Column Slenderness and Charts for Design

By RICHARD W. FURLONG

The iterative problem of selecting a column cross section to meet strength and long column slenderness requirements which are themselves dependent upon the cross section selected, is greatly simplified by the use of graphs that are derived and demonstrated. The graphical aids incorporate a slenderness index and moment magnifying function for use with ultimate load-moment-interaction diagrams.

Keywords: columns (supports); frames; interaction diagrams; long columns; reinforced concrete; slenderness ratio; stability.

■ THE SELECTION OF A column cross section for a specified combination of ultimate thrust P_u and ultimate moment M_u can proceed readily after appropriate modifications are made to each in order to account for any possible strength-reducing influence of slenderness. Minimum standards of ACI 318-63[1] recommend the application of long column reduction factors R to increase values of P_u and M_u for which cross sections are selected. There exists in addition Section 916(d) that permits "an analysis . . . taking into account the effect of additional deflections on moments in columns." Since the "analysis" is not specified, relatively little use has been made of this section. However, applications of a simplified form of the secant modulus theory to obtain moment magnification factors have appeared in European[2] and American structural steel design[3] specifications for column design. Such applications represent a form of analysis that would fulfill the requirements of Section 916(d).

Analytical procedures that take into account secondary moments caused by deflections in columns can promote for designers an understanding of column behavior in framed structures better than can R factor procedures of ACI 318-63. Indeed, the tentative recommendations proposed by ACI-ASCE Column Committee 441[4] refer exclusively to the use of a rational analysis for secondary deflections due to column deformation in order to account for the slenderness of columns. The use of a rational analysis can be made quite simple with the aid of graphs that are to be derived and demonstrated. The graphs will be useful for applications of the proposed new rules for column design, but they are applicable already in fulfillment of section 916(d). The discussion that follows and the representative samples of design charts that are included are presented to promote a more thorough understanding of secondary moments and to simplify any subsequent modification of a trial cross section. The analytic procedure will be based on recommendations of ACI-ASCE Committee 441, as adopted by ACI Committee 318 for the proposed Building Code.[5]

GENERAL CONSIDERATIONS

Since the selection of cross sections is the goal of column design, it will be assumed for this discussion that primary values of ultimate thrust and ultimate moment (uncorrected for the effects of column deformation) are available from a complete analysis of the structural frame. The column designer should be cautioned to use for frame analysis a stiffness EI for columns at least as high as the initial stiffness of an uncracked gross cross section of concrete. If the relative flexural stiffness of columns is undervalued for frame analysis, the apparent moments will be less than those likely to exist in the columns. It will be assumed further that the material properties f_c' and f_y to be used for design are known, as are the column shape and the story height.

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The two common design conditions remaining for the designer involve (a) the selection of the appropriate steel area necessary for a specified column size, or (b) the selection of an optimum column size. Condition (a) exists when several columns of the same size are subjected to different loads. Design condition (b) occurs when the gross size of columns for a particular structure or level is to be established. The selection of size is usually made on the basis of an estimated limit to the crowding of longitudinal steel within a cross section. If bars are to be spliced, the bar area A_{st} should be restricted to about 5 percent of the gross column area, but higher percentages of the gross area can be occupied by unspliced bars.

Interaction charts that display graphs of limiting combinations of axial and flexural capacity represent a familiar and efficient design aid for column cross sections. Families of curves applicable to all columns of a specified shape and material composition can be superimposed on one diagram, each curve representing a specific percent of the cross section occupied by longitudinal steel. Typical interaction curves are given in the upper right quadrant of Fig. 4 through Fig. 7. The ordinates to the interaction charts are gross axial stress obtained by dividing the ultimate load P_u by the product of the capacity reduction factors ϕ and the gross area of the column cross section, usually a circle or a rectangular shape. Abscissas are nominal flexural stresses equal to the ultimate moment M_u divided by the appropriate gross section modulus $bt^2/6$ for rectangles or $0.1D^3$ for circles of diameter D.

INFLUENCE of COLUMN SLENDERNESS

Moment magnification

A common procedure for determining the effect of secondary deformation in columns involves the use of a moment magnifier approximately the same as that obtained from the classical secant formula.[6] The secant formula, derived for a beam column in symmetric single curvature, results in a multiplier for the symmetrical end moments. The multiplier α is almost the same as that obtained from the simple ratio:

$$\alpha = \frac{1}{1 - P_u/P_c\,\phi} \tag{1}$$

The theoretical limit of a concentric load P_c on an elastic column of height Kh between end hinges is:

$$P_c = \frac{\pi^2 EI}{(Kh)^2} \tag{2}$$

In Eq. (2) the product EI is the flexural stiffness of the elastic cross section. ACI 318-63 requires that estimates of EI for concrete columns use a concrete stiffness not greater than half the initial tangent modulus stiffness E_c determined from the relationship:[10]

$$E_c = w^{1.5}\, 33\sqrt{f_c'}\ \text{psi}$$

or

$$E_c = w^{1.5}\, 4270\sqrt{f_c'}\ \text{kg/cm}^2$$

The density of concrete in 1b per cu ft or t/m^3 is w, and f_c' is the unconfined compressive strength of standard cylinders in psi or kg/cm^2. The proposed Code recommends that EI be taken as:

$$EI = (E_s I_s + 0.2 E_c I_g)/(1 + R_m) \tag{3a}$$

or

$$EI = E_c I_g / 2.5\ (1 + R_m) \tag{3b}$$

The ratio R_m is the ratio between dead load moment and total load moment on the column. It has the effect of reducing apparent stiffness if dead load generates a major part of flexural load and creep is likely to occur. E_s is the modulus of elasticity of steel, I_s is the moment of inertia of steel about the column centroid, and I_g is the moment of inertia of the gross concrete cross section. Either Eq. (3a) or (3b) fulfills the requirements of ACI 318-63.

Eq. (3b) is the simpler of the two and for rectangular sections of width b and thickness t, it can be written:

$$EI = \frac{E_c bt^3}{30\,(1 + R_m)} \tag{4}$$

The ratio P_u/P_c can then be expressed:

$$\frac{P_u}{\phi P_c} = \frac{30\,(1 + R_m)\ (Kh)^2}{\phi \pi^2 E_c bt^3}\, P_u$$

$$= \frac{3\ (1 + R_m)}{E_c}\left(\frac{Kh}{t}\right)^2\left(\frac{P_u}{\phi bt}\right) \tag{5a}$$

The term in the last parenthesis is the same term used for ordinates to the interaction curve, and the values in the second set of terms can be computed without knowledge of the reinforcement area. The ratio P_u/P_c then varies linearly with the axial stress ratio $P_u/(\phi bt)$ multiplied by the slenderness parameters in the second ratio of Eq. (5a). Families of graphs that use values of a slenderness parameter:

$$S_1 = \frac{(1 + R_m)}{E_c}\left(\frac{Kh}{t}\right)^2 \tag{5b}$$

to convert from axial stress ratios to P_u/P_c are shown in the upper left quadrants of Fig. 4 through 7.

