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# Design for Shear Based on Loading Conditions

by Michael D. Brown, Oguzhan Bayrak, and James O. Jirsa

*From published reports, a database of 1200 tests was compiled to examine the effects of loading type and position of the load on the shear strength of reinforced concrete beams. Twenty-four additional tests were conducted to examine the differences in shear response due to concentrated and uniform loads where data were lacking. Experimental results indicated that shear strength can be affected by the type of loading. It was observed that a significant number of beams subjected to concentrated loads applied between 2*d *and 6*d *from the face of the support failed at loads below the nominal strengths calculated using current design provisions. A simple change to the current ACI 318 shear design procedure is proposed for beams subjected to concentrated loads.*

**Keywords:** load; shear; structural concrete.

## INTRODUCTION

In 1962, Joint ACI-ASCE Committee 326 published a report[1] regarding the design and behavior of beams failing due to shear and diagonal tension. To develop safe design recommendations, a database of 194 beam tests without shear reinforcement was compiled. The database consisted of 130 laboratory specimens tested under single- and double-point loads and 64 beams subjected to uniformly distributed loads. Based on those data, the following design equation was developed (Fig. 1) and is included in ACI 318-05 (or ACI 318M-05 for the SI equation)[2] as Eq. (11-5)

$$V_c = \left(1.9\sqrt{f_c'} + 2500\rho_w \frac{V_u d}{M_u}\right) b_w d \leq 3.5\sqrt{f_c'}\, b_w d \text{ U.S.} \quad (1)$$

$$V_c = \left(\sqrt{f_c'} + 120\rho_w \frac{V_u d}{M_u}\right) \frac{b_w d}{7} \leq 0.3\sqrt{f_c'}\, b_w d \text{ SI}$$

where

$V_c$ = nominal shear strength provided by concrete;
$f_c'$ = specified compressive strength of concrete;
$\rho_w$ = $A_s/b_w d$;
$V_u$ = factored shear force at section;
$M_u$ = factored moment at section;
$b_w$ = web width;
$d$ = effective depth of section; and
$A_s$ = area of nonprestressed tension reinforcement.

For a given amount of flexural reinforcement, as the distance between a concentrated load and the support decreases, the ratio $Vd/M$ increases and the allowable shear strength of the member increases. For a simply supported member with a single concentrated load at midspan, the quantity $Vd/M$ varies from infinity at the supports to zero at midspan. To circumvent any problems, ACI Committee 326 calculated $Vd/M$ at the section where shear failure occurred in the laboratory specimen. Because the location of shear failure is unknown to the designer, the correct value of $Vd/M$ is also unknown. By neglecting the term involving $Vd/M$, a

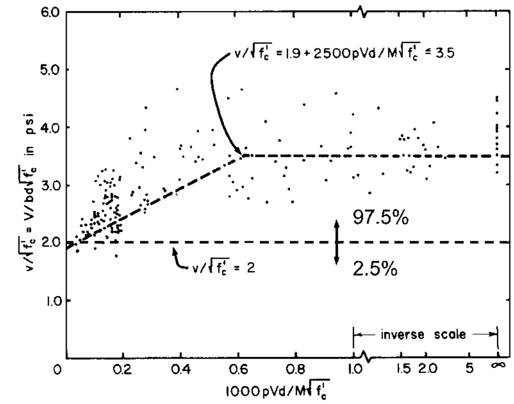


*Fig. 1—Database used in Joint ACI-ASCE Committee 326 Report.*[1]

simplified, conservative version of Eq. (1) could be derived (Eq. (2)).

$$V_c = 2\sqrt{f_c'}\, b_w d \text{ U.S.} \quad (2)$$

$$V_c = \frac{1}{6}\sqrt{f_c'}\, b_w d \text{ SI}$$

Using Eq. (2), 2.5% of the test results in the 1962 database failed at shear values less than those computed as can be seen in Fig. 1.

Two years after the ACI Committee 326 report appeared, Kani[3] published a paper in which the shear span-to-depth ratio ($a/d$) was used to determine the shear strength of a beam. Specifically, he quantified a range of $a/d$ in which a beam would fail at a moment less than the flexural capacity of the beam. The strength envelope Kani developed is shown in Fig. 2. The vertical axis of Fig. 2 is the ratio of the measured flexural strength to the calculated flexural strength of the beam. The range where reduced shear strength can occur is shown in Fig. 2 between $1.1 < a/d < 6.3$. In this range, the measured capacity of the beam is less than the calculated flexural capacity.

Kani defined two critical values of $a/d$: $(a/d)_{MIN}$ and $(a/d)_{TR}$. The first, $(a/d)_{MIN}$ is the $a/d$ at which the minimum strength of the beam occurs, and $(a/d)_{TR}$ is the $a/d$ at which the full flexural capacity of the beam can be reached. The values of these two critical $a/d$ depend on the material properties and geometry of the cross section.

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*ACI member* **Michael D. Brown** *is an Assistant Professor of Civil Engineering at Ohio University, Athens, Ohio. He received his BSCE, MSE, and PhD from the University of Texas at Austin, Austin, Tex., in 2000, 2002, and 2005, respectively. He is a member of Joint ACI-ASCE Committee 445, Shear and Torsion.*

*ACI member* **Oguzhan Bayrak** *is an Assistant Professor of Civil Engineering at the University of Texas at Austin. He is a member of ACI Committee 341, Earthquake-Resistant Concrete Bridges, and Joint ACI-ASCE Committee 445, Shear and Torsion. He is also a member and Chair of Joint ACI-ASCE Committee, 441, Reinforced Concrete Columns, and E 803, Faculty Network Coordinating Committee.*

**James O. Jirsa**, *FACI, holds the Janet S. Cockrell Centennial Chair in Engineering at the University of Texas at Austin. He is a Past President of ACI and is a member of the Concrete Research and Education Foundation (ConREF), the Concrete Research Council (CRC), the International Committee, the Committee on Personal Awards, the Standards Board, International Partnerships, and ACI Committees 318, Structural Concrete Building Code; 318-SC, Steering Committee; 318-WA, Workshop Attendees; and 408, Bond and Development of Reinforcement.*

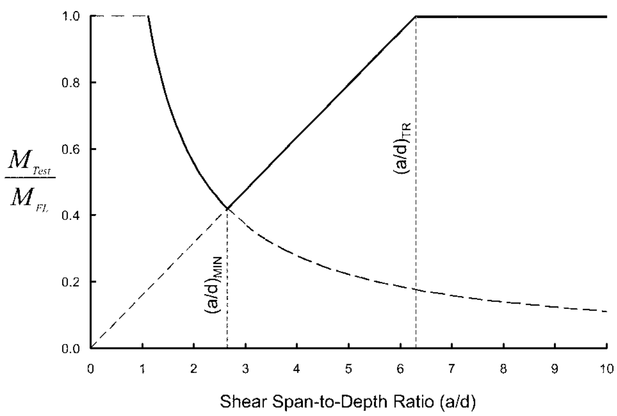


*Fig. 2—Kani's strength envelope.[3]*

Kani's strength envelope was developed using mechanics-based models of shear failure. Kani later confirmed the results of the mechanical models with extensive experimental research. He subjected several hundred beams, with and without shear reinforcement, to two point loads.

It has been evident for over 40 years that the type of loading and the location of the loads have an influence on the behavior of shear-critical reinforced concrete beams. The 1962 ACI Committee 326[1] report specifically cited differences in the cracking behavior of beams with concentrated or uniform loads. The relationship between the location of an applied concentrated load and shear strength is presented in textbooks by MacGregor and Wight,[4] Ferguson et al.,[5] and Collins and Mitchell.[6] Figure 3 is taken from Ferguson et al.[5] to illustrate the observed relationship between *a*/*d* and shear strength. No simple method to include these parameters in design equations in the ACI code has been adopted, however. With the increase in test data reported since 1962, it seems an opportune time to examine those data.

The current provisions of ACI 318 recognize this fact qualitatively in Section 11.5.6.1(a) and (b) which exempt slabs, footings, and concrete joist construction from the minimum shear reinforcement requirements. These exceptions address structural components that are typically subjected to uniformly, or nearly uniformly, distributed loads. Slabs are typically subjected to loads that are spread over wide areas. While footings may support concentrated loads, the footing itself is supported by a distributed load. In joist construction, the joists are generally closely spaced (less than 30 in. [762 mm]). These closely spaced joists are loaded through a slab that is monolithic with the joists. The slab serves to distribute forces to the joists and the beams supporting the

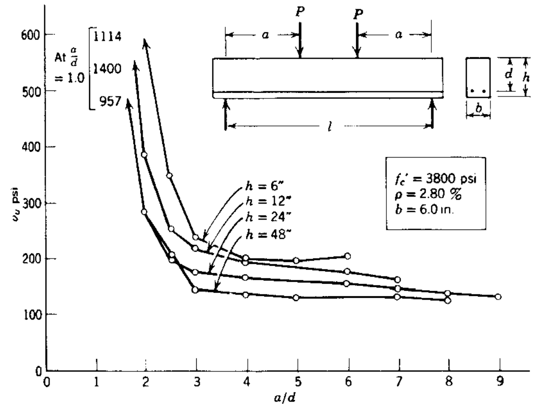


*Fig. 3—Relationship between shear span-to-depth ratio and shear strength.[5]*

joists. Because these structural components are exempt from the minimum shear reinforcement requirements, the code provisions implicitly recognize the increased shear strength of these types of members. Furthermore, when designing a beam subjected to uniformly distributed loads, the design shear is less than the peak shear carried by the beam. Again, within ACI 318 there is a qualitative understanding that there is a difference between the shear strength of a member subjected to uniform loads and a member subjected to concentrated loads.

When performing tests regarding the shear strength of reinforced concrete beams in a laboratory, a single concentrated load or a pair of concentrated loads is typically applied to the test specimens. Distributed loads have been used in relatively few tests. Therefore, many researchers have presented results that show the ACI 318 provisions for shear strength to be alarmingly unconservative. In practice however, failures of concrete structures are exceedingly rare. In the laboratory, concentrated loads are used; but in field conditions, many loads are distributed in some manner.

## RESEARCH SIGNIFICANCE

Tests show[7-27] that shear failure can occur in reinforced concrete beams at levels of load that are lower than indicated by ACI 318-05.[2] In this study, tests were conducted to examine the effect of type of loading and *a*/*d*, and the results were added to a database that was compiled to identify the effects of longitudinal reinforcement, transverse reinforcement, and cross-sectional dimensions on shear strength. Based on this study, a modification to current design procedures for shear is proposed to address member geometry and load configurations in the range of most concern.

## EXPERIMENTAL INVESTIGATION

The majority of published shear tests consist of beams with one or two concentrated loads placed symmetrically on the specimens. The results of relatively few tests with uniformly distributed loads have been published. None of the tests incorporated into the database included specimens with loads that were placed asymmetrically on the specimens. To fill gaps in the technical literature, tests of beams loaded with asymmetric concentrated and distributed loads were conducted. Symmetric tests were also conducted to examine the differences between the shear strength of members subjected to concentrated and distributed loads.

A total of 24 specimens were designed, detailed, and tested under various loading types and configurations. Ten

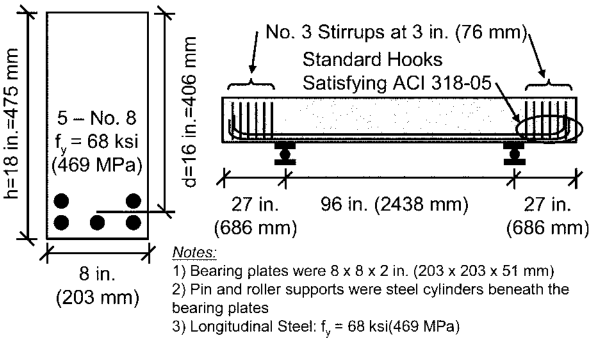


*Fig. 4—Details of test Specimens 1, 2, 4, and U.*

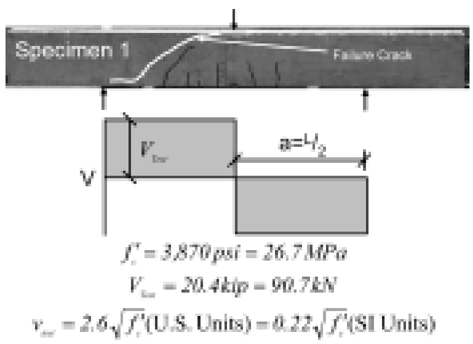


*Fig. 5—Failure conditions of Specimen 1.*

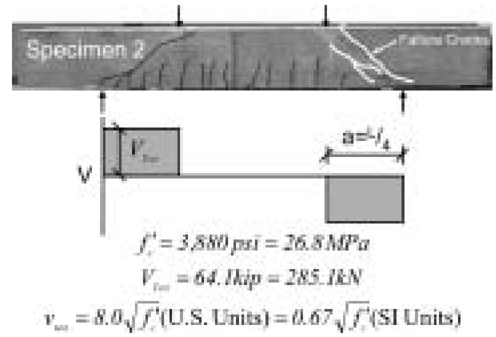


*Fig. 6—Failure conditions of Specimen 2.*

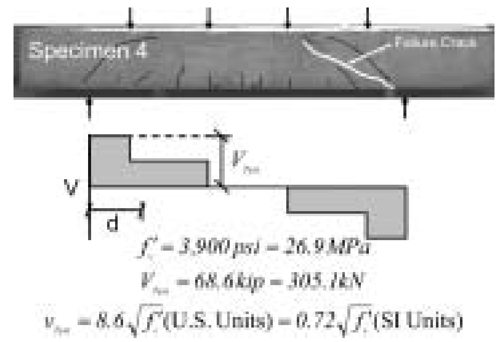


*Fig. 7—Failure conditions of Specimen 4.*

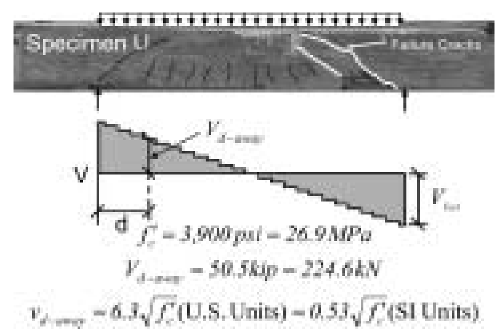


*Fig. 8—Failure conditions of Specimen U.*

tests are discussed in detail in this paper to describe the effects of loading type and *a*/*d* on shear strength of reinforced concrete beams. Data from the 24 tests conducted in this study and 1200 tests extracted from the literature are used in evaluating the current ACI 318 provisions for shear strength.

The concrete used to cast all specimens contained 3/8 in. (10 mm) aggregate (river gravel). The beams were cast in the same orientation as they were later tested. The specimens were wet cured for 7 days under layers of saturated burlap and plastic sheeting. The beams were then exposed to normal atmospheric conditions until the time of testing. The beams were tested between 30 and 60 days from the time of casting.

## Effects of loading type

To study the effects of loading type, four nominally identical beams were constructed. The details of the test specimens are shown in Fig. 4. Each of the beams was subjected to a different type of load (Fig. 5 through 8). Specimen 1 had a single point load at midspan, Specimen 2 had two point loads applied at *L*/4 and 3*L*/4, Specimen 4 had four point loads applied at *L*/8, 3*L*/8, 5*L*/8, and 7*L*/8; and Specimen U was subjected to a uniform load. The uniform load for Specimen U was produced with 24 hydraulic rams connected to a single hydraulic manifold and acting on 24 identical bearing plates. All four beams were constructed without stirrups between the supports. The compressive strength of the concrete was slightly less than 4000 psi (27 MPa) when the beams were tested.

The single concentrated load applied to Specimen 1 created equal *a*/*d* on either side of the load (Fig. 5). The *a*/*d* for Specimen 1 was 3.0 which is near the minimum point of Kani's shear strength envelope (Fig. 2). Therefore, the relatively low shear strength (20.4 kip [90.7 kN]) of this beam should not be surprising. The peak shear carried by Specimens 2, 4, and U increased as the load distribution became more uniform. The increase in strength, however, was most dramatic between Specimens 1 and 2. Specimen U carried the greatest peak shear (75.8 kip [337.2 kN]) of the four tests. In accordance with ACI 318 procedures, however, the shear force on the beam should be calculated at a distance *d* away from the face of the support (50.5 kip [224.6 kN]) for design purposes. It is important to note that the reported shear strengths of all four beams include the shear due to self-weight. The shear due to self-weight was calculated at the critical section as defined by ACI 318-05.

