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TESTS OF A TWO-WAY REINFORCED CONCRETE FLOOR SLAB

By William L. Gamble,<sup>1</sup> A. M. ASCE, Mete A. Sozen,<sup>2</sup> M. ASCE,  
and Chester P. Siess,<sup>3</sup> F. ASCE

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INTRODUCTION

A quarter-scale two-way reinforced concrete floor slab was constructed and subjected to 40 applications of various levels and patterns of uniform load as part of an investigation into the strength and behavior of floor slab construction. The two-way slab described in this report was the third in a series of five structures which were tested. The general aims of the investigation and results of two other test structures have been presented previously (3, 4, 7).<sup>4</sup> The purpose herein is to describe the strength and behavior characteristics of the two-way slab, test structure T1. Details of the construction, instrumentation, and test results are contained in Ref. 2.

DEFINITIONS

A two-way slab is a structure in which each panel is supported along all four edges by beams or walls, and a panel is an area of a structure bounded by column centerlines. The term, beam, indicates the portion of the beam extending below (or above) the slab plus the portion of the slab assumed to be acting as the T-beam flange, while slab section refers to the portion of a

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<sup>1</sup> Asst. Prof. of Civ. Engrg., Univ. of Illinois, Urbana, Ill.

<sup>2</sup> Prof. of Civ. Engrg., Univ. of Illinois, Urbana, Ill.

<sup>3</sup> Prof. of Civ. Engrg., Univ. of Illinois, Urbana, Ill.

<sup>4</sup> Numerals in parentheses refer to corresponding items in the Appendix I.—References.

critical section for moment extending between the edges of the assumed T-beam flanges.

### DESCRIPTION OF TEST STRUCTURE

*Design.*—The principal considerations governing the design of the two-way slab test structure T1 have been outlined in a previous paper (7). A prototype slab having nine-panels, each 20 ft square, was designed, and then scaled by multiplying all dimensions by 1/4 to obtain the dimensions of the test structure.

The design of the prototype slab was prepared in accordance with the 1956 ACI Building Code (318-56), using Method 1 of the design of two-way slabs. The design live load was 70 psf, and the dead load was 75 psf for the prototype 6-in. slab. The spandrel beams were designed for an additional wall load of 400 lb per lin ft. The assumed prototype story height was 9 ft.

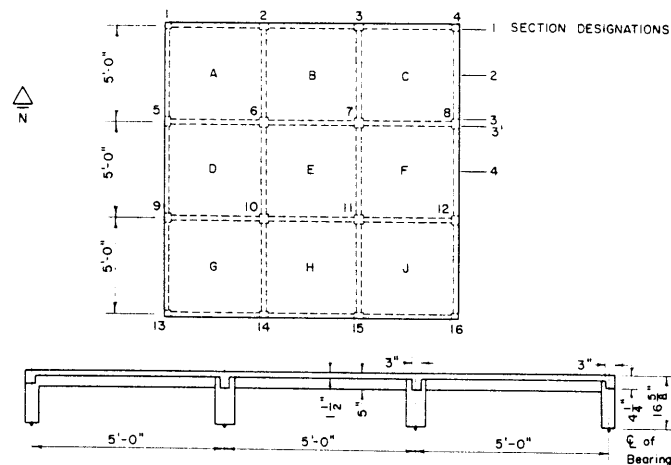


FIG. 1.—PLAN AND SECTION OF TEST STRUCTURE

The design concrete strength was 3,000 psi, the allowable concrete stress was 1,350 psi, and the allowable steel stress was 20,000 psi.

*Dimensions.*—The layout and section of the test slab are shown in Fig. 1. The length of the supporting pin-ended columns was chosen to approximate the flexural stiffness of fixed-ended columns extending a full story both above and below the floor. The column sections are shown in Fig. 4. The nominal concrete cover was 3/8-in. for the beam and 3/16-in. for the slab reinforcement.

*Reinforcement.*—The reinforcement in both the slabs and beams of the prototype structure was of 1/2-in. square bars so that 1/8-in. square bars could be used in the slab portions of the test structure on a direct substitution basis. The 1/8-in. square bars were chosen to facilitate mounting of electric strain gages. The beams of the test structure were reinforced with No. 2

plain bars and 1/8-in. square bars, and the columns with No. 3 deformed bars.

The slab positive and negative moment reinforcement is shown in Fig. 2. Figures 3 and 4 provide information on the beam and column reinforcement.

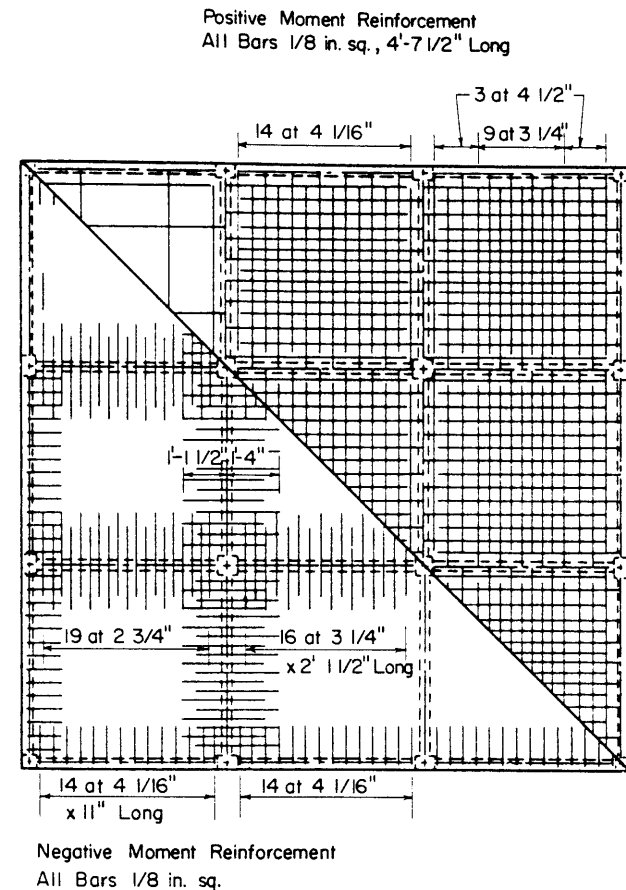


FIG. 2.—ARRANGEMENT OF SLAB REINFORCEMENT

stirrups were not required by the original design but were supplied in anticipation of torsional distress.

### MATERIALS AND CONSTRUCTION

*Reinforcing Bars.*—The reinforcement used in the slab was 1/8-in. square AISI B-1113 cold drawn steel. Since the yield stress of this steel was about 75,000 psi, it was annealed in order to obtain a lower yield stress and a "flat-top" yield range. After annealing, the average yield stress was 42,000 psi.

Strain-hardening began at a strain of about 0.035. The average ultimate stress was 59,000 psi.

The No. 2 plain bars used for the beam reinforcement had a yield stress of 50,000 psi. The yield stress of the No. 3 deformed bars used in the columns was 55,000 psi. The stirrups were of annealed No. 10 gage wire with a yield stress of 40,000 psi. All plain bars were rusted to improve bond.

