cracked concrete as θ varies from 0 to 90° is almost a constant value of 200 psi, the same value as previously reported (3) for concrete with reinforcement normal to the shear plane.

Shear Transfer Behavior of Initially Cracked Concrete With Parallel Reinforcement at Angle θ to the Shear Plane

The behavior of initially cracked concrete with parallel reinforcement crossing the crack at angle θ is similar to that of orthogonally reinforced concrete, providing the angle θ is large enough so that tension is developed in the reinforcement. It was observed that for small values of θ the relative displacement of the concrete on opposite sides of the crack was in a direction approximately at right angles to the bars, i.e. at $(90+\theta)$ to the crack. It appears as if this displacement corresponds to a rotation of the bars as they tear themselves out of the opposite faces of the crack. Under these conditions the shear transfer resistance will be developed primarily by the dowel action of the reinforcing bars, since there is little or no component of displacement in the direction of the reinforcing bars to cause a significant compressive strain in those bars.

The same assumptions will be made in order to calculate the ultimate shear transfer strength of initially cracked concrete reinforced with a parallel array of bars at angle θ to the crack, as were made in the case of orthogonally reinforced, initially cracked concrete, except as follows:

1. The relative displacement δ_u is assumed to take place in a direction at angle $(90+\theta)$ to the crack when θ is less than $(90-\phi)$ and in a direction at angle ϕ to the crack when θ is equal to or greater than $(90-\phi)$.

2. The resistance to shear along the crack due to dowel action is assumed to vary directly with $\sin^2\theta$, for reasons already cited in the discussion of the contribution of dowel action to the shear transfer strength of orthogonally reinforced, initially cracked concrete.

The stress in the reinforcement f_s is then given by:

$$\left.\begin{array}{ll} 0 < \theta < (90-\phi) & f_s = 0 \\ 90-\phi \leq \theta < 90 & f_s = -1.60 f_y \cos(\theta+38.7) \\ 90 \leq \theta \leq 180 & f_s = f_y \end{array}\right\} \qquad (26)$$

$$(90-\phi) = 51.3^\circ$$

The use of $f_s = 0$ when θ is less than 51.3° will be conservative since there will be a small indeterminate amount of compression in the bars. Making use of Equations (26) and assumption (2) we may write:

$$V_u = 400lw \sin^2\theta + 0.8F + F_v \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (27)$$

$$\text{where } F = \frac{A_s l}{s} f_s \sin^2\theta \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (28)$$

$$\text{and } F_v = -\frac{A_s l}{2s} f_s \sin 2\theta \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (29)$$

$$\therefore \quad v_u = V_u/lw$$

$$= 400 \sin^2\theta + 0.8 \frac{A_s f_s}{sw} \sin^2\theta - \frac{A_s f_s}{2sw} \sin 2\theta$$

$$\text{or} \quad v_u = 400 \sin^2\theta + \frac{A_s f_s}{sw} [0.8 \sin^2\theta - 0.5 \sin 2\theta] \quad \ldots. \quad (30)$$

In Fig. 12 a comparison is made between the shear transfer strengths measured in the tests of Series 4 and the calculated values obtained using Equations (30) and (26). It can be seen that the trend of the experimental data is predicted reasonably closely by the calculated relationship. The average value of v_u(test)/v_u(calc.) for the tests of Series 4 is 1.18. The conservatism in the calculated values for small values of θ probably reflects the neglect of the bar force component parallel to the crack implicit in the use of Equations (26). The conservatism when θ is greater than 90° may be due to the assumption of a too rapid reduction in dowel force effects as θ increases beyond about 100°, but it is difficult at present to be sure of the reason.

CONCLUDING REMARKS

It has been shown that the previously developed (4) hypotheses for shear transfer behavior in both initially cracked and initially uncracked concrete reinforced with bars normal to the shear plane can be extended to the case of concrete reinforced with orthogonal reinforcement or parallel reinforcement inclined at an arbitrary angle to the shear plane, provided the component of the bar forces parallel to the shear plane is also taken into account in calculating shear resistance.

