Design of Slender Concrete Columns—Revisited

by J. G. MacGregor

Revisions are proposed to Sections 10.10 and 10.11 of the ACI Building Code to simplify the design of slender columns and to recognize the use of second-order analyses. These changes are undergoing letter ballot in ACI Committee 318 and if accepted will appear in the 1995 ACI Code. Major changes include the listing of a series of EI values for use in second-order frame analyses, a test for sway and nonsway frames, a flat $q$ value for stability calculations, new slenderness limits, and the method of combining and magnifying the nonsway and sway moments.

Keywords: columns (supports); frames; reinforced concrete; stability; structural design.

The current ACI Building Code provisions for the design of slender columns were developed in the late 1960s and were incorporated in the Code in 1971. Additions to the Commentary in 1977 and 1983 amplified the calculation of $k$ factors and the differentiation between braced and unbraced frames. A major change in 1983 distinguished between sway and nonsway moments and magnified these separately.

In the last decade, second-order analysis programs have become widely available. This proposed revision to the slenderness provisions gives guidance for the use of such analyses in column design.

The proposed revision, presented in an appendix to this paper, has the following major subdivisions:

1. Slenderness effects in compression members—This section allows either a general method or a moment magnifier method for the design of slender columns.

2. Magnified moments—General—This section gives general rules applicable in the moment magnifier method given in Sections 3 and 4. These include values of $E$ and $I$ for use in frame analyses and a test of whether frames are sway or nonsway frames.

3. Magnified moments—Nonsway frames—This section gives rules for designing columns in nonsway frames. Nonsway frames have been separated to make the application of the procedures more evident. The major changes are a new slenderness limit equation and a requirement that the moments in the bracing elements be magnified.

4. Magnified moments—Sway frames—Major changes in this section include the method of calculation of the magnified sway moments and the method by which these are combined with the nonsway moments. A stability check under gravity loads is also required.

The rest of this paper will discuss the individual changes.

SECTION PROPERTIES FOR FRAME ANALYSIS

Traditionally engineers have used the gross moments of inertia of the columns and beams in frame analyses. With the advent of second-order analyses as design tools, however, it is important that the computed lateral deflections closely resemble the anticipated deflections so that realistic $PA$ moments are obtained. This requires realistic $EI$ values.

Member stiffness $EI$ values are used for three things in the proposed code sections on slenderness: (a) in frame analysis, (b) when calculating the effective length factor $k$, and (c) in the design of individual columns. Two different sets of $EI$ values are given. Since the lateral deflections from the frame analysis are affected by the stiffnesses of all the members in the structure, the $EI$ values used in frame analysis should approach the mean values for the individual members. On the other hand, when dealing with the stability of a single isolated member the value used should be a safe lower bound estimate to the $EI$ value for a single column. As a result, the $EI$ values for columns given in Section 2.1 for frame analysis are larger than those for member design given in Section 3.3.

The $EI$ values for second-order frame analysis should be representative of the member stiffnesses immediately before the ultimate condition. At this stage, parts of the beams, slabs, and walls will be cracked in flexure. It is too conservative to base the $EI$ on the cracked moment of inertia because the beam will not be completely cracked at all sections. Instead the $EI$ should be back-calculated from the member stiffness, $K = 4EI/l$, taking into account the distribution of cracking along the member. When dealing with a 20-story building with more than 1000 members and more than 2000 critical sections, it is not economically feasible for designers to go
through such calculations, and simplified methods must be used to compute $EI$.

Kordina and Hage have studied the variation of stiffness for various types of frame members subjected to gravity load moments, lateral load moments, and combinations of the two. Based on these studies, MacGregor and Hage concluded that a reasonable estimate of $EI$ for second-order analysis would be based on the ACI value of $E$ and $I = 0.4 I_e$ for beams and 0.8 $I_e$ for columns.

Fig. 1, taken from Hage, shows the variation in the effective $EI$ for a T-beam as the load level is increased.

Fig. 1(a) considers gravity load moments. The term $\eta$ is the ratio of the fixed end moment to the nominal moment capacity of the end of the beam

$$\mu = \frac{wL^2}{12Mn}$$

For small loads (small $\mu$), the effective $EI$ slightly exceeds $E I_e$ due to the presence of the reinforcement. As $\eta$ increases, parts of the beam crack and the effective $EI$ approaches $0.4E I_e$. Fig. 1 is plotted for one particular cross section. Similar trends were obtained for other sections including rectangular sections.

