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CODE PREVIEW PAPER

Background to material being considered
for the next ACI Building Code

Design of Slender Concrete Columns*

By JAMES G. MAC GREGOR, JOHN E. BREEN, and
EDWARD O. PFRANG

Offers a proposal for revising the slender column design procedures of the 1963 ACI Code. Proposes the use of a rational second-order structural analysis wherever possible or practical. In place of such an analysis, an approximate design method based on a moment magnifier principle and similar to the procedure used under the AISC Specifications is proposed. An outline of the normal range of variables in column design and a lower limit of applicability is proposed which will eliminate over 90 percent of columns in braced frames and almost half of columns in unbraced frames from consideration as slender columns. Through a series of comparisons with analytical and test results, the accuracy of the approximate design procedure is established.

Keywords: bending moments; building codes; columns (supports); frames; long columns; reinforced concrete; slenderness ratio; strength; structural analysis; structural design; ultimate strength.

■ SINCE THE ESTABLISHMENT OF ACI-ASCE Committee 441, Reinforced Concrete Columns, in 1961, a major amount of attention has been given to design provisions for slender columns. Numerous papers outlining and probing many facets of the problem were published in a Committee-sponsored ACI Symposium volume in 1966. A number of individual committee members have presented proposals for revising Sections 915 and 916 of ACI 318-63.[1] The main arguments for a revision have generally been based on the following shortcomings of the present reduction factor design method:

1. The reduction factor method implies maintenance of the same eccentricity in both the slender and analogous short column. This is contradictory to the actual behavior of slender columns where the reduction in load-carrying capacity is caused by the increased eccentricity due to secondary deflection moments. This is a severe shortcoming in the case of unbraced frames, since the magnitude of the secondary moments is extremely important and should be included in the design of the restraining beams.

2. Due to practical considerations, many important variables had to be neglected in trying to express the reduction factor method in a designer-usable form. Because of this, the reduction factor expressions were based on extreme lower bounds and are unduly conservative for many practical cases.[2]

3. The entire treatment of the slender column problem lacks rationality in ACI 318-63. Little encouragement or direction is given to the designer to use a more comprehensive method of analysis which considers the secondary moments and actual member response. In view of the rapidly developing capacity for improved structural analysis using computers, the design method for slender columns should actively encourage the use of a highly accurate second-order structural analysis wherever possible.

This paper presents a proposal for revising the long-column design procedures of ACI 318-63.[1] It is proposed that, wherever possible, columns should be proportioned to carry the forces and moments from a rational second-order structural analysis. Where such an analysis is not possible or practical, it is proposed that the cross sections of slender columns be proportioned for the

ACI member **James G. MacGregor** is professor of civil engineering, University of Alberta, Edmonton, Canada. Currently, he is secretary of ACI-ASCE Committee 441, Reinforced Concrete Columns; vice-chairman of ACI-ASCE Committee 426, Shear and Diagonal Tension; a member of ACI Committee 115, Research, and a Canadian representative to Comite Europeen du Beton.

ACI member **John E. Breen** is professor of civil engineering, The University of Texas at Austin, Tex. Currently, he is chairman of ACI-ASCE Committee 441, Reinforced Concrete Columns; a member of ACI-ASCE Committee 334, Concrete Shell Design and Construction; of Comite Europeen du Beton Commission VIII, Buckling, and of the ASCE research council on performance of full scale structures.

ACI member **Edward O. Pfrang** is chief, Structures Section, Building Research Division of the National Bureau of Standards, Gaithersburg, Md. Currently, he is a member of ACI-ASCE Committee 441, Reinforced Concrete Columns; of ACI Committee 118, on computers and of ACI Committee 318, Standard Building Code. In addition, he is secretary of the ASCE Research Council on performance of full scale structures and active in numerous other technical committees.

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axial load from a nominal structural analysis* combined with the moment from a similar analysis multiplied by a moment magnifier F. The moment magnifier F is similar to the one used in the 1963 AISC Specifications for structural steel design.[3] It is a function of the ratio of the axial load to the critical load of the column, the ratio of column end moments, and the deflected shape of the column.

Lower slenderness limits are proposed to identify columns in which the secondary moments can be disregarded. Comparison with surveys of existing practice indicates that over 90 percent of the columns in braced frames and over 40 percent of the columns in unbraced frames fall into this classification.

This paper consists of six distinct parts: (1) the range of variables encountered in the design of columns in buildings is outlined; (2) the behavior of slender columns is reviewed briefly; (3) the philosophy and basic requirements of a long column design procedure are considered; (4) the general concepts of three possible approximate column design methods are presented along with the criteria used to select the design methods proposed in this paper; (5) the proposed revisions to Sections 915 and 916 of ACI 318-63 are presented and explained, clause by clause; and (6) the proposed design procedure is compared to data from analyses and tests of hinged and restrained columns. Although Committee 441 hopes that more and more columns will be designed according to the more rational second-order analysis, the majority of this paper deals with the assumptions, simplifications, and limitations of the more approximate moment magnifier method.

The revisions proposed in this paper grew out of a set of four different design procedures prepared by the Design Subcommittee of Committee 441. These proposals were discussed in lengthy rounds of correspondence and at five meetings of the Column Committee over a period of 3 years. The final draft of the proposal for revising ACI 318-63 was approved by letter ballot of the entire membership of Committee 441 in January 1968 and was submitted to Subcommittee 10 of ACI Committee 318 (Flexure and Axial Loads) in March 1968. The final arrangement and wording of the proposals contained in this paper were largely developed by Subcommittee 10.

THE NORMAL RANGE OF VARIABLES ENCOUNTERED IN THE DESIGN OF BUILDING COLUMNS

During 1967 a subcommittee of ACI-ASCE Committee 441 surveyed typical reinforced concrete buildings to determine the normal range of variables encountered in the design of columns in buildings. The buildings studied ranged from single story frames to 33-story office towers, including an unbraced 20-story high-rise tower in a high seismic intensity zone. Concrete strengths ranged from 3750 to 5000 psi,† steel strengths from 50 to 75 ksi, reinforcement percentages from 0.5 to 8.3, and both ties and spirals were used for lateral reinforcement. The reporting sheets indicate substantial numbers of both typical interior and exterior columns, and include data for representative lower, midheight, upper, and roof stories in the multistory structures. The total number of columns studied was in excess of 20,000. While it would be desirable to augment the study with reports from more geographical regions, the present results should give a reasonable picture of the column in general usage.

Frequency distributions of slenderness ratios h/t; relative stiffness factors r'; and maximum eccentricity ratios e/t were plotted for both braced and unbraced columns. Studies of these distributions indicated that 98 percent of the columns in braced frames had h/t less than 12.5, r' less than 22.5, and e/t less than 0.64, while 98 percent of the columns in unbraced frames had h/t less than 18, r' less than 5.3, and e/t less than 0.84. In general, these ratios provide an idea of the limits necessary on any approximate design procedure for columns in frames.

Further studies were made of the interrelationships reported between h/t, e/t, and r'. These studies indicated that the upper practical limit on the slenderness ratio kh/r is about 70 in building columns, and that the large r' values only occur with very short columns. The study further indicated that slender columns very seldom have e/t values in excess of about 0.4. The reason for this might not be immediately apparent, but the investigation confirmed yet unpublished results found in a current Reinforced Concrete Research Council sponsored University of Texas study of columns as frame members. In this study it has been noted that an interrelationship between beam flexural capacity, beam deflection slenderness limits, and relative column-beam stiffness factors imposed a limit on the range of practical column variables. The major trend of the effect of beam capacity can be seen in Fig. 1. Results are shown superimposed on a set of long column interaction curves used in the Committee's studies for single curvature columns. All points in the shaded area inside the broken line represent column strengths which could not be attained without yielding of the floor system occurring prior to column failure for $r' = 1$ and a beam slenderness limit L/t of 30. Since present design procedures are based on elastic analysis, it is im-

*A "nominal" analysis is any conventional elastic analysis (such as moment distribution) which uses stiffnesses based on gross sections and ignores secondary deflection effects and the effect of axial load on member stiffnesses.

†See Appendix D for SI (metric) conversion factors.

plicit that actual columns in braced frames will be restricted to those cases where the floor system does not yield prematurely. Confirmation of this can be seen from the heavy lines superimposed on the radial e/t lines in Fig. 1. These heavy lines represent the building survey range of h/t values which exist for a given e/t. The agreement between the range of actual h/t and e/t found in the survey of actual structures and the usable nonyielding range found in the computer study is very good and indicates the range of greatest interest in column design.

From this study the Column Committee has concluded that an approximate design procedure for slender columns should be most accurate in the compression failure range for columns up to kh/r of about 70.

BEHAVIOR OF SLENDER REINFORCED CONCRETE COLUMNS

In this paper the term "short column" is used to denote a column which has a strength equal to or greater than that computed for the cross section using the forces and moments obtained from a nominal analysis and the normal assumptions for combined bending and axial load. A "slender column" is defined as a column whose strength is reduced by second-order deformations. By these definitions a column with a given slenderness ratio may be a short column under one set of restraints and loadings and a slender column under another combination of restraints.

The effect of slenderness on a slender column is illustrated in Fig. 2. The maximum moment in the column occurs at Section A-A, due to the combination of the initial eccentricity e in the column and the deflection Δ at this point. Two types of failure can occur. First, the column may be stable at the deflection Δ_1, but the axial load P and the moment M at Section A-A may exceed the strength of that cross section. This type of failure, known as a "material failure," is illustrated by Column 1 in Fig. 2(c) and is the type which will generally occur in practical building columns which are braced against sway. Second, as shown for Column 2 in Fig. 2(c), if the column is very slender it may reach a deflection Δ_2 due to the axial force P and the end moment Pe, such that the value of $\delta M/\delta P$ is zero or negative. This type of failure is known as a "stability failure" and may occur in slender columns in sway frames.

In discussing the effect of variables on the slenderness effect it is often convenient to use slender column interaction diagrams. The behavior of a typical slender column, as illustrated in Fig. 2, is also shown for a specific case with h/t of 30 in Fig. 3a. Although actual failure occurs when the axial load and amplified moment line for the critical section in this column inter-

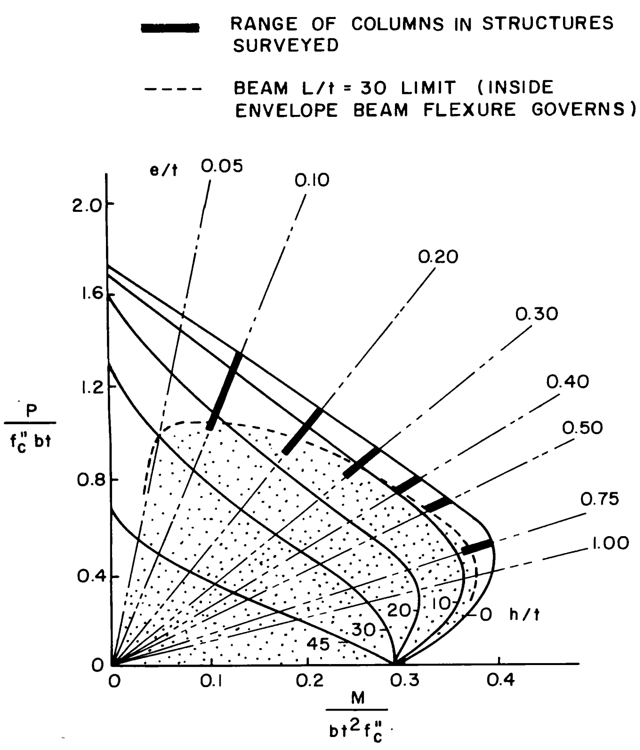


Fig. 1—Long column interaction curve limits

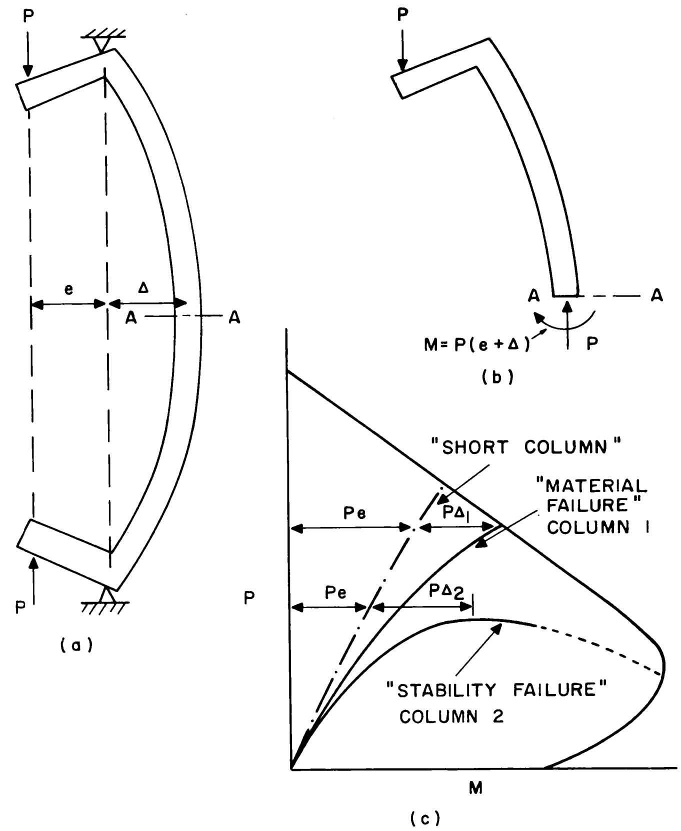


Fig. 2—Load and moment in a slender column

sects the cross section interaction diagram at Point B, the designer may be interested in expressing this failure in terms of the axial load and the primary or nominal moment at Point A, which corresponds to the load and maximum end moment from a nominal structural analysis.

