TITLE NO. 67-54

CODE BACKGROUND PAPER

Background material used in developing the proposed ACI Code

Equivalent Frame Analysis For Slab Design

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A completely changed design procedure for slabs was proposed in the February 1970 ACI JOURNAL. In addition to providing a single design procedure applicable to all types of concrete slab systems reinforced in more than one direction, the revised Code contains major changes in the assumptions required to determine slab design moments by use of a frame analysis.

This paper presents background for the equivalent frame analysis and gives an example of its application. In addition, moments calculated by the proposed frame analysis are compared with those measured in test slabs. Finally, tables giving frame design constants for common structures are presented in the Appendix.

Keywords: building codes; concrete slabs; flexural strength; frames; moments; reinforced concrete; structural analysis; structural design.

■ CHAPTER 13 OF THE PROPOSED REVISIONS of ACI 318-63[1] contains entirely new design requirements that are applicable to all slab systems reinforced in more than one direction, with or without beams between supports.

Two design procedures are described in Chapter 13 of the proposed revision. These are the direct design method (Section 13.3) and the equivalent frame method (Section 13.4).

This paper describes the background of the equivalent frame method and presents a numerical example of its application.* It is shown that the elastic analysis of previous ACI Codes is identical to the proposed frame analysis except in the definition of section properties of the equivalent frame. To aid in design, a list of constants for calculating stiffness, fixed-end moment, and carry-over factors for beam elements is provided in Appendix B.

Computed moments using the proposed frame analysis are shown to compare well with measured moments for several test structures. Comparisons reported elsewhere have shown satisfactory agreement between moments calculated by the equivalent frame analysis and moments calculated on the basis of the theory of flexure for plates. Consequently, it is concluded that the proposed equivalent frame method provides an improved design procedure that may be used to proportion structures that do not satisfy limitations necessary for application of the direct design method.

BACKGROUND

Purpose of frame analysis

In early ACI Building Codes, the empirical method of slab design was the only one permitted. Since this design method was permitted only for slabs with dimensions similar to those that had been built near the turn of the century, it soon became apparent that a method was needed for analyzing and designing slabs having dimensions, shapes, and loading patterns different from those to which empirical method was applicable.

Based on a 1929 study made by a committee working on the California Building Code,[3] an equivalent frame analysis for slabs was first codified in the 1933 Uniform Building Code, California Edition. Following this, the 1941 ACI Building Code adopted a similar method of analysis, but modified[4,†] to give the same results as the empirical design method. With some additional modifications, this same procedure was used in ACI 318-63.[5]

The equivalent frame analysis discussed in this paper is very similar to that previously used. Only the definitions of stiffness of the frame members are substantially modified. Where changes are made, they are intended to better reflect the behavior of slab structures and provide designs in better agreement with the direct design method proposed for the 1971 ACI Building Code.[1] These

*The provisions described in this paper were developed in cooperation with ACI-ASCE Committee 421, Design of Reinforced Concrete Slabs.

†DiStasio, J., and van Buren, M. P., "Background of Chapter 10, 1956 ACI Regulations on Flat Slabs," Private Communication, Distasio and van Buren, Consulting Engineers, New York City.

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modifications are described in more detail and are compared with analytical studies elsewhere.[6,7]

Description of analysis

The proposed method of analysis may be applied to flat slabs, flat plates, and to two-way slabs. The following description applies to a flat slab, the most complex case. Modifications applicable to elements of other types of slabs are discussed.

The first step in the frame analysis requires that a section one panel wide be considered. The cross section of an interior bay of a flat slab and the areas considered in calculation of the moments of inertia of the sections along the equivalent frame used in the analysis of this structure are shown in Fig 1. The $1/EI$ diagram for the slab may be used to determine moment distribution constants and fixed-end moments.*

For a two-way slab supported on columns, the moment of inertia I_s is the sum of the moment of inertia of a T-beam section and the moments of inertia of the rectangular slab sections extending from the edge of the assumed T-beam to the panel

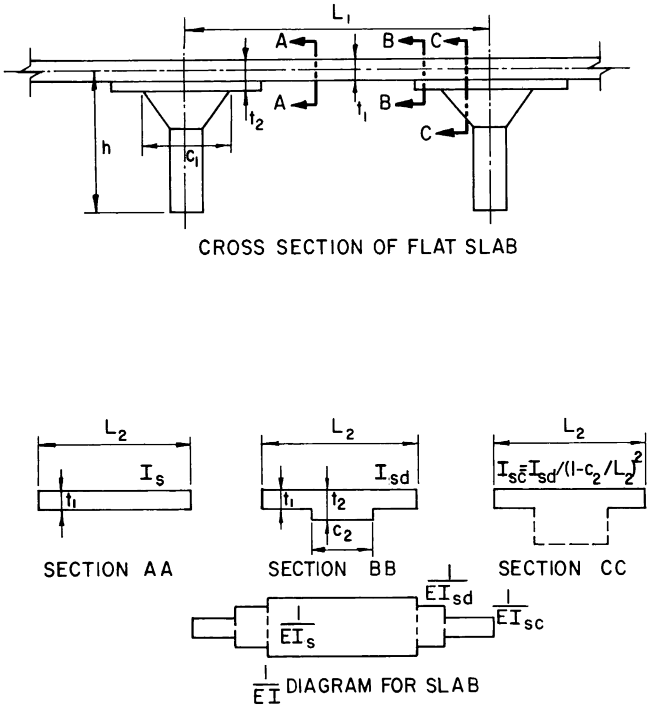


Fig 1 — Cross sections for calculating stiffnesses of equivalent frame

center lines.† In making this calculation, it is assumed that the flanges of the T-beam extend on each side of the beam stem a distance equal to the projection of the beam above or below the slab but not greater than four times the slab thickness as provided in Section 13.1.5.[1] In cases where the beam stem is short, the T-beam is assumed to have a width equal to that of the support.

The moment of inertia I_{sc} of the slab over the support (from the face of the support to the column center line) is based on the moment of inertia I_{sd} of the slab immediately surrounding the column. It is given by the following equation:

$$I_{sc} = I_{sd}/(1 - c_2/L_2)^2 \qquad (1)$$

where

c_2 = size of rectangular column, capital, wall or bracket measured transverse to the direction moments are being determined

L_2 = length of span transverse to L_1, measured center to center of supports

L_1 = length of span in the direction moments are being determined, measured center to center of supports

Eq. (1) serves two functions. It increases the stiffness of the equivalent beam to a level consistent with that determined by a three-dimensional slab analysis and verified by tests. At the same time, this equation covers the condition where a slab is supported on very wide columns. If the slab is supported on a reinforced concrete wall, $c_2/L_2 = 1.0$, and I_{sc} becomes very large. It should be noted, however, that this increase in moment of inertia is present only when the slab is constructed monolithically with the supports.