For the specimen with a concentrated load, a single strut formed between the load point and the support reaction. This strut focused the forces applied to the beam to the small volume of concrete within the strut. Once that strut reached peak capacity, the beam failed. For specimens with more uniform loads, the volume of concrete subjected to large stress was greater. Therefore, the capacity of these specimens was not based solely on the capacity of a single strut.

Specimens 4 and U had similar shear strengths and failure crack orientations. The similarity in cracking reflects the fact that four point loads distribute load nearly uniformly along the span. As a result, Specimens 4 and U had greater shear strength than Specimen 1.

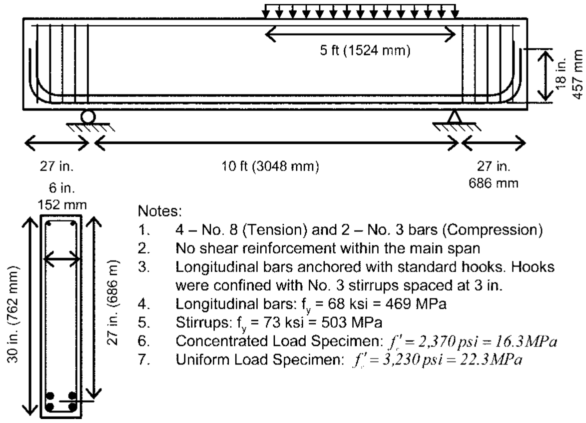


*Fig. 9—Specimens used to examine strain distribution.*

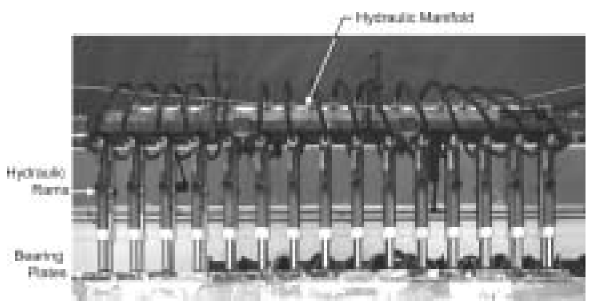


*Fig. 10—Hydraulic rams used for uniform load.*

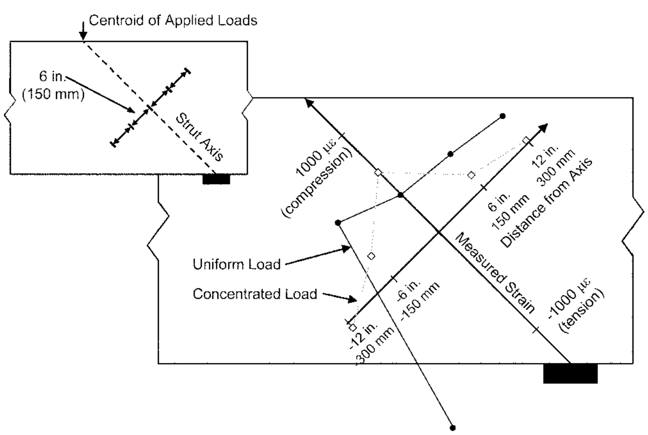


*Fig. 11—Location of strain gauges (inset top left) and measured strain values.*

## Concentrated versus uniform loads

Ten tests were conducted to examine the effect of load type on the strain distributions within a cross section. The details of two of the specimens are given in Fig. 9. The first of the specimens was subjected to a uniform load over half of its span and the second specimen was subjected to a single concentrated load. The single load was placed at the centroid of the uniform load used for the companion specimen. Note that only the uniformly loaded specimen is shown in Fig. 9. Figure 10 shows a photograph of the hydraulic rams used to produce the uniform load. The uniform load was produced by 30 identical hydraulic rams connected to a single hydraulic manifold. A similar hydraulic system was used to develop the uniform loads in the previous series of tests (Specimen U).

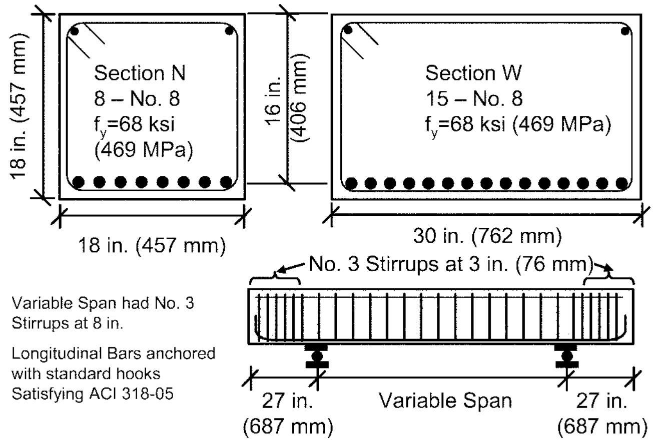


*Fig. 12—Details of asymmetric concentrated load tests.*

Strain gauges were placed on the surface of the specimens as shown in Fig. 11. The strain gauges were placed in exactly the same location on both specimens. This layout of gauges was intended to capture the distribution of stresses in the strut that forms between the load point and the support for the concentrated load specimen. Afterwards, the same data were collected from the specimen with uniform load for comparison.

The measured strain distributions from both specimens at maximum load are shown in Fig. 11. The magnitude of the peak strain in both specimens was similar (≈600 με). The distributions, however, were quite different. The distribution corresponding to the specimen with a concentrated load shows a distinct peak at the center of the plot (and therefore the center of the strut). The measured strains then decay to approximately zero as one moves from the center of the strut. For the specimen with uniform load, the peak is located closer to the tension face of the beam ($x$ = –6 in. = –150 mm). Also, the peak is less pronounced than in the concentrated load case. The strain distribution for the uniform load is much more uniform than for the concentrated load. A large tensile strain was recorded only during the uniform load test. The strain distributions presented in the figure are from only two specimens, but those distributions are typical of additional specimens not reported herein.

These strain distributions further reinforce the statements made regarding the four specimens used to examine effects of load type. There appears to be a significant difference between uniform and concentrated loads. The ACI Committee 326 report[1] noted that for specimens loaded with a uniformly distributed load, the shear cracks occurred some distance from the face of the support but for concentrated loads the cracks occurred at the face of the support. The strain distributions shown in Fig. 11 show that the peak strain for the beam subjected to uniform load was below the strut axis while the peak strain for the specimen with concentrated load was directly on the axis. The strain distributions agree with the locations of crack observed in the ACI Committee 326 report.

## Asymmetric concentrated load tests

Four tests were conducted with a single concentrated load applied asymmetrically with respect to the supports so that the behavior of two different shear spans could be observed in the same test. The segment of the beams with smaller shear span was subjected to higher shear force than that with a larger shear span. Other variables (concrete strength, longitudinal reinforcement, and stirrup spacing) were kept constant within a given test.

Two different cross sections were used in this stage of testing (Fig. 12). Before Specimen N-1 was tested, it was expected that shear failure would occur in the segment of the specimen with the greater shear force, that is, the end with the smaller shear span. However, failure occurred on the side of the beam with the longer shear span and smaller shear force. With *a*/*d* of 1.7 and 5.8, the applied shear force on the short span was 3.4 times that on the long span, yet shear failure occurred on the longer portion of the span. Failure of all four specimens occurred in the region of lower shear. Photographs of the specimens after failure are shown in Fig. 13. Only a portion of Specimen N-1 is visible in Fig. 13. The left reaction is not shown in the photograph, because it is blocked by the loading apparatus. The results of all four asymmetric tests are summarized in Table 1.

In the shorter shear span, a direct strut formed between the reaction and the load point; however, in the longer shear span, a more complex truss mechanism formed. These two mechanisms resulted in drastically different shear strengths. The strength of the more complex mechanisms (the longer shear span) was low enough to produce shear failure at much lower levels of load than the direct strut mechanism (the shorter shear span).

Specimens N-1, W-1, and W-2 failed in shear at a shear force lower than that determined using the provisions for $V_c$ in ACI 318-05. Because these specimens contained shear reinforcement, the concrete contribution was calculated as $V_c = V_{test} - V_s$. Only Specimen W-3 reached shear strength in excess of the design shear strength of the beam. The long shear span of all four beams was within the limits of Kani's shear strength envelope, thus low strengths are not surprising. By examining each the two different shear spans within these four specimens, the data suggests that the *a*/*d* is an important parameter in determining concrete contribution to shear strength.

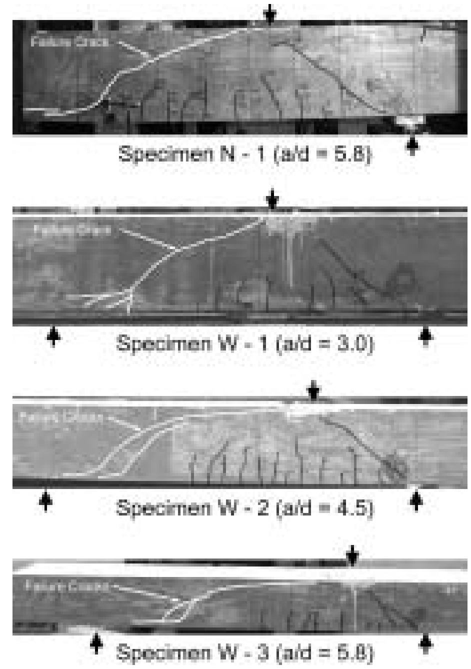


*Fig. 13—Photographs of asymmetric tests after failure (for all specimen* a/d *= 1.7 for right side;* a/d *ratios listed within figure refer to left side).*

**Table 1—Results of asymmetric concentrated load tests**

| Specimen | $f'_c$, ksi | Span, in. | $V_s$,* kip | South portion of span†: Distance from support to load, in *d* | South portion of span†: $V_{Test}$, kip | South portion of span†: $\frac{V_{Test} - V_s}{\sqrt{f'_c}\, b_w d}$ | North portion of span: Distance from support to load, in *d* | North portion of span: $V_{Test}$, kip |
|---|---|---|---|---|---|---|---|---|
| N-1 | 2.85 | 120 | 32 | 5.8 | 42 | 0.64 | 1.7 | 153 |
| W-1 | 3.11 | 75 | 32 | 3.0 | 84 | 1.78 | 1.7 | 149 |
| W-2 | 3.57 | 99 | 32 | 4.5 | 82 | 1.75 | 1.7 | 236 |
| W-3 | 3.65 | 120 | 32 | 5.8 | 101 | 2.58 | 1.7 | 266 |

*$V_s = A_v f_y d/s$, No. 3 stirrups with $f_y$ = 73 ksi.

†Failures occurred on south portion of span (Fig. 9, left side).

Note: 1 in. = 25.4 mm; 1 kip = 4.448 kN; 1 ksi = 6.895 MPa.

## DATABASE OF SHEAR TESTS

To examine the differences in the measured shear strengths of beams subjected to concentrated loads and beams subjected to uniform loads, a database of published test results was compiled. A brief description of the beams included in this database is included in Table 2. The database comprises tests that represent the last 50 years of research into the shear strength of reinforced concrete beams. Beams that were described by the original authors as having a failure mode other than shear were not included in the database. The shear due to the self-weight of the test specimens in the database has been included in the calculation of the failure shear.

Some limitations were placed on specimens included in the database. Only rectangular cross sections supported on simple spans, without axial loads, were considered. Normal-weight concrete and conventional steel reinforcing bars were used to construct all beams. These limitations were imposed to assure simple, well-defined geometry that would permit relatively easy determination of the concrete contribution to shear strength $V_c$.

The provisions for shear strength developed by ACI Committee 326 in 1962 were based on a database of 194 tests. Of those tests, 18 were T-beams and 15 were continuous beams while in practice most beams are continuous and T-sections. Therefore, the limitations placed on the specimens compiled into the database presented herein seem justified. The flanges of T-sections are not generally believed to significantly affect the shear strength of the sections. Additional study is needed, however, to examine the impact of continuity on the shear strength of reinforced concrete beams.

Of the 1200 tests which comprise the database, 104 were beams subjected to uniform load. For the cases of beams subjected to uniform load, the measured shear capacity $V_{test}$ is taken as the shear occurring at a distance *d* away from the face of the support in accordance with ACI 318 design procedures.

### Nominal shear strength provided by concrete $V_c$

For evaluating the concrete contribution to the shear strength in the database, only beams without shear reinforcement were considered. Of the 1200 tests, 758 beams had no web reinforcement. To determine the shear force contributed by web reinforcement, the stirrups must be instrumented and the number of stirrups bridging the shear crack must be known. By and large, the test specimens included in the database did not contain such instrumentation or strain measurements

**Table 2—Components of database**

| Reference | No. | $f'_c$, ksi | $\rho_w$, % | $d$, in. | $a/d$ |
|---|---|---|---|---|---|
| Ahmad and Lue[7] | 54 | 8.8 to 9.7 | 0.35 to 6.64 | 7.3 to 8.4 | 1.0 to 4.0 |
| Angelakos et al.[8] | 21 | 3.0 to 14.4 | 0.50 to 2.09 | 36.4 | 2.9 |
| Bažant and Kazemi[9] | 27 | 6.7 to 6.8 | 1.62 to 1.65 | 0.8 to 13.0 | 3.0 |
| Bresler and Scordelis[28] | 12 | 3.3 to 5.6 | 1.80 to 3.66 | 18.1 to 18.4 | 3.9 to 7.0 |
| Cao[10] | 4 | 4.0 to 4.5 | 0.36 to 1.52 | 74.3 | 2.8 to 2.9 |
| Chang and Kesler[29] | 25 | 2.2 to 5.6 | 1.86 to 2.89 | 5.4 | 1.7 to 3.5 |
| Clark[11] | 62 | 2.0 to 6.9 | 0.98 to 3.42 | 13.0 to 16.0 | 1.1 to 2.3 |
| de Paiva and Siess[12] | 19 | 2.9 to 5.6 | 0.46 to 2.58 | 6.0 to 12.0 | 0.7 to 1.3 |
| de Cossio and Siess[30] | 7 | 3.1 to 4.6 | 0.34 to 3.36 | 9.9 to 10.0 | 2.0 to 7.0 |
| Ferguson[13] | 4 | 3.5 to 4.3 | 2.15 | 7.1 to 7.4 | 1.5 to 3.2 |
| Foster and Gilbert[14] | 16 | 11.2 to 17.4 | 1.25 to 2.15 | 27.6 to 47.2 | 0.7 to 1.7 |
| Hsiung and Frantz[31] | 4 | 6.2 | 1.82 | 16.5 | 3.0 |
| Johnson and Ramirez[15] | 8 | 5.3 to 10.5 | 0.25 | 21.2 | 3.1 |
| Kani et al.[16] | 190 | 2.2 to 5.3 | 0.48 to 2.89 | 5.2 to 43.2 | 1.0 to 9.1 |
| Kong and Rangan[32] | 48 | 9.2 to 13.0 | 0.34 to 4.47 | 7.8 to 21.3 | 1.5 to 3.3 |
| Kong et al.[33] | 35 | 2.7 to 3.8 | 0.49 to 1.47 | 10.0 to 30.0 | 0.3 to 1.0 |
| Krefeld and Thurston[17] | 195 | 1.6 to 7.0 | 0.34 to 5.01 | 9.4 to 19.0 | 2.3 to 9.7 |
| Laupa et al.[18] | 13 | 2.1 to 4.7 | 0.34 to 4.11 | 10.3 to 10.8 | 5.0 to 5.2 |
| Lubell et al.[19] | 1 | 9.3 | 0.76 | 36.0 | 3.0 |
| Moody et al.[34] | 42 | 0.9 to 6.0 | 0.80 to 4.25 | 10.3 to 21.0 | 1.5 to 3.4 |
| Morrow and Viest[20] | 38 | 1.6 to 6.8 | 0.57 to 3.83 | 13.9 to 14.8 | 0.9 to 7.9 |
| Oh and Shin[35] | 53 | 3.4 to 10.7 | 1.29 to 1.56 | 19.7 | 0.5 to 2.0 |
| Ozcebe et al.[36] | 13 | 8.4 to 11.9 | 1.93 to 4.43 | 12.2 to 12.8 | 1.9 to 5.0 |
| Rajagopalan and Ferguson[37] | 10 | 3.4 to 5.3 | 0.25 to 1.73 | 10.2 to 10.6 | 3.8 to 4.3 |
| Ramakrishnan and Ananthanarayana[38] | 26 | 1.5 to 4.1 | 0.12 to 0.60 | 15.0 to 30.0 | 0.2 to 0.9 |
| Rigotti[39] | 12 | 2.4 to 5.0 | 4.14 | 12.0 | 1.8 to 2.3 |
| Rogowsky et al.[21] | 13 | 3.8 to 6.3 | 0.40 to 1.80 | 19.7 to 39.4 | 0.8 to 1.6 |
| Roller and Russel[22] | 10 | 10.5 to 18.2 | 1.64 to 6.97 | 22.0 to 30.0 | 2.5 to 3.0 |
| Sarsam and Al-Musawi[40] | 14 | 5.7 to 11.6 | 0.22 to 3.51 | 9.2 | 2.5 to 4.0 |
| Shin et al.[41] | 30 | 7.6 to 10.6 | 3.77 | 8.5 | 1.5 to 2.5 |
| Shioya[23] | 8 | 3.1 to 4.1 | 0.39 | 7.9 to 118.1 | 6.0 |
| Smith and Vantsiotis[24] | 47 | 2.3 to 3.3 | 1.94 | 14.0 | 0.9 to 1.8 |
| Subedi et al.[42] | 8 | 4.1 to 7.5 | 0.14 to 1.09 | 17.7 to 35.4 | 0.4 to 1.4 |
| Tan and Lu[43] | 12 | 4.5 to 7.1 | 0.26 | 17.5 to 61.4 | 0.6 to 1.1 |
| Tan et al.[44] | 13 | 6.4 to 8.5 | 1.23 | 18.2 | 0.3 to 1.1 |
| Tan et al.[45] | 19 | 8.1 to 12.5 | 2.58 | 17.4 | 0.9 to 1.7 |
| Tan et al.[46] | 3 | 9.4 to 10.1 | 2.58 to 4.08 | 16.5 to 17.4 | 0.3 to 0.6 |
| Uribe and Alcocer[47] | 2 | 5.1 | 1.58 | 43.3 | 1.3 |
| Uzel[25] | 14 | 4.0 to 6.2 | 0.76 to 2.16 | 9.1 to 36.4 | 1.9 to 4.9 |
| Van Den Berg[48] | 44 | 2.6 to 11.2 | 1.72 to 4.35 | 14.1 to 17.6 | 2.1 to 4.9 |
| Watstein and Mathey[49] | 9 | 3.3 to 3.9 | 0.75 to 3.05 | 13.1 to 15.9 | 1.5 to 2.1 |
| Xie et al.[50] | 15 | 5.8 to 15.8 | 0.21 to 4.54 | 7.8 to 8.5 | 1.0 to 4.0 |
| Yang et al.[51] | 8 | 4.6 to 11.4 | 0.90 to 1.00 | 14.0 to 36.8 | 0.5 to 0.6 |
| Yoon et al.[26] | 12 | 5.2 to 12.6 | 2.49 | 25.8 | 3.3 |
| Yoshida[27] | 4 | 4.9 to 5.0 | 0.74 | 74.3 | 2.9 |
| Current investigation | 24 | 2.4 to 3.9 | 2.0 to 3.1 | 16 to 27 | 1.5 to 6.0 |
| Complete database | 1200 | 0.9 to 18.2 | 0.1 to 7.0 | 0.8 to 118.1 | 0.2 to 9.7 |