Fig. 1(b) gives the effective $EI$ for beam moments due to lateral loads. The term $\mu$ in Fig. 1(b) is the ratio of the end moment due to lateral load to the nominal moment capacity. Again $EI$ approaches 0.4 $E I_e$ as $\mu$ approaches 1.0. Fig. 1(c) considers combinations of $\mu$ and $\eta$. Similar graphs are obtained for rectangular cross sections. Hage proposed $EI$ for beams equal to $0.4E I_e$.

Once the effective $EI$ of beams had been obtained, Hage obtained the value of the effective $EI$ for columns by back calculating from the lateral deflection of laboratory tests of reinforced concrete frames. This gave $EI = 0.8E I_e$.

Furlong proposed that the $EI$ of T-beams be taken as the gross $EI$ of the stem but not less than $0.5 E I_e$ where $I_e$ is for the T-shaped cross section. For lower floor columns he suggested $EI = 0.6 E I_e$, for upper floor columns $0.3 E I_e$.

Dixon back-calculated $EI$ for columns in 13 frame tests using a second-order analysis program. Based on Hage's work, he assumed the $EI$ of the beams as $0.5E I_e$. Using this beam stiffness, the column stiffness which gave the best conservative estimate of the measured lateral deflections was $0.5 E I_e$.

McDonald generated moment-end rotation relationships for T-beams, one-way slabs, and columns. For T-beams with 1.2 percent steel, he found $EI$ ranged from 0.37 to 0.44 $E I_e$. For one-way slabs with 0.5 percent steel, $EI$ varied from 0.16 to 0.22 $E I_e$. For columns $EI$ varied from 0.66 to 0.89 $E I_e$. MacDonald proposed $EI$ values of 0.42 $E I_e$, 0.20 $E I_e$, and 0.7 $E I_e$ for T-beams, one-way slabs, and columns, respectively.

A strength reduction factor $\varphi$ should be included in the second-order analysis to account for the variability in the predicted lateral deflections resulting from simplifications in modeling the structure and the assumed values of $E$ and $I$. Later in this paper a single value of $\varphi = 0.75$ is proposed for use in the moment magnifier equations. This is related to the probability that an individual column will be understrength. The variability of the lateral deflections of a frame are related to the variability of the mean $E$ and $I$ values of all the members of the frame. Since this is considerably less than the variability of an individual member in the frame, the $\varphi$ factor applied to the second order analysis should be closer to 1.0 than that for an individual member. A value of 0.875 is proposed.

The $EI$ values proposed by MacGregor and Hage are recommended for use in frame analysis and are incorporated in the proposed revisions. When these are multiplied by $\varphi = 0.875$ they become:

a. Modulus of elasticity = $E$, from Section 8.5.1.

b. Moment of inertia

Beams: $0.35 I_e$
Columns: $0.70 I_e$

Fig. 1—Variation of beam stiffness with load
SWAY AND NONSWAY FRAMES

Traditionally, frames have been classified as braced frames and unbraced frames when evaluating effective length factors. Since all practical frames deflect laterally under lateral loads there is no such thing as a truly braced frame. For this reason the revised slenderness provisions refer to sway and nonsway frames rather than unbraced and braced frames. A nonsway frame is defined as one in which the second-order magnification of sway moments is 5 percent or less. This is checked by determining if

\[ Q = \frac{\sum P_u \Delta_0}{V_{dlc}} \]  

(2)

is equal or less than 0.05 where \( V_u \) is the lateral shear in the story and \( \Delta_0 \) is the first-order relative lateral deflection of the top and bottom of the story due to \( V_u \). As shown in References 5 and 10, the sway magnifier \( \delta_0 \) is approximated closely by \( 1/(1 - Q) \) giving rise to the limit on \( Q \) of 0.05 for nonsway frames. The Commentary to ACI 318-89 set a limit of 0.04 on \( Q \) corresponding to a 4 percent permissible increase in sway moments. A slightly more liberal limit is given in the proposed revisions because the lateral deflection \( \Delta_0 \) is based on the values of \( EI \) given earlier. The 1990 CEB-FIP Model Code requires consideration of second-order effects if lateral deflections result in more than a 10 percent increase in sway moments.

