A Procedure to Evaluate the Potential for Drying Shrinkage Cracking of Concrete under Restraint

A basis for selecting durable concrete mixtures

by Emmanuel K. Attiogbe

Drying shrinkage cracking of concrete under restraint is a major cause of deterioration and reduction in service life of concrete elements in applications such as floors, walls, pavements, and bridge decks. Such shrinkage cracking typically occurs early in the life of the concrete structure.[1] Therefore, as part of the process for selecting the concrete mixture to use in a project, it would be beneficial to evaluate candidate concrete mixtures for their potential to resist early-age restrained shrinkage cracking. This article provides a procedure that involves simple calculations, using concrete properties obtained from standard tests, to evaluate the shrinkage cracking potential of concrete under restraint. The procedure offers engineers and concrete producers a simple tool for evaluating concrete mixtures during prequalification testing to select mixtures that would enhance the long-term durability of concrete structures.

Background

The analysis to evaluate the potential of concrete mixtures for restrained shrinkage cracking as presented in this article is based on studies that have shown that as concrete dries under restraint, tensile stresses build up over time.[2-6] The rate of stress development at the time of cracking is inversely related to the age at which cracking occurs, as shown in Fig. 1.[4,6] The figure reflects the effects of direct tensile stresses that are assumed to be uniformly distributed over the cross section, as the ring test setups used do not allow for vertical deformation or curling of the concrete elements. The rate of stress development is directly proportional to the rate of drying or rate of shrinkage.[6] Therefore, Fig. 1 indicates that, regardless of the magnitude of drying shrinkage, a slower rate of drying would enable tensile stresses to build up slowly and thereby prolong the time-to-cracking. A slower rate of drying would also permit the concrete a longer period to relax stresses (that is, lower effective elastic modulus) and thereby reduce its susceptibility to early-age cracking. Figure 1 has been found to be due to moisture gradient effects caused by differential drying[6] and forms the basis for classifying the potential for restrained shrinkage cracking of concrete per ASTM C1581/C1581M, "Standard Test Method for Determining Age at Cracking and Induced Tensile Stress Characteristics of Mortar and Concrete under Restrained Shrinkage."

In quantifying the moisture gradient effects in concrete under restraint,[6] as per the equations given in a subsequent section of this article, the ratio of the calculated tensile stress at cracking (or residual tensile stress) to the splitting tensile

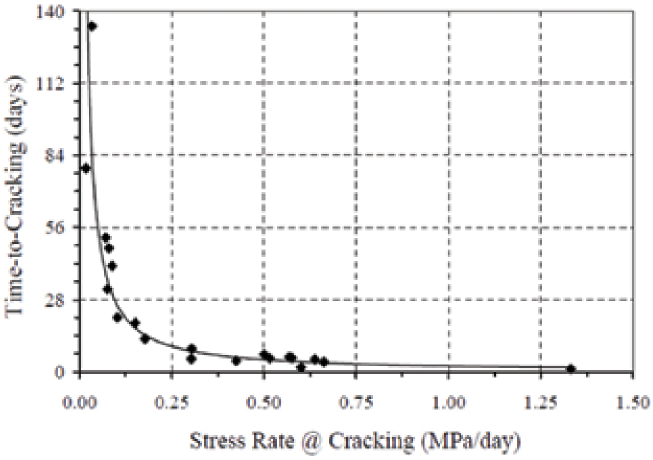

Fig. 1: Relationship between time-to-cracking and stress rate at cracking (from References 4 and 6). Note: 1 MPa = 145 psi

strength is found to decrease with longer drying times. The reduction in strength at which cracking occurs is attributed to accumulation of internal damage as it relates to tensile creep strain, and consequently stress relaxation, under the gradually increasing restrained stresses.[5,6] Figure 2 was obtained from test data for ring specimens (with a degree of restraint, *R*, of about 0.7) that were dried in the laboratory at a temperature of about 22°C (72°F) and a relative humidity (RH) of about 50%.[2-4] The figure indicates that below a residual tensile stress-to-splitting tensile strength ratio (σ_r/σ_{sp}) of about 0.50, restrained shrinkage cracking is unlikely. That is, when $\sigma_r/\sigma_{sp} \geq 0.50$, the potential for early-age restrained shrinkage cracking is high, and the potential is low when $\sigma_r/\sigma_{sp} < 0.50$. Given that the direct tensile strength of concrete is about 80% of the splitting tensile strength,[5] the limiting value of the residual tensile stress to direct tensile strength ratio below which the potential for cracking is low would be about 0.60.

