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Flexural Behavior of High-Strength Concrete Beams

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Flexure tests are reported on 12 under-reinforced rectangular beams with f_c' ranging between 9300 and 11,800 psi (64 to 81 MN/m²). Serious loss of ductility is evident even when the reinforcement ratio ρ is less than $0.50\rho_b$. The currently used rectangular stress block (from the ACI Building Code) does not accurately predict the beam behavior when concrete strengths exceed 8000 psi (55 MN/m²). Pending further test results, the use of a triangular stress block seems prudent.

Keywords: beams (supports); deflection; ductility; flexural strength; flexural tests; high-strength concretes; reinforced concrete; reinforcing steels; stress block; stress-strain relationships; structural analysis.

■ THE EQUIVALENT RECTANGULAR STRESS BLOCK permitted by the ACI Building Code[1] was based on beam tests with concrete strengths in the range of 3000 psi (21 MN/m²) to 6000 psi (41 MN/m²). For beams with concrete strengths in excess of 21,000 psi (145 MN/m²), the current Code provisions would call for a zero-depth rectangular stress block—an obvious fallacy. Hence, there is a strong need to evaluate the concept of the rectangular stress block for high-strength concrete. This paper reports the results of flexure tests on 12 rectangular under-reinforced beams with concrete strengths varying between 9300 and 11,800 psi (64 and 81 MN/m²).

BEAM DETAILS AND TEST SETUP

The concrete consisted of Type I cement, crushed limestone aggregate of maximum size ½ in. (12.7 mm), common sand, and a small quantity of retarder (18 oz (532 cc) to 24 oz (710 cc) per cu yd). The cement content varied between 7.5 to 9 sacks per cu yd (705 to 846 lb/cu yd or 305,000 to 365,000 kg/cu m) with the water-cement ratio by weight ranging between 0.364 and 0.410. Table 1 shows the specimen details, the reinforcement, and the concrete strengths. The reinforcement was ASTM A 615-60 grade steel. The specimen details and the test setup are shown in Fig. 1.* Shear

*For more detailed information refer to "Flexural Behavior of Rectangular Beams of Extra High Strength Concrete," by Keith E. Leslie, Unpublished MSc Thesis, the University of Texas at Arlington, 1975, 101 pp.

