conditions of curvature determined by either of the following:

"(a) When the actual computed eccentricities are less than the specified minimum, the computed end moments may be used to evaluate the conditions of curvature."

"(b) If computations show that there is no eccentricity at both ends of the member, conditions of curvature shall be based on a ratio of M_1/M_2 equal to one."

Section 11.11.1

Amend to read:

"**11.11.1** — Shear reinforcement consisting of bars or wires anchored in accordance with Section 12.13 may be provided in slabs. For design of such shear reinforcement, shear stresses shall be investigated at the critical section defined in Section 11.10.2 and at successive sections more distant from the support; and the shear stress v_c carried by the concrete at any section shall not exceed $2\sqrt{f_c'}$. Where v_u exceeds v_c, the shear reinforcement shall be provided according to Section 11.6."

Section 13.4.3

Revise so that the last three lines read as follows " . . . each frame may be distributed to the column strips, middle strips, and beams as specified in Section 13.3.4 if the requirement of Section 13.3.1.6 is satisfied."

Section 13.5.1

Delete the last sentence in Section 13.5.1.

Section 14.2(f)

Place period after "wall" in fifth line. Reword the remainder of the section to read: "These values may be reduced to 0.0020 and 0.0012, respectively, if the reinforcement is not larger than ⅝ in. in diameter and consists of either welded wire fabric or deformed bars with a specified yield strength of 60,000 psi or greater."

Section 15.5.1

Add: "The location of the critical section for shear shall be measured from the face of a column, wall, or pedestal or, in the case of a member on a steel base plate, from the section described in Section 15.4.2 (c)."

Section 18.9.3

Revise the first sentence of the amendment to this section, as published in the September 1970 JOURNAL, to read:

18.9.3 — The minimum amount of bonded reinforcement A_s in two-way slabs shall be that required by Section 18.9.1. This requirement for bonded reinforcement in two-way slabs may be decreased where the tension in the precompressed tensile zone at service loads does not exceed zero.

Compression Steel Effect on Long-Time Deflections

By DAN E. BRANSON*

Discusses concrete beam deflections relating to ACI 318-63, ACI 318-71, the Unified British Code, and the references cited. It is written in response to a study by M. R. Hollington, which indicates that the restraining effect of compression steel on time-dependent deflections is less than the ACI 318-63 (similar to ACI 318-71) and the Unified Code provisions predict for beams with low steel percentages. The ACI 318-71 procedure is evaluated and found to be within reasonable limits in most cases for such a grossly simplified approach to this rather complex problem. However, the procedure does somewhat overestimate the effect of compression steel for beams with low steel percentages (approximately 1 percent and less) when A_s'/A_s is high (as about 1.0). An alternate method for predicting the effect of compression steel on long-time deflections, as a function of the compression steel percentage, p', rather than the steel area ratio, A_s'/A_s, is presented. Reference is also made to the corresponding prediction of shrinkage warping, and to the variation in creep and shrinkage effects with time.

Keywords: beams (supports); creep properties; deflection; moments of inertia; reinforced concrete; reinforcing steels; shrinkage; structural design; warpage.

It should first be noted that the method of computing initial deflections, to which the time-dependent factor is applied in computing additional long-time deflections, is different in ACI 318-71[2,3,†] as compared to ACI 318-63, and also different than the Unified British Code. The effects of load level and degree of cracking are included in ACI 318-71 by Eq. (1) and the Unified British Code by Eq. (2).

$$I_{eff} = (M_{cr}/M_{max})^3\, I_g + [1 - (M_{cr}/M_{max})^3] I_{cr} \quad (1)$$

Eq. (1) is to be used with an equation such as:

$$\Delta_i = K\, M\, L^2/E_c\, I_{eff}$$

For additional load increments, such as live load, I_{eff} must be computed for the total moment, and the de-

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†ACI Committee 435, Subcommittee 4, "Recommendation for ACI Building Code Provision on Deflections," Report approved by ACI Committee 435 and submitted to ACI Committee 318, Oct. 1967.

flection increment computed from the total deflection, as indicated by Eq. (29) to (32) in References 4 and 5.

$$\Delta_i = K\,L^2 \left[\frac{M - M_{cr}}{0.85\,E_c\,I_{cr}} + \frac{M_{cr}}{E_c\,I_o}\right] \tag{2}$$

where I_o is the uncracked transformed section I.

