SECONDARY MOMENT AND MOMENT REDISTRIBUTION IN CONTINUOUS PRESTRESSED CONCRETE BEAMS

Explains the 1971 ACI Building Code provisions for moment redistribution in continuous prestressed concrete beams. Emphasizes, by means of examples, when the secondary moments due to prestress force are to be included in determining the ultimate capacity of continuous beams.

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The subject of moment redistribution in prestressed continuous beams has received careful study[1] for beams with concordant cables*, but not for those with non-concordant cables. In the latter case, what happens to the moment produced by prestressing (the secondary moment) in the plastic or elasto-plastic stage has become a subject of much discussion[2,3].

The purpose of this paper is to bring up the serious nature of the problem, particularly in the light of the 1971 ACI Building Code. The existence of secondary moments in the post-elastic range will be clarified. One simple and conservative solution is suggested, but no exact method is proposed.

1971 ACI Building Code[4] limits the percentage of moment redistribution in prestressed concrete continuous beams in Sect. 18.12, somewhat similar to that for conventional reinforced concrete in Sect. 8.6. In addition, it states, "The effect of moments due to prestressing shall be neglected when calculating the design moments." In the 1971 Code Commentary[5] to Sect. 18.12, it is further explained, "The secondary bending moments produced by the prestress force in a non-concordant tendon disappear at the capacity at which, due to plastic hinge formation, the structure becomes statically determinate. Therefore, the design load moments at the critical sections of a continuous prestressed beam are only those due to dead and live loads."

The above statement on the disappearance of secondary moments, while correct by itself if properly interpreted, has been quite misleading to many engineers, especially when taken together with the limitations on moment redistribution.

It is well known that secondary moments are produced by prestressing a continuous beam with a non-concordant c.g.s. line. It is also well known that when plastic hinges form in a continuous beam, converting it into a statically determinate structure, the moments in the beam can be computed, taking into account only the external dead and live loads and the moment capacity at the critical sections. To these moments, the secondary moments due to prestressing need not be added. In fact, whether or not to add the secondary moments will yield the same load carrying capacity for the beam, if complete moment redistribution can take place.

It has been stated[6], "Linear transformation of the c.g.s. line does not change the ultimate load-carrying capacity of a continuous beam." This means a non-concordant c.g.s. line can be linearly transformed into a concordant c.g.s. line without changing the ultimate load capacity. Since this beam will now possess a concordant c.g.s. line, it will have no secondary moment. This phenomenon led to the statement in the ACI Building Code Commentary that the design (ultimate) load moments are only those due to dead and live loads.

All of the above, however, is predicated on full moment redistribution with complete development of plas-

* *A concordant cable is so located to produce a line of compressive force in the concrete at each section that coincides with the center of gravity of the steel (c.g.s.).*

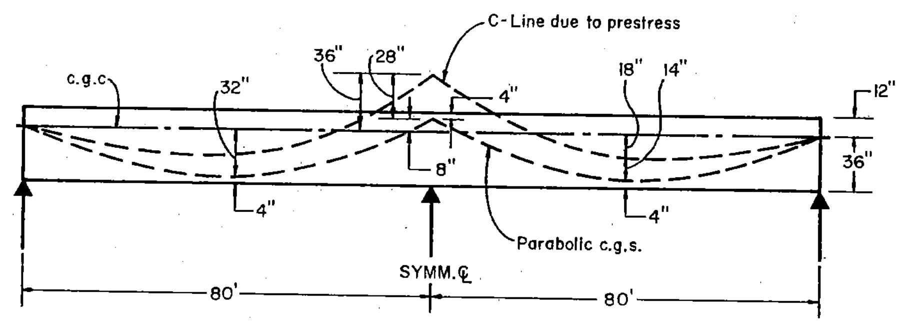


Fig. 1. Beam elevation for Example 1

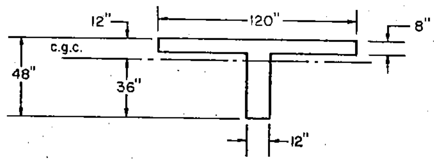


Fig. 2. Beam section for Example 1

tic hinges. If redistribution is not permitted at all, i.e., if the beam is considered to be totally elastic, then the elastically computed secondary moments due to non-concordant cables are obviously there and should not be neglected. However, the Code and its Commentary seem to say that in calculating the design moments, full moment redistribution is not permitted and secondary moments must be neglected at the same time. This interpretation will lead to grossly incorrect answers, which could be conservative or non-conservative.