Note: 1 ksi = 6.895 MPa; 1 in. = 25.4 mm.

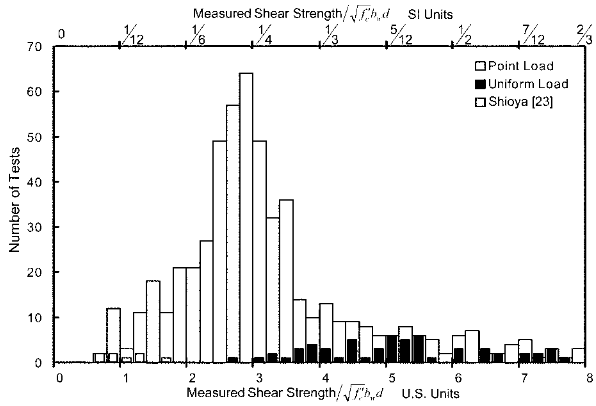


*Fig. 14—Shear strength of specimens without web reinforcement.*

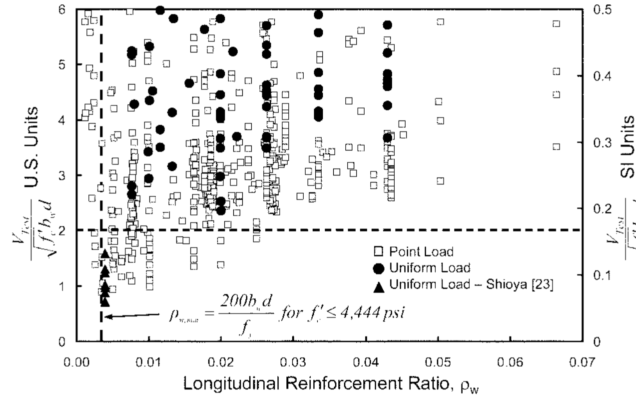


*Fig. 15—Concrete contribution to shear strength versus longitudinal reinforcement ratio.*

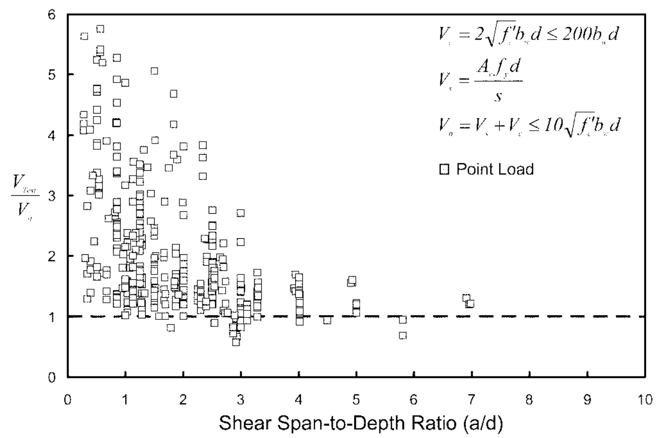


*Fig. 16—Shear strength of specimens with web reinforcement.*

were not reported in the papers. For the relatively few tests where strain instrumentation was present, not all stirrups were instrumented so that an accurate estimate of the steel contribution to the shear strength is difficult to determine. Consequently, only beams without transverse reinforcement were considered in evaluating $V_c$. Specimens that included transverse reinforcement will be discussed in a later section to evaluate the nominal shear capacity ($V_n = V_c + V_s$).

Of the 758 specimens without web reinforcement, 57 failed at loads less than that given by Eq. (11-3) of ACI 318-05 (Eq. (2)). The current strength reduction factor of ACI 318-05 ($\phi$) is insufficient to address the number of unconservative tests. The test specimens that failed below the strength allowed by ACI 318 were confined to tests of beams with a concentrated load acting between 2*d* and 6*d* from the support (Fig. 14).

In Fig. 14, the difference between the response of beams subjected to uniform loads and concentrated loads is apparent. The ACI 318 provisions for the concrete contribution to shear strength result in conservative estimates of strength for all of the uniform load tests with the exception of the tests conducted by Shioya.[23] Those tests will be discussed in detail later. The only factors common to the tests that failed at $V_c < 2\sqrt{f'_c}\, b_wd$ (U.S. units) ($V_c < 1/6\sqrt{f'_c}\, b_wd$ [SI units]) are *a*/*d* and the loading type, that is, concentrated loads.

### Uniform tests by Shioya[23]

The 13 tests conducted by Shioya[23] constitute a series of carefully conducted, large-scale tests intended to examine size effect and the influence of maximum aggregate size on overall strength. The results were thoroughly analyzed and reasons for the low capacities can be explained. Three of the beams failed in flexure and are not included in the database. Two of the remaining beams failed due to "abnormal diagonal tension," as per Shioya.[23] These beams have no apparent diagonal or shear cracks, but the flexural reinforcement did not yield during the test, hence they are referenced as "abnormal." These two beams were also omitted from the database.

For six of the eight remaining specimens that failed in shear, the longitudinal reinforcement was not constant along the length of the beam. The location where the longitudinal bars were cut was 1.5*d* from the support. Six beams failed at a shear crack that initiated near that cut-off point. It has been established that shear strength may be reduced at the location of a longitudinal bar cut-off.[5] The factored self-weight of the largest beam in the series (*d* = 118.1 in. [3000 mm]) produced a moment greater than the factored moment capacity ($\phi M$) using the design provisions in ACI 318-05. Therefore, the beam did not have sufficient capacity to carry its self-weight. All of the beams in the test series had minimal longitudinal reinforcement. The beams tested by Shioya had longitudinal reinforcement ratios ($\rho_w$ = 0.4%) that were only slightly greater than the minimum allowed by Section 10.5.1 of ACI 318-05 ($\rho_{min}$ = 0.33%). For the strengths of concrete used by Shioya,[23] the minimum reinforcement ratio is governed by $200b_wd/f_y$ (U.S. units) or $1.4b_wd/f_y$ (SI units) rather than

$$A_{s,\,min} = \frac{3\sqrt{f'_c}}{f_y}b_wd \text{ (U.S.)} \tag{3}$$

$$A_{s,\,min} = \frac{\sqrt{f'_c}}{4f_y}b_wd \text{ (SI)}$$

where $f_y$ = specified yield strength on nonprestressed reinforcement.

The link between longitudinal reinforcement ratio and shear strength can be seen in Fig. 15. Therefore, the parameters of the Shioya tests were considered to be near or outside the limits for reinforcement details and minimum capacity that are given in ACI 318.

### Effect of transverse reinforcement

In Fig. 16, the capacities of 444 test specimens with shear reinforcement are plotted. From this figure it is apparent that

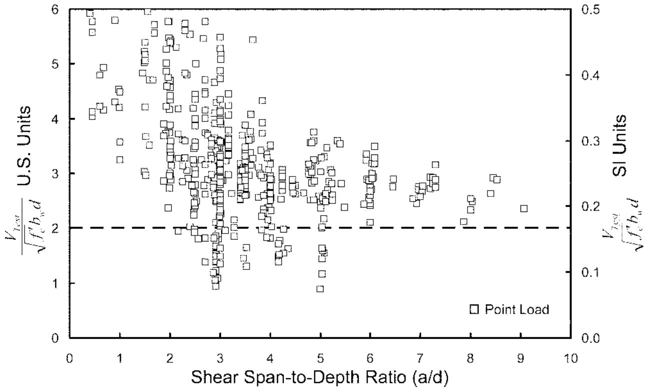


*Fig. 17—Shear strength of specimens without web reinforcement.*

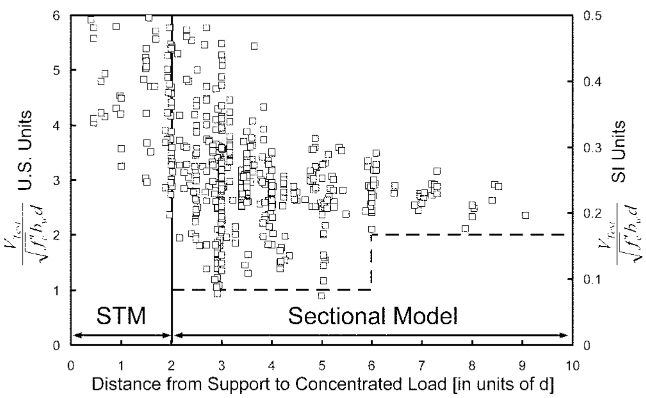


*Fig. 18—Proposed shear strength provisions for sectional models for members subjected to concentrated loads.*

the conclusions regarding specimens without web reinforcement hold for specimens with web reinforcement. The similarity between Fig. 16 and 17 implies that unconservative estimates of the concrete contribution to shear strength ($V_c$) are the primary cause of low strength of beams subjected to concentrated loads.

In Fig. 16, low-strength values for tests with an $a/d$ less than 2.0, are in the range that according to ACI 318-05 provisions must be designed using Appendix A, "Strut-and-Tie Models." Consequently, the shear strength of specimens with $a/d$ between 2.0 and 6.0 are of importance for a sectional shear model ($V_n = V_c + V_s$).

The number of unconservative test results for specimens with shear reinforcement is 22 of 442 such specimens. The corresponding number of unconservative test results for specimens without shear reinforcement is 57 (of 758 tests). If only specimens that satisfy the maximum spacing requirement for shear reinforcement are considered, the number of results that are unconservative are reduced from 22 (of 442 tests) to 12 (of 269 tests).

In Fig. 19, the strength of the specimens is plotted as a function of the ratio of $V_s$ to $V_c$. The majority of unconservative results are from tests with low levels of shear reinforcement ($V_s/V_c < 1$). For specimens that satisfy the transverse spacing requirements of ACI 318-05 (Fig. 20), many unconservative test results are still present. In both figures, the vertical line at $V_s/V_c = 4$ represents the maximum steel contribution to shear strength allowed by ACI 318. Using maximum allowable shear strength of a beam as $10\sqrt{f'_c}\, b_w d$ (U.S. units) ($5/6\sqrt{f'_c}\, b_w d$

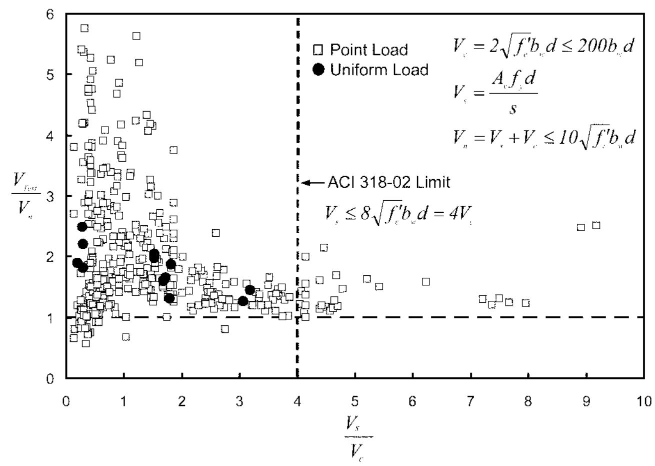


*Fig. 19—Effect of transverse reinforcement for all specimens with* $V_s > 0$.

[SI units]) and $V_c = 2\sqrt{f'_c}\, b_w d$ (U.S. units) ($V_c = 1/6\sqrt{f'_c}\, b_w d$ [SI units]), the steel contribution is equal to four times the concrete contribution. For the data in these figures, the upper limit on concrete strength has been used in determining the nominal shear capacity.

## DESIGN RECOMMENDATIONS

The current ACI 318-05 code provisions for shear yield unconservative strength estimates only for beams subjected to concentrated loads applied between 2$d$ and 6$d$ from the support. From Section 11.3.1.1 of ACI 318-05:

For members subject to shear and flexure only

$$V_c = 2\sqrt{f'_c}\, b_w d \quad \text{(U.S.)} \tag{2}$$

$$V_c = \frac{1}{6}\sqrt{f'_c}\, b_w d \quad \text{(SI)}$$

To include the effects of loading type and shear span to depth ratio into the current code provisions, the following statement should be added to that provision:

For members in which more than 1/3 of the factored shear at the critical section results from a concentrated load located between 2$d$ and 6$d$ of the face of the support

$$V_c = 1\sqrt{f'_c}\, b_w d \quad \text{(U.S.)} \tag{4}$$

$$V_c = \frac{1}{12}\sqrt{f'_c}\, b_w d \quad \text{(SI)}$$

Such a reduction in shear strength will substantially reduce the number of tests that fall below code values (Table 3 and Fig. 18). By implementing the proposed provision only one test result in the database is unconservative (of 269 specimens that satisfy transverse spacing requirements) compared with 12 (of 269 specimens) using the current provisions. Similar changes result for specimens with no transverse reinforcement. The distribution of the ratio of measured strength to nominal strength calculated using the proposed provisions is shown in Fig. 21. The entire database (specimens with and without shear reinforcement) is included in Fig. 21.