The computation of column stiffness is somewhat more complicated. Previous studies[7] have shown that the positive moment in a slab increases under pattern loads even if rigid columns are used. However, if a two-dimensional frame analysis is applied to a structure with infinite column stiffness, pattern loads will have no effect. To account for this difference in behavior between frames and slab structures, the section at the columns is considered as a beam-column combination in which the beam across the column can rotate even though the column is infinitely stiff. The resulting section may physically be likened to a hammerhead, as shown in Fig. 2.

In the case of an edge beam, the behavior mechanism is easily visualized. Some of the moment is transferred from the slab directly to the column while the remainder is transferred first to the beam, then to the column. It can be seen that a rigid column does not prevent rotation of the beam with respect to the columns.

*For convenience, fixed end moments, stiffnesses, and carry-over factors for flat plates and for a common configuration of flat slab are tabulated in Appendix B.

†Notation is given in Appendix A.

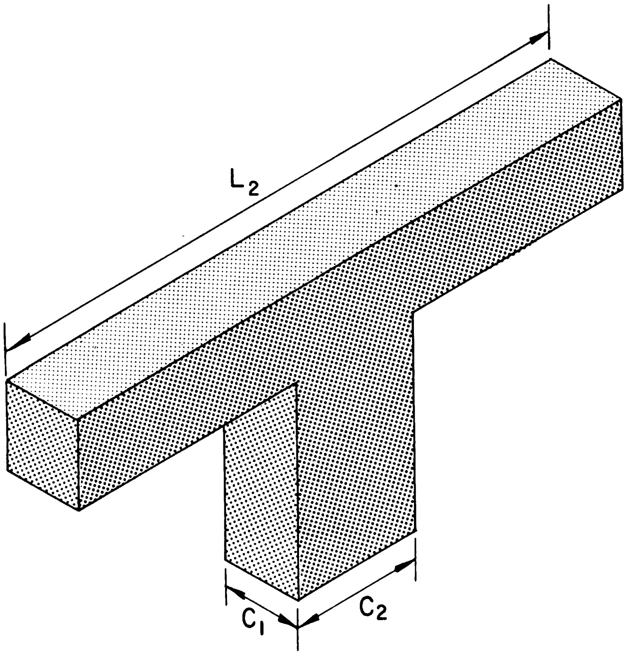


Fig. 2 — Hammerhead

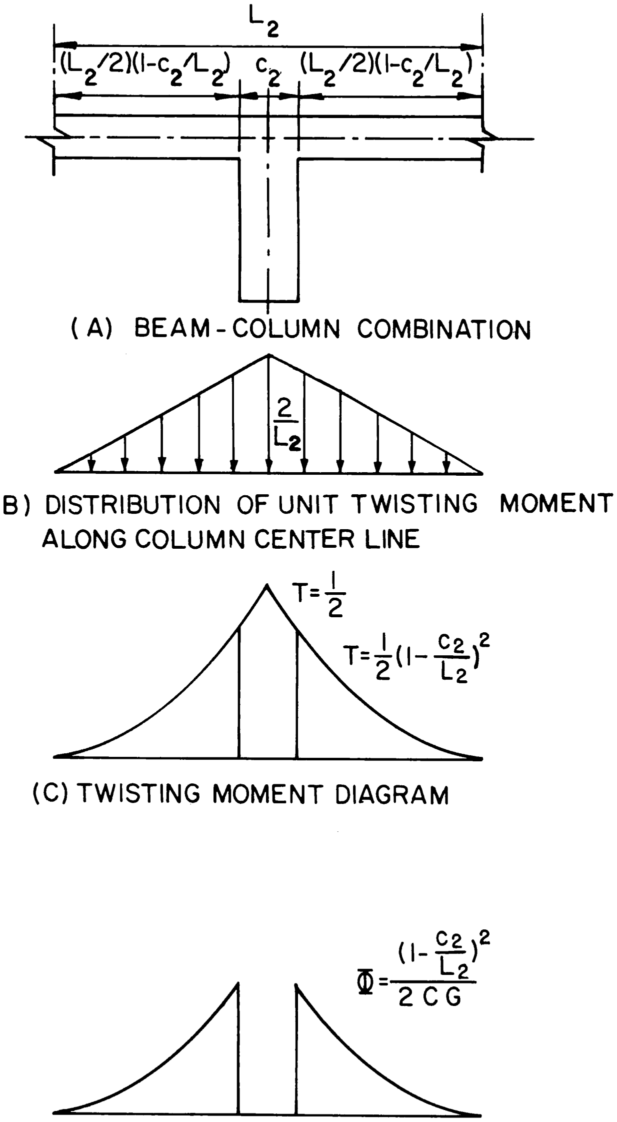


Fig. 3 — Rotation of beam under applied unit twisting moment

For use with the Cross distribution procedure,[8] the flexibility (inverse of the stiffness) of the beam-column combination, hereafter called the equivalent column, is defined as:

$$\frac{1}{K_{ec}} = \frac{1}{K_c} + \frac{1}{K_t} \tag{2}$$

where

K_{ec} = flexural stiffness of the equivalent column, moment per unit rotation

K_c = flexural stiffness of column, moment per unit rotation

K_t = torsional stiffness of torsional member, moment per unit rotation

Eq. (2) provides that the stiffness of the equivalent column is a function of both the flexural stiffness and the torsional stiffness of the slab or beams framing into the column transverse to the direction moments are being determined.

The value of K_c is independent of the distribution of torque along the beam or of the beam torsional stiffness since the total applied torque ultimately is resisted by the column. The moment of inertia of the column is computed on the basis of gross cross section below the capital (if one exists) and then is assumed to vary linearly from the base of the capital to the base of the slab. The column is assumed to be infinitely stiff over the depth of the slab.

The computation of K_t requires several simplifying assumptions. If no beam frames into the column, a portion of the slab equal to the width of the column is assumed to offer the torsional resistance. If a beam frames into the column, T-beam action is assumed with flanges extending on each side of the beam a distance equal to the projection of the beam above or below the slab. It is assumed that no rotation occurs in the beam over the width of the support.