Since the problem cannot be easily discussed in formulas, which could become quite lengthy and complicated, the nature and magnitude of the problem will be illustrated by two examples.

EXAMPLE 1

Consider a two-span continuous beam, Figs. 1 and 2, prestressed with a non-concordant parabolic c.g.s. line and bonded tendons; beam properties are:

1. Section properties

$A_c = 1440$ in.^{2}
$I = 253{,}340$ in.^{4}
$y_t = 12$ in.
$y_b = 36$ in.
$S_t = 21{,}100$ in.^{3}
$S_b = 7{,}030$ in.^{3}

2. Steel properties

A_s of tendons = 4.0 in.^{2}
Ultimate strength of steel
$f'_s = 250$ ksi
Effective prestress $f_{se} = 150$ ksi
Total effective prestress
$F_e = 600$ k.

3. Concrete properties

Ultimate compressive strength
$f'_c = 5$ ksi
Tensile stress at cracking =
$6\sqrt{f'_c} = 424$ psi

4. Secondary moments and reactions

With the c.g.s. line located as shown, it can be determined by inspection[6] that the C-line under prestress alone can be located by linearly transforming the parabolic c.g.s. line 28 in. upward at center support, and 14 in. upward at midspan. This results in a net eccentricity of 36 in. up at center support and 18 in. down at midspan. The secondary moment due to the effective prestress is

$$600 \times 28/12 = 1400 \text{ k. ft.}$$

over center support, and 700 k. ft. at midspan. Each end reaction due to prestressing is

$$\frac{1400}{80} = 17.5 \text{ k. upward}$$

and the corresponding secondary reaction at center support is

$$2 \times 17.5 = 35 \text{ k. downward}$$

To simplify our discussion throughout this example, we will consider the controlling $+M$ at midspan simultaneously with the $-M$ over center support. It is realized that, to get an accurate computation of the utlimate load, the exact location of controlling $+M$ away from midspan should be considered.

5. Balanced load due to prestressing

Since the parabolic c.g.s. has an effective sag of

$$h = 32 + 8/2 = 36 \text{ in.} = 3 \text{ ft.}$$

uniform load balanced by prestress is

$$w = \frac{8Fh}{L^2} = \frac{8 \times 600 \times 3}{80^2} = 2.25 \text{ k./ft.}$$

Thus, under an external load of 2.25 k./ft., the beam has a uniform compressive stress of $\frac{600{,}000}{1440} = 417$ psi along its entire length. Internal moment $= 600 \times \frac{8}{12} = -400$ k. ft. over center support and $600 \times \frac{32}{12} = 1600$ k. ft. at midspan.

6. $-M$ section at center support

To obtain zero tension at top fiber, it is necessary to negate the precompression of 417 psi by an additional moment,

$$M = f_t S_t = \frac{417 \times 21{,}100}{12{,}000} = 736 \text{ k. ft.}$$

which corresponds to a uniform load of

$$w = \frac{8M}{L^2} = \frac{8 \times 736}{80^2} = 0.92 \text{ k./ft.}$$

Adding this to the balanced load of 2.25 k./ft., we have

$$w_T = 0.92 + 2.25 = 3.17 \text{ k./ft.}$$

which is the total uniform load producing zero tension at top fiber over center support. The internal moment

$$= -736 - 400$$
$$= -1136 \text{ k. ft.}$$

To obtain a top fiber tension of 424 psi, additional uniform load can be computed by direct proportioning from the above 0.92 k./ft.