Figure 3 is a log-log plot of the data in Fig. 2 and shows a strong linear relationship between σ_r/σ_{sp} and the time-to-cracking, which supports using σ_r/σ_{sp} as a cracking index for concrete under restrained shrinkage. The figure indicates that very low values of σ_r/σ_{sp} (such as $\sigma_r/\sigma_{sp} = 0.25$) would imply cases where early-age restrained shrinkage cracking is highly unlikely. It should be noted, however, that as Fig. 1 indicates, whether the concrete cracks or not, and when it cracks under field conditions would depend on environmental factors such as ambient RH, which would control the rate of drying, and hence control the rate at which stress builds up in the concrete. The rate of stress build-up, hence the time-to-cracking, would also depend on the degree to which the concrete element is restrained. Figure 3 is valuable because it indicates that when comparing the potential performance of two concrete mixtures under specific conditions of 50% ambient RH and $R = 0.7$, the concrete with the lower value of σ_r/σ_{sp} would be more resistant to restrained shrinkage cracking as it will take a longer time to crack, if cracking were to occur in both concretes. It is reasonable to expect that this relative performance of the concrete mixtures would be the same under any other specific conditions of ambient RH and restraint. Therefore, Fig. 3 provides the basis to rank candidate concrete mixtures with respect to their potential to resist restrained shrinkage cracking.

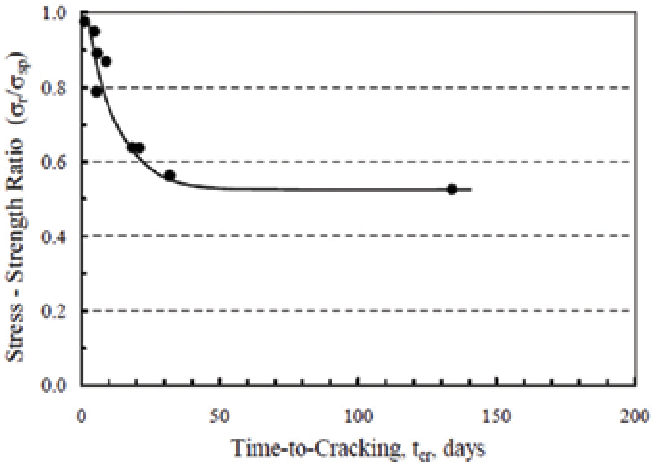

Fig. 2: Residual stress-splitting tensile strength ratio versus time-to-cracking (from Reference 6)

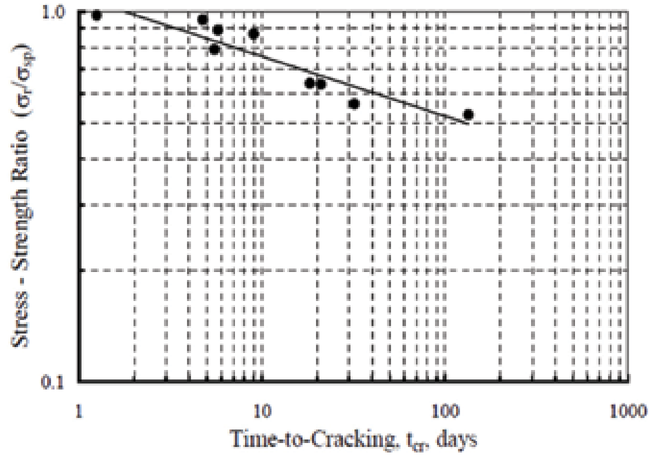

Fig. 3: Log-log plot of residual stress-strength ratio versus time-to-cracking (from Reference 6)

The long-term durability of concrete is enhanced through appropriate concrete mixture design, proportioning, and placement,[7] which may include using shrinkage-reducing admixtures (SRAs) or internal curing to reduce both the magnitude and rate of drying shrinkage,[3,8,9] and using macrofibers to keep crack widths small.[9-11] In addition, reducing the rate of drying by protecting the concrete from windy and low RH conditions during early ages—with measures such as use of curing blankets, plastic sheeting, curing compounds, or periodic wetting—would minimize the potential for restrained shrinkage cracking.

Concrete Properties and Analysis Parameters

This analysis for restrained drying shrinkage cracking is considered approximate because values of the tensile creep coefficient on which the analysis is based are generalized from previous studies and applied to three classes of concrete strength. In addition, the residual tensile stress calculations are based on a single value of the degree of restraint, and any potential curling stresses caused by the moisture gradient are not accounted for. Stresses induced by direct tension only, the effects of which yield the results in Fig. 1 to 3, are considered a sufficient basis for an analysis to determine the relative potential performance of different concrete mixtures.

