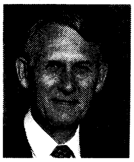
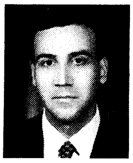
ACI STRUCTURAL JOURNAL TECHNICAL PAPER

Title no. 92-S20

## Code Background Paper:*

# Design for Torsion

by J. G. MacGregor and M. G. Ghoneim

*In the 1995 ACI Building Code, the section on design for torsion will be completely revised based on a thin-walled tube, space-truss analogy. This paper explains the origin of the new procedure, derives the design equations, and compares the method to test results.*

**Keywords:** beams (supports); prestressed concrete; reinforced concrete; shear; **structural design; torsion.**

The torsion provisions in the ACI Building Code (318-89) were proposed in a series of papers by ACI Committee 438, Torsion, in 1968 and 1969[1,2] and adopted in the 1971 ACI Building Code. Shortly thereafter, a radically different design procedure based on a thin-walled tube, space-truss analogy was proposed in Switzerland.[3,4] This paper gives the background for revisions recently adopted by ACI Committee 318 to replace the current ACI design method for torsion with one based on the thin-walled tube, space-truss analogy. This design method is currently included in the CEB Model Code[5] and the Canadian code,[6] among others.

The change is proposed primarily because the new method is considerably simpler to understand and apply, and is equally accurate. The new method can also be used for prestressed concrete loaded in torsion, a case not covered by ACI 318-89. There are a number of more "accurate," but more complex, design procedures in the literature.[7,8] The method presented here has intentionally been kept as simple as possible while maintaining ratios of measured to computed strengths for test specimens that are equivalent to or better than those for ACI 318-89.

## RESEARCH SIGNIFICANCE

This paper reviews the derivation of a design procedure for reinforced and prestressed concrete beams loaded in torsion, which will be included in the 1995 ACI Building Code. The method is compared to test data from the literature.

## BASIC THEORY

The test data[1] for solid and hollow beams plotted in Fig. 1 suggest that, once cracking has occurred, the concrete in the center of the member has little effect on the torsional strength of the cross section and can be ignored. The beams can, in effect, be considered to be equivalent tubular members. This observation is the basis of the design procedures for torsion presented here.

The new torsion design provisions are based on a thin-walled tube, space-truss analogy in which the beam cross section is idealized as a tube, as shown in Fig. 2(a). After cracking, the tube is idealized as a space truss consisting of closed stirrups, longitudinal bars in the corners, and concrete compression diagonals approximately centered on the stirrups. The diagonals are at an angle $\theta$, generally taken as 45 deg for reinforced concrete beams, as shown in Fig. 2(b). Although the tube analogy is less obvious before cracking, it will be used to predict cracking to maintain the consistency of the model.

### Shear stresses in thin-walled hollow tube

Textbooks on mechanics of materials explain that, for any shape of thin-walled hollow member with continuous walls, the shearing stress $\tau$ due to torsion can be calculated from

$$\tau = \frac{T}{2A_o t} \tag{1}$$

where $T$ is the torque, $A_o$ is the area enclosed by a line around the tube at the midthickness of the wall, and $t$ is the thickness of the wall of the tube.

For solid cross sections, it is necessary to define the wall thickness of the equivalent thin-walled tube. Prior to cracking, the wall thickness is defined as a function of the area and perimeter of the uncracked concrete section. Expressions will be given later. After cracking, the resistance to torsion comes from the stirrups, longitudinal bars, and outer con-

*ACI Committee 318 has endorsed early publication of this paper which serves as background to upcoming code revisions.

*ACI Structural Journal*, V. 92, No. 2, March-April 1995.
Received Nov. 1, 1994, and reviewed under Institute publication policies.  Pertinent discussion will be published in the January-February 1996 *ACI Structural Journal* if received by Sept. 1, 1995.

*James G. MacGregor, FACI, is University Professor Emeritus of Civil Engineering, University of Alberta, Edmonton, Canada. He is a Past President of ACI. Dr. MacGregor is a member of ACI Committees 318, Standard Building Code; and joint ACI-ASCE Committees 441, Reinforced Concrete Columns; and 445, Shear and Torsion. He is also Chairman of the Standing Committee on Structural Design of the National Building Code of Canada.*

*Mashour G. Ghoneim is an assistant professor of structural engineering, Cairo University, Egypt. He received his BSc and MSc degrees from Cairo University and his PhD from the University of Alberta. Prior to joining Cairo University, he worked as a research engineer at the University of Alberta. His research interests relate to stability and behavior of reinforced and prestressed concrete structures.*

crete skin, $A_o$ is empirically taken as 0.85 times the area $A_{oh}$, enclosed by the outermost closed stirrups in the section, and $t$ is taken as $A_{oh}/p_h$ where $p_h$ is the perimeter of the centerline of the closed stirrups.

In this paper, the design method is referred to as the thin-walled tube analogy because this is the terminology used in mechanics of materials textbooks. The thickness of the walls of the tube representing a solid member is actually on the order of one-sixth to one-quarter of the minimum width of a rectangular member.

## DERIVATION OF TORSION DESIGN EQUATIONS

The proportioning equations presented in this section of the paper were derived about 20 years ago, and the reader should refer to the earlier publications for more discussion.[3,4]

Torsion causes inclined cracks, which tend to extend around the member in a spiral fashion. After cracking, a rectangular beam subjected to pure torsion can be idealized, as shown in Fig. 2(b). The beam is modeled by a space truss consisting of longitudinal bars in the corners, closed stirrups, and concrete compression diagonals, which spiral around the member between the torsional cracks. The width and height of the truss are $x_o$ and $y_o$, respectively, measured center-to-center of the sides of the closed stirrups. The angle of the cracks is $\theta$, which initially is close to 45 deg for nonprestressed beams, but may deviate from 45 deg at high torques.

