Toward the Performance-Based Design of Confined Concrete

by S.A. Sheikh and Y. Li

Synopsis: This paper summarizes results from a comprehensive research program that aims at developing rational guidelines for the design of confinement reinforcement in concrete columns. The first part of the paper briefly introduces an analytical model for confined concrete in tied columns. The model is based on the results of testing 24 square columns with various tie configurations under concentric compression. The second part presents results from square columns tested under cyclic flexure and shear, and constant axial load simulating earthquake loads. The specimens tested included normal-strength concrete (NSC) and high-strength concrete (HSC) columns confined by steel and NSC columns confined by fiber-reinforced polymers (FRP). Performance-based procedures for the design of confinement reinforcement in these columns are proposed in light of the experimental results and analytical models. The design procedures incorporate various ductility parameters that include energy dissipation capacity, ductility factors, and cumulative ductility indices in addition to the type, amount, and configuration of the confinement reinforcement and the level of axial load. The areas in which further research is needed are also discussed.

Keywords: columns; confined concrete; confinement; ductility; earthquake; energy dissipation; fiber-reinforced polymers; lateral reinforcement
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INTRODUCTION

Research in the area of concrete confinement dates back to 1903 when Considère first introduced the use of spirals as confinement reinforcement in concrete columns. Over the last century, a large number of experimental and analytical studies have been carried out to study the behavior of confined concrete. These studies have significantly improved the understanding of confined concrete and resulted in the development of many stress-strain models and design procedures. Despite of these research efforts, how to design and detail confinement reinforcement remains a somewhat puzzling issue. The confinement requirements of the current ACI Code (ACI 318-02) and Canadian Code (CSA A23.3-94) are still based on the philosophy that the axial load carrying capacity of a column should be maintained after spalling of the cover concrete. In practice, however, the confinement is required to produce ductile behavior of the columns subjected to a combination of forces (Sakai and Sheikh 1989). Hence a rational design approach should relate the ductile behavior of a column to the confinement requirements, with due considerations given to those factors that have significant effects on column ductility.

To establish rational guidelines for the design and detailing of confinement reinforcement for confined columns, Sheikh and Uzumeri initiated a comprehensive research program on concrete confinement in 1970s. At the early stage of this program, 24 square columns with various tie configurations were tested under concentric compression, and an analytical stress-strain model was developed for confined concrete in tied columns (Sheikh and Uzumeri 1980; Sheikh and Uzumeri 1982). At the second stage, a series of tests (Sheikh and Yeh 1990; Patel and Sheikh 1992; Sheikh and Khoury 1993; Sheikh et al. 1994) were conducted on tied columns subjected to flexure and axial loads. Based on these results, Sheikh and Khoury (1997) proposed a performance-based procedure for the design of confining steel in tied columns. Bayrak and Sheikh (1997; 1998) further conducted tests on high-strength concrete (HSC) tied columns and proposed modification to this procedure to make it applicable to HSC square columns. Following these efforts, Sheikh and Yau (2002), Iacobucci et al. (2003), and Memon and Sheikh (2002) conducted tests on FRP-confined columns and Li and Sheikh (2003) developed a procedure for the design of confining FRP in square columns following the
This paper presents the significant results from this research program following a critical review of the confinement provisions in current North American codes. Some of the areas in which further research is needed are also discussed. Full details of these studies may be seen in the literature.

CODES’ PROVISIONS FOR CONFINEMENT

ACI 318-02 Code and CSA A23.3-94 Code

The provisions for confinement reinforcement are similar in both the codes. The basic philosophy behind these provisions is that the increase in strength of the core concrete due to confinement should offset the loss in strength caused by spalling of the cover concrete, thus maintaining the axial load carrying capacity of the columns. For circular columns, both Codes specify that the minimum volumetric ratio of spiral steel, $\rho_s$, shall be given by the larger amount given by Equations 1 and 2.

$$\rho_s = 0.45 \left( \frac{A_g}{A_c} - 1 \right) \frac{f_c'}{f_{yh}}, \quad (1)$$

$$\rho_s = 0.12 \frac{f_c'}{f_{yh}} \quad (2)$$

Eq. (1) was derived on the basis of the strength gain of core concrete due to confinement as suggested by Richart et al. (1929),

$$f_{cc} = f_{cp} + 4.1 f_t, \quad (3)$$

while the lower limit provided by Eq. (2) is mainly applicable to large columns in which $A_g/A_c$ is less than 1.27.

The Codes’ requirements for the confinement reinforcement in tied columns are expressed in terms of the total cross sectional area of rectilinear ties, with the implied efficiency of rectilinear ties ranging from 67 to 75 percent of that of spirals. The total cross sectional area of ties is given by the larger amount from Equations 4 and 5:

$$A_{sh} = 0.3 s_h t \left( \frac{A_g}{A_c} - 1 \right) \frac{f_c'}{f_{yh}}, \quad (4)$$

$$A_{sh} = 0.09 s_h t \frac{f_c'}{f_{yh}} \quad (5)$$

The confinement requirements of the ACI 318-02 and the CSA A23.3-94 Codes only aim to maintain the axial load capacity of a column section after spalling of the cover concrete. These provisions ignore the most important parameter, ductility, when a column is subjected to seismic loading. Since the primary objective of concrete confinement is to enhance the ductility of columns, the rational design of confinement
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reinforcement should take into account both strength and ductility. Moreover, many experimental studies (Sheikh and Uzumeri 1980; Sheikh and Yeh 1990; Patel and Sheikh 1992; Sheikh and Khoury 1993; Sheikh et al. 1994) have confirmed that both the level of axial load applied to the column and the steel configuration have great effects on the column behavior. Column ductility decreases with the increase in axial load. Given the same amount of longitudinal and transverse reinforcement and the same level of axial load, column ductility varied significantly from one steel configuration to another. Column sections designed in accordance with the codes’ provisions thus display behavior that varies from very ductile to brittle depending on the arrangement of steel and the nature of loads.

BEHAVIOR OF CONFINED CONCRETE

Experimental program

To study the behavior of confined concrete in tied columns, Sheikh and Uzumeri (1980) tested 24 square columns under increasing concentric compression to failure. All columns were 305 mm square and 1.96 m long. Sections were gradually enlarged to 305 x 508 mm at both ends to avoid failure in the end zones. Variables studied were the distribution of longitudinal steel around the core perimeter and the resulting tie configuration, the amount of longitudinal steel, and the amount, spacing, and characteristic of lateral steel. The longitudinal steel content varied between 1.72% and 3.67% of the gross section area, while the amount of lateral steel ranged from 0.76% to 2.39% of the core volume. The core area was 267 x 267 mm measured from the centerline of the exterior tie.

The test results indicated that the distribution of the longitudinal steel around the core perimeter and the resulting tie configuration have a significant effect on the behavior of confined concrete. Strength and ductility of the concrete increased as the number of laterally supported longitudinal bars increased. Moreover, reduction in tie spacing and an increase in the confining steel content resulted in an increase in concrete strength and significant improvement in the ductility.

Sheikh and Uzumeri Model for tied columns

Based on their results, Sheikh and Uzumeri (1982) proposed an analytical model for confined concrete in tied columns. The stress-strain curve for this model is shown in Fig.1, which consists of a second-degree parabola and three straight lines. The curve can be defined completely by four parameters, namely $f_{cc}$, $\varepsilon_{s1}$, $\varepsilon_{s2}$, and $\varepsilon_{s85}$.

One of the important features of this model is the concept of the concrete effectively confined within the concrete core defined by the centerline of the perimeter ties. As illustrated in Fig. 2, the model assumes that at the tie levels, the separation between the effectively confined concrete and the unconfined concrete is in the form of a series of arcs spanning between the laterally supported bars with an initial tangent slope of 45 degrees. Between tie levels, the area of the effectively confined concrete core reduces and is minimum midway between two tie sets. For square sections with
uniformly distributed longitudinal steel, the area of the effectively confined concrete core at the critical section, \( A_{ec} \), can be calculated from

\[
A_{ec} = \left( B^2 - \frac{nC^2}{5.5} \right) \left( 1 - \frac{S}{2B} \right)^2
\]  

(6)

The strength gain factor \( K_s \) is calculated as

\[
K_s = 1.0 + \frac{B^2}{140P_{occ}} \left[ \left( 1 - \frac{nC^2}{5.5B^2} \right) \left( 1 - \frac{S}{2B} \right)^2 \right] \sqrt{\rho_s f_s}
\]  

(7)

Strain values to define the complete stress-strain curve can be calculated from the following equations.

\[
\varepsilon_{s1} = 80K_s f'_c \times 10^{-6}
\]  

(8)

\[
\varepsilon_{s2} = \left( 1 + \frac{248}{C} \left[ 1 - 5.0 \left( \frac{S}{B} \right) \right] \rho_s f'_s \right) \varepsilon_{\infty}
\]  

(9)

\[
\varepsilon_{s85} = 0.225 \rho_s \sqrt{(B/S) + \varepsilon_{s2}}
\]  

(10)

According to this model, for the same amount of steel in the column, better distribution of longitudinal steel around the core perimeter and smaller tie spacing would result in larger \( A_{ec} \) and higher strength and ductility of concrete. The model was applied to predict the results of the tests reported by various researchers. The comparison between the experimental and analytical results showed good agreement.

**PERFORMANCE-BASED DESIGN OF CONFining STEEL IN COLUMNS**

**Experimental program**

**Test setup and procedure** — To investigate the behavior of confined concrete columns under earthquake loads, a series of tests have been conducted at the University of Houston and the University of Toronto. The specimens tested included normal-strength concrete (NSC) and HSC columns confined by steel and NSC columns confined by fiber-reinforced polymers (FRP). Fig. 3 shows the test setup used. To facilitate the direct comparison of the column behavior, all the columns tested were 1.47m long with a 510 × 760 × 810 mm stub that represented a beam-column joint or a footing. The square columns had a cross section of 305 × 305 mm while the circular columns had a 356 mm diameter. All specimens were tested horizontally under cyclic shear and flexure while subjected to constant axial load to simulate earthquake loads. The lateral displacement excursion regime consisted of one cycle to a displacement of 0.75 \( \Delta_1 \) followed by 2 cycles each of \( \Delta_1, 2 \Delta_1, 3 \Delta_1 \), and so on until the specimen was
unable to sustain the applied axial load. A nalytical yield displacement $A_1$ was the lateral deflection corresponding to the estimated maximum lateral load along a load-deflection line that represented the initial stiffness of the column without the effect of axial load.

**Ductility Parameters** — In evaluating the seismic performance of the columns, ductility and toughness parameters defined in Fig. 4 were used. These include curvature ductility factor $\mu_\Phi$, cumulative ductility ratio $N_\Phi$, and energy-damage indicator $E$. Wherever used, subscripts $t$ and $80$ indicate, respectively, the value of the parameter until the end of the test (total value) and the value until the end of the cycle in which the moment is dropped to 80 percent of the maximum value. Energy parameter $e_i$ represents the area enclosed in cycle $i$ of the $M-\Phi$ loop.