Construction of the corresponding family of slender column interaction diagrams is shown in Fig. 3b, where interaction diagrams for members of varying slenderness ratios are presented in terms of the maximum loads and moments that can be applied at the ends of the columns. For the typical column shown in Fig. 3a, actual failure occurs with a combination of axial load P and amplified moment M_{max}, as shown at Point B, although the nominal end moment is considerably different as shown at Point A. Fig. 3b indicates that this failure case is represented at A on a slender column interaction diagram by plotting the point for the combination of axial load P and nominal end moment M which produced the failure case.

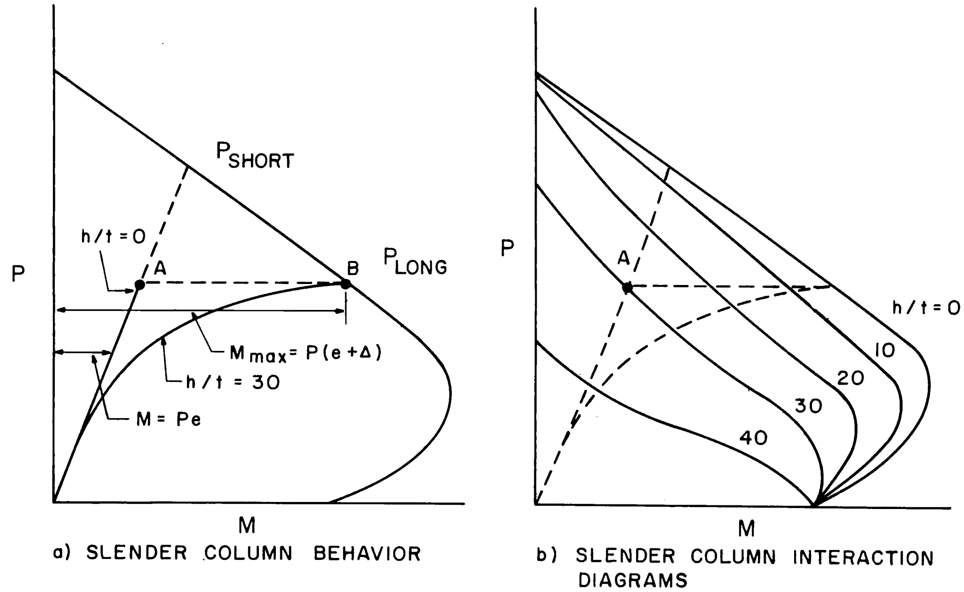


Fig. 3—Construction of slender column interaction diagrams

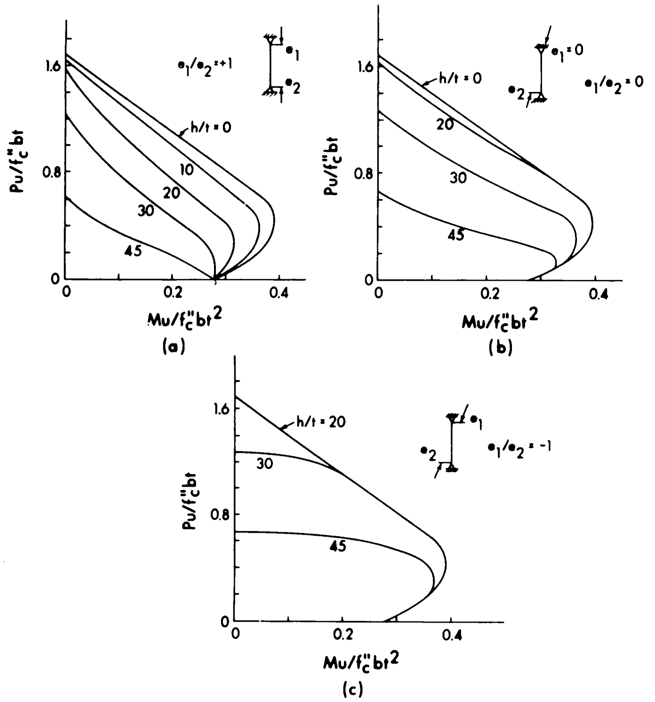


Fig. 4—Effect of deflected shape on interaction diagrams for long hinged columns

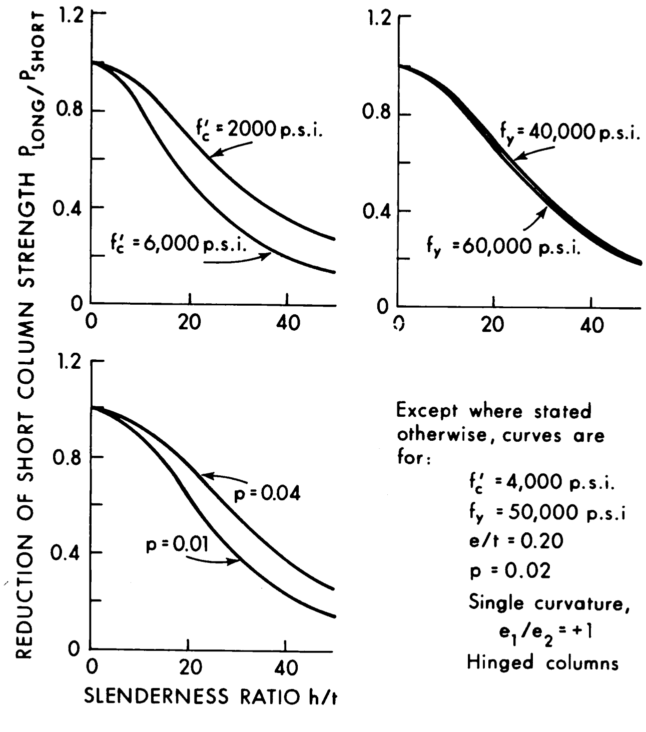


Fig. 5—Effect of variables on strength of slender hinged reinforced concrete columns

Major Variables Affecting the Strength of Slender Hinged Columns

Broms and Viest[4] and, more recently, Pfrang and Siess[5] have presented comprehensive discussions of the effects of a number of variables on the strength of restrained and unrestrained columns. These two studies have shown that the three most significant variables affecting the strength and behavior of a slender hinged column are: the slenderness ratio h/t; the end eccentricity e/t; and the ratio of end eccentricities e_1/e_2. The effects of these variables are strongly interrelated, as illustrated in Fig. 4. This figure presents three series of load-moment interaction curves for hinged columns with various e_1/e_2 ratios. The interaction diagrams are presented in terms of the maximum loads and moments that can be applied at the ends of columns with various slenderness ratios, as discussed in the previous section.

A hinged column will be weakened if at any section the sum of the moments due to the end eccentricities or imperfections and the column deflections exceeds the maximum moment in the undeflected column. In a column subjected to symmetrical single curvature the column deflections will always increase the column moments. Thus, in Fig. 4a the interaction diagrams for all h/t values greater than zero fall inside the interaction diagram for the cross section $(h/t = 0)$.

In the case of double curvature, however, this will not always be true, since the maximum applied moment occurs at one or both ends of the column while the maximum deflection moments occur between the column ends. This is illustrated by the interaction diagrams for $h/t = 30$ in Fig. 4c. This column is weakened by the column deflections for small eccentricities where the sum of the deflection moments and the applied moments lead to maximum moments greater than the applied moments. For larger eccentricities, however, the maximum moments will always occur at the ends of the column and as a result there is no weakening due to length.

Broms and Viest showed that an increase in the proportion of the load carried by the reinforcement led to a more stable column. Thus, columns with high concrete strengths f_c' and/or low reinforcement percentages p tended to be most strongly affected by length, as shown in Fig. 5, which has been reproduced from Reference 4. In other words, as the p/f_c' ratio is increased, the column tends to be more stable.

Sustained loads tend to weaken a hinged slender column by increasing the column deflections. Columns bent in symmetrical single curvature will always be weakened by sustained loads. The effect of sustained loads on the strength of hinged columns bent in double curvature is much less pronounced, especially if the end eccentricities are large.

Behavior of columns in braced frames

The discussion of the strength and behavior of restrained columns will be broken into two parts, depending on whether the frame is laterally braced or is free to sway. To date all tests and analyses of laterally braced frames have assumed that no lateral deflection can occur.

Fig. 6 shows a restrained column bent in single curvature. An axial load P and an unbalanced moment M_{ext} are applied to the joint at each end of the column. The external moment M_{ext} is resisted by the restraints M_r and the column M_c as shown in Fig. 6b. The maximum moment in the column is the sum of the column end moment M_c and the deflection moment M_d as shown in Fig. 6c and Eq. (1):

$$M_{max} = M_c + M_d = M_{ext} - M_r + M_d \qquad (1)$$

The restraint moment is a direct function of the relative stiffnesses of the column and beam. As the column deflects laterally under axial load and moment, the column stiffness is reduced so that more of the external moment at the joint is resisted by the beams and less by the column. Inelastic action in the column tends to hasten this reduction in the column stiffness thus re-

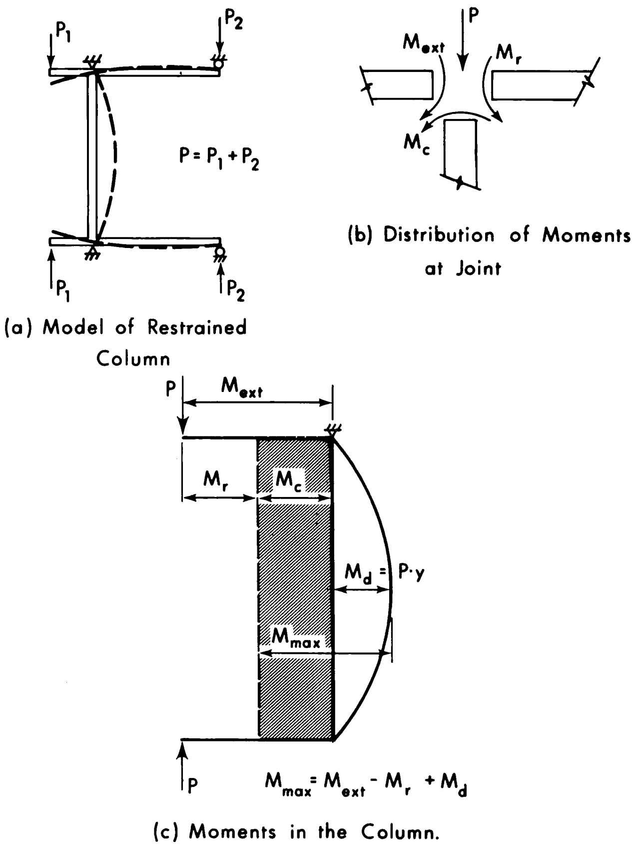


Fig. 6—Moments in a restrained column

ducing the moment developed at the ends of the column. On the other hand, inelastic action in the beam tends to throw moment back into the column.

The deflection moment is affected by the same variables that affect the column stiffness. An increase in the deflection moment tends to weaken the column.

For short restrained columns, the reduction in the column end moment due to axial load may be larger than the increase in moment due to deflections. As a result, the maximum moment in the column is decreased below the value from a first-order analysis and the axial load capacity of the column increases. For a slender restrained column, however, the deflection moments tend to increase more rapidly than the restraint moments and the over-all effect is a weakening of the column because the maximum moment is increased.