The Commentary to ACI 318-89 also defined a braced story as one in which the sum of the lateral stiffnesses of the bracing elements exceeded six times the sum of the lateral stiffnesses to the columns. This definition can be unconservative if \( \sum P_u/P_{crf} \) is high, where \( P_{crf} \) is the critical load of the entire frame.

DESIGN OF NONSWAY FRAMES

Slenderness limit

Section 10.11.4.1 of ACI 318-89 allows the effects of slenderness to be neglected if

\[ k_{Lu}/r < 34 - 12M_{1b}/M_{2b} \]  

(3)

This equation was derived from Code Eq. (10-7), assuming \( \delta_0 \) was limited to 1.05. Two things are wrong with this equation. First, the original derivation was carried out in the late 1960s using a form of Eq. (10-7) which did not include the \( \psi \) factor. As a result, the code slenderness limit corresponds to a magnifier considerably greater than 1.05. Second, the equation ignores the effect of the axial load level on the moment magnification. In the proposed revision, Eq. (3) is replaced by Eq. (4)

\[ \frac{k_{Lu}}{r} \leq \frac{25 - 10(M_1/M_2)}{\sqrt{f_{c}^{'2}A_{g}}} \]  

(4)

The shaded bands in Fig. 2 show \( k_{Lu}/r \) values corresponding to a magnification factor of 1.05 for two axial load levels.
for braced hinged columns. Eq. (4) is shown by dashed lines and Eq. (3) by the broken line. For a column with \( f_c' = 3 \text{ ksi}, f_y = 60 \text{ ksi}, 2 \text{ percent steel}, \) and \( \gamma = 0.75, \ e/h = 0.10 \) corresponds to \( P_c/A_e f_c' = 0.68 \) and the balanced eccentricity corresponds to \( P_c/A_e f_c' = 0.27. \) As a result, the \( kl_e/r \) value beyond which a column is classed as slender will tend to increase compared to the 1989 Building Code.

**EI equations for slenderness calculations**

The \( EI \) equations in ACI 318-89 are

\[
EI = \frac{0.4 E_c l_g + E_s l_se}{(1 + \beta_d)}
\]

(5)

and

\[
EI = \frac{0.4 E_c l_g}{(1 + \beta_d)}
\]

(6)

Eq. (5) and (6) have been retained, but in the draft they are multiplied by \( \varphi, \) to maintain consistency with the format of the \( EI \) values proposed for structural analysis. For preliminary design of nonsway frames, Eq. (6) could be replaced with

\[
EI = 0.25 E_c l_g
\]

(7)

This is equivalent to assuming \( \beta_d = 0.60. \) When lateral load moments govern the design, \( \beta_d \) will be zero and Eq. (7) will be excessively conservative.

**Strength reduction factor \( \varphi_s \)**

The 1971 and subsequent codes have taken the strength reduction factor \( \varphi \) in the moment magnifier equations equal to 0.7 or 0.75 for tied and spiral columns. This increases to 0.9 for the pure moment case. These values were originally derived for axially loaded short columns.

Mirza, Lee, and Morgan\(^{12} \) suggest that for the practical range of variables for tied columns, \( \varphi \) could be taken equal to 0.80, while for the extreme range of variables, \( \varphi, \) should be between 0.7 and 0.75. A flat value of 0.75 has been used in the magnifier equations in the proposed revisions. This remains constant throughout the whole range of eccentricity ratios \( e/h \) and applies to tied or spiral columns alike. To distinguish it from the regular \( \varphi \) factors for columns it has been called \( \varphi_s. \)

**DESIGN OF SWAY FRAMES**

In the proposed revisions, the design of sway frames for slenderness consists of three steps:

1. The magnified sway moments \( \delta M_s \) are computed. This may be done in one of three ways which will be discussed in the next part of this paper.