The ACI 318-63 provision uses either the gross section I (I_g) or the fully cracked section I (I_{cr}) in computing deflections, depending on the value of pf_y. Eq. (1) provides a transition between these two limits of I_g and I_{cr}. The relation between the ACI 318-63 and ACI 318-71 provisions has been discussed[2,3,6] and statistically compared with a considerable amount of experimental data by ACI Committee 435.‡ The effective I method used in ACI 318-71 was developed[2] using the Newmark numerical method (also used by Hollington[1]), in which the effect of crack distribution along the span, as function of moment level, was evaluated in terms of the limiting second moments of the area—the fully cracked I and the uncracked I. The gross section I can be used instead of the uncracked transformed section I for most purposes, as in Eq. (1).

EFFECT OF NONTENSIONED STEEL IN PRESTRESSED BEAMS

This section discusses the theoretical development of an expression for the effect of nontensioned steel in prestressed beams and its possible relation to the effect of compression steel in reinforced beams.

Although not directly related, it is interesting to note than an equation, theoretically derived using an energy method,[7] for predicting the effect of nontensioned steel in reducing time-dependent camber of prestressed beams (and verified experimentally), yields approximately the same results as the ACI 318-71 provision (which was empirically determined) for the similar effect of compression steel in reinforced beams. These two procedures are given by Eq. (3) and (4) and compared with a modified procedure, and with various sets of data (basic data in Table 1[1,8,9]) for reinforced beams, in Table 2 [also see Eq. (23) to(25) in References 4 and 5].

Using the ACI 318-71 approach:

$$k_r = 1 - 0.60\,(A_s'/A_s) \tag{3}$$

However, from Reference 7:

$$k_r = 1/[1 + (A_s'/A_s)] \tag{4}$$

Eq. (4) was derived for the special case of equal eccentricities for both tensioned and nontensioned steel. The general solution is also given in Reference 7 which shows the effect of concrete strain, member size, steel percentage, etc., in addition to the ratio, A_s'/A_s. A possible modification of Eq. (4) to take into account the effect of steel percentage is discussed in the next section [see Eq. (5) to (10) and Table 2].

EFFECT OF COMPRESSION STEEL ON REINFORCED BEAMS

This section discusses the time-department deflections of reinforced concrete beams, including the effect of compression steel.

Hollington[1] has correctly pointed out that the effect of compression steel in restraining time-dependent deflections is less in beams with low steel percentages (say for a given A_s'/A_s). This can be seen in the theoretical procedure[7] mentioned in the previous section. It also seems to follow intuitively. The pertinent conclusions by Hollington regarding this question are the following:

> "The restraint provided by compression reinforcement was found to be considerably less than the calculated value based on ACI Code 318-63 or the draft Unified Code. For members containing a larger percentage of reinforcement, it is likely that the calculated and measured results would be compatible. For lightly reinforced floor slabs made

‡ACI Committee 435, Subcommittee 2, "The Variability of Deflections," ACI Publication in Progress.