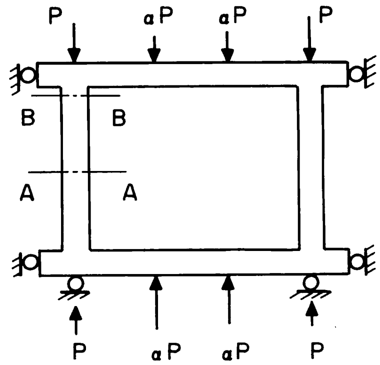
Within the outline provided by Eq. (1), it is possible to discuss the effects of other variables on the strength of a restrained column. Thus, sustained loads will increase column deflections but at the same time will reduce the column stiffness. For h/t greater than about 10 the net effect would be a reduction in strength, while for shorter columns the sustained loads may increase the strength of the column.[6]

Fig. 7a shows Frame F2 tested by Furlong and Ferguson.[7] This frame was loaded so that the columns deflected in a single-curvature mode. The columns had $h/t = 20$ ($kh/r = 57$) and an initial eccentricity ratio $e/t = 0.106$. Failure occurred at Section A at midheight of one of the columns. In Fig. 7b the load and moment history at Sections A and B is superimposed on an interaction diagram for that column. The moment at B corresponds to the moment M_c at the end of the column in Fig. 6c. Although the loads P and αP were proportionally applied, the variation in moment at Section B is not linear with increasing load because the column stiffness decreased more rapidly than the beam stiffness. The moment at Section A, the failure section, is equal to the sum of the moment at Section B plus the moments due to the column deflection, as indicated by Eq. (1) and Fig. 6c. The intersection of the interaction diagram and the line for Section A corresponded to column failure.

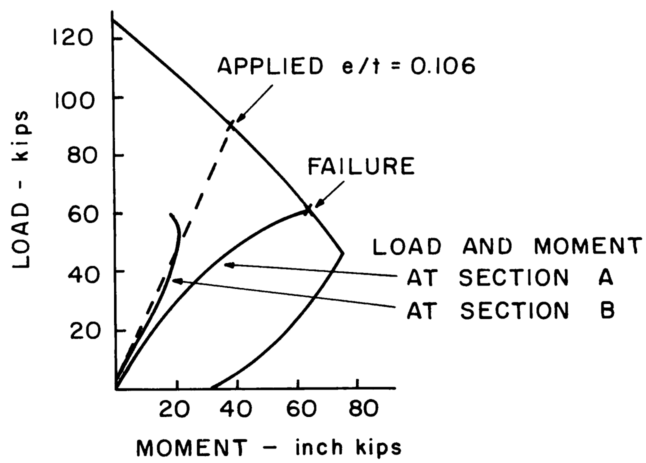
For restrained columns the effects of the slenderness ratio h/t; the eccentricity ratio e/t; and the ratio of end eccentricities e_1/e_2 are interrelated in much the same way as for hinged columns. This is illustrated in Fig. 8, which shows interaction diagrams for slender columns with an end restraint corresponding to an elastic effective length of $kh = 0.78h$ for a slenderness ratio h/t of 20.[5,8] The columns in Fig. 8a are bent in symmetrical single curvature, those in Fig. 8b in antisymmetrical double curvature. The interaction diagrams are presented in terms of the axial load and the column end moments assigned to the column by a first-order elastic analysis.

For large eccentricities and double curvature, the strength of a very slender column, such as the $h/t = 45$ column in Fig. 8b, may significantly exceed the short column strength for the first-order moments and loads. This is because the axial loads and cracking have reduced the rotational stiffness of the column and as a result the actual column end moments are less than the corresponding first-order moments. For small eccentricities the column deflections reduce the column strength below the short column value.

The effect of varying degrees of end restraint are shown by the solid lines in Fig. 9 for a series of columns bent in single curvature with $h/t = 20$.[5,8] The end restraints are expressed in terms of the ratio r' of the column stiffness to the restraint stiffness. The corresponding elastic effective lengths k are also shown. As expected, an



(a) TEST SPECIMEN



(b) MEASURED LOAD - MOMENT RESPONSE

Fig. 7—Behavior of a column in a frame

increase in the degree of end restraint increased the column capacity. The dashed lines will be discussed later.

In preparing Fig. 8 and 9 the end restraints were assumed to be linearly elastic for any required rotation. This is not the case, however, for the actual end restraints in reinforced concrete structures. It has been shown that in practical structures the end restraints will generally yield prior to the failure loads for columns with h/t values greater than about 15 to 20. The actual range where this may occur is shown by the shaded area in Fig. 1. Once the end restraints have yielded the additional load capacity of the column will generally be drastically curtailed.

Behavior of slender columns in sway frames

The behavior of a column free to sway is markedly different from that of a column restrained against sway. As the stiffness of the lateral restraints is decreased, there is a transition from the no-sway to the sway case.

A column in a frame without lateral restraint must depend largely on the beams to prevent sidesway collapse. If the beams are quite flexible, the column will behave essentially as a rigid body

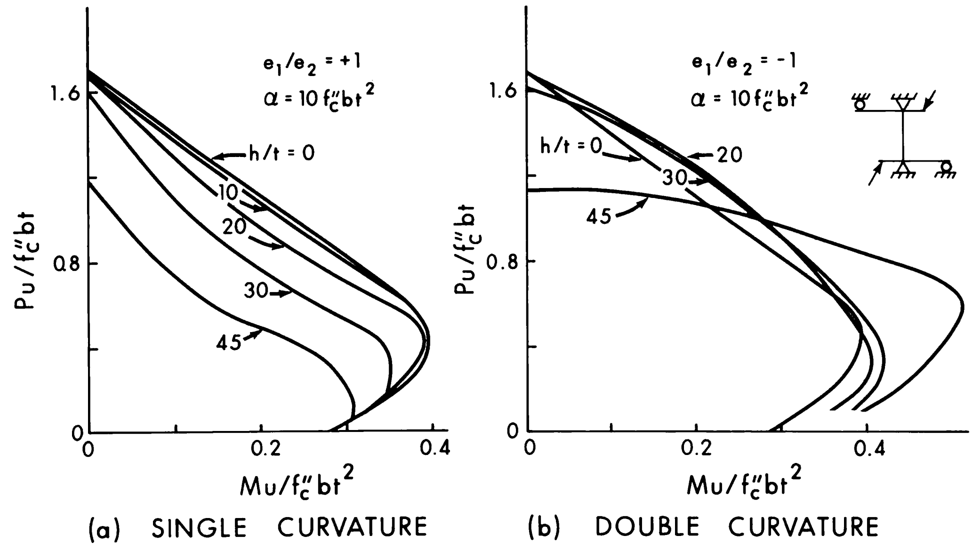


Fig. 8—Interaction diagrams for long restrained columns

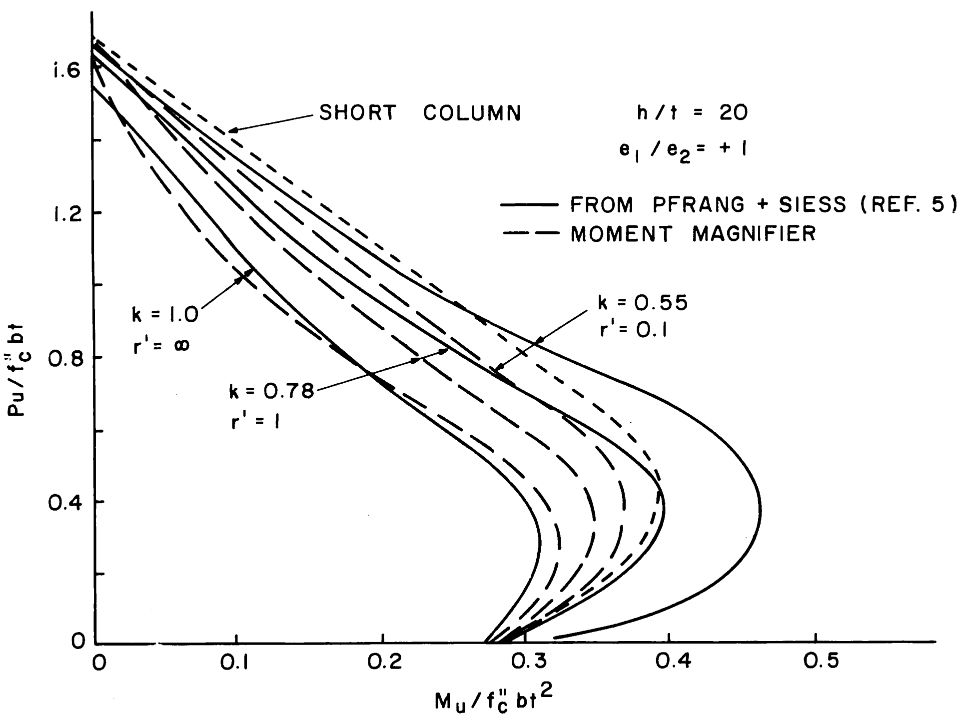


Fig. 9—Effect of effective length factor on theoretical and design interaction curves

and the frame will deflect laterally due primarily to the bending of the beams. If the restraints are stiff, the column will resist some lateral deflection by bending of the column itself. In either case, however, the frame will form an unstable mechanism if the beams yield and can no longer restrain the columns. An increase in the degree of rotational restraint will always increase the strength of a column that is free to sway, unless, of course, the restraints yield.

With the presence of even a small amount of lateral restraint the behavior and capacity of the column changes significantly.[9] Since part of the lateral load is resisted by the lateral restraint, the column no longer deflects as a rigid body and flexural deflections of the column begin to play an important role in the column behavior. However, since a laterally unrestrained column depends entirely on its rotational restraints for stability, if these restraints yield the column becomes unstable. Because of this potentially large reduction in the strength of the frame due to yielding of the restraining beams, it is essential that design of these beams reflect the magnified column end moments which they will be called on to resist.

Summary of the primary factors affecting the strength of slender columns

This brief review of column behavior suggests that the principal variables affecting the strength of slender columns are:

1. The degree of rotational end restraint. An increase in the degree of end restraint will increase the capacity of a column. The effect of the restraints is less marked if they yield under the moments transferred to them by the column.

2. The degree of lateral restraint. A completely unbraced column is significantly weaker than a braced column, but a relatively small amount of bracing is enough to increase the strength almost to that of a completely braced frame. The strength of an unbraced column is strongly dependent on the rotational capacity of the restraining beams.

3. The slenderness ratio h/t, the end eccentricity e/t, and the ratio of end eccentricities e_1/e_2. All have a significant and strongly interrelated effect.

4. The ratio p/f_c'. An increase in this ratio tends to increase the stability of a column.

5. Sustained loads. These loads increase the column deflections and usually decrease the strength of slender columns.

GENERAL REQUIREMENTS OF A DESIGN METHOD FOR SLENDER COLUMNS

At the present time the design of a slender reinforced concrete column consists of three stages:

1. The analysis of the structure to determine the forces and moments in each member.

2. The modification of the forces and moments computed in Stage 1 to account in some way for the column slenderness.

3. The proportioning of the cross section to resist these forces and moments.

The first of these stages involves an ordinary structural analysis and is described in Sections 904, 905, and 914 of ACI 318-63.[1] Much of the difficulty various investigators have encountered in studying column length effects is due to the highly approximate analysis methods permitted in these sections. Since the analysis errors are most significant for cases with very low slenderness ratios, corrective action should properly be applied to these sections and not to the column slenderness provisions alone. The third stage involves the choice of the size and shape of a cross section for a given set of moments and is described in Chapters 14 and 19 of ACI 318-63. The basic procedures on which Chapter 19 are based seem adequate with minor revisions. This brief discussion considers primarily the second stage listed above, which is described in Sections 915 and 916 of ACI 318-63. In doing this it is tacitly assumed that the analysis in Stage 1 and the design of the cross section in Stage 3 will be carried out as accurately as possible.

The proposed revisions to Sections 915 and 916 allow two methods of carrying out this procedure. The first of these involves a rational second-order structural analysis which includes both Stages 1 and 2, of the three-stage design procedure described above. The term "rational" analysis is used to describe an analysis that considers the effects of column and frame deflections on moments and the effects of axial loads on the member stiffnesses. Such an analysis should be carried out for the frames in combination with any bracing members or shear walls that exist.

In the second proposed design method, design is based on the axial loads and moments from a conventional structural analysis and the effects of slenderness on column strengths are approximated by multiplying the column moments by a "moment magnifier" F. In this expression F is a function of the ratio of the axial load in the column to the assumed critical load of the column, the ratio of column end moments, and the deflected shape of the column. This procedure is similar to the beam column design procedure in the 1963 AISC Specifications[3] and will be referred to as the "moment magnifier method" in the rest of this paper.

It cannot be emphasized too strongly that, for unbraced or partially braced frames, the design procedure based on a rational frame analysis is preferable because it adequately estimates the behavior of the structure. It is the authors' understanding that at least one American consulting firm is using such an analysis today and the authors believe that during the 1970's, when the next ACI Code is in use, such analyses will be readily available to consulting firms.

REVIEW OF APPROXIMATE SLENDER COLUMN DESIGN PROCEDURES CONSIDERED BY ACI-ASCE COMMITTEE 441

Three different procedures for designing slender columns were considered by ACI-ASCE Committee 441 in their studies. These included several variations of the moment magnifier method,[3,10-13] the complementary moment method,[14] and the long column reduction factor method.[1] All of these design methods involve a nominal elastic frame analysis to compute design forces and moments. These nominal forces and moments are then modified for each individual column to account for the slenderness and, finally appropriate column cross sections are chosen for the modified forces and moments. The basic concept of each of these methods and the reasons for the Committee's choice will be presented in the following sections.

