Ref. 11.4

Behavior and Design of Prestressed Concrete Beams With Large Web Openings

Reports the results of tests on 18 full-size precast prestressed tee beams (13 long-span beams and 5 short-span beams) to determine the effects of large web openings on the performance of prestressed concrete members.

The results showed that large web openings can be accommodated in prestressed concrete members while maintaining their full strength. Also, serviceability requirements can be satisfied.

A design procedure based on the results of these tests is presented and a fully worked numerical example illustrates the applicability of the proposed design method.

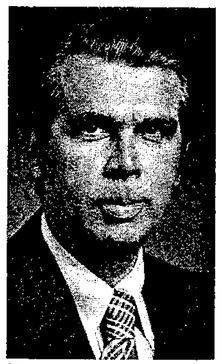
George B. Barney
Structural Engineer
Structural Development Section
Portland Cement Association
Skokie, Illinois

W. Gene Corley
Director
Engineering Development Department
Portland Cement Association
Skokie, Illinois

John M. Hanson
Vice-President of Operations
Wiss, Janney, Elstner and Associates
Northbrook, Illinois

Richard A. Parmelee
Professor of Civil Engineering
Northwestern University
Evanston, Illinois


George B. Barney


W. Gene Corley


John M. Hanson


Richard A. Parmelee

The trend in recent years toward the systems approach to building has generated a need for web openings in structural members. Mechanical and electrical services in most buildings are carried in the space within the floor-ceiling sandwich.

Passing these services through openings in the webs of the floor beams eliminates a significant amount of dead space and results in a more compact and often more economical design. However, the effect of the openings on the strength and serviceability of the floor beams must be considered.

Background

Only limited research has been done to determine the effects of web openings in prestressed concrete tee beams, a member widely used in the precast concrete industry. Ragan and Warwaruk[1] at the University of Alberta were the first to conduct a series of tests on prestressed concrete tee beams with multiple web openings. They found that sizeable web openings could be accommodated without sacrificing strength. Deflections for beams with openings were not significantly greater than those for beams without openings.

Tests[2] were also conducted to determine the effect of both vertical and longitudinal reinforcement in tee beams with multiple openings. It was found that increasing the vertical reinforcement in the posts between openings increased shear capacity of the specimens. Additional tests of prestressed beams with multiple parallelogram-shaped web openings[3] indicated that these beams were stronger than similar beams with rectangular openings.

The investigation reported in this paper was carried out at the Structural Development Laboratory of the Portland Cement Association.

Design Recommendations

Based on the results of the investigation reported here, a design procedure is recommended for prestressed, pretensioned concrete beams with large rectangular web openings. The procedure is applicable for beams having straight strands.

To avoid slip of the prestressing strands, openings must be located outside the required strand embedment length. This length may be calculated using the provisions of Section 12.11 of the 1971 ACI Building Code.[4] The value of the force to be transferred by each strand can be estimated as its breaking strength.

Vertical stirrups must be provided adjacent to both sides of all web openings. These stirrups should be proportioned to carry the total shear force at the section where they are located.

The analytical procedure described later in this report may be utilized to determine axial forces, shear forces, and moments in the struts above and below openings. The capacity of the struts to resist flexure and axial loads may conveniently be determined from interaction curves.

Slenderness effects in the compressive strut should be considered in accordance with Section 10.10 of the ACI Code.[4] When the section being analyzed is a tee beam, the effective width of the flange in determining properties and capacities of the compressive strut should not exceed the limits established in Section 8.7 of the ACI Code.[4]

Axial forces should be accounted for in the shear design of the struts. The shear capacity of concrete in the compressive strut can be determined from the provisions of Section 11.4.3 of the ACI Code[4] for members subjected to axial compression.

Design of the tensile strut for shear depends on whether the net axial force is tension or compression. For net axial compression, the shear capacity of the concrete section without web reinforcement can be determined from the provisions of Section 11.5.2 of the ACI Code.[4] Eq. (11-11) will usually govern the design. In terms of notation used in this report, this equation becomes:

$$v_{ci} = 0.6\sqrt{f'_c} + \frac{V_t M_{cr}}{b_w d\, M_u} \qquad (1)^*$$

where

b_w = minimum width of tensile strut

d = distance from extreme compressive fiber to centroid of prestressed reinforcement but not less than $0.8h$

f'_c = compressive strength of concrete

h = overall depth of tensile strut

M_u = maximum moment in tensile strut at section considered due to superimposed loads. Note that $M_u = V_t l/2$ where l is the effective strut length

M_{cr} = bending moment causing flexural cracking at section considered due to superimposed loads

v_{ci} = shear stress at diagonal cracking due to all design loads, when such cracking is result of combined shear and moment

V_t = shear force in tensile strut

and

$$M_{cr} = \frac{I_g}{y_t}\left[6\sqrt{f'_c} + f_{pe} - \frac{M}{d_s A_{gt}}\right] \qquad (2)$$

In Eq. (2):

A_{gt} = gross area of tensile strut

d_s = distance between centroidal axis at tensile and compressive struts

f_{pe} = compressive stress in concrete due to prestress only after all losses, at extreme fiber of section at which tensile stresses are caused by applied loads. Note that $f_{pe} = P(d_s + \Delta d)/(d_s A_{gt})$ where P is the effective prestress force and Δd is the distance of the effective prestress force resultant below the centroidal axis of the tensile strut

I_g = moment of inertia of uncracked section transformed to concrete

M = moment at center of opening

y_t = distance from centroidal axis of uncracked section to extreme fiber in tension

When the tensile strut is in net axial tension, the shear capacity of concrete may be determined from the provisions of Section 11.4.4 of the ACI Code.[4]

Shear reinforcement in the struts, when required, should be proportioned using the provisions of Section 11.6 of the ACI Code.[4]

Results of the tests indicate that cracking in the struts occurred in most specimens prior to reaching service load. However, this did not appear to have a significant effect on deflections. In view of this finding, it is recommended that the allowable tensile stress of $6\sqrt{f'_c}$ specified in Section 18.4.2(b) of the ACI Code[4] be increased to $7.5\sqrt{f'_c}$ psi ($0.62\sqrt{f'_c}$ MPa) for concrete in the struts.

Furthermore, it is likely that the provisions of Section 18.4.2(c) will often apply for prestressed beams with web openings. The allowable tensile stress would then be increased to $12\sqrt{f'_c}$ psi ($1.00\sqrt{f'_c}$ MPa). Allowable tensile stresses at sections away from openings should not be increased above those allowed in the ACI Code.[4]

*Note that in SI units, $\sqrt{f'_c}$ psi = $0.08304\sqrt{f'_c}$ MPa.

Synopsis

Results of tests on 18 full-size prestressed, pretensioned concrete tee beams representing one-half of a structural double tee section are reported. The variables investigated were opening size, location of opening along the span, type and amount of web shear reinforcement, and amount of primary flexural reinforcement.

Behavior of beams with openings was found to be similar to that of a Vierendeel truss. For the sizes of openings studied, distribution of shear force above and below an opening was dependent on the relative flexural stiffnesses of the struts. Based on these findings, a method of analysis was established. Criteria for strength design are presented and a fully worked numerical example is included to illustrate the application of the proposed design method.

The tests indicate that large web openings can be accommodated in pretensioned double tees while maintaining required strength and serviceability. However, the openings must be located outside the required strand embedment length and adequate shear reinforcement must be provided adjacent to the openings.

The experimental work was carried out at the Portland Cement Association.

Table 1. Variables and measured material properties for short-span beams.

Specimen	Opening** Location ft	Concrete Properties f'_c psi	f'_{sp} psi	E_c ksi	Equivalent Load kips/ft.	% of Calculated Ultimate Load+
P1-P*	6	6040	570	3570	5.36	61
P2-P	3	5990	570	3530	3.69	42
P3-W	6	5920	550	3290	8.52	97
P4-R	3	6040	560	3350	5.38	61
P5-W	6	6200	590	3750	8.25	94

*Symbols indicate reinforcement as follows:

P - no web reinforcement
W - welded wire fabric to provide minimum shear reinforcement
R - welded wire fabric and stirrups near openings

**Distance shown as X in Figure 1a

+Ultimate load for flexure calculated with ϕ = 1.0

1 ft = 0.305 m
1 psi = 0.006895 MPa
1 ksi = 6.895 MPa
1 kip/ft = 14.6 kN/m

Experimental Investigation

The experimental investigation was conducted to determine whether prestressed concrete beams could accommodate large web openings while maintaining adequate strength and serviceability. Details and results of the investigation are presented below.

Short-span beam tests

A series of short-span beams tested at the Structural Development Laboratory of the Portland Cement Association indicated that slip of the prestressing strands limited strength of beams containing openings in the region required for strand embedment.

In this investigation, five 26-in. (660 mm) deep tee beams, each containing five ½-in. (12.7 mm) diameter, 7-wire, prestressing strands with a breaking strength of 286 ksi (1972 MPa) were tested. Forces were applied to simulate conditions in a uniformly loaded beam. Note that the cross section of these beams was similar to that for the long-span beams, except that the flange was monolithic.

Rectangular openings 10-in. (254 mm) deep and 30-in. (762 mm) long were placed symmetrically about midspan. These openings were centered at either 6 ft (1.8 m) or 3 ft (0.9 m) from each end of the 18-ft (5.5 m) span.

Welded wire fabric and U-shaped No. 3 stirrups with yield strengths of 80.6 and 67.3 ksi (556 and 464 MPa), respectively, were used as shear reinforcement in three specimens. The other two specimens contained no shear reinforcement. Variables and concrete material properties of the short-span beam specimens are shown in Table 1.

Strand slip occurred in all five specimens causing a premature loss of strength. Vertical stirrup reinforcement along each side of the openings did not delay the occurrence of strand slip in these beams. However, Specimen P3-W with welded wire fabric for minimum shear reinforcement and with openings centered at 6 ft (1.8 m) from each support, carried a load corresponding to 97 percent of the calculated flexural capacity for a beam without openings.

Table 2. Details of test specimens.