TABLE 1 — MEASURED DEFLECTIONS SHOWING EFFECT OF COMPRESSION STEEL

Beams		Measured deflection ratios						
		Duration of load, months	p, percent	$A_s'/A_s = 0$	$A_s'/A_s = 0.5$		$A_s'/A_s = 1.0$	
				Δ_t/Δ_i Column 1	Δ_t/Δ_i Column 2	Column 2 / Column 1 A	Δ_t/Δ_i Column 3	Column 3 / Column 1 B
Washa and Fluck[8] Rectangular solid beams from Reference 2, Table A2.4	Series							
	A	30	1.63	1.09/0.67 = 1.63	0.65/0.62 = 1.05	0.64	0.40/0.53 = 0.76	0.47
	B	30	1.67	2.36/1.04 = 2.27	1.58/0.98 = 1.61	0.71	1.09/0.92 = 1.18	0.52
	C	30	1.67	3.66/1.88 = 1.95	2.25/1.71 = 1.31	0.67	1.57/1.58 = 0.99	0.51
	D	30	1.67	1.21/0.70 = 1.73	0.74/0.56 = 1.32	0.76	0.62/0.47 = 1.32	0.76
	E	30	1.59	4.80/2.48 = 1.94	2.87/2.20 = 1.30	0.67	2.54/2.34 = 1.09	0.56
Yu and Winter[9] T-beams from Reference 2, T. A2.4	A,B,C	6	1.01	1.31/1.34 = 0.98	0.99/1.24 = 0.80	0.82	1.05/1.19 = 0.88	0.90*
Hollington[1] Rectangular solid and hollow box beams from Table 9	Line 14, 17, 20 (Beams 13-21)	26	0.59	0.70/0.46 = 1.52	0.55/0.47 = 1.17	0.77	0.64/0.47 = 1.36 †0.53/0.43 = 1.23	0.89* 0.81
	Line 35, 38 (Beams 34-39)	26	0.59	1.29/0.57 = 2.26			0.98/0.61 = 1.61	0.71
	Line 47, 50 (Beams 46-51)	25	0.77	0.56/0.45 = 1.24			0.48/0.48 = 1.00	0.81

Δ_i = initial deflection or short-time deflection
Δ_t = time-dependent deflection or total deflection minus initial deflection
p = (A_s/bd)

Note: Each time-dependent deflection was normalized with respect to its initial deflection. Each ratio showing the effect of compression steel content for a given series of tests referred to the same duration of loading, although the duration of loading varied for the different tests. However, this should not materially affect the comparisons from test to test.

*This figure is somewhat inconsistent with Column A and other data in Column B. However, these and the other data in Column B do justify a word of caution when p is low and A_s'/A_s is high.
†Line 20[A].

TABLE 2 — REDUCTION FACTORS AND TIME-DEPENDENT FACTORS

Method of determination		Reduction factors k_r: $\frac{A_s'}{A_s} = 0$*	$\frac{A_s'}{A_s} = 0.5$	$\frac{A_s'}{A_s} = 1.0$	Time-dependent factors = 2 × reduction factor T: $\frac{A_s'}{A_s} = 0$*	$\frac{A_s'}{A_s} = 0.5$	$\frac{A_s'}{A_s} = 1$
Measured values from Table 1, Columns A and B	All data, except as noted	1.00 1.00	Range 0.64-0.82 Avg 0.72	Range 0.47-0.81† Avg 0.64†	2.00	1.44	1.28
	Washa and Fluck[8] data $p = 1.6$ percent	1.00 1.00	Range 0.64-0.76 Avg 0.69	Range 0.47-0.76 Avg 0.56	2.00	1.38	1.12
	Yu and Winter,[9] and Hollington[1] data, $p = 0.6$ percent to 1.0 percent	1.00 1.00	Range 0.77-0.82 Avg 0.80	Range 0.71-0.81† Avg 0.78†	2.00	1.60	1.56
ACI 318-71 (see Footnote ‡ for comparisons): $[2 - 1.2(A_s'/A_s)]$		1.00	(0.60)‡ 0.70	0.40	2.00	(1.2)‡ 1.40	0.80
Theoretically derived[7] equation for effect of nontensioned steel in reducing time-dependent camber in prestressed concrete beams: $1/[1 + A_s'/A_s)]$		1.00	0.67	0.50	2.00	1.33	1.00
Possible modification of theoretical equation[7] similar to Reference 13, but here for effect of p: $1/[1 + C(A_s'/A_s)]$ where $C = 50p$, $p = A_s/bd$	$p = 0$, $C = 0.00$	1.00	1.00	1.00	2.00	2.00	2.00
	p = 0.005, C = 0.25	1.00	0.89	0.80	2.00	1.78	1.60
	p = 0.006, C = 0.30	1.00	0.87	0.77	2.00	1.74	1.54
	p = 0.010, C = 0.50	1.00	0.80	0.67	2.00	1.60	1.34
	p = 0.014, C = 0.70	1.00	0.74	0.59	2.00	1.48	1.18
	p = 0.015, C = 0.75	1.00	0.73	0.57	2.00	1.46	1.14
	p = 0.016, C = 0.80	1.00	0.71	0.56	2.00	1.42	1.12
	p = 0.020, C = 1.00	1.00	0.67	0.50	2.00	1.34	1.00
	p = 0.030, C = 1.50	1.00	0.57	0.40	2.00	1.14	0.80
	p = 0.040, C = 2.00	1.00	0.50	0.33	2.00	1.00	0.67