Moment magnifier design method

A good approximation of the maximum moment in an elastic beam-column bent in single curvature is given by Eq. (2):[12]

$$M_{max} = M_o + \frac{P\Delta_o}{1 - (P/P_c)} \tag{2}$$

where M_o and Δ_o are the first-order moment and deflection, respectively, P is the column load, and P_c is the critical load of the column. It can be shown[12] that Eq. (2) can be approximated by Eq. (3):

$$M_{max} = \frac{M_o}{1 - (P/P_c)} \tag{3}$$

Eq. (3) is reasonably accurate for a column bent in uniform single curvature because in this case the maximum moment and maximum deflection occur at the same point. In the more usual case where the end moments are not equal, the maximum moment may be estimated using an "equivalent uniform moment" C_mM_o, which would lead to the same long column strength as the actual moment diagram. Thus, Eq. (3) becomes:

$$M_{max} = \frac{C_mM_o}{1 - (P/P_c)} \geq M_o \tag{4}$$

where C_m is the ratio of the equivalent uniform end moment to the numerically larger end moment. Values of C_m are presented for several common design cases in References 10 and 12. The AISC Specifications[3] call for the working stress design of eccentrically loaded steel columns using Eq. (5):

$$\frac{P}{P_u} + \frac{M_{max}}{M_u} \leq 1 \tag{5}$$

where M_{max} is defined using Eq. (4).

For reinforced concrete columns, the design can be based on the axial load P from a first-order analysis and the moment M_{max} computed from Eq. (4).[15] This design procedure closely approximates the actual case shown in Fig. 2, in which the most highly stressed section, Section A-A, is loaded with axial load P and a moment, $Pe + P\Delta$ equivalent to M_{max}.

The measured behavior for a column in a frame tested by Furlong[7] has been shown in Fig. 7b. The load and moment developed at Failure Section A during the test are shown by the solid line. The intersection of this line and the interaction curve corresponded to column failure.

Fig. 10 shows the way in which this behavior is represented in the moment magnifier method. The column is designed for the axial load P and a magnified moment shown as FM. Thus, the "moment magnifier" based load-moment path shown by the solid line in Fig. 10a closely approximates the test results [which are repeated as a dashed line in Fig. 10(a)] and intersects the interaction curve at approximately the same combination of

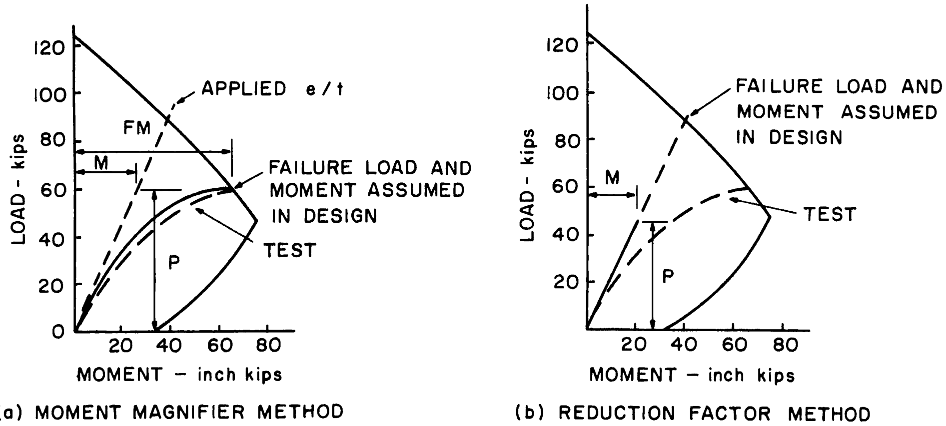


Fig. 10—Representation of column behavior by approximate design procedures

load and moment as the test. Fig. 10(a) is based on the revisions to Sections 915 and 916 proposed in this paper.

Complementary moment design method

The moment at the failure section in a column could also be taken equal to the sum of the applied moment M and a complementary moment M_c equal to the load times a complementary eccentricity e_c. The complementary moment represents the moment induced by column deflections. The column is designed for the axial load P and the moment $M + M_c$. The value of M_c may be based on the 1963 Comité Européen du Béton Recommendations[14] for the design of slender columns. These recommendations are based on the use of a complementary moment which is assumed to be a function of the slenderness ratio kh/r and the eccentricity ratio e/t. Applying the CEB complementary moment expression to the Furlong test frame case yields a design load slightly higher than the test value.

Long column reduction factor design method

ACI 318-63 requires that the axial load and moment computed from an ordinary analysis be divided by a long column reduction factor R. This factor is a function of the slenderness ratio, h/r or h'/r, and the deflected shape of the column. This procedure is illustrated in Fig. 10(b), which was computed using Eq. (9-3) of ACI 318-63. It can be seen that the conditions assumed in design do not approximate those at the failure section of the test specimen.

Basis for choice of an approximate design method

At the 1967 Des Moines meeting of ACI-ASCE Committee 441 the moment magnifier method was chosen for development as the basis for the approximate design method for the next ACI Code. A wide range of factors was considered in choosing this approximate design method. The more significant of these were:

Rationality—A truly satisfactory design procedure should be rational so that it can be extended easily to design cases not envisaged in its derivation. The design method should also correctly reflect the significant behavior of a slender column. The Committee felt the moment magnifier method was the most rational and the long column reduction factor method the least rational of the three methods considered.

Accuracy—To be satisfactory, a design procedure must give a reasonable prediction of the experimental strength of slender columns or the strength indicated by more exact analyses. A comparison with test results showed that the complementary eccentricity method predicted failure loads slightly better than the moment magnifier method and much better than the reduction factor method. Studies indicated that in certain cases of unbraced frames, the reduction factor method was very unsafe. By increasing the complexity of the definition of the critical load P_c, the accuracy of the moment magnifier method can be increased.

Ease of use—Since the effect of slenderness on the strength of columns is a secondary design problem for columns in most building frames, the procedure for including the effect of slenderness in design should be simple. In addition, since an approximation to the behavior is sought, there are distinct advantages to a simple approximation for everyday design use. Of the three methods, the reduction factor method seemed easiest to use with the complementary eccentricity and the moment magnifier methods only slightly harder to apply. The ease with which the moment magnifier method could be used was related to the complexity of the definition of the critical load term P_c (see Appendix B). The practical degree of difficulty in application for all methods is minimized by a careful choice of factors which identify columns in which secondary effects can be disregarded. Design procedures for all methods can be minimized with the availability of a few relatively simple design aids.

Familiarity—The fourth point considered in choosing the design methods was the need to reeducate designers to use the proposed method. The reduction factor method is most familiar to reinforced concrete designers in this country, although extensive revisions would be required in this method before incorporating it in a revised Code. No satisfactory revision of the reduction factor method could be agreed upon to treat unbraced columns with high slenderness ratios and it was apparent that the moment magnifier method would probably be required for this case, even if a reduction factor method was used for braced frames. The moment magnifier concept is also very familiar to American engineers, who currently use important elements of the same concept in designing steel columns under the AISC Specifications.[3,10] The complementary eccentricity method is relatively unknown in North America, although it is used in Europe.[14]

EXPLANATION OF PROPOSED REVISIONS TO SECTIONS 915 AND 916

This part of the paper will consist of a paragraph by paragraph review of the revisions proposed to implement the moment magnifier design method. These paragraphs would replace Sections 915 and 916 of ACI 318-63. The complete text of the proposed revisions is included in Appendix A. (Styling conforms to that being used in the proposed ACI Code draft.)

Section 10.10—Slenderness effects in compression members

10.10.1—The design of compression members shall be based on forces and moments determined from an analysis of the structure. Such an analysis shall take into account the influence of axial loads and variable moment of inertia on member stiffness and fixed end moments, the effect of deflections on the moments and forces, and the effects of the duration of the loads.

10.10.2—In lieu of the procedure described in Section 10.10.1, the design of compression members may be based on the approximate procedure presented in Section 10.11. The detailed requirements of Section 10.11 do not need to be applied if design is carried out according to Section 10.10.1.

Sections 10.10 and 10.11 of the proposed Code will deal with the effects of slenderness on the strength of columns in structures. As shown previously, this is an extremely complex problem involving a large number of variables, including the geometry of the structure, the geometry and properties of the member, and the duration and type of loading. Two alternate design procedures are outlined in Sections 10.10 and 10.11. Section 10.10.1 describes the requirements for a rational frame analysis. Alternately, the designer can use the relatively simple but approximate design rules presented in Section 10.11. These rules have been derived to be conservative but to follow the trends indicated by more complex column analyses.

Committee 441 unanimously endorsed the position that the slenderness effect provisions should encourage improvement in the structural analysis, since the basic need for any slenderness effect provision stems from weaknesses in conventionally used methods of frame analysis. Many of the analysis shortcomings affect short columns as much or more than slender compression members. Wherever possible the designer is urged to use a computer-oriented analysis of the type described in Section 10.10. The ideal structural analysis would be a historical second-order elastoplastic analysis of the entire structure considering load history.

The following elements are regarded as minimum requirements for an adequate rational frame analysis for design of compression members under Section 10.10.1:

1. The structure may be idealized as a plane frame of linear elements. In structures containing structural walls, a better estimate of moments and deflections will be obtained if the stiffness of the wall is considered in the analysis.

2. Realistic moment-curvature relationships must be used to provide accurate values of deflections and secondary moments. A linear approximation of the moment-curvature relationship defined by Eq. (D) in Section 10.11.5 of the proposed Code statement will be acceptable, although use of a more accurate relationship is encouraged. The effect of the duration of loads on deformations must be considered.

3. The analysis must consider the influence of the axial load on the rotational stiffness of the member.

4. The maximum moments in the compression members must be determined considering the effects of member and frame deflections. The possibility of having a maximum moment occur at sections other than the ends of the member must be considered.

5. Because of the complexity of the problem, any proposed analysis used under the provisions of Section 10.10.1 should be checked against the limited test results available and should show accuracy at least comparable with the more approximate provisions of Section 10.11.

Paragraph 10.10.2 allows the use of the approximate and more conservative design procedure presented in Section 10.11. It also points out that it is not necessary to consider the detailed limitations and requirements in Section 10.11, if a more complete rational design procedure under Section 10.10.1 is used.

Section 10.11—Approximate evaluation of slenderness effects

A satisfactory approximate design procedure for slender columns should consider at least five major points:

1. A basic procedure for calculating the effects of slenderness on structural members subjected to bending and axial compression must be stated. In the proposed revisions to the slender column provisions, the basic approximate procedure is specified in Section 10.11.5.1.

2. Where necessary, modifications must be provided to extend the basic design procedure to columns in braced or unbraced frames. The basic moment magnifier procedure was derived for hinged end columns and is extended to columns in braced or unbraced frames, using effective length factors defined in Section 10.11.1. The method of computing the basic moment magnifier is modified for the sidesway case in Section 10.11.5.2.

3. The need to distinguish between braced and unbraced frames should be indicated. The design method differs for these two cases. However, no specific procedure for distinguishing between these cases is contained in the Code statement.

4. Upper and lower limits of the range of slenderness ratios for which the procedure can be applied should be specified. A lower limit is desirable to exclude those columns which are too short to be significantly affected by slenderness in order to simplify design, and an upper limit is necessary to exclude those columns whose slenderness requires a more accurate analysis. This is done in Section 10.11.4.

5. Design rules are also required for the beams restraining slender columns. This is especially true in unbraced buildings, where the columns can only resist the sway moment if the beams restraining them can also resist this moment. This is done in Section 10.11.7.

Sections 10.11.1 and 10.11.2

10.11.1—The unsupported length h of a compression member shall be taken as the clear distance between floor slabs, girders, or other members capable of providing lateral support for the compression member. Where capitals are present, the unsupported length shall be measured to the lower extremity of the capital or haunch in the plane considered.

10.11.2—The radius of gyration r may be taken equal to 0.30 times the over-all depth in the direction in which stability is being considered for rectangular compression members, and 0.25 times the diameter for circular compression members. For other shapes, r may be computed for the gross concrete section.

Sections 10.11.1 and 10.11.2 are taken from Sections 915(a) and 916(a)4 of ACI 318-63, with some abbreviation and a rearrangement of the order. They are included in the section on the approximate design procedure because these requirements are of significance in this procedure only.

Section 10.11.3

10.11.3—For compression members braced against sidesway, the effective length factor k shall be taken as 1.0, unless an analysis shows that a lower value may be used. For compression members not braced against sidesway, the effective length factor k shall be determined by a rational method with due consideration of cracking and reinforcement on relative stiffness, and shall not be less than 1.0.