$$0.92 \times \frac{424}{417} = 0.93 \text{ k./ft.}$$

Thus, the total uniform load at cracking is

$$w_T = 3.17 + 0.93 = 4.10 \text{ k./ft.}$$

and internal moment

$$= -1136 - 736 \times \frac{424}{417}$$
$$= -1880 \text{ k. ft.}$$

To obtain ultimate moment capacity with $A_s = 4.0$ in^2., $b = 12$ in., and $d = 44$ in., compute

$$p = \frac{4}{12 \times 44} = 0.756\%$$

which indicates an over-reinforced section. Using 1963 ACI Code

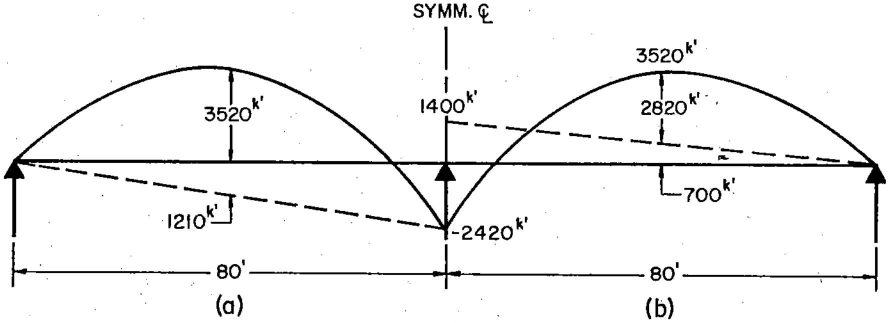
$$-M_u = 0.25 f'_c\, bd^2$$
$$= 0.25 \times 5 \times 12 \times \frac{44^2}{12}$$
$$= -2420 \text{ k. ft.}$$

Since no moment redistribution is permitted for this over-reinforced section, the elastic moment over the center support, $-M = wL^2/8$, must be used for calculating the ultimate load capacity. If, as per 1971 ACI Code, moment due to prestressing is neglected, we will have an ultimate uniform load

$$w_u = \frac{8\,M_u}{L^2}$$
$$= \frac{8 \times 2420}{80^2}$$
$$= 3.03 \text{ k./ft.}$$

Note that this is less than $w_T = 4.10$ k./ft. at start of top fiber cracking, and even less than $w_T = 3.17$ k./ft. under zero tension. This is obviously an incorrect answer as a result of the above interpretation of the 1971 ACI Code. The mistake results from the inconsistent assumption that, on the one hand, the beam is elastic under external load while, on the other hand, it is not elastic under the effect of prestress.

If it is agreed that secondary moments do exist, since the beam still



(a) With no secondary moments (b) With secondary moments

Fig. 3. Ultimate load capacity assuming full moment redistribution

behaves elastically, then the ultimate load capacity should be computed including the effect of the secondary moments. If secondary moment due to effective prestress is used, we have

$$w_u = \frac{8\,(M_{\text{ult.}} + M_{\text{sec.}})}{L^2}$$

$$= \frac{8\,(2420 + 1400)}{80^2}$$

$$= 4.78\ \text{k./ft.}$$

which is correct if no moment redistribution takes place.

7. $+M$ section at midspan

To obtain zero tension at bottom fiber, additional moment is

$$M = f_t S_b = \frac{417 \times 7030}{12{,}000}$$

$$= 244\ \text{k. ft.}$$

which for $+M = wL^2/16$ at midspan, means a uniform load of

$$w = \frac{16M}{L^2} = \frac{16 \times 244}{80^2}$$

$$= 0.61\ \text{k./ft.}$$

$$w_T = 0.61 + 2.25 = 2.86\ \text{k./ft.}$$

$$\text{Internal moment} = 1600 + 244 = 1844\ \text{k. ft.}$$

To obtain a bottom fiber tension of 424 psi, additional uniform load is computed

$$w = 0.61 \times \frac{424}{417} = 0.62\ \text{k./ft.}$$

$$w_T = 2.86 + 0.62 = 3.48\ \text{k./ft.}$$

aad internal moment

$$= 1844 + 244 \times \frac{424}{417}$$

$$= 2092\ \text{k. ft.}$$

To obtain ultimate moment capacity with $A_s = 4.0$ in.², $b = 120$ in., and $d = 44$ in., we have