### Area of stirrups

The shear force per unit length of perimeter, at any point on the perimeter of the tube, is referred to as shear flow $q = \tau t$, where $\tau$ is the shear stress due to torsion, and $t$ is the thickness of the wall of the tube at the point under consideration. From Eq. (1)

$$q = \tau t = \frac{T}{2A_o} \tag{2}$$

The total shear force due to torsion in a given side of the tube is $q$ times the length of the side. Thus, the shear force in one vertical side is

$$V_2 = \frac{T}{2A_o} y_o \tag{3}$$

Similar forces act on all four sides, as shown in Fig. 2(b). These forces are oriented to cause a torque about the axis of the member to resist the applied torque.

A portion of one of the vertical sides is shown in Fig. 3. The inclined crack cuts $n = y_\theta \cot\theta / s$ stirrups, where $s$ is the stirrup spacing. The force in the stirrups crossing the crack must equilibrate $V_2$. Assuming all the stirrups yield at ultimate

$$V_2 = \frac{A_t f_{yv} y_o}{s} \cot\theta \tag{4}$$

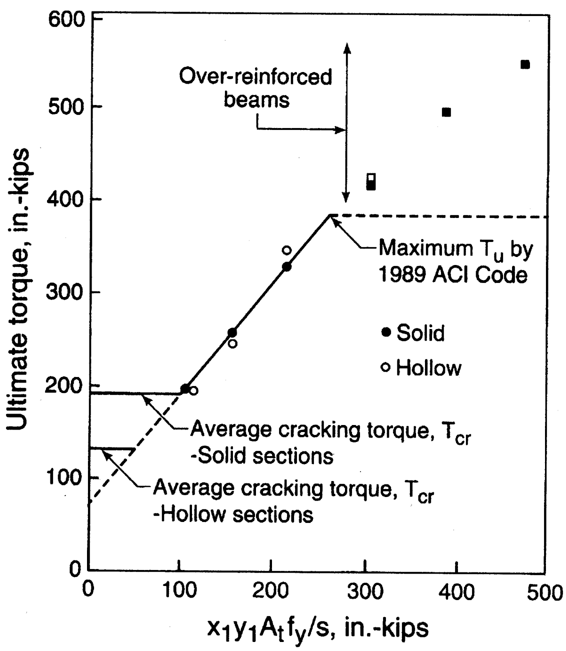



*Fig. 1—Comparison of torsional strengths of solid and hollow beams*

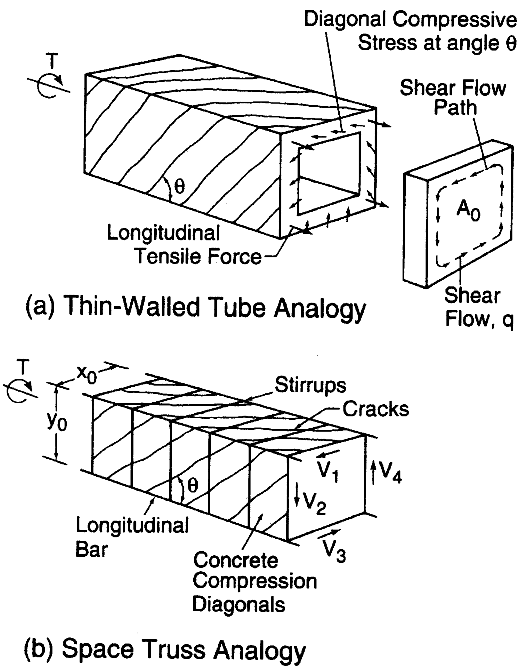



*Fig. 2—Thin-walled tube and space-truss analogies*

where *fyv* is the yield strength of the stirrups. Substituting the value of $V_2$ from Eq. (3) and taking $T$ equal to $T_n$, the nominal torsion capacity, the design equation for computing the required area of closed stirrups, is

$$T_n = \frac{2A_o A_t f_{yv}}{s} \cot\theta \quad (5)$$

where $\theta$ may be taken as any angle between 30 and 60 deg. For nonprestressed concrete, the new code statement suggests that $\theta$ be taken as 45 deg. This corresponds to the value assumed in the equations for the design of stirrups for shear in the ACI Building Code.

For prestressed concrete, $\theta$ is considerably less than 45 deg, due to the effect of the axial compression stress $f_{pc}$ on the inclination of the principal tensile stresses, and also because prestressed members frequently have more longitudinal steel than required for torsion. The new code statement suggests $\theta = 37.5$ deg in this case, arbitrarily taken halfway between the lower limit on $\theta$, 30 deg, and the value used to design stirrups for shear, 45 deg.