The basic slender compression member design equations in Section 10.11.5 were derived for hinged columns. For restrained columns the basic equations must be modified to account for the effect of the end restraints. This is done by using an "effective length" kh in the computation of slenderness ratios, as has been used for beam column design in the AISC Specifications[3,10] since 1963. Indeed, the wording of Section 10.11.3 is based on that in the AISC Specifications. The use of an effective length in the design of restrained steel beam columns is discussed by Winter[12] who has stated:

> For eccentrically loaded, end-restrained columns Dr. Annabel Lee and Professor P. P. Bijlaard have shown, respectively for initial yield and for ultimate strength, that a safe design results from the following method: The restrained column is replaced by an equivalent, hinged column whose length is equal to the effective length for concentric compression of the real, restrained column; and this equivalent column is analyzed for compression plus that part of the total end moments which is resisted by the column alone.

And later in the same article:

> This simplified procedure would result in the following:
>
> (i) For rigid frames determine the end moments of the column by conventional moment distribution, without regard to change of effective stiffness caused by axial load. Then dimension the column for the normal force and the moments so obtained, but use kL instead of the real length L in determining the slenderness ratio. This method is applicable regardless of the provision specified for compression plus bending in the particular code (secant formula, interaction formula or any other rational device acceptable in the design specification).
>
> (ii) For real eccentricities . . . .

Yura and Galambos[16] show that the AISC moment-magnifier procedure gives good correlation with theory for steel columns in single story sway frames when the effective length factor is used in computing the critical load. Comparisons with more precise computer solutions indicate that this procedure is especially accurate for the unbraced frame.

Committee 441 proposed that the effective length be computed in a more or less standard way by use of the Jackson and Moreland Alignment Charts.[12] These charts allow graphical determination of the value of k for a typical interior column of constant cross section in a high multibay frame. The column is assumed to be elastic and axially loaded. The effect on k of other frame arrangements is discussed by Wakabayashi.[17] The assumptions made in the derivation of these factors are reviewed and discussed by Grier.[18] A copy of the charts is presented in Fig. 11. The effective length is a function of relative stiffnesses at each end of the compression member. Studies have indicated that the effects of widely varying beam and column reinforcement percentages and of beam cracking should be considered in determining these relative stiffnesses. Long column interaction curves computed using the proposed moment-magnifier method are plotted as broken lines in Fig. 9 to show the effect of using the effective length factor k.

Because the behavior of braced and unbraced frames is so different, it is difficult to derive a single all-encompassing design rule for columns in both types of frames. For this reason it has been traditional to present one set of design rules or effective length factors for completely braced frames and another set of rules or factors for completely unbraced frames. This distinction has been retained in Paragraph 10.11.3 of this Code proposal.

In actual fact, there is very rarely such a thing as a completely braced or a completely unbraced frame. For all intents and purposes, structures with reasonably heavy shear walls can be considered completely braced, since the second-order lateral deflections of such structures are not large enough to significantly affect the column strength.

Analyses of the amount of lateral bracing required to prevent sidesway instability in building frames have been presented in References 19 to 24. Unfortunately, none of these procedures are entirely suitable in the case of a multistory reinforced concrete building braced with a shear wall.

For the purposes of applying Section 10.11.3, a compression member braced against sidesway is defined as a member in a story in which the bracing elements such as shear walls, shear trusses, or other types of lateral bracing have a total translational stiffness sufficiently greater than the sum of the translational stiffness of all the columns in the story under consideration, so that the lateral deflections of the story are not large enough to significantly affect the column strength. What constitutes adequate bracing in a given case must be left to the judgment of the engineer, depending on the arrangement of the structure in question.

Sections 10.11.4 and 10.11.5

10.11.4—For compression members braced against sidesway, the effects of slenderness may be neglected when kh/r is less than $34\text{-}12M_1/M_2$. For compression members not braced against sidesway, the effects of slenderness may be neglected when kh/r is less than 22. For all compression members with kh/r greater than 100, a rational analysis as defined in Section 10.10.1 shall be made.

10.11.5—Compression members shall be designed using the design axial load from a conventional frame analysis and a magnified moment M, defined by Eq. (A).

$$M = FM_2 \qquad \text{(A)}$$

where

$$F = \frac{C_m}{1 - P_u/P_c} \geqq 1.0 \qquad \text{(B)}$$

and

$$P_c = \frac{\pi^2 EI}{(kh)^2} \qquad \text{(C)}$$

In lieu of a more precise analysis EI may be taken either as:

$$EI = \frac{E_c I_g/5 + E_s I_s}{1 + R_m} \qquad \text{(D)}$$

or

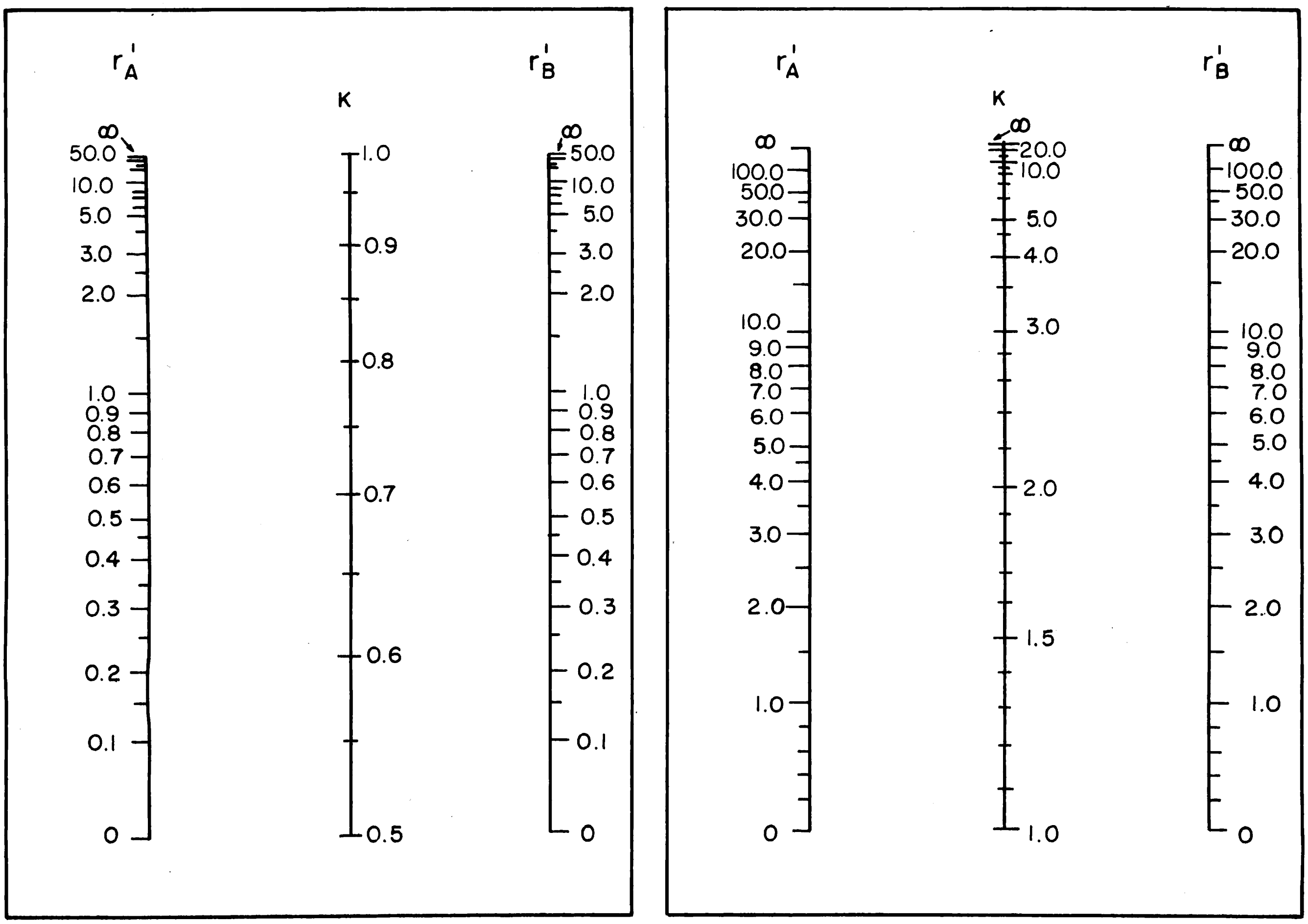
$$EI = \frac{E_c I_g/2.5}{1 + R_m} \qquad \text{(E)}$$

In Eq. (B), for members braced against sidesway and without transverse loads between supports C_m may be taken as:

$$C_m = 0.6 + 0.4(M_1/M_2) \qquad \text{(F)}$$

but not less than 0.4.

For all other cases C_m shall be taken as 1.0.



(a)
Braced Frames

(b)
Unbraced Frames

Fig. 11—Effective length factors

The actual slender compression member approximate design equations and their limits of applicability are presented in these sections. The definitions of the EI value, the equivalent moment term C_m and the definition of a short column in Section 10.11.4 are discussed in detail in the following subsections of this paper.

Definition of EI value—The moment magnifier is computed using Eq. (B) which has been discussed earlier. The main problem in applying the moment magnifier concept to an inelastic, nonhomogeneous material such as reinforced concrete is the manner in which the critical load of the column is defined. In particular, it is difficult to choose a value of the stiffness parameter EI which will reasonably approximate the variations in stiffness due to cracking, creep, and the nonlinearity of the concrete stress-strain curve. Several investigators have proposed values for EI for use in defining the critical load of concrete columns. For the design of columns in bridges, Reference 11 presents a method of computing the critical load for columns based on EI as given by Eq. (6):

$$EI = E_c I_t \, 1.6 \frac{P_u}{P_o}\left(1 - \frac{P_u}{P_o}\right) \tag{6}$$

where

E_c = modulus of elasticity of concrete
I_t = transformed moment of inertia
P_u = ultimate load column is to be designed for
P_o = capacity of column section in pure compression

In Reference 13, Parme presented a table of EI values for tied columns with bars in two faces. For columns failing in compression this table gives values of EI ranging from 0.167 to 1.625 times $(1000\ f_c'I_g)$. In the table EI is given as a function of $q = pf_y/f_c'$, P/P_o, and the ratio of the distance between the reinforcement to the column thickness g.

Spang[15] proposed that for columns with 4 percent steel:

$$EI = 1000\, f_c' I_g \tag{7}$$

and for other steel percentages:

$$EI = \frac{E_c I_c}{A} + \frac{E_s I_s}{B} \tag{8}$$

where A and B are 4.1 and 1.0, respectively.

The authors, serving as the Design Subcommittee of ASCE-ACI Committee 441, recommended that where more precise values are not available EI be defined by Eq. (D) and (E). These equations approximate the lower limits to EI for practical cross sections and, hence, are conservative for secondary moment calculations. Three procedures were followed to arrive at these two expressions for EI:

1. EI values were estimated from almost a hundred theoretical load-moment-curvature diagrams, computed by the method described in Reference 25, for columns of various dimensions, strengths, and steel percentages.

2. The effective EI values were computed for each of the Universty of Texas frame tests.[7,26-28]

3. Finally, similar effective EI values were computed for a series of frames simulated by the computer.

Because slender columns generally fail in compression with small e/t values as shown in the survey of actual building cases, and because the

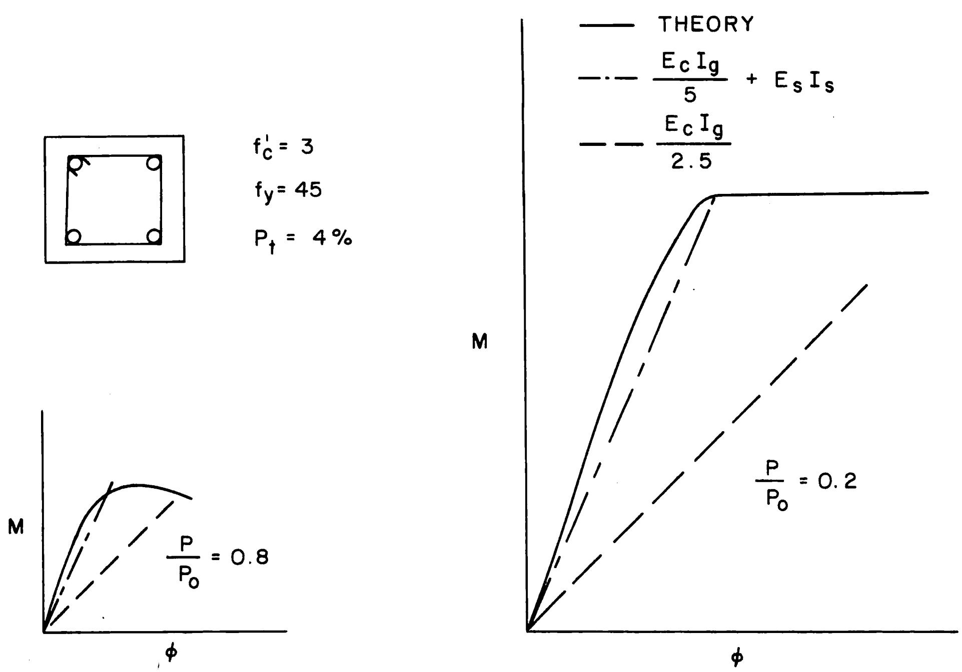


Fig. 12—Theoretical and approximate values of *EI*

effect of axial loads is more pronounced for small e/t values and high P/P_o values, Eq. (D) and (E) were chosen to fit this region.