Specimen	Beam Concrete f'_c psi	f'_{sp} psi	E_c ksi	Topping Concrete f'_c psi	f'_{sp} psi	E_c ksi	Opening Size in.	Opening** Location ft.
B1-W*	6820	590	4150	2990	370	3150	10x45	6
B2-W	7610	650	4260	2820	370	2680	-	-
B3-W	7600	650	4320	2840	380	3080	10x45	12
B4-W	7820	690	4340	2800	360	3100	10x45	9
B5-W	7410	630	4320	2830	380	3110	10x60	9
B6-W	7500	650	4230	2860	360	3050	10x30	9
B7-R	8020	660	4220	2860	390	3010	10x60	9
B8-P	7660	650	4190	3110	440	3220	10x60	9
B9-W	8140	670	4300	3680	420	3330	10x60	15
B10-W	7910	670	4310	3710	380	3440	10x60	12
B11-R	8000	670	4530	3760	500	3740	10x60	9 & 15
B12-R+	7170	690	3770	3460	400	3580	10x60	9
B13-R+	7460	680	4130	3110	400	3250	14x60	9

*Symbols indicate reinforcement as follows:

P - no web reinforcement
W - welded wire fabric to provide minimum shear reinforcement
R - welded wire fabric and stirrups near openings

**Distance shown as X in Figure 1a

+Specimens B12 and B13 contained 2 strands, all others contained 3 strands

1 psi = 0.006895 MPa
1 ksi = 6.895 MPa
1 in. = 25.4 mm
1 ft = 0.305 m

Long-span beam tests

Test beams in the long-span series represented one-half of a standard double tee section. Dimensions and details of the test specimen are shown in Figs. 1 and 2.

The web and the lower 2 in. (50.8 mm) of the flange were cast with normal weight concrete designed to have a compressive strength of 4000 psi (27.6 MPa) at the time of initial prestress. After transfer of prestress, a 2-in. (50.8 mm) topping of normal weight unreinforced concrete was cast. The topping had a design compressive strength of 3000 psi (20.7 MPa) at 14 days, normally the age at time of testing.

A span of 36 ft (11.0 m) was selected so that openings could be placed at several locations outside the required strand embedment length. This was intended to decrease the likelihood of bond failures that were observed in the short-span test series. The overall length of each specimen was 37 ft (11.3 m). Specimen details and concrete material properties are shown in Table 2.

Prestressing was provided with ½-in. (12.7 mm) diameter, 270K Grade, 7-wire straight strand with a breaking strength of 278 ksi (1917 MPa). Most specimens contained three strands spaced at 4 in. (102 mm), as shown in Fig. 2b. This arrangement permitted 10-in. (254 mm) deep openings to be placed in the web while still maintaining adequate concrete cover. One specimen was tested with only the top and bottom strands in place. A specimen containing 14-in. (356 mm) deep openings was tested with only the two bottom strands.

Web shear reinforcement, when provided, was of two types. Minimum shear reinforcement as specified by the 1971 ACI Building Code[4] was

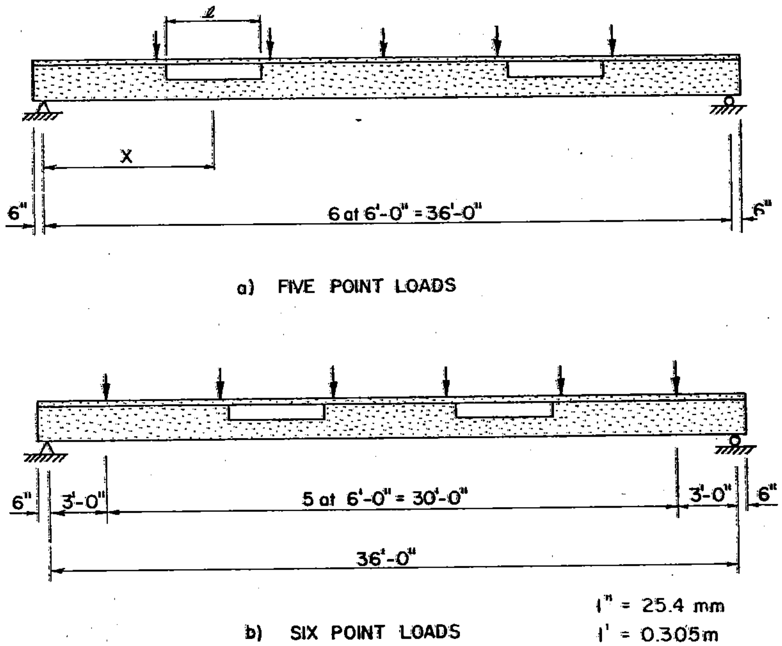


Fig. 1. Loading for long-span beams.

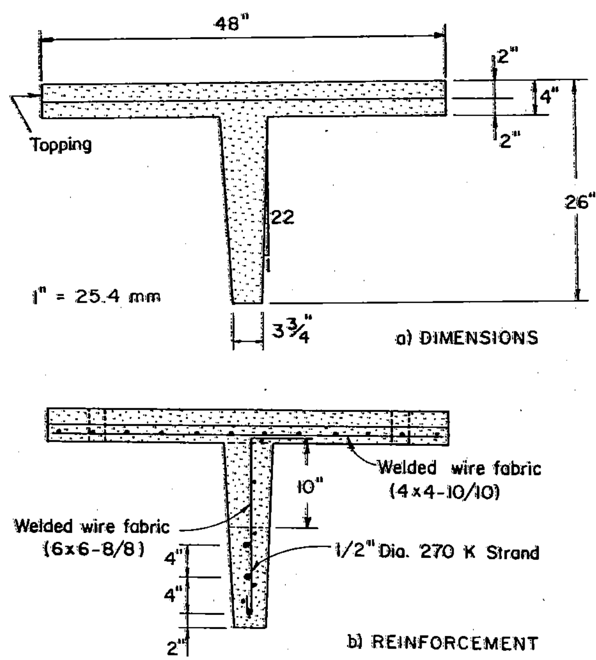


Fig. 2. Properties of long-span beams.

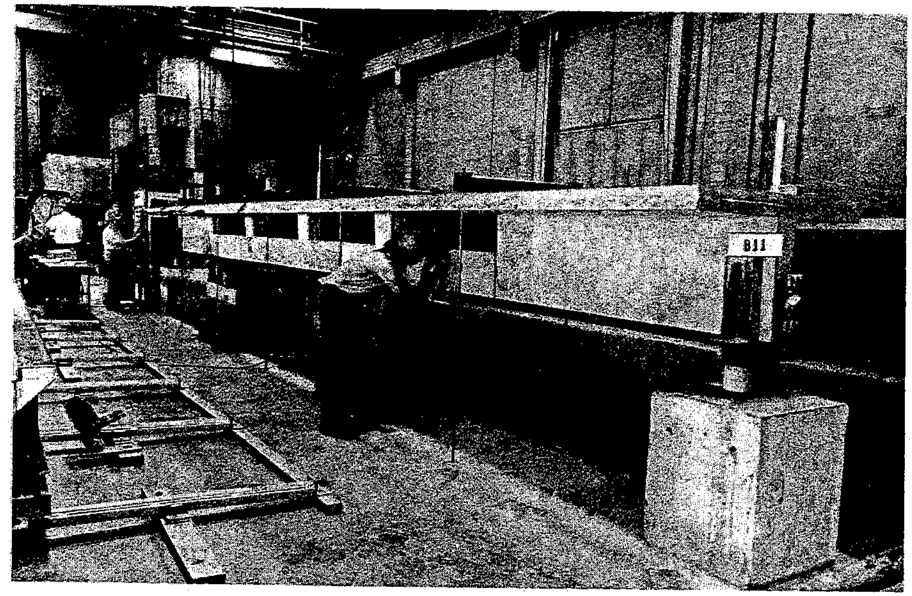


Fig. 3. Test setup for prestressed beam with four openings.

provided by welded wire fabric in most specimens with three strands. This type of reinforcement had measured yield strengths between 68.2 and 75.2 ksi (470 and 519 MPa). Specimens with two strands, although having a lower minimum shear reinforcement requirement, were provided with the same amount of web steel.

One additional U-shaped No. 3 stirrup was placed at each side and adjacent to each opening in four specimens. Distance from the centroid of the stirrups to the edge of the opening was 1 in. Welded wire fabric was provided in the flange of each specimen to satisfy temperature reinforcement requirements.

The tests were limited to beams with rectangular-shaped openings. Opening depths in 12 beams were 10 in. (254 mm). Specimen B13 had 14-in. (356 mm) deep openings. The openings were long enough, 30, 45, and 60 in. (762, 1143, and 1524 mm), to cause failures in the struts of some test specimens. Consequently, information was obtained for the development of strength design criteria.

Openings were placed symmetrically about midspan with the distance from the supports to the center of the openings, shown as "X" in Fig. 1a, being either 6, 9, 12, or 15 ft (1.8, 2.7, 3.7, or 4.6 m). Web shear reinforcement in most specimens consisted of at least the minimum amount required by the 1971 ACI Building Code.[4] Some specimens contained additional stirrup reinforcement adjacent to the openings. Specimen B8 was tested with no shear reinforcement.

Specimen B2 was tested with no openings and with minimum shear reinforcement. This provided a standard with which to compare the results of other specimens. The beam was designed to be under-reinforced.

Specimen B11 contained four openings. This beam was tested to determine the effect of closely spaced openings on forces in the struts (see Fig. 3).

Test procedure—The test setup is shown in Fig. 3. Concentrated loads located directly over the web were applied at 6-ft (1.8 m) intervals along the span to simulate conditions in a uniformly loaded beam.

To avoid having load points directly over openings, a five-point loading scheme was used for specimens having openings at 9 and 15 ft (2.7 and 4.6 m) from supports, and a six-point loading scheme was used for specimens having openings at 6 and 12 ft (1.8 and 2.7 m) from supports. Positions of loads are shown in Fig. 1.

During each test, the design service load was reached with the application of seven to twelve equal load increments. The beams were then unloaded in one increment. Next, the service load was reapplied in one increment. The beams were then unloaded before being tested to destruction. This sequence provided data in the service load range both before and after the specimens cracked.

Instrumentation—To determine the distribution of forces in the vicinity of the openings, parallel lines of electrical resistance strain gages were attached to the compressive struts over the openings. Additional strain gages were placed on the prestressing strands in the tensile strut.

Gage points for a Whittemore mechanical strain gage were attached to the test specimens at the level of the prestressing steel at the center of the span. These gage points were used to determine strains due to prestress both after transfer and at the time of testing.

In addition to strain readings, deflections were measured at midspan and at points directly under the edges of one of the openings. Load cells measured the applied forces and reactions. Dial gages placed at the tip of the strands extending from the ends of the beam were used to detect slip.

Test Results

Observed behavior

Observed behavior of the test specimens can be assigned to three different categories. Examples of each are shown in Fig. 4.

Specimens with adequate strength at the openings, and the standard specimen with no openings, reached their capacity in flexure. These tests ended when the prestressing strands fractured at midspan. Fig. 4a illustrates this behavior.