*Concrete.*—The concrete used in the test structure was a small-aggregate mix having a design strength of 3,000 psi. The aggregate was composed of

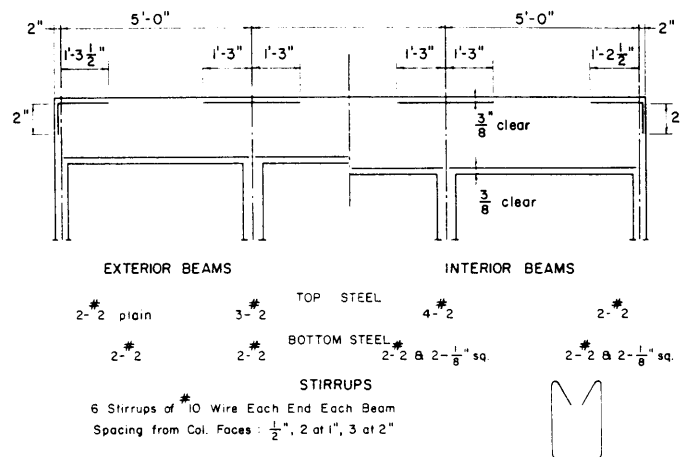


FIG. 3.—ARRANGEMENT OF BEAM REINFORCEMENT

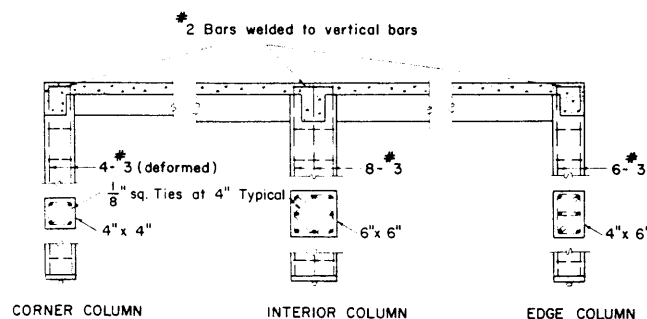


FIG. 4.—ARRANGEMENT OF COLUMN REINFORCEMENT

80% Wabash River sand and 20% fine lake sand. The fineness modulus of the blended aggregate was about 2.5 and the maximum aggregate size was 1/8-in. Type I cement was used.

The water-cement ratio was 0.75, and the aggregate-cement ratio was 5.4, both by weight. The 2 x 4-in. test cylinders were cast from each of the 11 batches of concrete required for the slab. Additional 4 x 8-in. cylinders were

cast from some batches, and 16 small modulus of rupture specimens were cast. At the beginning of the testing program (76 days after casting), the average compressive strength of the concrete was 3,420 psi with a standard deviation of 230 psi as indicated by 16 tests on 4 x 8-in. cylinders and 2,830 psi with a standard deviation of 310 psi as indicated by 34 tests on 2 x 4-in. cylinders.

The initial tangent modulus at 76 days was indicated to be  $3 \times 10^6$  psi by tests on both the smaller and larger cylinders. The average modulus of rupture at 80 days was 590 psi with a standard deviation of 58 psi.

## TEST EQUIPMENT AND INSTRUMENTATION

*Loading System.*—The reaction frame consisted of 16 concrete piers, one under each column of the test structure. The five-foot tall piers were tied together with a grid of steel beams. The 3 parallel loading frames extended across the slab. Each frame held three 20-ton hydraulic jacks, each over the center of a panel.

The load from a hydraulic jack was distributed equally to 16 loading pads in each panel by means of loading trees. Each loading pad was 8-in. sq and rested on a 3/8-in. thick grey sponge rubber sheet. Any combination of the nine panels could be loaded from one hydraulic pump.

*Load and Reaction Measurement.*—A set of dynamometers, with sensitivities of about 1.0 psf per dial division deflection, were used to measure the applied loads. The vertical and two horizontal reaction components were measured at each column with a tripod dynamometers. The sensitivity of the tripod dynamometers was about 20 lb vertical load per dial division deflection on the strain indicator.

*Strain Measurements.*—Reinforcement strains were measured at 315 locations in the test structure, using SR-4 strain gages. The gages were mounted on the reinforcement after the concrete had been cast and cured. The 26 strain gages on the concrete were on the upper surface of the slab.

*Deflection Measurements.*—The vertical deflections of the test structure were measured at each of 33 locations. Dial gages sensitive to 0.001 in. were located at the center of each panel and the center of each beam.

*Crack Detection.*—Seven-power magnifying lenses were used whenever the slab was examined for cracks. Crack widths were measured with a 50-power portable microscope.

## TEST PROGRAM

*Chronology.*—A total of 40 tests were carried out on the structure over the period of Sept. 29 to Dec. 17, 1959. The test series is summarized briefly in Table 1. At each new load level, the first test was with all panels uniformly loaded and the later tests were with various combinations of panels loaded in order to produce the maximum moments at the various sections. The partial loadings consisted of "checkerboard" loading patterns rather than "strip" loadings since the supporting beams were quite stiff.

*Test Procedure.*—In each test the load was applied in several increments, with the size and number of increments depending on the maximum load level to be reached, the previous loading history, and the expected behavior of the slab.

The deflections and strains were read and recorded before loading began in each test and after each load increment was applied to the structure. After the maximum load readings were taken, the structure was unloaded and the zero readings were taken. The structure was then reloaded to the maximum load reached, in one increment, and the maximum load and zero load strain and deflection readings repeated.

The strain indicator readings were recorded semiautomatically and punched into IBM cards and tabulated by an electric typewriter. It took 30 min to 40 min to complete the readings for one load increment and change the load for

TABLE 1.—TEST SERIES

Test Number (1)	Total load, in pounds per square foot (2)	Description (3)
300	41 total (22 applied)	Assembly of load distributing system
301-306	100	Pre-cracking
307-313	145	1 DL + 1 LL
314-334	215	1 DL + 2 LL
335-337	355	1 DL + 4 LL
338-339		Failure

the next increment except when there were large numbers of cracks to be marked.

### BEHAVIOR

The behavior of the test structure is presented in terms of deflections, stresses in the reinforcement, and cracking of the concrete. The stresses and deflections reported for any test include the effects of the dead load of the structure and residual deformations resulting from earlier tests. The tests to be described are presented in Table 2. The cumulative load-deflection curves may be explained by reference to Fig. 5. For each plot, 4 solid curves are shown, representing the load-deflection relationships obtained in 4 individual tests. The deflection corresponding to the dead load of the structure was obtained by extrapolation of the results of tests 301 and 307. The dead load deflection and load (41 psf), serve as the initial point for the deflections observed during test 307. The starting points of the following curves are the dead load and the summation of the net residual deflections between test 307 and the test considered.

The broken line connecting the upper portions of the load-deflection curves represents the curve that should have been obtained had the loading to failure been carried out in one continuous operation. Cumulative load-steel strain relationships have also been obtained, and were used in constructing Figs. 9 and 10.

If the reported quantities are to be projected up to a "full-size" structure, it should be remembered that for a given unit load intensity, the slab stresses are the same while the deflections vary as the span. The crack widths are reported only as a measure of the degree of yielding and should not be scaled.