$$p = \frac{4}{120 \times 44} = 0.076\%$$

Using 1963 ACI Code

$$f_{su} = f'_s\left(1 - 0.5p\,\frac{f'_s}{f'_c}\right)$$

$$= 250\left(1 - 0.5 \times 0.00076 \times \frac{250}{5}\right)$$

$$= 245\ \text{ksi}$$

$$M_u = A_s f_{su} d\left(1 - 0.6\,p\,\frac{f'_{su}}{f'_c}\right)$$

$$= 4 \times 245 \times 44 \times \left(1 - 0.6 \times 0.00076 \times \frac{245}{5}\right)$$

$$= 3520\ \text{k. ft.}$$

As far as the ultimate moment capacity of this section is concerned, neglecting both moment redistribution and the secondary moment, i.e. simply using elastic moment coefficients, we would have a theoretical ultimate load of

$$w_T = \frac{16M}{L^2}$$

$$= \frac{16 \times 3520}{80^2}$$

$$= 8.80\ \text{k./ft.}$$

If the secondary moment of -700 k. ft. produced by prestressing still exists, it must be deducted from the 3250 k. ft. to give a theoretical ultimate load of

$$w_T = \frac{16(3520 - 700)}{80^2}$$

$$= 7.05\ \text{k./ft.}$$

again assuming elastic moments, with no moment redistribution.

8. Ultimate load capacity with full moment redistribution

If full moment redistribution were possible, whether to include secondary moments does not make any difference. This is shown by the following two simple calculations, illustrated by Fig. 3 for this two-span continuous beam. The left half, Fig. 3 (a), assumes no secondary moments. Corresponding to $-M = -2420$ k.ft. and $+M = 3520$ k. ft., the load capacity is

$$w_u = \frac{8(3520 + 1210)}{80^2}$$

$$= 5.91\ \text{k./ft.}$$

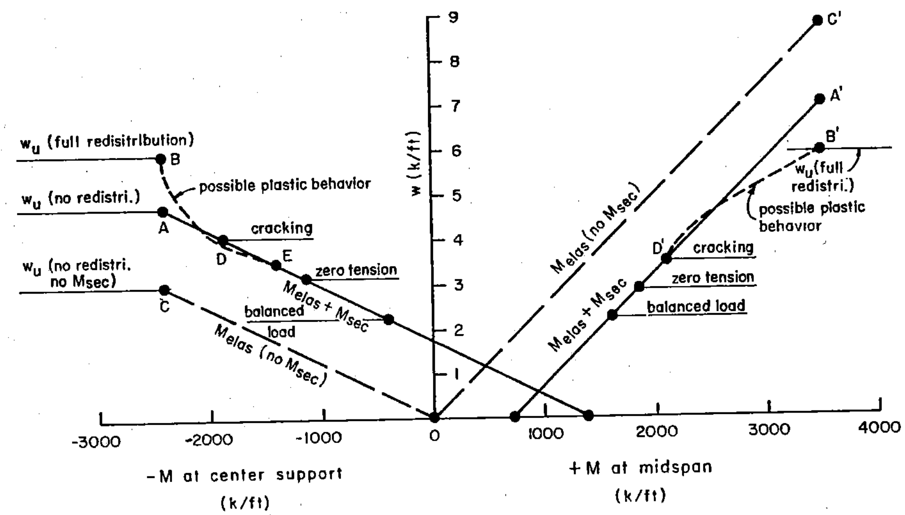


Fig. 4. Load and moment relationships for Example 1

Table 1. Loads and moments for critical conditions (Example 1)