The tests reported did not include a normal stress acting across the shear plane and therefore they strictly only validate the hypotheses for that situation. However, it has previously been shown (4) that for the case of reinforcement crossing the shear plane at right angles, an externally applied compressive stress acting transversely to the shear plane is additive to ρf_y in calculations of the ultimate shear transfer strength of both initially cracked and uncracked concrete. It is felt that the same will be true in the case of orthogonal reinforcement and inclined parallel reinforcement. The validity of the hypotheses when tension acts across the shear plane is not proved and experimental studies of this combination of stresses should be made.

For a given *normal* spacing of the reinforcing bars, the maximum value of ultimate shear transfer stress will be developed when the bars (or one set of bars in the case of orthogonal reinforcement) are inclined at approximately 110° to the shear plane. This is true for both initially cracked and initially uncracked concrete. It should be noted, however, that if the bar spacing along the shear plane is constant and θ is varied, calculations indicate that the maximum ultimate shear stress will be developed when θ is 135°.

As indicated earlier, this study is presented as a step toward solution of the general problem of shear transfer across a crack encountered in the design of concrete reactor containment vessels for lateral forces. It should be noted that the experimental work reported involved only single direction loading and relatively small diameter reinforcing bars. Further work is necessary to determine whether cyclic reversal of shear results in a degradation of shear transfer strength. It is also necessary that studies should be made using large size reinforcing bars to determine whether their effectiveness in dowel action resistance to shear is limited by local splitting of the concrete, and whether the slip displacements necessary to mobilize the dowel resistance of very large bars, become excessive. Studies of these problems are currently under way or planned at Cornell University and the University of Washington.

ACKNOWLEDGEMENTS

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APPENDIX 1

NOTATION

A_s = Cross sectional area of one rebar, $in.^2$

A_{sa} = Cross sectional area of one of the rebars A, $in.^2$

A_{sb} = Cross sectional area of one of the rebars B, $in.^2$

C = Compression force in concrete strut, lb.

C_1, C_2 } = Coefficients relating strain and displacement.

D = Dowel action shear transfer resistance when $\theta = 90°$, lb.

F = Component normal to the shear plane of the total rebar force at ultimate, lb.

F_v = Component parallel to the shear plane of the total rebar force at ultimate, lb.

f'_c = Concrete compression strength measured on 6x12-inch cylinders, psi.

f_s = Stress in rebar at ultimate, psi.

f_{sa} = Stress in rebars A at ultimate, psi.

f_{sb} = Stress in rebars B at ultimate, psi.

f_t = Concrete split cylinder tensile strength measured on 6x12-inch cylinders, psi.

f_y = Yield strength of rebar, psi.

K = Coefficient relating maximum and average shear stresses.

l = Length of shear plane, in.

N = Externally applied force acting normal to the shear plane, lb.

s = Space between rebars, in.

s_a = Space between rebars A inclined at angle θ to the shear plane, in.

s_b = Space between rebars B inclined at angle $90+\theta$ to the shear plane, in

T = Component normal to shear plane of rebar force acting at one end of a strut, lb.

T_v = Component parallel to shear plane of rebar force acting at one end of a strut, lb.

V = Applied shear acting along shear plane, lb.

V' = Shear in strut, lb.

V_u = Ultimate shear force, lb.

v_u = Ultimate shear stress, psi. ($= V_u/lw$).

w = Width of shear plane, in.

α = Angle between diagonal tension cracks and the shear plane, degrees.
δ_u = Relative displacement at ultimate of the concrete on opposite sides of the shear plane, in.
ε_s = Strain in rebar at ultimate, in./in.
ε_y = Yield strain of rebar, in./in.
θ = Angle of inclination of rebars to the shear plane, degrees.
μ = Coefficient of friction.
ρ = Shear transfer reinforcement ratio (= A_s/sw)
σ = Direct stress, psi.
τ = Shear stress, psi.
ϕ = $\mathrm{Tan}^{-1}\mu$