**Design procedure for NSC tied columns**

Sheikh and Khoury (1993) and Sheikh et al. (1994) tested eleven square columns under simulated earthquake loads. The variables examined were the level of axial load, the concrete strength, and the amount and configuration of the lateral ties. The details and ductility parameters of the columns are listed in Table 1. From these results, Sheikh and Khoury (1997) found a reasonable correlation between different ductility parameters, as shown in Fig. 5. Data from nine specimens that were tested under similar conditions were used in the construction of this figure. From the best-fit curves, it can be shown that for $\mu_{\Phi 80}$ of 16, the values of $N_{\Phi 80}$ and $E_{80}$ are 64 and 575, respectively. A column section with this level of deformability was defined as highly ductile. With a $\mu_{\Phi 80}$ value of 8 to 16, the section was defined as moderately ductile and the low ductility column has $\mu_{\Phi 80} < 8$. With this correlation between ductility parameters, the results of the columns tested by Sheikh and Yeh (1990) and Patel and Sheikh (1992) that were tested under monotonic flexure, as listed in Table 1, were also used to derive the procedure for the design of confining steel in tied columns with concrete strength up to 55 MPa.

**Effect of axial load level** — Increased axial load reduces ductility significantly (Sheikh and Yeh 1990; Sheikh and Khoury 1993; Sheikh et al. 1994). Level of axial load is generally measured by indices $P/f'_cA_0$ and $P/P_o$. For columns with similar $f'_c$, both these indices provide similar comparison. For columns with different $f'_c$, however, the comparison using $P/f'_cA_0$ may not remain valid (Sheikh and Khoury 1997). Hence in the design procedure $P/P_o$ instead of $P/f'_cA_0$ was used to measure the level of axial load.

Effect of a change in axial load on the column behavior can be evaluated by comparing the moment-curvature responses of Specimens AS-3 and AS-17 (Fig. 6), which are almost identical in every other regard. Increase in axial load from $0.5P_o$ to $0.63P_o$ resulted in significantly less ductile behavior. Curvature ductility factor $\mu_{\Phi 80}$ was reduced by about 45%.

**Steel configuration** — The effectiveness of confining steel primarily depends on the area of the effectively confined concrete and the distribution of confining pressure, which, in turn, are highly affected by the distribution of longitudinal and lateral steel and
the extent of lateral restraint provided to the bars (Sheikh and Uzumeri 1980; Sheikh and Uzumeri 1982; Sheikh and Yeh 1990). With larger number of longitudinal bars laterally supported by tie bends, the area of effectively confined concrete is increased considerably. Fig. 7 shows the moment-curvature responses of two specimens ES-13 and FS-9. These specimens and Specimen AS-17 are almost identical in all regards except steel configuration. Specimen AS-17 displayed more ductile behavior (see also Table 1) than Specimen FS-9 that in turn is tougher than Specimen ES-13.

Based on this concept and extensive experimental data (Sheikh and Uzumeri 1980; Sheikh and Yeh 1990; Patel and Sheikh 1992; Sheikh and Khoury 1993; Sheikh et al. 1994), Sheikh and Khoury (1997) divided steel configurations into the following three main categories (Fig. 8):

- **Category I**: Only single-perimeter hoops are used as confining steel
- **Category II**: In addition to the perimeter hoops supporting four corner bars, at least one middle longitudinal bar at each face is supported at alternate points by hooks that are not anchored in the core. At other points the supporting hooks are anchored in the core.
- **Category III**: A minimum of three longitudinal bars are effectively supported by tie corners on each column face and hooks are anchored into the core concrete.

**Limiting conditions for steel configurations** — Sheikh and Khoury (1997) suggested that for earthquake design, columns should be designed and detailed with high or moderate ductility. Based on the experimental evidence, they suggested that Category I configuration not be used for high ductility columns. The use of Configuration E in moderately ductile columns should be limited to lower range of axial load ($P < 0.40P_o$). For conservative design, the Category I configurations are recommended for moderate ductility columns only if the applied axial load is less than the balanced load $P_b$. With regard to Category II configurations, tests (Sheikh and Yeh 1990; Sheikh and Khoury 1993; Sheikh et al. 1994) on columns with Section F (Fig. 8) under high levels of axial load showed that the use of 90 degrees hooks not anchored in the core provided sufficient restraint to the middle bars up to a certain stage of loading, but at large deformations the 90 degrees hooks tended to open, resulting in a loss of confinement. Therefore, it was recommended that the use of Category II configurations to produce high-ductility columns be limited to cases with low levels of axial load. These columns can be used for moderate ductility if axial load dose not exceed $0.4P_o$. The limiting conditions under which the three categories of steel configurations may be reliably used for moderate and high ductility columns are outlined in Fig. 8.

**Proposed approach** — The relationship between the amount of lateral steel as recommended by the current ACI Code, $A_{sh,c}$, and the suggested amount of lateral steel $A_{sh}$ was taken as:

$$A_{sh} = A_{sh,c} \cdot \alpha \cdot Y_p \cdot Y_\phi$$  (11)

Parameter $\alpha$ is assumed to be equal to unity for Category III configurations. This factor is expected to be greater than unity for Category I configurations even for their use under
limiting conditions prescribed earlier. For such a case, the value for \( \alpha \) is discussed later in this section. Use of Category II configurations is subjected to the imposed limitations because some of the hooks are not anchored in the core. It is reasonable to assume a value of \( \alpha \) equal to unity for these configurations in situations where opening of these hooks does not take place until sufficient ductility is exhibited (Sheikh and Yeh 1990; Sheikh and Khoury 1993; Sheikh et al. 1994). In the event of high axial load levels, the value of \( \alpha \) would be much greater than unity; however such an application is not recommended and should be avoided.

With these values of \( \alpha \), Eq. (11) for sections with at least three longitudinal bars restrained on each face \((\alpha = 1)\) reduces to

\[
A_{sh} / A_{sh,c} = Y_p Y_\phi
\]  

(12)

After investigating several possible forms of expressions for \( Y_p \) and \( Y_\phi \), the following simple forms were selected,

\[
Y_p = a_1 + a_2 (P / P_o)^{a_3}
\]  

(13)

\[
Y_\phi = b_1 (\mu_{\phi})^{b_2}
\]  

(14)

where \( a_1, a_2, a_3, b_1, \) and \( b_2 \) are constants to be determined empirically.

As a starting point, since the two parameters \( Y_p \) and \( Y_\phi \) are independent of each other, the value of \( Y_\phi \) was assumed to be unity for highly ductile sections with \( \mu_{\phi00} \) equal to or greater than 16. Specimens meeting this requirements are AS-3, AS-18, AS-19, AS-20H, A-3, and F-4. Using the results from these specimens, a least square analysis was performed to find constants \( a_1 \) and \( a_2 \) for selected values of \( a_3 \) that ranged from 1 to 6. Corresponding to each chosen value of \( a_3 \), and consequently obtained values for \( a_1 \) and \( a_2 \), the constants \( b_1 \) and \( b_2 \) in the expression for \( Y_\phi \) were then determined using the test results for those 16 specimens in which \( \alpha = 1 \). These included all the specimens with A and D configurations (Category III) and Specimens F-4 and F-12 (Category II) from Table 1. Specimen A-3 was not included in the analysis since its \( \mu_{\phi00} \) was unusually large compared with other similar specimens.

Minimization of the total cumulative error for all the 16 specimens was the only criterion used to select the final values of the empirical constants. The expressions for parameters \( Y_p \) and \( Y_\phi \) are given below:

\[
Y_p = 1 + 13 (P / P_o)^5
\]  

(15)

\[
Y_\phi = (\mu_{\phi00})^{1.15} / 29
\]  

(16)

The correlation coefficients for Equations (15) and (16) are 0.99 and 0.93, respectively. The high coefficients indicate excellent agreement between the analytical and the
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experimental values. Based on the above, the amount of lateral steel in tied columns may be calculated using the following expression.

$$A_{sh} = \alpha \left\{ 1 + 13 \left( \frac{P}{P_0} \right)^5 \left( \frac{\mu_{gb0}}{29} \right)^{1.15} A_{sh,c} \right\}$$  \hspace{1cm} (17)

The simplified versions of the expressions for $Y_p$ and $Y_\phi$ can be taken as

$$Y_p = 6(P / P_0) - 1.4 \geq 1.0 \hspace{1cm} (18)$$
$$Y_\phi = \mu_{gb0} / 18 \hspace{1cm} (19)$$

and the required amount of lateral steel may be calculated as

$$A_{sh} = \alpha \left[ 6 \frac{P}{P_0} - 1.4 \right] \frac{\mu_{gb0}}{18} A_{sh,c} \geq \alpha \frac{\mu_{gb0}}{18} A_{sh,c} \hspace{1cm} (20)$$

Factor $\alpha$ is unity for Category III configurations and for Category II configurations as long as the prescribed limiting conditions are met; whereas $\alpha$ value for Category I configurations may be estimated by using the experimental results. Values for $\alpha$ were calculated using Eq. (17) for all the specimens with Configuration E. The average value of $\alpha$ is about 2.70.

The factor $\alpha$ for Category I configurations may also be estimated by adopting the concept of “effectively confined concrete core area” as shown in Fig. 2. The ratio between the area of effectively confined concrete and the total concrete area $\lambda$ at tie level is given by:

$$\lambda = 1 - \frac{\sum_i^n C_i^2}{5.5B^2} \hspace{1cm} (21)$$

It may be reasonably assumed that the configuration parameter $\alpha$ is proportional to $1/\lambda$. Since $\alpha = 1$ for Category III configurations, $\alpha$ for Category I configurations ($\alpha_i$) may be written as $\alpha_i = \lambda_{III} / \lambda_1$ where $\lambda_{III}$ and $\lambda_1$ can be calculated using Eq. (21). For the specimens in which the longitudinal bars are uniformly distributed around the core perimeter, the $\lambda$ values for Configurations A and O are 0.636 and 0.273, respectively. Hence, $\alpha_i = 2.33$. It may be reasonable, therefore, to conclude that the factor $\alpha$ for Category I configurations may range from 2.3 to 2.7. An average value of 2.5 is thus assumed for all configuration types in this category.

The above procedure, for the sake of simplicity, does not include tie spacing as an active parameter. However, it should be recognized that the test data on which the equations are based were obtained from specimens in which tie spacing varied from 0.20B to 0.43B (or 3.4d_b to 7.2d_b). Experimental and theoretical evidence (Sheikh and
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Uzumeri 1980; Sheikh and Uzumeri 1982) shows that hoop spacing plays a significant role in the mechanism of confinement. Larger hoop spacing will result in smaller area of effectively confined concrete in the core and may result in premature buckling of longitudinal bars. Hence for a conservative design the limit to the tie spacing is suggested to be the smallest of $B/3$, $6d_b$, and 200 mm.

**Design procedure for FRP-confined columns**

In recent years, FRP jacketing has emerged as a promising retrofitting technique to provide additional confinement to existing columns due to the lightweight, high strength, and excellent corrosion resisting property of FRP. As this technique has been increasingly used in the field, it is imperative to develop appropriate design procedure for practicing engineers to implement this new technology with confidence.