It is possible to express a slenderness parameter S_2 that includes the influence of the reinforcement ratio p using Eq. (3a). The value of S for Eq. (3a) is:

$$S_2 = \frac{(1 + R_m)}{5pg^2 E_s + 0.5 E_c}\left(\frac{Kh}{t}\right)^2 \tag{5c}$$

Initially, for design, the value of S_1 should be used to determine the required reinforcement ratio p. The value of S_1 will be greater than S_2 unless reinforcement ratio p is greater than:

$$p = \frac{E_c}{10g^2E_s} \tag{5d}$$

When p is greater than the value determined from Eq. (5d), S_2 will permit a higher stiffness for the cross section. The lines of the upper left quadrant of design charts (the slenderness quadrant) are valid for values of S_1 and also for S_2. The smaller of the two values will provide the more economical section, but a value of p must be known to compute S_2 values.

For circular cross sections of diameter D, the moment magnifier parameter $P_u/P_c\phi$ can be expressed:

$$\frac{P_u}{P_c\phi} = \frac{4\ (1 + R_m)}{E_c}\left(\frac{Kh}{D}\right)^2 \tag{6a}$$

and the slenderness parameter S' for circular sections becomes:

$$S' = \frac{(1 + R_m)}{E_c}\left(\frac{Kh}{D}\right)^2 \tag{6b}$$

The corresponding slenderness parameter S_2' including the influence of the reinforcement ratio in Eq. (3a) becomes:

$$S_2' = \frac{(1 + R_m)}{2.5\,pg^2E_s + 0.5E_c}\left(\frac{Kh}{D}\right)^2 \tag{6c}$$

The value of S_2' will be greater than that of S_1' if the reinforcement ratio p is greater than:

$$p = \frac{E_c}{5\ g^2E_s} \tag{6d}$$

Again it should be noted that the slenderness quadrants of the design charts may be used with either S_1' or S_2' values, but a good estimate of p must be available to compute S_2'.

EFFECTIVE HEIGHT

Frames braced against sidesway

The elastic buckling phenomenon reflected by the magnitude of P_c was derived for a concentrically loaded column in which the distance between end hinges is Kh. The shape of the deformed ideal member is proportional to one-half of a sine wave if the half period is taken as Kh, the column height between hinges. The presence of rotational restraints at column ends can alter the deflected shape of the elastic column into various combinations of sine waves. Several examples for which the effective height Kh may be less than h are illustrated in the laterally braced frame of Fig. 1.

Precise magnitudes of a coefficient K for modifying story heights h to represent effective heights can be computed if relative beam and column rotational stiffnesses are known at each joint.[7] In general, any sophistication for determining effective heights of columns in braced frames may be impractical. Estimates based on judgment regarding possible buckling modes are adequate for design, since K can vary only between values of 0.5 and 1.0 in braced frames. The magnitudes suggested in Fig. 1 are reasonable adaptations from standards suggested for steel columns in rigid frames.[8]

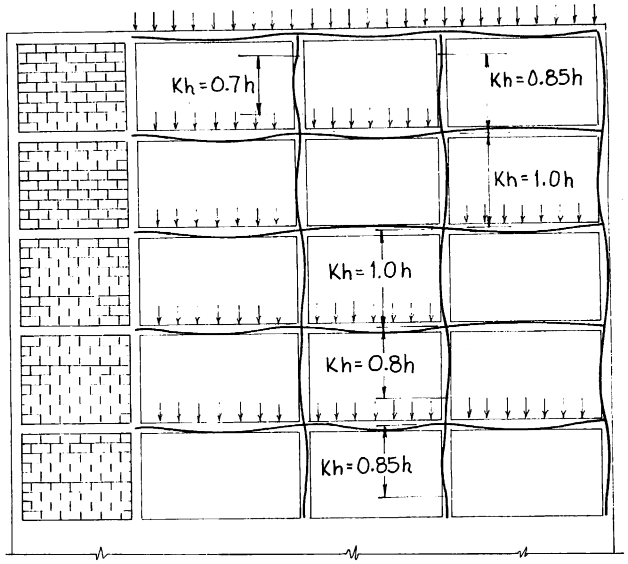

Fig. 1—Typical estimates of effective height

In braced frames, rotational restraints at column ends must impose moments on the columns in order to restrain rotations, and proposed adaptations of the secant formula include a coefficient C_m to approximate the influence of moments that act at the ends of columns. (A discussion of end moments follows.) In lieu of any attempt to approximate effective height coefficients for braced frames, the writer prefers to accommodate end effects, both from imposed forces as well as from potential restraints, by using $K = 1$ and computing the end moment coefficient C_m from design values of end moments.

Frames that depend on columns for lateral stiffness

In the absence of shear walls or other specific lateral restraints in a frame, the flexural stiffness of columns and beams must be relied upon for resistance to lateral deformation of the frame itself. A simple analytic model that has been used to determine the relationship between frame stability, flexural beam stiffness EI_{beam}/L and column stiffness EI_{col}/h, is a rectangular frame consisting of two identical beams and two identical columns, as sketched in Fig. 2 (a). Under the action of some horizontal force H, the frame deforms as suggested in Fig. 2 (c). If secondary effects of axial force along deflected members are neglected, the horizontal displacement Δ can be related to

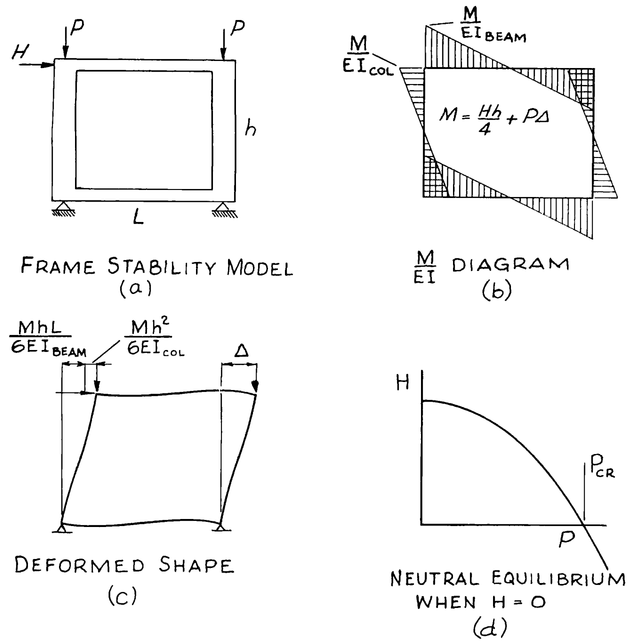

Fig. 2—Analytic model for frame stability

beam and column flexural stiffness, and column end moments M:

$$\Delta = \frac{M}{6}\left(\frac{h^2}{EI_{col}} + \frac{Lh}{EI_{beam}}\right) \tag{7a}$$

The equilibrium equation for moments about the base of a column [Fig 2(c)] can be written:

$$\frac{Hh}{4} + P\Delta - 2M = 0. \tag{7b}$$

The frame is in a condition of neutral equilibrium when the horizontal force H, required to create lateral displacement, is zero. For such a condition of instability, the column load P becomes the critical thrust P_{cr}, associated with frame instability. Eq. (7b) can be solved with H equal to zero:

$$P_{cr} = \frac{12\, EI_{col}}{h^2 \sqrt{1 + \dfrac{L}{EI_{beam}} \dfrac{EI_{col}}{h}}} \tag{8a}$$

The ratio of column stiffness,

$$K_{col} = \frac{EI_{col}}{h}$$

to beam stiffness,

$$K_{beam} = \frac{EI_{beam}}{L}$$

is called r' in ACI 318-63.