Assumptions for determining the value of K_t are illustrated in Fig. 3. The length L_2 is the distance between slab center lines. Unit twisting moment is assumed to vary from a maximum at the column center line to zero at the slab center. This triangular distribution is used since the moment in the slab tends to be attracted toward the column. The twisting moment diagram is parabolic as shown in Fig. 3 (C). Once the twisting moment is known at each section, the unit rotation Φ can be expressed by the relationship $\Phi = T/CG$. For the beam considered in Fig. 3, the ordinate to the unit rotation diagram at the face of the column is:

$$\Phi = \frac{(1 - c_2 L_2)^2}{2CG} \tag{3}$$

where

Φ = angle of twist per unit of length

G = shearing modulus of elasticity,

$$= \frac{E_{cs}}{2\,(1+\mu)}\ ,\ \mu = 0$$

C = cross-sectional constant to define the torsional properties of edge beams and attached torsional members

The constant C may be evaluated for any shape of cross section [9] by dividing it into separate rectangular parts and carrying out the following summation:

$$C = \Sigma \left(1 - 0.63\ \frac{x}{y}\right)\frac{x^3y}{3} \tag{4}$$

where

x = shorter over-all dimension of a rectangular part of a cross section

y = longer over-all dimension of a rectangular part of a cross section

The rectangular parts should be chosen to minimize the length of the common boundaries.

For the beam-column combination shown in Fig. 3, the average effective angle of rotation of the torsional beam θ_t is the area of one of the parabolas shown in Fig 3(D). Since the stiffness K_t is equal to the torque along the beam axis per unit of rotation, the value of K_t for a unit torque is given by the relationship:

$$\frac{1}{K_t} = \theta_t\,\frac{L_2\,(1-c_2/L_2)^3}{36\,GC} \tag{5}$$

If a panel contains a beam parallel to the direction moments are being determined, the assumption of a triangular distribution of applied twisting moments may lead to equivalent column stiffnesses that are too low. Although a different distribution of applied torque could be assumed, a simpler approach is to increase K_t as follows:

$$K_t' = K_t\ \frac{I_{sb}}{I_{sa}} \tag{6}$$

where

K_t' = increased torsional stiffness due to presence of parallel beam

I_{sa} = moment of inertia of slab away from support and without parallel beam

I_{sb} = moment of inertia of slab including composite parallel beam

After the values of K_c and K_t have been calculated, the equivalent column stiffness K_{ec} can be determined and the distribution constants computed for the frame. Using the moment distribution procedure, moments at the column center lines on the line frame are then determined.

According to the provisions in Chapter 13 of the proposed 1971 ACI Code,[1] the design section may be taken at the face of square or rectangular sections. Consequently, negative design moments may be taken as those calculated at the face of the column.

Although the proposed equivalent frame analysis was developed primarily for an interior strip of panels, the necessary assumptions have been given for extending the analysis to a strip parallel to the edge of a structure having a width of one-half panel.

Comparison of moments from frame analysis with measured moments

The procedure outlined in the preceding sections was applied to an interior equivalent frame of each of five structures tested at the University of Illinois and one tested at the PCA Laboratories. Full descriptions of the test structures are available elsewhere.[2,7,10] On each test at the University of Illinois, moments were measured under both uniform and pattern loads. The pattern loads consisted of panel strips loaded to produce maximum moments at particular sections. In analyzing the structures, the ratio of movable to permanent load w_m/w_p has been considered. Values of w_m/w_p are listed in Tables 1, 2, and 3. No strip loads were applied to the PCA structure.

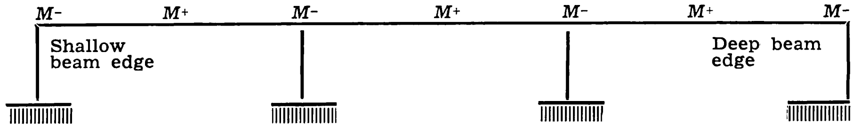
The measured uniform and strip load moments for each slab are listed in Tables 1, 2, and 3. Both the ratio of maximum to uniform load moments and the ratio of computed to measured uniform load moments are given. The values of measured moments in the University of Illinois flat plate (F1) and flat slab structures (F2 and F3) and in the PCA flat plate structure were obtained by combining middle and column strip moments.

In the University of Illinois two-way slab structures (T1 and T2), the measured moments were obtained by summing the interior beam moments and the interior slab moments. Moments were not measured under strip loads in the two-way slabs. Maximum moments were obtained by combining the measured maximum beam moments (under checkerboard patterns) with the uniform load slab moments. Since the beams in the two-way slab were relatively stiff, the differences between slab moments for strip load and for uniform load would have been insignificant.

In making comparisons between absolute moments at a section, it must be remembered that the frame analysis is based on statics and the full static moment (the sum of positive and average negative moments) is always present in any given bay. Due to experimental limitations, the measured moments vary somewhat from the static

TABLE 1 — COMPARISON OF MEASURED WITH COMPUTED MOMENTS (FLAT PLATE STRUCTURES)

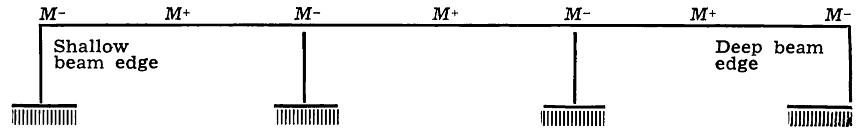
Section



University of Illinois structure, F1 (1/4 scale), $w_m/w_p = 2.5$	Moment coefficients, 1000 M/WL_1								
Calculated uniform load design moment	47	44	72	66	34	67	73	44	46
Calculated maximum design moment	54	50	75	73	45	73	76	50	52
Ratio maximum to uniform load moment	1.15	1.14	1.04	1.11	1.32	1.09	1.04	1.13	1.13
Measured uniform load moment	27	49	65	64	40	58	58	47	34
Measured maximum moment	21	52	68	67	44	63	63	48	26
Ratio maximum to uniform load moment	—	1.06	1.04	1.05	1.10	1.09	1.09	1.02	—
Ratio design to measured uniform load moment	1.74	0.90	1.11	1.03	0.85	1.16	1.26	0.94	1.35
PCA structure (3/4 scale)									
Calculated uniform load design moment	44	48	67	62	38	62	68	49	43
Measured uniform load moment	37	47	68	68	31	73	73	42	31
Ratio design to measured uniform load moment	1.19	1.02	0.99	0.91	1.22	0.85	0.85	1.16	1.39