## Longitudinal reinforcement

As shown in Fig. 4, the shear force $V_2$ in Side 2 can be resolved into a diagonal compressive force $D_2$, parallel to the inclined compression struts, and an axial tensile force $N_2$, where $D_2$ and $N_2$ are given by

$$D_2 = V_2/\sin\theta \quad (6)$$

$$N_2 = V_2 \cot\theta \quad (7)$$

Since the shear flow $q$ is constant along the side of the member in a thin-walled tube, $D_2$ and $N_2$ act at midheight of the side. For a beam with longitudinal bars in the top and bottom corners of the side as shown, half of the tension force $N_2$ in Side 2 will be resisted by each corner bar. Similar force components $N_1$, $N_3$, and $N_4$ exist in the other three sides. For a rectangular member, as shown in Fig. 2(b), the total longitudinal force is

$$N = 2(N_1 + N_2) \quad (8)$$

Substituting Eq. (3) and (7) into Eq. (8) and taking $T = T_n$ gives

$$N = \frac{T_n}{2A_o} 2(x_o + y_o) \cot\theta \quad (9)$$

where $2(x_o + y_o)$ is the perimeter of the closed stirrup $p_h$. Longitudinal reinforcement must be provided for the entire axial force $N$. Assuming the longitudinal steel yields at failure, the required area is given by setting $A_\ell f_{y\ell} = N$ which gives

$$A_\ell = \frac{T_n p_h}{2A_o f_{y\ell}} \cot\theta \quad (10)$$

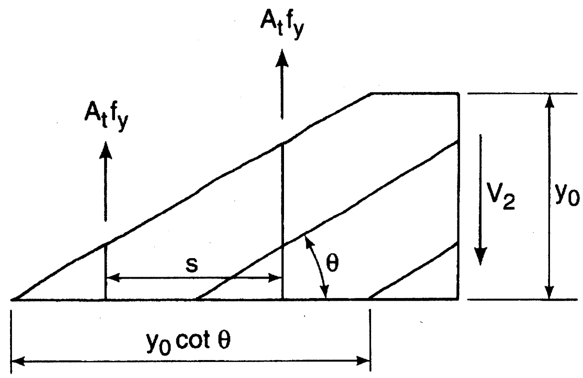



*Fig. 3—Portion of vertical side of space truss*

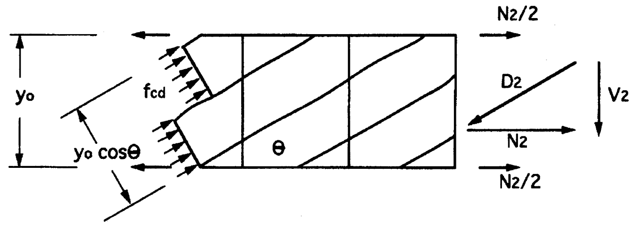



*Fig. 4—Resolution of shear force in Side 2 of space truss*

For convenience in design, $A_\ell$ can be expressed in terms of the area of the torsional stirrups. Substituting Eq. (5) into Eq. (10) gives

$$A_\ell = \left(\frac{A_t}{s}\right) p_h \left(\frac{f_{yv}}{f_{y\ell}}\right) \cot^2\theta \quad (11)$$

If a value of $\theta$ less than 45 deg is used in Eq. (5), the area of stirrups $A_t/s$, required for a given torque $T_n$ decreases. This saving is offset by an increase in the required area of longitudinal steel $A_\ell$, as shown by substituting $\theta$ less than 45 deg into Eq. (10). The revisions allow the use of $\theta$ from 30 to 60 deg to allow designers to optimize the relative amounts of stirrups and longitudinal reinforcement if they wish. However, the same value of $\theta$ must be used in Eq. (5) and (10) or (11) when designing a given member for torsion.

Because the forces $N_1$ through $N_4$ each act at the middle of one of the walls, the resultant force $N$ acts along the centroidal axis of the cross section of the space truss. The line of action of the force in the longitudinal bars $A_\ell f_{y\ell}$ should coincide with that of $N$. As a result, the longitudinal torsional steel must be distributed around the perimeter of the section, essentially as required in ACI 318-89.

## Combined shear and torsion

In ACI codes prior to 1995, a portion of both the shear and the torsion are resisted by the concrete terms $V_c$ and $T_c$. A significant part of the complexity of the previous ACI design procedure arises from the assumed circular interaction between $V_c$ and $T_c$. In the new design method, $V_c$ is assumed to be unaffected by the presence of torsion, and $T_c$ is always taken equal to zero. This greatly simplifies the calculations. Design comparisons prepared by the subcommittee that developed these revisions showed that for combinations of low

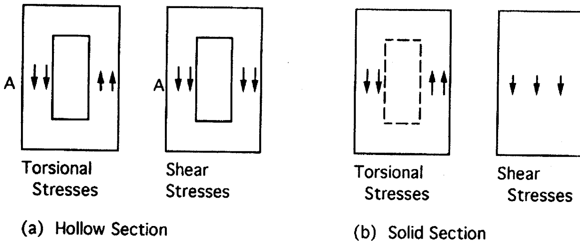


*Fig. 5—Addition of torsional and shear stresses*

$V_u$ and high $T_u$, with $v_u$ less than about $0.8(\phi 2\sqrt{f_c'})$, the new method requires more stirrups than ACI 318-89. For $v$ greater than this value, the new method requires marginally fewer stirrups than ACI 318-89.

For prestressed concrete, ACI 318-89 Eq. (11.10), (11.11), and (11.13) are used to compute $V_c$ and Eq. (11.17) is used to compute $V_s$. These have been retained because ACI Committee 318 did not want to change the shear design provisions for prestressed concrete at this time. The design of stirrups for shear is based on Eq. (11.17), which assumes that the crack angle $\theta$ is 45 deg. On the other hand, the stirrups and longitudinal steel for torsion are computed using Eq. (5) and (11), in which one is permitted to take $\theta = 37.5$ deg. Despite this inconsistency in angles, the nominal strength computed by the new method compares well with test results, as shown later in this paper.

## Combined torsion and moment

Torsion leads to an axial force $N$, shown in Fig. 4, which must be resisted by longitudinal reinforcement. If torsion occurs in a region of a reinforced concrete beam where moment also acts, the longitudinal torsion reinforcement in the flexural tension zone is added to the flexural reinforcement. In the flexural compression zone, the compression due to flexure reduces the need for longitudinal torsion reinforcement. This allows a reduction in the area of the longitudinal torsion reinforcement in the compression zone by the area of steel corresponding to the flexural compressive force $M_u/(\theta f_y 0.9d)$.