To investigate of behavior of FRP-confined columns, Iacobucci et al. (2003) and Memon and Sheikh (2002) tested 11 square FRP-confined columns under simulated earthquake loads. The test setup, loading sequences, testing procedures, and configurations of the specimens are similar to those used by Sheikh and Khoury (1993). The specimens were designed to model typical pre-1971 seismic design details and were externally retrofitted by different amounts of continuous carbon fiber-reinforced polymer (CFRP) or glass fiber-reinforced polymer (GFRP) wraps to provide additional confinement in the potential plastic hinge regions. The details and the ductility parameters of these specimens are listed in Table 2. Based on these results, Li and Sheikh (2003) have developed a procedure for the design of confining FRP reinforcement following the design philosophy proposed by Sheikh and Khoury (1997).

**Ductility requirements for FRP-confined columns** — To develop the ductility requirements for FRP-confined columns, the relationships between different ductility parameters of steel-confined columns are compared with those of FRP-confined columns in Fig. 5. The relationships of FRP-confined columns were constructed using the results of ten FRP-confined specimens as listed in Table 2. From Fig. 5, it can be seen that for a certain value of $\mu_{\phi_b}$, the values of $N_{\phi_b}$ and $E_{\phi_b}$ of the FRP-confined columns are significantly higher than those of the steel-confined columns, which indicates that for dissipating equal amounts of energy, the curvature ductility factors of the FRP-confined columns are smaller than those of the comparable steel-confined columns. This may be attributed to the different curvature distributions in the plastic hinge regions and different plastic hinge lengths in these two types of columns. For steel-confined columns, the equivalent plastic hinge length is reported to be approximately equal to the dimension of the cross section (Sheikh and Khoury 1993; Sheikh et al. 1994), whereas the equivalent plastic hinge length of most of the FRP-confined columns is larger than the dimension of the cross section (Iacobucci et al. 2003; Memon and Sheikh 2002).

For steel-confined columns, a $\mu_{\phi_b}$ value of 16 corresponds to $E_{\phi_b} = 575$, while a $\mu_{\phi_b}$ value of 8 corresponds to $E_{\phi_b} = 123$. From the best-fit curve for FRP-confined columns, it can be shown that at $E_{\phi_b} = 575$, the corresponding value for $\mu_{\phi_b}$ is 13.2; while at $E_{\phi_b} = 123$, the corresponding value for $\mu_{\phi_b}$ is 8.2. Therefore it is suggested that
for FRP-confined columns, the behavior of a column section with $\mu_{\Phi 0} = 13$ can be considered as highly ductile. The section with a $\mu_{\Phi 0}$ value of 8 to 13 can be defined as moderately ductile and the low ductility column has $\mu_{\Phi 0} < 8$.

Amount of confining FRP—The effect of the amount of confining FRP on column ductility can be evaluated by comparing the moment vs. curvature behavior of Specimens ASC-4NS, ASC-3NS, and ASC-5NS, as presented in Fig. 9. These three columns were tested under an axial load of 0.56 $P_o$ and were confined by 1, 2, 3 layers of CFRP, respectively. As can be seen from Fig. 9, the behavior of the columns improved progressively as the amount of confining CFRP increased. These three columns were able to sustain 8, 11, and 15 load cycles, respectively. The enhancements in curvature ductility of the columns were approximately proportional to the amount of confining FRP provided. Comparisons of the behavior of Specimens ASC-2NS and ASC-6NS that were tested under an axial load of 0.33 $P_o$ and the behavior of the GFRP-confined columns (Table 2) also lead to the same conclusion.

Axial load level—The effect of the level of axial load on column behavior can be evaluated by comparing the behavior of Specimens ASC-2NS and ASC-4NS (Fig. 9), which were almost identical in every other respect except for the different levels of axial load applied. An increase in axial load from 0.33 $P_o$ in ASC-2NS to 0.56 $P_o$ in ASC-4NS resulted in significantly less ductile behavior. Curvature ductility factor was reduced by about 36 percent. Similar observations can be made by comparing the behavior of Specimens ASC-6NS and ASC-3NS and that of Specimens ASG-2NSS and ASG-4NSS.

Proposed approach—From the above discussion, it is evident that for square FRP-confined columns, the column ductility increases as the amount of confining FRP increases, whereas an increase in axial load level reduces column ductility. These effects are similar to those in steel-confined columns. For square columns confined by FRP, the lateral confining pressure provided by the FRP, $f_l$, can be calculated as

$$ f_l = 2 \cdot n \cdot f_{FRP} / h \tag{22} $$

Since the actual average strains of the FRP at the failed sections at column failure are difficult to measure, the corresponding confining pressures at column failure are difficult to obtain. However, from the reported experimental results listed in Table 2, it was found that a reasonable relationship existed between the curvature ductility factors of the columns and the theoretical maximum lateral confining pressure, $f_{l,max}$, provided by the FRP, which can be defined as

$$ f_{l,max} = 2 \cdot n \cdot f_u / h \tag{23} $$

The $f_{l,max}$ of the ten specimens were calculated and are listed in Table 2. The relationship between the curvature ductility factor and theoretical maximum lateral confining pressure provided by the FRP is shown in Fig. 10. In this figure, the theoretical maximum lateral confining pressures are normalized with respect to the concrete
compressive strength of the columns, which can be defined as the confinement ratio for the FRP-confined columns.

Fig. 10 indicates that, for columns confined by FRP, the curvature ductility factor, $\mu_{80}$, increases almost linearly with the increase in the confinement ratio for a certain level of axial load. This relationship can be expressed by the following equation:

$$f_{l,\text{max}}/f_c' = \gamma \cdot Y_p \cdot Y_\phi$$

(24)

Substituting Eq. (23) into Eq. (24) and rearranging the equation give

$$n \cdot f_u = \beta \cdot f_c' \cdot h \cdot Y_p \cdot Y_\phi$$

(25)

where $\beta = \gamma/2$. In the application of Eq. (25) and the expressions for $Y_p$ and $Y_\phi$ (Eq. [15] and Eq. [16]) for steel-confined columns to the aforementioned FRP-confined columns, it was found that the expressions for $Y_p$ and $Y_\phi$ for steel-confined concrete columns were applicable to FRP-confined columns. Substituting the expressions for $Y_p$ and $Y_\phi$ into Eq.(25) gives the following form of the design equation:

$$n \cdot f_u = \beta \cdot h \cdot f_c' \cdot \left[1 + 13 \left(\frac{P}{P_o}\right)^5 \left(\frac{\mu_{80,\text{in}}}{20}\right)^{1.15}\right]$$

(26)

The experimental results of the ten FRP-confined columns were used to calculate the value for $\beta$. The average value of $\beta$ was found to be 0.25 with the standard deviation of 0.03.

**Design procedure for HSC tied columns**

The procedure developed by Sheikh and Khoury (1997) was derived from the test results of the columns with concrete strength up to 55 MPa. To check the applicability of this procedure to HSC columns, Bayrak and Sheikh (1997; 1998) tested eight HSC and ultrahigh-strength concrete (UHSC) specimens under simulated earthquake loads. The test setup, loading sequences, testing procedures, and configurations of the specimens are similar to those used by Sheikh and Khoury (1993). Concrete strength varied between 71.7 and 102.2 MPa. The effectiveness of two tie configurations, i.e. Type A and Type E in Fig. 8, was examined. The details and the ductility parameters of these specimens are listed in Table 3.

**Effects of confinement parameters on column behavior** — Test results indicate that HSC and UHSC columns with concrete strength up to 102 MPa can be made to behave in a ductile manner under high levels of axial load, provided that sufficient amount of lateral reinforcement is used in an efficient configuration. As in the case of
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NSC specimens, an increase in the amount of lateral reinforcement significantly improved the cyclic behavior of HSC and UHSC specimens; whereas an increase in axial load caused substantial reductions in column ductility. The specimens with Type E steel configuration behaved in a considerably less ductile manner than the comparable specimens with Type A configuration. From the results, Bayrak and Sheikh (1998) also found that specimens behaved in similar manner as long as they had the same reinforcement configuration and the same $R_{\text{IP}}$ ratio, which was defined as $R_{\text{IP}} = \left( \frac{A_{\text{sh}}}{A_{\text{sh,c}}} \right) / \left( \frac{P}{P_{\text{o}}} \right)$.

Confinement reinforcement requirements for HSC columns — Bayrak and Sheikh (1998) checked the accuracy of the design procedure suggested by Sheikh and Khoury (1997) for the HSC and UHSC test data. It was found that the average of all eight ductility predictions made by Eq. (17) was 37% larger than the average experimental results. Standard deviation from the mean was 29%. To improve the predictions, a regression analysis was performed using the results from the eight specimens listed in Table 3. In the application of Eq. (17) to specimens having concrete strengths varying between 72 and 102 MPa, it was found that constants $a_1$, $a_2$, and $a_3$ in Eq. (13) remained the same and constants $b_1$ and $b_2$ in Eq. (14) changed. In other words, the effect of axial load on the lateral reinforcement demand is not influenced significantly by concrete strength. Section ductility demand measured by $\mu_{\phi_{80}}$, on the other hand, is influenced by concrete strength. This is because higher strength concrete specimens have lower deformability and energy absorption capacity initially. These properties improve considerably during the latter part of the displacement excursions. Curvature ductility factor $\mu_{\phi_{80}}$ does not reflect the total behavior of a columns section. Instead, it focuses only upon the part of a moment-curvature relationship in which the strength loss is smaller than or equal to 20% of the maximum bending moment. For NSC columns $b_1$ and $b_2$ were found to be equal to 1/29 and 1.15, respectively. Using the results of the eight specimens in Table 3, a least-square analysis yielded $b_1 = 1/8.12$ and $b_2 = 0.82$. With these new constants, the average of the predicted curvature ductility ratios is roughly equal to the average of the experimental values, and the standard deviation from the mean is 10%. Furthermore, the value of configuration efficiency factor $\alpha$ for E-type HSC and UHSC columns is calculated to be equal to 1.35. The following equation summarizes the suggested design procedure for the design of confinement reinforcement for HSC columns with $f'_{c}$ in the range of 55 MPa to over 100 MPa:

$$A_{\text{sh}} = \alpha \cdot \left\{ 1 + 13 \left( \frac{P}{P_{\text{o}}} \right)^5 \right\} \cdot \left( \frac{\mu_{\phi_{80}}}{8.12} \right)^{0.82} \cdot A_{\text{sh,c}} \quad (27)$$

RESEARCH NEEDED FOR CIRCULAR COLUMNS

It is generally recognized that the confinement efficiency of circular lateral steel is higher than that of rectilinear ties. Li and Sheikh (2003) conducted a review on the available experimental results on circularly confined columns tested under simulated earthquake loads. This review shows that only limited test data is available in the
literature. Moreover, the test setups, loading histories, instrumentations, and ductility parameters used in the available experimental investigations were different from one program to another, making it difficult to evaluate the results on a common platform. Yau and Sheikh (2002) tested four circular steel-confined columns and eight FRP-confined columns under cyclic flexure and shear, and constant axial load. Two of the steel-confined columns meet the ACI 318-02 Code’s confinement requirements and were tested under different levels of axial load. The test results of these two columns indicated that an increase in axial load resulted in reduced ductility and deformability of the columns. The ACI 318-02 Code’s confinement requirements for circular columns are not sufficient to provide ductile behavior under an axial load level of 0.54\(P_o\). A reexamination of these provisions is needed. However, due to the limitations of the available test data, it is difficult to suggest a rational design procedure at this stage. More data is needed to investigate the effects of various confinement parameters on the behavior of circular columns and develop rational design procedure.

