Analysis of Cracked Prestressed Concrete Sections: A Practical Approach

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This paper presents a practical approach for analyzing the elastic behavior of cracked prestressed concrete sections of any shape, using existing section property software. The use of the results for estimating deflection and crack control is presented. The method is applicable to sections with any degree of prestress, from no prestress to full prestress. Examples are given, including the analysis of cracked composite sections. The procedural steps for analyzing cracked prestressed concrete sections are summarized.

To fully understand the behavior of a prestressed concrete member cracked at service load, an analysis of the cracked prestressed section should be made. This analysis is needed in order to find the change in steel stress after cracking (for use in evaluating crack control at service load), and for finding the appropriate flexural stiffness for use in deflection calculations.

The analysis of cracked prestressed sections requires, at best, the solution of a cubic equation.[1,2,3,4] The complexity of this solution, requiring the use of charts, tables, or special software, has impeded the use of prestressed concrete members with tensile stresses beyond the code limits for nominal tensile stress.

The purpose of this paper is to present an analysis method using conventional section property software. The solution requires iteration, but the bulk of the work is done by an existing section property program. The iteration may be done manually or a small additional program may be written that will do the iteration, using an existing section property program to do the computation inside an iteration loop.

The iterative procedure consists of assuming a depth c of the neutral axis, computing section properties of the net cracked section, checking stresses at the assumed neutral axis location, and revising c as necessary to make the concrete stress equal to zero at the assumed neutral axis location.

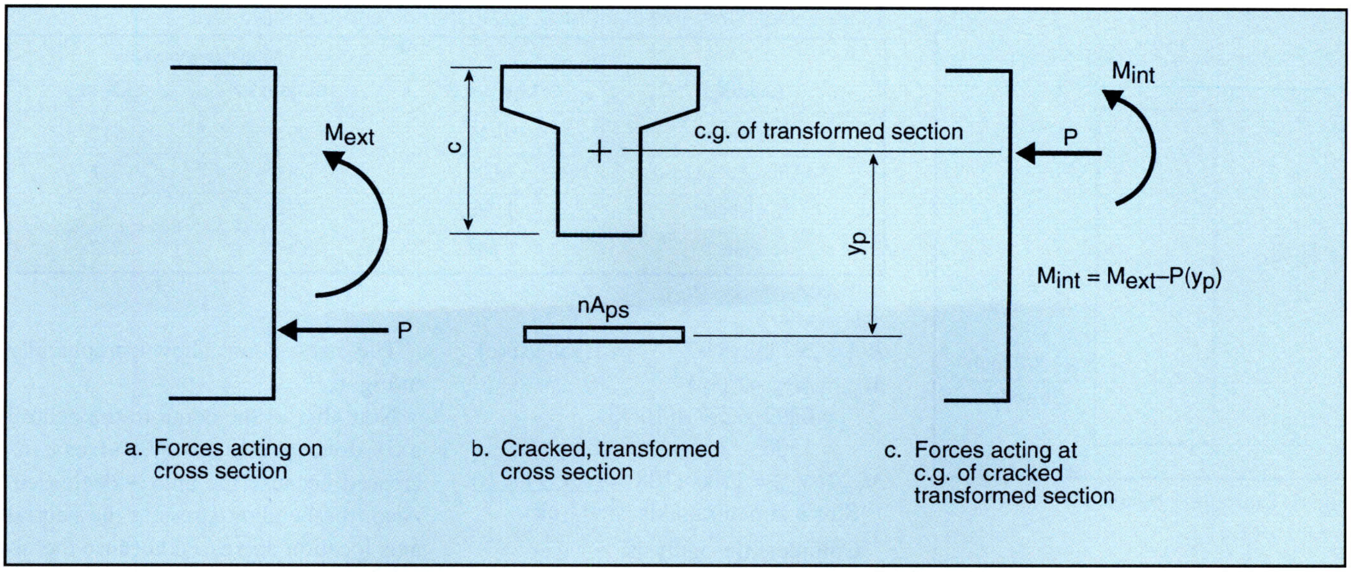

Fig. 1. Forces acting on cracked transformed section.

DESIGN IMPLICATIONS

Today, there is a trend to unify the design of nonprestressed and prestressed concrete members, and to permit designs with any combination of nonprestressed and prestressed reinforcement. In order to accomplish this goal, it will be necessary to replace the nominal tensile stress limits in the current ACI Code with requirements limiting cracking and deflection at service load. This paper describes a practical method of performing the needed cracked section analysis.

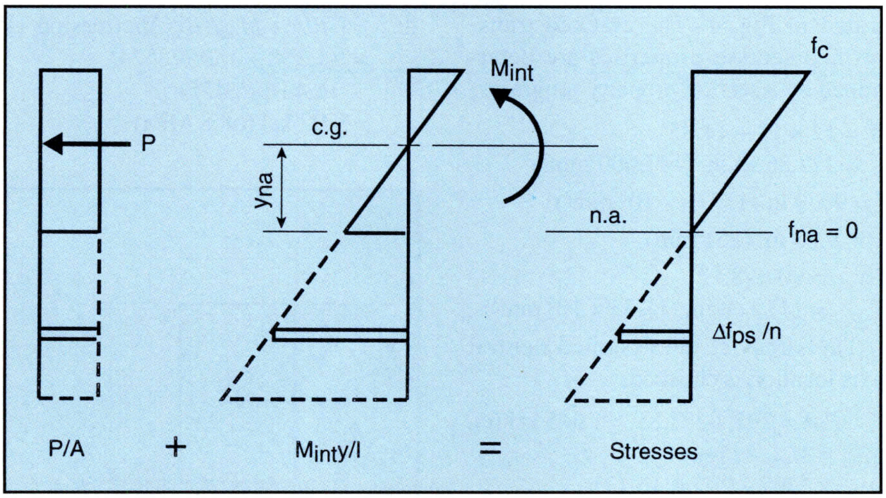

Fig. 2. Stresses in cracked transformed section.

SOLUTION STRATEGY

The analysis will make use of cracked transformed section properties, like those used in the past days of working stress analysis of ordinary (nonprestressed) reinforced concrete. The area of steel elements is replaced by a "transformed" area of concrete equal to n times the actual steel area, where n is the ratio of the modulus of elasticity of steel to that of concrete.

To begin the analysis, assume a trial depth c of the neutral axis of the cracked transformed section. The forces acting will be the prestress force P acting at the level of the tendons and the bending moment M_{ext} caused by external loads (all loads except prestress) (see Fig. 1a).

The forces may be resolved into an axial force P acting at the center of gravity of the cracked transformed section and an internal bending moment M_{int} acting about the center of gravity of the cracked transformed section. The internal bending moment M_{int} is the external bending moment M_{ext} reduced by the amount P times y_P, where y_P is the distance P moved upward from the location of the tendons to the center of gravity of the cracked transformed section, as shown in Fig. 1c.

The forces shown in Fig. 1c may be applied to the cracked transformed section, producing the stresses shown in Fig. 2. The stress at the neutral axis must be zero. That is:

$$f_{na} = P/A - M_{int}\, y_{na}/I = 0$$

On the first try for the neutral axis depth c, f_{na} will doubtless not be zero. If it is positive (compressive), c must be increased; c must be decreased if f_{na} is negative.