*Reduction factors and time-dependent factors are based on or normalized with respect to the case of no compression steel.
†Excluding two apparently inconsistent data points as noted in Table 1.
‡The ACI 318-63 provisions for including the effect of compression steel in deflection computations is the same as the Unified British Code for 14 days at loading, and these are the same as ACI 318-71, except as noted above—(0.60) and (1.2) for $A_s'/A_s = 0.5$.

with a low-shrinkage concrete, it is more advantageous to position any additional restraint reinforcement in the tensile zone."

On this last point, the effect of additional tensile steel is included in the value of I_{cr}, and could be checked against the effect of additional compression steel, using Eq. (5), (8), or (9), for example.

It should be noted that the factor to be multiplied by the initial deflection in computing additional long-time deflection is a grossly simplified provision for taking into account not only the effect of compression steel, but *all* time-dependent effects for different concretes, loading conditions, environments, etc., and including the effects of downward movement of the neutral axis due to creep strain distribution, upward movement of the neutral axis due to progressive cracking under sustained loading, and any effect of repeated live load cycles in an actual structure, warping, etc. These have been discussed by ACI Committee 209[4] and Branson,[5] following previous reports by ACI Committee 435.[3]*

The simplified Code expression assumes a time-dependent factor of 2.0 for the case of no compression steel in computing additional long-time deflections that take place after attachment of nonstructural elements (where appropriate), for average conditions. The ACI Committee 209 paper[4,10] shows a variation in the creep factor alone (exclusive of shrinkage) of 0.9 to 2.9, with an average value of 1.6, for 70 percent average relative humidity, and 3 weeks average loading age. These tend to be the dominate effects on creep. Also, the effects of humidity and loading-age variations are not as marked as the statistical variation in the basic creep behavior. The writer personally believes that, after all factors are considered, a time-dependent (including shrinkage effects) parameter of 2.5 would be better than 2.0 for the Codes in attempting to minimize deflection problems.

A possible modification of the theoretical Eq. (4), similar to that by Shaikh[11] but here for the effect of steel percentage, is given by Eq. (5). Results computed by Eq. (5) are shown in Table 2 to fit all of the average-value data for beams with different steel percentages quite well:

$$k_r = 1/[1 + C(A_s'/A_s)] = 1/(1 + 50\,p') \qquad (5)$$

where $C = 50\,p$, $p = A_s/b\,d$, and $p' = A_s'/bd$.

It appears that no limits need be placed on Eq. (5). A study of the results in Table 2 indicates that the single best equation for all steel percentages seems to be Eq. (6):

$$k_r = 1/[1 + 0.70(A_s'/A_s)] \qquad (6)$$

Based on the results shown in Table 2, Eq. (4) appears to be somewhat better than Eq. (3), and Eq. (6) about the best single expression for all steel percentages. With a time-dependent factor T of 2.0 for $A_s'/A_s = 0$, the following observations can be made with respect to Eq. (3), (5), (6), (7), and Table 2:

$$T = 2.0/[1 + 0.70(A_s'/A_s)] \qquad (7)$$

1. For A_s'/A_s of 0.5; from Eq. (3), $T = 1.40$, and from Eq. (7), $T = 1.48$. For the data in Table 2, the cor-

*ACI Committee 435, Subcommittee 4, "Recommendation for ACI Building Code Provisions on Deflections," Report approved by ACI Committee 435 and submitted to ACI Committee 318, Oct. 1967.

responding average value for all beams is 1.4, for the higher p beams is 1.4, and for the lower p beams is 1.6.