	Stress and loading condition	Over center support		At midspan	
		$-M$ (k. ft.)	w_T (k./ft.)	$+M$ (k. ft.)	w_T (k./ft.)
Considering M_{sec}	Prestress only	+1400	0	+ 700	0
	Balanced load	− 400	2.25	+1600	2.25
	Zero tension	−1136	3.17	+1844	2.86
	Cracking at 424 psi	−1880	4.10	+2092	3.48
	Ultimate—no redistribution	−2420	4.78	+3520	7.05
	Ultimate—full redistribution	−2420	5.91	+3520	5.91
Neglecting M_{sec}	Prestress only	0	0	0	0
	Ultimate—no redistribution	−2420	3.03	+3520	8.80
	Ultimate—full redistribution	−2420	5.91	+3520	5.91

The right half, Fig. 3(b), considers secondary moments, $-M = 1400$ k. ft. and $+M = 700$ k. ft., and the load capacity is

$$w_u = \frac{8\left[2820 + \dfrac{2420 + 1400}{2}\right]}{80^2}$$

$$= 5.91 \text{ k./ft.}$$

This simple phenomenon justifies the ACI Code Commentary that the secondary moments disappear, but it should be emphasized that this is true only at full moment redistribution. If the elastic moments are not fully redistributed, then the secondary moments must also remain.

9. Summary of load and moment relationships

The above values of critical moments and corresponding uniform load intensity are now summarized in Table 1 and plotted in Fig. 4. Along the Y-axis of Fig. 4 is plotted the uniform load w. Along the X-axis are the $-M$ over center support and the $+M$ at midspan. The solid lines show the actual elastic moments produced by external loads including the effect of prestressing (M_{sec}). The dotted lines show the elastic moments, without considering the effect of prestressing (these, of course, do not represent the actual internal moments, but are plotted for the purpose of comparison).

Now let us look at Fig. 4 and try to visualize the actual ultimate load capacity. Let us assume that secondary moments do remain, and let us use elastic moments with no redistribution at all, the ultimate load is $w_u = 4.78$ k./ft. Then let us assume full moment redistribution, the capacity is $w_u = 5.91$ k./ft. (with or without secondary moments). For this particular example, because of the high value of $p = 0.756\%$ for the $-M$ section, plastic hinging action

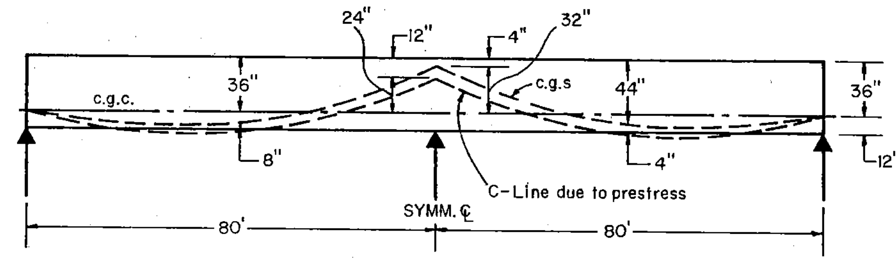


Fig. 5. Beam elevation for Example 2

cannot easily develop, and w_u is probably only slightly higher than 4.78 k./ft. However, if this section is not so highly over-reinforced, some hinging action will develop and moments will be redistributed. Thus, w_u will fall between 4.78 and 5.91 k./ft. It can be shown that the ultimate load capacity will not fall below 4.78 k./ft., since the $+M$ section is very much under-reinforced in this beam.

The cracking of the $-M$ and $+M$ sections starts at points D and D′ respectively. Since the $+M$ section will crack first (at $w = 3.48$ k./ft.), its moment will be gradually redistributed to the $-M$ section starting at point E. Then when the $-M$ section also starts to crack, moments will be redistributed back to the $+M$ section. Hence, the load-moment curve will follow the dotted line EDB, approaching point B as a limit, if full redistribution is achieved. If moment redistribution is not complete, this dotted line will not end at point B, but at some point between A and B[7]. It is just not possible for this line to dip downward toward point C, as an improper interpretation of the 1971 ACI Code would indicate.