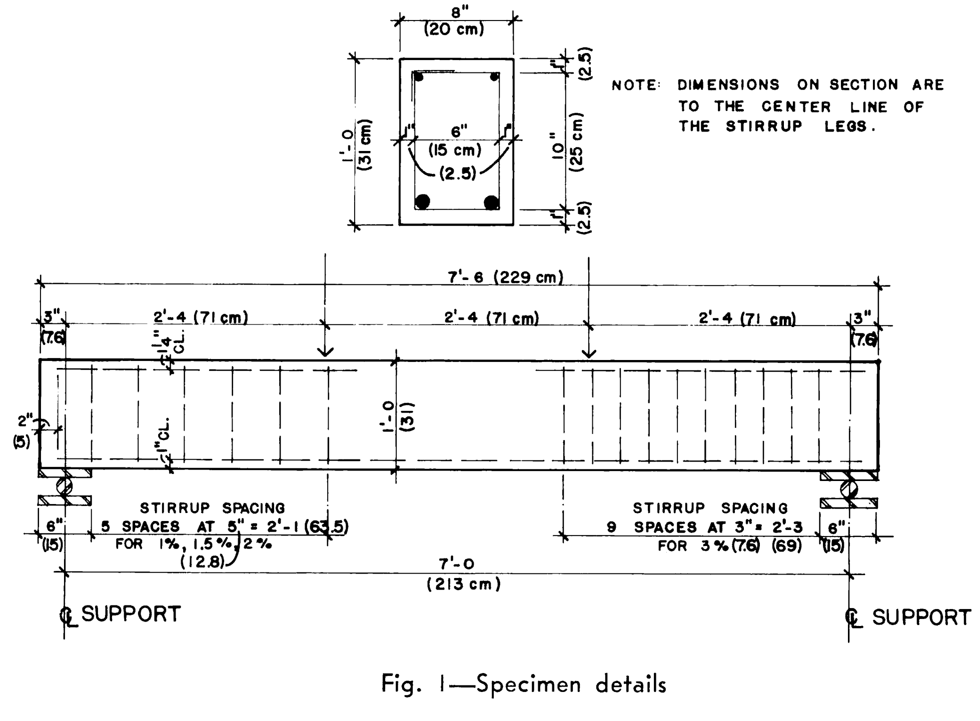

Fig. 1—Specimen details

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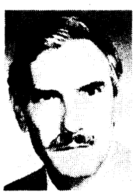

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failures were precluded by use of heavy stirrups in the shear spans. The curing period between the time of casting and the time of the tests varied from 291 to 519 days, and the tabulated values of f_c' are concrete cylinder strengths on the day of the test. Cylinders and beams were cured under the identical air curing conditions.

TEST RESULTS AND ANALYSIS

The results dealing with the ductility are discussed first followed by those pertaining to strength, because of the severe problems of brittle failures associated with beams in high-strength concrete.

Load-deflection curves

Fig. 2 shows the load-deflection diagrams for four beams having a cement content of 9 sacks/cu yd (846 lb/cu yd or 365,000 kg/cu m) but with longitudinal steel ratio ρ varying from 0.01 to 0.027. The concrete strengths for these beams range from 11,210 psi (77 MN/m^2) to 11,780 psi (81 MN/m^2). The total load and the central deflection are represented in the y- and the x-axes in Fig. 2, where it can be seen that as ρ increased, (a) the maximum ultimate deflection decreased, and (b) the ductility index u (the ratio of maximum ultimate deflection to the deflection at the end of the initial linear portion of the load-deflection curves) decreased drastically. Table 2 shows the variation of the ductility indexes with increasing ρ for all beams tested in this series. Note that even beams with ρ of 0.019 had a ductility index of 2.9 only and for those with ρ of 0.027, this index dropped to 1.8.