2. For A_s'/A_s of 1.0; from Eq. (3), $T = 0.80$, and from Eq. (7), $T = 1.18$. For the data in Table 2, the corresponding average value for all beams is 1.3, for the higher p beams is 1.1, and for the lower p beams is 1.6. This latter case when p is low and A_s'/A_s is high (discrepancy between computed $T = 0.8$ or 1.2, and measured value of 1.6) would seem to justify the use of Eq. (5) rather than Eq. (3), (6), or (7).

3. No limits appear to be needed for Eq. (5), (6), or (7).

In summary, Eq. (8), or preferably Eq. (9) [these from Eq. (5)], as a function of p', is recommended by the author as an appropriate Code provision:

$$T = 2.0/[1 + C(A_s'/A_s)] = 2.0/(1 + 50\,p') \qquad (8)$$

$$T = 2.5/[1 + C(A_s'/A_s)] = 2.5/(1 + 50\,p') \qquad (9)$$

where $C = 50\,p$, $p = A_s/bd$, $p' = A_s'/bd$.

Also, Eq. (7), or preferably Eq. (10), would perhaps be satisfactory as an expression for all steel percentages. Again, it appears that no limits need be placed on the parameters p, p', and A_s'/A_s in these equations. This can be seen in Table 2.

$$T = 2.5/[1 + 0.70(A_s'/A_s)] \qquad (10)$$

Eq. (5), (8), or (9) are seen to be satisfactory for all steel percentages, although the differences between results by these equations and Eq. (3), (4), (6), (7), and (10) are not great in most cases. The principal exception to this [especially for Eq. (3)] is found in the case of beams with low steel percentages (approximately 1 percent and less) when A_s'/A_s is high (as about 1.0). Also, Eq. (5), (8), and (9) should be preferable to Eq. (3), (4), (6), (7), and (10) for typical T-beams and hollow-box beams, which usually have smaller steel percentages.

Additionally, perhaps the effects of age at loading (which is included in the Unified Code) and average relative humidity are two dominate effects[4,10] that should be included, or at least mentioned, in any code provision on long-time deflections, along with the basic statistical variation in the concrete behavior itself. In large structures, the effect of member size can also be a significant factor.[4,10,12]

If the larger time-dependent factor of 2.5 [as in Eq. (9) and (10)] is to be used, it would also be appropriate to use a modulus of rupture of about $7.8\sqrt{f_c'}$ to $8.0\sqrt{f_c'}$, rather than $7.5\sqrt{f_c'}$ as in ACI 318-71 for normal weight concrete deflections. The modulus of rupture ranges from $7.5\sqrt{f_c'}$ to $12\sqrt{f_c'}$, for normal weight concrete, in Reference 3; and average values of $7.8\sqrt{f_c'}$ to $8.4\sqrt{f_c'}$ are given in Reference 4, for normal weight concrete. Using the constant of 7.8, the corresponding expression[4] for concretes of different weight is given by Eq. (11).

$$f_r = 0.65\sqrt{w\,f_c'} \qquad (11)$$

where f_r is in psi, w in 16 per cu ft, and f_c' in psi.

A consistent refinement that may be appropriate with these suggested changes is the use of I_o, rather than I_g, in Eq. (1). These changes would all tend to follow the philosophy of "zero safety factor" in predicting serviceability conditions, as advocated by ACI Committee 435, Subcommittee 1.[13] The variability of the problem can then be taken into account in the limitations placed on the predicted results.

Shrinkage Warping

A principal part of the difficulty of any simplified code approach for lumping creep and shrinkage effects together in estimating additional long-time deflections is the effect of shrinkage warping. Hollington[1] has discussed this in relation to his tests and Miller's work.[14] The author also believes Miller's concept is helpful in studying the subject of shrinkage warping, since it avoids the use of any quasi-elastic procedure, such as the equivalent tensile force method. The same advantage is found in the I_{eff}-k_r-C_t method[4] (similar to ACI 318-71), as compared to the sustained modulus method, for computing creep deflections, for example.