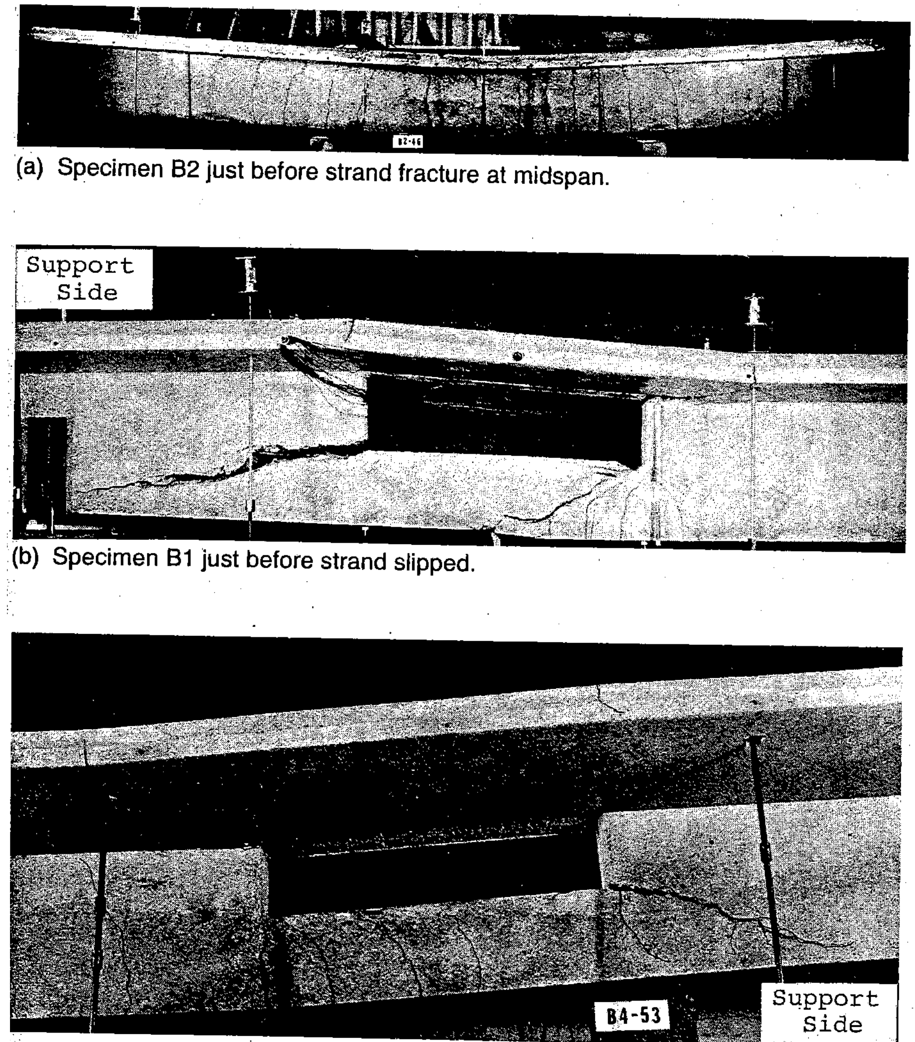
The capacity of several specimens having openings in high shear regions was limited by an unrestrained shear crack extending from the low moment side of an opening toward the support. These cracks normally propagated along the prestressing strands. In some beams the cracks extended into the region required for strand embedment causing the strand to slip.

The capacity of the tensile strut to carry shear was reduced as the crack lengthened. As a result, additional shear was transferred to the compressive strut. Capacity was reached when a hinging mechanism formed in the compressive strut. An example of this behavior is shown in Fig. 4b.

Some tests ended with fracture of the prestressing strands beneath an opening. For specimens having openings in high shear regions, strand fracture frequently coincided with the formation of a hinging mechanism in both struts. This mechanism is illustrated in Fig. 4c.

Findings

The tests clearly established that large web openings can be accommodated in prestressed concrete beams without decreasing their strength. However, this is possible only when cracking at an opening is not allowed to extend into the required strand embedment length.



(a) Specimen B2 just before strand fracture at midspan.

(b) Specimen B1 just before strand slipped.

(c) Specimen B4 just before strand fracture beneath opening.

Fig. 4. Behavior of beams under overload (Specimens B2, B1, and B4).

To satisfy this requirement, openings must be located outside the required embedment length. Additionally, vertical stirrup reinforcement must be provided adjacent to openings in an amount sufficient to carry the full design ultimate shear force.

Principal test results are presented in Table 3. Listed are the maximum load carried by each specimen, the percentage of design ultimate load attained, the type and cause of failure, and the location of damage.

In most specimens with openings, cracking occurred in the struts at less than service load. Therefore, design of these members may be controlled by serviceability requirements when cracking is not allowed.

In all tests, measured service load

Table 3. Principal test results.

Specimen	Equivalent Load kips/ft	% of Design Load*	Failure Type	Failure Initiated by	Failure Location
B1	0.98	77	Shear-secondary Hinging**	Strand Slip	Opening
B2	1.36	107	Flexure	Strand Fracture	Midspan
B3	1.41	110	Flexure	Strand Fracture	Midspan
B4	1.35	106	Flexure	Strand Fracture	Opening
B5	1.18	92	Shear-secondary Hinging**	Shear Cracking	Opening
B6	1.38	108	Flexure	Strand Fracture	Midspan
B7	1.34	105	Flexure	Strand Fracture	Opening
B8	0.94	73	Shear-secondary Hinging**	Strand Slip	Opening
B9	1.34	104	Flexure	Strand Fracture	Opening
B10	1.36	105	Flexure	Strand Fracture	Opening
B11	1.33	103	Flexure	Strand Fracture	Opening
B12	0.91	106	Flexure	Strand Fracture	Opening
B13	0.86	91	Flexure	Strand Fracture	Opening

*Ultimate load for flexure based upon $\phi = 1.0$

**Indicates shear failure of tensile strut followed by hinging in compressive strut

1 kip/ft = 14.6 kN/m

deflections were well within those allowed in the 1971 ACI Building Code.[4] A comparison of load versus deflection for Specimen B2 with no openings and Specimen B11 with four openings is shown in Fig. 5. It may be concluded that the influence of openings on deflection is minor in properly detailed beams.

Behavior of the test specimens was analogous to that of a Vierendeel truss. Analysis of recorded strains indicated that points of contraflexure existed in the compressive struts of specimens with openings. For openings in high shear regions, the point of contraflexure was near the midlength of the strut.

Forces determined from strain readings indicated that shear in the compressive and tensile struts was carried in proportion to their flexural stiffnesses. It was also found that cracking in the struts had a significant effect on the distribution of shear. This is illustrated in Fig. 6. Axial forces in the struts were close to those calculated on the basis of a Vierendeel truss analogy. Cracking had little effect on axial forces, as seen in Fig. 7.

Specimen B11 was tested with 10 x 60-in. (254 x 1524 mm) openings

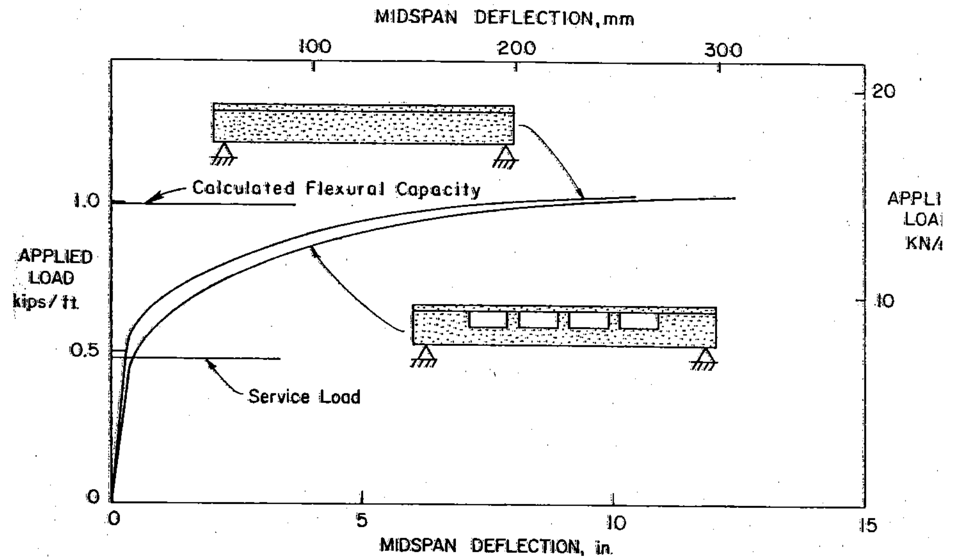


Fig. 5. Load versus deflection for beams with and without openings.

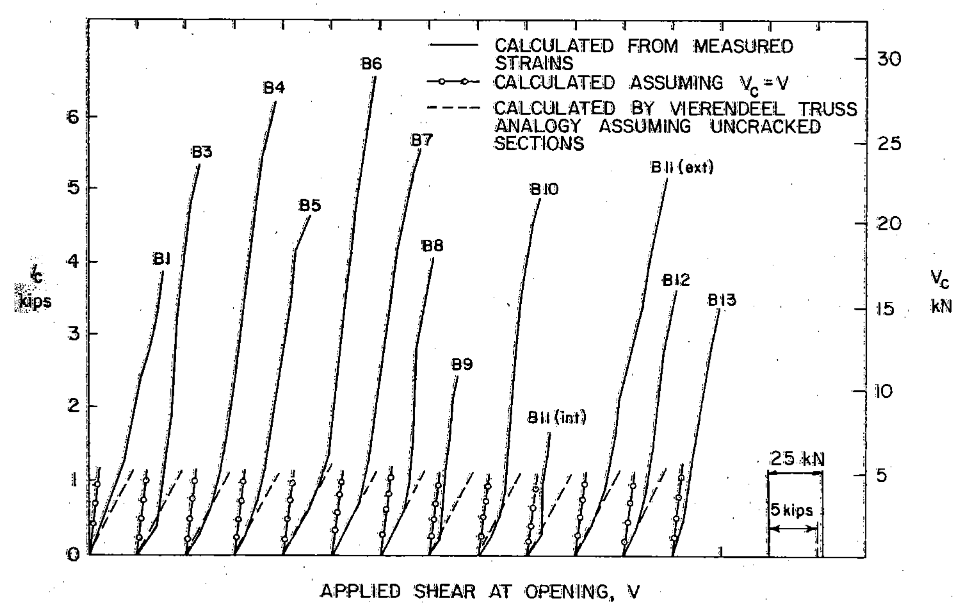


Fig. 6. Shear force in compressive strut.