Modifying the interpretation of the frame buckling load P_{cr} to be a buckling load on the limit elastic condition for the same column with an equivalent height Kh:

$$P_{cr} = \frac{12}{\pi^2} \frac{\pi^2 EI_{col}}{h^2} \sqrt{\frac{1}{(1 + r')}} = \frac{\pi^2 EI_{col}}{(Kh)^2} \tag{8b}$$

Solving Eq. (8b) for K, the effective height coefficient K becomes:

$$K = \frac{\pi}{\sqrt{12}} \sqrt{1 + r'} \approx 0.9 \sqrt{1 + r'} \tag{8c}$$

Eq. (8c) is accurate for frames in which the ratio r' is greater than 2 because the influence of axial thrust along the deflected column is negligible when the column stiffness is as much as twice the beam stiffness. Alignment charts that give effective length factors K for stiffness ratios r' are available in popular references.[8] The charts are simple, but not always available to the designer. The following formulas give K values within 2 percent of those given by alignment charts:

$$\begin{aligned} K &= \frac{20 - r'}{20} \sqrt{1 + r'} \qquad (r' < 2) \\ K &= 0.9 \sqrt{1 + r'} \qquad (r' \geqq 2) \end{aligned} \tag{9}$$

The effective height h' of a column in a frame that requires beam and column flexural stiffness to resist sidesway should be taken as the product of the proper ratio K and the story height h. If different ratios r' occur at ends of the same column, the average K value should be used.

End moment coefficient C_m

The ratio between moments at each end of a column indicates the shape of the deformed column. In moment multiplier equations a factor C_m has been used to reflect the influence of end moments:

$$C_m = 0.6 + 0.4\, M_{small}/M_{large} \tag{10}$$

The algebraic sign to be used for moments is positive if each moment induces compression on the same face of the column. For example, if end moments are equal, C_m would be unity, but if one end of a column were pin-connected, C_m would be only 0.6. If the base of a column were built in, and a carryover moment were taken as minus one-half the top moment, C_m would be 0.4.

Analysis of an elastic beam column reveals the theoretically correct relationships between moment magnifiers, ratios of end moments, and the slenderness index P/P_c.[9] The solid line graphs of Fig. 3 represent "exact" curves obtained from the cumbersome trigonometric functions of elastic theory. Dashed lines of Fig. 3 illustrate the approximate curves for a moment multiplier F taken simply as the product of α from Eq. (1) and C_m from Eq. (10):

$$F = \alpha C_m = \frac{C_m}{1 - \dfrac{P_u}{P_c \phi}} \tag{11}$$

Revised column design rules for the proposed ACI Code include Eq. (11).

The effective length parameter K for columns in frames that sway has been illustrated to represent a distance between hinges on a pin-ended column, so $C_m = 1$ must be used for frames that depend on column flexure for resistance to lateral force.

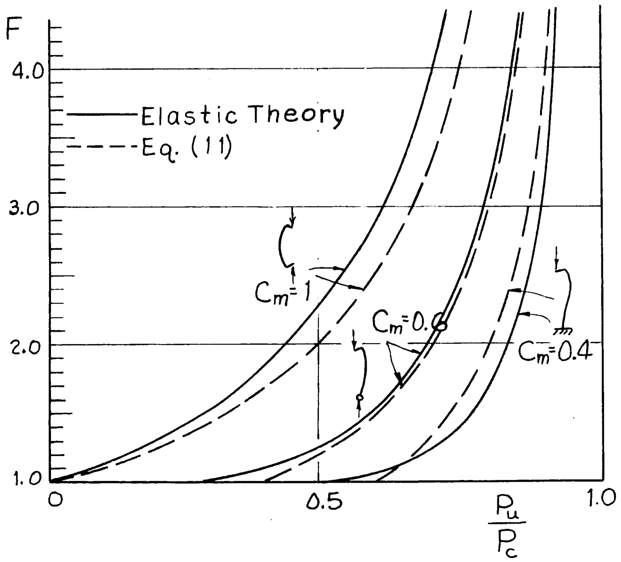

Fig. 3—Moment magnifiers for beam columns

MOMENT MULTIPLIER GRAPHS

After the ratio P_u/P_c is determined from the graphs of Eq. (5a) or (5b), shown in the upper left quadrant of Fig. 4 through Fig. 7 the magnitude of moment multipliers F can be shown as negative ordinates in the graphs of Fig. 4 through Fig. 7 for various values C_m shown in the lower left quadrant of each. The lower right quadrant of Fig. 4 through 7 has been used as a multiplier quadrant by expanding the abscissas scale for increasing ordinates F. Moving horizontally from the F scale to the value of the initial moment stress index, the magnified moment stress can be read vertically on the moment stress scale. More appropriately, the required steel percentage needed for the magnified moment acting on the cross section can be read by moving vertically above the moment stress axis to the intersection with the initial value used for axial stress $P_u/\phi bt$. Incidentally, in unbraced frames, the moment magnifier F must be applied to design moments for beams as well as those for columns.

SAMPLE PROBLEMS

Example 1

Given: P_u = 460 kips (208 t); M_u = 140 ft-kips (19.4 t-m); R_m = 0.2; no sidesway; $h = Kh$ = 13.3 ft (4.1 m);