EXAMPLE 1

How the solution strategy works can best be illustrated by a simple example. Fig. 3 shows the cross section of a beam, with design parameters given below:

12 x 32 in. (305 x 813 mm) beam

f_c' = 6000 psi (41.4 MPa)

Twelve $^1/_2$ in. (12.7 mm) 270K strands

Depth d_p = 26 in. (660 mm)

Prestress level f_{dc} = 162 ksi (1117 MPa)

Span = 40 ft (12.2 m)

Dead and live loads, and midspan moments are given in Table 1.

$P = A_{ps}(f_{dc}) = 1.836\ (162)$
$= 297.4$ kips (1508 kN)

$n = E_{ps}/E_c = 28{,}500/4415.2 = 6.455$

$A_t = A_{ps}(n) = 1.836(6.455)$
$= 11.85$ sq in. (7645 mm^2)

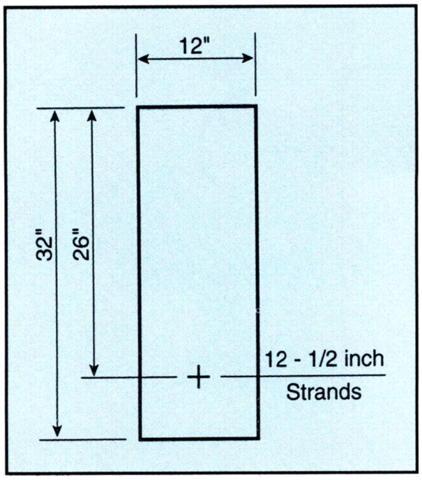

Fig. 3. Example 1 beam.

For a first try, assume the neutral axis depth c is 18 in. (457 mm). The cracked transformed section is illustrated in Fig. 4. The cracked transformed section properties are determined by a section property program.

$A = 12 \times 18 + 11.85$
$= 227.85$ sq in. (147,000 mm^2)

$I = 9079$ in.^{4} (3779×10^6 mm^4)

$y_t = 9.88$ in. (251 mm)

$I/y_{na} = 9079/8.12$
$= 1119$ cu in. (18.33 x 10^6 mm^3)

The stress at the assumed neutral axis location is checked.

$P/A = 297.4/227.85 = 1.305$ ksi (c)
$M_{int} = M_{ext} - P(y_P)$
$= 6392 - 297.4(16.12)$
$= 1598$
$M_{int}/(I/y_{na}) = 1598/1119 = \underline{-1.428 \text{ ksi (t)}}$
Stress at neutral axis = -0.123 ksi (t) (-0.848 MPa)

The stress at the neutral axis must be zero. Reduce c, to reduce tension at the assumed neutral axis location. After a few more trials (not shown), the solution at $c = 17.26$ in. (438 mm) is found. Fig. 5 illustrates the cracked transformed section for the correct solution. The calculation for determining the stress at the assumed neutral axis location follows.

The cracked transformed section properties from the section property program are:

$A = 218.97$ sq in. (141,270 mm^2)

$I = 8524$ in.^{4} (3548×10^6 mm^4)

$y_t = 9.57$ in. (243 mm)

$I/y_{na} = 8524/7.69$
$= 1108$ cu in. (18.16 x 106 mm^3)

Table 1. Dead and live loads, and midspan moments (Example 1).

Loadings	w kips per ft	Midspan moments in.-kips	Midspan moments kN-m
Self weight	0.413	992	112
Additional dead load	1.000	2400	271
Live load	1.250	3000	339
Sum	2.663	6392	722

$P/A = 297.4/218.97 = 1.358$ ksi (c)
$M_{int} = M_{ext} - P(y_P)$
$= 6392 - 297.4(16.43)$
$= 1506$
$M_{int}/(I/y_{na}) = 1506/1108 = \underline{-1.359 \text{ ksi (t)}}$
Stress at neutral axis $\cong$ 0 (ok)

Complete the analysis.

$f_c = P/A + M_{int}/(I/y_t)$
$= 1.358 + 1506/(8524/9.57)$
$= 3.048$ ksi (21.0 MPa)

$\Delta f_{ps} = [-P/A + M_{int}/(I/y_{ps})]n$ (tension +)
$= [-1.358 + 1506/(8524/16.43)]\ (6.455)$
$= 9.97$ ksi (68.8 MPa)

The stresses are shown graphically in Fig. 6.

Note that as the depth to the neutral axis decreased, the P/A stress increased because the area A decreased. Also, the bending stress at the neutral axis location decreased because the internal moment M_{int} decreased as the shift in the location of the center of gravity of the composite section increased.

Equilibrium may be checked manually, without the use of computers. Refer to Fig. 7.

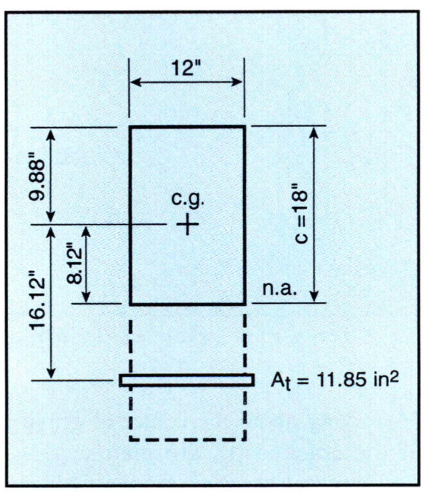

Fig. 4. Trial value of c.

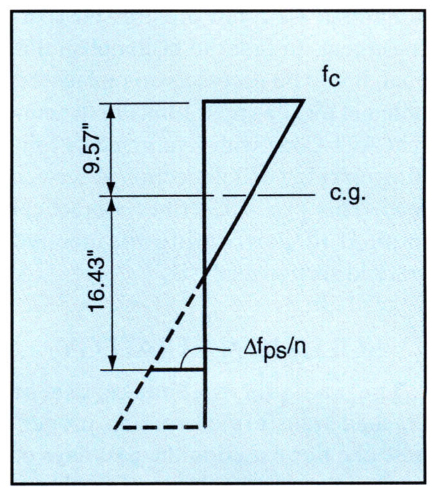

Fig. 6. Stresses in cracked section.

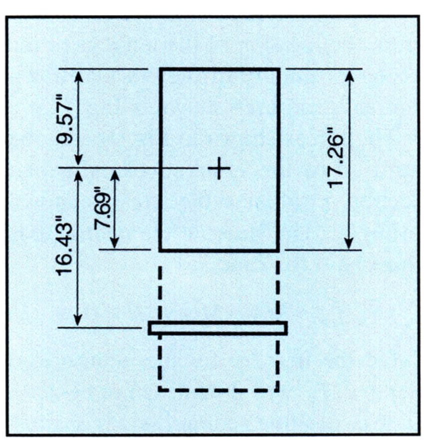

Fig. 5. Solution at $c = 17.26$ in. (438.4 mm).

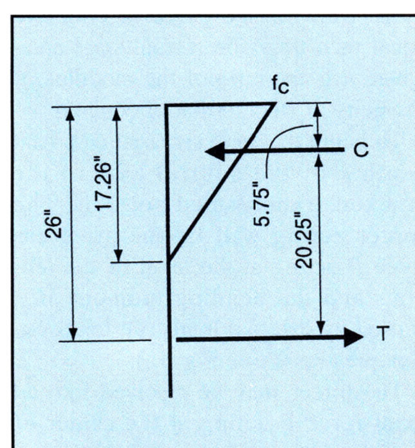

Fig. 7. Equilibrium check.