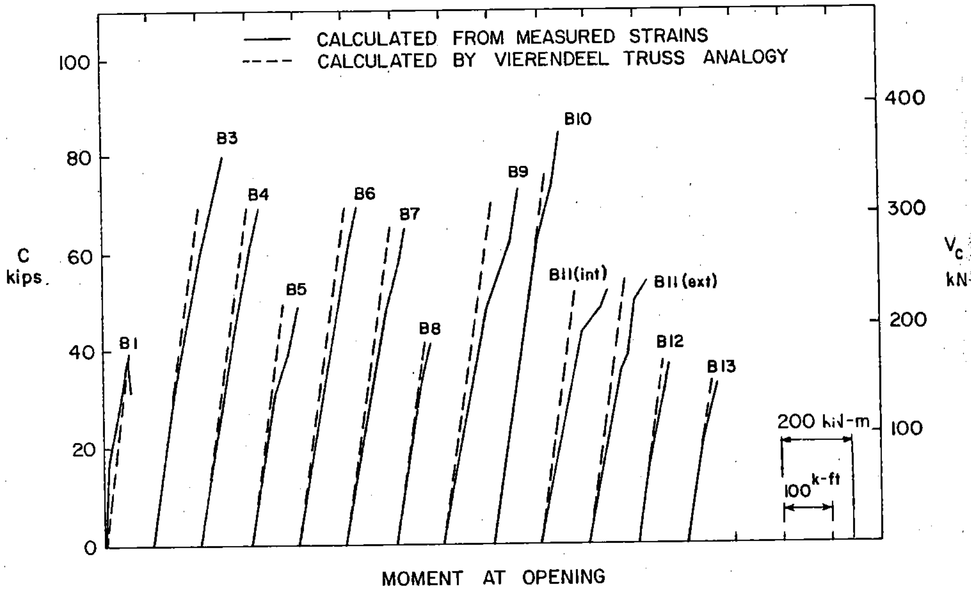


Fig. 7. Axial force in compressive strut.

centered at 9 and 15 ft (2.7 and 4.6 m) from each support. The openings were separated by 1-ft (0.31 m) thick web elements, referred to here as posts. Forces in the struts of Specimens B7 and B9, having isolated openings centered 9 and 15 ft (2.7 and 4.6 m) from supports, respectively, were compared to corresponding forces in Specimen B11. It was determined that cracking of the posts affected strut forces. Nominal shear stresses in excess of $7\sqrt{f_c'}$ psi ($0.58\sqrt{f_c'}$ MPa) were calculated from experimental data. However, the cracks in the posts did not reduce the strength of the specimen. Further research is needed to identify the behavior of posts between closely spaced openings.

Effect of variables

Of the variables considered in this investigation, those having the greatest effect on specimen strength and behavior were the location of the web openings along the span and the amount of web shear reinforcement. The effect of web reinforcement and opening length on beam strength is shown in Fig. 8a. Only specimens with openings centered 9 ft (2.7 m) from the supports are compared in this figure.

Minimum shear reinforcement provided adequate strength for specimens having opening lengths of 45 in. (1143 mm) or less. For specimens with 60-in. (1524 mm) openings, additional stirrup reinforcement at the openings was required to prevent strength reduction. Stirrup forces for Specimen B7 are shown in Fig. 9.

The effect on strength of opening location and opening size for specimens containing minimum shear reinforcement is shown in Fig. 8b. No loss was found for specimens with openings centered 12 ft (3.7 m) or more from the supports. However, 10 x 60-in. (254 x 1524 mm) openings located 9 ft (2.7 m) from the supports decreased the strength of Specimen B5.

Specimen B1, with 10 x 45-in. (254 x 1143 mm) openings centered 6 ft (1.8 m) from the supports, exhibited a substantial loss of strength. Cracks extending from the openings into the regions required for strand embedment in this specimen caused the strands to slip. This led to a premature failure.

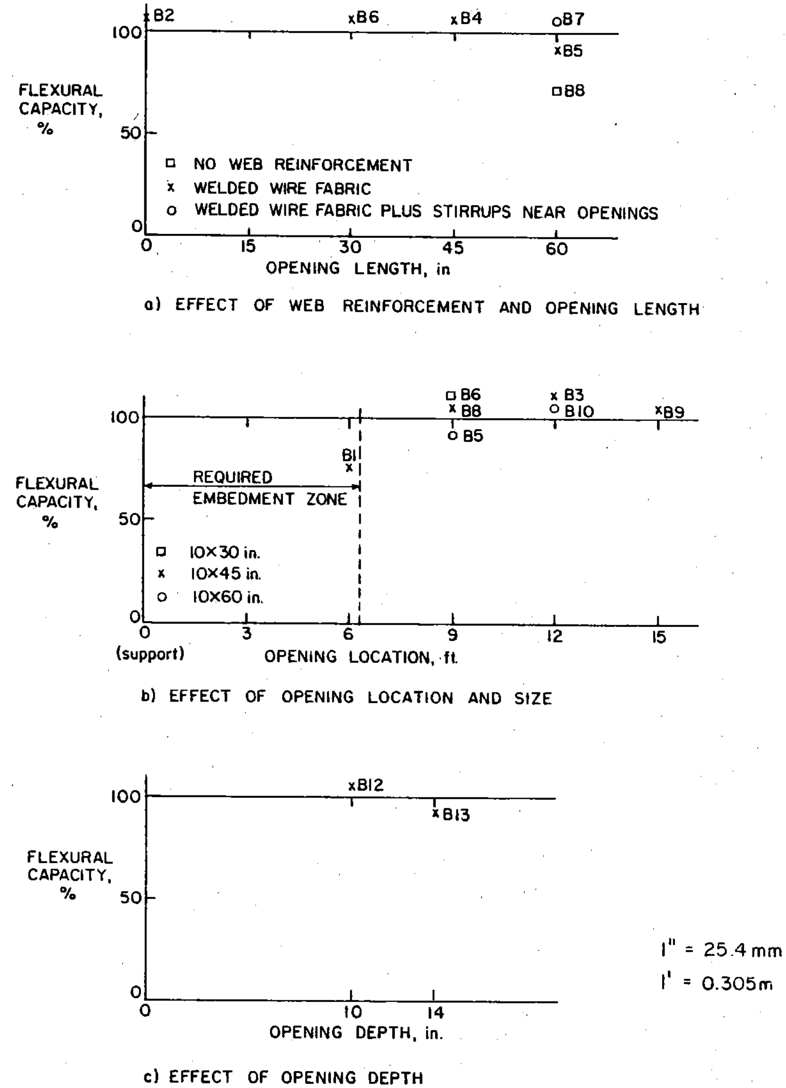


Fig. 8. Effect of variables.

A comparison of test results for Specimens B12 and B13 provides an indication of the effect of opening depth on behavior. Increased opening depth in Specimen B13 was provided by decreasing the depth of the tensile strut.

As shown in Fig. 8c, Specimen B12 with an opening depth of 10 in. (254 mm) carried a load 6 percent greater

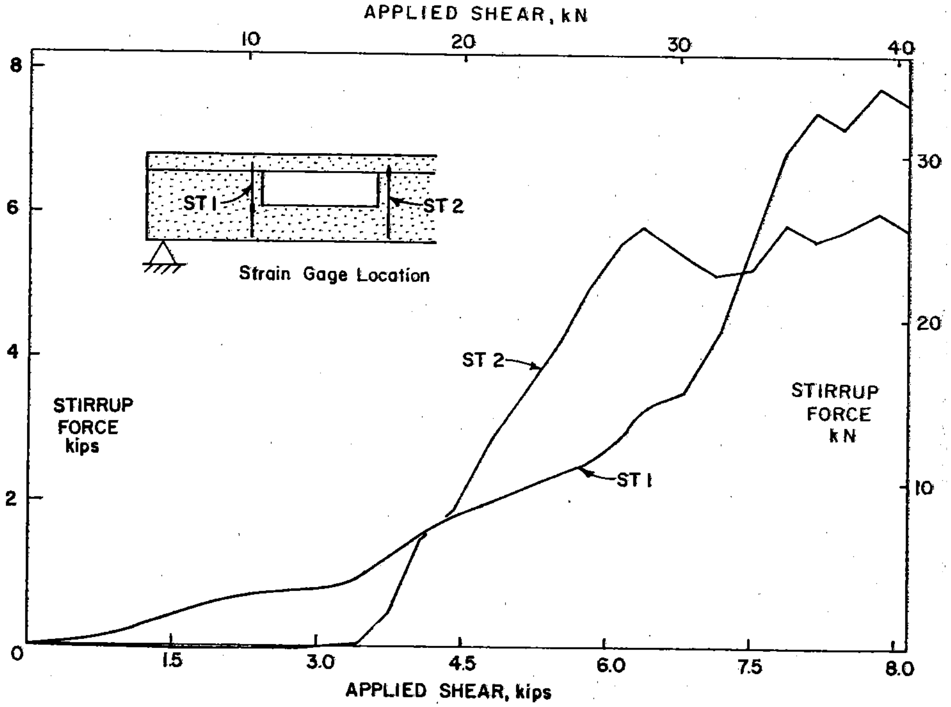


Fig. 9. Stirrup forces in Specimen B7.

than that corresponding to its calculated flexural capacity. The capacity of Specimen B13 with an opening depth of 14 in. (356 mm) was 9 percent below its calculated flexural strength.

Varying the amount of primary flexural reinforcement did not significantly change specimen behavior. Specimens B7 and B12 were similar except for the number of prestressing strands. Since Specimen B7 had three strands, its calculated flexural capacity was greater than that of Specimen B12 with only two strands.

Therefore, the shear at the opening for Specimen B7 was much more severe. However, the behavior of the two specimens was virtually identical. Both carried loads exceeding their calculated flexural capacities. Fracture of the prestressing strands beneath the openings occurred in both specimens at ultimate load.

Analytical Procedure

In this section, an analytical procedure for determining forces in the struts of prestressed beams with large web openings is presented.

Analysis of recorded strain data indicated that the behavior of the test specimens was similar to that of a Vierendeel truss. Cracking was observed to have a significant effect on the shear distribution in the struts. Before cracking, shear was distributed to the struts in proportion to their gross moments of inertia.

Forces acting on a beam with an opening are illustrated in Fig. 10. Loads on the beam produce shear, V, and moment, M. Moment is resisted by the two struts acting together as integral parts of a beam. This results in the primary stress condition indicated.

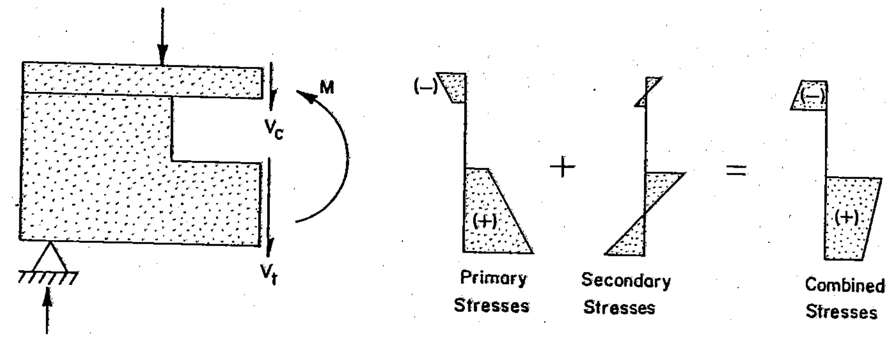


Fig. 10. Stresses at opening.