EXAMPLE 2

In order to illustrate a different situation, when neglecting secondary moments can result in a mistake on the non-conservative side, we will now discuss another 2-span continuous beam, Figs. 5 and 6. This beam is the same as the one in Example 1, except with the section upside down, changing into an inverted T-section. The section properties for the beam in Example 1 can be reversed and used for this section and will not be listed again.

1. Secondary moment and reactions

It can be shown that the secondary moment at center support due to a prestress of 600 k. is

$$600 \times \frac{8}{12} = -400 \text{ k. ft.}$$

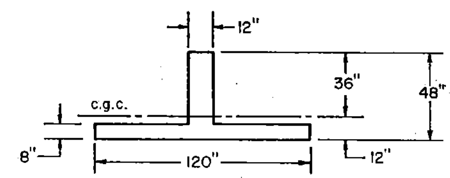


Fig. 6. Beam section for Example 2

Secondary exterior reaction $= \frac{400}{80}$ $= 5$ k. downward.

2. Balanced load due to prestressing

Effective parabolic sag

$$h = 8 + \frac{32}{2} = 24 \text{ in.}$$

$$w = \frac{8Fh}{L^2}$$

$$= \frac{8 \times 600 \times (24/12)}{80^2}$$

$$= 1.50 \text{ k./ft.}$$

Thus under an external load of 1.50 k./ft., the beam has a uniform compressive stress of 417 psi along its entire length. Internal moment $= 600 \times \frac{32}{12} = -1600$ k. ft. over center support, and $600 \times \frac{8}{12} = 400$ k. ft. at midspan.

3. −M section at center support

Using calculations similar to Example 1, the following critical moment and load values are obtained.

For zero tension at top fiber, additional moment is

$$M = \frac{7030 \times 417}{12{,}000} = 244 \text{ k. ft.}$$

$$w = \frac{8M}{L^2} = \frac{8 \times 244}{80^2} = 0.30 \text{ k./ft.}$$

$$w_T = 0.30 + 1.50 = 1.80 \text{ k./ft.}$$

Internal moment $= -244 - 1600 = -1844$ k. ft.

For tension = 424 psi at top fiber

$$w_T = 1.80 + 0.31 = 2.11 \text{ k./ft.}$$

Internal moment $= -1844 - 248 = -2092$ k. ft.

Ultimate moment capacity = 3520 k. ft. (from Example 1). If no moment redistribution is permitted and secondary moment neglected, the ultimate load capacity is

$$w_u = \frac{8M}{L^2} = \frac{8 \times 3520}{80^2} = 4.40 \text{ k./ft.}$$

If the secondary moment of −400 k. ft. is considered, actual capacity left for load-carrying is 3520 − 400 = 3120 k. ft., which, with no redistribution, yields

$$w_u = \frac{8 \times 3120}{80^2} = 3.90 \text{ k./ft.}$$

This indicates that neglecting secondary moment gives a non-conservative value of 4.40 k./ft. for this beam.

4. +M section at midspan

For zero tension at bottom fiber, additional moment is

$$M = f_t S_t = \frac{417 \times 21{,}100}{12{,}000} = 736 \text{ k. ft.}$$

$$w = \frac{16M}{L^2} = \frac{16 \times 736}{80^2} = 1.84 \text{ k./ft.}$$

$$w_T = 1.50 + 1.84 = 3.34 \text{ k./ft.}$$

Internal moment $= 400 + 736 = 1136$ k. ft.

For ultimate moment = 2420 k. ft., neglecting secondary moment, (considering only this section)

$$w_u = \frac{16 \times 2420}{80^2} = 6.05 \text{ k./ft.}$$

If secondary moment of −200 k. ft. is considered

$$w_u = \frac{16(2420 + 200)}{80^2} = 6.55 \text{ k./ft.}$$

5. Ultimate load capacity with full moment redistribution

Similar to Example 1, it can be shown that, assuming full moment redistribution, the ultimate load capacity is

$$w_u = \frac{8(2420 + 3520/2)}{80^2} = 523 \text{ k./ft.}$$

Table 2. Loads and moments for critical conditions (Example 2)