*Test 307 (1 DL + 1 LL).*—The design load of the structure was 145 psf, and the maximum load reached in this test was 150 psf with all panels loaded. The most important observation that may be made is that the deformations of the structure at the full design load were extremely small. The maximum deflec-

TABLE 2.—TEST PRESENTATION

Test number (1)	Total load, in pounds per square foot (2)	Description (3)
307	150	Design load
314	213	1 DL + 2 LL
335	353	1 DL + 4 LL
338	537	Failure, all panels loaded
339	829	Failure of interior panel

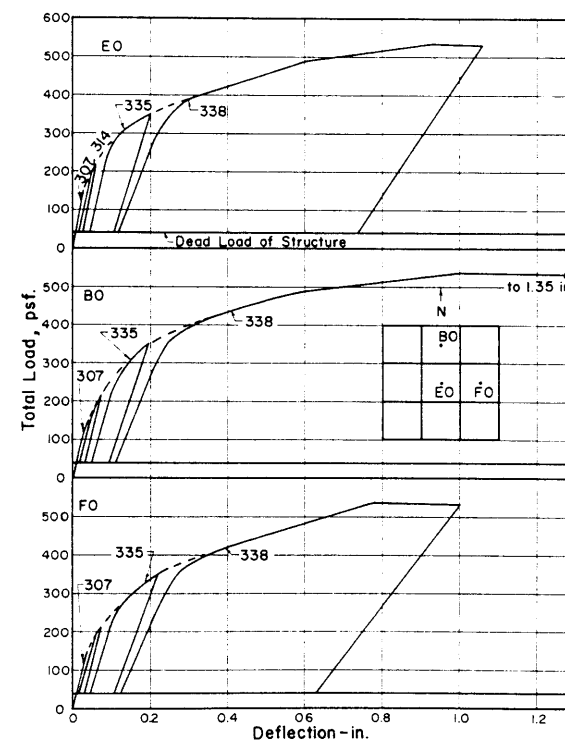


FIG. 5.—MIDPANEL LOAD-DEFLECTION CURVES

tion was 0.046 in., including the dead load deflection. This is a deflection of only  $L/1,300$ , in which  $L$  = the span, center-to-center of the columns.

The deflection of 0.046 in. was measured in a corner panel. The interior

panel deflection was 0.027 in., and the maximum beam deflections were 0.024 in. and 0.015 in. in the end spans of the interior and spandrel beams, respectively. A number of load-deflection curves are shown in Figs. 5, 6 and 7, and the deflections at the load of 150 psf are shown graphically in Fig. 8.

The maximum stresses in the reinforcement in the slab sections, obtained by multiplying the measured strain by the Young's modulus of  $30 \times 10^6$  psi, were 2,500 psi and 3,900 psi in the positive and negative moment reinforcement. The maximum stress in the beam reinforcement, 6,000 psi, occurred at the center of the end span of an interior beam. The distribution of the steel stresses at the 5 critical sections of the slab marked in Fig. 1 are depicted in Figs. 9 and 10 at the load of 150 psf.

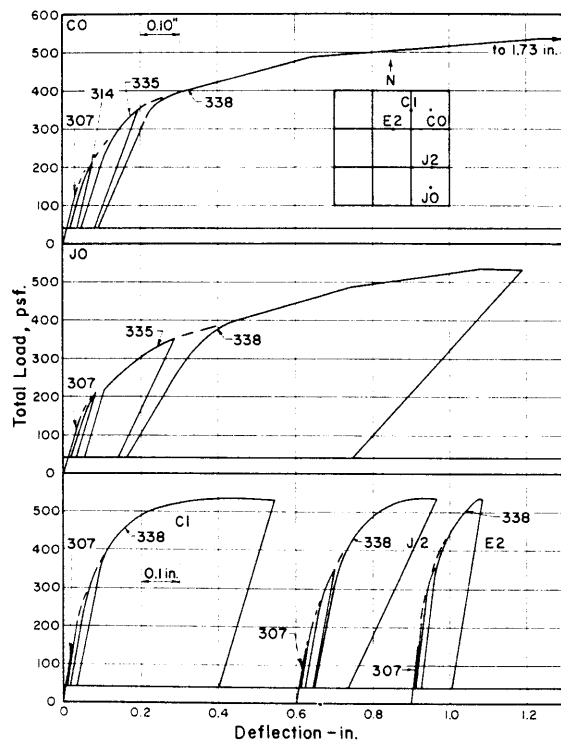


FIG. 6.—BEAM MIDSPAN LOAD-DEFLECTION CURVES

It must be emphasized that low stresses in the reinforcement do not necessarily mean low bending moments, since the contribution of the tensile stresses in the concrete is not negligible in sections where the reinforcement ratios are low. The nature of the moment-reinforcement strain relationship is presented later.

No cracks were found in the test structure at a total load of 150 psf. A few measured strains were comparable to the expected cracking strains, but there were no definite indications that cracking had occurred.