In prestressed concrete beams, the total longitudinal reinforcement, including tendons, at any section must be able to resist the factored moment at the section, plus an additional concentric longitudinal tensile force equal to $A_\ell f_{y\ell}$ from Eq. (11) for the factored torsion at that section. In the flexural compression zone, the longitudinal torsion steel may be reduced as a result of the flexural compressive force in the same manner as for reinforced concrete beams. Here, also, the longitudinal torsion reinforcement must be spaced around the perimeter of the section.

## Maximum shear and torsion

A member loaded in torsion may fail by yielding of the stirrups or the longitudinal steel, or by crushing of the concrete diagonals between the cracks. A serviceability failure may result if the inclined crack widths are excessive at service loads. The limitation on shear stresses in Section 11.5.6.8 of ACI 318-89 was originally derived to limit shear crack widths and will be used for that purpose here. The shear stress due to direct shear is $V_u/(b_w d)$. From Eq. (1) with $A_o$ after torsional cracking taken equal to $0.85A_{oh}$ and $t = A_{oh}/p_h$, the shear stress due to torsion is $T_u p_h/(1.7A_{oh}^2)$. In a box girder, these are directly additive on Side A, as shown in Fig. 5(a), and the limit is given by

$$\frac{V_u}{b_w d} + \frac{T_u p_h}{1.7A_{oh}^2} \leq \phi\,(v_c + 8\sqrt{f_c'}) \tag{12}$$

If a box girder has a wall thickness less than $A_{oh}/p_h$, the second term on the left-hand side of Eq. (12) becomes $T_u/(1.7A_{oh}t)$ or more correctly, $T_u/(2A_o t)$.

In a solid section, the shear stresses due to direct shear are assumed to be distributed uniformly across the width of the section, while the torsional shears only exist in the walls of the assumed thin-walled tube, as shown in Fig. 5(b). For this reason, the direct summation of the two terms tends to be conservative, and a root-square summation is used

$$\sqrt{\left(\frac{V_u}{b_w d}\right)^2 + \left(\frac{T_u p_h}{1.7A_{oh}^2}\right)^2} \leq \phi\,(v_c + 8\sqrt{f_c'}) \tag{13}$$

Failure can also occur due to crushing of the concrete in the web. In the next few paragraphs, it will be shown that this gives a higher limit on the stresses than Eq. (12) and (13).

The diagonal compressive force in a vertical side of the member shown in Fig. 2(b) is given by Eq. (6). This force acts on a width $y_o \cos\theta$, as shown in Fig. 4. The resulting compressive stress is

$$f_{cd} = \frac{V_2}{t y_o \cos\theta \sin\theta} \tag{14}$$

Substituting Eq. (3), again taking $A_o$ equal to $0.85\,A_{oh}$, and approximating $t$ as $A_{oh}/p_h$ gives

$$f_{cd} = \frac{T_u p_h}{1.7A_{oh}^2 \cos\theta \sin\theta} \tag{15}$$

The compressive stresses due to shear may be calculated in a similar manner as

$$f_{cd} = \frac{V_u}{b_w d \cos\theta \sin\theta} \tag{16}$$

Because the compressive stress due to shear may be assumed to be distributed uniformly across the width of the web, while that due to torsion exists only in the walls of the assumed thin-walled tube, a root-square summation is used. Thus, for a solid section

$$f_{cd} = \sqrt{\left(\frac{V_u}{b_w d \cos\theta \sin\theta}\right)^2 + \left(\frac{T_u p_h}{1.7A_{oh}^2 \cos\theta \sin\theta}\right)^2} \tag{17}$$

The value of $f_{cd}$ from Eq. (17) should not exceed the crushing strength of the cracked concrete in the web $f_{ce}$. Collins and Mitchell[7] have related $f_{ce}$ to the strains in the longitudinal and transverse reinforcement in the web of the beam. For $\theta = 45$ deg, and longitudinal and transverse strains equal to the yield strain of Grade 60 steel, $\varepsilon = 0.002$, Collins and Mitchell predict $f_{ce} = 0.549 f_c'$. Setting $f_{cd}$ in Eq. (17) equal to $0.549 f_c'$ and evaluating $\cos\theta \sin\theta$ for $\theta = 45$ deg, the upper limit on the shears and torques as determined by crushing of the concrete in the web becomes

$$\sqrt{\left(\frac{V_u}{b_w d}\right)^2 + \left(\frac{T_u p_h}{1.7 A_{oh}^2}\right)^2} = 0.275 f_c' \qquad (18)$$

The limit given in Eq. (13) has been set at $v_c + 8\sqrt{f_c'}$ to limit crack widths. For reinforced concrete with $v_c$ taken equal to $2\sqrt{f_c'}$, the shear stresses necessary to cause crushing, as given by Eq. (18), will exceed the crack width limitation for $f_c'$ greater than 1324 psi. For prestressed concrete, assuming $\theta = 37.5$ deg and $v_c = 5\sqrt{f_c'}$, the crushing limit in Eq. (18) becomes $0.20 f_c'$, which exceeds the crack width limit in Eq. (13) for $f_c'$ greater than 4225 psi. Since reinforced concrete members will always have $f_c'$ exceeding 1324 psi, and the majority of prestressed members will have $f_c'$ exceeding 4225 psi, only the crack width limit has been included.