Moment-curvature diagrams are shown in Fig. 12 for a tied column. The two solid curved lines represent the theoretical moment-curvature relationships for two ratios of axial load P_u to the pure axial load capacity P_o. The value of EI can be computed from a moment-curvature diagram using the following relationship:

$$EI = M/\phi \tag{9}$$

The dashed lines in Fig. 12 represent the two approximations given by Eq. (D) and (E). It can be seen that Eq. (D) closely approximates the initial slope of the theoretical curves, while Eq. (E) is more conservative.

In Fig. 13, Eq. (D) and (E) are compared to the EI values derived from the load-moment-curvature diagram for the case of no sustained load ($R_m = 0$). Eq. (D) represents the lower limit of the practical range of column EI values as is shown in Fig. 13(a). This is especially true for heavily reinforced columns. Because lightly reinforced columns tend to be affected more strongly by creep and inelastic action in the concrete, the greater conservatism of Eq. (D) for 1 percent reinforcement is acceptable.

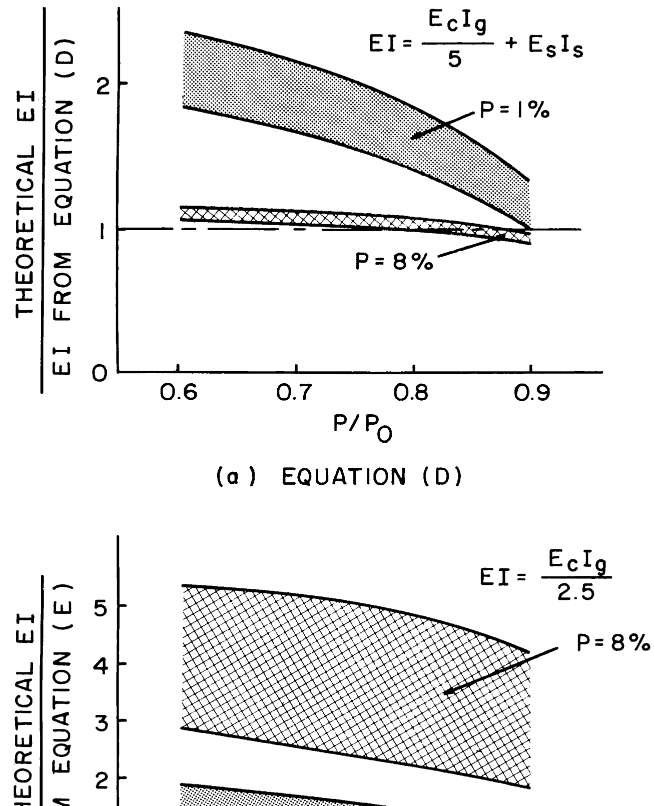


Fig. 13—Comparison of equations for *EI* with *EI* values from moment-curvature diagrams

The simpler, but more approximate, Eq. (E) is compared to the same set of theoretical EI values in Fig. 13(b). Eq. (E) is not unreasonable for lightly reinforced columns, but greatly underestimates the effect of the reinforcement in heavily reinforced columns. However, in many cases where reinforcement percentages are low or slenderness effects not very substantial, its relative simplicity may be desirable (see Appendix B).

The creep due to sustained loads tends to reduce the effective value of EI. This is taken into account by dividing the EI term by $(1 + R_m)$ where R_m is the ratio of dead load moment to total load moment. This factor has been chosen to give the correct trend when compared to analyses[6] and tests of columns under sustained loads.[7,26-28]

The equivalent uniform moment term, C_m—The column is designed for the maximum end moment M_2 multiplied by the moment magnifier F and by an equivalent moment correction factor C_m. The derivation of the moment-magnifier term F assumes that the maximum moment is at or near the midheight of the column. If the maximum applied moment occurs at one end of the column, the maximum moment along the length of the column cannot be found by adding the maximum end moment and the maximum deflection moment, since the two occur at different sections in the column. Massonnet,[29] Horne,[30] and others have shown that the design of such columns can be based on an equivalent symmetrical single curvature bending moment diagram which would give rise to the same maximum moment as occurs under the actual loading. In the equivalent moment diagram each end of the column is subjected to a moment equal to C_mP times the maximum end eccentricity e_2. However, the maximum moment from such an analysis cannot be less than the maximum end moment Pe_2.

In the original derivations both in-plane and lateral-torsional buckling were considered. The resulting value of C_m is included in the AISC Specifications by means of Eq. (10):

$$C_m = 0.6 + 0.4\, e_1/e_2 \tag{10}$$

but not less than 0.4.

Austin[31] and Galambos[32] have both shown that, if lateral-torsional buckling is not a problem, the lower limit of 0.4 is not necessary. Although lateral-torsional buckling is not a problem with reinforced concrete columns, this limit was continued in Eq. (F) of Section 10.11.5.1, because of the uncertainty of frame action when values of e_1/e_2 are between -0.5 and -1.0.

Limits of applicability of the approximate procedure—A great many columns are sufficiently stocky or sufficiently well-restrained that they can essentially develop the full cross section strength (assumed here as anything up to a 5 percent strength loss). The designer's job is considerably simplified if these columns are excluded from the range of slenderness ratios to be considered in design. In this proposal this has been done by the relationship given in Section 10.11.4. For the symmetrical single curvature case, and for the sidesway case, no long column reduction is required if kh/r is less than 22. This corresponds to h/t values of about 7.5 for hinged columns, 9 for restrained columns in braced frames, and 4.5 for restrained columns in unbraced frames. For a column in double curvature with one end moment equal to minus one-half the other, the column is assumed to be short if kh/r is less than 40. This corresponds to h/t values of about 13 and 16, respectively, for hinged and restrained columns in braced frames. As shown in Fig. 14, the survey of the normal range of column variables suggests that less than 10 percent of the columns in braced frames and less than 60 percent of the columns in sway frames qualify as slender columns according to Section 10.11.4.

The relationship for minimum kh/r in Section 10.11.4 was derived by considering a tied column cross section with $p_tm = 0.1$ and $d/t = 0.8$, and was checked for a wide range of other p_tm ratios. This section was selected because it is probably more strongly affected by slenderness than any other tied column section commonly used. The proposed design Eq. (A), (B), (C), (D), and (E) were used to compute the slenderness ratios corresponding to a slender column strength equal to 95 percent of the cross-sectional strength. These slenderness ratios are compared in Fig. 15 to the expression in Section 10.11.4.

An upper limit is imposed on the slenderness ratio of columns designed by the approximate moment-magnification method presented in Section 10.11. No similar limit is imposed on the slenderness ratio if design is carried out according to Section 10.10.1. The limit on slenderness ratio of kh/r of 100 corresponds to h/t equal to 30 for hinged columns and h/t of about 40 for restrained columns braced against lateral instability. The survey of actual practices indicates this limit would not restrict usual building design. The limit of $kh/r = 100$ represents the upper range of actual tests of slender columns in frames.

10.11.5.2—In frames not braced against sidesway, the value of F shall be computed for the entire story assuming all columns to be loaded. In Eq. (B), P_u and P_c shall be taken as the summation of P_u and P_c for all of the columns in the story. In designing each column within the story, F shall be taken as the larger value computed for the entire story or computed for the individual column assuming its ends to be placed against sidesway.

10.11.5.3—When compression members are subject to bending about both principal axes, the moment about each axis shall be amplified by F, computed from the corresponding conditions of restraint about that axis.

When a story of a structure fails in a lateral instability mode, one floor translates relative to another as a unit. Thus the deflections and hence the moment magnification must be related for all

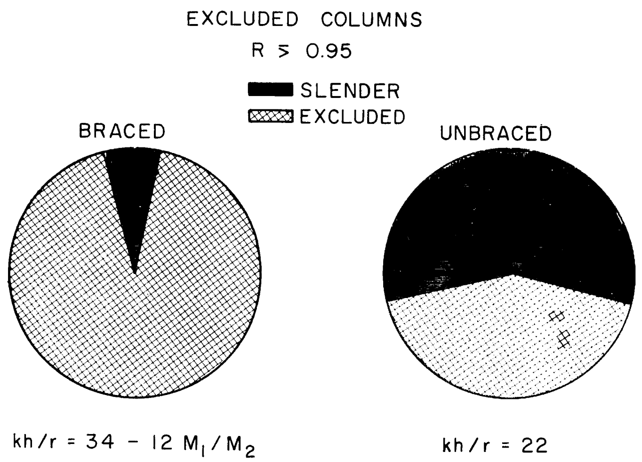


Fig. 14—Ratio of excluded columns in actual structures

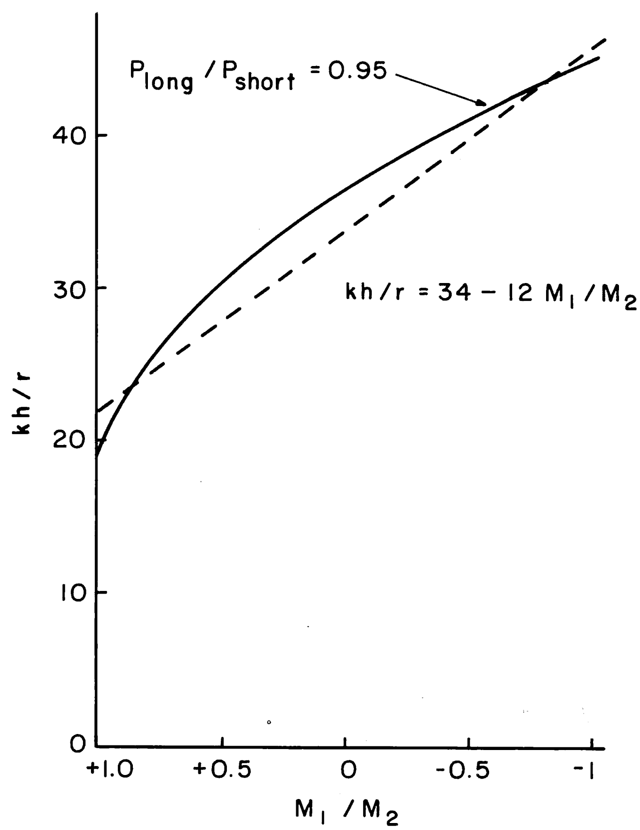


Fig. 15—Slenderness ratios corresponding to 5 percent strength reduction

compression members in the story. This section provides a procedure for computing an effective moment magnifier for the entire story. This approximation to the failure load of the story is based on the observation that the sidesway instability load for a single story frame is affected more by the total load and total translational stiffness of the columns than it is by the distribution of the column stiffnesses.[13,33]

If a very slender compression member can be heavily loaded while being braced against lateral instability by the other members in the story, it should also be checked individually to see if it could fail as a braced column.

Section 10.11.6

10.11.6—When the actual computed eccentricities are less than the minimum eccentricity specified in 10.3.7, M_2 in Eq. (A) shall be based on the minimum eccentricity, and the value of C_m shall be taken as 1.0.

These sections are a restatement of Section 916(c) of ACI 318-63, rewritten in the context of this design method.

Section 10.11.7

10.11.7—In structures which are not braced against sidesway, the flexural members shall be designed for the total magnified end moments of the compression members at the joint.

The strength of a laterally unbraced frame is governed by the stability of the columns and by the degree of end restraint provided by the beams in the frame. If plastic hinges form in the restraining beams, the structure approaches a mechanism and its axial load capacity is drastically reduced. In the absence of a comprehensive structural analysis of the type suggested in Section 10.10.1 the designer is required to make certain that the restraining flexural members have the capacity to resist the amplified column moments and hence can restrain the columns in the manner assumed in the derivation of the slender column design equations. The ability of the moment-magnification method to provide a good approximation of the actual magnified moments at the column ends in a sway frame is a significant improvement over the present reduction factor method. The Committee feels that this provision is essential for unbraced frames. This is one of the major arguments for the adoption of the moment-magnifier approach to the design of slender columns.

COMPARISON OF APPROXIMATE DESIGN METHOD WITH TESTS AND ANALYSES

Two design procedures are allowed in the proposed Code revisions. Wherever possible the designer is urged to use a rational structural analysis, as provided for in Section 10.10.1. It is assumed that if the assumptions made in this analysis are realistic, the designer will have realistic loads and moments for the design of the columns and beams in the structure. It would seem proper that the engineers who might develop or use such a complex analytical method would be required to demonstrate that it could treat available test results as well as the more approximate provisions of Section 10.11.