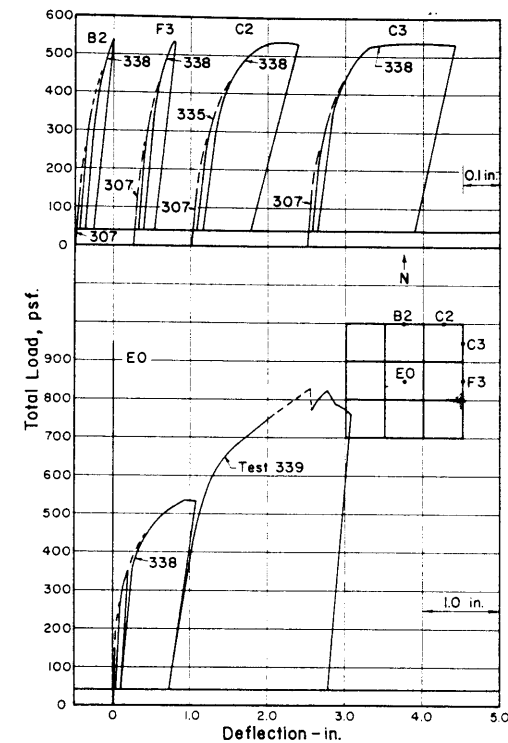


FIG. 7.—BEAM AND SLAB LOAD-DEFLECTION CURVES

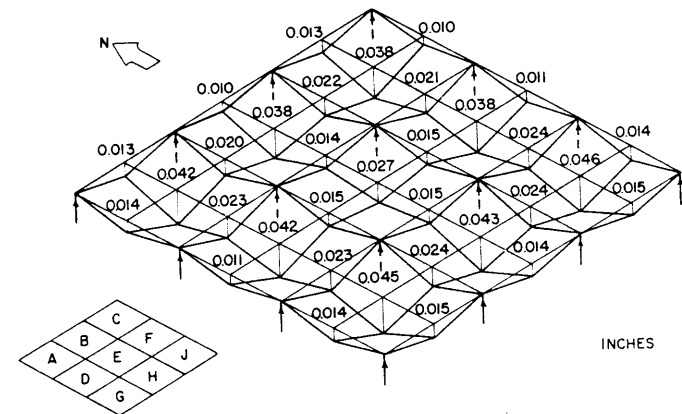


FIG. 8.—DEFLECTIONS AT DESIGN LOAD, 150 PSF

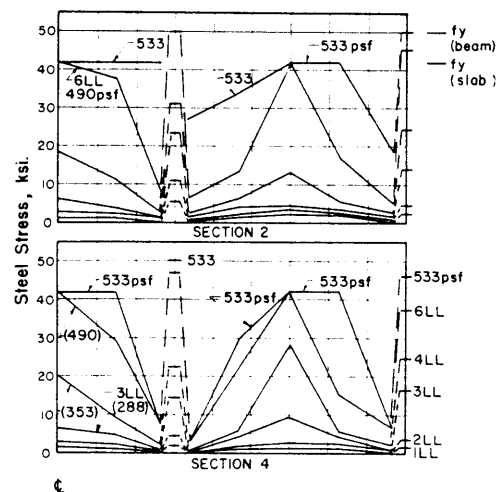


FIG. 9.—MEASURED STEEL STRESS DISTRIBUTIONS, POSITIVE MOMENT SECTIONS

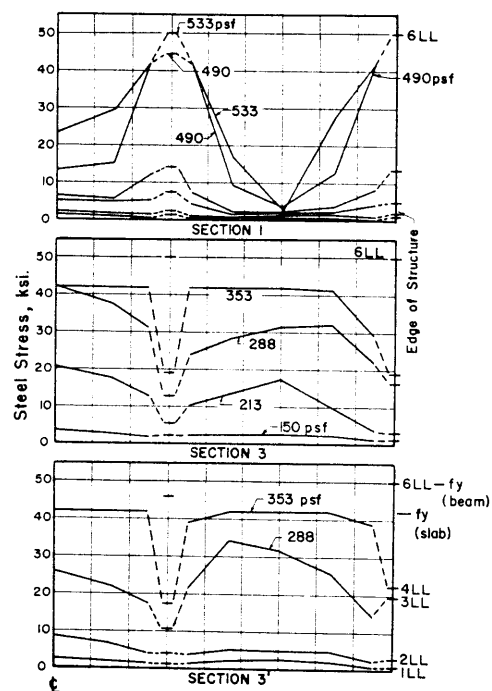


FIG. 10.—MEASURED STEEL STRESS DISTRIBUTIONS, NEGATIVE MOMENT SECTIONS

*Test 314(1 DL + 2 LL).*—At a total load of 213 psf, corresponding to the application of twice the design live load, the structure showed no signs of distress. The stresses were less than 20,000 psi in nearly all cases and the deflections were small.

The deflections measured at a number of points in the test structure are shown on the cumulative load-deflection curves of Figs. 5, 6 and 7. The curves are essentially linear up to 213 psf, with only a few of the curves indicating

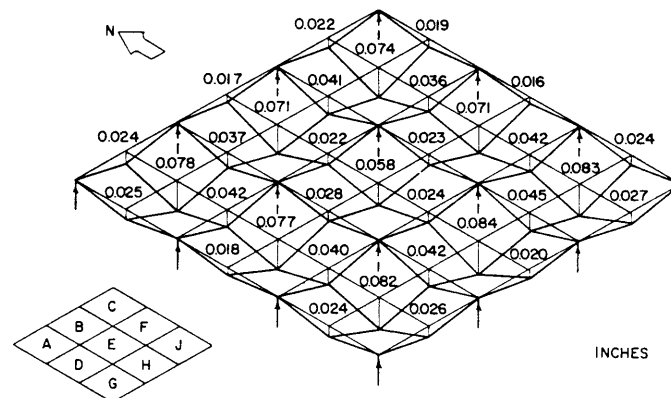


FIG. 11.—DEFLECTIONS AT  $DL + 2LL$ , 213 PSF

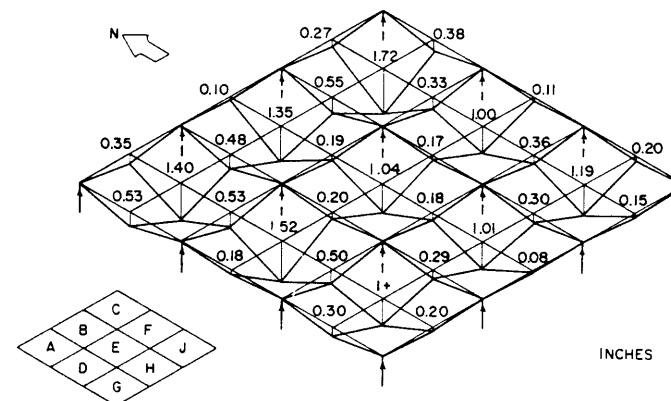


FIG. 12.—DEFLECTIONS AT 533 PSF

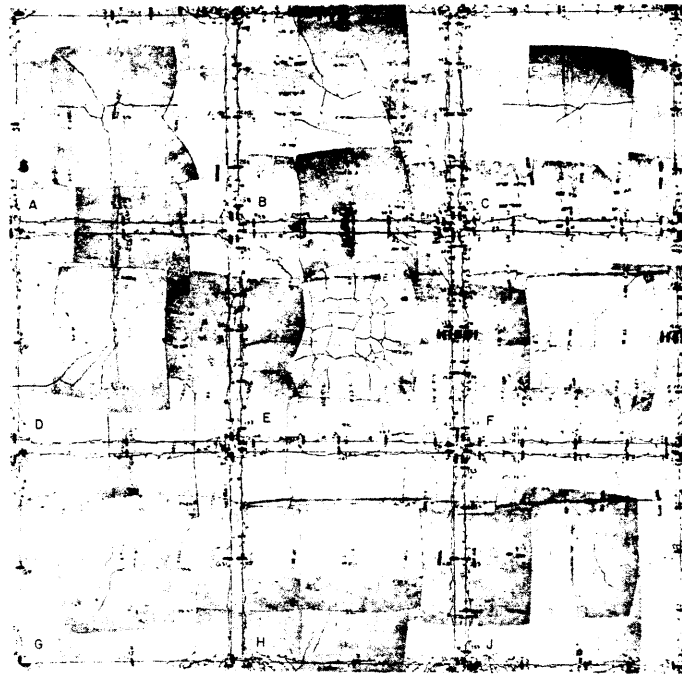
nonlinear response above 175 psf. The deflections are also shown in Fig. 11. The greatest deflection, 0.083 in., or  $L/720$ , was measured at the center of a corner panel.

The stresses in the reinforcement at the 5 critical sections are shown in Figs. 9 and 10. The maximum stress reached was 20,500 psi in the slab negative moment reinforcement. The maximum stress in the slab positive mo-

ment reinforcement was only 5,200 psi. The maximum stress in the beam reinforcement was 15,100 psi, at the center of the end span of an interior beam.