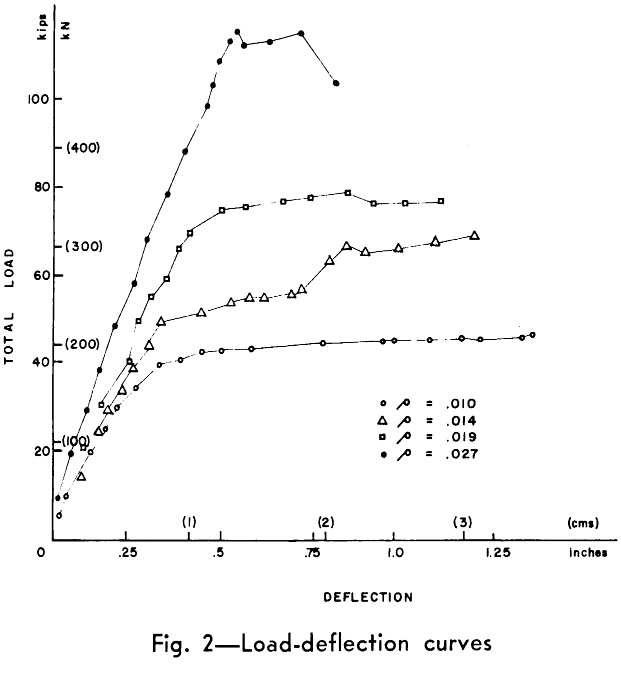

Fig. 2—Load-deflection curves

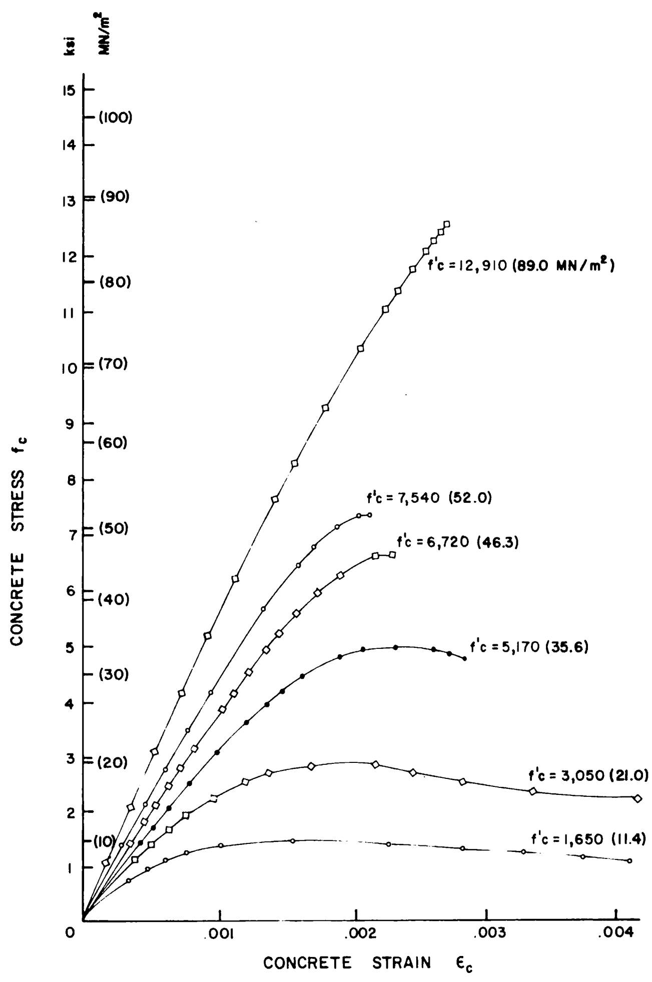

Fig. 3—Stress-strain relationships for concrete—PCA tests (Reference 2). Note that the test with f_c' = 12,910 psi (89 MN/m^2) is by Nedderman.

TABLE 1—SPECIMEN DETAILS

Specimen*	Cement, sacks per yd³	Content, lb/yd³ (kg/m³)	Water-cement ratio, by weight	f_c', psi (MN/m²)	Specimen dimensions b, in. (cm)	Specimen dimensions d, in. (cm)	A_s†	ρ
7.5-1	7.5	705 (305,000)	0.410	9,310 (64.1)	8.25 (21.0)	10.63 (27.0)	2-#6	0.01
8.0-1	8.0	752 (325,000)	0.388	10,660 (73.5)	8.25 (21.0)	10.63 (27.0)	2-#6	0.01
9.0-1	9.0	846 (365,000)	0.364	10,620 (73.2)	8.25 (21.0)	10.63 (27.0)	2-#6	0.01
7.5-1.5	7.5	705 (305,000)	0.410	9,720 (67.0)	8.00 (20.3)	10.56 (26.8)	2-#7	0.014
8.0-1.5	8.0	752 (325,000)	0.388	11,400 (78.6)	8.13 (20.7)	10.56 (26.8)	2-#7	0.014
9.0-1.5	9.0	846 (365,000)	0.364	11,630 (80.2)	8.50 (21.6)	10.56 (26.8)	2-#7	0.013
7.5-2	7.5	705 (305,000)	0.410	10,850 (74.8)	8.50 (21.6)	10.50 (26.7)	2-#8	0.018
8.0-2	8.0	752 (325,000)	0.388	10,610 (73.1)	7.88 (20.0)	10.50 (26.7)	2-#8	0.019
9.0-2	9.0	846 (365,000)	0.364	11,780 (81.2)	8.13 (20.7)	10.50 (26.7)	2-#8	0.019
7.5-3	7.5	705 (305,000)	0.410	11,650 (80.3)	8.38 (21.3)	10.50 (26.7)	3-#8	0.027
8.0-3	8.0	752 (325,000)	0.388	11,730 (80.9)	8.25 (21.0)	10.50 (26.7)	3-#8	0.027
9.0-3	9.0	846 (365,000)	0.364	11,210 (77.3)	8.25 (21.0)	10.50 (26.7)	3-#8	0.027

*The first number indicates the cement contents in sacks/cu yd. The second number indicates the nominal percentage of longitudinal reinforcement.