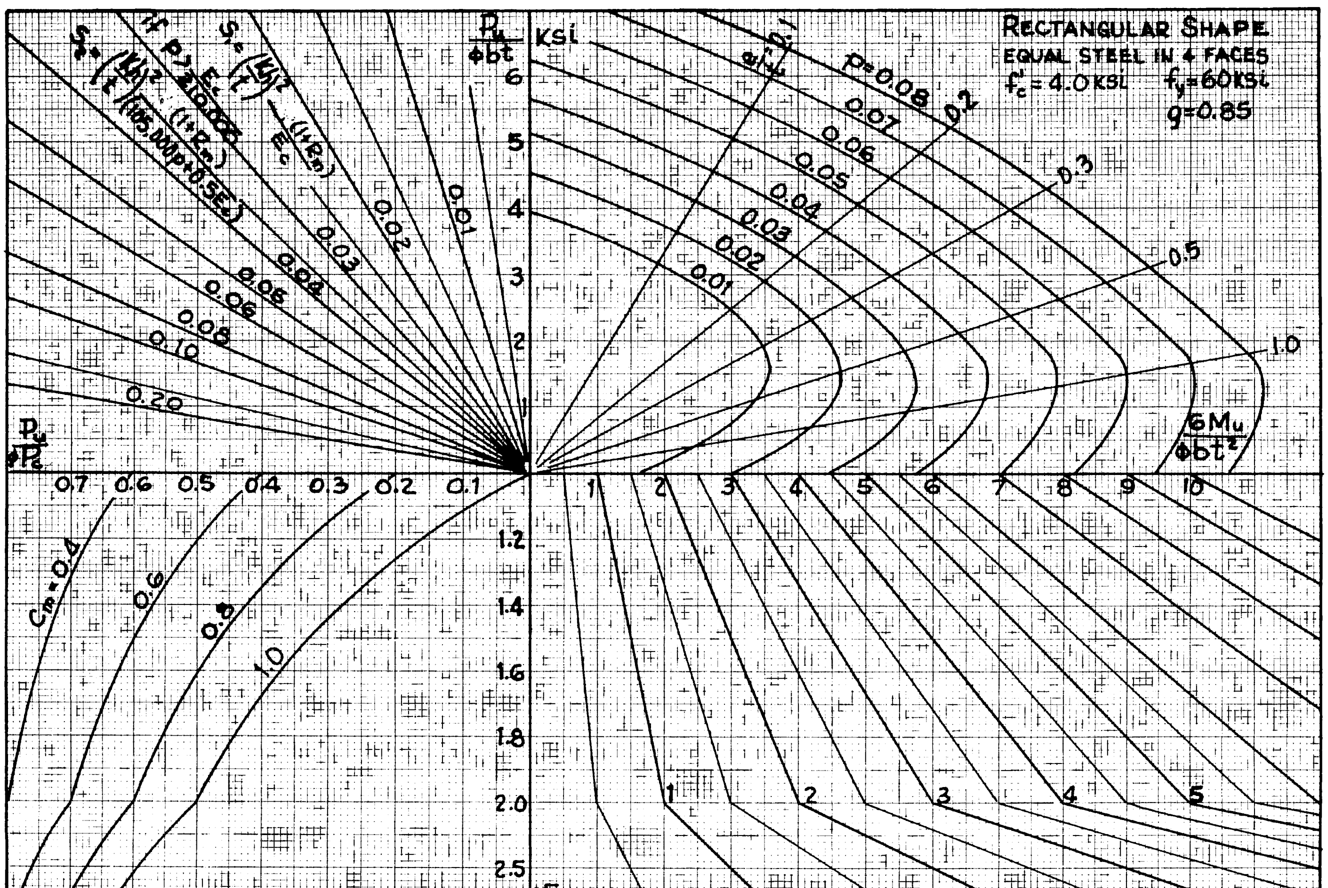

Fig. 4—Design aid—Rectangular column

$C_m = 0.8$; $f_c' = 4.0$ ksi (281 kg/cm²); $f_y = 60$ ksi (4219 kg/cm²); and $E_c = 3600$ ksi (253 t/cm²).

Required: Find the square cross section with p_t approximately 4 percent.

$$\text{Eccentricity } e = \frac{140 \times 12}{460} = 3.65$$

(*a*) *Choose trial shape*

Estimate $e/t = 0.20$, $g = 0.7$, and use Fig. 5 with p = 0.04, to obtain $P_u/(\phi bt) = 3.5$ ksi (246 kg/cm²).

Thus, the required:

$$bt = \frac{460}{3.5\phi} = \frac{460}{3.5\ (0.7)} = 188\ (1212\ \text{cm}^2)$$

For a 14 in. square column, $e/t = 3.65/14 = 0.26$; initial estimate of e/t was too small, so try a 15 in. square column (38 cm square column):

$$g \approx \frac{15 - 2\tfrac{1}{2} - 2\tfrac{1}{2}}{15} = 0.67$$

Hence, Fig. 5 applies.

(*b*) *Find slenderness parameter P_u/P_c*

Compute:

$$\frac{P_u}{\phi bt} = \frac{460}{0.7\ (15 \times 15)} = 2.92\ (206\ \text{kg/cm}^2)$$

and locate Point A on Fig. 5a.

To find the ratio P_u/P_c, a value of S is needed. Compute:

$$S_1 = \left(\frac{Kh}{t}\right)^2 \frac{(1 + R_m)}{E_c} = \left(\frac{13.3 \times 12}{15}\right)^2 \frac{(1 + 0.2)}{3600} = 0.038$$

Note that if S_2 governs, it can be computed after P_t is known.

Use upper left quadrant with $P_u/\phi bt = 2.92$ and $S_1 = 0.038$ to find $P_u/P_c\,\phi = 0.0335$.

(*c*) *Find moment magnifier F*

C_m was given as 0.8, and $P_u/P_c\,\phi = 0.0335$.

Use lower left quadrant to find $F = 1.21$.

(*d*) *Find required steel percentage*

Compute:

$$\frac{6M_u}{\phi bt^2} = \frac{6\ (140 \times 12)}{0.7\ (15)^3} = 4.27\ \text{ksi}\ (300\ \text{kg/cm}^2)$$

Use lower right quadrant with $F = 1.21$ and move directly above the flexural stress index of 4.27 to find Point B in the upper right quadrant at $P_u/P_c\,\phi = 2.92$ for which the required steel percentage $p_t = 0.046$.

Check:

$$\frac{E_c}{142{,}000} = \frac{3{,}600}{142{,}000} = 0.0254$$

which is less than 0.046, so S_2 should be used instead of S_1. Estimate $p_t = 0.04$, and compute:

$$S_2 = \left(\frac{13.3 \times 12}{15}\right)^2 \times \frac{(1 + 0.2)}{71000 \times 0.04 + 0.5 \times 3600} = 0.0292$$