The concrete properties required for the analysis are drying shrinkage strain (ε_{sh}), compressive strength (f_c'), from which modulus of elasticity (E_c) can be estimated, splitting tensile strength (σ_{sp}), and tensile creep coefficient (C_r) under restrained shrinkage. The 28-day values of f_c', E_c, and σ_{sp} are taken as the characteristic values for the concrete. The residual tensile stress (σ_r) depends on the degree to which the concrete element is restrained and on the ultimate shrinkage strain of the concrete (ε_{shu}).[6] The ε_{shu} is estimated using Eq. (2-9) in ACI 209R-92[12] based on drying shrinkage strain at 28 days after the start of drying, which is obtained per ASTM C157/C157M, "Standard Test Method for Length Change of Hardened Hydraulic-Cement Mortar and Concrete." The splitting tensile strength at 28 days is determined per ASTM C496/C496M, "Standard Test Method for Splitting Tensile Strength of Cylindrical Concrete Specimens." If flexural strength testing is performed per ASTM C78/C78M, "Standard Test Method for Flexural Strength of Concrete (Using Simple Beam with Third-Point Loading)," instead of splitting tensile strength testing, the splitting tensile strength is taken to be in the range of 60 to 70% of the flexural strength depending on the strength of the concrete.[13] The ACI 318-14 equation (Eq. (19.2.2.1.b)[14]) is used to calculate E_c from 28-day compressive strength data obtained per ASTM C39/C39M, "Standard Test Method for Compressive Strength of Cylindrical Concrete Specimens." Alternatively, E_c at the specimen age of 28 days can be determined per ASTM C469/C469M, "Standard Test Method for Static Modulus of Elasticity and Poisson's Ratio of Concrete in Compression." The test data of concrete properties used to illustrate the analysis procedure are presented in Table 1 for four concrete mixtures (C1, C2, C3, and C4). All the concrete mixtures contained an SRA except C4.

Values of C_r under restrained shrinkage have been estimated in studies involving concrete mixtures with and without SRAs.[2-5,8,15] It was found that tensile creep strains are reduced when SRA is used. The level of R in most of these studies (0 being free shrinkage and 1 being complete restraint) was about 0.7 or higher. The C_r values reflect the likelihood that microcracking caused by restraint significantly contributes to the relaxation effect.[5,16] This means that the magnitude of R would influence the magnitude of C_r.[16] Values of C_r at cracking from the studies are generalized and summarized in Table 2 as a function of concrete compressive strength, with $R = 0.7$. When the concrete is reinforced with macrofibers, the values of C_r shown in Table 2 are applicable because macrofibers do not seem to have significant influence on the magnitude of creep at the time of visible cracking under restrained shrinkage.[5] The importance of macrofibers is to delay the development of visible cracks by keeping crack widths small.

While the value of R used in this analysis for the relative comparison of the potential performance of different concrete mixtures is 0.7, in general, R can be calculated for a specific case using the restraint factor equation (Eq. (5-1)) in ACI 207.2R-07.[17] For a case where the relative section stiffness of the restrained and the restraining concrete elements is 1, R is 0.5. If the concrete elements are reinforced, R would be greater than 0.5 and increase with the reinforcement ratio. Detailed analyses have been performed to show that R can range in value from about 0.5 to about 0.8 for slabs, roofs, and walls.[18,19] Therefore, the R value of 0.7 used in this analysis represents a reasonable level of restraint and is deemed to be appropriate for a relative comparison of the potential performance of different concrete mixtures. The information

Table 1:
Test data of concrete properties to illustrate the analysis procedure

Concrete properties		Concrete mixtures			
		C1	C2	C3	C4
28-day compressive strength, f_c', MPa		39.5	40.7	60.6	31.5
28-day splitting tensile strength, σ_{sp}, MPa		3.97	3.52	4.28	3.15
28-day drying shrinkage strain, ε_{sh}, %	with SRA*	0.013	0.019	0.020	—
	no SRA	—	—	—	0.048

*Typical commercially available SRA at about 0.75 to 1.00% by weight of cementitious content of the concrete

Note: 1 MPa = 145 psi

Table 2:
Values of tensile creep coefficient, C_r, and degree of restraint, R, for use in the analysis procedure

Concrete compressive strength level at 28 days	R = 0.7	
	SRA*	C_r at cracking
$f_c' \leq 42$ MPa	No	1.50
	Yes	1.25
$42 < f_c' < 50$ MPa	No	1.05
	Yes	0.85
$f_c' \geq 50$ MPa	No	0.60
	Yes	0.45

*Typical commercially available SRA at about 0.75 to 1.00% by weight of cementitious content of the concrete

Note: 1 MPa = 145 psi

in Table 2 may be used to evaluate any given set of concrete mixtures.