$C = f_c bc/2 = 3.048(12)(17.26)/2$
$= 315.7$ kips (1404 kN)

C acts at the top kern of compression zone equals $d_c/3$ for a rectangular area

$17.26/3 = 5.75$ in. (146 mm)

$T = P + \Delta f_{ps}(A_{ps}) = 297.4 + 9.97(1.836)$
$= 315.7$ kips (1404 kN) $= C$ Check

$M = C$ or $T \times$ lever arm
$= 315.7 \times 20.25$
$= 6392$ in.-kips (722 kN-m) Check

The beam of the example was chosen for simplicity. It is a rectangular beam with only one level of tendons and no unstressed reinforcement. Nevertheless, the method is general and will work for any section, no matter how complicated, for which the cracked transformed properties may be calculated.

THE PRESTRESS FORCE P

For use in a transformed analysis, the prestress level in pretensioned tendons should be taken as the stress that would exist in the tendons when the stress is zero in the adjacent concrete at the same level. This is called the decompression stress. The use of the transformed area of prestressing steel in the section properties will automatically account for the fluctuation of stress in the tendons when the stress in the adjacent concrete is not zero. The consideration of decompression stress was not apparent in the working stress analysis of nonprestressed concrete, because it was assumed (neglecting shrinkage) that it would be zero.

How should the decompression stress f_{dc} be calculated? It does not make sense to calculate it more accurately than the loss calculations used to estimate the effective prestress f_{se}. If the estimate of prestress loss is done using the method described in Section 4.7 of the PCI Design Handbook,[5] the decompression stress may be estimated as the effective prestress f_{se} plus an amount equal to $(f_{cir} - f_{cds})E_{ps}/E_c$. The quantities f_{cir} and f_{cds} are defined in the PCI Design Handbook, and their difference represents an estimate of the stress in the concrete adjacent to the tendons under sustained dead loads.

The most simple estimate of the decompression stress f_{dc} is to assume it is equal to the effective prestress f_{se}. The error will almost always be on the conservative side when computing crack control and deflections at service loads.

More precise methods of computing prestress losses and the decompression stress are available.[2,6,7]

NONPRESTRESSED REINFORCEMENT

In the design of cracked nonprestressed sections, it is the usual practice to neglect shrinkage and assume that the decompression stress is zero. In beams without prestress, the error introduced is probably small. But, in beams with a large amount of prestress, creep due to compressive stresses at the level of the nonprestressed reinforcement can magnify the error in neglecting the decompression stress in the nonprestressed reinforcement.

If the prestress losses are calculated by the PCI Design Handbook[5] method, the same calculations may be used to estimate the decompression stress in the nonprestressed reinforcement. Using the notation of the PCI Design Handbook, the decompression stress in the nonprestressed reinforcement could be estimated as a compressive stress equal to the creep losses CR plus the shrinkage losses SH computed by the method given in the PCI Design Handbook.

More comprehensive methods of estimating the decompression stress f_{dc} in nonprestressed reinforcement are given in Refs. 6, 7, and 8. These methods show that substantial compressive stresses can be built up in the nonprestressed reinforcement, and that these stresses can have a significant effect on the cracking moment and the post-cracking behavior.

Once the decompression stress (usually compressive) in the nonprestressed reinforcement is estimated, it should be combined with the decompression tensile force in the tendons to produce a resultant force P and a resultant location y_P of this force, for use in the cracked section calculations.

DEFLECTION

When designing a prestressed member intended to be cracked at service load, it is necessary to check deflections. Where the prestress and dead load produce stresses below the cracking strength, deflection may be calculated in the usual manner. The incremental deflection due to live load may be found using the bilinear behavior method or the effective moment of inertia method described in Sections 4.8.3 and 4.8.4 of the PCI Design Handbook.[5]

The results of the cracked section analysis provide another way of computing deflection. One normally thinks of deflection as a function of M/EI. But M/EI is equal to the curvature K. And the curvature K is simply equal to the maximum concrete strain ε_c divided by the depth c to the neutral axis. The computation of curvature K for Example 1 is shown below:

$K = \varepsilon_c/c$

$\varepsilon_c = f_c/E_c = 3.048/4415 = 0.000690$

$K = 0.000690/17.26$
$= 0.400 \times 10^{-4}$/in. (0.157×10^{-5}/mm)

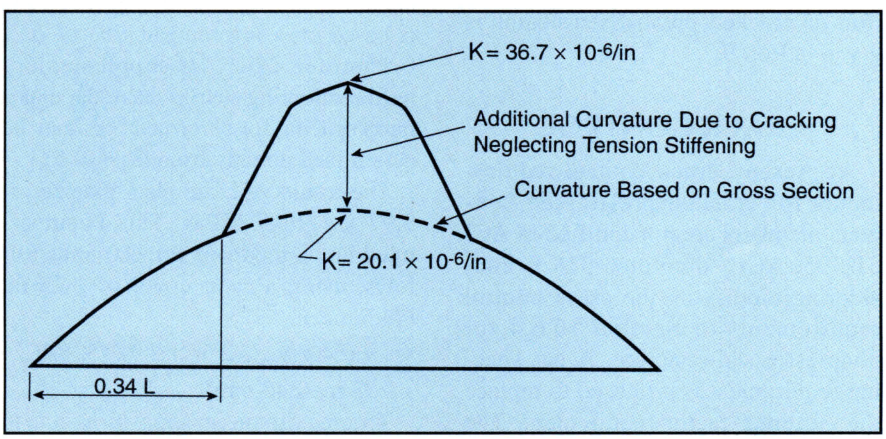

Fig. 8. Live load curvature diagram.

In order to obtain the incremental curvature due to live load, the curvature due to prestress and dead load must be subtracted. This curvature (calculated using gross transformed section, and the same E_c as in the cracked section analysis) was equal to 0.034×10^{-4}/in. Thus, the net curvature K_L due to live load is 0.366×10^{-4}/in. (0.144×10^{-5}/mm).

For an approximate value of midspan deflection, a parabolic curvature diagram may be assumed. The live load deflection is:

$$\Delta_L = 5/48 K_L L^2$$
$$= 5/48(0.366 \times 10^{-4})(480)^2$$
$$= 0.88 \text{ in. (22 mm)}$$

This result is conservative because the beam is not cracked throughout its length. Fig. 8 shows the curvature diagram for this beam, obtained by calculating the curvature at 1/20 points. Integration of this curvature diagram produces a live load deflection of 0.69 in. (17.5 mm). Even this number is conservative because tension stiffening is neglected.

For comparison, the deflection was calculated using the methods given in the PCI Design Handbook. Cracking was found to occur at 79 percent of full live load and the cracked moment of inertia I_{cr} was found to be 5513 in.^{4} (2.295×10^9 mm^4). The results are given below.

Bilinear behavior:
$\Delta_L = 1.00$ in. (25 mm)

Effective moment of inertia:
$\Delta_L = 0.86$ in. (22 mm)

It may be seen that all of the above methods give a conservative estimate of the instantaneous live load deflection. Additional information on deflection of cracked prestressed beams is given in Ref. 9.