**SUMMARY AND CONCLUSIONS**

The test results from an extensive research program that aims at investigating the behavior of confined concrete columns have been briefly summarized. An analytical model for rectilinear confinement of concrete is briefly presented that highlights the effects of the distribution of longitudinal and lateral steel on column behavior. The paper also outlines procedures for the design of steel and FRP confinement reinforcement in square columns. Design of steel reinforcement is provided for normal as well as high strength concrete columns while only normal strength concrete columns were considered for FRP confinement. The procedures directly relate the confinement requirements to a column’s ductile performance, the level of axial load applied, and the configuration of reinforcement. A review of the available research on circularly confined columns suggests that further test data is needed to evaluate the effects of different variables on the performance of these columns, and develop and corroborate analytical models and design procedures.

**NOTATIONS**

\[\begin{align*}
A_c &= \text{cross sectional area of the core concrete, mm}^2 \\
A_{ec} &= \text{area of effectively confined concrete core at critical sections, mm}^2 \\
A_g &= \text{gross cross-sectional area of column, mm}^2 \\
A_{sh} &= \text{total cross-sectional area of rectilinear ties perpendicular to dimension } h_c, \text{ mm}^2 \\
B &= \text{center-to-center distance between the outer legs of perimeter ties, mm} \\
C &= \text{center-to-center distance between laterally supported longitudinal bars, mm} \\
d_b &= \text{diameter of longitudinal bars, mm} \\
E &= \text{energy damage indicator} \\
f_c' &= \text{compressive strength of concrete as measured from standard } (150 \times 300 \text{ mm}) \text{ cylinders, MPa} \\
f_{cc} &= \text{compressive strength of confined concrete, MPa} \\
f_{cp} &= \text{compressive strength of plain concrete, MPa} \\
f_{FRP} &= \text{tensile stress of FRP, MPa}
\end{align*}\]
\[ f_l = \text{lateral confining pressure, MPa} \]
\[ f_{s'} = \text{stress of lateral reinforcement, MPa} \]
\[ f_y = \text{yield strength of longitudinal steel, MPa} \]
\[ f_{yh} = \text{yield strength of lateral steel, MPa} \]
\[ f_u = \text{ultimate tensile strength of FRP, MPa} \]
\[ h = \text{cross-sectional dimension of cross section, mm} \]
\[ h_c = \text{cross-sectional dimension of concrete core measured center-to-center of rectilinear ties, mm} \]
\[ K_s = \text{strength gain factor} \]
\[ L_r = \text{length of most damaged region of column, mm} \]
\[ n = \text{layers of FRP laminates or number of arcs} \]
\[ N_f = \text{cumulative ductility ratio} \]
\[ P = \text{applied axial load on column, kN} \]
\[ P_b = \text{balance load, kN} \]
\[ P_o = \text{unconfined axial load capacity of column} = 0.85f_{c'}(A_g - A_c) + A_s f_{yh}, \text{kN} \]
\[ P_{oc} = 0.85f_{c'}(B^2-A_g), \text{kN} \]
\[ s = \text{spacing of rectilinear ties, mm} \]
\[ Y_p = \text{parameter to take into account the effect of axial load level on column ductility} \]
\[ Y_{\phi} = \text{parameter to take into account section ductility demand} \]
\[ \alpha = \text{confinement efficiency factor} \]
\[ \beta \]
\[ \epsilon_{oo} = \text{strain corresponding to the maximum stress in plain concrete} \]
\[ \epsilon_s = \text{strain of confined concrete} \]
\[ \rho_s = \text{ratio of the volume of total lateral steel to the volume of concrete core} \]

**REFERENCES**


“Building Code Requirements for Reinforced Concrete (ACI 318-02) and Commentary (ACI 318R-02).” (2002). American Concrete Institute, Detroit.


## Table 1 — Details and ductility parameters of NSC steel-confined specimens

<table>
<thead>
<tr>
<th>Researchers</th>
<th>Specimen</th>
<th>$f'_c$ (MPa)</th>
<th>$s$ (mm)</th>
<th>$\rho_s$ (%)</th>
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* Lightweight aggregate concrete specimens
### Table 2 — Details and ductility parameters of NSC FRP-confined specimens

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<th>Researchers</th>
<th>Specimen</th>
<th>$f_c'$ (MPa)</th>
<th>Layers &amp; type of FRP</th>
<th>$p_1$</th>
<th>$\mu$</th>
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<th>Energy indicator</th>
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* Reduction in capacity less than 20% for completed cycles

<sup>a</sup> Control steel-confined specimens

### Table 3 — Details and ductility parameters of HSC steel-confined specimens (Bayrak and Sheikh 1997; 1998)

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<thead>
<tr>
<th>Specimen</th>
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<th>$f_c'$ (MPa)</th>
<th>Lateral steel</th>
<th>Size @ spacing (mm)</th>
<th>$\rho_s$ (%)</th>
<th>$f_{\text{min}}$ (MPa)</th>
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<th>$\mu$</th>
<th>$N_{\theta_0}$</th>
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Figure 1 — Sheikh and Uzumeri (1982) model for confined concrete in tied columns
Figure 2 — Concept of effectively confined concrete area

Figure 3 — Schematic of test setup

Figure 4 — Ductility parameters
Figure 5 — Relationships between ductility parameters

Figure 6 — Behavior of Specimens AS-3 and AS-17

Figure 7 — Behavior of Specimens FS-9 and ES-13
Figure 8 — Categories and limiting conditions for steel configurations

Figure 9 — Behavior of FRP-confined columns

Figure 10 — Relationship between curvature ductility factor and confinement ratio
Seismic Performance of Confined Concrete Bridge Columns

by J.P. Moehle and D.E. Lehman

Synopsis: A current focus in earthquake engineering research and practice is the development of seismic design procedures whose aim is to achieve a specified performance. To implement such procedures, engineers require methods to define damage in terms of engineering criteria. Previous experimental research on bridge columns has focused on component failure, with relatively little attention to other damage states. A research program was undertaken to assess the seismic performance of well-confined, circular-cross-section, reinforced concrete bridge columns at a range of damage states. The test variables included aspect ratio, longitudinal reinforcement ratio, spiral reinforcement ratio, axial load ratio, and the length of the well-confined region adjacent to the zone where plastic hinging is anticipated. The experimental results are used to identify important damage states and to link those states to engineering parameters.

Keywords: bridges; columns; confined concrete; earthquake engineering; performance-based earthquake engineering; reinforced concrete
Moehle and Lehman

INTRODUCTION

Most current seismic design codes do not require the engineer to explicitly assess performance. Instead, seismic design actions are specified through an analysis of the structure, the structure is proportioned to resist those actions, and prescriptive details are provided. Although damage is anticipated in future earthquakes, the extent of damage is not an explicit consideration in design. Furthermore, little attention is paid to performance levels other than life safety. New performance-based seismic design approaches aim to provide more direct consideration of a broader range of performance objectives to meet the needs of individual owners or society.

The Pacific Earthquake Engineering Research Center is developing an integrated methodology for performance-based earthquake engineering [Moehle, 2003], illustrated qualitatively in Figure 1. The methodology begins with representation of seismic demand through an Intensity Measure, simulation of soil-foundation-structure response in terms of Engineering Demand Parameters (such as displacement, acceleration, strain), computation of Damage Measures (such as cracking, spalling, reinforcement fracture) given the engineering demand parameters, and interpretation of the damage in terms of aggregated Decision Variables (repair cost, downtime, casualties) that are relevant to owners and decision-makers. The methodology requires a probabilistic description of each of the components shown in Figure 1, with uncertainties of the entire problem carried consistently from one step to the next.

A key part of the development of performance-based earthquake engineering is to relate damage to engineering demand parameters. An experimental program was developed to improve analytical capabilities for relating damage to engineering demand parameters. The program was limited to circular-cross-section, reinforced concrete bridge columns detailed for ductile flexural response. Damage measures of interest, because they relate to different degrees of repair effort, include crack width, onset of spalling, onset of reinforcement buckling and fracture, and loss of lateral load resistance. This paper describes the study and its principal findings.
EXPERIMENTAL PROGRAM

Overview

The experimental program was designed to obtain performance data for bridge columns having details typical of those currently in use in regions of high seismicity in the United States. The columns had circular cross sections and were reinforced with well-distributed longitudinal reinforcement and closely spaced spiral reinforcement (Figure 2). Columns were fixed to a stiff foundation and were proportioned so that flexure would dominate inelastic response under lateral loading. Column dimensions were selected to represent typical bridge column dimensions scaled to one-third of full scale.

Table 1 lists important properties of the test specimens. Each specimen has an alphanumeric designation. The last two numbers relate to the longitudinal reinforcement ratio while the preceding number(s) relate to the aspect ratio (length/diameter); e.g., Column 415 has aspect ratio of 4 and longitudinal reinforcement ratio of 0.015.

The ten columns are organized in five series (Table 1). Test Series LR included Columns 407, 415, and 430, the main variable being longitudinal reinforcement ratio. (Column 430 was detailed with bundled bars, while the others in this test series were not.) Test Series AR (Columns 415, 815, and 1015) studied effect of varying aspect ratio. Test Series AL (Columns 415 and 415P) had axial loads of $0.1A_g f'_c$ and $0.2A_g f'_c$, where $A_g$ is the gross cross-sectional area of the column and $f'_c$ is the specified concrete compressive strength. Test Series SR (Columns 415 and 415S) examined effects of doubling the spiral spacing. Lastly, Test Series CL included Columns 328T, 828T, and 1028T; this series featured variable spacing of the spiral along the length, with smaller spacing near the column end where inelastic flexural response was anticipated.

Specimen design and materials

Column spiral reinforcement was designed considering recommendations and requirements for shear and confinement (Caltrans 1991, ATC 1996, AASHTO 2000). The confinement requirements of AASHTO, ATC 32, and Caltrans are expressed by Equations (1), (2), and (3), respectively. Table 2 presents spiral reinforcement ratios obtained by these expressions for the test columns. With the exception of Column 415S, all of the columns meet or exceed these provisions.

$$\rho_s = 0.45 \left( \frac{A_g}{A_c} - 1 \right) \frac{f'_c}{f_{yh}} \text{ but not less than } \rho_s = 0.12 \frac{f'_c}{f_{yh}}$$  (1)

$$\rho_s = 0.16 \frac{f'_c}{f_{yh}} \left( 0.5 + \frac{1.25P}{f'_c A_g} \right) + 0.13(\rho_l - 0.01)$$  (2)
\[
\rho_s = 0.12 \frac{f'_{c}}{f_{yh}} \left( 0.5 + \frac{1.25P}{f'_{c} A_{c}} \right) \tag{3}
\]

In the preceding expressions, \( \rho_s \) is the spiral reinforcement ratio, \( \rho_l \) is the longitudinal reinforcement ratio, \( A_c \) is the cross-sectional area of the concrete core, \( P \) is the applied axial load, and \( f_{yh} \) is the specified yield strength of the spiral reinforcement.