Each strut also carries a statically indeterminate portion of the total shear force acting at the section. The shear carried by the compressive strut is designated V_c and that carried by the tensile strut is designated V_t. These produce secondary flexural stresses in the struts.

At some section near the left edge of the opening, the secondary stresses before cracking are similar to those shown. Superposition of these two states of stress results in a combined stress condition as shown.

Distribution of forces

Simplified method—An idealized model of a beam with an opening is shown in Fig. 11a. The length of the struts, shown as l, is conservatively taken as the distance between vertical stirrups on each side of the opening. In practice, these stirrups must be provided to contain cracking.

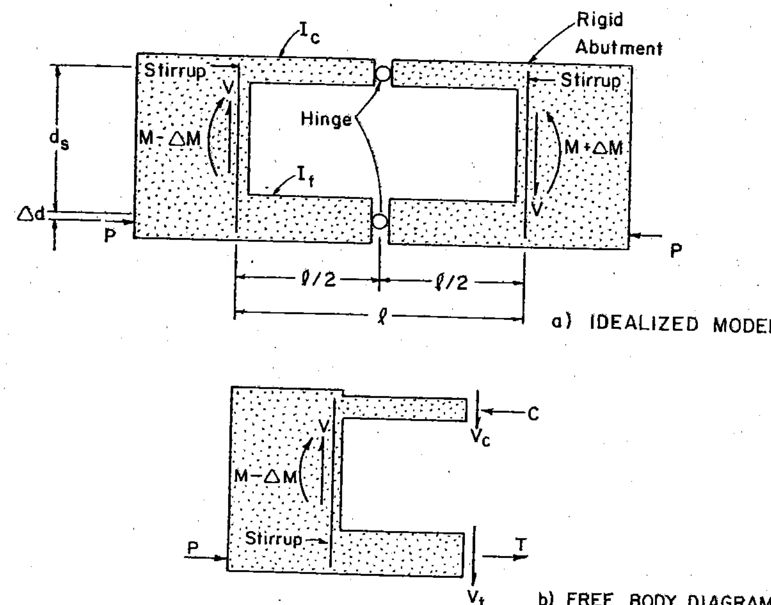


Fig. 11. Idealized model and free body diagram at beam opening.

The compressive and tensile struts are assumed to frame into rigid abutments on each side of the opening. To reflect the Vierendeel truss action observed in the tests, hinges are assumed at the midlength of each strut. Moments of inertia for the compressive and tensile struts are shown as I_c and I_t, respectively.

Shear, moment, and prestress are introduced into the system through the rigid abutments. For strength design, the shear, V, and moment, M, at the center of the opening are determined from beam forces at ultimate.

When the opening length, l, is small compared to the span length of the beam, V can be assumed constant over the length of the opening. Moment then varies linearly across the opening from $M - \Delta M$ to $M + \Delta M$, where:

$$\Delta M = \frac{Vl}{2} \quad (3)$$

and ΔM denotes the change in moment over one-half of the strut length.

The prestress force, P, acts at a distance Δd below the centroidal axis of the tensile strut. The distance between the centroidal axes of the struts is shown in Fig. 11a as d_s.

Forces acting at a section through the center of the opening are shown in Fig. 11b. With respect to the applied loads, the axial forces in the struts are calculated as:

$$C = \frac{M - P\,(\Delta d)}{d_s} \quad (4)$$

$$T = \frac{M - P\,(d_s + \Delta d)}{d_s} \quad (5)$$

For design purposes a simplified procedure for estimating shear forces in the struts has been derived. When no cracking has occurred in the struts, shear is carried in proportion to the uncracked moments of inertia. This will often be the case at transfer of prestress and at service load.

Once cracking has occurred, a redistribution of forces takes place in the struts. For this case, the design procedure is dependent on the extent of cracking in the tensile strut. When Eq. (5) results in:

$$T \geq 6\,A_{gt}\sqrt{f_c'} \quad (6)$$

a crack extending the full depth of the tensile strut is likely to have occurred. For this condition, it is recommended that the struts be designed for:

$$V_c = V \quad (7)$$

$$V_t = 0 \quad (8)$$

For values of T satisfying the condition:

$$T < 6\,A_{gt}\sqrt{f_c'} \quad (9)$$

a full-depth crack has not occurred. For this case the tensile strut must be designed to carry some of the shear. The recommended design forces are:

$$V_c = V\left[\frac{I_c}{I_c + I_{t(cr)}}\right] \quad (10)$$

$$V_t = V\left[\frac{I_t}{I_c + I_t}\right] \quad (11)$$

where

I_c = moment of inertia of uncracked compressive strut

I_t = moment of inertia of uncracked tensile strut

$I_{t(cr)}$ = moment of inertia of fully cracked tensile strut

The use of this simplified method for determining strut shear forces at ultimate load results in a conservative design. This should be satisfactory for most design applications.

Iterative method—When an accurate determination of strut shear forces is required, analysis using the modified idealized model shown in Fig. 12 is recommended. Variable effective moments of inertia I_{c1}, I_{c2}, I_{t1}, and I_{t2} are assumed in each strut segment to allow the effects of cracking to be included in the analysis.

Axial forces in the struts are calculated from Eqs. (4) and (5).

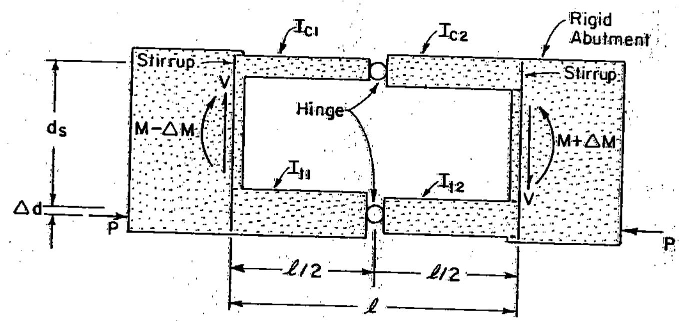


Fig. 12. Modified idealized model.

In terms of applied loads and strut moments of inertia, the shear forces are defined as:

$$V_c = \frac{V}{1 + \dfrac{I_{t2}\left[\dfrac{I_{c2}}{I_{c1}} + 1\right]}{I_{c2}\left[\dfrac{I_{t2}}{I_{t1}} + 1\right]}} \quad (12)$$

$$V_t = V - V_c \quad (13)$$

When the struts do not crack, the gross moments of inertia may be used. However, when the forces applied to the system are large enough to cause cracking in either strut, the values of V_c and V_t must be determined by an iterative procedure. Values of moment of inertia for cracked strut segments are approximated in each cycle by the following equation from the ACI Code:[4]

$$I_{eff} = \left(\frac{M_{cr}}{M_a}\right)^3 I_g + \left[1 - \left(\frac{M_{cr}}{M_a}\right)^3\right] I_{cr} \quad (14)$$

where

M_a = maximum moment in strut segment

I_{cr} = moment of inertia of cracked section transformed to concrete

Although Eq. (14) is intended for use with uniformly loaded beams, its use here is justified since only the relative stiffnesses of the struts are important in determining shear distribution. The analytical procedure compensates for inaccuracies in moments of inertia resulting from the use of Eq. (14).

The effect of axial forces must be considered when determining cracking moments. Axial compressive forces increase cracking moment, while axial tensile forces decrease cracking moment.

This analytical procedure is essentially a method for determining forces in the linear-elastic range of structural response. However, the results of an investigation using a more complex analytical model[5] indicate that the use of such a procedure also gives good correlation with experimental results in the nonlinear range of response.

The iterative analytical procedure is applicable for any load up to that causing full depth cracks in the tensile strut. Results from the test program indicate that additional shear applied after full depth cracking occurs is carried entirely by the compressive strut.

The analytical procedure can be extended to include prestressed beams with concrete toppings. For this case it becomes necessary to distinguish between the loads in the untopped system and loads in the composite system. Dead load and prestress forces are initially resisted by the untopped system. However, once cracking occurs, some of the dead load and prestress forces along with all of the load applied after the topping is cast are redistributed to the composite system.

The two systems must be analyzed separately to satisfy compatibility. However, the effects of forces in both systems must be considered together in determining the properties of the struts after they have cracked. More detail on the aspects of this analysis are contained elsewhere.[5] A computer program for this analysis is available.

The analysis applies only when the struts behave primarily as flexural members. As a guide in this regard, the analysis is not recommended when overall length-to-depth ratios of the struts are less than 2.5. Furthermore, to ensure that the posts behave rigidly, it is recommended that adjacent web openings be separated by web elements (posts) having overall width-to-height ratios of at least 2.0 where the width of the posts is the distance between adjacent stirrups. A limit on nominal total design shear stress, v_u, of $2\sqrt{f_c'}$ psi ($0.17\sqrt{f_c'}$ MPa) is advised for the posts.

Acknowledgments

The investigation described in this report was carried out in the Structural Development Laboratory of the Portland Cement Association in Skokie, Illinois. B. W. Fullhart, A. G. Aabey, and W. Hummerich, Jr. of the technician staff of the Structural Development Section fabricated and tested the beams. Photographic services were provided by P. J. Walusek. Astaire M. Parisi provided the secretarial services. Figures in the report were prepared by Louise S. Masten.

Concluding Remarks

Tests were carried out on 18 full-size tee beams containing large rectangular web openings. The beams were loaded to simulate conditions in a uniformly loaded beam.

Principal variables in the test program were size and location of openings, type and amount of web shear reinforcement, and amount of primary flexural reinforcement.

The behavior of beams with openings was similar to that of a Vierendeel truss. Test results indicate that large web openings can be placed in prestressed concrete beams without sacrificing strength or serviceability. However, openings must be located outside the required strand embedment length. Adequate shear reinforcement must be provided adjacent to openings.

An analytical procedure has been established for determining forces and moments in struts above and below openings. Design criteria have been presented.

References

1. Ragan, H. S. and Warwaruk, J., "Tee Members With Large Web Openings," PCI JOURNAL, V. 12, No. 4, August 1967, pp. 52-65.
2. Sauve, Jacques Germain, "Prestressed Concrete Tee Beams With Large Web Openings," MS Thesis, University of Alberta, Fall 1970.
3. LeBlanc, Eric P., "Parallelogram Shaped Openings in Prestressed Concrete Tee Beams," MS Thesis, University of Alberta, Fall 1971.
4. ACI Committee 318, "Building Code Requirements for Reinforced Concrete (ACI 318-71)," American Concrete Institute, Detroit, Michigan, 1971, 78 pp.
5. Barney, G. B., "Design of Prestressed Concrete Beams with Large Web Openings," PhD Thesis, Northwestern University, June 1975.