CRACK CONTROL

At present, crack control requirements for cracked prestressed concrete members are not codified in ACI 318-95. ACI Committee 318 is considering modifying the crack control requirements of Section 10.6.4 for nonprestressed concrete. A bar spacing requirement is proposed to replace the current z factor requirement. The proposed spacing requirement is:

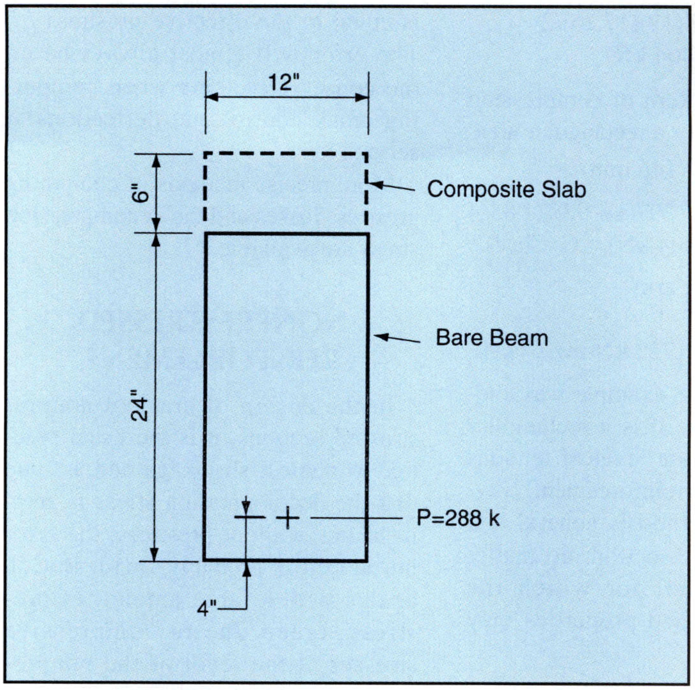

Fig. 9. Example 2 beam.

$$s \leq 540/f_s - 2.5c_c \leq 432/f_s \qquad \text{Eq. (10-5)}$$

where

s = center-to-center spacing of flexure tension reinforcement, in.

f_s = calculated stress in reinforcement at service loads, ksi

c_c = clear cover from nearest surface in tension to surface of flexural tension reinforcement, in.

A similar approach to crack control for cracked prestressed concrete members is under consideration. For pretensioned strands, the above equation needs to be modified in two ways:

1. The change in stress Δf_{ps} needs to be substituted for f_s.

2. For seven-wire strand, the spacing s needs to be reduced by a factor of two-thirds to account for the bond properties of strand being different from those of deformed reinforcement. This is based on a recommendation of ACI Committee 224.[10] [If supplementary mild reinforcing steel is used, the maximum spacing for deformed bars may be determined directly from Eq. (10-5).]

The results of Example 1 give Δf_{ps} = 9.97 ksi (68.7 MPa). This is substituted into a modified Eq. (10-5) as follows, using a clear cover of 1.75 in. (44.5 mm):

$$s \leq {}^2/_3(540/\Delta f_{ps} - 2.5c_c) \leq {}^2/_3(432)/\Delta f_{ps}$$
$$s \leq 33 \text{ in. (840 mm)}$$

The maximum spacing turns out to be very large because Δf_{ps} is so low compared to the f_s of 36 ksi (248 MPa) used in nonprestressed beams. Locating one strand in each corner of the tension side easily satisfies crack control requirements.

COMPOSITE SECTIONS

For composite sections, the situation is more complicated. The prestress and some dead load bending are usually applied to the bare noncomposite beam. This creates stress in the bare beam, but not in the composite slab. This causes a discontinuity in stress and strain at the interface and this discontinuity remains while additional loads are applied to the composite beam.

How does one find a section property of a cracked composite section when some of the forces and moments were applied to a different bare beam section? The solution is to work with section properties of the composite beam and apply all forces and moments to the composite section. This requires modifying the forces and moments applied to the bare beam to an equivalent force and moment applied to the composite beam. This equivalent force and moment must produce stresses in the bare beam portion of the composite beam that are equal to the actual stresses in the bare beam.

The process can best be illustrated by Example 2, which is purposely

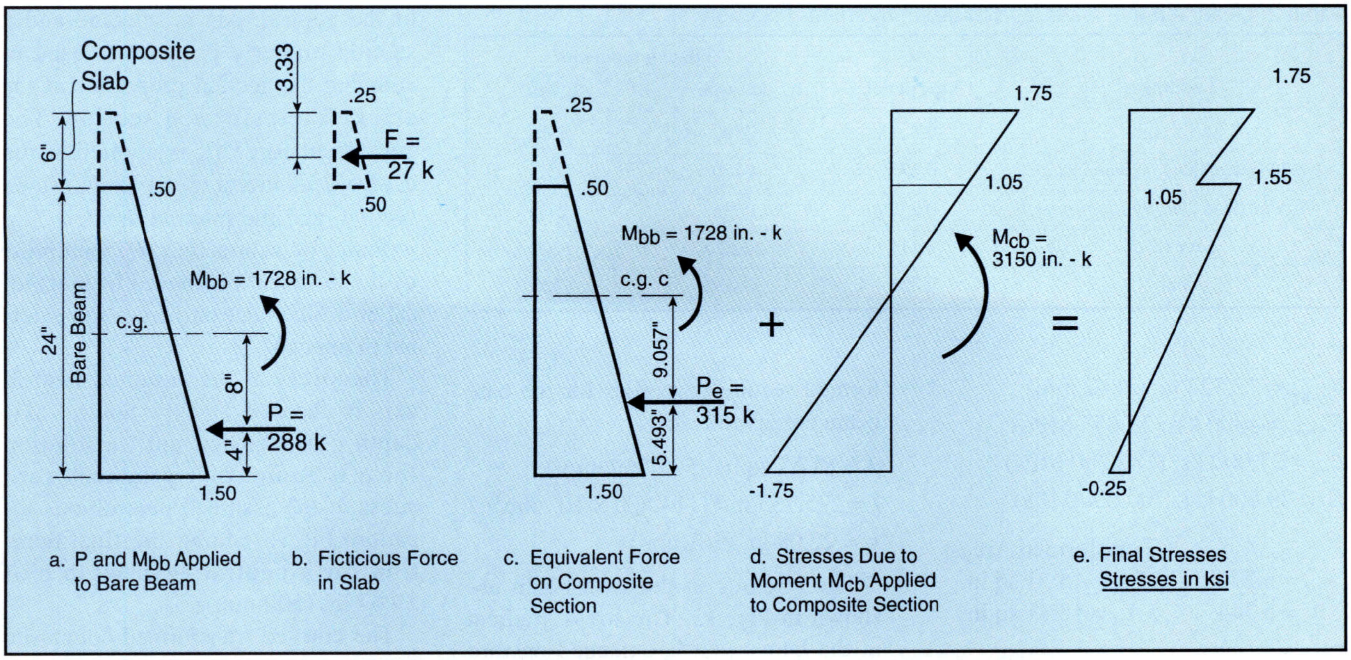

Fig. 10. Composite section analysis.

made simple in order to illustrate the process.