### Lower limit on when torsion should be considered

Section 11.6.1 of ACI 318-89 allows torsion to be neglected if the torque is less than a quarter of the cracking torque in pure torsion. The same reasoning has been retained in the new method. In pure torque, the principle tensile stress $\sigma_1$ due to torsion is equal in magnitude to the shear stress due to torsion. Thus, for a thin-walled tube

$$\sigma_1 = \tau = \frac{T}{2A_o t} \qquad (19)$$

To apply this to a solid section, it is necessary to define the wall thickness of the equivalent tube prior to cracking. The Euro-International Concrete Committee (CEB)[5] approximates $t$ as $A_{cp}/p_{cp}$, where $p_{cp}$ is the perimeter of the concrete section, and $A_{cp}$ is the area enclosed by this perimeter. The Canadian Code[6] assumes that, prior to cracking, the wall thickness is $0.75\, A_{cp}/p_{cp}$ and the area enclosed by the tube centerline is $2A_{cp}/3$. Substituting the latter values into Eq. (19) gives

$$\sigma_1 = \tau = \frac{T p_{cp}}{A_{cp}^2} \qquad (20)$$

Setting $\sigma_1$ equal to the tensile strength of concrete in biaxial tension-compression, taken as $4\sqrt{f_c'}$, gives the torque at cracking

$$T_{cr} = 4\sqrt{f_c'}\left(\frac{A_{cp}^2}{p_{cp}}\right) \qquad (21)$$

If $T_u$ exceeds a quarter of $T_{cr}$, or

$$\phi\sqrt{f_c'}\left(\frac{A_{cp}^2}{p_{cp}}\right) \qquad (22)$$

torsion must be considered in design. The limit given for prestressed concrete is obtained in a similar way, allowing for the effect of the prestress on the principal tensile stress, derived using a Mohr's circle. The resulting limit is

$$\phi\sqrt{f_c'}\left(\frac{A_{cp}^2}{p_{cp}}\right)\sqrt{1 + \frac{f_{pc}}{4\sqrt{f_c'}}} \qquad (23)$$

### Minimum torsional reinforcement

The minimum closed stirrup reinforcement for shear and torsion given in the new procedure is the same as in ACI 318-89. For pure torsion, this is equivalent to $A_{t,min} = 25 b_w s / f_{yv}$.

In Hsu's tests[1] of rectangular reinforced concrete members subjected to pure torsion, two beams failed at the torsional cracking load. In these beams, the total ratio of the volume of the stirrups and longitudinal reinforcement to the volume

**Table 1—Comparison of tests to design procedure**

| Loading | No. | Test/calculated strength: ACI, Mean | Test/calculated strength: ACI, Coefficient of variation | Test/calculated strength: New method, Mean | Test/calculated strength: New method, Coefficient of variation |
|---|---|---|---|---|---|
| Reinforced concrete | | | | | |
| Pure torsion | 100 | 1.028 | 0.165 | 1.276 | 0.161 |
| Combined bending and torsion | 42 | 1.332 | 0.227 | 1.383 | 0.168 |
| Combined bending, shear, and torsion | 38 | 1.382 | 0.156 | 1.359 | 0.106 |
| Prestressed concrete* | | | | | |
| Pure torsion | 48 | — | — | 1.417 | 0.180 |
| Combined bending and torsion | 49 | — | — | 1.642 | 0.200 |
| Combined bending, shear, and torsion | 63 | — | — | 1.773 | 0.234 |

*The ACI method does not apply to prestressed concrete. Values given for the new method were evaluated using $\theta = 37.5$ deg.

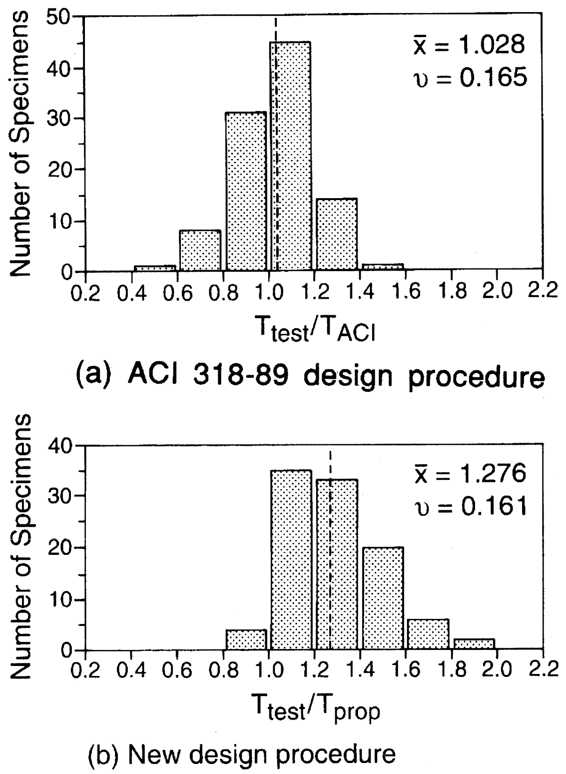



*Fig. 6—Comparison of measured and computed failure torsions for 100 reinforced concrete beams—Pure torsion*