TABLE 2 — COMPARISON OF MEASURED WITH COMPUTED MOMENTS (FLAT SLAB STRUCTURES)

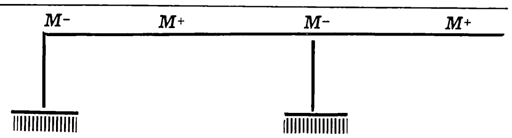
Section



University of Illinois structure, F2 (1/4 scale), $w_m/w_p = 5.5$	Moment coefficients, 1000 M/WL_1								
Calculated uniform load design moment	21	44	57	50	26	49	57	44	21
Calculated maximum design moment	28	53	63	60	44	60	62	53	29
Ratio maximum to uniform load moment	1.33	1.20	1.11	1.20	1.69	1.22	1.09	1.20	1.38
Measured uniform load moment	25	42	68	62	29	61	65	38	25
Measured maximum moment	27	49	79	72	33	67	71	42	25
Ratio maximum to uniform load moment	1.08	1.17	1.16	1.16	1.18	1.10	1.09	1.11	1.00
Ratio design to measured uniform load moment	0.84	1.05	0.84	0.81	0.90	0.80	0.88	1.16	0.84
University of Illinois structure, F3 (1/4 scale), $w_m/w_p = 3.5$									
Calculated uniform load design moment	21	44	57	50	26	49	57	44	21
Calculated maximum design moment	28	52	62	59	42	59	62	52	29
Ratio maximum to uniform load moment	1.33	1.18	1.09	1.18	1.62	1.20	1.09	1.18	1.38
Measured uniform load moment	29	38	57	55	23	58	60	34	24
Measured maximum moment	34	42	60	58	37	60	61	39	27
Ratio maximum to uniform load moment	1.17	1.11	1.05	1.05	1.60	1.03	1.02	1.15	1.12
Ratio design to measured uniform load moment	0.72	1.16	1.00	0.91	1.13	0.85	0.95	1.30	0.88

TABLE 3 — COMPARISON OF MEASURED WITH COMPUTED MOMENTS (TWO-WAY SLAB STRUCTURES)

Section



University of Illinois structure, T1 (1/4 scale), $w_m/w_p = 4.02$	Moment coefficients, 1000 M/WL_1				
Calculated uniform load design moment	35	47	79	66	34
Calculated maximum design moment	39	51	80	72	43
Ratio maximum to uniform load moment	1.11	1.09	1.01	1.09	1.26
Measured uniform load moment	43	46	79	71	36
Measured maximum moment	57	54	90	83	42
Ratio maximum to uniform load moment	1.33	1.17	1.14	1.17	1.17
Ratio design to measured uniform load moment	0.79	1.02	1.00	0.93	0.95
Univesity of Illinois structure, T2 (1/4 scale), $w_m/w_p = 1.09$					
Calculated uniform load design moment	46	44	74	66	34
Calculated maximum design moment	49	47	76	70	42
Ratio maximum to uniform load moment	1.07	1.07	1.03	1.06	1.24
Measured uniform load moment	36	56	69	61	45
Measured maximum moment	41	60	77	64	47
Ratio maximum to uniform load moment	1.14	1.07	1.12	1.05	1.05
Ratio design to measured uniform load moment	1.28	0.79	1.07	1.08	0.76

TABLE 4—COMPARISON OF MEASURED WITH COMPUTED MOMENTS (ELASTIC ANALYSIS USING ACI 318-63)

Section	Shallow beam edge								Deep beam edge
	M^-	M^+	M^-	M^-	M^+	M^-	M^-	M^+	M^-
University of Illinois, Flat Plate F1	Moment coefficients, 1000 M/WL_1								
Calculated uniform load design moment	51	35	70	61	31	61	70	35	51
Calculated maximum design moment	53	36	71	64	32	64	71	36	53
Ratio maximum to uniform load moment	1.04	1.03	1.01	1.05	1.03	1.05	1.01	1.03	1.04
Measured uniform load moment	27	49	65	64	40	58	58	47	34
Measured maximum moment	21	52	68	67	44	63	63	48	26
Ratio maximum to uniform load moment	—	1.06	1.04	1.05	1.10	1.09	1.09	1.02	—
Ratio design to measured uniform load moment	1.89	0.72	1.08	0.95	0.78	1.05	1.21	0.75	1.50
University of Illinois, Flat Slab F2									
Calculated uniform load design moment	23	28	51	46	19	46	51	28	23
Calculated maximum design moment	28	34	53	52	30	52	53	34	28
Ratio maximum to uniform load moment	1.22	1.21	1.04	1.13	1.58	1.13	1.04	1.21	1.22
Measured uniform load moment	25	42	68	62	29	61	65	38	25
Measured maximum moment	27	49	79	72	33	67	71	42	25
Ratio maximum to uniform load moment	1.08	1.17	1.16	1.16	1.18	1.10	1.09	1.11	1.00
Ratio design to measured uniform load moment	0.92	0.67	0.75	0.74	0.66	0.75	0.78	0.74	0.92

Note: Computed moments not reduced to M_o [318-63 Section 2102 (a)].

moment. In general, the greatest variation between measured and computed moments is found in end bays where the exterior negative slab moment is difficult to determine experimentally. Therefore, the basic criterion for judgment is whether the frame analysis provides sufficient moment capacity at a section to provide for uniform or strip loads while avoiding an overdesign.

The moments in the interior row of panels of the flat plate structures are given in Table 1. Comparison of ratios of calculated to measured moments indicates that the frame analysis is in satisfactory agreement with measured moments for pattern loads. The computed values of uniform load moment are within 15 percent of the measured values at most sections. Only at the exterior negative sections is there a serious discrepancy. This may be partially the result of a general reduction of stiffness due to cracking in the beam-column connection at the exterior column of the test structures.[7]

Calculated and measured moments for the flat slab structures are listed in Table 2. Calculated moment increases due to pattern loads compare favorably with those measured. Absolute moment comparisons between Structures F2 and F3 show that the measured moments are less in Structure F3 than in Structure F2. The test results indicate that the sum of positive and negative moments provided in each span is adequate.