The ATC 32 provisions require a well-confined region adjacent to expected plastic hinge locations, and allow the spiral reinforcement ratio to decrease outside that region. If column axial load is less than \( 0.3 A_{c} f'_{c} \), the length of the well-confined region, \( L_c \), must satisfy Equation (4).

\[
L_c \geq \text{minimum of } (0.2L, D) \tag{4}
\]

In the expression, \( L \) is the column length and \( D \) is the column diameter. The three columns of Test Series CL were designed using these recommendations. The well-confined region was equal to the column diameter for the shortest column and 20 percent of the column length for the two taller columns. Outside the well-confined region, the spiral reinforcement was designed to suppress shear failure.

Longitudinal reinforcement met ASTM A 706. Spiral reinforcement was 6 mm in diameter. As this reinforcement was not available with specified yield strength of 414 MPa (60 ksi), ASTM A 82 reinforcement was selected. The 28-day strength of concrete was specified as 28 MPa (4 ksi). Tests were done on 150mm by 300mm cylinders stored with the test columns. Table 3 summarizes the results.

**Test details**

Figure 3 shows a drawing of a column with an aspect ratio of 4 in the test apparatus. Similar apparatus was used for other tests. Manually controlled jacks applied axial load through a spreader beam attached to the top of a test column. A horizontally aligned actuator at the top of the test column applied the lateral displacement history.

Instrumentation monitored global response quantities (e.g., applied lateral load and displacement) as well as local ones (e.g., steel strains and column segment rotations). Local response instrumentation was concentrated where significant inelastic action was anticipated, and included external instruments to monitor shearing, bending, and expansion of local segments of the column (Figure 3) and foil strain gauges on the longitudinal and spiral reinforcement.

Axial load was maintained constant during testing. Figure 4 shows the target lateral displacement history. The history was based on nominal displacement ductility because histories based on drift or displacement would have resulted in very different inelastic demands for columns of different heights. Three cycles were run at each amplitude, followed by a single cycle at one-third of the preceding amplitude.
OBSERVED PERFORMANCE

The sequence of damage was similar for all columns. The most notable observations, in sequence of first occurrence, were concrete cracking, longitudinal reinforcement yielding, initial spalling of the concrete cover, complete spalling of the concrete cover, spiral fracture, longitudinal reinforcement buckling, and longitudinal reinforcement fracture. These damage states are described in the following text and in the photographs of Figures 5 through 9. The first occurrence of each damage state was identified for each of the force-displacement histories (see example in Figure 10) and values are summarized in Tables 4 and 5.

Cracking and yielding

Initial damage was in the form of horizontal cracks. Initially, crack spacing was equal to approximately half the column diameter (Figure 5). In general, new cracks were not observed for subsequent displacement cycles at the same displacement amplitude; instead, new cracking occurred mainly when displacement amplitude increased.

Initial yielding of longitudinal reinforcement was measured using a strain gauge attached to the extreme longitudinal bar immediately above the column-footing interface. The geometry of a circular-cross-section column is such that initial yielding occurs only in a small number of bars at the extremity of the section; therefore, softening of the load-displacement response occurs only gradually as yielding spreads to adjacent bars around the column circumference. Measured initial yield displacement, $\Delta y$, is listed in Table 4.

Initial spalling and core damage

Initial spalling of concrete cover was recorded visually at the peaks during the first cycle at a given displacement amplitude (Figure 6). For displacement cycles after spalling but before apparent crushing of the core concrete, the extent of concrete spalling increased with increasing displacement amplitude but remained essentially constant for cycles at constant amplitude. After the core concrete had sustained apparent damage (defined when the inner spiral diameter was visible), cycling at constant amplitude resulted in additional visible damage. Figure 7 shows two columns after the height of spalling had stabilized. Height of the spalled region was greater for taller columns. Table 4 records the onset of initial cover and core spalling.

Bar buckling, bar fracture, and loss of lateral-load-carrying capacity

Typically, longitudinal bar buckling was observed after spalling had fully exposed the spiral and longitudinal reinforcement (Figure 8). The buckled portion of the bar always spanned several spiral spacings. The lateral displacement of the buckled bar increased during subsequent displacement cycles at a given displacement level, and spiral fracture occurred where it had been extended and kinked by the buckled bar (Figure 8).
The length of the buckled region was similar for all columns and was centered approximately 8 percent of the column height above the base (Table 5).

Fracture of longitudinal bars occurred after bar buckling (Figure 8). The bars did not fracture in Column 430 (bundled bars) or Column 415S (reduced spiral reinforcement ratio), both contrary to expectations. Figure 9 shows conditions at end of testing.

RELATIONS OF DAMAGE AND ENGINEERING DEMAND PARAMETERS

The damage states identified in the preceding section are quantified in terms of engineering demand parameters (Figure 1) in the following paragraphs.

Cracking

Cracking is important in the context of performance-based earthquake engineering because residual cracks may require repair by epoxy injection. Some engineering criteria (e.g., ATC 32 1996) relate residual crack width to maximum strain in the longitudinal reinforcement.

Residual crack widths, $w_{res}$, were measured for the eight columns of Test Series LR, AR, and CL. Residual crack widths were measured at the end of the three cycles at a displacement level (zero displacement). In most cases, the residual force was small. For Test Series LR and AR, the measured residual crack widths were located approximately 150 mm (6 in.) and 300 mm (12 in.) above the top of the footing. For Test Series LR and AR, the measured residual crack widths were at a height of approximately 300 mm (12 in.). Corresponding maximum longitudinal strains were measured on the extreme bars using foil strain gauges glued to the longitudinal reinforcement.

Figure 11 shows the relation between residual crack width and maximum previous longitudinal reinforcement strain. Figure 12 shows cumulative distributions of residual crack widths exceeding 0.5 mm, 0.13 mm and 0.25 mm. For each curve, the abscissa indicates the percentage of the crack width data that exceed the value indicated in the legend. The corresponding ordinate indicates the maximum measured strain.

Concrete cover spalling and core crushing

Onset of spalling signals a point at which more costly, time-consuming, and disruptive repairs likely will be required. As crushing becomes more severe, damage to core concrete may occur, perhaps requiring more extensive repair measures (Lehman et al. 2001). The extent of spalling along column height also is important, as it determines the minimum length over which confinement by spiral reinforcement is required.
Concrete compressive strains were approximated using external deformation measurements. As shown in Figure 3, steel instrumentation rods passed through the column core at selected elevations. Vertically oriented displacement transducers spanned between rods at one elevation and those at the next and measured deformations along the span between rods, enabling derivation of average rotation, average curvature, and average strain relations between sets of rods. Actual peak local values would be equal to or greater than the averages along the gauge lengths. Table 4 presents the cycle displacement, average strains at the extreme compression fiber of the cross section at spalling, $e_{spall}$, and the average strains in the core concrete (defined at the inside diameter of the spiral) when core crushing was apparent, $e_{core}$.

Initial spalling occurred over a wide range of strains (-0.0039 to -0.011), with a mean value of -0.066 and COV of 0.33 (Table 4). ATC 32 (1996) cites an extreme fiber concrete compression strain of -0.004 before spalling, which is approximately equal to the mean minus one standard deviation of the recorded data. Figure 13 presents distribution and cumulative probability curves for the data.

Spalling did not initiate at the interface between the column and footing, but instead at a distance $h_{spall}$ above the interface. The reported strain values correspond to the elevation $h_{spall}$. Figure 14a shows the relation between spalling strain and $h_{spall}$. The trend is for $e_{spall}$ to increase with decreasing $h_{spall}$. One explanation is that footing concrete may be confining column concrete close to the footing. It also is possible that spalling strain was influenced by moment gradient — for steep moment gradients in columns with small aspect ratios, sections with lower strain demand can confine adjacent sections with higher demand. As shown in Figure 14b, columns with higher moment gradients tended to have larger initial spalling strains. Other study parameters such as axial load, reinforcement ratio, and confinement ratio did not have significant influence on spalling strain.

Onset of core damage was defined as the point at which the inside diameter of the spiral reinforcement was fully visible. Strains corresponding to initial crushing of the core concrete are indicated in Table 4. Figure 15 shows the distribution and cumulative probability curves. For the range of the study parameters, there was no apparent effect of axial load, reinforcement ratio, or aspect ratio.

Figure 16 plots maximum spall height (highest location of spalled concrete) as a function of column aspect ratio and axial load ratio. Maximum spall height increases approximately linearly with aspect ratio and increases with axial load. Moehle et al (1996) proposed Equation (5) to define length of confined region.

$$L_c = D, \text{ but not less } \frac{P}{A_g f_c} L$$

The first term recognizes that kinematics requires inelastic response to spread over a length of column related to the diameter. The second term is based on the assumption that confinement is required over full height for pure compression, and a fraction of full
height for lower axial load. Equation (5) is plotted for three axial load ratios in Figure 18. It may be acceptable that Equation (5) specifies confinement over height less than the maximum spall height, as full confinement is not required at the extremity of the spalled region. Reduced confinement is adequate at that height, as demonstrated by the results for Column 1028T.

**Bar buckling, bar fracture, and loss of lateral-load-carrying capacity**

Degradation of column moment strength may adversely affect seismic response and may signal the onset of bridge column failure. For the columns tested, loss of strength resulted from bar buckling, spiral fracture, and longitudinal bar fracture, usually in this order. The onset of this type of damage led rapidly to significant loss of lateral-load strength. An arbitrary but convenient point to represent failure, then, is when lateral-load strength drops by more than 20 percent of the peak value.

Table 5 reports average tensile strain at the center of the extreme longitudinal bar and average compressive strain at the extreme fiber of the core at time of failure. (Column failure was not achieved for Column 828.) Both compressive and tensile strains were obtained using the external instrumentation attached to rods. Average tensile strains were obtained between rods located at 0.25\(D\) and 0.5\(D\) from the top of the foundation, where \(D\) is the column diameter. The measurement between 0 and 0.25\(D\) was not used because it primarily measured slip of the reinforcement from the foundation.

Previous research has shown that cyclic history has a stronger influence on bar buckling than on damage states such as initial spalling (Kunnath et al. 1997). This is partly because tensile fracture is influenced by prior buckling, which in turn is influenced by the maximum prior tensile strain in the bar. The displacement histories used in the experimental program described here were intended to be similar for all columns tested. Therefore, the results may not be generally applicable. Consequently, cumulative probability curves are not presented for these damage states.

Some procedures for evaluation and design (e.g., ATC 1996) define failure according to the compressive strain at which the hoop reinforcement ruptures. These procedures are based on models developed from pure compressive tests of confined concrete cross sections in which hoop rupture due to concrete dilation was a predominant failure mode (Mander et al. 1981). Under reversed cyclic loading it is more common for hoop rupture to be controlled by local strains induced as the longitudinal reinforcement buckles and bears against the hoops. New models for compressive strain limits, based on longitudinal reinforcement buckling, are needed.