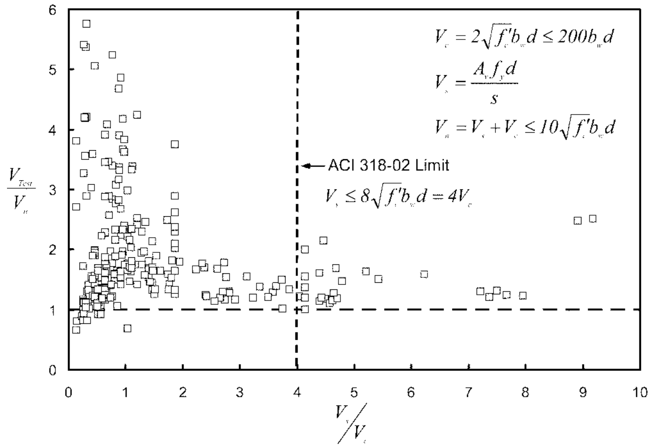


*Fig. 20—Effect of transverse reinforcement for specimens with point loads that satisfy ACI 318-05 spacing requirements and minimum shear reinforcement requirements.*

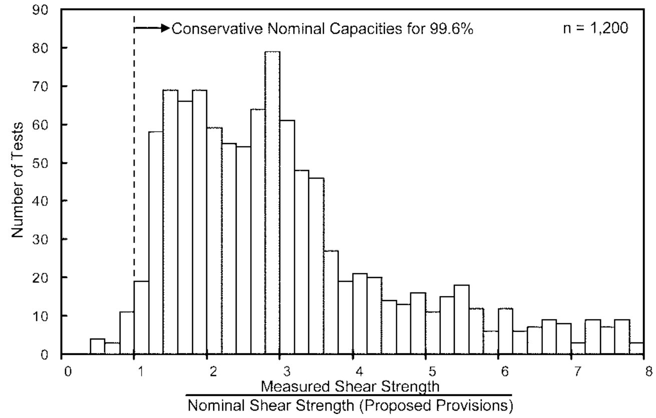


*Fig. 21—Distribution of shear strengths calculated using proposed provisions for full database.*

**Table 3—Percentage of unconservative test results**

| | No. of unconservative test results | |
|---|---|---|
| | ACI 318-05 provisions | Proposed provisions |
| Specimens without shear reinforcement ($V_s = 0$) 758 total tests | 57 | 11 |
| Specimens with shear reinforcement ($V_s > 0$) 442 total tests | 22 | 5 |
| Specimens with shear reinforcement and satisfy ACI 318-05 transverse spacing requirements 269 total tests | 12 | 1 |

The maximum shear strength allowed by ACI 318-05 is $10\sqrt{f'_c}\, b_w d$ (U.S. units) ($5/6\sqrt{f'_c}\, b_w d$ [SI units]). The data plotted in Fig. 22 are from test specimens with concentrated loads located between 2*d* and 6*d* of the support. Additionally, the data shown in Fig. 22 are from test specimens that satisfy the current limits for the maximum spacing of transverse reinforcement and minimum amount of shear reinforcement in ACI 318-05. Using the proposed shear provisions for $V_c$, the maximum allowable steel contribution $V_s$ is nine times that of $V_c$, so that the maximum shear strength remains at $10\sqrt{f'_c}\, b_w d$ (U.S. units) ($5/6\sqrt{f'_c}\, b_w d$ [SI units]) as indicated by the vertical line in Fig. 22. Nearly all data in Fig. 22 exhibit strengths greater than that indicted by ACI 318-05, even for specimens with large amounts of transverse reinforcement.

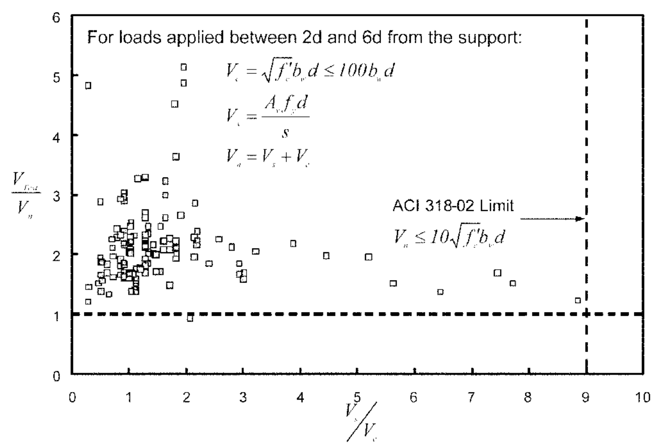


*Fig. 22—Maximum shear strength using proposed shear provisions for specimens that satisfy ACI 318-05 spacing requirements and minimum shear reinforcement requirements and subjected to concentrated loads.*

## SUMMARY AND CONCLUSIONS

In this study, the results of 1200 beams tests were examined. By identifying the effects of loading type and the distance from the applied load to the support, some simple changes in code provisions were developed. These proposed provisions are applicable only to structural members subjected to a narrowly-defined-type of loading. The shear design of many structural components is left unchanged.

1. The shear strengths of members subjected to uniform, or near uniform, loads are higher than those of member subjected to concentrated loads. Current code provisions provide safe estimates of strength for beams subjected to uniform loads;

2. Test specimens that exhibit shear strengths less than that permitted by ACI 318-05 are by and large limited to specimens subjected to concentrated loads that are applied between 2*d* and 6*d* from the face of the support;

3. The primary impact of the proposed provisions will be to increase the size of transfer girders or other elements under concentrated loads and hence increase the shear strength of such critical structural elements;

4. Most beams in a reinforced concrete building are loaded via a slab or a series of joists. Such loads are much closer to a uniform loading, and the shear design provisions for these members will remain unchanged; and

5. The current upper limit on shear strength, $10\sqrt{f'_c}\, b_w d$ (U.S. units) ($5/6\sqrt{f'_c}\, b_w d$ [SI units]), should remain in place if the proposed provisions are adopted, that is, if Eq. (4) is used for concentrated loads acting at distances between 2*d* and 6*d* from the face of the support.

## ACKNOWLEDGMENTS

The authors would like to thank J. Breen and S. Wood for their assistance with the interpretation of the data. The authors would also like to thank the Texas Department of Transportation for providing financial support for this research program, I. Ornelas and H.-J. Shin for their assistance with the experimental program and T. Okazaki for assisting with the interpretation of the Shioya report. Opinions, findings, conclusions, and recommendations in this paper are those of the authors.

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Disc. 103-S56/From the July-Aug. 2006 *ACI Structural Journal*, p. 531

**Punching of Reinforced and Post-Tensioned Concrete Slab-Column Connections.** Paper by Thomas H.-K. Kang and John W. Wallace

### Discussion by Robert E. Englekirk

*Englekirk Partners Consulting Structural Engineers, Inc., Los Angeles, Calif.*

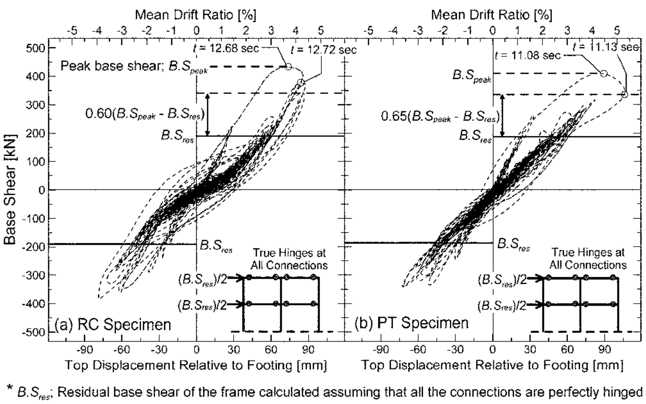


*Fig. A—Base shear versus mean drift ratio.*

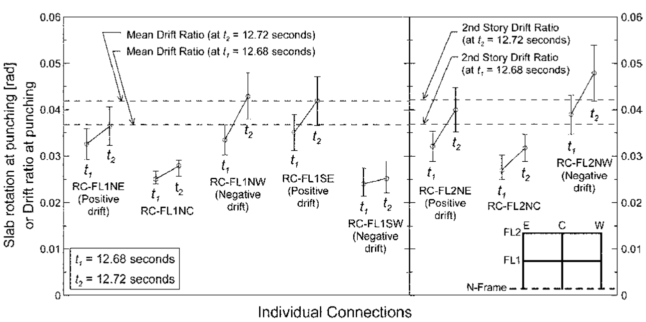


*Fig. B—Slab rotation capacities at punching of individual connections (reinforced concrete specimen).*

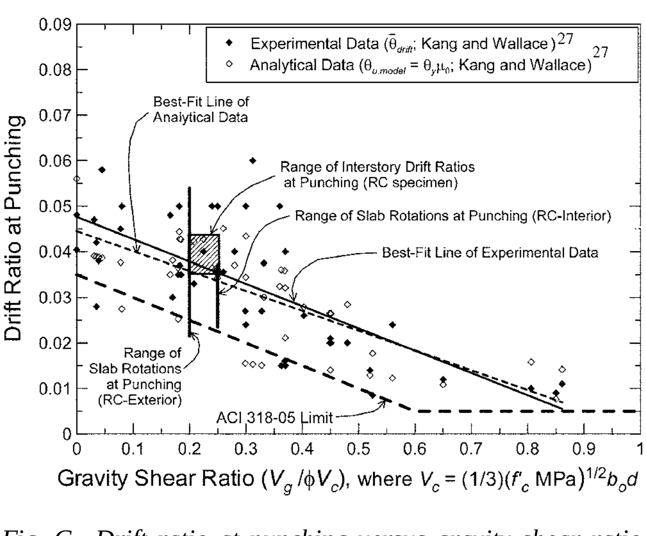


*Fig. C—Drift ratio at punching versus gravity shear ratio (reinforced concrete interior connections without shear reinforcement and reinforced concrete shaketable specimen).*

This article is in support of ACI 318-05, Section 21.11.5. It suggests that the identified limit states (Fig. R21.11.5) are quite conservative. The discusser raises four questions for the authors.

*Question 1*—Figures 2(a) and 8 identify $V_c$ as $(1/3)f'^{1/2}_c b_o d$. The ACI limit state for slab shear $V_c$ is slightly more than $2f'^{1/2}_c b_o d$. Please explain.

*Question 2*—Drift limits contained in the ACI 318-05 referenced codes are collapse threshold events. Is a punching shear failure consistent with this design objective?

*Question 3*—Did any of the referenced test specimens result in a collapse or complete failure of the slab?

*Question 4*—If the designer of a post-tensioned deck provides shear reinforcement, must he or she still pass at least two strands through the column?

*Comment*—The cost and time required to build concrete residential buildings has doubled in the last 10 years—compliance with this provision adds another 5%.

### AUTHORS' CLOSURE

The authors would like to thank the discusser for his interest in the paper and the opportunity to clarify and comment on the issues raised. In addition, the authors use this opportunity to correct an error in data reduction that impacts the results presented in the paper. Responses to the questions and comments posed by the discusser are provided, followed by the correction.

First of all, the authors would like to clarify that the article was neither for nor against the provisions of Section 21.11.5 of ACI 318-05. Rather, the article provided background, data, and analysis to assess the impact of the provisions as well as to provide context.

In response to Question 1, the authors note that the units used for $f'_c$ are MPa, not psi, and slab shear stress (in psi) for a square critical section is typically $4\sqrt{f'_c}$ psi, not $2\sqrt{f'_c}$ psi. Therefore, the 1/3 multiplier in this case is equivalent to ($0.33\sqrt{f'_c}$ MPa = $4\sqrt{f'_c}$ psi).

In response to Questions 2 and 3, the intent of the ACI 318-05, Section 21.11.5, requirements is to reduce the likelihood of punching failure (damage) in the design-basis earthquake (DBE), and not the maximum considered earthquake, which is generally associated with collapse. As well, at least two continuous bottom bars are required to pass within the column (Section 13.3.8.5) to support gravity load after punching failure. Therefore, the requirements appear to be focused more on improved performance under the DBE versus collapse prevention. The apparent focus on improved performance produced substantial debate within ACI Committee 318 prior to the approval of this code change; however, consensus was apparently achieved because the provision provides both improved performance and safety at relatively low cost.

The shaketable specimens tested by the authors were designed according to the ACI 318-02 code, and thus included continuous bottom (integrity) reinforcement; therefore, no collapse was observed during the tests. Furthermore, allowing complete collapse is not feasible for shaketable tests. Of the prior, quasi-static, lateral load tests referenced, the drift levels at punching failures (that is, substantial loss of lateral load capacity) are reported. None of the tests produced complete collapse, either because testing was stopped or continuous bottom reinforcement was provided to prevent complete collapse.

**Table A—Interstory drift capacities at punching**

| | Time, seconds | $V_s/\phi V_c$ | $\bar{\theta}_{drift}$ |
|---|---|---|---|
| Reinforced concrete mean drift | 12.68 | 0.25 | 0.0368 |
| | 12.72 | 0.25 | 0.0419 |
| Reinforced concrete second-story drift | 12.68 | 0.25 | 0.0369 |
| | 12.72 | 0.25 | 0.0421 |
| Post-tensioned mean drift | 11.08 | 0.33 | 0.0439 |
| | 11.13 | 0.33 | 0.0521 |
| Post-tensioned second-story drift | 11.08 | 0.33 | 0.0458 |
| | 11.13 | 0.33 | 0.0559 |

Note: $\phi = 1$.

In reference to Question 4, the use of shear reinforcement reduces the extent of the damage and, in particular, prevents the dropping of the slab observed in reinforced concrete connections where shear reinforcement is not provided.[12] Because the shear reinforcement commonly used in construction practice does not pass through the column, it may not be effective in preventing collapse and continued use of current requirements is prudent. The lack of slab damage adjacent to the column could improve gravity load transfer (for example, improved dowel action), however, potentially reducing the quantity of reinforcement that must pass within the column core.

During data reduction, the authors mistakenly removed the contribution of rigid body rotation of the load cells mounted under the footings to the story drift ratio, which impacted Fig. 5, 6, and 8 and Table 4, but not the findings. The corrected figures and table are provided as Fig. A, B, and C and Table A, respectively.

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Disc. 103-S57/From the July-Aug. 2006 *ACI Structural Journal*, p. 541

**Design for Shear Based on Loading Conditions.** Paper by Michael D. Brown, Oguzhan Bayrak, and James O. Jirsa

### Discussion by Himat T. Solanki

*ACI member, Professional Engineer, Building Department, Sarasota County Government, Sarasota, Fla.*

The authors have presented an interesting paper. The discusser would like to offer the following comments:

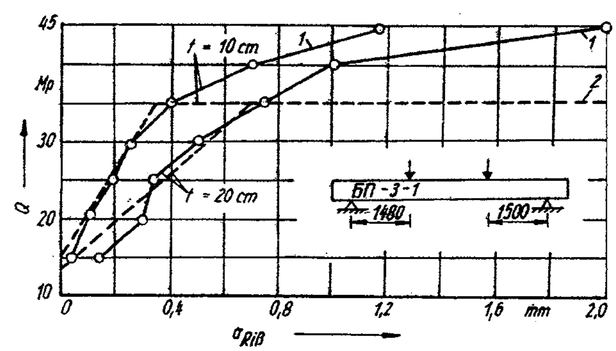
1. Based on the cited references by the authors, it appears that the authors are either unaware of the previously published work or may not have reviewed the work.[54-56]

2. Based on the cited references it appears that the authors have not considered the beams such as I-beams, double T-beams with symmetrical and unsymmetrical flange width, beams having an opening(s) within the web (beams having hole(s)), variable (tapered/hunched) depth, or circular beams in their study. Though these beams would not make any difference in the ACI code limitation, they do have an impact on their strength ratio, that is, test values versus calculated values.

3. From the paper, it is very difficult to judge how the analysis (strut-ties model [STM]) was performed, particularly for single-point/two-point/uniformly-distributed loads. For example, in a beam having a single-point concentrated load, was an STM considered as a one-unit truss or a multi-unit truss? (Based on the space truss theory, the STM could be rearranged for a given loading condition.) Though a single-truss versus multi-truss model has no impact on its ultimate load-carrying capacity, it does have an impact on the crack pattern (that is, crack width and crack spacing).

4. The authors have not addressed the crack pattern such as the crack width in their analysis. For example, when all parameters of beams were kept constant, but only the stirrup spacing had changed, what impact would there be on the beam behavior? Borischanskij[57] has tested two beams (Fig. A) with a change in stirrup spacing, and he observed different crack widths for a given load on both beams. From Fig. A, it can be seen that the crack width increases with the stirrup spacing increases for a given constant load condition.