On the other hand, the approximate design procedure described in Section 10.11 is based on a number of approximations and simplifications. For this reason it is desirable to check it against more accurate analyses and against test results.

Comparison with analytical results

Fig. 16 and 17 compare interaction curves for slender columns computed using the proposed moment magnifier method to the more accurate slender column interaction curves previously presented in Fig. 4 and 8. The theoretical interaction curves were derived by Pfrang and Siess[5] and MacGregor[8] for columns with $p_t m$ of 0.70. (The figures are based on $f_y = 45{,}000$, $f_c' = 3000$, $g = 0.8$, and 4 percent reinforcement.)

The figures can be separated into two basic groups. Fig. 16(a) and 17 illustrate the accuracy of the proposed revision when the more comprehensive stiffness value of Eq. (D) is used. Fig. 16b shows one of the same cases when the more approximate value from Eq. (E) is used.

Detailed examination of Fig. 16a for hinged columns indicates that the proposed design method, using $EI = E_c I_g/5 + E_s I_s$, is extremely accurate for this column section through $h/t = 20$ ($kh/r = 67$) becoming more conservative when $h/t = 30$ ($kh/r = 100$). Conservatism at this extreme slenderness is not unwarranted. Fig. 17 indicates even better accuracy for restrained columns in both the single and the reversed curvature cases through h/t of 30 ($kh/r = 78$). (The degree of restraint indicated corresponds to $r' = 1$.) For the restrained column cases shown, there are only a few points where the variation amounts to as much as 10 percent. These are usually in the tensile failure range at impractically large eccentricities when viewed in the light of the beam yielding limits shown in Fig. 1.

In contrast to the good agreement reported in the previous paragraph, Fig. 16(b) indicates that the proposed design method using the cruder stiffness approximation, $EI = E_c I_g/2.5$, is very inadequate (extremely conservative) for hinged columns with h/t of 20 or greater and moderate or high reinforcement percentages. Similar curves for 1 to 2 percent reinforcement indicate much better agreement. This approximation is more usable for restrained columns, but only for slenderness ratios up to kh/r of about 50 for single curvature and about 70 for reversed curvature. However inaccurate this method might seem, it is still a very useful approximation for lightly reinforced columns in the moderate slenderness ranges and may be useful as a design simplification, although designers should be aware of the extreme conservatism of this expression with higher reinforcement ratios.

An important facet of the comparisons with analytical results is a comparison of the proposed design approximation with the ACI 318-63 reduction factor method as well as the more accurate theoretical interaction curve results. A very extensive series of comparisons of the ACI 318-63 method with theoretical interaction curves was given in Reference 2. The Code procedures were compared with analytical bands of curves considering extreme practical values of end eccentricity ratios, end restraint ratios r' varying from 1.0 to 10.0, and both short-term and sustained loading effects. Generally speaking, the ACI reduction factor method results were found to be a conservative lower bound of all possible design cases represented by the broad analytical band. In the present study a sample case is presented in the form of reduction factors, R, to illustrate the basic differences. Fig. 18 presents the results of a comparison of two series of slender columns in which the primary variables are the ratios p/f_c' and g. As pointed out previously, an increase in the ratio of p/f_c' tends to increase the strength and stability of a column. This ratio is actually more sensitive than $p_t m = pf_y/0.85f_c'$, since it is a more effective measure of the bending stiffness contributed by the reinforcement. Because the moment of inertia of the reinforcement about the centroidal axis is proportional to the square of g, the bending stiffness is also quite affected by the differences in the concrete cover index shown. Parme[34] has shown that comparisons with the ACI 318-63 reduction factors are greatly affected by variations in g. Since the

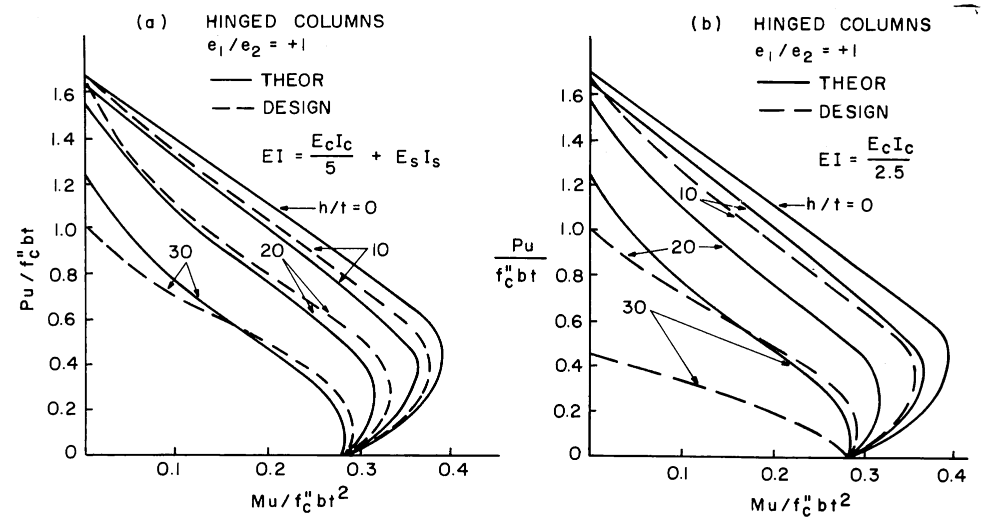


Fig. 16—Comparison of theoretical and design interaction curves

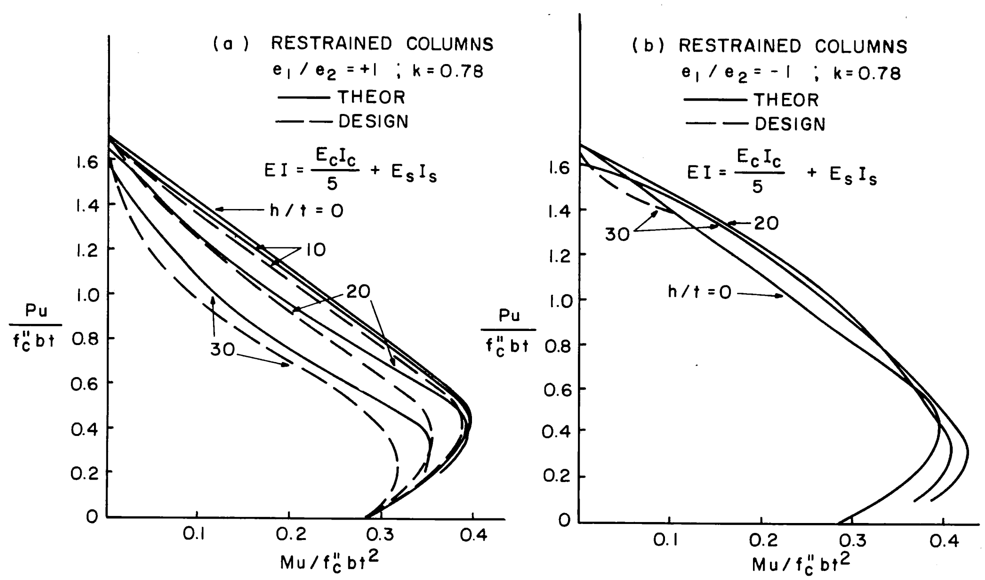


Fig. 17—Comparison of theoretical and design interaction curves

bending stiffness is an extremely important variable in slender column behavior, it is not surprising that the computer-based theoretical curves indicate a substantial difference in reduction factors for these two sets of columns. The proposed approximate design method considers the reinforcement index and cover ratio and also indicates the same type of behavioral difference. The present reduction factors given in ACI 318-63 are conservative lower bounds which tend to ignore the effects of frame restraint and penalize the stiffer columns more than the others. ACI 318-63 Eq. (9-3) includes an allowance for sustained load effects and the theoretical and design results shown are both for "short-time" loading ($R_m = 0$). However, if they are compared to the 1963 ACI Code "short-time" loading expression given by Eq. (9-5), there is still the same basic variation.

This comparison indicates the main type of economic benefit possible from the proposed design method. Slender columns would be designed more realistically by considering many of the variables neglected in the present design procedure. Since the design procedures continued in ACI 318-63 are largely based on very conservative

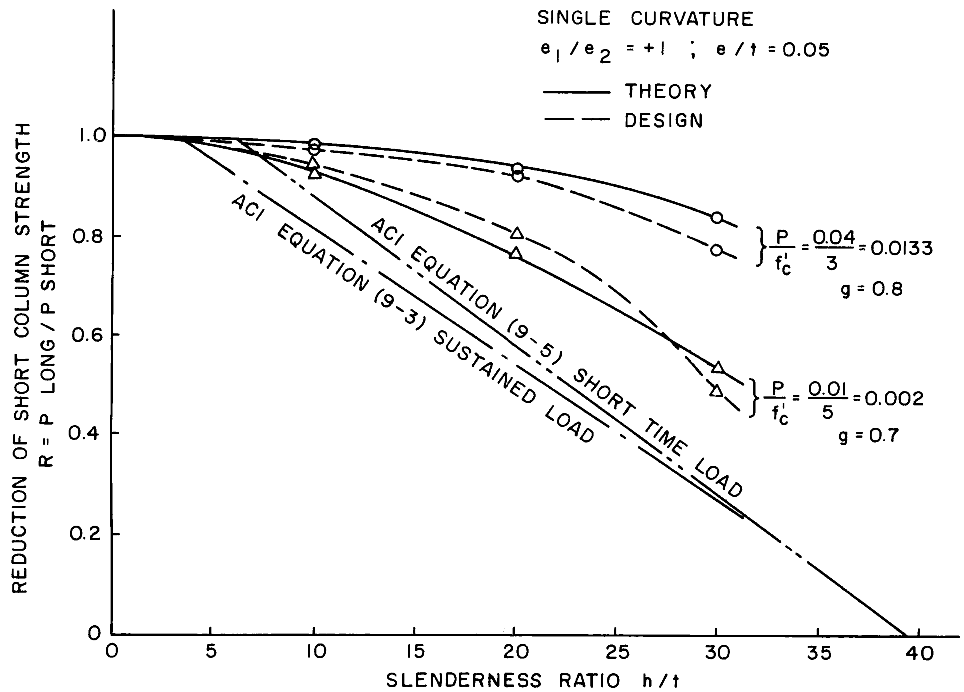


Fig. 18—Comparison of Section 10.11 with analyses and ACI 318-63 method

TABLE 1—COMPARISON OF TEST DATA AND SECTION 10.11

Type of column	Reference	Number	e_1/e_2	kh/r	P_{test}/P_{comp} Maximum	Minimum	Average
Reinforced concrete (Short-time loads)							
Hinged tied columns	28, 39, 40	11	+1.0	18 to 102	2.00	0.97	1.34
	35	4	−1.0	91	1.23	0.95	1.10
	36	6	−0.5	133	2.07	1.23	1.67
Hinged channel columns	38	4	+1.0	102	1.80	1.28	1.53
Cantilever columns	*	10	—	33 to 200	1.60	1.03	1.19
Restrained columns Sway prevented	7	6	+1.0	46 to 57	1.25	0.92	1.08
	26	5	−0.33	42 or 77	1.59	1.06	1.37
	35	4	−1.0	69	1.11	0.86	0.95
Sway frames	27	6	—	54 to 106	1.38	0.86	1.05
Reinforced concrete (Sustained loads)							
Hinged columns	28	2	+1.0	62	(1.40)	(1.18)	
Restrained columns	7, 26	2	+1.0, −0.33	46 or 77	(1.47)	(1.16)	
Sway frames	27	1	—	89		(1.11)	
Lightweight concrete (Short-time loads)	39	4	+1.0	102	1.43	1.22	1.34
Prestressed concrete (Short-time loads)	37	36	+1.0	67 to 133	1.53	0.87	1.14
Over-all for columns with $kh/r < 100$ and short-time loads		60			1.71	0.86	1.13

Coefficient of variation = 16.9 percent

*Breen, J. E., and Ferguson, Phil M., "The Tall Bridge Pier Subject to Longitudinal Bridge Forces and Tilting of the Base," unpublished report, The University of Texas at Austin, 1966, 84 pp.

lower bound values, a more rational design procedure will result in decreases in the penalties imposed on the columns with greater flexural rigidities, as indicated in Fig. 18.