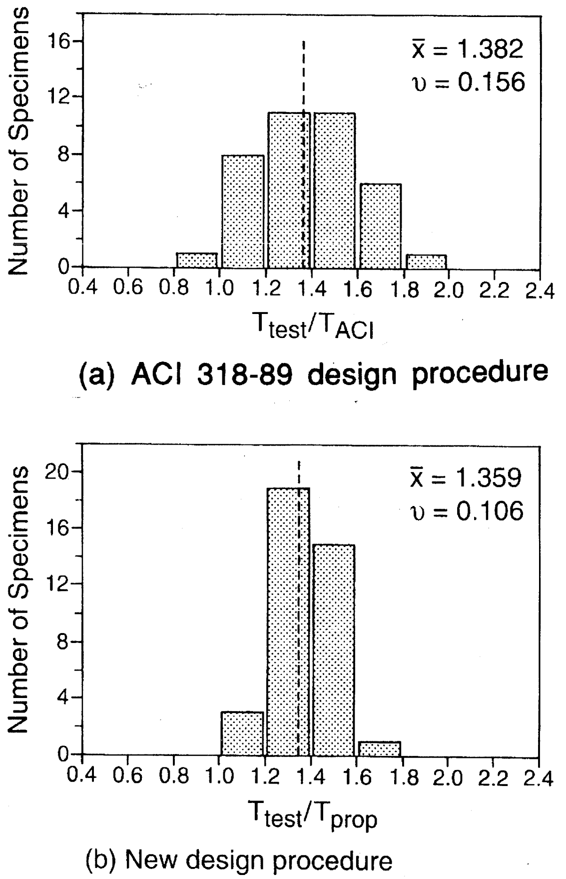



*Fig. 7—Comparison of measured and computed failure torsions for 38 reinforced concrete beams—Combined torsion, shear, and moment*

of the concrete was 0.802 and 0.88 percent, respectively. A beam with a volumetric ratio of 1.07 percent failed at 1.08 times the cracking torque. All other beams had reinforcement ratios of 1.07 percent or greater and failed at torques in excess of 1.2 times the cracking torque. This suggests that beams with similar concrete and steel strengths loaded in pure torsion should have a minimum volumetric ratio of reinforcement on the order of 0.9 to 1 percent. Thus

$$\frac{A_{\ell,min}s}{A_{cp}s} + \frac{A_t p_h}{A_{cp}s} \geq 0.01 \tag{24}$$

or

$$A_{\ell,min} = 0.01A_{cp} - \frac{A_t p_h}{s} \tag{25}$$

If the constant 0.01 is assumed to be a function of the material strengths, Eq. (25) can be rewritten as

$$A_{\ell,min} = \frac{7.5\sqrt{f_c'}}{f_{y\ell}}A_{cp} - \frac{A_t p_h}{s}\left(\frac{f_{yv}}{f_{y\ell}}\right) \tag{26}$$

In the 1971 and subsequent ACI Building Codes, a transition was provided between the amount of steel required by the equation for $A_{\ell,min}$ for pure torsion and the much smaller minimum reinforcement required for beams subjected to shear without torsion. This was accomplished by multiplying the first term on the right-hand side of Eq. (26) by $\tau/(\tau + v)$. During the committee ballot process to approve the new procedure, Professors Mattock and Hsu developed simplified expressions approximating $\tau/(\tau + v)$ as a function of $v$ only, and as a function of $\tau$ only, respectively. Hsu showed that the practical range of behavior of beams with $A_\ell$ satisfying Eq. (11) could be represented by taking $\tau/(\tau + v)$ equal to 2/3, which gave Eq. (27) for the minimum longitudinal reinforcement

$$A_\ell = \frac{5\sqrt{f_c'}A_{cp}}{f_{y\ell}} - \left(\frac{A_t}{s}\right)p_h\left(\frac{f_{yv}}{f_{y\ell}}\right) \tag{27}$$

Comparison with the results of tests of prestressed concrete beams indicates the same amount of minimum longitudinal reinforcement was needed in prestressed beams.*

## COMPARISON OF STRENGTH PREDICTIONS BY NEW DESIGN PROCEDURES TO TEST RESULTS

The strengths predicted by the new design procedure were compared to reinforced and prestressed concrete test data from the literature. Many of the test specimens in the literature had unrealistic reinforcement ratios, excessive stirrup or longitudinal reinforcement spacings, or torsional reinforcement combinations, which would not occur if design were carried out by the new design method. Criteria similar to

*Mattock, A. H., Personal communication, Jan. 1993.

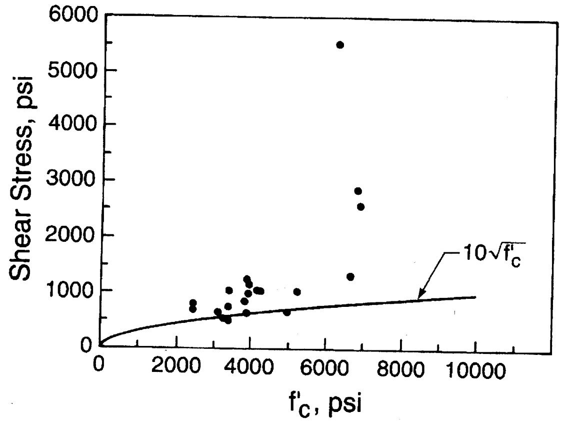



*Fig. 8—Comparison of shear stress limit in Eq. (13) to shear stresses at failure of reinforced concrete beams failing due to web crushing—Pure torsion*

those proposed by Hsu and Mo[8] have been adopted for rejecting unrealistic test specimens. Beams having stirrup spacing in excess of the code limit of $p_h/8$, having longitudinal reinforcement spacing in excess of the code limit of 12 in., or having torsional reinforcement content less than the amount required to develop the cracking moment given by the new design procedures, have been excluded.

## Reinforced concrete

The results of tests of reinforced concrete beams were compared to the failure torsions predicted by the ACI 318-89 code and the new design procedures. The results are summarized in Table 1.

Histograms of the test-to-calculated strength ratios are given in Fig. 6 for 100 beams loaded in pure torsion. The lowest strength ratio for these 100 beams was 0.453 for ACI 318-89 and 0.895 for the new design method.