5. As shown in Fig. B, the discusser has analyzed 2381 test specimens, including a large number of the authors' specimens (except References 25, 27, 39, and 47) and also beams such as I-beams, double T-beams with symmetrical and unsymmetrical flange width, beams having an opening(s) within the




1 Versuche von M. S. Borischanskij
2 Rechnung

| b | d | R | Längsbewehrung | | | Querbewehrung | | | |
|---|---|---|---|---|---|---|---|---|---|
| b cm | d cm | R kp/cm² | Anzahl ø | $\sigma_s$ kp/cm² | $\mu_e$ % | Anzahl ø | $\sigma_s$ kp/cm² | t cm | $\mu_{eB}$ % |
| ,23 | 62,5 | 344 | 8ø25 | 6 000 | 2,73 | 4ø7,9 | 3 240 | 20 | 0,425 |
| | | | | | | 2ø7,9 | | 10 | |

*Fig. A—Load versus crack width.*[57]

web (beam having hole(s)), variable (tapered/hunched) depth, circular beams from various publications using an STM, as well as considering multi-unit truss elements[58,59] for single-point and two-point loading conditions found to be consistent with Hedman and Losberg.[54] These beams were described by the original authors as having a shear failure mode.

6. Because the authors have concentrated on the ACI code formula and its limitation for $V_c$, the discusser would like to request a clarification based on the following concept:

To calculate the real shear strength of concrete

$$f'_c = P_u/\{A_s\} \quad \text{or} \quad P_u = f'_c A_s$$
$$P_u = 2D^2 f'_c \quad \text{or} \quad P_u = 72 f'_c \tag{5}$$

where $P_u$ equals the failure axial load on cylinder; $D$ equals the diameter of a cylinder equal to 6.0 in. (152.4 mm) (ACI code); and $A_s$ equals the real single plane maximum sheared cross section (maximum probable value of ideal sheared cross section).

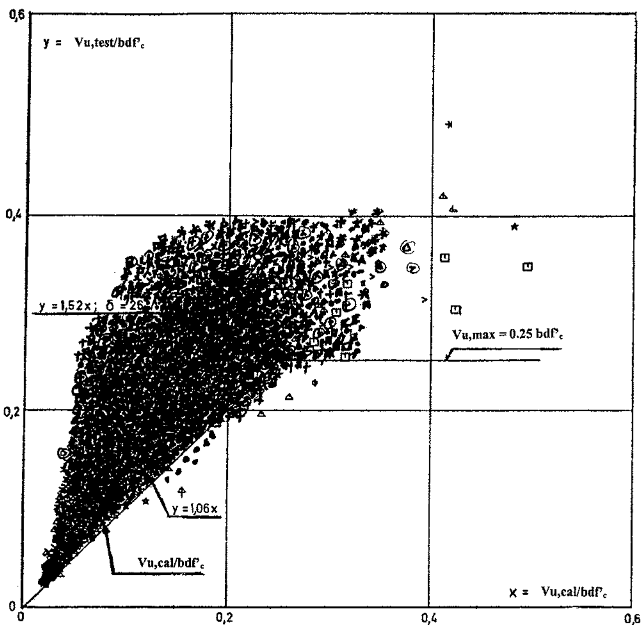



*Fig. B—Test shear failure* $V_{u,test}$ *compared with calculated shear capacity* $V_{u,cal}$.[54]

Because of the shear strength due to a single concentrated load in the beam along the line of 45 degrees, the shear strength per Joint ACI-ASCE Committee 326[1] and ACI Committee 318[2]

$$v_c = (P_u \text{ or } V_u/2)/bd = 2\sqrt{f_c'}$$
$$\text{or} \quad P_u = 2\sqrt{f_c'}bd \tag{6}$$

Equating Eq. (5) and (6)

$$72 f_c' = 2\sqrt{f_c'}bd$$
$$bd = 36\sqrt{f_c'}$$

or cross-sectional area of beam = $36\sqrt{f_c'}$

This means the cross-sectional area is directly proportional to the square root of the concrete compressive strength and, hence, the square root of the concrete compressive strength controls the beam dimensions/geometries. Is this true? If it is true, how can a dimension for all other beam geometries be established? Should it be based on concrete compressive stress block?

## ACKNOWLEDGMENTS


The discusser gratefully acknowledges S. Unjoh, Leader, Earthquake Engineering Team, Public Works Research Institute; R. K. Dhir, University of Dundee; K. Reineck, University of Stuttgart; P. Chanda, British Cement Association; J. Clarke, The Concrete Society; M. W. Braestrup, Ramboll Hannermann Hojlund; R. Heffernan, Institution of Civil Engineers; R. Thomas, Institution of Structural Engineers; and G. Sabnis for providing the publications related to the shear strength of beams.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Disc. 103-S57/From the July-Aug. 2006 *ACI Structural Journal*, p. 541


**Design for Shear Based on Loading Conditions.** Paper by Michael D. Brown, Oguzhan Bayrak, and James O. Jirsa

### Discussion by Ivan M. Viest


*ACI member, IMV Consulting, Bethlehem, Pa.*


The authors should be congratulated on the significant contribution to shear research. The discusser would like to add a historical perspective.

Equations (1) and (2) are based on research performed at the University of Illinois half a century ago. One of the enduring contributions of that research was expressing the shear strength of concrete as a function of the square root of its compressive strength. In the 1963 issue of the ACI code, the square root relationship replaced an earlier linear one. It has been retained to this day. It first appeared in print in an internal report issued in Dec. 1955 and in the ACI JOURNAL, *Proceedings* in March 1957.[60] In both publications, the shear strength was shown as a function of the ratio of moment to shear in the form $M/Vd$; and Eq. (2) was suggested as the lower-bound design limit for shear at ultimate load in reinforced concrete members without web reinforcement.

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Disc. 103-S57/From the July-Aug. 2006 *ACI Structural Journal*, p. 541

**Design for Shear Based on Loading Conditions.** Paper by Michael D. Brown, Oguzhan Bayrak, and James O. Jirsa

## Discussion by Evan C. Bentz

*ACI member, Associate Professor, University of Toronto, Toronto, Ontario, Canada.*

The authors have presented a paper that makes a case for improving the safety of the shear provisions of the current ACI code. While the discusser fully agrees with this goal, he has serious concerns with the first conclusion presented in the paper. This conclusion is that the shear strength of a member subjected to a uniformly distributed load (UDL) is inherently higher than that of a member subjected to concentrated loads, perhaps twice as high on average. This conclusion is contradicted in previous technical literature and does not appear to be supported by the new tests in the authors' paper. A total of four arguments are used in the paper to support the conclusion and these are each discussed in the following.

The authors note that the current code exempts slabs, footings, and joist construction from the requirement to provide minimum shear reinforcement when the shear exceeds $0.5V_c$. They suggest that this higher allowable stress provides implicit support for their conclusion as these member types are often subjected to uniform loads. It is important to note that the commentary to the code states that these member types "are excluded from the minimum shear reinforcement requirement because there is a possibility of load sharing between weak and strong areas." That is, it indicates a different explanation than that provided by the authors and thus some care is warranted in interpreting any assumed implicit meaning. The technical report on which the current shear strength provisions of the ACI code are based is the "326 Report" from 1962, included in the authors' paper as Reference 1. In this reference, tables show that the average ratio of experimentally-observed shear strength to ACI code predicted shear strength was 1.180 for 430 test results without stirrups. For the subset of 64 experimental results of uniformly loaded members, the average ratio was 1.192. Thus, the report on which the current code provisions are based indicates that the UDL member may be stronger than point-loaded members, but only by approximately 1% on average.

The authors' second argument in favor of their conclusion is in new test results presented on four experiments loaded with a variable number of point loads (refer to Fig. C). The authors suggest that, as the number of point loads is increased on the span, the shear strength increased. Figure D plots the failure shear at the critical section for shear $d$ from the support with respect to the distance to the centroid of the forces causing that shear. While there is no clear trend of the shear strength with respect to the loading type, there is a clear trend compared with the shear span. This trend is the same as that shown in Fig. 1 of the authors' paper where Kani showed that shorter shear spans result in higher shear strengths. With regard to this, it is relevant to note that Kani himself, in his 1966 paper on shear,[61] stated that "the behavior of reinforced concrete beams under a uniformly distributed load appears to be essentially the same as under point loads."

The third argument in support of uniformly loaded members being different from point-loaded members is that the distribution of internal concrete strains is different. These results are for members with a shear span-to-depth ratio of 1.0, and thus provide some evidence for behavior associated with an Appendix A strut-and-tie analysis, but their relevance to the "beam shear" equations of Chapter 11, which the authors propose to change, is unclear. Perhaps the authors can explain.

The final argument used is based on the database of shear test results as presented in Table 2 of the paper. The discusser has serious concerns about this comparison primarily due to clear mistakes in the database. Consider that the first data series listed in the table indicates that 54 members were used by the authors, yet the original reference[7] clearly indicates that 18 of these specimens failed in flexure rather than shear. It is not appropriate to compare the ACI shear strength equations with members that did not fail in shear. With a brief examination of Table 2, the reinforcement values for at least seven of the test series were also found to be wrong, often with the lower bound of $\rho_w$ being incorrect by a factor of 10. Overall, the table mixes three failure modes: strut-and-tie failures, beam action shear failures, and flexural failures. These should be compared with the ACI Appendix A strut-and-tie equations, Chapter 11 shear equations, and Chapter 10 flexural equations, respectively. By putting them all together and only comparing them with the simplest shear equation in the

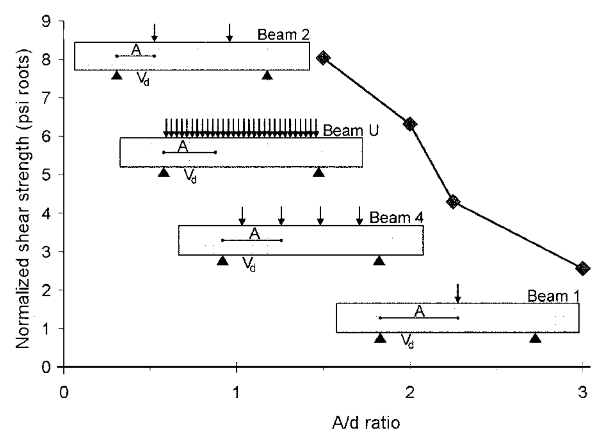


*Fig. C—Shear strength of authors' tests with respect to shear span.*

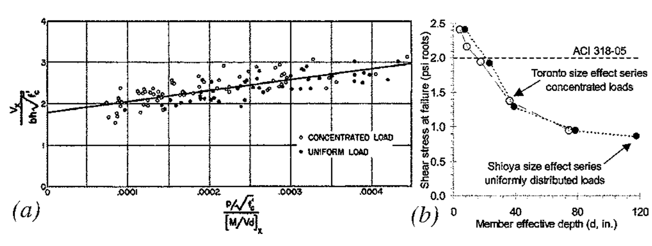


*Fig. D—Comparison of shear strengths of uniformly loaded and point loaded members for: (a) small heavily reinforced members; and (b) large lightly reinforced members.*

code, it is, perhaps, not surprising that the authors failed to note clear trends in the database.

Conclusions based on Fig. 14 to 22 in the paper should be treated with caution as many make "apples to oranges" comparisons. As an example of an "apples to apples" comparison, Fig. D(a) shows results from tests in the authors' database[17] on UDL and point-loaded members. This clearly shows that small heavily-reinforced beams produce similar shear strengths regardless of loading type. The one set of shear experiments performed to date on uniformly loaded, large lightly-reinforced members is the famous Shioya series from Japan,[23] also included in the authors' database. It appears that the section of text on the bottom half of page 547 in the authors' paper is intended to discredit these tests presumably as they directly refute the authors' conclusions about the safety of uniformly loaded members. These tests were intended to determine the shear strength of the base footing slabs of large in-ground liquid natural gas (LNG) vessels and thus represented members supported on soil. Soil supported structures do not show any shear forces due to self weight and thus it is simply irrelevant that the largest member may not have been able to support its own self weight. Figure D(b) compares these Japanese tests to other tests performed at the University of Toronto, also included in the authors database,[19] in another "apples to apples" comparison. These two experimental series had a similar value of the term $\rho_w Vd/M$, as used in Eq. (1) of the authors' paper and thus the ACI code would suggest that the member should show similar shear behavior. As is clear from the figure, the point-loaded members and the uniformly-loaded members did show very similar behavior. This figure supports the conclusions that: a) the results of the Japanese tests are in no way inconsistent with others and should not be ignored; b) the shear strength of UDL and point-loaded members is essentially the same across different depth ranges; and c) the ACI code has problems with estimating the shear strength of large lightly-reinforced members regardless of loading type.

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## AUTHORS' CLOSURE

### Authors' closure to discussion by Solanki

The discusser provided six specific discussion points. Each is addressed in turn in the following:

1. The authors thank the discusser for calling attention to additional references.[54-56]

2. As stated in the paper, "Some limitations were placed on specimens included in the database. Only rectangular cross sections supported on simple spans, without axial loads, were considered. Normalweight concrete and conventional steel reinforcing bars were used to construct all beams. These limitations were imposed to assure simple, well-defined geometry that would permit relatively easy determination of the concrete contribution to shear strength $V_c$."

3. The conclusions of this paper are based on and applicable to sectional shear design provisions of ACI 318. The discussion of the experimental results is partially based on strut-and-tie models because strut-and-tie modeling allows the complex behavior of reinforcement concrete elements to be explained in relatively simple terms. No strut-and-tie analyses were presented in this paper.

4. The authors observed that, in general, cracks tend to form at the location of the stirrups. Hence crack spacing was approximately equal to stirrup spacing. If two beams are identical except for the stirrup spacing and are subjected to the same moment, it is likely that fewer, more widely spaced cracks develop in the beam with larger stirrup spacing. Because the beams are subjected to the same curvature, the average bottom fiber strain must be identical; therefore, the beam with greater stirrup spacing could be expected to have larger and more widely spaced cracks than its companion beam with smaller stirrup spacing, as appears to be indicated in the figure showing the work of Borischanskij. However, crack widths were not considered by the authors for the work presented in the paper.

5. The discusser is to be commended for calling attention to test data that extends the data base to include a wide variety of variables not included in the paper or the database used.

6. The authors are unable to follow the derivation for the cross-sectional area of a beam as a function of concrete strength. However, the equation derived indicates that, as the concrete strength increases, the cross-sectional area will also increase. This result does not seem reasonable or consistent with test results or with design practice.

### Authors' closure to discussion by Viest

The authors would like to thank the discusser for his kind words regarding the research effort presented in this paper. The authors also thank the discusser for his pioneering research efforts in shear that have endured the test of time since their development 50 years ago.

### Authors' closure to discussion by Bentz

The authors wish to thank the discusser for his comments and for his thorough review of the paper. Prior to addressing his concerns, it is important to state the primary goal of the paper which was to ensure that the nominal shear strength a designer determines using the simple expression ($V_c = 2\sqrt{f_c'}\, b_w d$) given in ACI 318-05 provisions can in fact be realized.

The questions posed by the discusser regarding the conclusions will be discussed individually.

*Load distribution*—In a Bernoulli Beam, a truss consisting of a number of diagonal struts and horizontal and vertical ties may form between the applied loads and supports. For loads that are not far away from the supports (within approximately 2*d*), a direct strut may form between the loads and the support. In both instances, the dispersion of stress through the depth of the member triggers the formation of truss mechanisms through cracking and redistribution of stresses. Figure 11 was intended to graphically depict such redistribution in members subjected to uniform loads as compared with members with distributed loads. In Fig. 11, the distribution of measured strains from a beam subjected to uniform loads is quite different from the distribution associated with a concentrated load. On average, the measured strains from the uniformly loaded beam are much higher than those measured during the concentrated load test. These two strain distributions show clear evidence of stress redistribution. The discusser indicates that slabs, footings, and joist construction are exempt from the minimum shear reinforcement requirements because these types of construction have a significant capacity to redistribute stresses from strong areas to weak areas. As stated in our paper, the authors agree with this statement. Furthermore, the authors suggest that, in the case of a beam subjected to uniformly distributed load, multiple load paths between the applied load and support exist. With multiple load paths, redistribution is possible. The strain distributions in Fig. 11 show the results of that redistribution. Strain is migrating from the peak value (as shown for the concentrated loads) to a more uniform distribution (as shown for the distributed load).