TABLE 1 - DETAILS OF SPECIMENS WITH ORTHOGONAL REINFORCEMENT

Specimen No.	Angle θ (degrees)	Stirrup spacing, s (in.)	Number of stirrups @ θ° to the shear plane	Number of stirrups @ (90+θ)° to the shear plane	Length of shear plane (in.)	f'_c (1) (psi)	f_t (2) (psi)
1.1*	0	5	0	2	10.00	4370	420
1.2*	26.5	5	1	2	11.17	4370	438
1.3*	45.0	5	2	2	14.14	4015	367
1.4*	63.5	5	2	1	11.17	4120	439
2.1	0	5	0	2	10.00	4175	425
2.2	26.5	5	1	2	11.17	4245	407
2.3	45.0	5	2	2	14.14	3990	362
2.4	63.5	5	2	1	11.17	4270	395
3.1*	0	2.5	0	4	10.00	3920	326
3.2*	26.5	2.5	2	4	11.17	4300	436
3.3*	45.0	2.5	4	4	14.14	4010	383
3.4*	63.5	2.5	4	2	11.17	4470	-

* Cracked along shear plane before test.
(1) Concrete compressive strength measured on 6 x 12-inch cylinders.
(2) Concrete splitting tensile strength measured on 6 x 12-inch cylinders.

To convert inches (in.) to centimeters (cm.) multiply by 2.540

To convert psi to kilogram-force/square centimeter (kgf/cm^2) multiply by 0.0703.

TABLE 2 - DETAILS OF SPECIMENS WITH PARALLEL REINFORCEMENT

Specimen No.	Angle θ (degrees)	Stirrup spacing s (in.)	Number of stirrups at θ° to the shear plane	Length of shear plane (in.)	f'_c (1) (psi)	f_t (2) (psi)
4.1*	26.5	-	1	11.17	3975	406
4.2*	45.0	5	2	14.14	4275	381
4.3*	63.5	5	2	11.17	3880	419
4.4*	90.0	5	2	10.00	4370	420
4.5*	116.5	5	2	11.17	4050	314
4.6*	135.0	5	2	14.14	4040	360
4.7*	153.5	-	1	11.17	4090	419
5.1	90.0	5	2	10.00	4175	425
5.2	116.5	5	2	11.17	3885	358
5.3	135.0	5	2	14.14	3805	381
5.4	153.5	-	1	11.17	3825	405

* Cracked along shear plane before test

(1) Concrete compressive strength measured on 6 x 12-inch cylinders

(2) Concrete splitting tensile strength measured on 6 x 12-inch cylinders

To convert inches (in.) to centimeters (cm.) multiply by 2.540
To convert psi to kilogram-force/square centimeter (kgf/cm^2) multiply by 0.0703

TABLE 3 - TEST RESULTS

Specimen No.	V_u, Ultimate Shear (kips)	v_u* Ultimate Shear Stress (psi)	Specimen No.	V_u Ultimate Shear (kips)	v_u* Ultimate Shear Stress (psi)
1.1	45.8	654	4.1	7.4	95
1.2	62.5	799	4.2	23.75	240
1.3	65.0	657	4.3	28.15	360
1.4	47.5	607	4.4	45.8	654
			4.5	60.0	767
2.1	70.0	1000	4.6	67.5	682
2.2	79.0	1010	4.7	25.1	321
2.3	80.0	808			
2.4	69.5	889	5.1	70.0	1000
			5.2	75.0	959
3.1	52.75	754	5.3	81.0	818
3.2	101.75	1301	5.4	49.0	627
3.3	96.0	970			
3.4	72.5	927			

* Nominal ultimate shear stress obtained by dividing the ultimate shear by the area of the shear plane.

To convert kips to kilogram-force (kgf) multiply by 453.6.

To convert psi to kilogram-force/square centimeter (kgf/cm^2) multiply by 0.0703.