†The yield points for #6, #7, and #8 are 60.22 ksi (415 MN/m²), 55.83 ksi (385 MN/m²), and 66.88 ksi (461 MN/m²), respectively.

Compressive stress block and ultimate strength

Compressive stress-strain curves for axially loaded concrete specimens of progressively higher strengths have been reported by Hognestad[2] (Fig. 3). Two characteristics are to be noted in this figure:

(1) With increasing concrete strengths, the maximum concrete strain becomes progressively smaller. It seems possible that the maximum concrete strain values may have to be redefined in the code for higher strength concrete. Further research work in this area seems warranted.

(2) For concrete strengths in excess of 6000 psi (41 MN/m²) the ascending portion of the stress-strain curve is almost linear up to the maximum strain. This second phenomenon is also typically noticed in Nedderman's* specimens (see Specimens A3, B3, and D3, Fig. 4).

Fig. 3 and 4 indicate that for high-strength concretes, the stress-strain curve is steeply ascending, almost linearly, to the maximum strain in marked contrast to the stress-strain curves of lower strength concretes, which have a descending branch past maximum stress. Based on this behavior, it seems conservative to use an equivalent triangular stress block with maximum concrete strength at the extreme fiber, and zero stress at the neutral axis.

TABLE 2—COMPARISON OF DUCTILITY INDEXES

Reinforcement ratio ρ	Deflection at the end of initial strength portion, in. (cm)	Ultimate deflection, in. (cm)	Ductility index
0.01	0.28 (0.71)	1.70 (4.32)	6.0
0.014	0.32 (0.81)	1.18 (3.00)	3.7
0.019	0.36 (0.91)	1.05 (2.67)	2.9
0.027	0.55 (1.40)	1.00 (2.54)	1.8

Table 3 compares the values of ultimate moments reached in these tests with those predicted by the ACI Building Code rectangular stress block and the triangular stress block indicated in the last paragraph. For these under-reinforced beams, the predictions based on triangular stress block are comparable to the predictions based on ACI rectangular stress block. In the authors' opinion, the triangular stress block will predict the be-

*Nedderman, Howard, "Flexural Stress Distribution in Very High Strength Concrete," Unpublished MSc Thesis, the University of Texas at Arlington, 1973, 183 pp.

TABLE 3—ULTIMATE STRENGTH PREDICTIONS

Specimen	f_c', psi (MN/m²)	M_u (test), ft-kips (km-N)	M_u (calc), ft-kips (km-N)	M_u (test)* / M_u (calc) (ACI)	M_u (Δ), ft-kips (km-N)	M_u (test)† / M_u (Δ)
7.5-1	9310 (64.1)	60.6 (82.2)	45.1 (61.2)	1.34	44.9 (60.9)	1.39
8.0-1	10655 (73.5)	60.2 (81.6)	45.5 (61.7)	1.34	45.3 (61.4)	1.33
9.0-1	11620 (73.2)	54.6 (74.0)	45.4 (61.6)	1.20	45.3 (61.4)	1.21
7.5-1.5	9720 (67.0)	64.86 (87.9)	56.6 (76.7)	1.14	55.8 (75.7)	1.16
8.0-1.5	11490 (78.6)	73.40 (99.5)	56.6 (76.7)	1.30	56.3 (76.3)	1.30
9.0-1.5	11630 (80.2)	80.2 (108.8)	56.7 (76.9)	1.41	56.5 (76.6)	1.42
7.5-2	10850 (74.8)	88.5 (120.0)	87.6 (118.8)	1.01	85.7 (116.2)	1.03
8.0-2	10610 (73.1)	84.3 (114.3)	86.9 (117.8)	.97	85.1 (115.4)	.99
9.0-2	11780 (81.2)	91.4 (123.9)	87.7 (118.9)	1.04	86.0 (116.6)	1.06
7.5-3	11650 (80.3)	129.3 (175.3)	125.0 (169.5)	1.03	124.6 (169.0)	1.04
8.0-3	11730 (80.9)	131.1 (177.8)	124.9 (169.4)	1.05	124.2 (168.4)	1.04
9.0-3	11210 (77.3)	136.3 (184.8)	124.3 (168.6)	1.10	123.6 (167.6)	1.10