The discusser also indicates that the 1962 Committee 326 report[1] did not find any difference between concentrated and uniform loads because the average values of ratio of measured to calculated values of shear were within 1% of each other. The discusser's assertion is based on average values. The authors focused solely on the lower-bounds to the data. While on average there may be little difference between concentrated and uniform loads, the lower-bounds of these two types of members are quite different. In data where significant scatter exists, both accuracy and safety cannot be assured simultaneously. An average value of tested to calculated strength of 1.00 indicates that 50% of the test specimens would have a failure load less than the nominal capacity. Such an approach is not appropriate for a design code. Conclusions in the paper are based on a lower-bound to strength rather than the average. Furthermore, the 1962 ACI Committee 326[1] report specifically cited differences in the cracking behavior of beams with concentrated or uniform loads.

*Use of small datasets*—Establishing trends by passing a line through a small number of data points may result in conclusions that have limited or no use in the development of expressions for design codes. The plot of the authors' data in Fig. C is not correct. The corrected version is show in Fig. E. ACI 318-05, Section 11.1.3c, allows the critical section of a beam to be calculated a distance *d* from the support only if there are no concentrated loads applied within a distance *d* from the support. The authors' Beam 4 does not meet that criterion and, therefore, the shear at the critical section should be twice the value at which the discusser has shown it to be in Fig. C. When plotted in the correct location (Fig. E), the trend described by the discusser is no longer present. Note that the specimen subjected to a single concentrated load has much less shear strength than the remaining three specimens that were subjected to multiple loads. Figures C and E highlight the potential for errors that arises when attempting to base wide-ranging conclusions on only a few data points. For this reason, the authors based all of their conclusions on a combination of their own experimental work and a large database of published work. To reinforce this point, the authors would like to quote from Reineck et al.,[62] "Year by year, different proposals are put forward by researchers all over the world for predicting the shear capacity of members without transverse reinforcement. The proposed relationships are usually empirical and designed to fit the limited set of shear test results that are most familiar to the researcher(s)…This limited amount of information is insufficient for the development of comprehensive and reliable expressions for estimating the shear strength of concrete members."

The discusser has presented a figure from the Committee 326 report in Fig. D(a). Based on this figure, the discusser concludes that there is no significant difference in shear strength based on loading type. Based on the work presented by Leonhardt[63] and Uzel,[25] the authors disagree. Leonhardt reasoned that, in the portion of a beam beneath a load or above a reaction, a vertical stress acts on the beam due to the compression induced by the loads. These vertical stresses reduce the principle tensile stresses in the member. By reducing principle tensile stresses, the external loads or reactions restrain the formation of a diagonal tension crack and shear strength is thereby enhanced. Uzel[25] identified, through both experimental and analytical investigation, the same phenomenon in footings that were subjected to concentrated loads and supported by uniform loads. Uzel described these compressive stresses induced by supports and loads as clamping stresses. In both cases, the beneficial effects of distributed loading were clearly observed and noted.

The discusser included a quote from Kani[61] and the readers should note that in the paragraph following the one from which the discusser quoted, Kani further states that, "A comparison of…point loading tests shows, as could be expected, that a uniformly distributed loading produces somewhat more favorable results. Thus, it is slightly conservative if the design requirements for beams under point loadings are extended to beams under uniform distributed loads." Herein, Kani's language ("…as could be expected…") indicates that he was well aware of a difference between uniform loads and concentrated loads.

The discusser indicated that the Shioya tests were intended to simulate the foundations for LNG tanks and, as such, code provisions that apply to beam designs should not be used to check the validity of those tests. This point is consistent with our impression of the Shioya tests. While they may provide valuable data for certain issues, their use in judging the beam shear or sectional shear provisions of ACI code is not appropriate. After stating that beams cannot be compared with footings due to the way the two types of members handle self-weight, Fig. D(b) is presented to identify parallels between research results from Toronto and research results from Shioya.[23] This would appear to be "comparing apples to oranges."

*Shear span-to-depth ratio*—The discusser questions the applicability of the specimens shown in Fig. 9 through 11 because these specimens would fall under the strut-and-tie provisions of ACI 318-05. Section 11.8.1 indicates that the ACI 318-05 limits for deep beams where nonlinear strain distributions or strut-and-tie models should be used as a basis for design. Currently, the ACI code suggests the use of strut-and-tie models for members in which the clear span is less than four times the overall depth of the member or if a concentrated is load is located within twice the member depth from the support. Even if a member is considered a deep beam by these provisions, strut-and-tie modeling is not required to design the member.

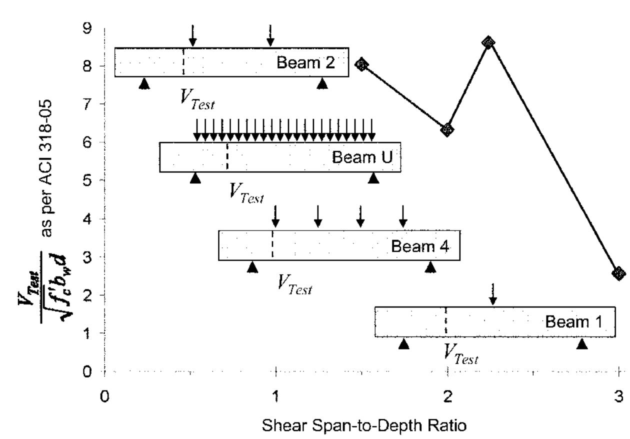


*Fig. E—Corrected version of discusser's Fig. C.*

The specimen subjected to uniformly distributed loads in this paper has a clear span equal to four times the overall depth and there are no concentrated loads. Hence, these specimens could be designed using the provisions of Chapter 11 of ACI 318-05; although, the authors would recommend the use of Appendix A for such a task. These specimens were included in the paper to highlight the differences in strains for members subjected to concentrated or uniformly distributed loads as per the previous discussion of redistribution.

In his discussion, the discusser indicated that the shear span-to-depth ratio for the specimen subjected to uniform loads over half the span was 1.0. The authors are unsure of the basis for that determination. Additionally, the authors are unsure of the basis for the calculation of shear span for Beam 4 as shown in the discusser's Fig. C. For specimens subjected to multiple loads, or distributed load, the definition of shear span becomes nebulous. Leonhardt and Walther[52] defined the shear span of a uniformly loaded beam as one-fourth of the span length. That decision was made to ensure that the test results from specimens with distributed loads resembled specimens with two concentrated loads. In the process of forcing the two sets of data to resemble one another, Leonhardt and Walther were successful. The definition proposed by Leonhardt and Walther, however, is completely inadequate for specimens such as those presented by the authors (distributed loading over half of the span).

The authors would like to further discuss the definition of shear span by calling attention to the conclusions of Bryant et al.[64] Bryant et al.[64] conducted a series of tests of two-span continuous beams with varying numbers of concentrated loads applied to the beams. Those tests consisted of members subjected to 1, 3, 5, or 11 concentrated loads per span. Eleven closely-spaced concentrated loads resemble a uniformly distributed load. Bryant et al. concluded, "As the number of loads on a beam increased, the failure section became impossible to predict. The material and geometrical properties of beam, viz., $\rho$, $f_c'$, and $M/Vd$, does not lead to a precise analysis of the failure section for these beams." Note that the quantity $M/Vd$ is equal to the shear span-to-depth ratio. Bryant et al.,[64] therefore, found that the shear span-to-depth ratio is an unreliable parameter for describing the failure of specimens subjected to distributed loadings. So, for specimens subjected to distributed loads, the shear span is difficult to define as evidenced by the inability to define a shear span for the specimen subjected to a partial distributed load but, at the same time, a precise definition may be unnecessary for such specimens based on the conclusions of Bryant et al.[64]

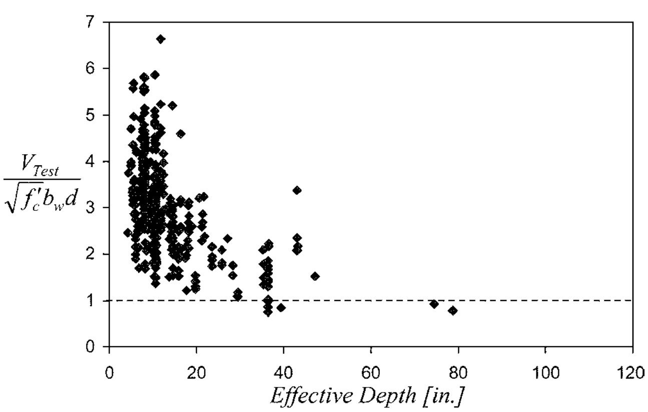


*Fig. F—Shear strength versus effective depth based on Reineck et al.[62] database.*

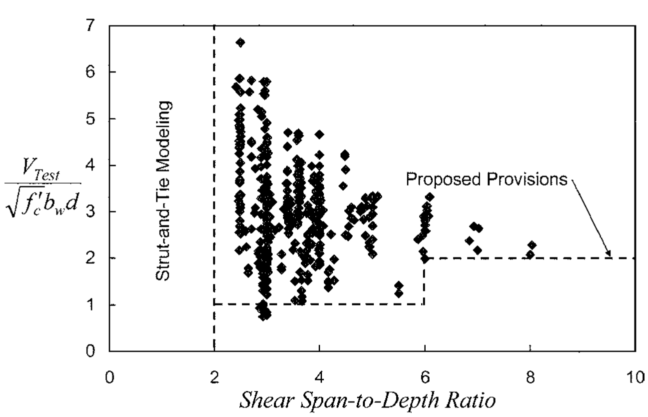


*Fig. G—Shear strength versus shear span-to-depth ratio based on Reineck et al.[62] database.*

While all data in the shear database assembled during the course of the research is presented in the paper, the only data that is used to arrive at the conclusions that apply to sectional shear provisions of ACI 318 were taken from test specimens with shear span-to-depth ratios greater than two. To be exact, the authors spent a substantial amount of time in studying the potential causes of the low-shear strength values that are limited to a narrow range of shear span-to-depth ratios ($2 < a/d < 6$). All specimens within this shear span-to-depth ratio were thoroughly examined prior to reaching the conclusions reported in the paper.

*Shear database*—The authors thank the discusser for identifying miscalculations in Table 2. Based on the discusser's comments, the following miscalculations were identified in Table 2:

1. de Cossio and Siess[30]: $\rho_w$ = 1.00 to 3.36%
2. Johnson and Ramirez[15]: $\rho_w$ = 2.49%
3. Kong and Rangan[32]: $\rho_w$ = 1.00 to 4.47%
4. Krefeld and Thurston[17]: $\rho_w$ = 0.80 to 5.01%
5. Ramakrishnan and Ananthanarayana[38]: $d$ = 13.8 to 28.8 in.
6. Rogowsky et al.[21]: $\rho_w$ = 0.90 to 1.12%
7. Roller and Russell[22]: $a/d$ = 1.8 to 2.5
8. Sarsam and Al-Musawi[40]: $\rho_w$ = 2.23 to 3.5%
9. Tan and Lu[43]: $\rho_w$ = 2.60%
10. Xie et al.[50]: $\rho_w$ = 2.07 to 4.54%

It is important to note that these miscalculations were confined solely to the summary table (Table 2) in the paper. The authors have examined the entries corresponding to those miscalculations in the originating database. The values stored in the database and used for analysis within the paper are correct. Therefore, the plots (Fig. 15 through 22) are correct as published. While the authors made every effort to produce a table without errors, some errors did make it through the review process into the final paper. The database that was assembled by the authors is intended to be updated with further developments in shear research. Readers who find errors in the database or test results that are missing are encouraged to contact the authors so that corrections can be made.

In Fig. F and G, the authors have presented data that were assembled as part of another database regarding the shear strength of reinforced concrete beams.[62] In Fig. F, it can be observed that the lower-bound to the data is essentially constant as a function of depth for beams in excess of 30 in. (762 mm). Furthermore, the authors have reproduced Fig. 18 from the original paper using the data collected by

Reineck et al.[62] The results are shown in Fig. G. Regardless of which database is used, the lower-bounds to Fig. 18 and E are essentially the same.

*Flexural failure*—The discusser has taken issue with the authors' choice to include some specimens in which yielding of flexural reinforcement took place prior to shear failure. In fact, the discusser refers to these specimens as "flexural failure." The authors would like to discuss the subtle, but important, difference between flexural failure and flexural yielding. Flexural yielding involves yielding of the longitudinal reinforcement in tension. Flexural failure involves the loss of equilibrium within a member. Flexural failure is caused by two distinct limit states: crushing of the concrete in the compression zone prior to or after the yielding of longitudinal reinforcement or rupture of the longitudinal reinforcement. Flexural yielding does not in any way imply or require flexural failure. The distinction between flexural yielding and flexural failure is an important one due to the philosophy or ACI 318 and strength design. In all the beams that are designed and detailed according to ACI 318 provisions flexure ought to be the "weakest link" in the chain. In other words, typical beams are designed to possess sufficient shear strength such that flexural yielding and redistribution of the moments takes place prior to shear failure. Throughout the redistribution process, the beams are expected to have sufficient shear strength. The shear strength of a beam that contains large amounts of flexural reinforcement is of limited use to evaluate the performance of beams that are designed using the ACI 318 code.

In short, the ACI 318 code encourages designers to seek ductile limit states (yielding in flexure) rather than brittle ones (shear failure). Therefore, if code documents are predicated on members that fail in shear after yielding in flexure, the code must be based on test specimens with those limit states. Hence, the decision to include members that failed in shear after yielding of the longitudinal reinforcement in the database is consistent with code intent.

In conclusion, researchers have historically favored the use of concentrated loads in tests for shear strength of reinforced concrete beams. Such tests resemble transfer girders more than any other building member. Extrapolating recently reported low shear strengths[19] of elements that are subjected to concentrated loads to beams that support slabs, joists, or other loads that are reasonably uniform should be done with caution. There are no reported instances of shear distress in elements subjected to uniform loads. Once again, we thank the discusser for allowing us to re-evaluate our conclusions and the code change proposal included in our paper through the use of another shear database developed by Reineck et al.[62]

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Disc. 103-S58/From the July-Aug. 2006 *ACI Structural Journal*, p. 551

**Steel Fiber Concrete Slabs on Ground: A Structural Matter.** Paper by Luca G. Sorelli, Alberto Meda, and Giovanni A. Plizzari

## Discussion by Shiming Chen

*Professor, School of Civil Engineering, Tongji University, Shanghai, China.*

The authors attempted to develop a tentative design method in assessment of the load-carrying capacity of steel fiber-reinforced concrete (SFRC) ground slabs. The discusser appreciates the authors' comprehensive work carrying testing and FE parametric analysis on SFRC ground slabs. Some findings are interesting to the discusser, however, were not well clarified. Discussion is drawn as follows.

### Experimental study

The tests demonstrated the significant enhancement of steel fiber to the bearing capacity and the ductility of concrete slabs on ground. Accordingly, it is indicated that the ultimate load was conventionally defined as corresponding to a sudden change of the monitored displacement that evidence the formation of a collapse mechanism full-developed crack surface along the medians or the diagonals. It looks likely that the maximum load levels illustrated in Fig. 4 and 5 of the paper are higher than the ultimate loads given in Table 6, and the SFRC ground slabs are capable of subjecting to further load even after the formation of a collapse mechanism. It is not clear what criterion is used in determining the ultimate load for each specimen. Is it judging by the sudden change of the monitored displacement or judging by the peak load level in the load-displacement curves?

To assess the effect of steel fiber on a ground slab, the load levels (where crack initiates in the ground slab) are very important, especially when the slab design is governed by crack control. Whereas it is not well clarified when the first crack initiated for each specimen, which was reinforced with different types of steel fibers and in different mixing dosages, are they inherent in a similar cracking load level, for example, 100 kN?

A comparison of the fracture properties given in Table 5 of the original paper demonstrates that there was a substantial increase in the fracture energy $G_F$ and crack opening $w_{ck}$ for SFRC specimens over the plain concrete specimen (S6), but the cracking stress levels ($\sigma_{ct}$) are almost the same. Figure A illustrates that the $F/F_0$ varies with the fracture energy $G_F$ derived from Tables 5 and 6, where $F$ is the collapse load of the ground slab and $F_0$ is the collapse load of the control specimen $S_0$. It appears that adding fibers in concrete enhances the collapse load of the ground slabs; however, the bearing capacity of the slabs decreases with the fracture energy for slabs with a single type of longer fiber (steel fiber 50/1.0), such as Slabs S4, S8, and S11, in a volume ratio of fiber 0.38 and 0.57%, respectively, but increases for Slabs S3 and S14, with hybrid longer and shorter fibers, in a volume ratio of 0.57%. It would be explained by the better efficiency of shorter fibers and likely that mixed fiber reinforcement is more effective.