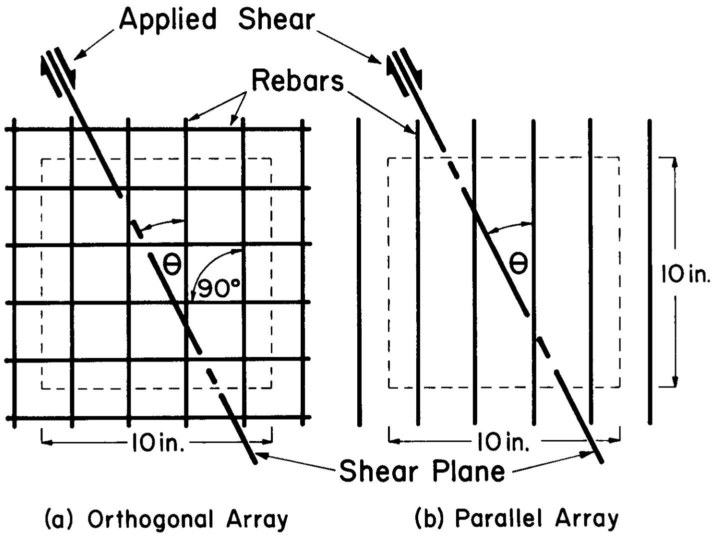


Fig. 1—Shear plane crossing reinforcement array at an arbitrary angle θ

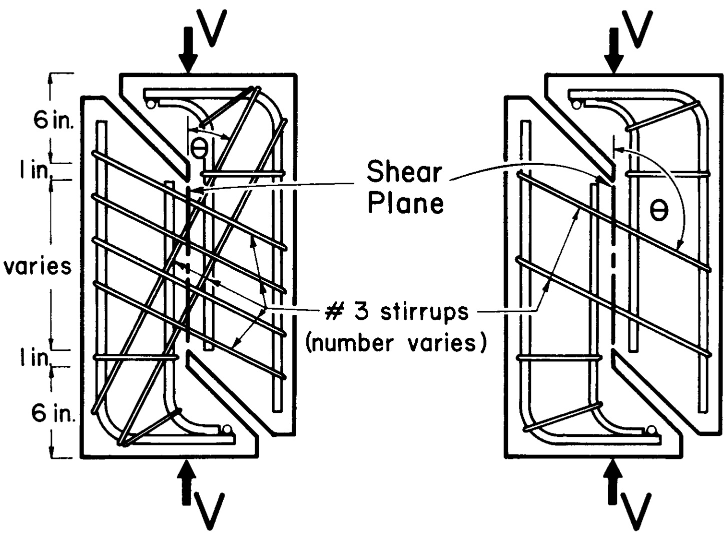


Fig. 2—Typical push-off specimens with orthogonal and parallel reinforcement

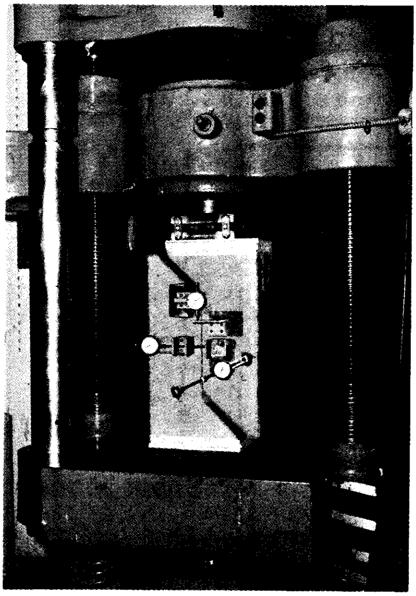


Fig. 3—Arrangement of push-off test

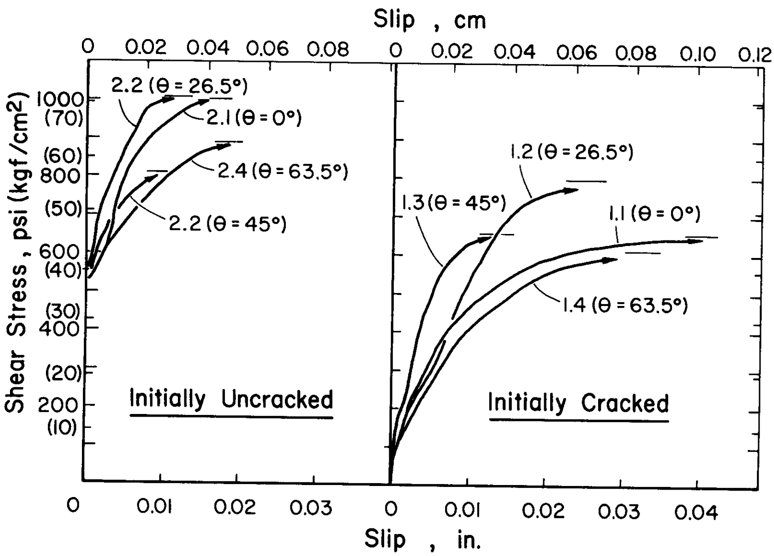