The ratios of the test to predicted torsional strength of beams subjected to combined shear, bending, and torsion are shown in Fig. 7. The extreme strength ratios for the beams examined were 0.940 and 1.827 for the 1989 ACI code procedure and 1.034 and 1.698 for the new design procedure.

In Fig. 8, the limit on shear stresses given by the right-hand side of Eq. (13), with $v_c$ set equal to $2\sqrt{f_c'}$ is compared to the shear stresses at failure of 20 reinforced concrete beams that failed due to web crushing. The fact that the computed values of the shear stresses at failure for all the beams, except one, are larger than the suggested limiting value of $10\sqrt{f_c'}$ indicated that these specimens failed at a higher level of shear stress than that predicted by the criteria given in Eq. (13). Although the limit $10\sqrt{f_c'}$ is extremely conservative for some cases, it is a safe and easily applied lower bound value for reinforced concrete members subjected to pure torsion and experiencing web crushing.

Comparison of the predicted strengths with test data for reinforced concrete beams suggests that, although the new design procedure is simpler to understand and apply, it predicts test strengths at least as well as the ACI 318-89 procedure.

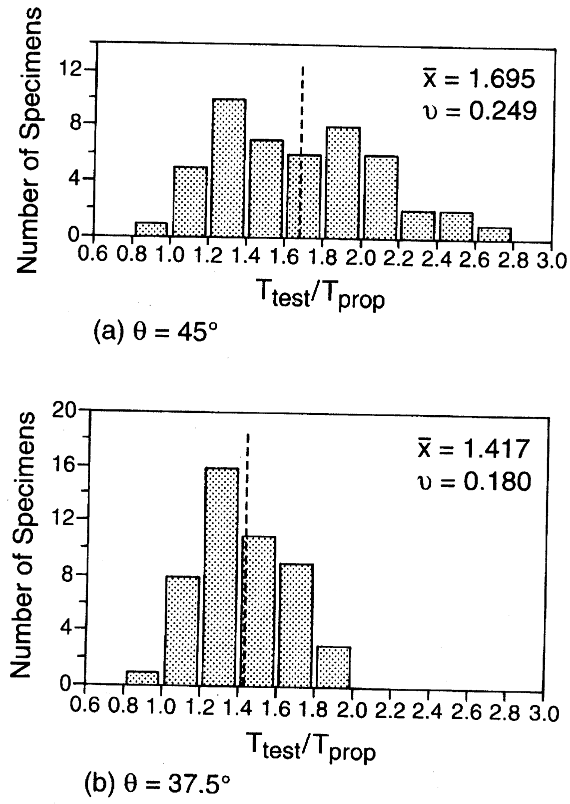



*Fig. 9—Comparison of measured and computed failure torsions for 48 prestressed concrete beams—Pure torsion*

## Prestressed concrete

The new design procedure is compared to prestressed concrete test data in Table 1. No comparisons are given for ACI 318-89 because that code did not treat prestressed concrete in torsion. In the new design procedure, the crack slope has been taken equal to 37.5 deg in calculations related to torsion.

Histograms of the test-to-calculated strength ratios are given in Fig. 9 for 48 prestressed beams loaded in pure torsion based on crack slopes of $\theta = 45$ deg and $\theta = 37.5$ deg. It can be seen that the predictions using $\theta = 37.5$ deg, although conservative, show much better agreement with the test results. This can be attributed in large part to the fact that, due to prestress, the crack angle is usually less than 45 deg. Fig. 10 shows a comparison between tests and the strengths computed using the new design procedure for $\theta = 37.5$ deg for 63 prestressed concrete beams under combined shear, bending, and torsion.

The results of 19 tests of prestressed beams in which web crushing failures occurred due to combined shear and torsion are compared to the design limits in Fig. 11. The two lines represent the shear stress limits given by the right-hand side of Eq. (13), with $v_c$ having the extreme high and low values from the ACI Building Code Section 11.4.1. Once again the web crushing criteria given from Eq. (13) are extremely conservative in some cases. However, the lack of ductility in web crushing failure makes it highly desirable to be conservative in this area.

## CONCLUSIONS

A design method for torsion in reinforced and prestressed concrete is proposed to replace Section 11.6 of the ACI

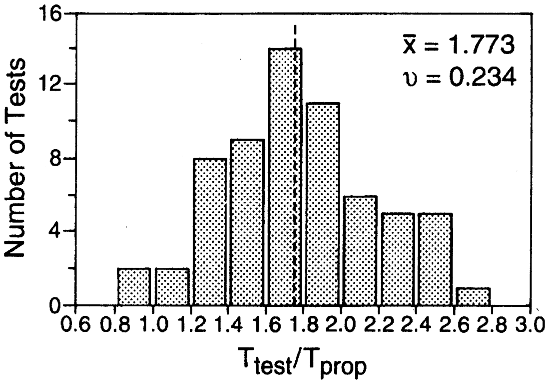



*Fig. 10—Comparison of measured and computed failure torsions for 63 prestressed concrete beams—Combined torsion, shear, and moment*

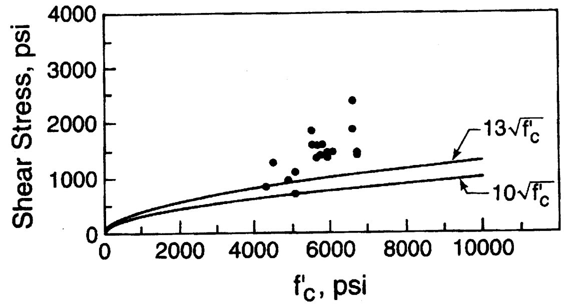



*Fig. 11—Comparison of shear stress limit in Eq. (13) to shear stresses at failure of prestressed concrete beams failing due to web crushing—Combined torsion and shear*

Building Code. The derivation of the design equations is presented. Comparison of predicted strengths with test data shows at least as good agreement as the current ACI Building Code procedure for reinforced concrete beams. Good agreement is also obtained for prestressed concrete.