2. The magnified sway moments \( \delta M_s \) are added to the unmagnified nonsway moments \( M_{ns} \) at each end of each column.

\[
M_1 = M_{1ns} + \delta_s M_{1s}
\]

(8)

\[
M_2 = M_{2ns} + \delta_s M_{2s}
\]

(9)

where \( M_2 \) is the larger of the two end moments.

3. If the column is slender and the loads on it are high, it is checked to see whether the moments between the ends of the column exceed these at the ends of the column. This is done using the nonway frame magnifier \( \delta_m, \) with \( P_c \) computed assuming \( k = 1.0 \) or less.

This is an extension of the separation of \( \delta_8 \) and \( \delta_1 \) and nonway and sway moments proposed by Ford, Chang, and Breen\(^{13} \) and introduced in the 1983 Code. The procedure has been changed because in most sway frames the possibility of the maximum moment occurring between the ends of the column is greatly reduced by the presence of the large double-curvature moments due to the lateral loads.

**Determination of whether maximum moment is at the end of the column**

In most columns in sway frames, the maximum column moment will occur at one end of the column, and the third step listed in the previous section will not be required. It is useful to have a simple way of determining when this will occur so that Step 3 is avoided when not required.

Galambos\(^{14} \) has shown that the maximum moment \( M_c, \) in an elastic beam column loaded with an axial load and end moments \( M_1 \) and \( M_2 \) is

\[
M_c = M_2 \delta
\]

(10)

where

\[
\delta = \sqrt{1 + \left( \frac{M_1}{M_2} \right)^2 - 2 \left( \frac{M_1}{M_2} \right) \cos \alpha \over \sin \alpha}
\]

(11)

and \( \alpha^2 = EI / P l^2. \) It will be assumed that stability effects can be disregarded if \( M_c \) is not more than 1.05\( M_2, \) i.e., \( \delta \leq 1.05. \) Eq. (11) can be solved for the combinations of \( M_1/M_2 \) and \( \alpha \) corresponding to \( \delta = 1.05. \) These are plotted with the solid line in Fig. 3. Combinations of \( M_1/M_2 \) and a falling below this line can be designed for the second-order end moments without further magnification. For sway frames the range of \( M_1/M_2 \) of interest is approximately −0.5 to −1.0, in the double curvature range. In this region, the solid line may be approximated by

\[
\frac{M_1}{M_2} = 0.6 - \frac{P a^2}{5.25 EI}
\]

(12)

Substituting \( EI = 0.4 E A r^2 \) and the ACI code value of \( E_c, \) and solving for the case of \( M_1/M_2 = −0.5 \) and \( f_c' = 8000 \text{ psi} \) (55 MPa) gives

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The classical case of sidesway buckling under gravity loads must be checked for sway frames. This is checked in different ways depending on which of the three methods was used to compute $\delta M$. When $\delta M$ is calculated using a second-order elastic frame analysis, the possibility of sidesway buckling is checked by analyzing the structure loaded with the factored dead and live loads plus an arbitrarily chosen lateral load applied to the frame. The designer is free to choose any lateral load or set of lateral loads he or she wishes provided the load is large enough that the increase in lateral deflections due to second-order effects is distinguishable in the results. Thus, for example, the lateral load could be the factored wind loads used in designing the frame or it could be a single load applied at the top of the frame. For unsymmetrical frames for which gravity loads cause a lateral deflection, the arbitrary lateral load should be applied in the direction that increases the gravity load deflection. The frame is analyzed twice for this lateral load, once using a first-order elastic analysis and again using a second-order elastic analysis, and the ratio of lateral deflections is computed. If this ratio exceeds 2.5, the frame is too flexible laterally and hence will close to failing due to sidesway buckling. The values of $EI$ used in these analyses should be divided by $(1 + \gamma_1)$ corresponding to the factored gravity loads.

When $\delta M$ is calculated using $\delta = 1/(1 - Q)$, the test for possible sidesway buckling is carried out by setting an upper limit on $Q$ where $Q$ is calculated using $\Sigma P$ for 1.4 dead load and 1.7 live loads. The shear $V_s$ is due to any assumed lateral loading (the wind loads used in design, for example), and $\Delta_s$ is the first-order relative lateral displacement of the top and bottom of the story caused by $V_s$, computed using $EI$ values divided by the corresponding $(1 + \beta_d)$ for the gravity-load case. The limit on $Q$ of 0.6 corresponds to a magnified deflection of 2.5 times the first-order deflection.