The final crack patterns of slabs demonstrated in Fig. 6 of the original paper are quite similar, characterized by cracks developed along the median lines and fewer along the diagonals. In terms of simple plastic analysis based on energy method, the load enabling different collapse mechanisms would be different. Can the authors explain why the numerical development of the crack patterns shown in Fig. 10 is different? For example, there are diagonal crack patterns in S0, S1, S4, S8, and S11, but median crack patterns in S3, S5, and S14. What governs these crack patterns?

## Finite element model

There was uplift at the slab corners, as shown in Fig. 11 of the original paper, and this phenomenon was also observed and discussed in References 26 and 27. To evaluate realistic ultimate and service loads of a ground slab, it is necessary to take into account this nonlinearity between the foundation and the slab. In numerical modeling, the elastic soil was modeled by 616 linear elastic truss elements, which would be sharply different from the realistic situations as the uplifts developed at the slab corners, which would introduce substantial downward forces on the slab. The unilateral nonlinear elastic-plastic curve for the Winkle-type model proposed by Cerioni[26] would be better.

It is noted that the load-displacement curves based on the finite element analysis agree well with the test curves. However, would the load in numerical curves increase further or drop when the collapse mechanism developed, as defined in the original paper?

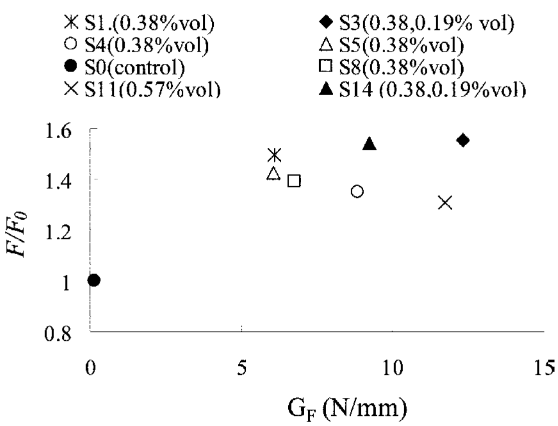


*Fig. A—F/$F_0$ varies with fracture energy $G_F$.*

## Table A—Comparison of collapse loads

| Slab no. | $F_{u,exp}$, kN | Steel fiber 50/1.0 %vol | Steel fiber 30/0.6 %vol | Steel fiber 20/0.4 %vol | Steel fiber 12/0.18 %vol | $1 + R_{e,3}$ |
|---|---|---|---|---|---|---|
| S0 | 177.0 | — | — | — | — | 1.0 |
| S1 | 265.0 | — | 0.38 | — | — | 1.497 |
| S3 | 274.9 | 0.38$^{(a)}$ | — | — | 0.19 | 1.553 |
| S4 | 238.6 | 0.38$^{(a)}$ | — | — | — | 1.348 |
| S5 | 252.3 | — | 0.38 | — | — | 1.425 |
| S8 | 246.2 | 0.38$^{(b)}$ | — | — | — | 1.391 |
| S11 | 231.9 | 0.57$^{(a)}$ | — | — | — | 1.310 |
| S14 | 273.0 | 0.38$^{(b)}$ | — | 0.19 | — | 1.542 |

Notes: $F_0$ is collapse load of control slab (S0); (a) and (b) refer to steel fiber type; 1 kN = 0.2248 kip.

## Design method

Although there appears to be good correlation between the collapse loads of the approximated equation (Eq. (1) of the original paper) and the NLFM model, the contribution and physical interpretation of each parameter is not clear and it is difficult to apply it in design practice. A unit scale analysis of Eq. (1) leads to $[N]^{0.999}[mm]^{0.001}$. The seven coefficients ($\alpha_1$, $\alpha_2$, $\alpha_3$, $\alpha_4$, $\alpha_5$, $c_1$, and $c_2$) should also be calibrated against test specimens whenever new fibers and different volume ratios are adopted.

The enhanced contribution of steel fiber on ground slabs is considered by introducing an equivalent flexural strength ratio or $R_{e,3}$ (at 3 mm [0.118 in.] deformation[28]). Let $F_0$ be the collapse load of a plain concrete ground slab; the collapse load of a SFRC ground slab is then expressed as

$$F_u = F_0\,(1 + R_{e,3}) \tag{4}$$

where $R_{e,3}$ is the equivalent flexural strength ratio based on the flexural toughness test in accordance with JSCE-SF4.[29] For a typical hooked-end steel fiber (35/0.55, 0.38%vol), $R_{e,3}$ is 0.62, and a similar hooked-end steel fiber (60/0.92, 0.35%vol[30]) $R_{e,3}$ is 0.43.

Basically, the equivalent flexural strength ratio $R_{e,3}$ will depend on the aspect ratio of the fiber and the minimum overlapped spacing of the fiber within the concrete. No values of $R_{e,3}$ were reported for the tested slabs in the original paper, so one might guess that $R_{e,3}$ for tested slabs would be approximately 0.3 to 0.5. A comparison of the collapse loads of SFRC ground slabs against plain concrete ground slab derived from Table 6 of the original paper is given in Table A. The final column demonstrates the $1 + R_{e,3}$ derived from Eq. (1) based on the test results.

Additionally, the discusser has noticed the following possible miscalculations; could the authors please comment?

- $k_w$ = 0.0785 kN/mm$^3$ (289.2 lb/in.$^3$) on page 555 and 0.21 kN/mm$^3$ (773.7 lb/in.$^3$) on page 556 should be $k_w$ = 0.0785 N/mm$^3$ (289.2 lb/in.$^3$) and 0.21 N/mm$^3$ (773.7 lb/in.$^3$).

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## AUTHORS' CLOSURE

The authors would like to thank the discusser for the interest and for the valuable discussion of the paper.

First of all, the authors would like to take this opportunity to underline an error in Tables 4 and 5 of the original paper, where S6 should be corrected in S0.

As far as the experiments are concerned, it should be observed that the load-carrying capacity of concrete slabs-on-ground is not exhausted even after the slab collapse because the elastic springs under the bottom surface can carry further load. Indeed, the experimental failure of the SFRC slabs was neither sudden nor catastrophic because the elastic foundation keeps carrying further load. Other researchers[31,32] defined the failure load based on the formation of a crack pattern compatible with a yield line plastic mechanism. The identification of such crack patterns (throughout the bottom surface of the slab) during a slab test, however, is not an easy task. In all the experimental results, the authors observed a sudden variation of the displacement field (monitored by 16 LVDTs), which was conventionally defined as the slab collapse mechanism. Figure 9 of the original paper clearly shows the identification procedure.

The first crack load of the ground slabs was very difficult to measure because the first crack formed on the bottom surface of the slab. In the authors' opinion, however, the first crack load in fiber-reinforced concrete (FRC) slabs only depends on the tensile strength of the concrete matrix and not on the fiber type and content because fiber reinforcement starts activating after cracking of the concrete matrix and does not significantly contribute to prior cracking.

The fracture energy $G_F$ is a significant parameter for material properties but may not be important in structures where the maximum crack opening at failure is very small (a few tenths of a millimeter), as in slabs-on-ground. In these structures, the fracture energy cannot fully develop in the cracked surfaces; however, FRC with shorter fibers develops more energy with smaller crack openings.[33]

As far as the numerical analyses are concerned, it should be noted that the collapse mechanisms can develop with cracks along the median or the diagonal lines. Previous numerical studies showed that the crack pattern depends on the slab stiffness related to the soils stiffness. In the slab specimens, these values are close to the border line so that cracks can develop either along the medial or the diagonal line.

Concerning the finite element model (FEM), it should be observed that all the numerical simulations of the slabs-on-ground stopped (no longer converged) at the slab collapse.

Furthermore, Belletti et al.[34] analyzed the experimental results by means of a multiple-crack model, which can be seen as an extension of the one proposed by Cerioni and Mingardi,[26] and accounted for the effect of the unilateral springs; their numerical results showed that the uplift at the slab corners has a minor influence on the ultimate load experimentally determined on the ground slabs.

As far as the design method is concerned, the authors would like to underline that the left term of Eq. (1) in the original paper is a force with the following fundamental physics dimensions: $[\text{Length}]^1$ $[\text{Mass}]^1$ $[\text{Time}]^{-2}$. The units calculated by the discusser as $[\text{N}]^{0.999}[\text{mm}]^{0.001}$ are likely due to the round-off error of the numerical solutions and should be reasonably approximated to the close integers (that is, 0.9999 ~ 1 and 0.001 ~ 0). The five coefficients ($\alpha_1$, $\alpha_2$, $\alpha_3$, $\alpha_4$, and $\alpha_5$) are the powers law exponents of quantities ($A_L$, $B$, $L$, $k_w$, $f_{res}$, and $f_{If}$), which do have a clear physical significance, as explained in the original paper. Moreover, Eq. (1) fits more than 1000 numerical simulations (based on nonlinear fracture mechanics) with remarkable accuracy. The fitting equation can be considered valid within a wide range of applications (fiber type and content, matrix strength, and slab geometry), as considered in the original paper.

The authors appreciate the comparison with the flexural strength ratio $R_{e,3}$; however, because the crack opening at collapse in real slabs is very small, according to authors' opinion, parameters associated with a smaller crack opening could be more representative of the slab-on-ground behavior.


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Disc. 103-S61/From the July-Aug. 2006 *ACI Structural Journal*, p. 577


**Strength of Struts in Deep Concrete Members Designed Using Strut-and-Tie Method.** Paper by Carlos G. Quintero-Febres, Gustavo Parra-Montesinos, and James K. Wight


## Discussion by Pedro R. Muñoz

*PhD, PE, Principal, PRM Engineering, Structural Consulting Engineers, Newburyport, Ma.*


The behavior of deep concrete members differs greatly from that of shallow concrete members. It appears that the load path for a point load applied at the top of a deep beam will follow a rather straight path from the point of application of the point load down to the points of support, which appears to deviate somewhat from that of a typical bending behavior for a point load applied at the top of a shallow concrete member.

That portion of the deep beam following the straight path of the axial compressive forces will behave very much like a strut in which case the strut-and-tie method of the ACI Building Code could be applied to evaluate the strength of the strut in the deep beam.

The authors of this paper have considered the main design variables for the experimental investigation of the strength of struts in deep concrete members: the angle between primary strut-and-tie axis, the amount of reinforcement crossing the strut, and the concrete strength, but failed to include any reinforcement in the section of the strut that will definitely have a significant increase effect in the total strength of the struts. The behavior of reinforced concrete columns has been investigated extensively, and it appears to be well understood for short and slender members under axial compressive forces and reinforced with longitudinal and transversal reinforcement. As part of the strength factors for struts in the strut-and-tie methods of the ACI Building Code, provisions shall be made to incorporate the contribution of steel reinforcement in the section of the strut that may become a reinforced strut. It is important to consider that not very well confined concrete members under axial compressive forces will crack and fail in shear rather prematurely for axial loads exceeding the capacity and strength of the unreinforced concrete member; therefore, the contribution of steel reinforcement in the strut section becomes significant and provides an added axial strength component to the total strength of a reinforced strut in a strut-and-tie model similar to the contribution of steel and concrete in a reinforced concrete short column.

Similar to what is done with concrete columns that are reinforced vertically with steel bars, the same could be done with this strut portion of the deep concrete member, providing a much more higher axial load capacity, becoming this portion of a reinforced strut where both the concrete section of the strut and the steel bar embedded in the middle of the strut both contribute to the axial load in proportion to their corresponding cross-sectional properties of concrete and steel.

The aforementioned concept can be described in the revised Fig. 2 from the paper (Fig. A herein), by adding a steel reinforcement bar or bars in the section of the strut that will resist part of the concentrated Load P. This bar is labeled as $A_{st}$ inside the strut section. Practically speaking, this should not be difficult to achieve in the field and it should not be any more difficult to install than any of the transverse or longitudinal reinforcement in the concrete member. It would be interesting to see if the authors of this paper could undertake another series of tests by adding the suggested steel reinforcement in the section of the struts and compare the ultimate achieved loads for the specimens with the modified reinforced struts. If possible, other steel reinforcement bars could also be added to the other strut sections of the entire strut-and-tie model of the deep concrete member. This could be a new trend to achieve higher loads in concrete members behaving in a manner consistent with the strut-and-tie model.

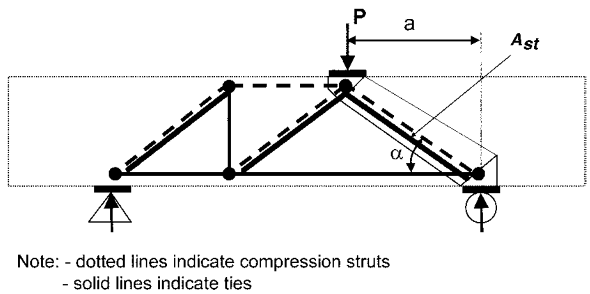


*Fig. A—Strut-and-tie model for deep beam design.*

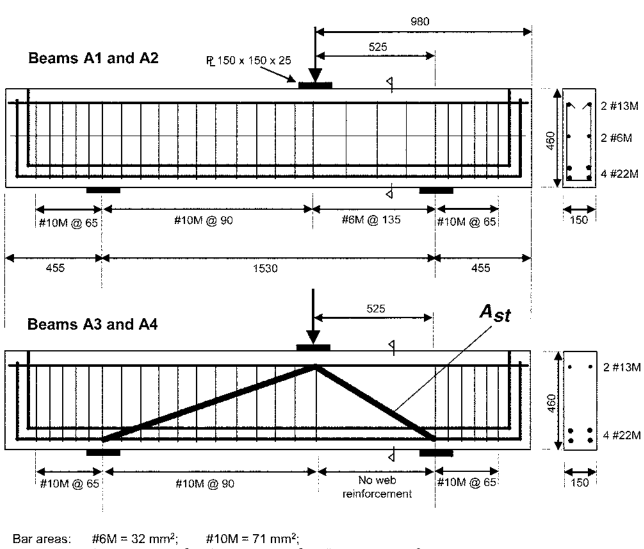


*Fig. B—Reinforcement details for Series A.*

The revised Fig. 3 from the paper (Fig. B herein) indicates the suggested additional steel reinforcement labeled $A_{st}$ in the sections of the struts. Equation (1) of the paper presents the strength of a concrete strut expressed as a function of the concrete compressive strength; again herein, it is suggested to add the contribution of the steel reinforcement to the strength of the strut and modify the equation to include the contribution of both steel and concrete. Other efficiency factors could be evaluated and incorporated into the final equation after a calibration of the tests and analytical studies are correlated.

The investigators of this paper have considered two amounts of reinforcement crossing the primary strut, It appears that a more effective contribution of steel reinforcement to the behavior and strength of the strut in deep concrete members could be achieved by incorporating a longitudinal steel reinforcement embedded right into the strut section—it will definitely prove to be more effective than the reinforcement crossing the strut.

Instead of very complicated expressions for the contribution of concrete in the strength of the strut, it would be more beneficial to incorporate the steel reinforcement and come up with expressions for the combined strength of the reinforced strut similar to what is currently done for reinforced concrete columns, with the appropriate modification and possibly efficiency factors suited for the case of deep concrete members. Therefore, Eq. (2) through (4) would have another term that would include the contribution of the steel reinforcement to the total strength of the reinforced strut.

It would be interesting to see how the load-displacement curves shown in Fig. 6 and 7 would look like after the specimens with the modified reinforced struts similar to what is shown in the revised Fig. 3 (Fig. B herein) are tested and the loads and deflections plotted for comparisons.

The cracking patterns most likely will change and the strains in the longitudinal and transverse reinforcement will be most likely lower than those in the specimens tested in this paper.

The authors have noted in the section Strains in web or strut reinforcement that, “the strain measurements and visual observations indicated that the web reinforcement was effective in controlling crack opening.” Having steel reinforcement in the strut sections will most likely reduce the cracking due to diagonal stresses along the path of the load through the strut to the support because the concrete alone will not carry all the combined stresses in the strut.