## ACKNOWLEDGMENTS


The concept of the new design method for torsion was developed by others, as indicated at the beginning of the paper. Code wording compatible with the ACI Building Code was developed by the authors, based initially on the 1977 Canadian code clauses, originally drafted by M. P. Collins and D. Mitchell. ACI 318 Subcommittee E, Shear and Torsion, and ACI Committee 318 voted on the proposal over a period of 2 years. Major contributions were made during this process by A. H. Mattock and T. T. C. Hsu. The calculation of the test-to-predicted strength ratios was financed by NSERC Operating Grant A1673.


## NOTATION

| | | |
|---|---|---|
| $A_o$ | = | gross area enclosed by the centerline of the shear flow path |
| $A_{oh}$ | = | gross area bounded by centerline of outermost closed stirrups |
| $A_{cp}$ | = | gross area bounded by outer perimeter of concrete cross section |
| $A_\ell$ | = | area of longitudinal reinforcement required for torsion |
| $A_t$ | = | cross-sectional area of one leg of closed stirrup |
| $b_w$ | = | width of rectangular cross section |
| $d$ | = | distance from extreme compression fiber to centroid of longitudinal flexural tension reinforcement |
| $D_2$ | = | diagonal compressive force in Side 2 of space truss |
| $f_c'$ | = | specified compressive strength of concrete |
| $f_{cd}$ | = | inclined compressive stress acting on compressive struts between inclined cracks |
| $f_{ce}$ | = | crushing strength of cracked concrete in web of beam |
| $f_{pc}$ | = | compressive stress due to effective prestress at centroid of section |
| $f_{y\ell}$ | = | yield strength of longitudinal torsional reinforcement |
| $f_{yv}$ | = | yield strength of stirrups |
| $M_u$ | = | factored flexural moment at section being designed |
| $N$ | = | total axial tension force due to torsion |
| $N_1, N_2$ | = | axial tension forces in Sides 1 and 2 of space truss |
| $p_{cp}$ | = | outer perimeter of concrete cross section |
| $p_h$ | = | perimeter of centerline of outermost closed stirrups |
| $q$ | = | shear flow |
| $s$ | = | spacing of stirrups in direction parallel to longitudinal axis of member |
| $t$ | = | wall thickness of thin-walled tube |
| $T_{ACI}$ | = | failure torque computed by ACI 318-89 |
| $T_c$ | = | torsion carried by concrete in ACI Building Code |
| $T_{cr}$ | = | torsional cracking torque under pure torsion |
| $T_n$ | = | nominal torsional strength of member |
| $T_{prop}$ | = | failure torque computed by new procedure |
| $T_{test}$ | = | measured failure torque |
| $T_u$ | = | factored torsional moment |
| $x_o$ | = | center-to-center length of shorter side of closed rectangular stirrup |
| $\bar{x}$ | = | mean value of $T_{test}/T_{prop}$ |
| $y_o$ | = | center-to-center length of longer side of closed rectangular stirrup |
| $v_c$ | = | shear stress carried by concrete |
| $V_c$ | = | shear force carried by concrete |
| $v_u$ | = | shear stress corresponding to factored shear force |
| $V_u$ | = | factored shear force |
| $V_1, V_2$ | = | shear forces in Sides 1 and 2 of space truss due to torsion |
| $\phi$ | = | strength reduction factor |
| $\sigma_1$ | = | principal tension stress |
| $\tau$ | = | shear stress due to torsion |
| $\theta$ | = | angle of inclination of diagonal compressive stresses to longitudinal axis of member |
| $\upsilon$ | = | coefficient of variation |

## REFERENCES


1. Hsu, Thomas, T. C., "Torsion of Structural Concrete—Behavior of Reinforced Concrete Rectangular Members," *Torsion of Structural Concrete*, SP-18, American Concrete Institute, Detroit, 1968, pp. 261-306.

2. ACI Committee 438, "Tentative Recommendations for the Design of Reinforced Concrete Members to Resist Torsion," ACI JOURNAL, *Proceedings* V. 66, No. 1, Jan. 1969, pp. 1-8.

3. Lampert, P., and Thurlimann, B., "Ultimate Strength and Design of Reinforced Concrete Beams in Torsion and Bending," *Publication* No. 31-1, International Association for Bridge and Structural Engineering, Zurich, 1971.

4. Thurlimann, B., "Torsional Strength of Reinforced and Prestressed Concrete Beams—CEB Approach," *Concrete Design: U.S. and European Practices*, SP-59, American Concrete Institute, Detroit, 1979, pp. 117-143.

5. Comité Euro-International du Beton, CEB-FIP Model Code 1990, Thomas Telford, London, 1991, 437 pp.

6. Design of Concrete Structures for Buildings, Canadian Standards Association, Rexdale, 1984, 281 pp.

7. Collins, M. P., and Mitchell, D., "Design Proposals for Shear and Torsion," *Journal of the Prestressed Concrete Institute*, V. 25, No. 5, Sept.-Oct. 1980, pp. 32-100.

8. Hsu, T. T. C., and Mo, Y. L., "Softening of Concrete in Torsional Members—Theory and Tests," ACI JOURNAL, *Proceedings* V. 82, No. 3, May-June 1985, pp. 290-303.