Only a limited amount of cracking occurred in the test to 213 psf. Four cracks found at the top of the structure at the interior negative moment sections of the edge panels. No cracks were observed on the lower surfaces of the slab, but cracks were found near the centers of some of the end spans of the interior beams.

*Test 335(1 DL + 4 LL).*—The maximum load reached in this test, 353 psf, corresponded to the design dead load plus four times the design live load for the structure. Though yield stresses were reached at several interior negative



TEST 339

FIG. 13.—PHOTO-MOSAIC OF TOP OF SLAB AFTER TESTS TO FAILURE

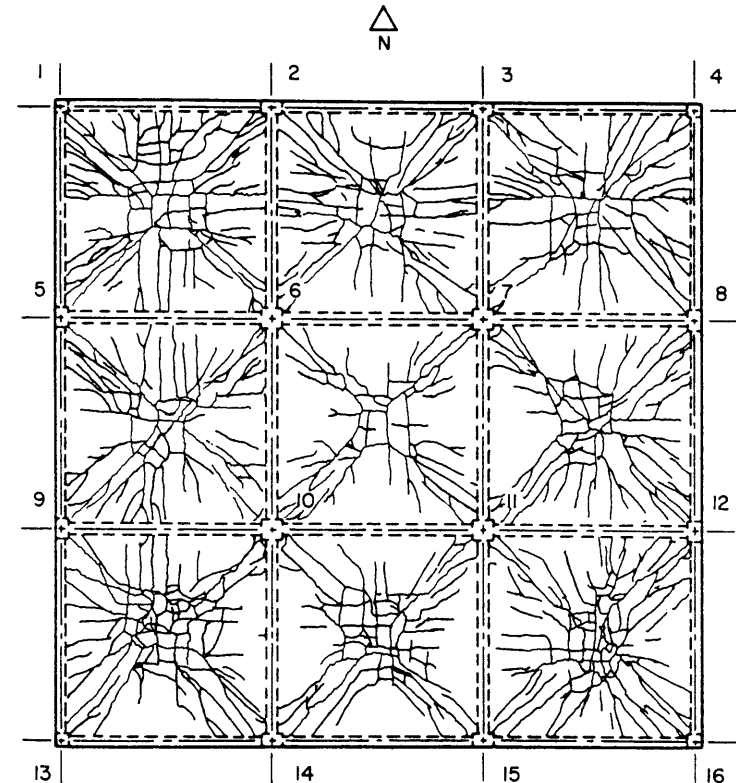
moment sections of the slab, the overall performance of the test structure was satisfactory, especially when it is considered that the test load was about 2.4 times the total design load.

The load-deflection curves remained linear until the previous maximum load level was exceeded. The deflections then increased at greater rates, as shown by the curves of Figs. 5, 6 and 7. The largest deflection, 0.286 in., or  $L/210$ , was in a corner panel. The distributions of the stresses in the reinforcement at the 5 critical sections of the structure are shown in Figs. 9 and 10 for the load levels of 288 psf and 353 psf.

Yield stresses were first reached at a load of 288 psf. Yielding began in section 3, and had spread across most of the slab interior negative moment sections at a load of 353 psf. At 353 psf, the stresses in the beam reinforcement were about 20,000 psi.

At the load of 288 psf the stresses in the positive moment slab reinforcement were generally less than 10,000 psi. At 353 psf, the maximum stress was about 28,500 psi, at the center of an edge panel (Fig. 9, Section 4).

At the exterior negative moment section, the high stresses were concen-



NOTE: Crack pattern as viewed from above slab.

FIG. 14.—CRACK PATTERN OF LOWER SURFACE OF SLAB AT 533 PSF

trated in the beam reinforcement and immediately adjacent slab reinforcement (Fig. 10, Section 1). At 353 psf, the cracking on the upper surface consisted of one crack on each side of each interior beam, and one additional crack over the top of corner column 13.

The first cracks on the lower surface of the slab were observed at 311 psf. At 353 psf, the positive moment cracks tended to form rectangular patterns near the center of each panel, and to follow the diagonals of the panels

toward the corners. None of the measured crack widths were greater than 0.005 in.

*Test 338 All Panels Loaded to Failure.*—In this test all panels of the structure were loaded to failure at a load of 537 psf, 3.7 times the total design load. The load was applied in 10 increments, with the maximum load being reached in the 9th increment. Further attempts to increase the load only deformed the structure, and the load was 533 psf when loading was stopped.

The measured deflections were quite large by the end of the test to failure. The largest deflection, 1.73 in., was measured at the center of corner panel C. The deflections of the centers of the panels were 1 in. or more in all cases.

The deflections, including dead load deflections and residuals, are shown in Fig. 12 for each of the 33 deflection points at the 10th increment of load (533 psf). The deflections were considerably greater in the north and west rows of panels than in the 4 south-east panels.

Cumulative load-deflection curves are shown in Figs. 5 to 7 for several points in the structure. Deflection curves are given at pairs of symmetrical

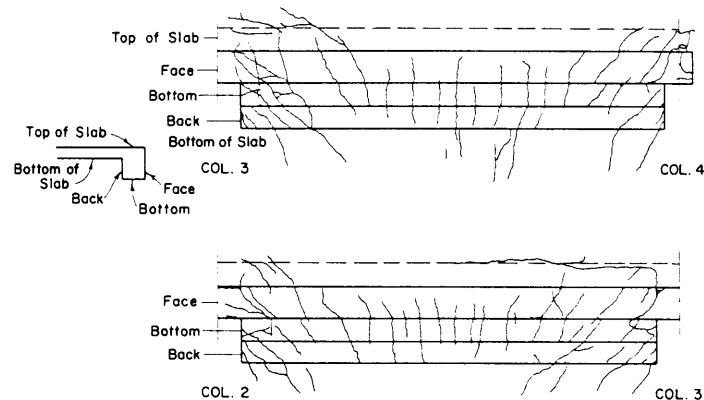


FIG. 15.—DEVELOPED VIEWS OF CRACKS IN EDGE BEAMS AT NORTH EDGE OF STRUCTURE

points, but in each case one of the points was in a failure zone and the other was not.

At the failure load, nearly all of the flexural reinforcement in the test structure had yielded, as shown in Figs. 9 and 10. With the exception of panel J, none of the exterior negative moment reinforcement at the centers of the slab sections yielded. The stresses in this reinforcement were directly dependent on the torsional stiffness and strength of the spandrel beams. That the torsional stiffnesses of the spandrel beams were low is evident in the fact that cracking occurred across the full width of the exterior negative moment section at only 4 of the 12 sections, and yielding of the reinforcement occurred at the centers of the sections only in panel J.

There were several instances of measured strain reduction at the exterior negative moment sections during the last 2 load increments. These reductions occurred in both the slab and beam reinforcement, and were caused by the failure of the spandrel beam in torsion.

Considerable additional cracking occurred during the test to failure. The additional cracking which occurred on the upper surface of the slab was concentrated near the edges of the structure. The cracks on the upper surface of the slab are shown in Fig. 13. Away from the edges of the structure the crack pattern was no different from that at 353 psf. The previously existing cracks were much wider than before, nearly 1/8-in. in some cases, and new cracking developed over the tops of the interior columns. In no case was there more than one crack parallel to each interior beam on each side of the beam.