Moments in the two-way structures, T1 and T2 are listed in Table 3. The calculated moment ratios for both structures are in reasonable agreement with measured values. While differences of 10 to 20 percent are found at some sections, it should be noted that the total moment is provided for in each bay. Consequently, adequate strength is provided when the calculated moments are used for design. Examination of the test results from Structure T2 again indicates satisfactory agreement between measured and calculated moments.

Table 4 shows a comparison of moment calculated according to the elastic analysis in ACI 318-63, Section 2103[5] with measured moments for University of Illinois Structures F1 and F2.[7] It can be seen that negative moments calculated by the 1963 Code are larger than measured values for Structure F1 while positive moments are about 25 percent less than measured. Applying the 1963 Code to Structure F2 shows that calculated design moments at all sections are about 30 percent less than those measured. Greater differences arise when calculated maximum moments are compared with measured maximum moments.

NUMERICAL EXAMPLE OF APPLICATION

Application of the equivalent frame analysis to calculation of design moments for an interior row of panels of a flat plate structure tested at the Portland Cement Association Structural Research Laboratories and described elsewhere[2] is illustrated. Dimensions of the structure are shown in Fig. 4. The columns of the test slab were supported to provide a stiffness equivalent to that of a structure having columns of the same dimensions framing into slabs 8 ft (244 cm) above and below the test slab.

The equivalent frame, as defined in Section 13.4.1.1,* is bounded by the panel center lines as indicated by the shaded area on the plan view of Fig. 4. Dimensions necessary for determining the

*All references in the numerical example are to the appropriate sections in the proposed 1971 ACI code.[1]