When $\delta M$ is calculated using the traditional sway magnifier equation, the sway magnifier must be positive and should not exceed 2.5, again with $\beta_d$ based on the ratio of factored axial loads. For higher $\delta$ values, the frame will be very susceptible to changes in $EI$, foundation rotations, and other weakening factors.

**SUMMARY**

Revisions are proposed to the slender column design provisions of the ACI Building Code to simplify the design of slender columns and to recognize the use of second-order analysis. The most significant change concerns the superposition and magnification of nonsway and sway moments.

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Chairman of ACI 318 Subcommittee D, who reviewed the various drafts to check if the code clauses followed a logical order comments. His design examples flagged sections needing change. The author also wishes to thank Professors R. W. Furlong and R. Green who worked with the author in developing the proposed revisions.

REFERENCES


NOTATION

$A_g$ = gross area of cross-section, in.

$E_e$ = modulus of elasticity of concrete, psi

$E_r$ = modulus of elasticity of reinforcement, psi

$EI$ = flexural stiffness of compression member

$f_c'$ = specified compressive strength of concrete, psi

$I_g$ = moment of inertia of gross concrete section about centroidal axis, neglecting reinforcement

$I_{re}$ = moment of inertia of reinforcement about centroidal axis of member cross section

$k$ = effective length factor

$K$ = flexural stiffness of a beam

$l_c$ = length of a compression member in a frame, measured from center to center of the joints in the frame

$l_u$ = unsupported length of compression member

$M_n$ = nominal moment capacity of cross section

$M_1$ = smaller factored end moment on a compression member, positive if member is bent in single curvature, negative if bent in double curvature

$M_{1ns}$ = factored end moment on a compression member at the end at which $M_1$ acts, due to loads that cause no appreciable side- way

$M_{1f}$ = factored end moment on compression member at the end at which $M_1$ acts, due to loads that result in appreciable sideways, calculated using a first-order elastic frame analysis

$M_2$ = larger factored end moment on compression member, always positive

$M_{2, min}$ = minimum value of $M_2$

$M_{2sd}$ = factored end moment on compression member at the end at which $M_2$ acts, due to loads that cause appreciable sideway, calculated by a first-order elastic frame analysis

$P_{cof}$ = critical load of a frame

$P_w$ = factored axial load in a column

$Q$ = stability index for a story

$r$ = radius of gyration of cross section of a compression member

$V_u$ = factored shear in a story

$w$ = uniform load on a beam

$\beta_d$ = a. for nonsway frames, $\beta_d$ is the ratio of the maximum factored axial dead load to the total factored axial load

b. for sway frames, except for gravity load stability checks, $\beta_d$ is the ratio of the maximum factored sustained lateral load to the maximum total factored lateral load in that story

c. for gravity load stability checks of sway frames, $\beta_d$ is the ratio of the maximum factored axial dead load to the total factored axial load

$\Delta_o$ = first order relative lateral deflection of the top and bottom of a story due to $V_u$ computed using the specified stiffness values

$\mu$ = ratio of each moment due to lateral loads to the nominal moment capacity

$\eta$ = ratio of maximum end moment due to gravity loads to the nominal moment capacity

$\phi$ = strength reduction factor

APPENDIX A—PROPOSED CHANGES TO ACI 318 SLENDERNESS PROVISIONS

1—Slenderness effects in compression members

1.1—Except as allowed in Section 1.2, the design of compression members, restraining beams, and other supporting members shall be based on the factored forces and moments from a second-order analysis considering material nonlinearity and cracking, as well as the effects of member curvature and lateral drift, duration of the loads, shrinkage and creep, and interaction with the supporting foundation. The dimensions of the cross sections used in the analysis shall be within 10 percent of the dimensions of the members shown on the design drawings, or the analysis shall be repeated. The analysis procedure shall have been shown to result in prediction of strength in substantial agreement with the results of comprehensive tests of columns in indeterminate reinforced concrete structures.

1.2—In lieu of the procedure prescribed in Section 1.1, it is permissible to base the design of compression members, restraining beams, and other supporting members on axial forces and moments from the analyses described in Section 2.