Fig. 4—Typical shear stress-slip curves for specimens with orthogonal reinforcement (s= 5 in.)

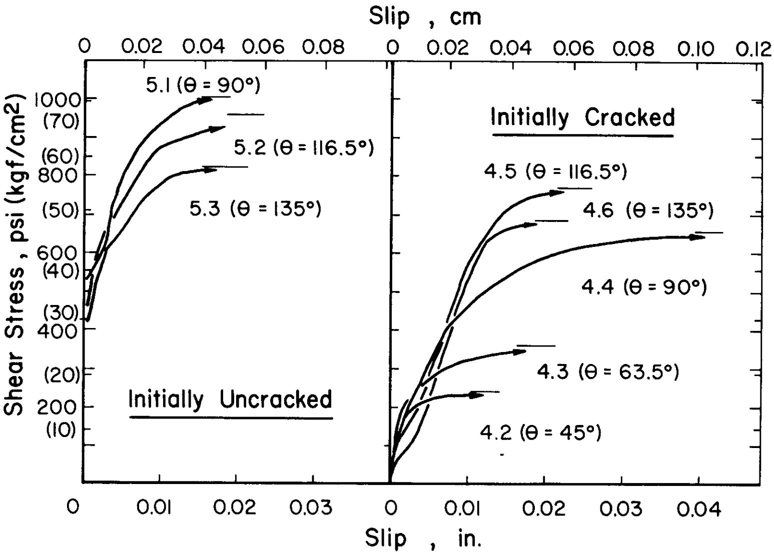
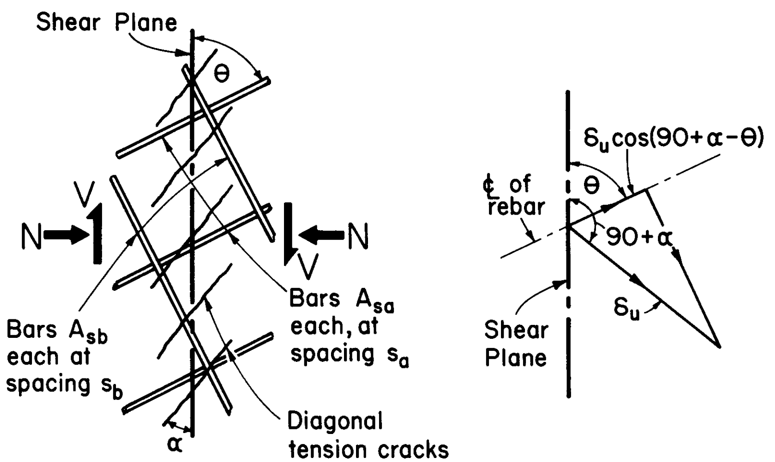


Fig. 5—Typical shear stress-slip curves for specimens with two parallel stirrups as reinforcement



(a) Diagonal tension cracking along the shear plane.

(b) Relative displacements

Fig. 6—Shear transfer in initially uncracked concrete with orthogonal reinforcement