**CONCLUSIONS**

Performance-based earthquake engineering requires procedures to translate engineering demand parameters to damage measures, as well as structural details that
An experimental study investigated the performance of ten spiral-confined, circular-cross-section columns. Primary test variables were column aspect ratio, longitudinal reinforcement ratio, spiral reinforcement ratio, axial load ratio, and length of the well-confined region. The columns were subjected to constant axial load and cyclic lateral loads. Within the limitations of the test program, the following conclusions were reached:

1. The progression of damage was similar for all columns: Concrete cracking, longitudinal reinforcement yielding, cover spalling, core crushing, longitudinal reinforcement buckling, spiral fracture, and (in most cases) longitudinal reinforcement fracture. In all cases, spiral fracture results from longitudinal bar buckling, and not solely from overall dilation of the core concrete.

2. Residual crack widths increased with increasing maximum measured longitudinal reinforcement strains. Residual crack width was not measurable if the strain was less than the yield strain. A fragility curve was presented to define the probability of exceeding a target residual crack width as a function of maximum prior steel strain.

3. Concrete spalling strains ranged from 0.0039 to 0.011 in compression. Lower values were associated with larger column aspect ratios. Axial load ratio, reinforcement ratio, and confinement ratio did not have a significant influence on spalling strain.

4. Height of spalling increased with column aspect ratio and axial load ratio.

5. Crushing of the concrete core was observed for compressive core concrete strains as low as 0.010 and as high as 0.029.

6. Column failure occurred rapidly following onset of longitudinal reinforcement fracture.

ACKNOWLEDGMENTS

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Table 1 - Specimen Properties

<table>
<thead>
<tr>
<th>Column</th>
<th>Series LR</th>
<th>Series AR</th>
<th>Series AL</th>
<th>Series SR</th>
<th>Length (mm)</th>
<th>Longitud. bars</th>
<th>Spiral Spacing (mm)</th>
<th>$\rho_s^{**}$</th>
<th>$\rho_v^{**}$</th>
<th>$L_c^{***}$ (mm)</th>
<th>Axial Load, $\gamma = P/A_d f_c$</th>
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<td>x</td>
<td>x</td>
<td>x</td>
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* An entry "x" indicates that the column was part of the test series indicated.
** Percent.
*** Spacing inside well-confined region/spacing outside well-confined region.
**** $L_c$ is the length of the well-confined region. n/a indicates full-length confinement.

Table 2 - Required and Provided Spiral Reinforcement Ratios

<table>
<thead>
<tr>
<th>Column</th>
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<th>Caltrans</th>
<th>Provided</th>
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Table 3 - Materials Properties

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*f_{cm}^*$ is concrete compressive strength. $f_{cm}^*$ and $f_{cm}^*$ are yield and ultimate strengths of longitudinal steel, respectively. $f_{chm}^*$ is yield strength of spiral steel. All MPa.

** $\varepsilon_{ch}$ and $\varepsilon_{u}$ are strains at onset of strain-hardening and ultimate strength, respectively.
### Table 4 - Concrete Damage Parameters

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<tr>
<th>Column</th>
<th>$\Delta_L$ (mm)</th>
<th>Initial Spalling</th>
<th>Initial Core Crushing</th>
<th>$L_{spall}$</th>
<th>$L_{spall}$/L</th>
<th>%L</th>
<th>$\varepsilon_{core}$</th>
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<td>97.5</td>
<td>254 559</td>
<td>-0.0043</td>
<td>889</td>
<td>22</td>
<td>-0.0175</td>
<td>1350 0.68</td>
</tr>
<tr>
<td>Mean</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>-0.0066 1.38</td>
</tr>
<tr>
<td>Coefficient of Variation</td>
<td>0.33</td>
<td>0.36</td>
<td>0.41</td>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>Minimum</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>-0.0039 0.68</td>
</tr>
</tbody>
</table>

$^1$\(\Delta_L\) = peak displacement for that cycle (mm); $^2$h_{spall} = height at which initial spalling was observed (mm); $^3$L_{spall} = length of spalled region (mm); $^4$\(\varepsilon_{spall}\) = compressive strain at circumference corresponding to $h_{spall}$; $^5$%L = $L_{spall}$/L; $^6$\(\varepsilon_{core}\) = compressive strain corresponding to $L_{spall}/2$; $^7$n/r = data were deemed not reliable.

### Table 5 - Data Measured at Bar Buckling and 20% Loss of Load

<table>
<thead>
<tr>
<th>Column</th>
<th>$h_{buckled}$</th>
<th>Bar buckling displacement (mm) /cycle</th>
<th>20% loss displacement (mm) /cycle</th>
<th>$\varepsilon_{comp}$</th>
<th>$\varepsilon_{ten}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>407</td>
<td>102</td>
<td>127 / 1</td>
<td>127 / 3</td>
<td>-0.0279</td>
<td>0.044</td>
</tr>
<tr>
<td>415</td>
<td>102</td>
<td>178 / 1</td>
<td>178 / 1</td>
<td>-0.0784</td>
<td>0.0496</td>
</tr>
<tr>
<td>430</td>
<td>76</td>
<td>178 / 1</td>
<td>178 / 2</td>
<td>-0.0531</td>
<td>0.0436</td>
</tr>
<tr>
<td>415P</td>
<td>102</td>
<td>127 / 2</td>
<td>178 / 1</td>
<td>-0.0476</td>
<td>0.0307</td>
</tr>
<tr>
<td>415S</td>
<td>102</td>
<td>127 / 2</td>
<td>127 / 3</td>
<td>-0.0559</td>
<td>0.0482</td>
</tr>
<tr>
<td>815</td>
<td>152</td>
<td>445 / 1</td>
<td>445 / 2</td>
<td>-0.0609</td>
<td>0.0512</td>
</tr>
<tr>
<td>1015</td>
<td>254</td>
<td>635 / 1</td>
<td>635 / 2</td>
<td>-0.0495</td>
<td>0.0491</td>
</tr>
<tr>
<td>328T</td>
<td>76</td>
<td>132 / 1</td>
<td>132 / 2</td>
<td>-0.0631</td>
<td>0.0442</td>
</tr>
<tr>
<td>1028T</td>
<td>305</td>
<td>889 / 2</td>
<td>889 / 3</td>
<td>-0.0447</td>
<td>0.0468</td>
</tr>
<tr>
<td>Mean</td>
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</tr>
<tr>
<td>Standard Deviation</td>
<td>0.014</td>
<td>0.006</td>
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</tr>
</tbody>
</table>

$^1$Bar buckling based on visual observations; $^2$h_{buckled} = mid-height of buckled bar estimated from measurements made during testing; $^3$Damage state corresponding to loss of lateral load calculated at peak displacements; $^4$Compression strains from external vertical displacement potentiometers, averaged over length 0D to 0.5D; $^5$Tensile strains from external vertical displacement potentiometers, averaged over length 0.5D to D.
Figure 1 – Performance-based earthquake engineering framework – damage to individual bridges determines the functionality of the bridge network

Figure 2 – Column geometry and reinforcement
Figure 3 – Test configuration and instrumentation

Figure 4 – Target displacement history
Figure 5 – Crack patterns for 76-mm cycle of Column 407

Figure 6 – Spalling at 76-mm cycle for Column 407

Figure 7 – Spalled regions for Columns 430 (left) and 1028T (right)
Figure 8 – Rebar buckling (left) and fracture (right) for Column 407

Figure 9 – Final damage states of Columns 407 (left) and 1015 (right)

Figure 10 – Force-drift relation, Column 415
Figure 11 – Residual crack width as function of maximum prior longitudinal reinforcement tension strain at crack location

Figure 12 – Cumulative probability relation for residual crack as function of maximum prior longitudinal reinforcement tension strain at crack location
Figure 13 – Distribution and cumulative distribution of compression strain at onset of spalling

Figure 14 – Dependence of initial spalling strain on height of spall and aspect ratio

Figure 15 – Distribution and cumulative probability relations for compression strain corresponding to core crushing
Figure 16 – Maximum spall height as function of aspect ratio and axial load ratio
Performance-Based Design of Confining Reinforcement: Research and Seismic Design Provisions

by S. Bae and O. Bayrak

Synopsis: In performance-based seismic design, evaluation of the deformation capacity of reinforced concrete columns is of paramount importance. The deformation capacity of a column can be expressed in several different ways: (1) curvature ductility, (2) displacement ductility, or (3) drift. Even though several performance-based confining reinforcement design procedures have been proposed, the relationship between different ductility factors is not clearly understood. The effect of concrete strength, longitudinal reinforcement ratio, volumetric ratio of confining reinforcement, shear span-to-depth ratio, and axial load level on the relationship between different ductility factors was studied. Finally, the confinement reinforcement design requirements of current design codes and recently proposed performance-based design methods were compared and critically examined.

Keywords: confinement; deformation capacity; drift; ductility; performance-based design
Recently, several performance-based confinement reinforcement design procedures have been proposed by Wehbe et al. (1995), Sheikh & Khoury (1997), and Saatcioglu & Razvi (2002). These procedures were developed to relate the target deformation capacity to the amount of confinement reinforcement. Curvature ductility, displacement ductility, or drift was employed by Sheikh & Khoury (1997), Wehbe et al. (1995) and Saatcioglu & Razvi (2002), respectively, in their design equations. In the absence of expressions relating various ductility parameters, the comparative evaluation of the aforementioned performance-based design approaches and the relevant code expressions is very difficult. Hence, the relationship between different ductility and deformation capacities was studied in this research. A software that is capable of modeling confinement of core concrete, buckling of longitudinal bars, rebar slip, shear deformations and \( P-\Delta \) effect was developed during the course of this research. The displacement ductility and drift capacity of columns tested by various researchers were predicted using the aforementioned software. Upon the verification of constitutive models and analytical procedures employed in the software, the effects of different variables on the relationships between ductility parameters were evaluated. The effects of concrete strength, longitudinal reinforcement ratio, volumetric ratio of confining reinforcement, shear span-to-depth ratio, and axial load level on the relationship between various ductility parameters were evaluated. In addition, the attainable deformation capacities of columns designed through the use of various codes and performance-based design proposals were also investigated.

**Review of performance-based design proposals**

Wehbe, Saiidi, Sanders, and Douglas (1995) – Wehbe et al. (1995) conducted tests on rectangular columns. The columns tested in the experimental program contained 46% to 60% of the minimum lateral reinforcement required by the AASHTO provisions. The applied axial loads were 10% and 20% of \( A_g f'_c \). The specimens were tested under constant axial loads and reversed cyclic lateral loads. The column specimens exhibited displacement ductilities, \( \mu_\Delta \), ranging between 5 and 7. Based on analytical and experimental results, the following equation was proposed to relate the amount of confining reinforcement to attainable displacement ductility.
\[ \frac{A_{sh}}{s_t h_t} = 0.1 \mu_A \left[ \frac{f'_{ce}}{f'_{ye}} \left( 0.5 + 1.25 - \frac{P}{f'_{ce} A_g} \right) + 0.13 \left( \rho_l - \frac{f_y}{f_{x,n}} - 0.01 \right) \right] \]  

\[ (1) \]

where,

- \( s_t \) = spacing of transverse reinforcement along the axis of the member
- \( h_t \) = cross-sectional dimension of column core measured center-to-center of confining reinforcement
- \( f'_{ce} \) = expected concrete strength
- \( f_{ye} \) = expected yield strength of transverse reinforcement
- \( f_y \) = expected yield strength of longitudinal reinforcement
- \( \rho_l \) = longitudinal reinforcement ratio
- \( f_{c,n} \) = 27.6 MPa (or 4 ksi)
- \( f_{s,n} \) = 414 MPa (or 60 ksi)

For the minimum amount of lateral steel in areas of high seismic risk, the use of \( \mu_A = 10 \) was recommended. The researchers also recommended that a displacement ductility value less than 10 could be used for moderate levels of ductility.