2—Magnified moments—General

2.1—The factored axial forces $P_w$, the factored moments $M_1$ and $M_2$ at the ends of the column and, where required, the first-order relative lateral story deflections $\Delta_o$, shall be computed using an elastic first-order frame analysis with the section properties determined taking into account the influence of axial loads, the presence of cracked regions along the length of the member, and effects of duration of the loads. Alternatively, it is permissible to use the following properties for the members in the structure:

a. Modulus of elasticity $E_e$ from Section 8.5.1

b. Moment of inertia

- Beams: $0.35 I_g$
- Columns: $0.70 I_g$
- Walls—Uncracked: $0.70 I_g$
- Cracked: $0.35 I_g$
Flat plates and flat slabs: 0.25 $I_g$

c. Area: $1.0 A_g$

The moments of inertia used in Section 2.1, and Section 4.3 shall be divided by $(1 + \beta_d)$ when (a) sustained lateral loads act, or for (b) stability checks made according to Section 4.5.

2.2.—It is permissible to take the radius of gyration $r$ equal to 0.30 times the overall dimension in the direction $V_o$ is considered for rectangular compression members and 0.25 times the diameter for circular compression members. For other shapes, it is permissible to compute the radius of gyration for the gross concrete section.

2.3.—Unsupported length of compression members

2.3.1.—The unsupported length $L_u$ of a compression member shall be taken as the clear distance between floor slabs, beams, or other members capable of providing lateral support in the direction being considered.

2.3.2.—Where column capitals or haunches are present, the unsupported length shall be measured to the lower extremity of the capital or haunch in the plane considered.

2.4.—Columns and stories in structures shall be designated as nonsway or sway columns or stories. It is permissible to assume a story within a structure is nonsway if

$$ Q = \frac{\sum P_u A_u}{V_o c} \leq 0.05 $$  \hspace{1cm} (A)

where $\sum P_u$ and $V_o$ are the total vertical load and the story shear, respectively, in the story in question, and $A_u$ is the first-order relative deflection of the top and bottom of that story due to $V_o$.

2.5.—The design of columns in nonsway frames or stories shall be based on the analysis given in Section 3.

2.6.—Frames or stories which do not satisfy the definition of nonsway frames in 2.4 shall be designed as sway frames or stories. The design of columns in sway frames or stories shall be based on the analysis given in Section 4.

2.7.—Where an individual compression member in the frame has a slenderness $k_{t_d}/r$ of more than 100, Section 1.1 shall be used to compute the forces and moments in the frames.

2.8.—For compression members subject to bending about both principal axes, the moment about each axis shall be magnified separately based on the conditions of restraint corresponding to that axis.

3.—Magnified moments—Nonsway frames

3.1.—For compression members in nonsway frames, the effective length factor $k$ shall be taken as 1.0, unless analysis shows that a lower value is justified. The calculation of $k$ shall be based on the $E$ and $I$ values used in Section 2.1.

3.2.—In nonsway frames it is permissible to ignore slenderness effects for compression members which satisfy

$$ \frac{k_{t_d}}{r} \leq \frac{25 - 12 (M_1/M_2)}{\sqrt{P_u/f_c^2 A_t}} $$  \hspace{1cm} (B)

where $M_1/M_2$ is not taken less than -0.5. The term $M_1/M_2$ is positive if the column is bent in single curvature.

3.3.—Compression members shall be designed for the factored axial load $P_u$ and the moment amplified for the effects of member curvature $M_c$ as follows

$$ M_c = \delta_{M_{11}} M_2 $$  \hspace{1cm} (C)

where

$$ \delta_{M_{11}} = \frac{C_m}{1 - P_u/Q_c} \geq 1.0 $$  \hspace{1cm} (D)

where $Q_c = 0.75$

$$ P_c = \frac{\pi^2 E I}{(k_{t_d})^2} $$  \hspace{1cm} (E)

$EI$ shall be taken as

$$ EI = \frac{0.2 E I_{c} + E I_{s}}{1 + \beta_d} $$  \hspace{1cm} (F)

or

$$ EI = \frac{0.40 E I_{c}}{1 + \beta_d} $$  \hspace{1cm} (G)

3.3.1.—For members without transverse loads between supports, $C_m$ shall be taken as

$$ C_m = 0.6 + 0.4 \frac{M_1}{M_2} \geq 0.4 $$  \hspace{1cm} (H)

where $M_1/M_2$ is positive if the column is bent in single curvature. For members with transverse loads between supports, $C_m$ shall be taken as 1.0.