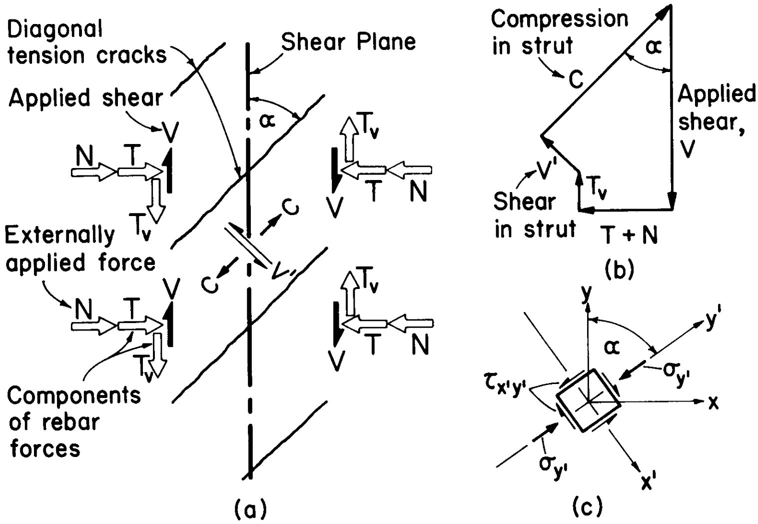


Fig. 7— The mechanics of shear transfer in initially uncracked concrete with orthogonal reinforcement

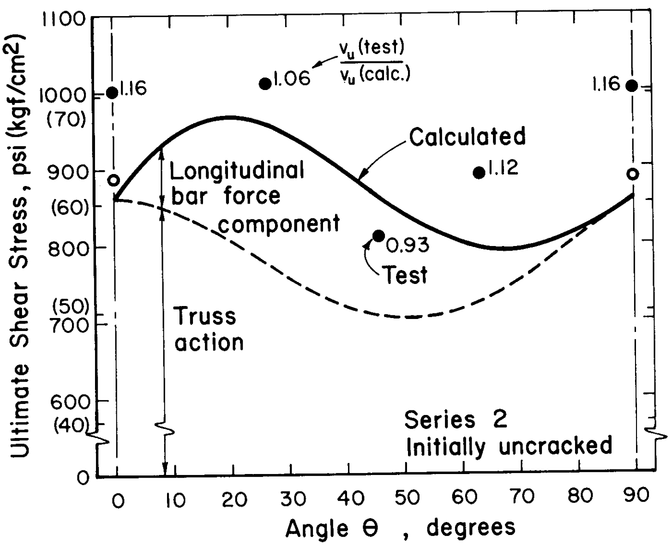


Fig. 8—Comparison of calculated and test shear transfer strengths of initially uncracked concrete with orthogonal reinforcement

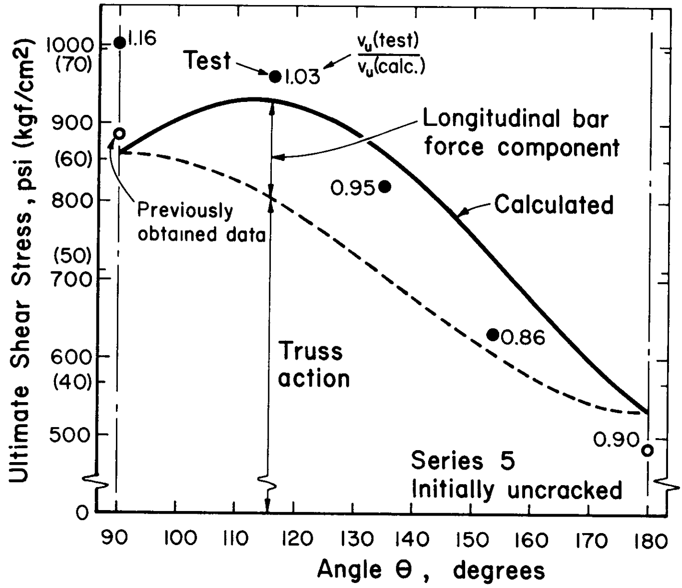


Fig. 9—Comparison of calculated and test shear transfer strengths of initially uncracked concrete with parallel reinforcement