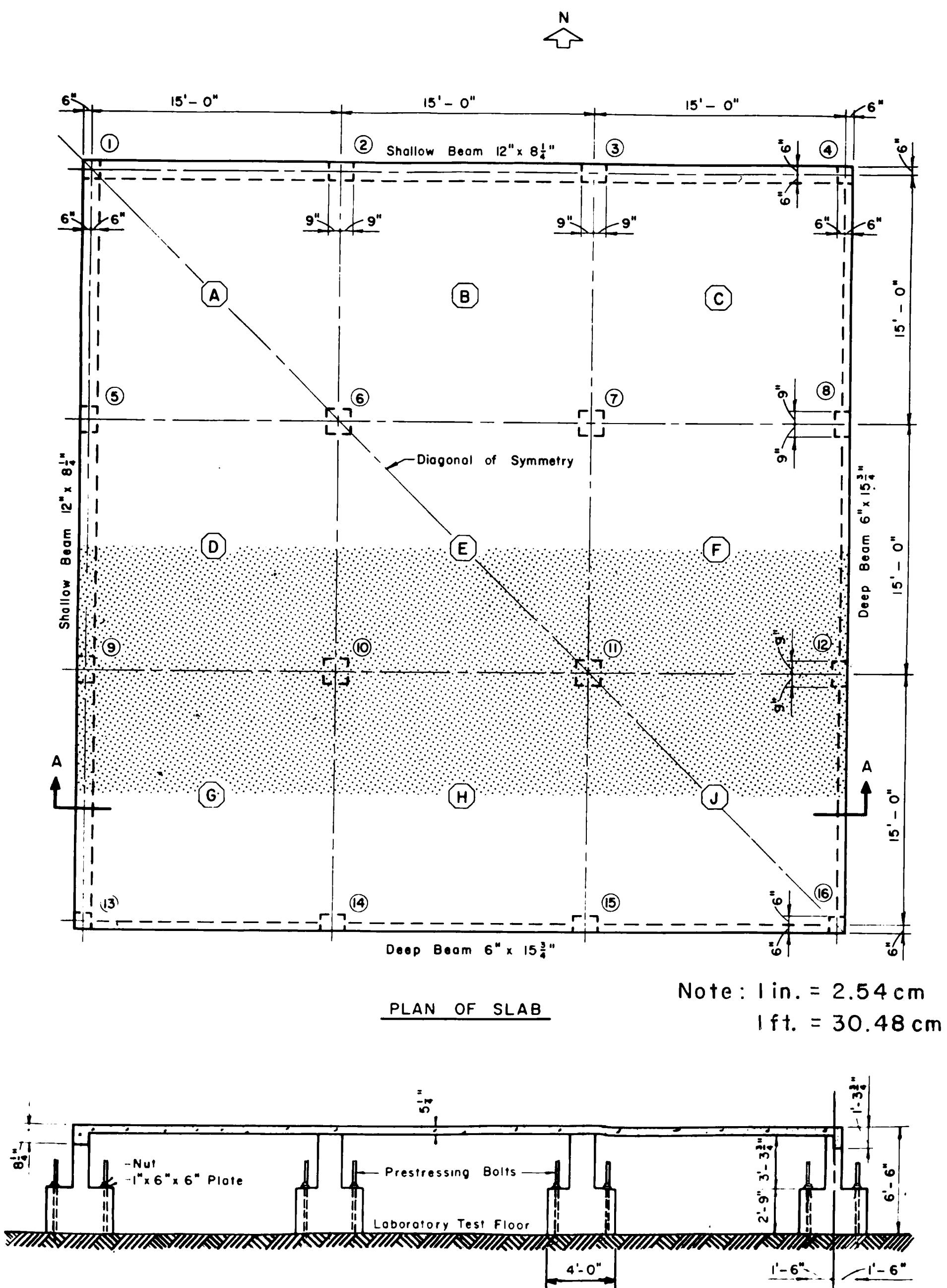


Fig. 4 — (at left) Layout of test structure

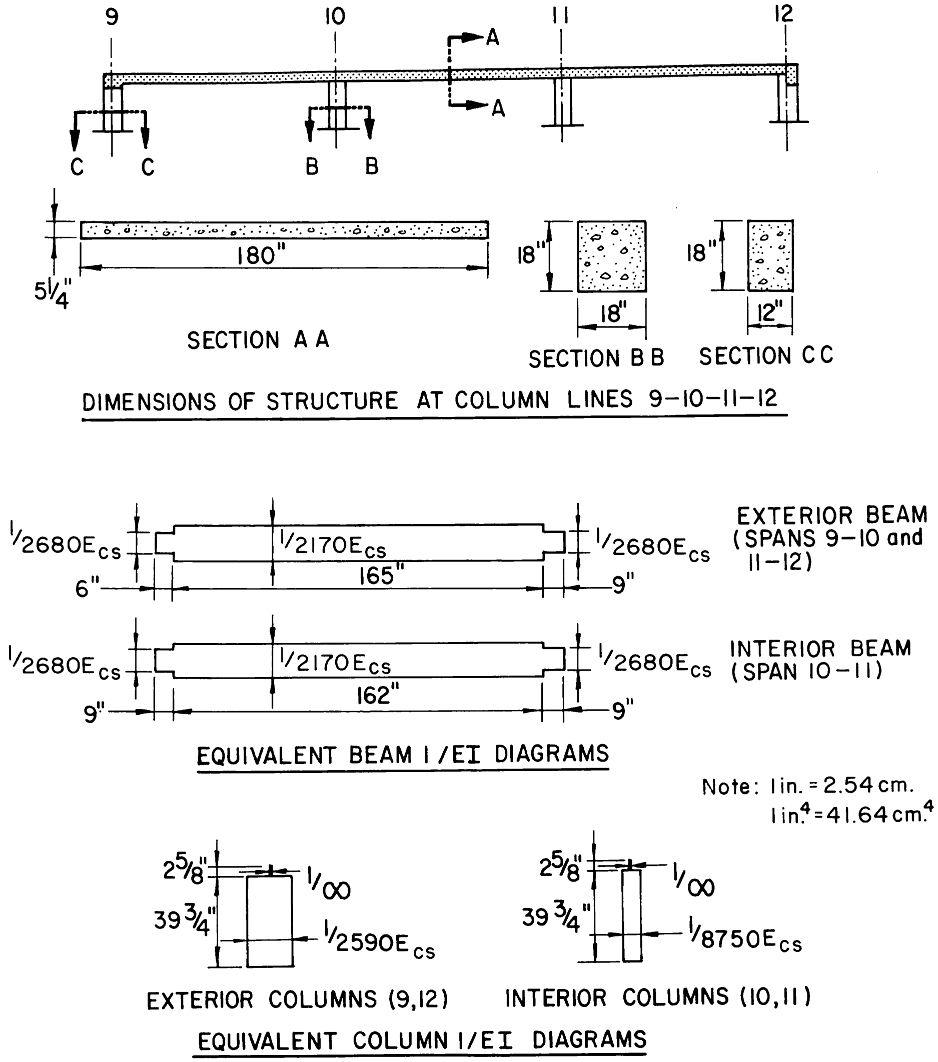


Fig. 5 — (above) Frame dimensions and I/EI diagrams

TABLE 5 — ANALYSIS OF FLAT PLATE*

	9		10			11			12
	M^-	M^+	M^-		M^+	M^-		M^+	M^-
Section	Shallow beam edge								Deep beam edge
Equivalent beams:									
Column-span ratio, c/L	0.0667		0.100			0.100			0.0667
Beam stiffness, KE_{cs}	49.9		50.4	50.3		50.3	50.4		49.9
Carry-over factor	0.513		0.507	0.513		0.513	0.507		0.513
Fixed-end moment, M/WL	0.0836		0.0853	0.0846		0.0846	0.0853		0.0836
Equivalent columns:									
Stiffness of actual column, K_c/E_{cs}	316		1060			1060			316
Stiffness of torsional members, K_t/E_{cs}	179		93			93			164
Stiffness of equivalent column K_{ec}/E_{cs}	114		86			86			108
Moments at column and panel center lines, M/WL_1	0.0597	0.0481	0.0941	0.0868	0.0380	0.0871	0.0944	0.0485	0.0586
Shear at column center lines, V/W	0.466		0.534	0.500		0.500	0.536		0.464
Distance from column center line to column face	0.033		0.050	0.050		0.050	0.050		0.033
Moments at column face M/WL_1	0.0442	0.0481	0.0674	0.0618	0.0380	0.0621	0.0677	0.0485	0.0432

*Stiffnesses are in in.^{4} 1 in.^{4} = 41.62 cm^4.

stiffnesses of the equivalent two-dimensional frame are shown in Fig. 5. In the analysis illustration, a moment distribution procedure is used to determine forces, however, other methods can be used to analyze the equivalent frame once the stiffnesses of the individual members are defined.

Determination of member stiffnesses

According to Section 13.4.1.3 the stiffnesses of the panels are determined from the moments of inertia of the gross cross-sectional areas. For the slab, the moment of inertia I_s is 2170 in.^{4} (90,300 cm^4). As defined in Section 13.4.1.4, the moment of inertia of the slab section over the columns is computed as $I_s/(1 - c_2/L_2)^2$. The c_2/L_2 ratios are the same for both the exterior and the interior columns. Consequently, the moment of inertia of the equivalent beam over each column is 2170 in.^{4}/$(1 - 0.1)^2$ = 2680 in.^{4} (111,500 cm^4).

Based on the slab moment of inertia, $1/EI$ diagrams for the equivalent two-dimensional beams are shown in Fig. 3. Note that the $1/EI$ diagrams for the exterior beams are not symmetrical since the c_1/L_1 ratios for the exterior and interior columns are different. From the $1/EI$ diagrams, the stiffnesses, carry-over factors, and fixed-end moments can be determined numerically. For convenience, these constants are listed for flat plates in Table B1 and for selected flat slabs in Table B2 of Appendix B.

A summary of the constants for the design example is given in Table 5. The fixed-end moments are in terms M/WL and stiffnesses are in terms of K/E_{cs} where E_{cs} is the modulus of elasticity of the slab concrete.

The moments of inertia for the interior and exterior columns are 8750 in.^{4} (364,000 cm^4) and 2590 in.^{4} (107,800 cm^4) respectively. The $1/EI$ diagrams for the columns are shown in Fig. 5. These diagrams are based on the assumption that the column is infinitely stiff over the full depth of the slab. The stiffness of the column K_c can be computed from the $1/EI$ diagrams. Values of calculated column stiffness are listed in Table 5.

Using Eq. (13-6) and (13-7), the stiffnesses K_t of the torsional members transverse to the direction of bending can be calculated. The cross sections of the torsional members as defined in Section 13.4.1.5 are shown in Fig. 6. At each edge column, a portion of the slab (equal to the projection of the beam below the slab) is assumed to act with the beam. Since no beams are present at the interior columns, a portion of the slab equal to the width of column is assumed to offer torsional resistance. Values of K_t are given in Table 5. The stiffnesses of the equivalent columns K_{ec} are computed using Eq. (13-5). These values are tabulated in Table 5.