APPENDIX A—NOTATION

A_{gt} = gross area of tensile strut

b_w = minimum width of tensile strut

C = axial force in compressive strut

d = distance from extreme compressive fiber to centroid of prestressed reinforcement but not less than $0.8h$

d_s = distance between centroidal axes of tensile and compressive struts

Δd = distance of effective prestressing force resultant below centroidal axis of tensile strut

E_c = modulus of elasticity of concrete

f_c' = compressive strength of concrete

f_{pe} = compressive stress in concrete due to prestress only after all losses, at extreme fiber of a section at which tensile stresses are caused by applied loads

f_{sp}' = splitting tensile strength of concrete

h = overall depth of tensile strut

I_c = moment of inertia of uncracked compressive strut

I_{cr} = moment of inertia of cracked strut section transformed to concrete

I_{c1}, I_{c2} = effective moments of inertia in compressive strut

I_{eff} = effective moment of inertia

I_g = moment of inertia of uncracked section transformed to concrete

I_t = moment of inertia of uncracked tensile strut

$I_{t(cr)}$ = moment of inertia of fully cracked tensile strut

I_{t1}, I_{t2} = effective moments of inertia in tensile strut

l = effective strut length

M = moment at center of opening

ΔM = change in moment over one-half of strut length

M_a = maximum moment in strut segment

M_{cr} = bending moment causing flexural cracking at section considered due to superimposed loads

M_u = maximum moment in tensile strut due to superimposed loads

P = effective prestress force

T = axial force in tensile strut

v_{ci} = shear stress at diagonal cracking due to all design loads, when such cracking is result of combined shear and moment

v_u = nominal total design shear stress

V = shear force at center of opening

V_c = shear force in compressive strut

V_t = shear force in tensile strut

X = distance from support to center of opening

y_t = distance from centroidal axis of uncracked section to extreme fiber in tension

ϕ = capacity reduction factor

APPENDIX B—DESIGN EXAMPLE

In this section a design example is presented for a prestressed concrete double tee beam with a 2-in. thick reinforced concrete topping. Part I of the example demonstrates strength design using the simplified method referred to in the text.

Part II illustrates procedures for checking stress and deflection requirements at service load.

Note: For the convenience of readers unfamiliar with the American system of units, a table of metric (SI) equivalents is included below.

Part I—Strength design

The simply supported prestressed concrete double tee beam shown in Fig. B1 has been designed without web openings to carry a live load of 50 psf. Material properties are as follows:

Concrete:
Beam $f'_c = 6000$ psi
Topping $f'_c = 3000$ psi

Metric (SI) Unit Equivalents

1 in. = 25.4 mm
1 ft = 0.305 m
1 in.^{2} = 645.16 mm^2
1 in.^{4} = 416,231 mm^4
1 psi = 0.006895 MPa
1 ksi = 6.895 MPa
1 lb = 4.448 N
1 kip = 4448 N
1 lb/ft = 14.594 N/m
1 kip/ft = 14.594 kN/m
1 in.-lb = 0.113 N • m
1 in.-kip = 113 N • m
1 ft-lb = 1.356 N • m
1 lb-in.^{2} = 0.00287 N • m^2
1 $\sqrt{f'_c}$ psi = 0.083036 $\sqrt{f'_c}$ MPa

Non-prestressed reinforcement:
$f_y = 60$ ksi
Prestressed reinforcement:
$f_{pu} = 270$ ksi

A requirement of two 10-in. deep by 36-in. long web openings for passage of mechanical and electrical services has been introduced. The centers of the openings are located 8½ and 15 ft from one support as shown in Fig. B1.

Design the beam to carry the required loads. Assume that the distance from the vertical edges of the openings to the centroid of the stirrup reinforcement adjacent to the openings is 1 in.

1. Calculate the required embedment length l_d for ½-in. diameter strand from the provisions of Section 12.11 of ACI 318-71. Assume a strand stress of $0.7f_{pu}$ immediately after transfer and 15 percent losses under service load conditions from the effects of creep, shrinkage, and strand relaxation:

$$l_d = (f_{ps} - \tfrac{2}{3} f_{se})\, d_b$$
$$= \{270 - \tfrac{2}{3}(270)(0.7)(1 - 0.15)\}(0.5)$$
$$= 81.5 \text{ in.}$$

Embedment length provided =
$6 + 102 - 18 - 1 = 89$ in. > 81.5 in.

2. Calculate the effective flange width from Section 8.7.2 of ACI 318-71:

¼ of span length =
(¼) (36) (12) = 108 in. > 48 in.
Allowable overhang
= 8 times slab thickness
= 8 (4) = 32 in.
or allowable overhang
= ½ clear distance to next beam
= (½) (48 − 5.75)
= 21.13 in.

Therefore, full flange width is effective.

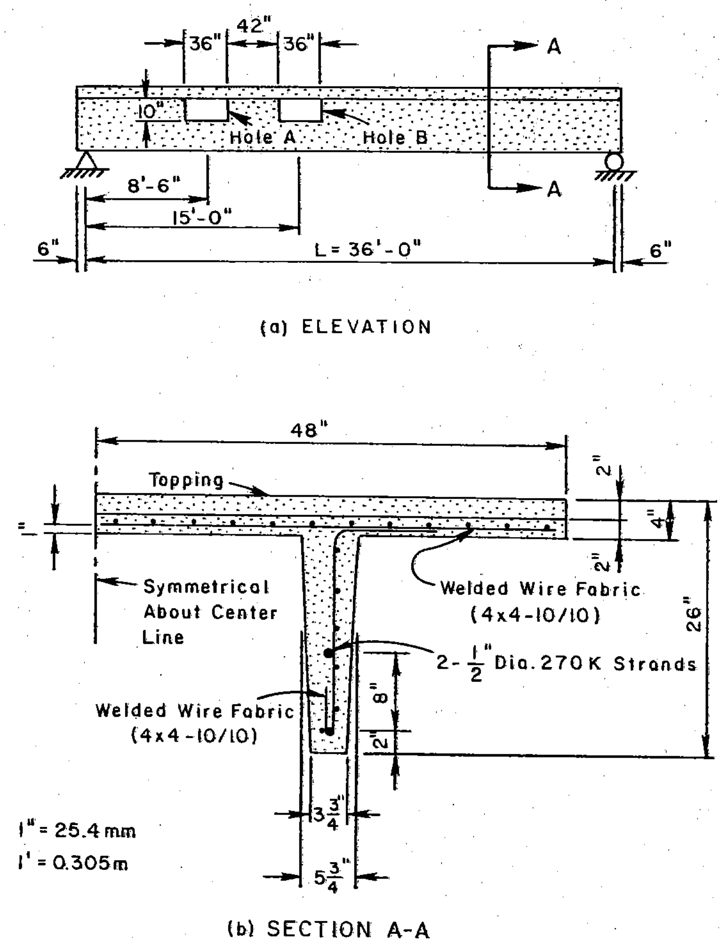


Fig. B1. Elevation and cross section of prestressed beam used in example.

3. Check the width-to-height ratio f post:

$$(42 - 2)/10 = 4.0 > 2.0$$

4. Calculate the beam design loads ssuming a uniformly distributed lead load. Since the double tee is ymmetric about its centerline, design or one-half of a standard double tee ection.

$$w_u = 1.4\, w_d + 1.7\, w_l$$
$$= 1.4\,(299.7) + 1.7\,(50)\,(4)$$
$$= 760 \text{ lb/ft}$$

5. Calculate the design moment and shear at the center of Hole A.

$$M_u = \tfrac{1}{2} w_u L X - \tfrac{1}{2} w_u X^2$$
$$= \tfrac{1}{2}(760)(36)(8.5) - \tfrac{1}{2}(760)(8.5)^2$$
$$= 88{,}825 \text{ ft-lb} = 1066 \text{ in.-kips}$$

$$V_u = \tfrac{1}{2} w_u L - w_u X$$
$$= \tfrac{1}{2}(760)(36) - (760)(8.5)$$
$$= 7220 \text{ lb} = 7.2 \text{ kips}$$

6. Calculate the size of stirrup reinforcement adjacent to the opening:

$$A_v = \frac{V_u}{\phi f_y}$$
$$= \frac{7200}{(0.85)(60{,}000)} = 0.14 \text{ sq in.}$$

Use U-shaped No. 3 stirrup, i.e., 0.22 sq in. of steel each side of opening.

7. Calculate the axial forces from Eqs. (4) and (5):

$$C = \frac{M - P\,(\Delta d)}{d_s}$$

where d_s = 17.57 in. based on transformed strut properties and Δd = 0.25 in.

$$C = \{\,1066 - (2)\,(270)\,(0.153)\,(0.7) \times (1 - 0.15)\,(0.25)\,\}\,/17.57$$
$$= 60 \text{ kips (compression)}$$

$$T = \frac{M - P\,(d_s + \Delta d)}{d_s}$$
$$= \{\,1066 - (2)\,(270)\,(0.153)\,(0.7) \times (1 - 0.15)\,(17.57 + 0.25)\,\}\,/17.57$$
$$= 10.8 \text{ kips (tension)}$$

8. Check the extent of cracking in the tensile strut from Eqs. (6) and (9).

The value of A_{gt} is calculated as 53.2 sq in.

$$6A_{gt}\sqrt{f'_c} = \frac{6\,(53.2)\sqrt{6000}}{1000}$$
$$= 24.7 \text{ kips}$$

T = 10.8 kips < 24.7 kips

Therefore, the tensile strut is not penetrated by a crack over its full depth.