The cracks in the centers of the edge and corner panels indicate tension cracking which had penetrated the full slab thickness when the deflections be-

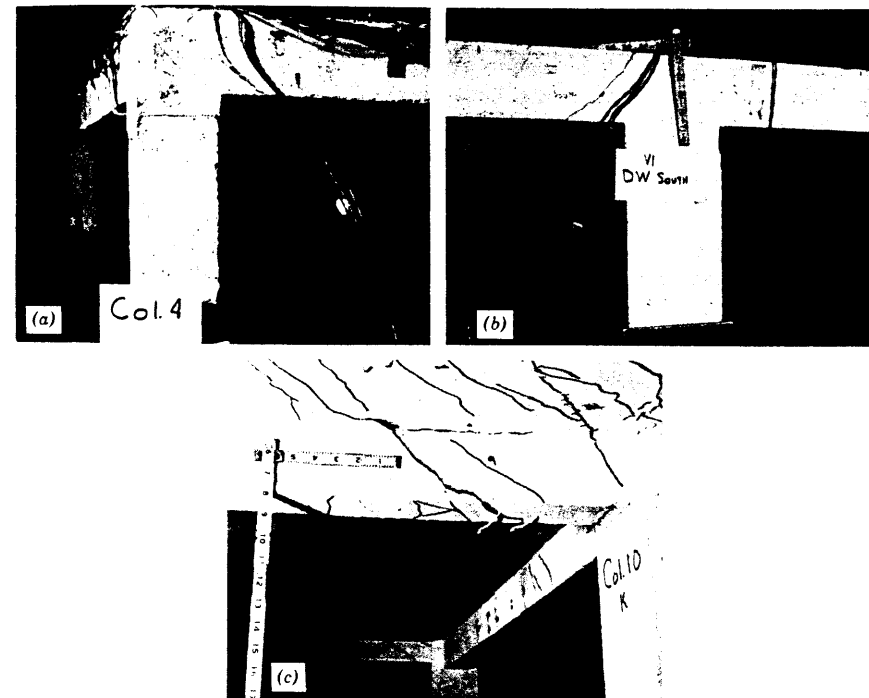


FIG. 16.—PHOTOGRAPHS OF BEAMS AFTER TESTS TO FAILURE: (a) CORNER COLUMN 4; (b) EDGE COLUMN 9; (c) INTERIOR BEAM EH

came large. The slab was apparently acting partially as a tension membrane when the deflections ranged from two-thirds to more than the slab thickness. The cracking in the center of Panel E occurred during test 339.

The cracks on the bottom of the slab at the end of the test to failure are shown in Fig. 14. Rectangular crack patterns developed first near the panel centers, with other cracks extending along the diagonals toward the corners of the panels. After this diagonal pattern was fairly well developed, the cracks extending along and parallel to the panel centerlines opened and then became the widest cracks as the failure load was approached.



Inclined cracking occurred at the ends of most of the interior beams, but this did not affect the behavior of the structure. The first inclined cracks were observed at a load of 395 psf. A large number of cracks occurred in and near the edge beams of the structure. These cracks may be roughly separated into the groupings of (a) cracks which are parallel to the beams, and (b) cracks which cross the beams. The cracks which cross the beams are, in general, negative moment cracks, but these cracks have been inclined by the influence of the torsional moments.

The cracking in the spandrel beams of the test structure was undoubtedly the most important cracking which occurred, from the point of view of the effect on the behavior of the structure. Torsional cracking of the edge beams was first observed at a load of 395 psf. On the inside faces of the spandrel beams the cracks looked like cracks caused by combined shear and negative bending moment except that the cracks reached the bottoms of the beams. In the developed views of the beams shown in Fig. 15, however, it can be seen that the cracks continue across the bottom of the beam and up the outside face of the beam, sloping toward the support and top of the beam. This slope is opposite that taken by a "shear" crack. These inclined cracks can also be traced across the tops of the spandrel beams.

As the load was increased the torsional cracks became larger, and at the final loading increment the distortion in the spandrel beams on the north and west sides of the structure was quite noticeable. Displacements across the cracks were as large as 1/8-in., and the tops of the beams were twisted toward the panel centers. The extent of the torsional cracking on the outside faces of the spandrel beams can be seen in the photographs of Fig. 16.

The torsional failures of the spandrel beams and accompanying disruption of the entire beam-column connection may be considered as a primary cause of failure at this load level. The useful negative moment capacities at the edge sections were severely reduced by these failures, and the slab was then unable to carry the load which had previously been applied. While the additional load capacity remaining had these failures not occurred must have been quite small, the ductility of the structure would have been greater if the edge beams had been stronger under the combined flexural, shearing, and torsional forces.

*Test 339 (Interior Panel Loaded to Failure).*—At the conclusion of test 338, the interior panel of the structure was still relatively intact. Therefore, this panel alone was loaded to failure in test 339. The load in panel E was applied in several increments while the corner panels of the structure were loaded to about 290 psf total load throughout the test.

A load-deflection curve for the center of panel E is plotted in Fig. 7. At 829 psf, both ends of the beam between panels E and H failed under combined shear and torsion and the load dropped to 770 psf. Under further jacking, a load of 821 psf was reached at which time the beam between panels D and E failed. The shear-torsion cracks in the beams went completely through the beam sections, and broke through to the top of the slab at the ends of the negative moment reinforcement in the beams. One of the torsional cracks in the beam between panels E and H can be seen in Fig. 16.

There was some additional cracking in the slab during this test. Several cracks near the center of the panel were more than 1/8-in. wide at the end of the test, and extended through the full thickness of the slab. A small amount of crushing occurred along one of the panel diagonals near a corner on the top of the slab.