Sheikh and Khoury (1997) – Sheikh & Khoury (1997) proposed a performance-based confining reinforcement design procedure. The researchers employed curvature ductility, \( \mu_D \), as the performance criterion in their design equations. The seismic performance of a column was classified to be in one of the following three categories: (1) highly ductile columns (\( \mu_D \geq 16 \)), (2) moderately ductile columns (16 > \( \mu_D \geq 8 \)), and (3) columns displaying low levels of ductility (\( \mu_D < 8 \)). The following equation was proposed and calibrated to relate the amount of lateral reinforcement in the potential plastic hinge regions of tied columns to axial load level and to curvature ductility.

\[ A_{sh} = \alpha \cdot \left\{ 1 + 13 \left( \frac{P}{P_0} \right)^5 \right\} \frac{(\mu_D)^{0.15}}{29} A_{sh,ACI} \]  

\[ (2) \]

The constant \( \alpha \) was used to take the arrangement of lateral and longitudinal steel into account. According to Sheikh & Khoury (1997), \( \alpha \) may be equal to one for tightly knit lateral reinforcement configurations in which effective lateral support to longitudinal bars is provided. In the presence of less efficient lateral reinforcement configurations and higher axial load levels, greater \( \alpha \) values are recommended. They concluded that the ACI 318-02 requirements (2002) for confining reinforcement may not be sufficient even in columns with efficient lateral reinforcement configurations to meet the high curvature ductility demands under moderate-to-high levels of axial loads. They recommended that at low axial load levels (\( P \leq 0.4P_0 \)) the code requirements may be relaxed. Bayrak & Sheikh (1998) modified the above equation for high-strength concrete columns with concrete strengths ranging between 55 MPa and 115 MPa:

\[ A_{sh} = \alpha \cdot \left\{ 1 + 13 \left( \frac{P}{P_0} \right)^5 \right\} \frac{(\mu_D)^{0.82}}{8.12} A_{sh,ACI} \]  

\[ (3) \]

Saatcioglu and Razvi (2002) – Saatcioglu & Razvi (2002) suggested that there was a direct correlation between lateral drift and concrete confinement based on their
analytical and experimental work. They concluded that the shear span to depth ratio ($L/h$) did not show a pronounced effect on drift capacity when the $P-\Delta$ effect was considered and that the amount of longitudinal reinforcement had a minor influence. In cases where the $P-\Delta$ effect was considered, the drift capacity increased by approximately 75%, when the $L/h$ ratio was changed from 2.5 to 5.0. Based on these findings, they derived the following relation for a $L/h$ ratio of 2.5 and a longitudinal reinforcement ratio of 2%.

$$\rho_c = 14 \frac{f_c'}{f_y} \left[ \frac{A_s}{A_c} - 1 \right] \left[ \frac{1}{\sqrt{k_2}} \frac{P}{P_0} \delta \right]$$

where,

$$k_2 = 0.15 \sqrt{\frac{b_c}{s}}$$

$$\frac{P}{P_0} \geq 0.2 \text{ and } \frac{A_s}{A_c} - 1 \geq 0.3$$

$b_c$ = core dimension, center-to-center of perimeter tie
$s$ = center-to-center spacing of transverse reinforcement along column height
$s_l$ = center-to-center spacing of longitudinal reinforcement, laterally supported by corner of hoop or hook of crosstie

A lateral drift ratio of 2.5% was recommended to ensure ductile performance.

**Definitions of ductility parameters**

Since the behavior of reinforced concrete sections and members is not elastic-perfectly plastic, several definitions for ductility are available in the literature. In this study, the ductility parameters are defined using idealized backbone curves shown in Figure 1. In defining the yield curvature (or displacement), a straight line joining the origin and a point on the ascending branch (where $M = 0.75M_{\text{max}}$ or $V = 0.75V_{\text{max}}$) is used. This line passes through the moment-curvature (or the load-displacement) curve at 75% of maximum moment (or load) and reaches the maximum moment (or load) to define the idealized yield curvature $\phi_1$ (or yield displacement $\Delta_1$), as shown in Figure 1. Failure of the column is conventionally defined when the post-peak curvature $\phi_2$ (or postpeak displacement $\Delta_2$) reaches a point at which the remaining column strength has dropped to 80% of the maximum moment (or load). The curvature and displacement ductilities are defined as follows:

$$\mu_\phi = \frac{\phi_2}{\phi_1} \text{; } \mu_\Delta = \frac{\Delta_2}{\Delta_1}$$

The drift capacity can be defined as the ratio of the maximum useful displacement capacity to the column height ($L$).
Relationship between various ductility factors

It was discussed earlier that different ductility factors were adopted by various researchers in the performance-based design of confining reinforcement. However, it is not clear how different ductility factors are related with each other under different conditions. It is likely that the level of axial load, concrete strength, amount, grade, spacing of longitudinal and lateral reinforcements, and shear span-to-depth ratio have different influences on various deformation parameters. The relationship between the curvature and displacement ductilities was previously investigated by Park & Pauley (1975). It should be noted that the $P$-$\Delta$ effect, rebar slip and shear deformations were neglected in this equation.

$$\delta = \frac{\Delta_2}{L}$$

Equation (9) indicates that the curvature and displacement ductilities have a linear relationship. These relationships are plotted in Figure 2 for various shear span-to-depth ratios ($L/h$) and equivalent plastic hinge lengths ($L_p$). Figure 2 illustrates that the displacement ductility increases as the shear span-to-depth ratio ($L/h$) decreases and the plastic hinge length ($L_p$) increases. It is interesting observe that despite the axial load has a very important role in the ductility, strength, stiffness, and energy dissipation characteristics of columns.

Figure 3 depicts a typical cantilever column subjected to an axial load, $P$, and a lateral load, $V$. The sectional behavior of this cantilever column under the given axial load is also shown in Figure 3. After reaching the peak lateral load ($V_{max}$), the loss of lateral load capacity can be attributed to two factors: the loss of moment capacity and the $P$-$\Delta$ effect (Figure 3). The loss of lateral load due to the $P$-$\Delta$ effect indicates that even though a column may have large curvature ductility, the maximum attainable lateral displacement ($\Delta_2$) of the column may be controlled by the $P$-$\Delta$ effects under high axial load.

ANALYTICAL PROGRAM

Displacement components that contribute to the tip displacement of a reinforced concrete column can be assumed to be: (1) flexural deformations along the column length, (2) shear deformations along the column length, and (3) fixed end rotation resulting from the slip of the longitudinal reinforcement out of the joint. The degradation of flexural strength becomes severe under high axial compression loads combined with high lateral drifts and this produces secondary moments, known as the $P$-$\Delta$ effect. This secondary moment may consume a significant portion of the total flexural resistance. The inelastic buckling behavior of reinforcing bars is also important to predict the accurate degradation of column behavior at the sectional and member levels.

A software that is capable of modeling confinement of core concrete, buckling of longitudinal bars, rebar slip, shear deformations and $P$-$\Delta$ effect was developed during the
course of this research. The analytical method was verified with the experimental results. The displacement ductility and drift capacity of columns tested by various researchers were predicted using the software developed during the course of this research. Figure 4 indicates that the analytical method provides reasonably good estimates for column deformation capacities. Detailed descriptions of the analytical program, the inelastic buckling behavior of reinforcing bars, and their verifications are reported elsewhere (Bae, Bayrak & Williamson (2004) and Bae, Mieses & Bayrak(2004)).

**OBSERVATIONS**

In order to examine the relationships between various ductility factors, the behavior of a typical column section (300 mm x 300 mm) was studied. The experimentally verified analytical procedure described earlier was used to conduct parametric studies. In all the analyses, longitudinal reinforcement was uniformly distributed along four faces of column sections and the center-to-center distance of extreme reinforcement layers was 240 mm, i.e. ($\gamma = 0.8$). The concrete strength was 40 MPa and the yield and ultimate strengths of the reinforcement were 415 and 620 MPa, respectively. The stirrup spacing was kept equal to six times the diameter of the longitudinal bars to avoid early bar buckling. It was assumed that enough anchorage length was provided to prevent excessive rebar slip. The height of the column was 1500 mm, resulting in a shear span-to-depth ratio of 5. The full plastic hinge length was assumed to be equal to the column depth ($h$).

**Modeling the behavior of confined concrete**

The influence of modeling the behavior of confined concrete on the relationship between various ductility parameters is illustrated in Figure 5(a). In all the analyses summarized in Figure 5(a) the axial load level was kept constant at 0.3$P_0$. As can be observed in this figure all concrete models provide similar results within the practical deformation range. The confinement model proposed by Razvi & Saatcioglu (1999) is used in the subsequent parametric studies as this model provided somewhat more accurate estimations for the experimental ductility parameters considered in the model verification stage.

**Level of axial load**

The effect of the level of axial load (0.1$P_0$, 0.3$P_0$, and 0.5$P_0$) on the relationship between various ductility factors was investigated, as shown in Figure 5(b). At this stage the $P$-$\Delta$ effect was considered in some analyses and neglected in others. When the $P$-$\Delta$ effect was not considered, the relationships were linear, as suggested by Park & Pauley (1975). As can be observed in Figure 5(b) the inclusion of the $P$-$\Delta$ effect changed the nature of the relationships between various ductility factors. With increasing axial load, the $P$-$\Delta$ effect became more pronounced and the attainable displacement ductilities and drift capacities reduced considerably. For high axial load levels, drastic increases in curvature ductility resulted in considerably smaller increases in displacement ductility and drift capacity. For high axial loads (~0.5$P_0$), the $P$-$\Delta$ effect had an important role in the loss of the lateral resistance. As the $P$-$\Delta$ effect is a function of the lateral displacement and the level of the axial load, improving the sectional behavior at high levels of axial load is not as effective as it is at the low axial load levels.
Concrete strength
The effect of concrete strength was studied using an axial load of 0.3$P_0$ and two different concrete strengths ($f'_c = 40$ and 80 MPa). Figure 5(c) illustrates that an increase in the concrete strength resulted in reduced displacement ductility and drift capacities for a given curvature ductility. To achieve the same level of displacement ductility or drift capacity in a high strength concrete column, the use of a larger amount of confining reinforcement was required.

Shear span-to-depth ratio
The effect of shear span-to-depth ratio ($L/h$) was studied by changing both the length of a column ($L$) and the depth of a column ($h$). The different depths of columns were modeled by keeping the same longitudinal reinforcement ratio and the same ratio of the center-to-center distance of extreme reinforcement layers to the column depth ($\gamma$). In this part of the parametric study the axial load was kept constant at 0.3$P_0$. The results are shown in Figures 5(d) and (e). Figures 5(d) and 5(e) indicate that both approaches resulted in similar trends. As the shear span-to-depth ratio decreased, the displacement ductility and drift capacity increased for a given curvature ductility. To achieve the same level of displacement ductility or lateral drift capacity, different levels of sectional performance are needed for various shear span-to-depth ratios.