Miller's method has been modified[2-4] to yield an expression as a function of steel percentage, and extended to the case of doubly reinforced beams as well. This procedure relates directly to the discussion herein. These three methods for computing warping are compared with experimental data in References 2 and 3.

HYPERBOLIC FORM OF CREEP AND SHRINKAGE EQUATIONS

It has recently been shown in an ACI Committee 209 paper,[4] following work in References 10, 15, 16, and 17, that the hyperbolic form of a time equation predicts shrinkage for both early and long-time periods, but not creep very well (see comments below on an extrapolation procedure from short-term tests). Primarily because the increase in creep after say 100 to 200 days is normally more pronounced than shrinkage, or in percent of ultimate value, shrinkage normally increases more rapidly than creep during the first few months, appropriate powers of t [see Eq. (12) to (14) below] were found[10,15] to be unity for shrinkage (flatter hyperbolic form) and 0.60 for creep (steeper curve for larger values of t).

It is also noted in connection with Fig. 4, 6, 7 of Reference 4 and Eq. (12) to (14) below, that these equations consist of a "time ratio" term which modifies an "ultimate" (in time) value for creep and shrinkage. The appropriate level of curve for a given case can thus be conveniently defined by the ultimate value, with the same time-ratio term used in general. The word "ultimate" is, of course, used here in a practical sense and not a philosophical sense.

This procedure has been extended somewhat for different weight concretes in Reference 18. Eq. (12) to (14) have also been shown[17] to extrapolate 28-day creep and shrinkage data to complete time curves quite well for creep, and reasonably well for shrinkage, for a wide variety of data.

Creep coefficient for moist and steam cured concrete

$$C_t = \frac{t^{0.60}}{10 + t^{0.60}} C_u \qquad (12)$$

Shrinkage for moist cured concrete

$$(\varepsilon_{sh})_t = \frac{t}{35 + t}(\varepsilon_{sh})_u \qquad (13)$$

Shrinkage for steam cured concrete

$$(\varepsilon_{sh})_t = \frac{t}{55 + t}(\varepsilon_{sh})_u \qquad (14)$$

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APPENDIX—NOTATION

A_s = area of tension steel in reinforced members and area of prestressed steel in prestressed members

A_s' = area of compression steel in reinforced members and area of nontensioned steel in prestressed members

b = width of compression face

C = coefficient defined by $C = 50p$, $p = A_s/b\,d$

C_t = creep coefficient defined as ratio of creep strain to initial strain at any time

C_u = ultimate (in time) creep coefficient

d = distance from extreme compression fiber to centroid of tension steel

E_c = modulus of elasticity of concrete

f_r = modulus of rupture of concrete

I = second moment of the area (moment of inertia)

I_{cr} = moment of inertia of cracked transformed section

I_{eff} = effective moment of inertia

I_g = moment of inertia of gross section, neglecting all steel

I_o = moment of inertia of uncracked transformed section

K = deflection coefficient

k_r = reduction factor to take into account effect of compression steel, movement of neutral axis, progressive cracking, etc., in reinforced beams; and effect of nontensioned steel in prestressed beams

L = span

M = total moment at midspan in Eq. (2), as defined in Reference 1

M_{cr} = cracking moment

M_{max} = maximum moment under service loads at stage for which deflection is computed

p = steel percentage defined as $p = A_s/bd$. Also $p' = A_s'/b\,d$

T = multiplier for additional long-time deflections due to creep and shrinkage

t = time in days in Eq. (12) to (14); also subscript denoting time-dependent, or at any time t

w = unit weight of concrete, lb per cu ft

Δ_i = initial deflection or short-time deflection

Δ_t = time-dependent deflection or total deflection minus initial deflection; also referred to as additional long-time deflection

$(\varepsilon_{sh})_t$ = shrinkage strain, in. per in. or cm/cm, etc., at any time

$(\varepsilon_{sh})_u$ = ultimate (in time) shrinkage strain, in. per in. or cm/cm, etc.