EXAMPLE 2

Consider a 12 x 24 in. (305 x 610 mm) beam shown in Fig. 9, subjected to a prestress force P of 288 kips (1281 kN) at an eccentricity of 8 in. (203 mm), and a bare beam bending moment M_{bb} of 1728 kip-in. (195.3 kN-m). This produces stresses in the bare beam as shown in Fig. 10a.

The beam is then made composite with a 6 in. (152 mm) slab of 12 in. (305 mm) width of the same concrete strength. It is now necessary to find the equivalent forces and moments applied to the composite section that will produce the same stresses in the bare beam portion of the composite beam.

This operation may be accomplished by extending the stress diagram for the bare beam up through the composite slab, as shown in Fig. 10a. This produces a fictitious force F in the slab, as shown in Fig. 10b. This fictitious force is combined with the prestress force P to produce the equivalent force P_e at a resultant location to be applied to the composite section. The magnitude and location of P_e combined with M_{bb} produce the desired stress in the composite section, as shown in Fig. 10c.

The stresses shown in Fig. 10c are then combined with the stresses shown in Fig. 10d due to moments M_{cb} applied to the composite beam. The fictitious stresses in the composite slab are subtracted in order to obtain the true stress in the composite slab. The final stresses are shown in Fig. 10e, and they are identical to stresses calculated in the usual manner.

Of course, when the composite beam is uncracked, this procedure is unnecessary. But, this procedure also works for cracked sections. The analysis of a cracked composite beam is similar to that of a cracked noncomposite beam, with the additional step of including the fictitious force in the composite slab due to bare beam stresses. Ref. 11 (pp. 201-204) and Ref. 12 give a more general method of analysis for cracked composite sections.

EXAMPLE 3

Example 3 illustrates the analysis of a cracked composite beam. The given parameters for Example 3 are identical to those for the example given by Al-Zaid and Naaman.[13] The beam is shown in Fig. 11, and the given parameters are summarized below.

E_{slab} = 3850 ksi (26,550 MPa)

n_{slab} = 0.89535

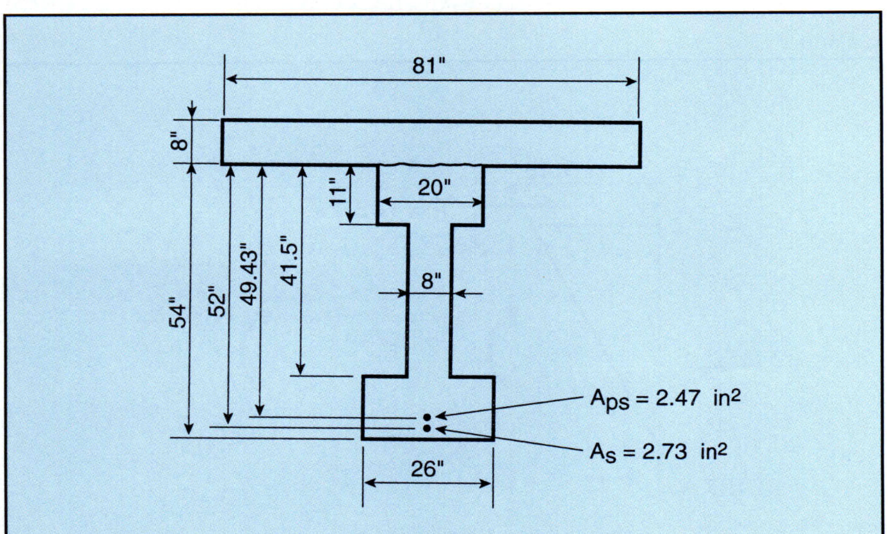

Fig. 11. Example 3 composite beam.

Table 2. Dead and live loads, and midspan moments (Example 3).

Loadings	w kips per ft	Midspan moments in.-kips	kN-m
Self weight	0.822	7890	892
Added dead load on bare beam	0.675	6480	732
Added dead load on composite	0.250	2396	271
Live load	1.111	10,670	1206
Sum	2.858	27,436	3100

b_{transf} = 72.523 in. (1842 mm)

E_{beam} = 4300 ksi (29,655 MPa)

E_{ps} = 27,000 ksi (186,200 MPa)

E_s = 29,000 ksi (200,000 MPa)

Transformed Areas

n_{ps} = 6.279 A_{pst} = 15.51 sq in.

n_s = 6.744 A_{st} = 18.41 sq in.

Decompression stress in tendons = 146.4 ksi (1010 MPa)

Decompression stress in reinforcement = 0

$P = A_{ps}(f_{dc}) = 2.47\ (146.4) = 361.6$ kips (1608 kN)

Dead and live loads, and midspan moments are given in Table 2.

The first step is to do an analysis of the bare beam for the moments applied to that beam. For consistency with the later cracked section analysis, transformed section properties are used with the areas of composite slab, tendons, and reinforcement transformed to an equivalent area of beam concrete. The analysis using gross transformed section properties shows that the beam is uncracked at the time it becomes composite. Gross transformed section properties for the bare beam are given below:

A = 817.7 sq in. (527,560 mm^2)

I = 275,755 in.^{4} (114,800 × 10^6 mm^4)

y_t = 22.08 in. (561 mm)

The stresses in the bare beam are shown in Fig. 12. The stress gradient in the bare beam is extended upward to the top of the future composite slab. The magnitude and location of the fictitious force F is computed. This is the force that would be in the composite slab, if it existed, without affecting the stresses in the bare beam. The fictitious force F is combined with the decompression force P and the magnitude and location of the equivalent force P_e are determined.

P:	361.6	at 57.43 =	20,777
F:	785.1	at 3.89 =	3,057
P_e:	1146.7		23,834

y_p = 23,834/1146.7
= 20.79 in. (528 mm)
(measured from top of composite)

The next step is the analysis of the cracked transformed composite section at full service load. A trial depth of the neutral axis is selected and a section property program is used to compute the section properties of the cracked transformed section. The equivalent force P_e is applied at the centroid of the cracked transformed section, and the internal moment M_{int} is found by subtracting (P_e multiplied by the distance it is moved from its location to the centroid) from the external moment M_{ext}.

The stress at the assumed neutral axis is checked and the neutral axis depth c is adjusted until a solution for c is found that produces zero stress at the assumed neutral axis location. Fig. 13 shows the final iteration for a neutral axis depth c of 19.67 in. (500 mm).

The cracked transformed composite section properties are as follows:

A = 839.5 sq in. (541,600 mm^2)

I = 109,079 in.^{4} (45,401 × 10^6 mm^4)

c = 19.67 in. (500 mm)

y_t = 8.80 in. (224 mm)

y_{na} = 10.87 in. (276 mm)

I/y_{na} = 10,035 cu in. (164.4 × 10^6 mm^3)

P_e/A = 1146.7/839.5 = 1.365 (c)

$M_{int} = M_{ext} - P_e\ (20.79 - 8.80)$
= 27,436 – 1146.7(11.99)
= 13,690

$M_{int}/(I/y_{cna})$ = 13,690/10,035
= -1.365 (t)

Stress at neutral axis = 0 (ok)

The results are identical to those given by Al-Zaid and Naaman.[13] The equilibrium checks are somewhat more complicated than for Example 1, but they can still be accomplished without the use of a computer, as shown in Fig. 14. Note that using the method described in this paper, it is not necessary to idealize the I-beam as three rectangles. The actual shape of the flanges should pose no problem for a section property program.