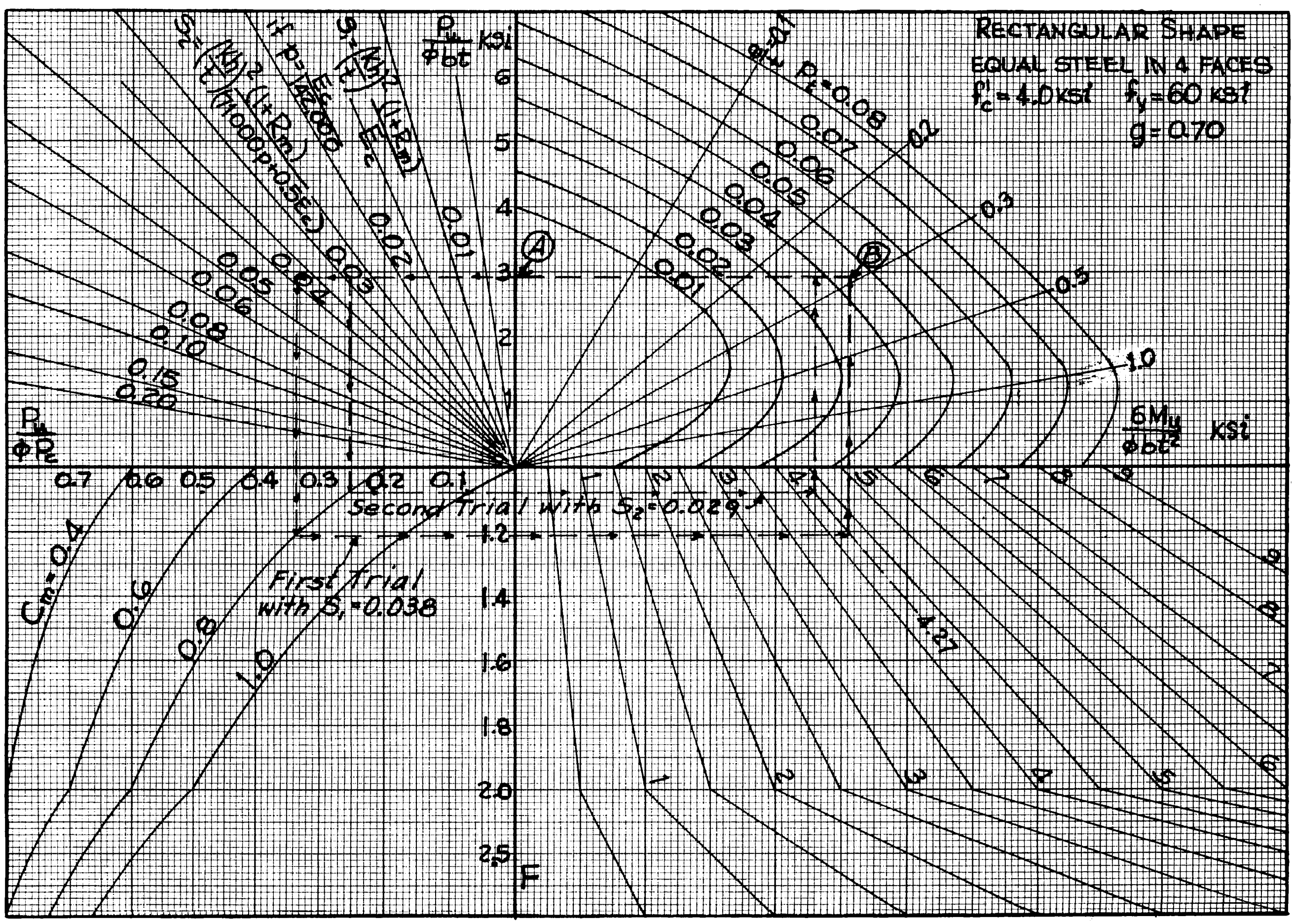

Fig. 5a—Design aid—Rectangular column, Example I

Again use the upper left quadrant with an axial stress of 2.92 ksi and $S_2 = 0.029$ to read $P_u/P_c\,\phi = 0.252$. Note that if $P_u/P_c\,\phi$ were less than 0.20, F would be 1.00.

Move downward to the value $C_m = 0.8$ in the lower left quadrant, to read a value $F = 1.08$. Move horizontally to the flexural stress of 4.27 ksi, and then vertically to the axial stress of 2.92, where the required steel ratio of 0.039 is read.

Use 15 in. (38 cm) square column.

$A_{st} = 0.039\ (225) = 8.78$ sq in. (56.6 cm^2)

Example 2

Given: For 15 in. (38 cm) square column, $b = t = 15$ in. ($g \approx 0.7$ and Fig. 5 applies); see Fig. 5b; no sidesway; $f_c' = 4.0$ ksi ($E_c = 3600$ ksi); $f_y = 60.0$ ksi; $h = 13.3$ ft; and $C_m = 1.0$.

Required: Select steel required for $P_u = 320$ kips (145 t), $M = 108$ ft-kips (14.9 t-m).

Compute:

$$\frac{P_u}{\phi bt} = \frac{320}{0.7\ (15 \times 15)} = 2.03 \text{ ksi } (143 \text{ kg/cm}^2)$$

As in Example 1:

$$\left(\frac{Kh}{t}\right)^2 \frac{(1 + R_m)}{E_c} = 0.038$$

Upper left quadrant shows $P_u/P_c\,\phi = 0.23$.

With $C_m = 1.0$, lower left quadrant shows $F = 1.31$.

With $F = 1.31$, and in lower right quadrant:

$$\frac{6M_u}{\phi bt^2} = \frac{6 \times 108 \times 12}{0.7\ (15)\ 225} = 3.29 \text{ ksi } (232 \text{ kg/cm}^2)$$

Move vertically to $P_u/(\phi bt) = 2.03$ to determine $p_t = 0.023$.

P_t is less than E_c /142,000, so S_1 governs slenderness.

$A_{st} = 0.023\ (15)^2 = 5.17$ sq in. (33.3 cm^2). Use four #10 in 15 in. (38 cm) square column.

Example 3

Given: $P_u = 372$ kips (169 t); $M_u = 112$ ft-kips (7.9 t-m); $h = 12$ ft (3.66 m); $f_c' = 4.0$ ksi (281 kg/cm^2); $E_c = 3600$ ksi (253 t/cm^2); $f_y = 60$ ksi (4219 kg/cm^2); and $R_m = 0.15$.

Required: Select reinforcement for an 18 in. (46 cm) diameter column (spirally reinforced) in a laterally *unbraced* frame for which beam stiffness $I/L = 24.8$ in.^{3} (406 cm^3) at the level above the column, and $I/L = 66.6$ in.^{3} (1090 cm^3) at the level below the column. Assume columns above and below are also 18 in. diameter. For the columns:

$$\Sigma \frac{I}{h} = 2 \times \frac{\pi (18)^4}{64} \times \frac{1}{12 \times 12} = 71.5 \text{ in.}^3 \ (1004 \text{ cm}^3)$$