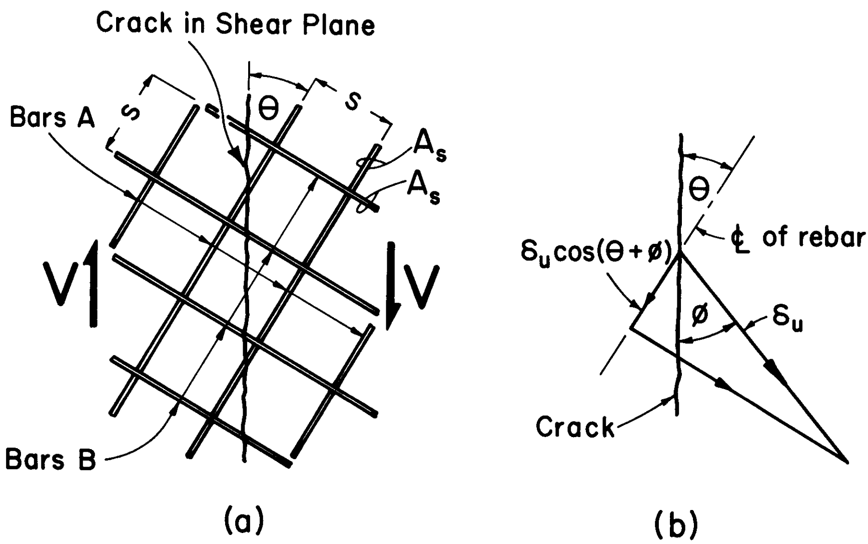


Fig.10—Shear transfer in initially cracked concrete with orthogonal reinforcement

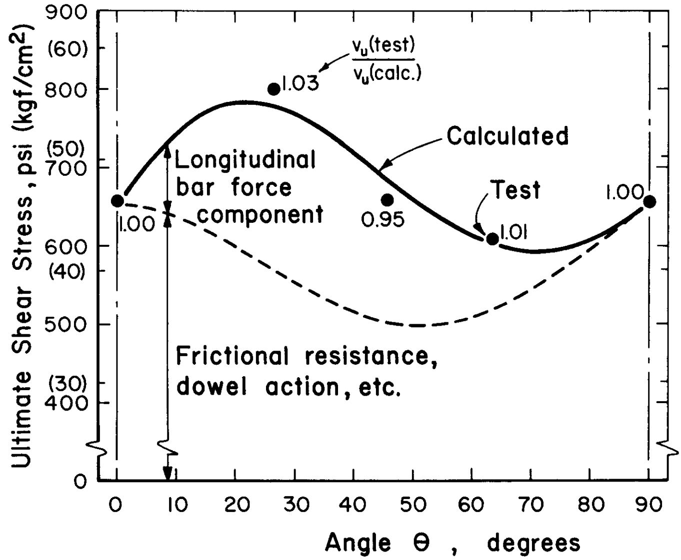


Fig.11—Comparison of calculated and test shear transfer strengths of initially cracked concrete with orthogonal reinforcement

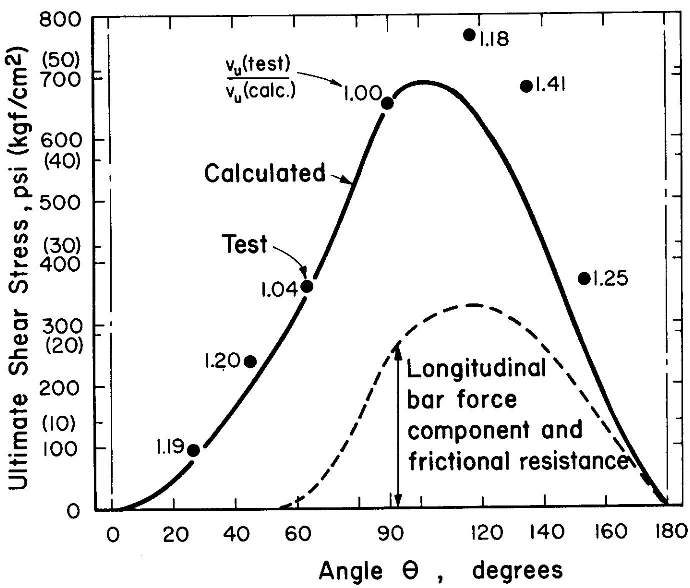


Fig.12—Comparison of calculated and test shear transfer strengths of initially cracked concrete with parallel reinforcement