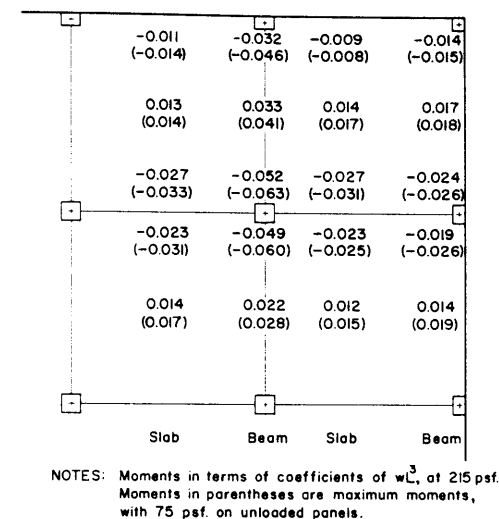


FIG. 17.—MEASURED BENDING MOMENTS IN SLAB AND BEAM SECTIONS

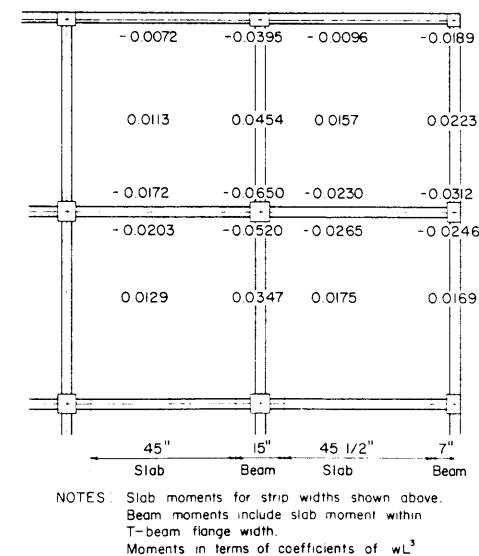


FIG. 18.—DESIGN MOMENTS FROM ACI METHOD 1

## MEASURED BENDING MOMENTS

*Relationship Between Moment and Reinforcement Strain.*—The moment-strain relationship for a reinforced concrete section can be approximated by 2 straight lines up to yield as shown in Fig. 17 of Ref. 3. The relationship is defined by points corresponding to the cracking of the concrete and yielding of the reinforcement. The moment-strain relationships used in converting the measured strains to moment were based on the results of tests of beams having cross sections similar to those in the test structure. The effective moduli of rupture used, 400 psi for the slab sections and 350 psi for the beams, were less than the average modulus of rupture of 590 psi measured in small unreinforced beams. This reduction occurred because the reinforcement present in the test structure restrained the shrinkage and created tensile stresses in the concrete under zero external load.

The beams in the test structure were assumed to be T- or L-beams, with flange widths equal to four times the slab thickness on one or both sides of the beam. The bending moments were obtained from the strains measured in the tests to the design dead load plus twice the design live load. All panels

TABLE 3.—MEASURED AND COMPUTED STATIC MOMENTS IN INDIVIDUAL PANELS

Static moments  (1)	MOMENT COEFFICIENT, $C = \frac{M}{wL^3}$			
	End Span		Interior Span	
	Edge panel (2)	Corner panel (3)	Interior panel (4)	Edge panel (5)
Measured	0.107	0.106	0.108	0.104
Computed	0.109	0.109	0.106	0.106

were loaded to 213 psf in test 314, and various combinations were loaded to produce the maximum moments in tests 315 to 334. The moments from test 307, to the design load of the structure, were not used since many of the strains were too small to be reliably measured. The load level of 213 psf corresponds to the load one would normally associate with the working load when it is considered that the strength of the structure was 537 psf.

It was assumed that there were no residual strains in the reinforcement at the beginning of test 314. The residuals in the later tests were obtained by summing the residuals from each separate test. The derived moments include the effects of the dead load.

*Measured Moments.*—The measured moments from the test with all panels loaded and the maximum moments produced by the various partial loading patterns are presented in Fig. 17. The most readily available check on the measured bending moments is the comparison of the total measured moments with the computed static moments. The total moment is the average of the negative moments acting on the ends of the spans plus the positive moment at midspan.

The static moments were computed by means of an analysis analogous to that by Nichols (6). The load on the panel was assumed to be transferred to

the columns entirely by the beams. The reactions were assumed to be concentrated at the faces of the supporting beams. The measured total moments in individual panels may be compared with the corresponding static moments,

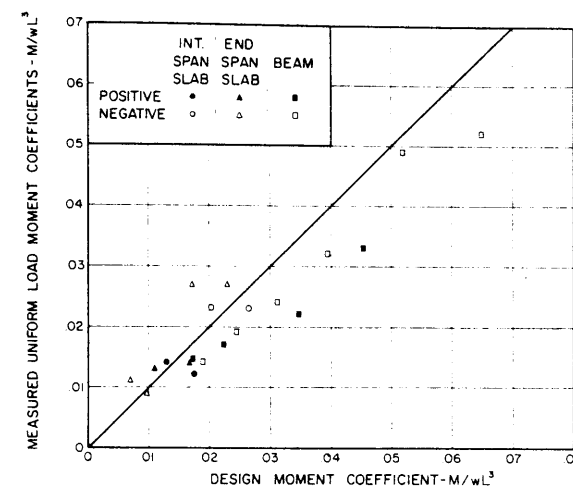


FIG. 19.—DESIGN MOMENTS VERSUS MEASURED MOMENTS FROM UNIFORMLY DISTRIBUTED LOAD

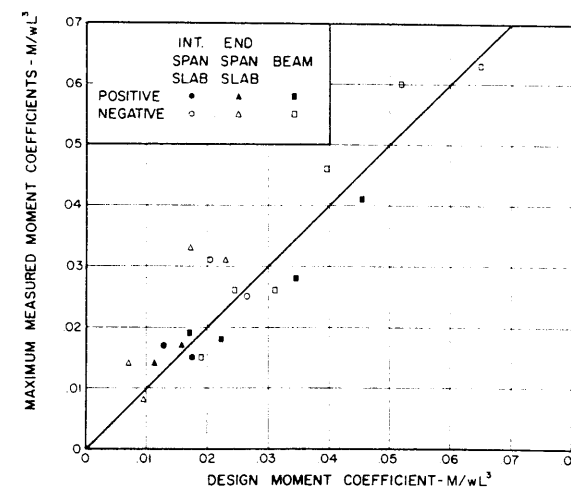


FIG. 20.—DESIGN MOMENTS VERSUS MAXIMUM MEASURED MOMENTS FROM PATTERN LOADINGS

shown in Table 3. In each case the correlation between the measured and computed total moments is quite good, and establishes a certain degree of confidence in the measured moments.

The design moments for the various beam and slab sections are shown in Fig. 18. The moments are shown for sections corresponding to those for which the measured moments are presented. The slab moments are for the central 45-in. or 45-1/2 in. width of the panel, the distance between the edges of the assumed L- and T-beam flanges. The beam design moments reported are the moments in the beam plus the moment normally assigned to the portions of the slab which are considered as the beam flanges.

The design moments were computed on the basis of the clear spans of the panels and are presented in terms of the coefficient  $C = M/wL^3$ , in which  $L$  = the span center-to-center of columns. The moments are those computed from ACI Method 1.

The total design moments in each individual panel and across the full width of the test structure in each span are considerably larger than the total measured moments or the computed static moments. This should be expected, however, as each of the critical slab and beam sections is designed implicitly for the maximum attainable bending moment.

The design moments for each section have been plotted against the measured uniform load moments in Fig. 19 and against the measured maximum moments in Fig. 20.

An examination of the measured and design moments reveals that none of the slab sections of the structure were grossly overdesigned. To the contrary, in several cases the slab sections were underdesigned in spite of the large difference between the total design and total measured moments. Every beam design moment was greater than the measured uniform load moment. The spandrel beam design moments were slightly greater than the measured moments; the interior beam design moments were considerably larger than the measured moments.

Since each of the critical moment sections of the structure was designed for the maximum moment attainable under some partial loading pattern, the maximum measured moments should be examined. The moment coefficients shown in Fig. 17 reveal increases in measured moments ranging from a small reduction to a maximum increase of 44%. At most of the sections the maximum moments were 15% to 25% greater than the uniform load moments.