*Mean = 1.16; Standard deviation = 0.021.
†Mean = 1.17; Standard deviation = 0.020.

TABLE 4—REINFORCEMENT RATIOS AND DUCTILITY INDEXES

Specimen	ρ	ρ_b (ACI)	ρ_b (Δ)	ρ/ρ_b (ACI)	ρ/ρ_b (Δ)	u
7.5-1	0.010	0.045	0.033	0.22	0.30	5.9
8.0-1	0.010	0.046	0.038	0.21	0.26	8.0
9.0-1	0.010	0.045	0.037	0.22	0.27	4.3
7.5-1.5	0.014	0.051	0.032	0.28	0.44	4.5
8.0-1.5	0.014	0.051	0.045	0.28	0.31	2.5
9.0-1.5	0.013	0.051	0.045	0.26	0.30	4.2
7.5-2.0	0.018	0.040	0.026	0.45	0.69	3.2
8.0-2.0	0.019	0.040	0.030	0.48	0.63	2.4
9.0-2.0	0.019	0.039	0.034	0.49	0.56	2.7
7.5-3.0	0.027	0.039	0.033	0.69	0.82	1.9
8.0-3.0	0.027	0.039	0.038	0.69	0.71	2.1
9.0-3.0	0.027	0.039	0.035	0.69	0.77	1.5

havior of over-reinforced beams better than the ACI Building Code rectangular stress block.

Balanced steel ratios and ductility

Table 4 shows the ratios of ρ/ρ_b where ρ_b is calculated both by current ACI Building Code rectangular stress block and by the triangular stress block. Measured ductility indexes are plotted against the $\rho/\rho_{b\Delta}$ in Fig. 5.

Furlong[3] recommends that for proper redistribution of moments the ductility index should satisfy the criterion:

$$u < 1 + 0.235\, l'/d$$

For common l/d ratios of 15 to 20, the u value would range from 4.5 to 5.7. Fig. 5 indicates that for this range of u values $\rho/\rho_{b\Delta}$ should be between 0.3 and 0.4. The present ACI Building Code limit of 0.75 for ρ/ρ_b needs reevaluation for beams with high-strength concrete. Pending further tests, it may be prudent to limit $\rho/\rho_{b\Delta}$ to 0.35 for concrete strength in excess of 8000 psi (55 MN/m²). Here again there is the possibility of defining a gradually decreasing limit on ρ/ρ_b as the concrete strength increases. When the $\rho/\rho_{b\Delta}$ is limited to

0.35, the steel ratio may fall below 0.01, a region in which v_c is less than $2\sqrt{f_c'}$, leading to reduced concrete shear capacity.

Also, the reinforcement ratio ω in prestressed concrete, now limited to 0.30 to assure ductility, may warrant further scrutiny.

RECOMMENDATIONS

Based on the limited number of tests on under-reinforced beams reported in this study, the following recommendations are made:

1. The ACI Building Code rectangular stress block does not predict the behavior of beams with f_c' above 8000 psi (55 MN/m^2).
2. Further research is warranted with respect to the maximum strain in the concrete for f_c' exceeding 8000 psi (55 MN/m^2).
3. Pending further tests, a triangular stress block with the extreme fiber stress at f_c' and zero stress at neutral axis is recommended as a conservative model for predicting the behavior of beams with f_c' above 8000 psi (55 MN/m^2).
4. To achieve the accustomed ductility in beams, $\rho/\rho_{b\Delta}$ values should be limited to 0.35, for f_c' in excess of 8000 psi (55 MN/m^2).
5. Further tests are needed to investigate the effect of the above recommendations on compression reinforcements, shear strengths, and beam ductility.