9. Calculate the strut shear forces from Eqs. (10) and (11). The section properties of the struts are:

I_c = 214 in.^{4} (with topping, based on transformed section)

I_{cu} = 32 in.^{4} (without topping)

I_t = 642 in.^{4}

$I_{t(cr)}$ = 72 in.^{4}

From Eq. (10) the shear force in the compressive strut is:

$$V_c = V\left|\frac{I_c}{I_c + I_{t(cr)}}\right|$$

$$V_c = 7.2\left|\frac{214}{214 + 72}\right|$$
$$= 5.4 \text{ kips}$$

To obtain a conservative estimate of shear force in the tensile strut, assume that cracking causes no redistribution of dead load shear, V_d, to the compressive strut.

$$V_d = \tfrac{1}{2}\,w_d L - w_d X$$
$$= \tfrac{1}{2}\,(299.7)\,(36) - (299.7)\,(8.5)$$
$$= 2847 \text{ lb} = 2.9 \text{ kips}$$

$$V_t = V_d\left|\frac{I_t}{I_{cu} + I_t}\right| + (V_u - V_d)\left|\frac{I_t}{I_c + I_t}\right|$$

$$= 2.9\left[\frac{642}{32 + 642}\right] + (7.2 - 2.9)\left|\frac{642}{214 + 642}\right|$$

$$= 6.0 \text{ kips}$$

10. Calculate the moments at ends of the struts using the free body diagram of Fig. B2. Assume strut lengths of 36 + 2 = 38 in. and hinges at midlengths of the struts:

In compressive strut

$$M_c = \pm\,5.4\,(19)$$
$$= \pm\,102.6 \text{ in.-kips}$$

In tensile strut

$$M_t = \pm\,6.0\,(19)$$
$$= \pm\,114.0 \text{ in.-kips}$$

11. Determine the magnified moments in the compressive strut to account for slenderness effects. Using Eq. (10-5) from Section 10.11.5 of ACI 318-71, for a frame not braced against sidesway:

$$\delta = \frac{C_m}{1 - \dfrac{P_u}{\phi P_c}}$$

where

C_m = 1.0 for unbraced member

P_u = C = 60 kips

ϕ = 0.7

Calculating dead load shear in compressive strut:

$$V_c = 2.9\left|\frac{32}{32 + 642}\right|$$
$$= 0.1 \text{ kips}$$

$$\beta_d = \frac{1.4\,(0.1)\,(19)}{(5.4)\,(19)}$$
$$= 0.03$$

Using Eq. (10-8):

$$EI = \frac{\dfrac{E_c I_c}{2.5}}{1 + \beta_d}$$

$$= \frac{\dfrac{(57\sqrt{6000})\,(214)}{2.5}}{1.03}$$

$$= 3.67 \times 10^5 \text{ lb-in.}^2$$

Using Eq. (10-6):

$$P_c = \frac{\pi^2 EI}{(kl_u)^2}$$

$$= \frac{\pi^2\,(3.67 \times 10^5)}{(1.0 \times 38)^2}$$

$$= 2508 \text{ kips}$$

$$\delta = \frac{1.0}{1 - \dfrac{60}{0.7\,(2508)}}$$

$$= 1.04$$

$$\delta M_c = \pm\,1.04\,(102.6)$$
$$= \pm\,106.7 \text{ in.-kips}$$

12. Check the positive and negative flexural capacities of struts with respect to interaction curves shown in Figs. B3 and B4 (see next page).

For C = 60 kips

$(\delta M_c)_{allow}$ = + 114 in.-kips
− 143 in.-kips (ok)

$(M_t)_{allow}$ = + 120 in.-kips
− 152 in.-kips (ok)

13. The net axial force in the tensile strut is 10.8 kips tension (from

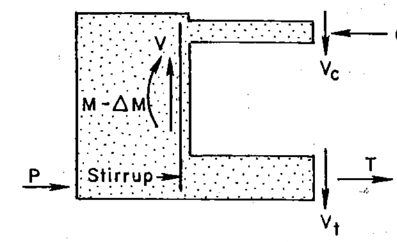


Fig. B2. Free body diagram.

Step 7). Determine the shear capacity from Section 11.4.4 of ACI 318-71.

$$v_u = \frac{V_u}{\phi b_w d}$$

$$= \frac{6.0}{(0.85)\,(3.75)\,(0.8)\,(12)}$$

$$= 0.20 \text{ ksi} = 200 \text{ psi}$$

$$v_c = 2\left|1 + 0.002\frac{N_u}{A_g}\right|\sqrt{f'_c}$$

$$= 2\left|1 + 0.002\frac{(-10800)}{53.2}\right|\sqrt{60}$$

$$= 92 \text{ psi} < 200 \text{ psi}$$

Hence, additional reinforcement is required. Proportion the shear reinforcement in tensile strut according to Eq. (11-13) in ACI 318-71. Maximum spacing by Section 11.1.4(b) of ACI 318-71 is $0.75h$ = 9 in.

$$A_v = \frac{(v_u - v_c)\,b_w s}{f_y}$$

$$= \frac{(200 - 92)\,(3.75)\,(9)}{60000}$$

$$= 0.06 \text{ sq in.}$$

Use No. 3 bar single leg stirrups at 9-in. centers in tensile strut to provide A_v = 0.11 sq in. Anchor stirrups around the strands using a 180-deg bend at each end.

14. Determine the shear capacity of compressive strut using the provisions

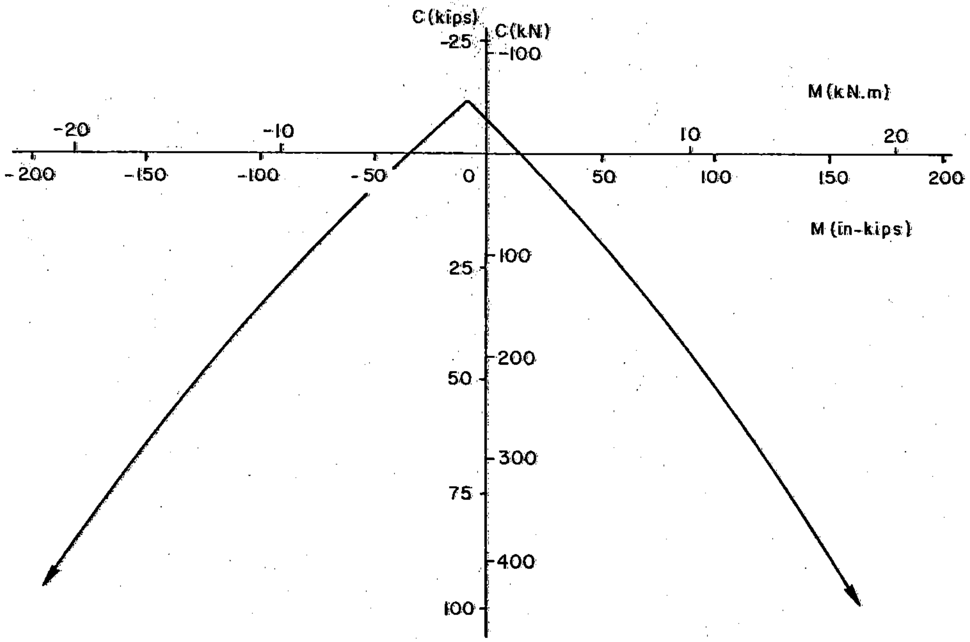


Fig. B3. Interaction curve for compressive strut.

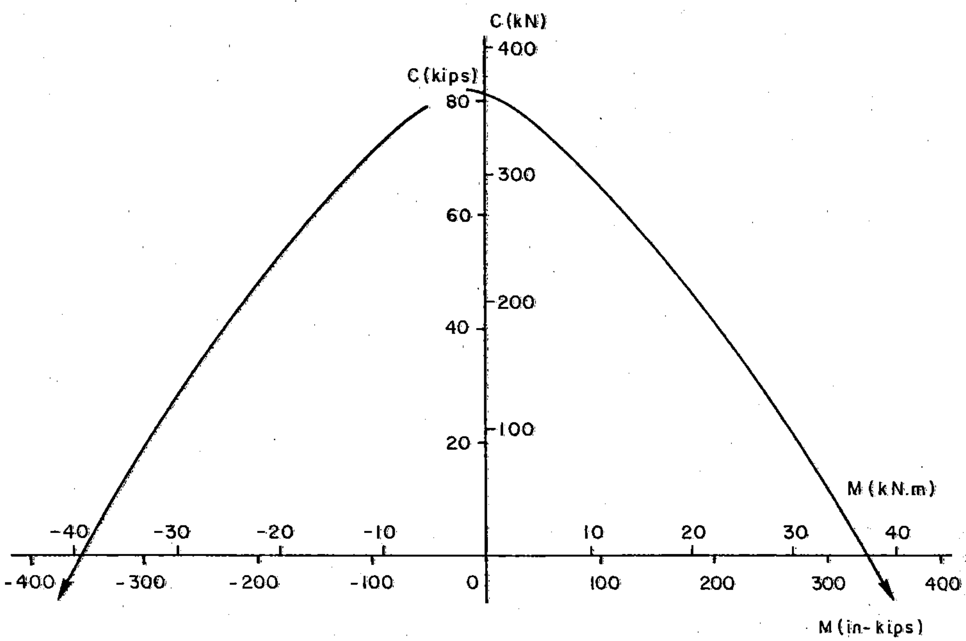


Fig. B4. Interaction curve for tensile strut.

of Section 11.4.3 of ACI 318-71. Since d is different at each end of the strut, check capacity at each end. For the end having $d = 1$ in.

$$v_u = \frac{V_u}{\phi b_w d}$$

$$= \frac{5.4}{(0.85)(48)(1)}$$

$$= 0.13 \text{ ksi} = 130 \text{ psi}$$

$$v_c = 2\left[1 + 0.0005\frac{N_u}{A_g}\right]\sqrt{f'_c}$$

where $A_g = 165$ sq in. is the gross area of the compressive strut transformed to 6000-psi concrete. Therefore:

$$v_c = 2\left[1 + 0.0005\frac{60000}{165}\right]\sqrt{6000}$$

$$= 183 \text{ psi} > 130 \text{ psi (ok)}$$

For the end having $d = 3$ in., assume the capacity is governed by topping concete having $f'_c = 3000$ psi since this is in compression. Thus, in Eq. (11-6) of ACI 318-71, $A_g = 193$ sq in.

$$v_u = \frac{5.4}{(0.85)(48)(3)}$$

$$= 0.04 \text{ ksi} = 40 \text{ psi}$$

$$v_c = 2\left[1 + 0.0005\frac{60000}{193}\right]\sqrt{3000}$$

$$= 127 \text{ psi} > 40 \text{ psi (ok)}$$

15. Check the horizontal shear capacity between flange and topping using Sections 17.5.3 and 17.5.4 of ACI 318-71.

$$v_{dh} = \frac{V_u}{\phi b_v d}$$

$$= \frac{5.4}{(0.85)(48)(3)}$$

$$= 0.04 \text{ ksi} = 40 \text{ psi}$$
$$< 80 \text{ psi (allowable)}$$

16. Following a similar procedure for Hole B results in the following:

Design moment at center of opening:

$M_u = 1436$ in.-kips

Design shear at center of opening:

$V_u = 2.3$ kips

Stirrup reinforcement adjacent to opening: U-shaped No. 3 bar

$C = 81$ kips [from Eq. (4)]

$T = 31.9$ kips [from Eq. (5)]

$6 A_{gt}\sqrt{f'_c} = 24.7$ kips
$< T = 31.9$ kips

Therefore, the tensile strut of Hole B is penetrated by a full-depth crack. The distribution of shear to the struts is determined from Eqs. (7) and (8).