	Stress and loading condition	Over center support −M (k. ft.)	Over center support w_T (k./ft.)	At midspan +M (k. ft.)	At midspan w_T (k./ft.)
Considering M_{sec}	Prestress only	− 400	0	− 200	0
	Balanced load	−1600	1.50	+ 400	1.50
	Zero tension	−1844	1.80	+1136	3.34
	Cracking at 424 psi	−2092	2.11	+1884	5.21
	Ultimate—no redistribution	−3520	3.90	+2420	6.55
	Ultimate—full redistribution	−3520	5.23	+2420	5.23
Neglecting M_{sec}	Prestress only	0	0	0	0
	Ultimate—no redistribution	−3520	4.40	+2420	6.05
	Ultimate—full redistribution	−3520	5.23	+2420	5.23

which value is good whether or not the secondary moments are considered.

6. Summary of load and moment relationships

The above values of critical moments and corresponding uniform load intensities are now summarized in Table 2 and plotted in Fig. 7, similar to Fig. 4. From Fig. 7 it can be observed that:

a. Point C is higher than Point A indicating that if secondary moments are neglected, a mistake is made on the non-conservative side.

b. When cracking starts at Point D, moment redistribution will begin. Depending upon the development of both +M and −M hinges, the ultimate load capacity may fall anywhere between Point A and Point B.

c. The + M section may pick up more moments than indicated by the elastic values, owing to hinging at the −M section. The amount of redistribution is subject to further study.

This example thus shows that neglecting secondary moments may yield a non-conservative result.

DESIGN METHOD

An approximate method for determining the ultimate load capacity of a continuous beam is proposed, based on a load-balancing approach which takes into account the secondary moments produced by the ultimate prestress. For convenience, this method will neglect moment redistribution caused by plastic hinging action.

This method assumes that, for un-

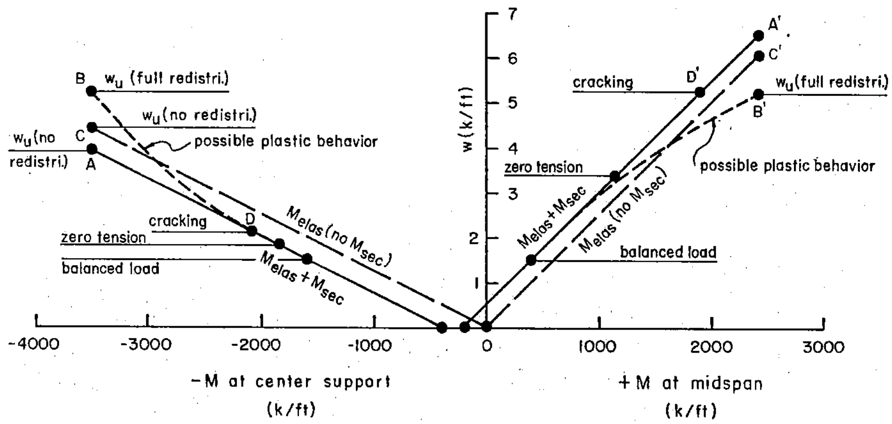


Fig. 7. Load and moment relationships for Example 2

bonded tendons, the ultimate stress capacity of the steel is at 1.20 × effective prestress (the coefficient 1.20 can be modified as required). It is further assumed that cable eccentricities in the elastic stage will be used for calculation. Hence, the load balanced by ultimate prestress is 1.20 × load balanced by the effective prestress. This load-balancing approach conveniently takes into account the effect of secondary moments at ultimate load. This method is quite accurate for unbonded tendons whose stress is more or less uniform along their length. For bonded tendons, high stresses are concentrated at points of hinging, and this method may not be accurate enough.

This method is applied to the above two examples.