$$r'_{top} = \frac{71.5}{24.8} = 2.88$$

$$K_{top} = 0.9\sqrt{1 + 2.88} = 1.78 \text{ [see Eq. (10)]}$$

$$r'_{bott} = \frac{71.5}{66.6} = 1.07$$

$$K_{bott} = \frac{20 - 1.07}{20}\sqrt{1 + 1.07} = 1.36$$

$$K_{avg} = \frac{1.78 + 1.36}{2} = 1.57$$

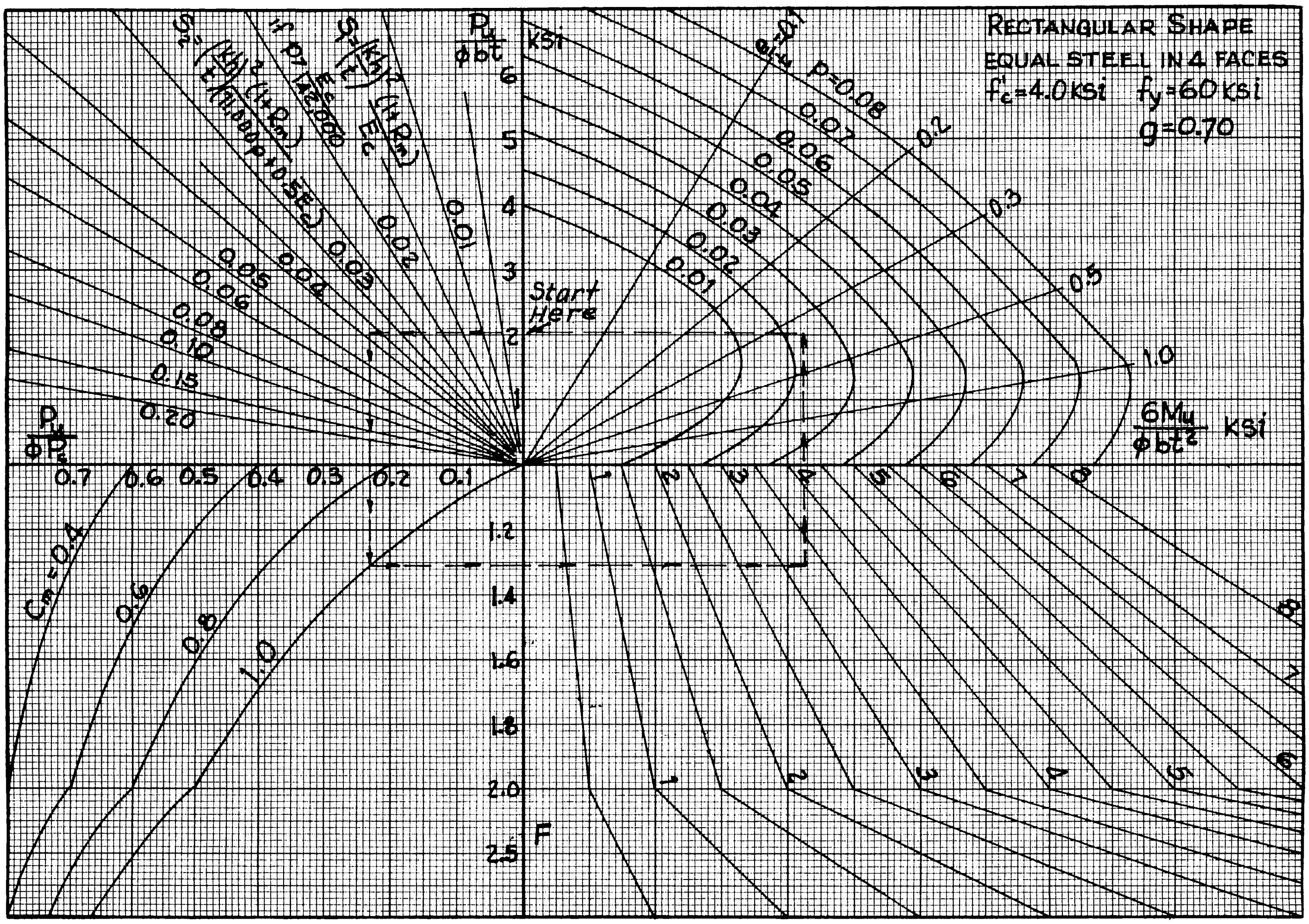

Fig. 5b—Design aid—Rectangular column, Example 2

$$Kh = 1.57 \times 12 = 18.9 \text{ ft } (8.92 \text{ m})$$

For 18 in. diameter, estimate $g = 0.7$ and use Fig. 7.

$$A_g = \frac{\pi (18)^2}{4} = 254 \text{ sq in. } (1640 \text{ cm}^2)$$

$$\frac{4P_u}{\phi \pi D^2} = \frac{372}{0.75\ (254)} = 1.95 \text{ ksi } (137 \text{ kg/cm}^2)$$

$$S_1' = \left(\frac{Kh}{D}\right)^2 \frac{(1 + R_m)}{E_c} = \left(\frac{18.9}{1.5}\right)^2 \frac{(1 + 0.15)}{3600} = 0.51$$

$$\frac{10M_u}{\phi D^3} = \frac{10 \times 112 \times 12}{0.75 (18)^3} = 3.07 \text{ ksi } (216 \text{ kg/cm}^2)$$

Starting at the axial stress of 1.95 ksi move left to $S_1' = 0.51$ and downward to $P_u/\phi\, P_c = 0.41$. Now using $P_u/\phi\, P_c = 0.41$ with $C_m = 1.0$ $F = 1.72$.

From the lower right quadrant with $F = 1.72$ and 3.07 ksi bending stress, read vertically to 1.95 axial stress and required steel ratio of 0.034.

The value of p is less than $E_c/71{,}000 = 0.051$, so S_1' gives stiffness greater than S_2'.

Use $A_{st} = 0.034 \times 254 = 8.64$ sq in. (55.5 cm²)

Use seven #10 in an 18 in. diameter column.

REFERENCES

1. ACI Committee 318, "Building Code Requirements for Reinforced Concrete (ACI 318-63)," American Concrete Institute, Detroit, 1963, 144 pp.
2. Levi, Franco, "Work of the European Concrete Committee," ACI Journal, *Proceedings* V. 57, No. 9, Mar. 1961, pp. 1041-1070.
3. *Manual of Steel Construction,* American Institute of Steel Construction, New York, Sixth Edition, 1963.
4. MacGregor, J. G.; Breen, J. E.; and Pfrang, E. O., "Design of Slender Concrete Columns," ACI Journal, *Proceedings* V. 67, No. 1, Jan. 1970, pp. 6-28.
5. ACI Committee 318, "Proposed Revision of ACI 318-63: Building Code Requirements for Reinforced Concrete," ACI Journal, *Proceedings* V. 67, No. 2, Feb. 1970, pp. 77-186.
6. Timoshenko, S., *Theory of Elastic Stability,* McGraw-Hill Book Co., New York, 1936, p 38.
7. Johnston, B. C. (Editor), *The Column Research Council Guide to Design Criteria for Metal Compression Members,* John Wiley and Sons, Inc., New York, Second Edition, 1966, 217 pp.
8. "Commentary on AISC Specification," *Manual of Steel Construction,* American Institute of Steel Construction, New York, Sixth Edition, 1963, pp. 5-117.
9. Ketter, Robert L., "Further Studies of Beam Columns," *Proceedings,* ASCE, V. 87, ST6, Aug. 1961, p. 140.
10. ACI Committee 318 "Building Code Requirements for Reinforced Concrete (ACI 318-63)," American Concrete Institute, Detroit, 1963, p. 51.