Determination of design moments

The moments at the column and panel center lines are computed from the constants determined above and are listed in Table 5. The negative moments must be reduced to values at the design sections as defined in Section 13.4.2. Assuming shear at the column center line (as determined from the equivalent frame analysis) to act at the face of the column, negative moments are reduced to values at this section. Values of shear at the column center lines, distances to the column face, midspan positive moments and reduced negative moments (design moments) are listed in Table 5.

CONCLUDING REMARKS

This paper shows that the equivalent frame method for design of reinforced concrete slabs provides a convenient method for proportioning structures that do not satisfy the limitations of the direct design method.[1] For convenience, a numerical design example illustrating application of the equivalent frame method is given.

Important findings are described in the Introduction of this paper.

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H. W. Conner, assistant to the manager, Computer Services Section, Portland Cement Association, developed the tables of constants for Appendix B.

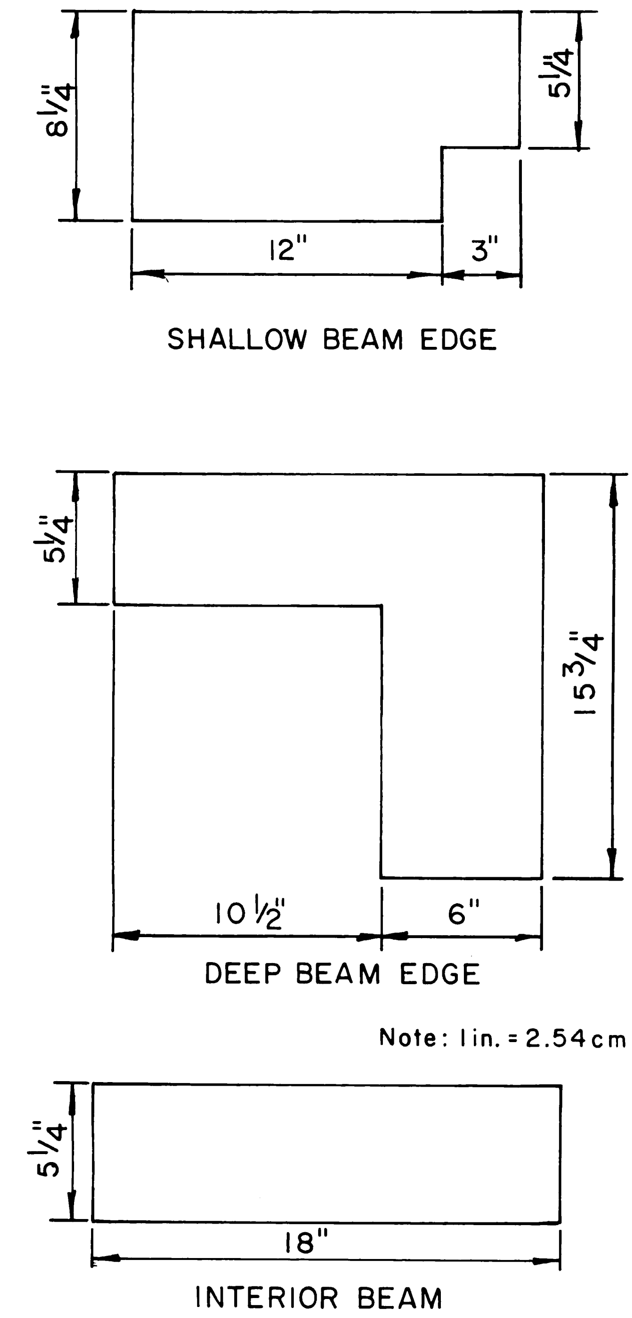

Fig. 6 — Dimensions of torsional members

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APPENDIX

APPENDIX A — NOTATION

c_1 = size of rectangular columns, capital, wall or bracket measured in the direction moments are being determined

c_2 = size of rectangular column, capital, wall or bracket measured transverse to the direction moments are being determined

C = cross-sectional constant used to define the properties of edge beams and attached torsional members

COF = carry-over factor for analysis by "Cross moment distribution"

E_{cs} = modulus of elasticity for slab concrete

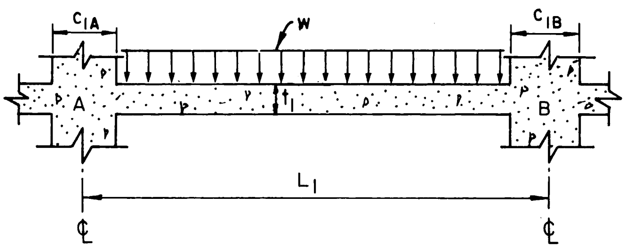
G = shearing modulus of elasticity, $G = E_{cs}/2(1 + \mu)$

I_s = moment of inertia of slab-beam away from support

I_{sa} = moment of inertia of slab-beam away from support and without parallel beam
I_{sb} = moment of inertia of slab-beam including composite parallel beam
I_{sc} = moment of inertia of slab-beam over the support from face of column or capital to column center line
I_{sd} = moment of inertia of slab-beam immediately surrounding the column
k = stiffness factor
K = flexural stiffness of equivalent beam or column
K_c = flexural stiffness of column moment per unit rotation
K_{ec} = flexural stiffness of equivalent column, moment per unit rotation
K_t = torsional stiffness of member, moment per unit rotation
K_t' = increased flexural stiffness of column due to presence of parallel beam, moment per unit rotation
L_1 = length of span in the direction moments are being determined, measured center to center of supports
L_2 = length of span transverse to L_1 measured center to center of supports
M = moment at section considered
t_1 = thickness of slab
t_2 = thickness of slab at drop panel
T = twisting moment
w_m = movable load per unit area
w_p = permanent load per unit area
W = total load on a panel
x = shorter over-all dimension of a rectangular part of a cross section
y = longer over-all dimension of a rectangular part of a cross section
Φ = angle of twist per unit of length
θ_t = average effective angle of rotation of torsional beam
μ = Poisson's ratio, assumed to be zero for slab concrete

APPENDIX B — TABLES OF CONSTANTS FOR "CROSS MOMENT DISTRIBUTION"