DEFLECTION AND CRACK CONTROL

The curvature K is found as ε_c/c, where ε_c at the top of the precast beam is equal to f_c/E_c = 1.467/4300 = 3.41 × 10^{-4}. This distance from the top of the precast beam to the neutral axis is 11.67 in. (296 mm). Dividing by this, K = 2.92 × 10^{-5}/in. Subtracting the dead load curvature of 0.70 × 10^{-5},

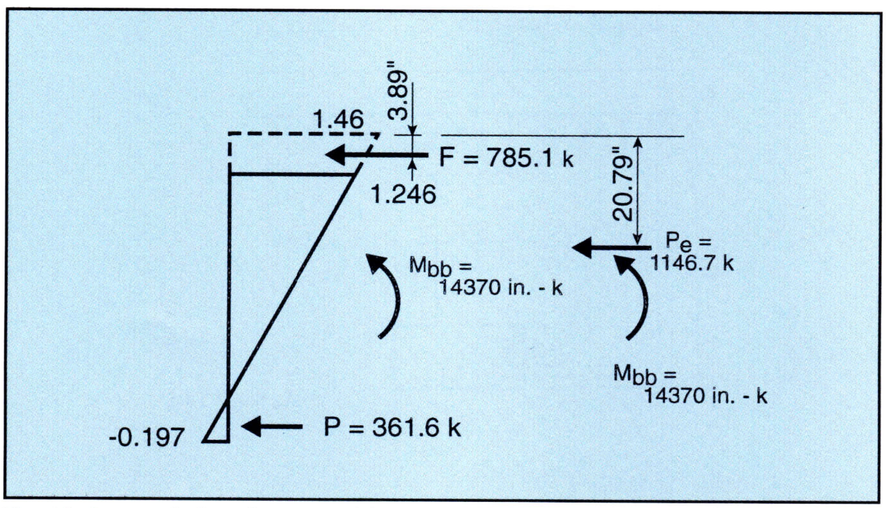

Fig. 12. Stresses in bare beam, and fictitious force F.

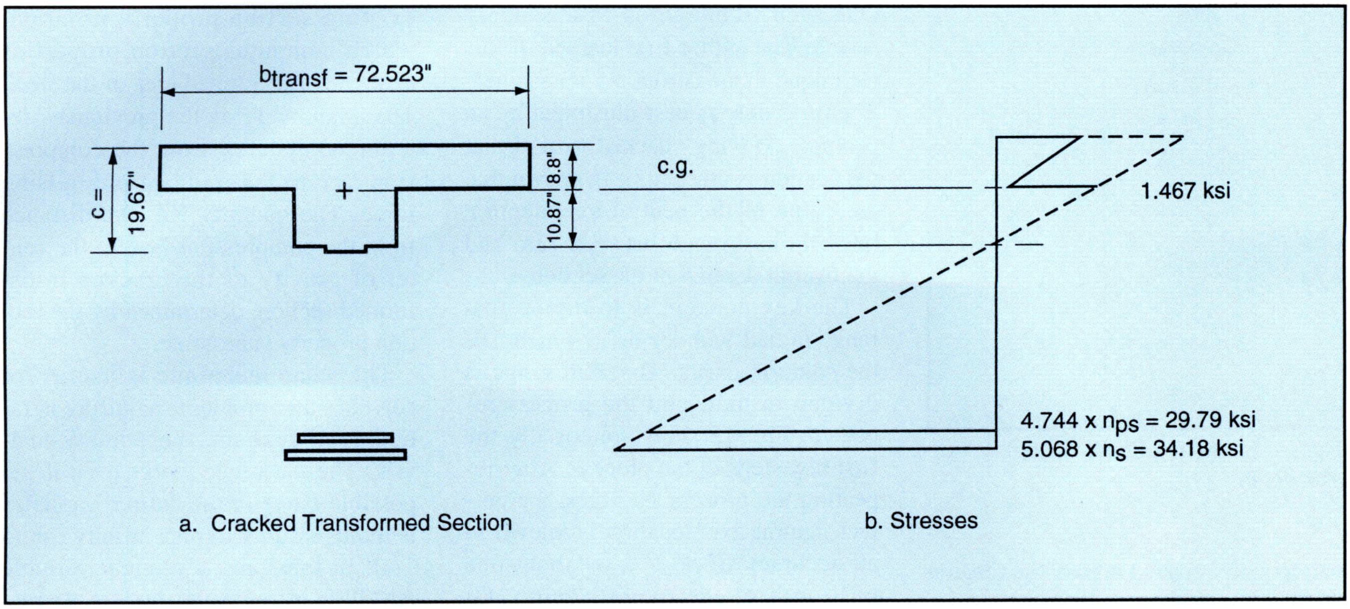

Fig. 13. Solution of cracked transformed composite section.

computed from a gross section analysis, the live load curvature $K_L = 2.22 \times 10^{-5}$/in. ($0.874 \times 10^{-6}$/mm). Substituting in the conservative and approximate formula $\Delta_L = (5/48)K_L L^2$, with L = 960 in. (24.4 m), Δ_L = 2.13 in. (54 mm). For a building, the allowable live load deflection would be $L/360$ = 2.67 in. (68 mm). Thus, the deflection is satisfactory.

The reinforcement nearest the tension face consists of deformed bars. The steel stress f_s is 34.18 ksi (235.7 MPa). Assuming a clear cover to the longitudinal steel of 1.5 in. (38 mm), the maximum spacing of reinforcement is given directly by Eq. (10-5) as 12 in. (306 mm). A minimum of three