Example 1

Under effective prestress, the balanced load is

$$w_{bal} = 2.25 \text{ k./ft.}$$

and the internal moments are

$$-M = -400 \text{ k. ft. over center support}$$

$$+M = 1600 \text{ k. ft. at midspan}$$

If ultimate prestress is 1.2 × effective prestress, the balanced ultimate load is

$$w_{bal} = 2.25 \times 1.2 = 2.70 \text{ k./ft.}$$

and the corresponding internal moments are

$$-M = -400 \times 1.2 = -480 \text{ k. ft. over center support}$$

$$+M = 1600 \times 1.2 = 1920 \text{ k. ft. at midspan}$$

The moment capacities left for loads above the balanced load are (since ultimate $-M$ capacity $= -2420$ k. ft., and $+M$ capacity $= 3520$ k. ft.)

$$-2420 + 480 = -1940 \text{ k. ft. over center support}$$

$$3520 - 1920 = 1600 \text{ k. ft. at midspan}$$

Obviously, $-M$ controls, and additional load is

$$w = \frac{8M}{L^2} = \frac{8 \times 1940}{80^2} = 2.43 \text{ k./ft.}$$

Total load capacity at ultimate is

$$w = 2.43 + 2.70 = 5.13 \text{ k./ft.}$$

As would be expected, this is slightly higher than the 4.78 k./ft. value previously obtained when considering the secondary moments produced by effective prestress.

Example 2

Similarly to the above, load balanced at ultimate prestress is

$$w_{bal} = 1.50 \times 1.2 = 1.80 \text{ k./ft.}$$

while the corresponding internal moments are

$$-M = 1600 \times 1.2 = -1920 \text{ k. ft. over center support}$$

$$+M = 400 \times 1.2 = 480 \text{ k. ft. at midspan}$$

Moment capacities left to carry additional load

$$-M = -3520 + 1920 = -1600 \text{ k. ft.}$$

$$+M = 2420 - 480 = 1940 \text{ k. ft.}$$

Again, $-M$ controls, and the additional load above balanced load is

$$w = \frac{8M}{L^2} = \frac{8 \times 1600}{80^2} = 2.00 \text{ k./ft.}$$

Total load capacity at ultimate is

$$w = 2.00 + 1.80 = 3.80 \text{ k./ft.}$$

This is slightly lower than the 3.90 k./ft. value previously obtained when considering the secondary moments produced by effective prestress. This is correct, because increasing the secondary moments by 20% decreases the load capacity, in this case.

This method is a conservative approach, since it neglects moment redistribution completely. It takes into account the secondary moments due to prestress without calculating for them. It correctly considers the secondary moments to increase instead of disappear at ultimate capacity. If it is desired to consider moment redistribution in the plastic range, adjustments can be best made by considering the load added beyond the balanced load. It should be noted that, under the balanced load, the beam has no curvature nor deflection anywhere.

CONCLUSIONS

A review of the above leads to the following conclusions with respect to continuous concrete beams prestressed with non-concordant cables. While tee and inverted-tee sections are used in the examples, rectangular or other sections will obviously follow similar reasoning. In fact, the following will apply generally to statically indeterminate structures prestressed with non-concordant cables.

1. If elastic moments are used to compute ultimate load capacity of a continuous prestressed beam, secondary moments produced by prestressing shall definitely be included in the calculations.

2. If plastic moments with full moment redistribution are used to compute ultimate load capacity of a continuous beam, then secondary moments may be either neglected or included, since the results will be the same.

3. If plastic hinges do not fully develop, then ultimate load capacity will lie between the two values computed in Items 1 and 2 above. The exact capacity can be determined by theoretical analysis, taking into account the moment-curvature relationships of the entire beam up to

the ultimate load, although this is not presented in this paper.

4. Approximate but conservative methods can be devised to consider the effect of secondary moments at ultimate load, but to neglect moment redistribution. This can be conveniently done using a load balancing approach.

5. More analytical and experimental research should be carried out concerning the plastic behavior of prestressed concrete continuous beams with non-concordant cables. However, it is abundantly clear that Sect. 18.12 of the 1971 ACI Building Code, if interpreted to neglect secondary moments when full moment redistribution does not occur, could lead to erroneous results.

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Discussion of this paper is invited. Please forward your comments to PCI Headquarters by May 1 to permit publication in the May-June 1972 issue of the PCI JOURNAL.