These increases were greater than would be expected in the prototype structure because of the differences in the ratio of moveable load to total load in the test and prototype structures. In the test structure the moveable load was 140 psf of a total of 215 psf; in the prototype structure at design load the moveable load was 70 psf of a total of 145 psf.

Since the ratio of moveable to total load in the prototype structure is about three-quarters that in the test structure, the increases in moment in the prototype would be about three-quarters of those measured in the test structure.

## STRENGTH

To consider the strength of the whole structure in terms of the yield-line analysis (5) is an oversimplification. Pure flexure seldom governs the strength of a typical slab structure. Even if the various components of the entire system would limit the failure to the slab, restraints in the plane of the slab and appreciable changes in geometry make an orthodox application of the yield-

line analysis somewhat irrelevant. Nevertheless, the study of the test results in the framework of the yield-line analysis is of significance to demonstrate that the analysis represents a safe lower bound to the flexural failure of the structure and that the observed yielding sequence in the structure was in accordance with the implications of the analysis.

Yield loads corresponding to various idealized failure mechanisms were computed. The results are shown in Table 4. On the basis of the magnitudes indicated (Table 4), it would be concluded that the flexural strength of the structure was limited to 426 psf. Furthermore, it would be reasoned that, should the interior panel sustain additional load, the other panels would yield individually.

In a slab bounded by strong and stiff elements, any capacity calculated ignoring the contribution of the in-plane forces and large deflections is bound to be grossly on the safe side. Consider the last test of this series when the interior panel was loaded to failure individually. It carried 829 psf, almost twice the calculated capacity, at which load not the slab but the supporting beams failed as a result of combined shear and torsion effects. Thus, it was

TABLE 4.—YIELD LOADS CORRESPONDING TO FAILURE MECHANISMS

Slab failure mechanisms, in pounds per square foot (1)	Failure mechanisms involving end spans, in pounds per square foot (2)
Interior panel, 426	Mechanism 1, 467 (excluding edge beams)
Edge panel, 442	Mechanism 2, 529 (including edge beams)
Corner panel, 435	

not unreasonable to expect that the loads corresponding to general yielding mechanisms would be reached. In fact, the individual slab mechanisms formed first. The structure continued to resist increases in load and an end-span mechanism including the interior end-span beams developed. Because the edge beams, which had not yielded, continued to provide a restraint around the structure, the load increased to 537 psf before failure was obtained. At failure, yield lines crossed the structure at the first interior column line, and near midspan. Torsional hinges developed in the edge beams. The torsional moment developed at the beam-column connections, based on reaction measurements, was virtually the same as the calculated flexural moment capacity in the slab. Consequently, the close comparison of the measured (537 psf) and calculated (529 psf) yield loads is significant.

## CONCLUSIONS

The results of the tests on the typical two-way slab structure demonstrated that the strength of the structure designed on the basis of the ACI Building Code was more than adequate. The ratio of failure to design load was 3.7, and would have been greater than 3.0 even if the yield stress of the reinforcement had been only 40,000 psi.

The behavior of the test structure at the design load level and at overloads to twice the design live load was quite satisfactory. Neither excessive deflections nor high stresses in the reinforcement resulting in objectionable cracking occurred.

A study of the moments measured in the slab panels showed that panels with various end restraints could be classified simply in two groups; end-span panels and interior-span panels. The bending moments in the corner panels were similar to those in the edge panels in the span perpendicular to the edge of the structure. The moments in the interior panel were about the same as those in the edge-panel span parallel to the edge of the structure.

The present design methods recognize 4 different types of panels according to the end restraints. The measured moments do not justify such distinctions, and in the case of the edge panels, indicate trends opposite those given in the design procedures.

The design of the structure on the basis that the beams are rigid is unreasonable and misleading. A result of this approach to the design of two-way slabs is that the beams, which cannot be made rigid, are overdesigned, and the slab sections are frequently underdesigned. A second result of designing a slab structure on the assumption of rigid beams is that, because of the large design moments, the beams become either very deep or heavily reinforced, and the use of beams of intermediate stiffness is not economically possible.

In the current (1) design specifications, no consideration of the torsional stiffness of the edge beams is made, and only nominal consideration is given to the torsional strength. A minimum torsional stiffness should be specified if the edge beams are to resist the exterior negative moments in the slab.

The ratios of maximum moments to the moments under uniform load were not excessive in the test structure, and would be smaller in the prototype structure because of the lower ratio of moveable to total load. The expected increases in moment in the prototype structure are so small that it is hard to justify designing each section for the maximum moment.

The measured and design moments were quite different at many design sections. However, the achievement of a design in which these moments match is not necessarily essential. The tests have demonstrated that a few sections of a structure may be appreciably overstressed without seriously affecting the overall behavior.

The results of the yield-line analysis were confirmed by the behavior of the test structure. The lowest computed failure load corresponded to a single panel failure; the various structural failure modes all gave higher collapse loads.

The tests confirm that single panel will not collapse under the yield line analysis load if there are stiff horizontal restraints available at the ends of the span. These horizontal restraints permit the development of in-plane forces in the slab as the panel deflects, enhancing the load carrying capacity of the panel. In the test structure, the increase was sufficient to allow the load to reach the capacity of the structure.

The yield-line analysis does not automatically lead to the correct final collapse mechanism and load for the structure because of the possible importance of such secondary effects as the influence of in-plane forces and warping of surfaces assumed to be plane. In the present test structure, the minimum yield-line analysis load was a conservative estimate of the strength

of the structure. The yield-line array corresponding to the observed yield-lines at failure gave good agreement between the observed and computed failure loads.

## SUMMARY

The results of loading tests of a nine-panel reinforced concrete two-way floor slab (Fig. 1) are described. The structure was designed for a total load of 145 psf by Method 1 of the ACI Building Code (1).

At the design load level, the response of the structure was essentially linear. No cracks were found, and the measured stresses and deflections were small. The short-time behavior of the test structure under the design dead load plus twice the design live load, 215 psf, was still acceptable. The maximum reinforcement stress was 20,000 psi. Yielding of the reinforcement began at a load of 288 psf, dead load plus 3 live loads.

The maximum load reached in the test to failure was 537 psf, the design dead load plus 6.6 design live loads, or 3.7 times the total design load. The failure mechanism included both beams and slabs in an end span. The yield lines crossed the entire width of the structure. The failure load computed on the basis of the yield line analysis was 529 psf for a mechanism which corresponded to the observed failure mechanism.

Bending moments were obtained from the strains measured in the reinforcement for the various critical sections of the structure. Good agreement between the total measured moments and the computed static moments was found.

The total design moments were appreciably larger than the total measured moments for the case of all panels loaded. Most of the excess design moment is concentrated in the beams of the structure, and few slab sections were overdesigned; some slab sections were definitely underdesigned.

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#### APPENDIX I.—REFERENCES

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#### APPENDIX II.—NOTATION

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The following symbols are used in this paper:

- $LL$  = the design unit live load, in pounds per square foot, for the prototype structure;  
 $DL$  = the unit dead load, in pounds per square foot, for the prototype structure;  
 $L$  = span of a panel, center-to-center of columns; and  
 $w$  = the load per unit area, in pounds per square foot.