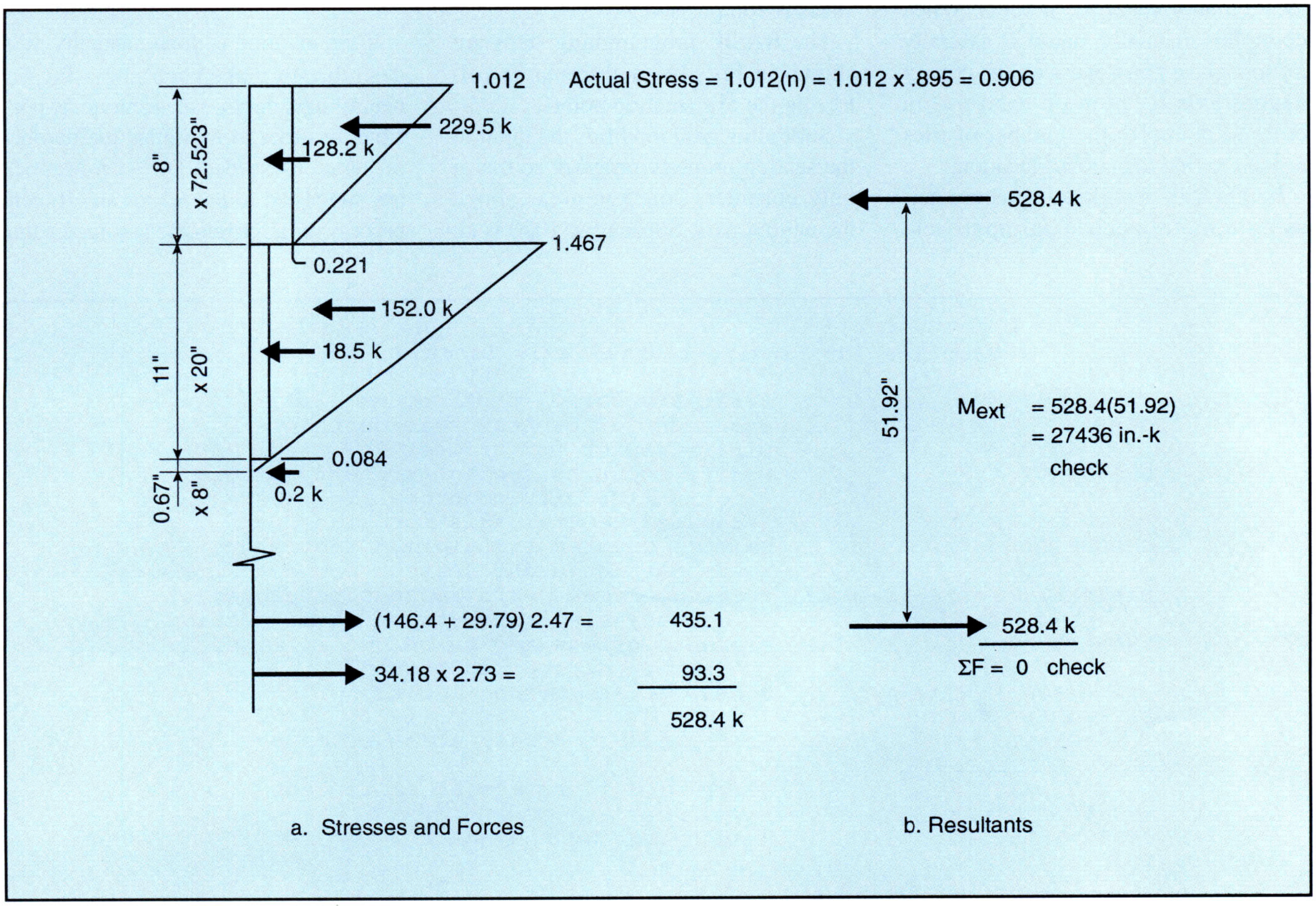

Fig. 14. Equilibrium check.

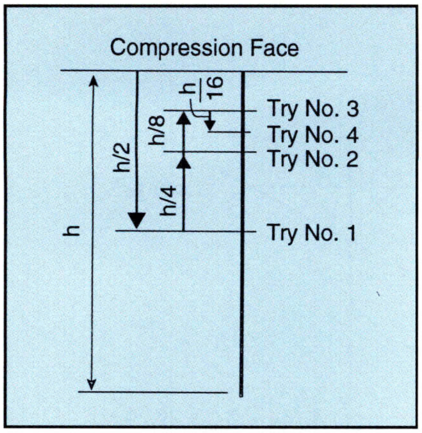

Fig. 15. Binary search for neutral axis depth.

bars equally spaced across the tension face are required.

ITERATION PROCESS

In order to find the correct location of the neutral axis of a cracked section, it is necessary to assume a neutral axis location and then check to see if the assumed location is correct. The first try will usually not be correct. It is then necessary to assume a new neutral axis location and repeat the process. When doing this manually, one will naturally try to make a good guess on the correct neutral axis location on each try, in order to minimize the number of tries needed to find the correct location.

Iteration is best done by computer, using a more mechanical approach. One such technique is called binary search. The author first learned of this technique in an astronomy magazine.[14] The procedure is best illustrated by an example. Having checked to find that the section is cracked, it is known that the value of the neutral axis depth c must lie in a range between zero and the overall depth h of the section.

The key concept is to divide that range in half and decide in which half the solution exists. That half range is divided in half, and the process repeated. Fig. 15 shows pictorially the first few steps of the process. After repeating the process 20 times, the correct neutral axis location is known to an accuracy of $h/(2^{20})$, or about one millionth of the overall depth. Of course, this degree of accuracy is not needed, but a personal computer does the calculation in an instant.

The decision of whether the neutral axis depth should be increased or decreased after each try is done in the same manner as was illustrated in Example 1. If the stress at the assumed neutral axis location is tensile, c must be decreased, and conversely if the stress is compressive.

The BASIC programming steps are shown in Fig. 16, with remarks following the exclamation points.

Subroutine 900 modifies the input to the section property program so that it only considers concrete areas above the neutral axis. Subroutine 1000 is an existing section property program, which computes section properties using the transformed area of the steel. The quantity YP is the previously determined distance from the compression face to the resultant prestressing force. The quantity Y is the distance from the compression face to the center of gravity of the cracked transformed section, determined by the section property subroutine.

The same technique is useful for solving other problems requiring iteration, such as strain compatibility analysis. The technique works even if the possible range of the desired variables is minus infinity to plus infinity ($-\infty$ to $+\infty$). In this case, a dummy variable ANGL is incremented, and the desired variable is equated to the tangent of ANGL. Then as ANGL is incremented in the range of (almost) ±90 degrees, the desired variable takes on values of (almost) $\pm\infty$. But, the process works only if the desired variable is continually increasing or continually decreasing throughout the range investigated.

MANUAL ITERATION

If the iteration is done manually, it is desirable to make each guess for the neutral axis depth c as accurate as possible, in order to minimize the number of iterations needed. Dr. Alan Mattock has suggested to the author an efficient process for adjusting the assumed value

```
!
!            Subroutine for finding neutral axis location
!
C=H/2                        !  First try for C, depth of na
DELTC=H/4                    !  Increment of neutral axis depth
FOR J=1 TO 20                !  Binary search for correct C
   GOSUB 900                 !  Modify input to section property program
                                  to include only concrete area above
                                  assumed neutral axis location
   GOSUB 1000                !  Compute cracked transformed section
                             !     properties using section property program
   MINT=MEXT-P*(YP-Y)        !  Internal moment with respect to center of
                             !     gravity of cracked transformed section
   YNA=C-Y                   !  Distance of assumed na below center of
                             !     gravity of cracked section
   FATNA=P/A-MINT*YNA/I      !  Stress at assumed na location
   IF FATNA>0 THEN
      C=C+DELTC              !  Increase C if stress at na is comp
   ELSE
      C=C-DELTC              !  Decrease C if stress at na is tens
   END IF
   DELTC=DELTC/2             !  Reduce increment by half
NEXT J
```

Fig. 16. BASIC programming steps for binary search technique.

of c after the first try. Find the depth of the zero stress fiber, and use this for the trial value of c for the next try.

For example, consider the first try in Example 1, with c assumed to be 18 in. (457 mm). Fig. 17 shows the stress diagram for this condition. The location of the zero stress fiber may be found by equating P/A to My_o/I and solving for y_o, the distance from the centroid of the cracked transformed section to the zero stress fiber. This results in y_o = 7.41 in. (188 mm) and a new trial value for c of 17.29 in. (439 mm). Repeating the process once produces a new trial value for c of 17.26 in. (438 mm), which is the correct solution.

To facilitate the computation, a spreadsheet may be used. The process converges rapidly; usually sufficient accuracy is obtained within three tries.