As the authors of this paper mention in one of the paragraphs before the Summary and Conclusions section, “Clearly additional experimental information needs to be generated to draw definite conclusions with regard to the minimum web reinforcement required in high-strength concrete members designed using strut-and-tie models.” It appears that perhaps the additional experimental information that could be under-taken in future research on this subject could be oriented toward having some type of reinforcement in the strut sections, which will clearly provide additional strength to the strut-and-tie models.

This comment addresses Item 2 of the Summary and Conclusions, where clearly the transverse reinforcement alone without any kind of steel reinforcement in the strut section will not provide a reliable strength capacity of the strut section.

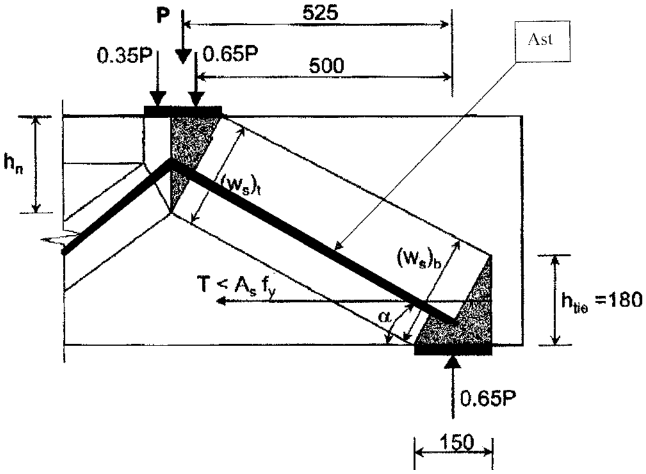


*Fig. C—Strut-and-tie model in critical span of Specimen A1.*

The revised Fig. A-1 (Fig. C herein) indicates the strut-and-tie model in the critical span section of the specimen tested and analyzed, with a steel reinforcement labeled $A_{st}$ in the middle of the strut section. Clearly, this could be treated as a short column, and, as such, the total strength provided by the combination of both the concrete in the strut and the added reinforcement will be much higher than what is calculated with the expressions and equation shown in the Appendix.

The diagonal steel reinforcement in the strut $A_{st}$ becomes a principal diagonal reinforcement of the deep concrete beam. This is not a current practice, but it is very effective and an inexpensive way to reinforce the diagonal strut in the deep concrete members and to enhance the load-carrying capacity of the deep concrete member.

It would be interesting to know what the authors think about the possibility of extending their research work by incorporating a reinforced strut section and studying the failure modes to see what enhancements could be achieved by reinforcing the compressive strut in the deep concrete beams.

A suggestion to the authors and future researches will be to look into incorporating some type of steel reinforcement in the strut section as shown in Fig. A; $A_{st}$ will be the steel bar in the concrete strut, making it a reinforced concrete strut.

## AUTHORS' CLOSURE

The authors would like to thank the discusser for his interest in the paper. The use of steel reinforcement in the longitudinal direction of the concrete diagonal struts was not investigated because the authors do not believe it represents typical practice for the design of deep concrete members. The discusser should notice, however, that the use of such reinforcement to increase the strength of concrete struts is discussed in Section A.3.5 of the 2005 ACI Building Code.

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Disc. 103-S64/From the July-Aug. 2006 *ACI Structural Journal*, p. 604

**Experimental Investigations on Punching Behavior of Reinforced Concrete Footings.** Paper by Josef Hegger, Alaa G. Sherif, and Marcus Ricker

**Discussion by Himat T. Solanki**

*ACI member, Professional Engineer, Building Department, Sarasota County Government, Sarasota, Fla.*

The authors have presented an interesting paper on punching behavior of reinforced concrete footings. However, the discusser would like to offer the following comments:

1. The discusser has reviewed several publications[12-22] regarding the punching shear failure cone angle. Based on the literature, the punching cone angle depends on the thickness of footing slabs, the amount and arrangement of reinforcement, strength of concrete, and the ground stiffness (Fig. A). The discusser believes that the range of cone angle should be between 25 to 60 degrees. Therefore, the authors conclusion No. 1 may be based on their limited data and cannot be generalized.

2. Though the ground stiffness has some influence[12,13,18,21,22] on the punching shear strength but it may be neglected.

3. The discusser believes a similar paper has been published in the German magazine *Beton und Stahlbetonbau*, V. 101, No.4, 2006, by the senior author and his colleagues.

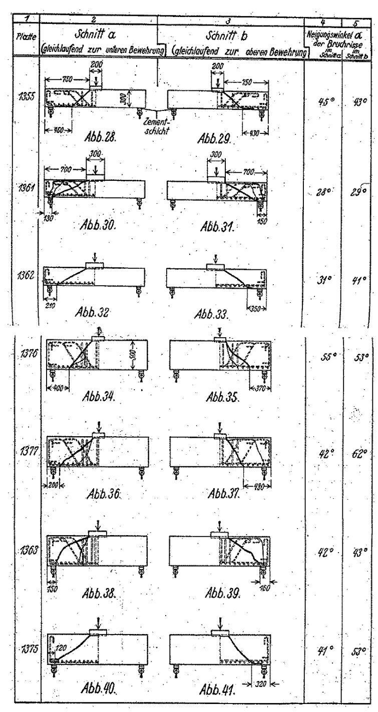

*Fig. A—Crack angles with respect to slab thickness, reinforcement, and concrete strength.[14]*

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Disc. 103-S65/From the July-Aug. 2006 *ACI Structural Journal*, p. 614

**Simplified Modified Compression Field Theory for Calculating Shear Strength of Reinforced Concrete Elements.** Paper by Evan C. Bentz, Frank J. Vecchio, and Michael P. Collins

### Discussion by Himat T. Solanki

*ACI member, Professional Engineer, Building Department, Sarasota County Government, Sarasota, Fla.*

1. The authors have presented an interesting paper. The discusser is somewhat confused about the intent of the paper. This paper does not provide any design-oriented or codified design concept. This paper is merely a theoretical approach of previously published papers by the senior authors on this subject, that is, modified compression field theory (MCFT). Normally, the modified version improves the mean and coefficient of variation (COV) values, but this paper presents a higher number of scatter results (–0.09 to +0.48) than the previously published papers. In the paper, all specimens have higher $v_{exp}/v_{predicted}$ values except Panels PV2, TP5, PP3, and VA4 in comparing the MCFT with the simplified MCFT, but no explanation was given by the authors. Also in this paper, the authors have primarily analyzed the University of Toronto and the University of Houston panels and approximately 50% Obayashi Corp., Japan, panels. The discusser has plotted a set of curves, as shown in Fig. A, and compares them with other curves. Based on Fig. A, it can be seen that the authors' curve falls quite far below all of the other curves; therefore, the authors' proposed method predicts scatter results.

2. The authors stated that "The MCFT β values for elements without transverse reinforcement depend on both $\varepsilon_x$ and $s_{xe}$." The discusser believes that the crack width is much more important[42,43] than the $s_{xe}$ values.

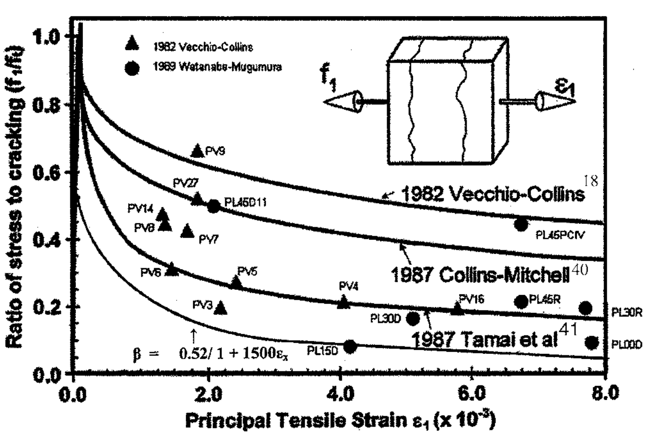


*Fig. A—Ratio or stress to cracking ($f_1/f_t$) versus principal tensile strain $\varepsilon_1$ ($\varepsilon_x$) ($\times 10^{-3}$).[39]*

3. The authors have developed Eq. (28) from the reinforcement in only the *z*-direction and 12 in. (304.8 mm) crack spacing, which is contrary to the published data. Also, if one can assume $\varepsilon_x = 0$ and $s_{xe} = 0$, an angle θ would become approximately 25.5 degrees, which is also contrary to Joint ACI-ASCE Committee 445.[44] Therefore, Eq. (28) has very limited applicability. Joint ACI-ASCE Committee 445[44] mentioned that the angle θ can be computed when the shear stresses are less than those causing first yield of a reinforcement

$$\tan^4\theta = (1+(1/n\rho_x))/(1+(1/n\rho_y)) \tag{33}$$

Hsu[45] proposed the following equation by assuming yielding of steel

$$\cot\theta = \sqrt{\rho_x f_{sxy}/\rho_z f_{szy}} \tag{34}$$

4. The discusser has reviewed and analyzed the University of Toronto reinforced panels, which the authors have not included in their analysis, having unsymmetrical reinforcement[36] and reinforced concrete panels with perforations[37] using the simplified MCFT and a similar performance, that is, scatter results, as outlined in the paper, was found. Is a proposed theory applicable to these types of structures? In practice, this type of condition always exists.

5. The discusser has analyzed the numerous panels using a very simple practical approach as outlined in the following. To calculate the shear stress $\tau_{xz}$, Sato and Fujii's[46] equations were simplified and rearranged as follows

$$\tau_{xz} = \sigma_{c1m}\tan\theta_m + \sigma_x f_{sxy}\tan\theta_m \tag{35}$$

$$\tau_{xz} = \sigma_{c1cr}\tan\theta_{cr} + \sigma_x f_{sxy}\tan\theta_{cr} \tag{36}$$

$$\tau_{xz} = \sigma_{cn}\tan\varphi - \tau_{ct} + \sigma_x f_{sxy}\tan\varphi \tag{37}$$

To calculate $\sigma_{c1m}$ and $\sigma_{c1cr}$, Kupfer[47] suggested the following equation for concrete subjected to biaxial tension-compression stresses

$$\sigma_{c1m} \text{ or } \sigma_{c1cr} = f_t(1 - 0.7(\sigma_{c2}/f'_c)) \quad (38)$$

To calculate the value $\sigma_{c2}/f'_c$, Schlaich et al.[48] have suggested several values for different effective stress levels in concrete struts and have outlined them in Table 1 of Reference 44. Therefore, $\sigma_{c2}/f'_c = Kf'_c$ could be taken.

The value $K$ equals the effective stress level constant and $f_t$ can be taken as Joint ACI-ASCE Committee 445.[45]

$$f_t = 0.33\sqrt{f_c'}/(1 + \sqrt{500\varepsilon_1}) \quad (39)$$

where $\varepsilon_1 = \varepsilon_x + (\varepsilon_x + \varepsilon_2)\cot^2\theta$; $\varepsilon_x$ equals the strain in the tension tie = 0.002; $\varepsilon_2$ equals the strain in the compression strut = 0.002; $\theta$ equals the angle between the strut and the tension tie.

To calculate $\tau_{ct}$ and $\sigma_{cn}$, Walraven[49] suggested the following formulas for shear stress $\tau_{ct}$ and compressive stress $\sigma_{cn}$

$$\tau_{ct} = (-f_{cube}/3) + [1.8\delta_n^{-0.8} - (0.234\delta_n^{-0.707} - 0.2)f_{cube}]\delta_t \quad (40)$$

when $\tau_{ct} > 0$

$$\sigma_{cn} = (-f_{cube}/20) + \left[1.35\delta_n^{-0.63} - \left(0.191\delta_n^{-0.552} - 0.15\right)f_{cube}\right]\delta_t \quad (41)$$

when $\sigma_{cn} > 0$

where $f_{cube} = 1.1f_c'$, $\delta_t$ equals the slip across crack, and $\delta_n$ equals the normal displacement across crack.

The crack width can be expressed as

$$w_{cr} = \sqrt{\delta_t^2 + \delta_n^2} \quad (42)$$

Also, concrete stresses at the crack can be related to[47]

$$\sigma_{c1cr} = \tau_{ct}\tan\theta + \sigma_{cn} \quad (43)$$

Assuming crack width as suggested by Beeby[42] and Walraven and Reinhardt[41] and considering Eq. (33) through (43), the discusser has analyzed 297 panels (including the authors 102 panels) and found them to be in very good agreement with the test values ($v_{exp}/v_{predicted}$ = 1.011 and COV 5.91%) as compared with the authors' scatter results from their simplified MCFT.

The discusser believes that the simplified concept could also be used for calculating the shear strength of reinforced concrete elements based on the crack width and can be applied to any code that recommends/limits the crack width as compared with the authors proposed theory/simplified MCFT.


## ACKNOWLEDGMENTS

The discusser gratefully acknowledges S. Unjoh, Leader, Earthquake Engineering Team, Public Works Research Institute, for providing Japanese publications related to the shear panels.


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## AUTHORS' CLOSURE

The discusser is thanked for his interest in the paper on the MCFT. The issues raised will be commented on in the order presented by the discusser.

1. The discusser seems to have missed the intent of the paper. As noted in the abstract, "this paper presents a new simplified analysis method that can predict the strength of…panels in a method suitable for 'back of the envelope' calculations." Thus, the authors were not trying to produce a method with improved statistical properties, but rather one that was easier to use. The MCFT requires that 15 nonlinear equations be solved simultaneously for any given load level. With the newly presented simplified MCFT, which has been implemented into the Canadian concrete code,[4] only four equations are required.

Figure A is derived from a paper by the first author and shows principal tensile stress on the vertical axis.[39] The discusser appears to have plotted shear stresses on this same axis, which results in an inappropriate comparison. As shown in Table 1 of the paper, the scatter associated with the full MCFT can be expressed as a coefficient of variation (COV) of 12.2%, whereas the simplified MCFT has a COV of 13.0%. Thus, the authors disagree that the new method produces significantly more scattered results than the more complex "full" MCFT.

2. The authors agree that crack width is the crucial concept in determining the parameter $\beta$. Crack widths can be estimated by multiplying the average spacing of the cracks by the average strain perpendicular to the cracks. In the simplified MCFT, the parameter $s_{xe}$ represents the crack spacing and $\varepsilon_x$ represents the strain. While these two parameters are determined in the $x$-axis direction rather than the diagonal direction, the concept is the same: the $\beta$ equation is based on an estimate of crack width.

3. Equation (28), which presents the angle $\theta$ for calculation of transverse reinforcement effectiveness and demand on longitudinal reinforcement, was derived based on the MCFT equations. The derivation of this equation was not presented in this paper as it is available in another paper published elsewhere.[50] The authors disagree that Eq. (28) is of limited applicability and simply note that in the preparation of Table 1, no significant residual trends were observed with respect to the different input variables as would be expected if it were

of limited applicability. Equation (33), as presented by the discusser, is not appropriate for methods like the simplified MCFT that assume the transverse reinforcement has yielded at shear failure. Equation (34) is based on plasticity and assumes that both directions of steel are yielding at failure. As shown in Fig. 9, specimens that fail in this way are modeled well by the simplified MCFT.

4. As noted in the paper, the simplified MCFT is directed toward members subjected to shear combined with uniaxial tension or compression as in a beam or column. All six elements loaded this way in Reference 36 were included in the paper. Reference 37 examined the effects of having a large opening in the shear panel and elements with such a hole were not included in the paper as the hole would produce a disturbed stress field. Of the two repeat experiments without openings in Reference 37 that were not biaxially loaded, the one with the lower strength was included in Table 1. Arguably, element PC1A should have been included instead of PC1 as PC1 suffered a premature edge failure,[37] but the lower strength was used in Table 1. No elements available to the authors that met the restrictions on loading were ignored or discounted in the preparation of the paper. It appears that the discusser has found additional tests from Japan and the authors look forward to testing the method against these results as well.

5. The analysis method presented in Item 5 appears to be a combination of strut-and-tie equations with equations from multiple other sources. Without the results of these calculations presented, or even the number of tests used in the discusser's statistical analysis, it is difficult to comment on the method as presented. The authors look forward to being able to examine it in more detail when it is published. It is clear, however, that the method proposed by the discusser requires the solution of at least 11 nonlinear equations, and thus is, again, aimed at a different user than the simplified MCFT.

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