TABLE B1 — MOMENT DISTRIBUTION CONSTANTS FOR FLAT PLATE*

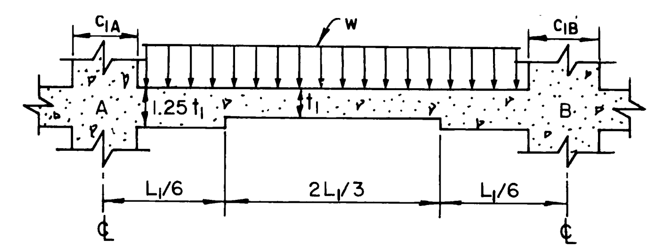


Column width		Uniform load FEM=Coef. (wL^3)		Stiffness factor†		Carry-over factor	
$\frac{c_1A}{L_1}$	$\frac{c_1B}{L_1}$	M_{AB}	M_{BA}	k_{AB}	k_{BA}	COF_{AB}	COF_{BA}
	0.00	0.083	0.083	4.00	4.00	0.500	0.500
	0.05	0.083	0.084	4.01	4.04	0.504	0.500
	0.10	0.082	0.086	4.03	4.15	0.513	0.499
0.00	0.15	0.081	0.089	4.07	4.32	0.528	0.498
	0.20	0.079	0.093	4.12	4.56	0.548	0.495
	0.25	0.077	0.097	4.18	4.88	0.573	0.491
	0.30	0.075	0.102	4.25	5.28	0.603	0.485
	0.35	0.073	0.107	4.33	5.78	0.638	0.478
	0.05	0.084	0.084	4.05	4.05	0.503	0.503
	0.10	0.083	0.086	4.07	4.15	0.513	0.503
	0.15	0.081	0.089	4.11	4.33	0.528	0.501
0.05	0.20	0.080	0.092	4.16	4.58	0.548	0.499
	0.25	0.078	0.096	4.22	4.89	0.573	0.494
	0.30	0.076	0.101	4.29	5.30	0.603	0.489
	0.35	0.074	0.107	4.37	5.80	0.638	0.481
	0.010	0.085	0.085	4.18	4.18	0.513	0.513
	0.15	0.083	0.088	4.22	4.36	0.528	0.511
0.10	0.20	0.082	0.091	4.27	4.61	0.548	0.508
	0.25	0.080	0.095	4.34	4.93	0.573	0.504
	0.30	0.078	0.100	4.41	5.34	0.602	0.498
	0.35	0.075	0.105	4.50	5.85	0.637	0.491
	0.15	0.086	0.086	4.40	4.40	0.526	0.526
	0.20	0.084	0.090	4.46	4.65	0.546	0.523
0.15	0.25	0.083	0.094	4.53	4.98	0.571	0.519
	0.30	0.080	0.099	4.61	5.40	0.601	0.513
	0.35	0.078	0.104	4.70	5.92	0.635	0.505
	0.20	0.088	0.088	4.72	4.72	0.543	0.543
0.20	0.25	0.086	0.092	4.79	5.05	0.568	0.539
	0.30	0.083	0.097	4.88	5.48	0.597	0.532
	0.35	0.081	0.102	4.99	6.01	0.632	0.524
	0.25	0.090	0.090	5.14	5.14	0.563	0.563
0.25	0.30	0.088	0.095	5.24	5.58	0.592	0.556
	0.35	0.085	0.100	5.36	6.12	0.626	0.548
0.30	0.30	0.092	0.092	5.69	5.69	0.585	0.585
	0.35	0.090	0.097	5.83	6.26	0.619	0.576
0.35	0.35	0.095	0.095	6.42	6.42	0.609	0.609

*Applicable when $c_1/L_1 = c_2/L_2$. For other relationships between these ratios, the constants will be slightly in error.

†Stiffness is $K_{AB} = k_{AB}\, E \frac{L_2 t_1^3}{12\ L_1}$ and $K_{BA} = k_{BA}\, E \frac{L_2 t_1^3}{12\ L_1}$

TABLE B2 — MOMENT DISTRIBUTION CONSTANTS FOR FLAT SLAB*



Column width		Uniform load FEM = Coef. (wL^3)		Stiffness factor†		Carry-over factor	
$\frac{c_1A}{L_1}$	$\frac{c_1B}{L_1}$	M_{AB}	M_{BA}	k_{AB}	k_{BA}	COF_{AB}	COF_{BA}
	0.00	0.088	0.088	4.78	4.78	0.541	0.541
	0.05	0.087	0.089	4.80	4.82	0.545	0.541
	0.10	0.087	0.090	4.83	4.94	0.553	0.541
0.00	0.15	0.085	0.093	4.87	5.12	0.567	0.540
	0.20	0.084	0.096	4.93	5.36	0.585	0.537
	0.25	0.082	0.100	5.00	5.68	0.606	0.534
	0.30	0.080	0.105	5.09	6.07	0.631	0.529
	0.05	0.088	0.088	4.84	4.84	0.545	0.545
	0.10	0.087	0.090	4.87	4.95	0.553	0.544
	0.15	0.085	0.093	4.91	5.13	0.567	0.543
0.05	0.20	0.084	0.096	4.97	5.38	0.584	0.541
	0.25	0.082	0.100	5.05	5.70	0.606	0.537
	0.30	0.080	0.104	5.13	6.09	0.632	0.532
	0.10	0.089	0.089	4.98	4.98	0.553	0.553
	0.15	0.088	0.092	5.03	5.16	0.566	0.551
0.10	0.20	0.086	0.094	5.09	5.42	0.584	0.549
	0.25	0.084	0.099	5.17	5.74	0.606	0.546
	0.30	0.082	0.103	5.26	6.13	0.631	0.541
	0.15	0.090	0.090	5.22	5.22	0.565	0.565
	0.20	0.089	0.094	5.28	5.47	0.583	0.563
0.15	0.25	0.087	0.097	5.37	5.80	0.604	0.559
	0.30	0.085	0.102	5.46	6.21	0.630	0.554
	0.20	0.092	0.092	5.55	5.55	0.580	0.580
0.20	0.25	0.090	0.096	5.64	5.88	0.602	0.577
	0.30	0.088	0.100	5.74	6.30	0.627	0.571
	0.25	0.094	0.094	5.98	5.98	0.598	0.598
0.25	0.30	0.091	0.098	6.10	6.41	0.622	0.593
0.30	0.30	0.095	0.095	6.54	6.54	0.617	0.617

*Applicable when $c_1/L_1 = c_2/L_2$. For other relationships between these ratios, the constants will be slightly in error.

†Stiffness is $K_{AB} = k_{AB}\, E \frac{L_2 t_1^3}{12\ L_1}$ and $K_{BA} = k_{BA}\, E \frac{L_2 t_1^3}{12\ L_1}$

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