APPENDIX—NOTATION

A_{st} = total area of longitudinal steel in column cross section

b = width of rectangular cross section

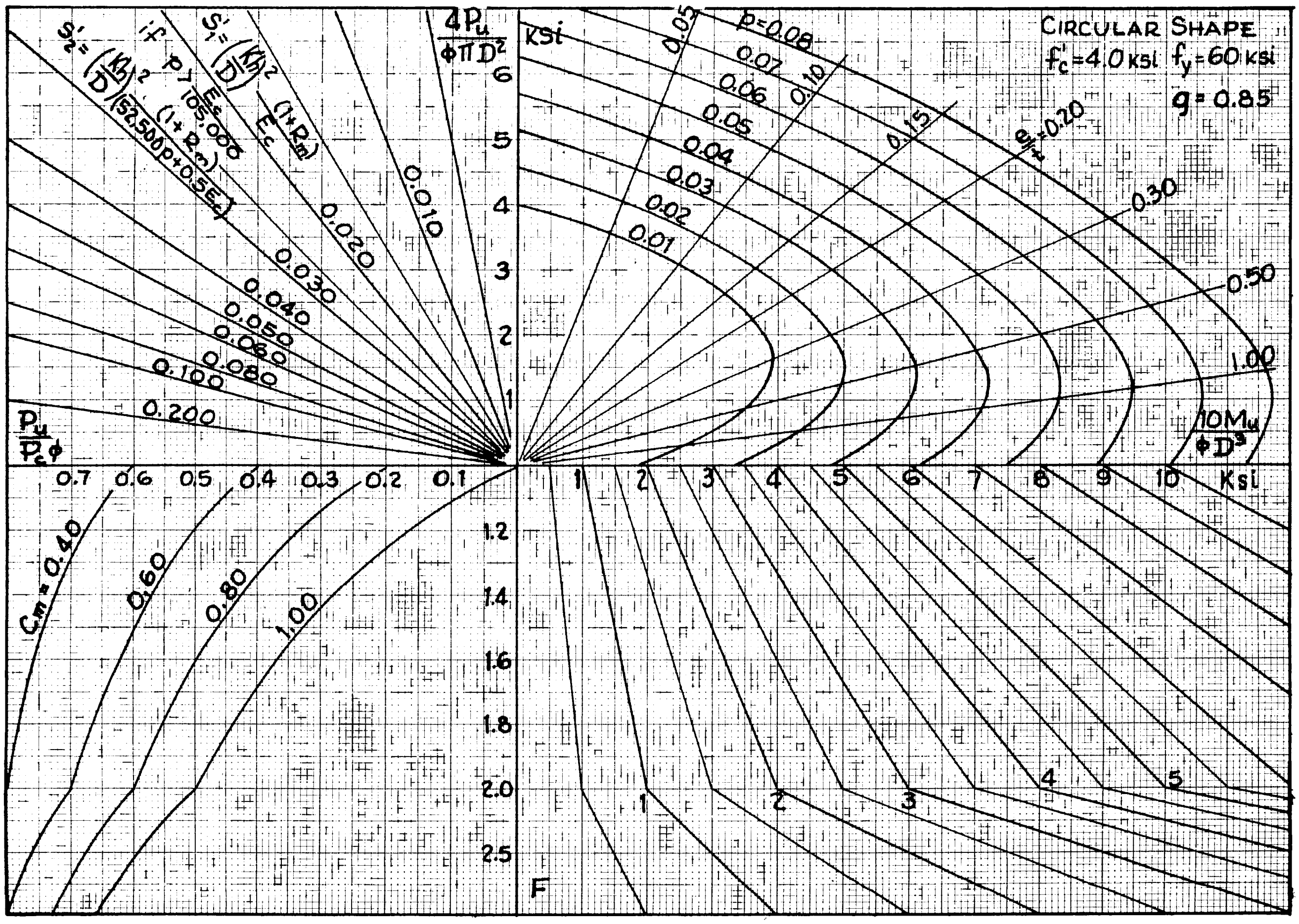

Fig. 6—Design aid—Circular column

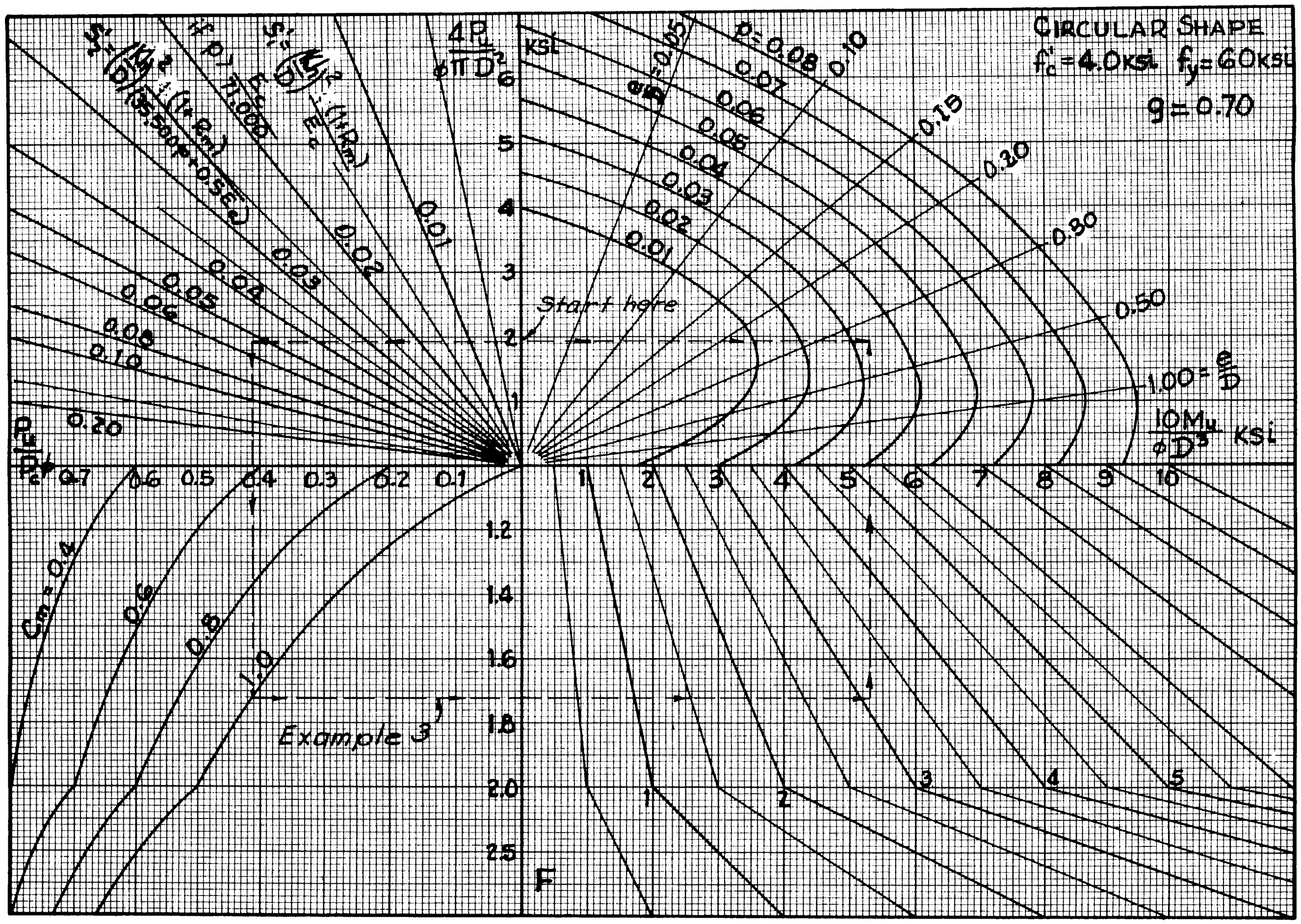

Fig. 7—Design aid—Circular column

C_m = coefficient reflecting ratio of end moments in moment magnifier equation, Eq. (11)
D = diameter of round shape
E = modulus of elasticity
E_c = modulus of elasticity of concrete
E_s = modulus of elasticity of steel
EI = flexural stiffness
F = moment magnifier factor, Eq. (11)
f_c' = compressive strength of concrete cylinders
f_y = yield strength of reinforcement
g = ratio between distance separating centroids of steel and total thickness of cross section
h = story height, centroid to centroid of horizontal members
H = story shear on unbraced frame
I = moment of inertia of cross sections
I_g = moment of inertia of gross cross section
I_s = moment of inertia of steel taken about centroid of cross section
K = coefficient used to modify story height to obtain effective height for columns in frames
K_{top} = effective height coefficient determined from stiffness of members connected at top of column
K_{bott} = effective height coefficient determined from stiffness of members connected at bottom of column
K_{col} = relative rotational stiffness at end of column
K_{beam} = relative rotational stiffness at the end of a beam
L = length of beam, center to center of columns
M_u = ultimate moment for which column is designed
M_{large} = larger of ultimate moments at ends of column
M_{small} = ultimate moment at column end opposite M_{large}
P = axial force
P_c = axial force representing Euler buckling load on column
P_{cr} = axial force representing buckling load on frame
P_u = ultimate axial force for which column is designed
R_m = ratio of ultimate dead load moment to ultimate total moment for which column is designed
r' = ratio between rotational stiffness of columns to rotational stiffness of beams at joint
r'_{top} = ratio r' at the top of column
r'_{bott} = ratio r' at the bottom of column
S = slenderness parameter
S_1 = slenderness parameter for rectangular cross section stiffness in terms of concrete only
S_2 = slenderness parameter for composite rectangular cross section stiffness, including steel and concrete
S_1' = slenderness parameter for round cross section stiffness in terms of concrete only
S_2' = slenderness parameter for composite round cross section stiffness, including steel and concrete

t = thickness of cross section
w = density of concrete
α = approximate values to secant formula for moment multipliers on symmetrically deformed columns
Δ = lateral displacement of one story with respect to story below
ϕ = capacity reduction factor
π = ratio between circumference and diameter of circle

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High Rise "Systems Building" in the Hudson Valley

By ROGER H. CORBETTA and ROBERT E. WILSON*

Describes the use of "systems building" techniques for an urban renewal project in Poughkeepsie, N. Y. The project consists of a central 18-story building surrounded by smaller buildings providing about 1000 dwelling units.