Comparison with test results

Numerous tests of slender columns with varied eccentricities and restraint conditions have been reported in recent years. The measured ultimate capacities of 65 hinged and restrained reinforced concrete columns and 36 hinged prestressed concrete columns are compared in Table 1 to the corresponding ultimate loads computed using Section 10.11. The columns used in this comparison are limited to eccentrically loaded columns since ACI 318-63 requires consideration of a minimum eccentricity. Eq. (D) was used to compute EI. For the 60 columns having kh/r values within the range for which the approximate method can be used ($kh/r \leqq 100$), the ratio of measured to computed ultimate loads ranged from 0.86 to 1.71 with a mean of 1.13 and a coefficient of variation of 16.9 percent. Columns tested under sustained loads have not been considered in computing the mean and coefficient of variation, since none of the tests considered involved more than 3 months of sustained load. The measured ultimate loads for those columns which failed during this period were probably greater than the loads they could have carried for the lifetime of a structure. Similarly, the ultimate capacity of test columns rapidly loaded to failure after a short period of sustained load probably exceeds the capacity of these columns under a sustained load of very long duration. For the entire group of 96 columns tested under short-time loads, the mean ratio was 1.22 and the coefficient of variation was 21.7 percent.

Fig. 19 is a histogram comparing the measured and computed loads for all 101 columns listed in Table 1. The shaded region includes all 60 columns with kh/r less than or equal to 100 and subjected to short-time loads. Since the design method was constructed to be conservative, the mean should be somewhat over 1.00 and the results skewed towards values greater than 1.00. This has been attained with only 3 percent of the test values less than 0.90. One of the two cases which are less than 0.90 has such a large e/t ratio (0.86) coupled with an extreme slenderness ratio ($h/t = 25$) that it is deep within the shaded restraint yield zone shown in Fig. 1 and thus is not a realistic case. As shown by the histogram the approximate design method is more conservative but less accurate for columns with kh/r greater than 100.

To place the coefficient of variation in proper perspective it is interesting to compare it to that reported by Broms and Viest[4] when they applied their more complex and more accurate long column analysis to tests of 61 slender hinged columns bent in symmetrical single curvature. For these tests they report the ratio of measured to computed ultimate loads ranged from 0.89 to 1.39 with a mean of 1.09 and a coefficient of variation of 11.6 percent. When it is observed that the data reported in Table 1 include hinged and restrained columns, prestressed and reinforced concrete columns, and both braced and unbraced columns all subjected to a wide range of eccentricity ratios, the coefficients of variation of 16.9 and 21.7 percent seem reasonable.

It should be noted that the physical tests reported cover a very limited range of reinforcement percentages and material properties. While considerably more test information is available than when ACI 318-63 was prepared, the main reliance still must be on comparison with theoretical analyses which check the test results where available.

CONCLUSIONS

This paper presents and documents a proposal for revising the slender column design procedures of ACI 318-63. It proposes use of a rational second-order structural analysis wherever possible or practical. In place of such an analysis it proposes an approximate design method based on a moment magnifier principle and similar to the procedure used under the AISC Specifications. It presents an outline of the normal range of variables in column design and proposes a lower limit of applicability which will eliminate over 90 percent of columns in braced frames and almost half of columns in unbraced frames from consideration as slender columns. Through a series of comparisons with analytical and test results, the accuracy of the approximate design procedure is established. It is shown that the proposed procedure is more rational, more accurate, and more con-

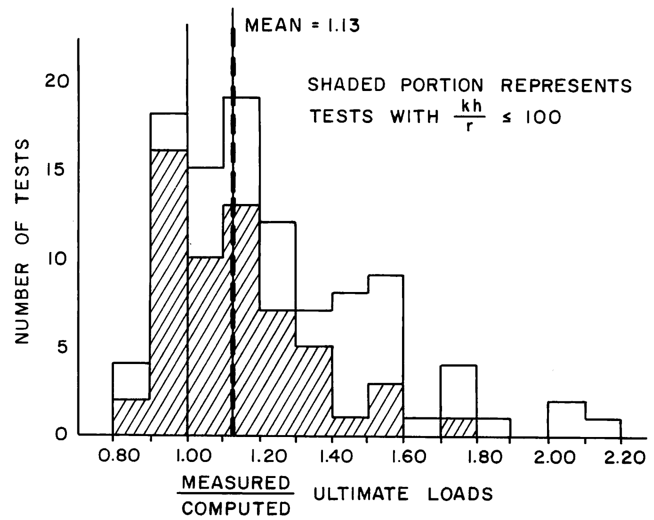


Fig. 19—Comparison of Section 10.11 with tests

sistent than the presently used procedure. Because the proposed method calls the attention of the designer to the basic phenomenon in slender columns and allows him to evaluate the additional moment requirements in restraining members, a superior and safer design results.

ACKNOWLEDGMENTS

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APPENDIX

APPENDIX A—PROPOSED BUILDING CODE TEXT (SLENDERNESS EFFECTS)

10.10—Slenderness effects in compression members

10.10.1—The design of compression members shall be based on forces and moments determined from an analysis of the structure. Such an analysis shall take into account the influence of axial loads and variable moment of inertia on member stiffness and fixed end moments, the effect of deflections on the moments and forces, and the effects of the duration of the loads.

10.10.2—In lieu of the procedure described in Section 10.10.1, the design of compression members may be based on the approximate procedure presented in Section 10.11. The detailed requirements of Section 10.11 do not need to be applied if design is carried out according to Section 10.10.1.

10.11—Approximate evaluation of slenderness effects

10.11.1—The unsupported length h of a compression member shall be taken as the clear distance, between floor slabs, girders, or other members capable of providing lateral support for the compression member. Where capitals or haunches are present, the unsupported length shall be measured to the lower extremity of the capital or haunch in the plane considered.

10.11.2—The radius of gyration r may be taken equal to 0.30 times the overall depth in the direction in which stability is being considered for rectangular compression members, and 0.25 times the diameter for circular compression members. For other shapes, r may be computed for the gross concrete section.

10.11.3—For compression members braced against sidesway, the effective length factor k shall be taken as 1.0, unless an analysis shows that a lower value may be used. For compression members not braced against sideway, the effective length factor k shall be determined by a rational method with due consideration of cracking and reinforcement on relative stiffness, and shall not be less than 1.0.

10.11.4—For compression members braced against sidesway, the effects of slenderness may be neglected when kh/r is less than $34\text{-}12M_1/M_2$. For compression members not braced against sidesway, the effects of slenderness may be neglected when kh/r is less than 22. For all compression members with kh/r greater than 100, a rational analysis as defined in Section 10.10.1 shall be made.

10.11.5—Compression members shall be designed using the design axial load from a conventional frame analysis and a magnified moment M, defined by Eq. (10-5):

$$M = FM_2 \tag{10-5}$$

where

$$F = \frac{C_m}{1 - P_u/P_c} \geq 1.0 \tag{10-6}$$

and

$$P_c = \frac{\pi^2 EI}{(kh)^2} \tag{10-7}$$

In lieu of a more precise analysis EI may be taken either as:

$$EI = \frac{E_c I_g/5 + E_s I_s}{1 + R_m} \tag{10-8}$$

or

$$EI = \frac{E_c I_g/2.5}{1 + R_m} \tag{10-9}$$

In Eq. (10-6), for members braced against sidesway and without transverse loads between supports C_m may be taken as:

$$C_m = 0.6 + 0.4\,(M_1/M_2) \tag{10-10}$$

but not less than 0.4.

For all other cases C_m shall be taken as 1.0.

10.11.5.1—For columns designed in accordance with 8.1.2.2, the term P_u in Eq. (10-6) shall be replaced by 2.5 times the design axial load.

10.11.5.2—In frames not braced against sidesway, the value of F shall be computed for the entire story assuming all columns to be loaded. In Eq. (10-6), P_u and P_c shall be taken as the summation of P_u and P_c for all of the columns in the story. In designing each column within the story, F shall be taken as the larger value computed for the entire story or computed for the individual column assuming its ends to be braced against sidesway.

10.11.5.3—When compression members are subject to bending about both principal axes, the moment about each axis shall be amplified by F, computed from the corresponding conditions of restraint about that axis.

10.11.6—When the actual computed eccentricities are less than the minimum eccentricity specified in 10.3.7, M_2 in Eq. (10-5) shall be based on the minimum eccentricity, and the value of C_m shall be taken as 1.0.

10.11.7—In structures which are not braced against sidesway, the flexural members shall be designed for the total magnified end moments of the compression members at the joint.

APPENDIX B—DESIGN EXAMPLE

To illustrate the use of the proposed Code provisions for slender columns, Problem 4 on Page 16 of *Ultimate Strength Design of Reinforced Concrete Columns,* ACI Special Publication 7, is recalculated here. The problem involves the design of a 12-ft high exterior column part way up a multistory building. The columns above and below are identical to the one to be designed. The floor system at the upper end of the column has a stiffness $K = I/L = 190$ in.^{3} and the one at the bottom has $K = 170$ in.^{3} The structure is not braced against sidesway. The column chosen is to be a circular spiral column with $f_c' = 3$ ksi and $f_y = 60$ ksi. The factored loads are $P_u = 672$ kips and $M_u = 269$ ft-kips, corresponding to combined dead, live, and wind loading. The solution presented here can be compared with the one presented in SP-7.

Solution

Assume a 24 in. round column

$e/D = M_u/(P_u D) = 269 \times 12/672 \times 24 = 0.20$

$r = (0.25)(24) = 6$ in.

$I_g = \pi D^4/64 = 16{,}290$ in.^{4}

$K_{col} = I/h = 16{,}290/144 = 113$ in.^{3}

At Joint B, $r_b' = K_{col}/K_b = 2(113)/170 = 1.33$

At Joint C, $r_c' = 2(113)/190 = 1.19$

From the nomograph in Fig. 11b, $k = 1.40$

$kh = (1.40)(144) = 202$ in.

Thus,

$$kh/r = (1.40[144])/6 = 33.7$$

Since kh/r exceeds 22, this is a slender column according to Section 10.11.4

As a first estimate, base EI on Eq. (E):

$$E = \frac{E_c I_g/2.5}{1 + R_m} = \frac{(3.2 \times 10^3)(16290)/2.5}{1 + 0}$$

$$= 20.8 \times 10^6 \text{ kip-in.}^2$$

where $R_m = 0$ because moments are due to wind loads and therefore are not sustained load moments.

$$P_c = \frac{\pi^2 (20.8 \times 10^6)}{(202)^2} = 5050 \text{ kips}$$

$$F = \frac{C_m}{1 - P_u/P_c} 1.0 = \frac{1.0}{1 - \frac{672}{5050}} = 1.16$$

where

$C_m = 1.0$ because the frame is unbraced

$M = (1.16)(269) = 311$ ft.-kips

$e/d = (311)(12)/(672)(24) = 0.231$

Chart 31 of SP-7 indicates that $p_t = 0.023$ is required.

Note: Beam moments must be increased by 16 percent of 269 ft-kips = 45 ft-kips to account for increased moment from this column and by a similar amount due to the magnified moments in the columns above or below.

This completes the design. If desired a check can be made using the value of EI given in Eq. (D). If this is done, EI increases to 24.2×10^6 kips-in.^{2}, F drops to 1.13, and p_t drops to 0.021.

APPENDIX C—NOTATION

b = width of rectangular section in direction at right angles to direction of bending
C_m = equivalent column correction factor
e = column end eccentricity; wherever end eccentricities ties are not equal, e_2 is taken as the larger numerical quantity and is always positive; e_1 is the numerically smaller eccentricity and can be positive or negative
E = modulus of elasticity
E_c = modulus of elasticity of concrete
E_s = modulus of elasticity of steel
f_c' = compressive strength of concrete determined from 28 day cylinder tests
f_c'' = compressive strength of concrete in columns
f_y = yield strength of reinforcing steel
F = moment magnifier coefficient
g = ratio of the distance between the reinforcement to the column thickness
h = unsupported length of a column
I = moment of inertia about the axis of bending
I_c = moment of inertia of concrete
I_g = column moment of inertia
I_s = moment of inertia of reinforcement
I_t = transformed moment of inertia
k = effective length factor
M = moment
M_c = column end moment
M_d = deflection moment
M_{ext} = external moment
M_{max} = maximum column moment
M_o = moment from a first order structural analysis
M_r = restraint moment
M_u = ultimate moment
M_1, M_2 = column end moments; whenever the column end moments are not equal, M_2 is taken as the larger numerical quantity and is always positive; M_1 is the numerically smaller of the end moments and may be positive or negative
p = ratio of reinforcement
P = column load
P_{long} = long column capacity
P_{short} = short column capacity
P_u = ultimate column capacity
r = radius of gyration of a column
r' = ratio of ΣK of columns to ΣK of floor members in a plane at one end of a column
R_m = ratio of dead load moment to total load moment
t = thickness of a column
Δ = column deflection
ϕ = curvature

APPENDIX D—SI (METRIC) CONVERSION FACTORS

multiply	by	to obtain
psi	0.0703	kgf/cm^2
ksi	70.3	kgf/cm^2
in.	2.54	cm
kip	453.6	kg
ft-kip	138.3	kgf-m
in.^{3}	16.387	cm^3
in.^{4}	41.623	cm^4
kip-in.^{2}	2925.8	kgf-cm^2

Based on a paper presented at ACI's 65th Annual Convention, Chicago, Ill., Apr. 4, 1969.