$V_c = V_u = 2.3$ kips
$V_t = 0$

Moments adjusted for slenderness effects become:

$\delta M_c = \pm 4.3$ in.-kips
$M_t = 0$

From the interaction diagram in Fig. B3, for $C = 81$ kips:

$(\delta M_c)_{allow} = +140$ in.-kips
-172 in.-kips (ok)

Shear design for the end of the strut with $d = 1$ in. results in

$v_u = 56$ psi
$v_c = 193 \text{ psi} > 56 \text{ psi (ok)}$

For the end of the strut with $d = 3$ in.

$v_u = 17$ psi
$v_c = 133 \text{ psi} > 17 \text{ psi (ok)}$

Checking horizontal shear between topping and flange leads to:

$v_{dh} = 17 \text{ psi} < 80 \text{ psi (ok)}$

17. The shear stress in the post is calculated as:

$$v_u = \frac{(C)_{Hole\ B} - (C)_{Hole\ A}}{\phi b_w d}$$

$$= \frac{81 - 60}{(0.85)(4.84)(42 - 1)}$$

$$= 0.12 \text{ ksi} = 120 \text{ psi}$$

$$2\sqrt{f'_c} = 155 \text{ psi (ok)}$$

Part II—Stress and deflection requirements

Check deflection and stresses around the openings at service load. Assume a strand stress of $0.7f_{pu}$ immediately following transfer of prestress. Also, assume that the entire prestress loss of 15 percent occurs after the topping is cast.

1. Service load stresses will be calculated at the extreme fibers of the struts at the four points shown in Fig. B5. Stresses will be calculated assuming that full dead load, including the weight of the topping, and the full prestress force are resisted by the untopped beam. After the topping becomes an integral part of the beam, additional stresses resulting from live load and loss of prestress will be calculated.

2. Referring to Fig. B2, the forces caused by dead load and prestress acting on the untopped beam at Hole A are calculated as follows:

$$P = A_s f_s$$
$$= 2\,(0.153)\,(0.7)\,(270)$$
$$= 57.8 \text{ kips}$$

$$M = \tfrac{1}{2}\, w_d L X - \tfrac{1}{2}\, w_d X^2$$
$$= \tfrac{1}{2}\,(199.7)\,(36)\,(8.5) - \tfrac{1}{2}\,(199.7)\,(8.5)^2$$
$$= 23340 \text{ ft-lb} = 280 \text{ in.-kips}$$

$$V = \tfrac{1}{2}\, w_d L - w_d X$$
$$= \tfrac{1}{2}\,(199.7)\,(36) - (199.7)\,(8.5)$$
$$= 1897 \text{ lb} = 1.9 \text{ kips}$$

$$V_c = V\left[\frac{I_{cu}}{I_{cu} + I_t}\right]$$
$$= 1.9\left[\frac{32}{32 + 642}\right]$$
$$= 0.1 \text{ kips}$$

$$V_t = V - V_c$$
$$= 1.9 - 0.1$$
$$= 1.8 \text{ kips}$$

The calculated values of d_s and Δd shown in Fig. 11a for the untopped beam are:

$$d_s = 16.75 \text{ in.}$$
$$\Delta d = 0.25 \text{ in.}$$

From Eqs. (4) and (5):

$$C = \frac{280 - 57.8\,(0.25)}{16.75}$$
$$= 15.9 \text{ kips}$$

$$T = \frac{280 - 57.8\,(16.75 + 0.25)}{16.75}$$
$$= -42.0 \text{ kips}$$

3. For the bottom extreme fiber at Point 1:

$$f = -\frac{C}{A} - \frac{V_c\left(\frac{l}{2}\right) y_b}{I_c}$$

$$= -\frac{15900}{97} - \frac{(100)\,(19)\,(1)}{32}$$

$$= -223 \text{ psi}$$

Stresses at other locations are calculated using a similar procedure.

4. Referring to Fig. 11b, the change in forces caused by live load and 15 percent loss of prestress at Hole A are calculated following the procedure in Step 2.

$$P = -2\,(0.153)\,(0.7)\,(0.15)\,(270)$$
$$= -8.7 \text{ kips}$$

$$M = \tfrac{1}{2}\,(50)\,(4)\,(36)\,(8.5) - \tfrac{1}{2}\,(50)\,(4)\,(8.5)^2$$
$$= 23375 \text{ ft-lb} = 281 \text{ in.-kips}$$

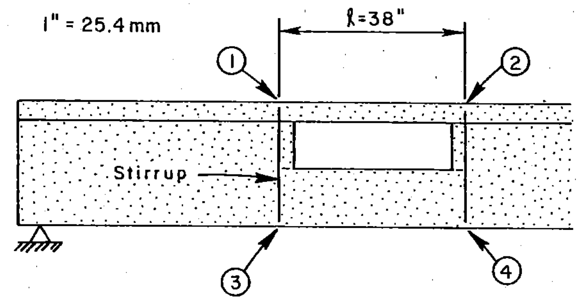


Fig. B5. Location of stresses.

$$V = \tfrac{1}{2}\,(50)\,(4)\,(36) - (50)\,(4)\,(8.5)$$
$$= 1900 \text{ lb} = 1.9 \text{ kips}$$

$$V_c = V\left[\frac{I_c}{I_c + I_t}\right]$$
$$= 1.9\left[\frac{214}{214 + 642}\right]$$
$$= 0.5 \text{ kips}$$

$$V_t = V - V_c$$
$$= 1.9 - 0.5$$
$$= 1.4 \text{ kips}$$

From Eqs. (4) and (5):

$$C = \frac{281 - (-8.7)\,(0.25)}{17.57}$$
$$= 16.1 \text{ kips}$$

$$T = \frac{281 - (-8.7)\,(17.57 + 0.25)}{17.57}$$
$$= 24.8 \text{ kips}$$

5. Stresses for live load and 15 percent loss of prestress acting on the topped beam are now calculated from first principles. A summary of service load stresses for Hole A is presented in Table B1.

Table B1. Stresses (psi) at service load.

Location	Hole A		Hole B	
	Top Fiber	Bottom Fiber	Top Fiber	Bottom Fiber
Calculated				
1	-48*	-455	-71*	-519
2	-92*	-241	-112*	-421
3	+648	-1052	+451	-279
4	-614	+478	-29	+241
Allowable				
1	-1350, +657*	-2700, +930	-1350, +657*	-2700, +930
2	-1350, +657*	-2700, +930	-1350, +657*	-2700, +930
3	-2700, +930	-2700, +930	-2700, +930	-2700, +930
4	-2700, +930	-2700, +930	-2700, +930	-2700, +930

*Stresses in topping

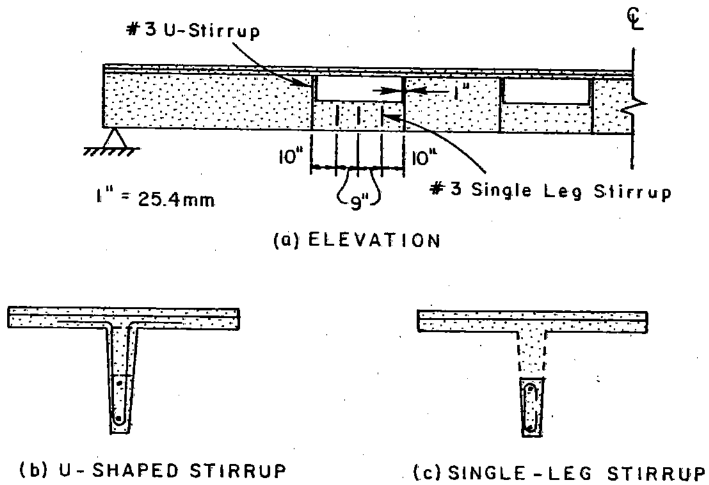


Fig. B6. Details of additional reinforcement.

6. Stresses at service load for Hole B are calculated in a similar manner. A summary of service load stresses for Hole B is given in Table B1.

7. Allowable stresses in tension and compression for 3000- and 6000-psi concrete from Section 18.4.2 of ACI 318-71 are as follows:

For 3000-psi concrete—
in compression,
$0.45 f'_c = 1350$ psi
in tension, $12\sqrt{f'_c} = 657$ psi

For 6000-psi concrete—
in compression,
$0.45 f'_c = 2700$ psi
in tension, $12\sqrt{f'_c} = 657$ psi

Modulus of rupture is determined from Section 9.5.2 of ACI 318-71 are as follows:

For 3000-psi concrete—
$f_r = 7.5\sqrt{f'_c} = 411$ psi

For 6000-psi concrete—
$f_r = 7.5\sqrt{f'_c} = 581$ psi

As indicated in Table B1, no allowable stresses are exceeded. However, cracking is indicated in the top extreme fiber of the tensile strut at Location 3.

8. Estimate the midspan deflection caused by live load and 15 percent loss of prestress. Assume that the component of deflection caused by live load shear at each opening is determined from the following expression derived from moment-area principles:

$$\delta_v = 2\,\frac{V\left(\dfrac{l}{2}\right)^3}{3E_c\,(I_c + I_t)}$$

where l is the opening length of 38 in. and E_c is the concrete modulus. For Hole A, conservatively estimate $I_t = 72$ in.^{4} This is the moment of inertia of the fully cracked strut.

Calculate the component of midspan deflection caused by loss of prestress from the expression:

$$\delta_p = -\frac{PeL^2}{8EI}$$

where $e = 13.27$ in. is the eccentricity of the prestressing steel.

Conservatively estimate $I = 12939$ in.^{4}, the moment of inertia of the beam at a section through an opening.

Total midspan deflection is calculated as:

$$\delta = \frac{5wL^4}{384E_cI} + (\delta_v)_{Hole\ A} + (\delta_v)_{Hole\ B} + \delta_p$$

$$= \frac{5\left(\dfrac{200}{12}\right)(36 \times 12)^4}{384\,(4415000)\,(12939)} + \frac{2\,(1900)\,(19)^3}{3\,(4415000)\,(214 + 72)} + \frac{2\,(600)\,(19)^3}{3\,(4415000)\,(214 + 642)} + \frac{(8700)\,(13.27)\,(36 \times 12)^2}{8\,(4415000)\,(12939)}$$

$$= 0.132 + 0.007 + 0.001 + 0.047$$

$$= 0.187 \text{ in.} < L/360 = 1.2 \text{ in. (ok)}$$

The above deflection compares to a calculated deflection of 0.168 in. for a similar beam with no web openings. Note that the increase in deflections caused by the holes is small. This finding was verified by the experimental program.

Design summary

1. Use U-shaped No. 3 stirrups adjacent to both edges of each opening to contain cracking within the struts. See reinforcement details in Fig. B6.
2. Use single-leg No. 3 stirrups at 9-in. centers as additional reinforcement in the tensile strut of Hole A. See reinforcement details in Fig. B6.

Discussion of this paper is invited. Please forward your comments to PCI Headquarters by May 1, 1978.