Paper describes construction of the central building which combines cast-in-place elevator and stairwell cores and shear walls with precast concrete elements, some of which were cast on-site.

Precasting operations are described and erection procedures discussed. In the latter, cast-in-place work and precast floor erection proceeded together floor by floor followed shortly by erection of precast exterior panels. Shear walls and floor panels were erected at a rate of one floor per week.

Keywords: concrete construction; industrialized buildings; multistory buildings; precast concrete.

Throughout Europe "systems building" has been adopted to produce prefabricated high rise apartment complexes for many thousands of new dwelling units. Systems building concepts were actually pioneered in Forest Hills, N.Y. in 1903 by Architect Grosvenor Atterbury. He built hundreds of precast concrete homes, which included precast floors, walls, stairways, porches, and chimneys. The first author became associated with Mr. Atterbury in 1919 and has had an interest in this approach to building ever since.

This economical method of construction, when it is applied to a minimum production of 500 to 1000 similar dwelling units in clusters or complexes, enables the builder to adopt the mass production technique of an automobile assembly line to construction. The rapid amortization of numerous costly casting beds or molds is one of many benefits. There is also a reduction in the number of trades employed.

So great are the practical aspects of systems building, that for an urban renewal program in Poughkeepsie, N.Y., it was decided to build a proposed 1000 dwelling-unit project of precast concrete units, mostly cast on the job site.

A mobile casting yard was established to produce concrete sections for as many as a dozen buildings, about 1000 dwelling units, alongside a central 18-story (179 unit) building. The techniques of systems building could then be applied to the central building.

CENTRAL BUILDING

The central building measures 242 x 42 ft (74 x 13 m), has 11 bays, and no columns (Fig. 1 and 2). There are two cast-in-place cores in the building: the elevator shaft and one stairwell.

A goal of building one floor per week was set and achieved after a few modifications in the construction sequence. The first thing to go up at every floor level were the cast-in-place shear walls. Five sets of steel forms for these cross walls were hoisted into location by a crawler crane and set in place.

Large precast concrete panels, with a light sandblast or exposed aggregate finish, form the skin of the building. They measure up to 20 x 14 ft (6 x 4 m) covering parts of three stories. Most floors were constructed of 6 x 24 in. (15 x 30 cm) factory precast hollow core slabs, which run parallel to the long axis of the building and, like the exterior wall panels, are tied in with the cast-in-place cross walls.

The vertical continuity of the cross walls was achieved by dowels, which were spliced with the wall steel at every floor, according to code requirements (see Fig. 3).

The slabs at every level became a continuous membrane by means of horizontal dowels, which tie them together across the top of the shear walls. Only every second void received a tie bar, which, according to structural conditions, varied in size from #4 to #6 (1.3 to 1.9 cm) and in length from 4 ft 6 in. to 7 ft 6 in. (1.4 to 2.3 m). The dowels were grouted into the voids using 4000 psi (280 kgf/cm^2) mortar. Intermittent control slots, starting at the edge of the slabs and continuing to the end of the embedded bars, provided control over the grout flow. Interior vibrators were used to

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Fig. 1—Central building

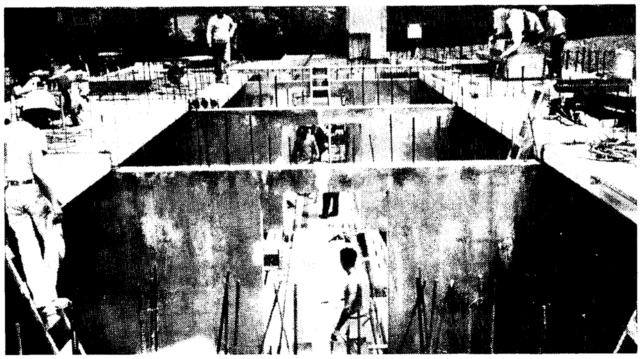

Fig. 2—Typical floor under construction

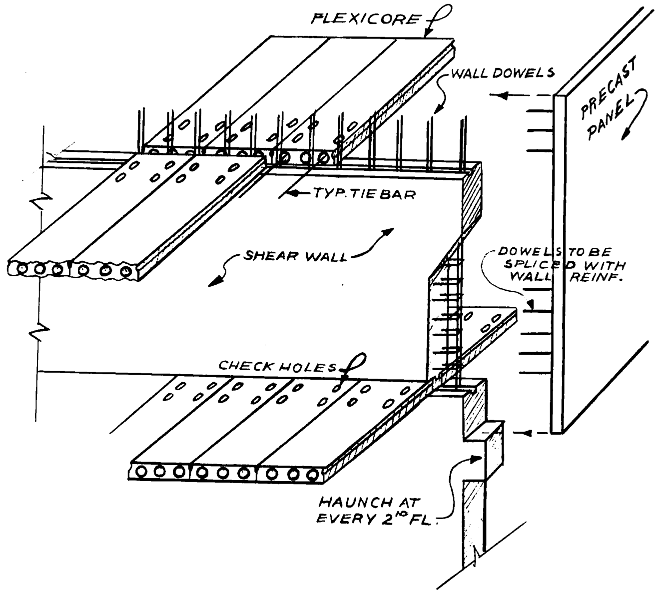

Fig. 3—Detail of connections between components

place the grout. The longitudinal joints between the slabs were filled flush with a 1:3 grout mix.

The shear walls have intermittent haunches to support the outside wall panels, the largest of which weigh about 12½ tons (11,400 kg).

The dowel bars protruding from the back of the precast panels were spliced at every floor with the wall steel in shear wall pockets.

Each panel was tied after erection to the shear walls by means of braces. The above-mentioned splices were welded and the braces released. The pockets were then filled with concrete, tying the panels and shear walls into an almost monolithic structure. Short rubber sleeves, with about ⅜ in. (1 cm) wall thickness, placed over the dowels at the back face of the precast units, allow the structure to breathe and yield to thermal influences within defined limits.

To avoid the problem of casting chases for service lines into the floor slabs, it was decided by the engineer to cast narrow floor slab strips in place in every bay. These strips were formed out to suit the individual chase requirements. Regular wood forms were used in this operation. The stairwell walls and elevator shaft were framed out with plywood.

Stripping of all forms took place within 24 hr.

The 28-day concrete strength for precast and cast-in-place members was above 4000 psi. The average slump was 3 in. (8 cm).

The roofs of all box panels received a three-coat silicone roofing membrane. Corresponding joints were caulked with a silicone sealant, all other independent joints with a two-component joint filler. A rigid foam was the back-up material.

The precast sections have two finishes. The light sandblast finish was achieved using a retarder. An extra heavy retarding agent was used for the exposed aggregate panels. Paint rollers were used to apply both materials to the mold surfaces.

All panels were stripped after 16 hr, using high-early-strength cement, and brushed and washed down with water jets. Diluted muriatic acid took care of the final cleaning.

The panels were moist cured by water spraying. Heat blankets were used during the winter season.

The storage yard for all precast sections was alongside the building, which helped to avoid any double handling.

The precasting operation, which took place in the open yard, consisted of the following steps: casting of the panels, lifting them off the casting beds, transferring them into the storage and finishing area, and finally lifting for erection.

A crawler crane moved and erected the precast sections.

Architects and engineers: a joint venture of Herbert Fleischer Associates and Associates Speyer and Dworkin (ASPAD). The structure is owned by Corbetta interests. Corbetta Construction Co. was the builder.