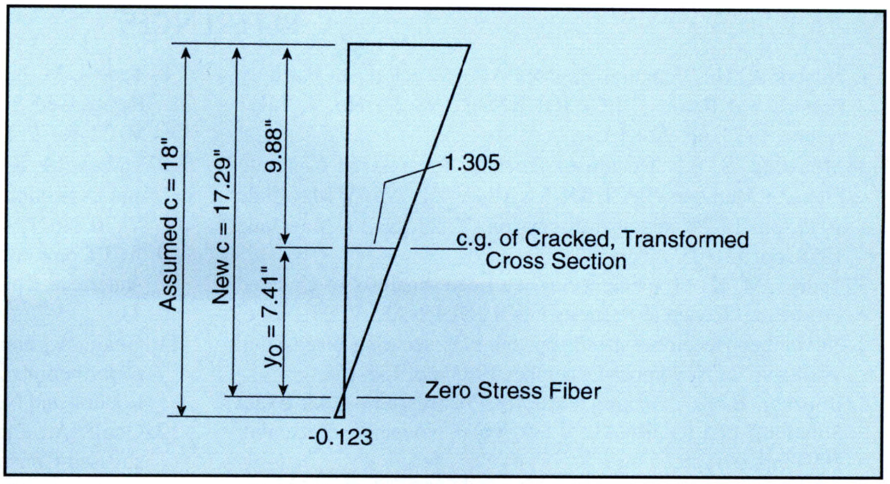

Fig. 17. Concrete stress block, first try, Example 1.

SUMMARY OF STEPS FOR CRACKED SECTION ANALYSIS

In general, the steps needed to carry out the cracked section analysis can be summarized as follows:

1. Perform a gross section analysis and determine if the section is cracked at service load.

2. Estimate the decompression force P in the prestressing steel. The decompression stress will usually be the effective prestress plus some (or most) of the elastic shortening loss added back in.

3. At the time of decompression, there will be a compression force in the unstressed reinforcement, approximately equal to the creep and shrinkage losses of the prestressing steel multiplied by the reinforcement area. This force may be combined with the decompression tensile force in the strands to obtain a resultant decompression force P, and a location for that resultant.

4. If the section is composite, compute the fictitious force in the composite slab created by extending the bare beam stress diagram through the composite slab. Combine this fictitious force with the decompression force P to obtain an equivalent force P_e and its location.

5. Compute the combined transformed section properties (A, I, center of gravity) of all of the steel elements, using the modular ratio to transform the steel area to an equivalent concrete area.

6. Select a trial depth c to the neutral axis. Compute the section properties (A, I, center of gravity) of the net concrete section between the compression face and the neutral axis.

7. Combine the net concrete cracked section properties with those of the transformed steel to obtain the combined section properties of the cracked transformed section.

8. Apply the decompression force P, or the equivalent force P_e for composite sections, and the external moment to the cracked transformed section. Apply the force at the center of gravity of the cracked transformed section and adjust the internal moment M_{int} to account for the shift in location of the force P or P_e.

9. Compute the location of the zero stress fiber (i.e., the neutral axis). If this agrees with the location assumed in Step 6, the solution is found. Otherwise, select a new trial depth to the neutral axis and repeat Steps 6 to 9 until the assumed and computed neutral axis locations agree with sufficient accuracy.

10. Compute the concrete and steel stresses by applying the decompression force P and the internal moment M_{int} to the cracked transformed section. Add the decompression stress in the prestressing steel to obtain the total stress in the prestressing steel.

11. Find the true stresses in the composite slab by deducting the stresses associated with the introduction of the fictitious force in the slab.

12. Find the midspan curvature of the cracked section and use this to estimate the deflection. Alternately, use other recognized methods for estimating the deflection of the cracked member.

13. Use the change in stress f_{ps} or f_s to evaluate strand or bar spacings required for reasonable crack control.

CONCLUDING REMARKS

A method is presented for the analysis of cracked prestressed concrete sections. Although the examples presented are simplified, the method is general and is applicable to the most complicated sections of cracked concrete members. These include members of any shape with a mix of reinforcement types at various depths and composite sections.

The method requires iteration and is best done by computer. The great bulk of the necessary computer code can be taken from existing section property software. The results can be checked without resorting to the use of computer software.

ACKNOWLEDGMENT

This paper is based on the works of many previous authors, several of whom are referenced. The purpose of the author in writing this paper is to synthesize all this work into a practical method for analyzing cracked prestressed concrete sections and, therefore, bring this methodology into more widespread use by structural engineers.

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APPENDIX — NOTATION

A = cross-sectional area of transformed section
A_{ps} = area of prestressed reinforcement
A_{pst} = transformed area of prestressed reinforcement
A_s = area of nonprestressed reinforcement
A_{st} = transformed area of nonprestressed reinforcement
A_t = transformed area of steel
b = width of compression zone
b_{transf} = transformed width of composite slab
C = compressive force in concrete
CR = loss of prestress due to creep of concrete
c = distance from extreme compression fiber to (assumed) neutral axis
d_c = depth of compression zone
d_p = distance from extreme compression fiber to centroid of prestressing steel in tension
E = modulus of elasticity
E_c = modulus of elasticity of concrete
E_{ps} = modulus of elasticity of prestressed reinforcement
E_s = modulus of elasticity of nonprestressed reinforcement
E_{slab} = modulus of elasticity of composite slab
e = eccentricity of prestress force
F = fictitious force in composite slab, needed in cracked section analysis of composite beams
f_c = concrete stress
f_c' = specified compressive strength of concrete at 28 days
f_{cds} = concrete stress at center of gravity of prestress force due to all permanent (dead) loads not used in computing f_{cir}
f_{cir} = concrete stress at center of gravity of prestressing force immediately after transfer
f_{dc} = decompression stress in prestressed tendons that exists when stress in adjacent concrete at same level is zero
f_{na} = concrete stress at (assumed) neutral axis location
f_{se} = effective stress in prestressing steel after losses
h = overall height of member
I = moment of inertia of transformed section
J = a counter used to count repetitions in binary search routine
K = curvature
K_L = curvature due to live load
L = span length
M = service load moment
M_{bb} = service load moment applied to bare beam
M_{cb} = service load moment applied to composite beam
M_{ext} = service load moment due to external loads (all loads except prestress)
M_{int} = internal bending moment acting about center of gravity of cracked transformed section
M_{msp} = service load moment at midspan
n = ratio of moduli of elasticity
n_{ps} = ratio of E_{ps}/E_c
n_s = ratio of E_s/E_c
n_{slab} = ratio of E_{slab}/E_c
P = prestress force at decompression
P_e = effective axial load, including prestress force at decompression and fictitious force in composite slab
SH = loss of prestress due to shrinkage of concrete
T = tension force in steel
w = unfactored load per unit length
y_o = distance between center of gravity of cracked transformed section and zero stress fiber
y_{na} = distance between center of gravity of cracked transformed section and (assumed) location of neutral axis
y_p = distance between center of gravity of cracked transformed section and location of axial force acting on section
y_t = distance from top fiber to center of gravity of transformed section
Δf_{ps} = change in stress in prestressed reinforcement between decompression stress and stress at full service load
Δ_L = deflection due to live load
ε_c = maximum concrete strain at service load