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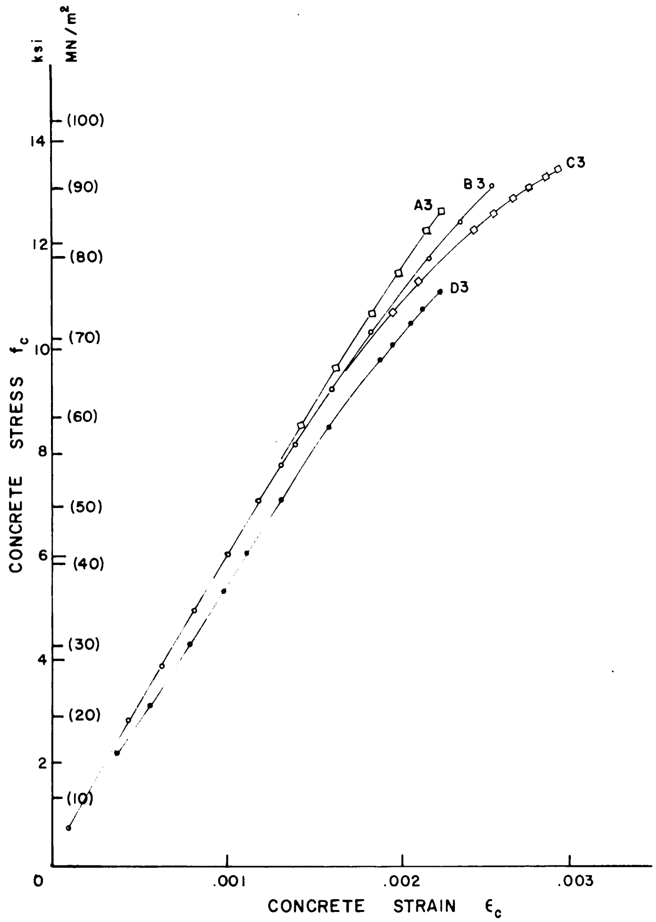

Fig. 4—Stress-strain relationships for concrete—Nedderman's tests

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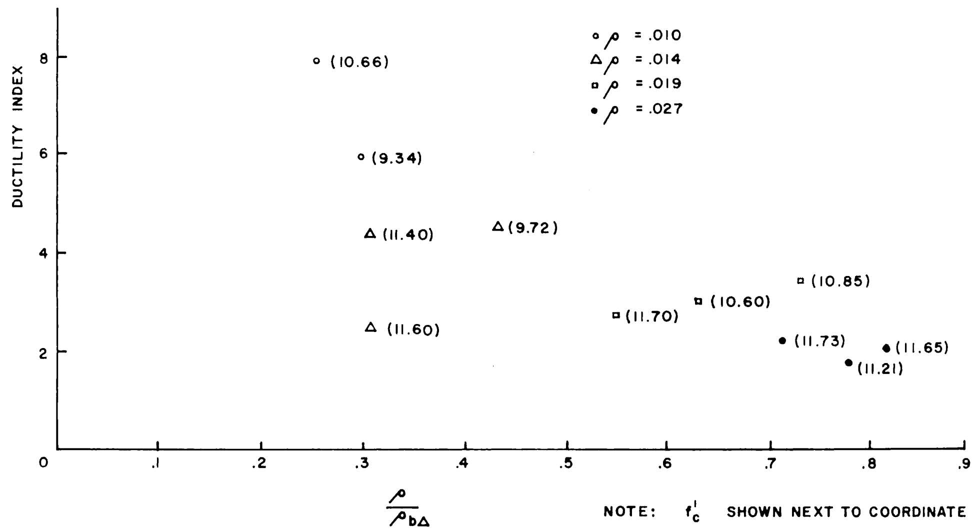

Fig. 5—Reinforcement ratio versus ductility index