Residual Tensile Stress and Cracking Potential Analysis

In applications such as concrete floors, walls, pavements, bridge decks, and water-retaining structures, differential drying is typical, in which drying occurs from the exposed surface toward the interior of the concrete element. Considering the moisture gradient that develops and controls cracking of the concrete, as previously explained with Fig. 1 to 3, the average σ_r across the section of the concrete element is approximated by[6]

$$\sigma_r = \frac{1}{4} R E_{ef} \varepsilon_{shu} \tag{1}$$

where E_{ef} is the effective or creep-adjusted modulus of elasticity of the concrete, and ε_{shu} is the ultimate shrinkage strain of the concrete. With E_c determined either from compressive strength or from testing, and values of C_r given in Table 2, E_{ef} is determined from the following equation[20]

$$E_{ef} = \frac{E_c}{1 + C_r} \tag{2}$$

The value of ε_{shu} is estimated from the measured 28-day shrinkage strain, ε_t, obtained per ASTM C157/C157M. For testing at an RH of 50% using 75 mm (3 in.) thick specimens that are moist cured for 7 days, ε_{shu} is estimated as[12,21]

$$\varepsilon_{shu} = \varepsilon_t \frac{35 + t}{t} \tag{3}$$

where $t = 28$ days. As indicated earlier, Eq. (1) to (3) were used with test data to obtain the relationships in Fig. 2 and 3.

Equations (1) to (3) are applied using the data in Tables 1 and 2 to obtain the results tabulated in Table 3. As previously noted, the E_c values in Table 3 were calculated per the equation in ACI 318-14 using the compressive strength values. Table 3 shows that the calculated values of σ_r/σ_{sp} are less than 0.50 for concrete mixtures C1, C2, and C3, and greater than 0.50 for concrete C4. This means that under conditions of 50% RH and $R = 0.7$, C1, C2, and C3 have a low potential for restrained shrinkage cracking, whereas C4 has a high potential per the previous discussions with respect to Fig. 2. Concrete C4 has been used in a concrete structure and has experienced early-age restrained shrinkage cracking. The values of σ_r/σ_{sp} in Table 3 increase from 0.17 for concrete C1 to 0.63 for concrete C4, implying that C1 would be the most resistant to restrained shrinkage cracking and C4 would be the least resistant under any specific conditions of ambient RH and restraint, as indicated by Fig. 3. It should be noted that SRA was used to achieve the lower shrinkage, and hence lower cracking potential, of C1, C2, and C3 compared to C4. As restrained shrinkage cracking is a major factor in determining the long-term durability of concrete, the use of SRAs to achieve both a low magnitude and a low rate of drying shrinkage is an important measure to mitigate such cracking.

Table 3:
Restrained shrinkage stress analysis results

Concrete mixtures	f_c', MPa	E_c, GPa	E_{ef}, GPa	ε_{shu}, %	σ_r, MPa	σ_r/σ_{sp}	Cracking resistance ranking
C1	39.5	29.5	13.1	0.029	0.66	0.17	1 (most resistant)
C2	40.7	30.0	13.3	0.043	1.00	0.28	2
C3	60.6	36.6	25.2	0.045	1.98	0.46	3
C4	31.5	26.4	10.6	0.108	2.00	0.63	4 (least resistant)

Note: 1 MPa = 145 psi; 1 GPa = 145 ksi

Concluding Remarks

Restrained shrinkage cracking caused by differential drying is a major factor in determining the long-term durability of concrete. An analysis procedure that involves simple calculations, using concrete properties obtained from standard tests, is presented to evaluate the potential for restrained shrinkage cracking as a basis for prequalification of concrete mixtures. Candidate concrete mixtures can be ranked with respect to their potential to resist restrained shrinkage cracking. The value of σ_r/σ_{sp} much less than 0.50 (such as 0.25 or lower), which indicates a very low potential for restrained shrinkage cracking, may be achieved with the use of SRAs or internal curing to reduce both the magnitude and rate of drying shrinkage. In addition, reducing the rate of drying by protecting the concrete from windy and low RH conditions during early ages would minimize the potential for restrained shrinkage cracking.

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Note: Additional information on the ASTM standards discussed in this article can be found at **www.astm.org**.

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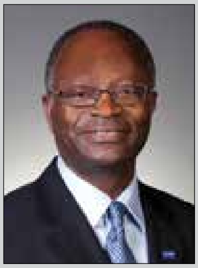

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