3.3.2.—The factored moment $M_2$ in Eq. (C) shall not be taken less than

$$ M_{2,\text{min}} = P_u (0.6 + 0.03 h) $$  \hspace{1cm} (J)

about each axis separately, where 0.6 and $h$ are in inches. For members for which $M_{2,\text{min}}$ exceeds $M_2$, the value of $C_m$ in Eq. (H) shall either be taken equal to 1.0, or shall be based on the ratio of the computed end moment $M_1$ to $M_{2,\text{min}}$. [In Eq. (J) becomes $M_{2,\text{min}} = P_u (15 + 0.03 h)$, where 15 and $h$ are in mm.]

4.—Magnified moments—Sway frames

4.1.—For compression members not braced against sway, the effective length factor $k$ shall be determined using $E$ and $I$ values in accordance with Section 2.1 and shall be greater than 1.0.

4.2.—The moments $M_1$ and $M_2$ at the ends of an individual compression member shall be taken as

$$ M_1 = M_{11} + \delta_{M_{11}} M_{12} $$  \hspace{1cm} (K)

$$ M_2 = M_{21} + \delta_{M_{21}} M_{22} $$  \hspace{1cm} (L)

where $M_{211}$ and $\delta_{M_{212}}$ shall be computed according to Section 4.3.

4.3.—Calculation of $\delta M_k$

4.3.1.—The magnified sway moments $\delta M_k$ shall be taken as the column end moments calculated using a second-order elastic analysis based on the member stiffnesses given in Section 2.1.

4.3.2.—Alternatively, it is permissible to compute $\delta M_k$ as

$$ \delta M_k = \frac{M_k}{1 - Q} \geq M_k $$  \hspace{1cm} (M)

where $Q = 0.75$

If $\delta M_k$ computed in this way exceeds 1.5, $\delta M_k$ shall be computed using Section 4.3.1 or 4.3.3.

4.3.3.—Alternatively, it shall be permissible to calculate the magnified sway moment $\delta M_k$

$$ \delta M_k = \frac{M_k}{1 - \frac{Q_u}{P_u}} \geq M_k $$  \hspace{1cm} (N)
where $\Sigma P_u$ is the summation for all the vertical loads in a story and $\Sigma P_c$ is the summation for all sway-resisting columns in a story. $P_c$ is computed using Eq. (E) using $k$ from Section 4.1 and $El$ from Eq. (F) or Eq. (G), and $\varphi_s = 0.75$.

4.4—If an individual compression member has

$$\frac{L_u}{r} > \frac{35}{\sqrt{\frac{P_u}{f_c' A_g}}}$$

(P)

it shall be designed for the factored axial load $P_u$ and the moment $M_c$, computed using Section 3.3 in which $M_1$ and $M_2$ are computed in accordance with Section 4.2, $\beta_d$ as defined for the load combination under consideration and $k$ as defined in Section 3.2.

4.5—In addition to load cases involving lateral loads, the strength and stability of the structure as a whole under factored gravity loads shall be considered.

a. When $\delta M_1$ is computed from Section 4.3.1, the ratio of second-order lateral deflections to first-order lateral deflections for 1.4 dead load and 1.7 live load plus lateral load applied to the structure shall not exceed 2.5.

b. When $\delta M_2$ is computed according to Section 4.3.2, the value of $Q$ computed using $\Sigma P_u$ for 1.4 dead load plus 1.7 live load shall not exceed 0.60.

c. When $\delta M_3$ is computed from Section 4.3.3, $\delta_1$ computed using $\Sigma P_u$ and $\Sigma P_c$ corresponding to the factored dead and live loads shall be positive and shall not exceed 2.5.

In the preceding Cases a, b, and c, $\beta_d$ shall be taken as the ratio of the factored sustained axial dead load to the total factored axial load.

4.6—In sway frames, flexural members shall be designed for the total magnified end moments of the compression members at the joint.