Longitudinal reinforcement
The amount of longitudinal reinforcement was varied ($\rho_l = 1$, 2 and 4%) to investigate its influence on the relationships between various ductility parameters. For the relationships shown in Figure 5(f), the axial load was kept constant at 0.3$P_0$. For a given curvature ductility, an increase in the amount of longitudinal reinforcement resulted in a decrease of displacement ductility, but in an increase of drift capacity. As the amount of longitudinal reinforcement increased, the lateral load carrying capacity, the yield displacement and the ultimate displacement capacity increased. However, the increase in the yield displacement was more pronounced than the increase in the ultimate displacement capacity. This explains the decrease in displacement ductility and the increase in drift capacity for a given curvature ductility as depicted in Figure 5(f).

Equivalent plastic hinge length
A review of technical literature revealed the fact that several researchers suggested different equivalent plastic lengths (Table 1). The suggested plastic hinge lengths varied from 0.5$h$ to $h$. Sakai & Sheikh (1989) stated that the plastic hinge length could be affected by the amount of transverse reinforcement, axial load level, and the aspect ratio of a column section. The influence of plastic hinge length ($L_p = 0.5h$ and $1.0h$) on the relationships between various ductility parameters was investigated (Figure 5(g)). A reduction in the plastic hinge length $h$ to 0.5$h$ did not change the relationships between various ductility parameters significantly. Therefore, it can be concluded that the equivalent plastic hinge length is very important in predicting the individual column behavior, especially for columns displaying limited ductility, but it does not affect the relationships of various ductility factors considered in this study.
DESIGN OF CONFINING REINFORCEMENT: CODES AND PROPOSALS

It was previously shown that different performance-based design methods used different ductility parameters as their performance criterion and the relationships of ductility parameters depended on many factors, such as the level of axial load, shear span-to-depth ratio, amount of longitudinal reinforcement, ratio of the center-to-center distance of extreme flexural reinforcement layers to the column depth, and concrete strength.

Current design code provisions for confining reinforcement design as well as performance-based design methods are critically examined in this section. The relevant provisions of ACI 318-02 (Equation 10), ATC-32 (Equation 11), and NZS 3101:1995(Equation 12) for tied columns are considered in this section:

**ACI 318-02:**
\[
\rho_c = 0.3 \frac{f'_c}{f_{yh}} \left[ \frac{A_g}{A_c} - 1 \right] > 0.09 \frac{f'_c}{f_{yh}}
\]

**ATC-32:**
\[
\rho_c = 0.12 \frac{f'_{cc}}{f_{yc}} \left( 0.5 + 1.25 \frac{P_e}{A_g f'_{ce}} \right) + 0.13(\rho_f - 0.01)
\]

**NZS 3101:1995:**
\[
A_{sh} = \frac{(1.3 - \rho_m) s_h h'' A_g}{3.3} \frac{f'_c}{A_c f_{yt} \phi f'_{ce}} N A_g - 0.006 s_h h''
\]

where \(A_g/A_c\) shall not be taken less than 1.2, \(\rho_m\) shall not be taken greater than 0.4 and \(f_{yc}\) shall not be taken larger than 800 MPa.

A comparison of confinement reinforcement provisions of various codes and performance-based design procedures is shown in Figure 6. For the comparative evaluation of these provisions and procedures, concrete strength was taken as 40 MPa and the yield strength of both longitudinal and lateral reinforcement was taken to be equal to 415 MPa. The ratio of the center-to-center distance of extreme flexural reinforcement layers to the column depth, \(\gamma\), was taken as 0.8. For performance-based design methods, the ductility parameters suggested for ductile performance were used: a curvature ductility factor of 16, a displacement ductility factor of 10, and a drift ratio of 2.5% were used for the procedures proposed by Sheikh & Khoury (1997), Wehbe et al. (1995) and Saatcioglu & Razvi (2002), respectively. Figure 6 shows that unlike the ACI code, other procedures consider the effect of axial load on the required amount of confinement reinforcement. For low axial load levels, the ACI code requires a greater amount of confining reinforcement than those required by the other codes and design procedures. However, for higher levels of axial load, the amount of confining reinforcement required by all other procedures increase to levels that are higher than that required by the ACI code. However, it should be noted that up to an axial load of 0.52\(P_0\), which is the maximum axial load permitted by the ACI code, the use of the performance-based design procedures proposed by Sheikh & Khoury (1997), Wehbe et al. (1995) and Saatcioglu &
Razvi (2002) result in small differences in the required amount of confinement reinforcement (0% to 20%) compared with the ACI code. However ATC-32 and NZS 3101:1995 codes require the use of 50% to 100% more confinement reinforcement than the amount required by ACI 318-02.

In order to establish a broader basis of comparison, a parametric study was conducted. A total of 16 cases involving various sectional dimensions, $\gamma$ values, column lengths, shear span-to-depth ratios, axial load levels and concrete strengths were considered (Table 2). The 16 cases, summarized in Table 2, were used in conjunction with two different longitudinal reinforcement ratios, resulting in the 32 cases summarized in Tables 3a and 3b. Hence, 192 (32 cases × 6 design procedures) column designs were performed using ACI 318-02, ATC-32, NZS 3101:1995 code provisions and performance-based design procedures proposed by Sheikh & Khoury (1997), Wehbe et al. (1995) and Saatcioglu & Razvi (2002). The performance of the columns was evaluated using the software developed during the course of this research. The drift capacities for the 192 columns were evaluated and results are listed in Table 3.

The seismic detailing and design provisions of the current design codes are aimed at achieving a story drift ratio of 2.0 to 2.5% for most concrete framed structures. Table 3(a) shows the drift capacities of the columns for a longitudinal reinforcement of 1%. The results summarized in Table 3(a) indicate that most problematic conditions involve normal or high strength concrete columns with small section sizes (where $\gamma = 0.8$) subjected to high axial loads where $P = 0.5P_0$ (Cases 3, 4, 11 and 12). Slender, high-strength concrete columns ($L/h = 6$) subjected to a moderate axial load level of $P = 0.3P_0$ (Case 10) constitute another problem area. Bearing in mind that high strength concrete is typically used to reduce the section size (and consequently decrease the $\gamma$ value while meeting the clear cover requirements) of columns, the importance of these findings can be appreciated. Columns designed in accordance with the Chapter 21 requirements of the ACI code could not achieve a drift level of 2% for all of the aforementioned cases (Cases 3, 4, 10, 11 and 12). It is important to note that the longitudinal reinforcement ratio in all of these cases was 1%. Interestingly, all other code provisions and performance-based design procedures failed to result in design that will produce “acceptable” drift capacities for these cases. For Case 10, the volumetric ratio of the confinement reinforcement ranged from 1.74% (ACI 318-02) to 2.25% (NZS 3101:1995). The drift capacity of the column studied in Case 10 practically remained unchanged ~1.5-1.6%. Hence small increases in the volumetric ratio of confining reinforcement (up to 29%) did not improve the drift capacity. Similar observations can be made for Case 12. However, for Case 12 NZS 3101:1995 provisions required the use of 2.5 times as much transverse steel as the that required by ACI 318-02. This drastic increase in the amount of confining reinforcement improved the drift capacity from 1.2% to 2.3%. Once again, the use of all other procedures did not improve the drift capacity of the column notably. Neither the performance-based design procedures, nor the code provisions studied here, consider the influence of column slenderness on the attainable drift capacity. Hence, when the behavior is dominated with slenderness effects, it is very difficult to improve the drift capacity of the columns. Conversely, in such cases, a small improvement will require the use of impractically large amounts of confining reinforcement.
When the longitudinal reinforcement ratio was 2% (Table 3(b)) the achievable drift levels improved. However, slender columns with $\gamma = 0.8$ under high axial load levels (Cases 4 and 12) proved to be problematic (i.e., drift capacity is smaller than 2%). It is interesting to recognize that the designs obtained through the use of ATC-32 and NZS 3101:1995 requirements provide satisfactory deformation capacities. This is a direct result of the use of very high volumetric ratios (up to 4.4%) of confining reinforcement. On the other hand, the practicality of using such great amounts of confining reinforcement needs to be investigated. It is equally interesting to note that performance-based design procedures proposed by Sheikh & Khoury (1997), Wehbe et al. (1995) and Saatcioglu & Razvi (2002) do not improve the drift capacities considerably in problematic cases (e.g. Cases 4 and 12).

NZS 3101:1995 provisions, ATC-32 provisions and the performance-based design procedure proposed by Sheikh & Khoury (1997) aim at relating the amount of confinement reinforcement to sectional ductility. Hence, in all the cases studied above the target sectional performances were achieved. However, this did not ensure satisfactory ductile performance at the member level. The procedure proposed by Wehbe et al. (1995) was used to produce a displacement ductility of 10. As displacement ductility is geometry dependent, in most cases the target displacement ductility of 10 was not achieved by the columns designed using this procedure. The procedure proposed by Saatcioglu & Razvi (2002) was used to produce 2.5% story drift. However, in 11 of the 32 columns, designed using the procedure suggested by Saatcioglu & Razvi (2002), the target drift level was not achieved.

CONCLUSIONS

The following conclusions can be drawn from the research reported here:

- The relationships between various ductility parameters (curvature ductility, displacement ductility and drift capacity) are affected by the level of axial load. As the axial load increases, the loss of lateral load carrying capacity becomes higher due to the $P-\Delta$ effect. Because of this, the attainable drift capacity or displacement ductility can be limited at high axial load levels regardless of the amount of confinement.
- An increase in the concrete strength results in reduced displacement ductility and drift capacities for a given curvature ductility. To achieve the same level of displacement ductility or drift capacity in a high strength concrete column, the use of a larger amount of confining reinforcement is required.
- As the shear span-to-depth ratio decreases, the displacement ductility and drift capacity increases for a given curvature ductility. To achieve the same level of displacement ductility or lateral drift capacity, different levels of sectional performance are needed for various shear span-to-depth ratios.
- It is extremely difficult, if not impossible, to generalize the relationships between various ductility parameters. The level of axial load, shear span-to-depth ratio, concrete strength, amount of longitudinal and transverse reinforcement influence the deformation capacity of a column.
REFERENCES

American Concrete Institute (2002), “Building Code Requirements for Structural Concrete (ACI 318-02) and Commentary (ACI 318R-02),” Farmington Hills, Michigan.


Table 1 – Equivalent Plastic Hinge Length

<table>
<thead>
<tr>
<th>Researcher(s)</th>
<th>Equivalent plastic hinge length ($L_p$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sawyer (1964)</td>
<td>0.25D + 0.075L</td>
</tr>
<tr>
<td>Corley (1966)</td>
<td>0.5D + 0.2\sqrt{D}\frac{L}{D}</td>
</tr>
<tr>
<td>Priestley &amp; Park (1987)</td>
<td>0.08L + 6d, ($\approx 0.5h$)</td>
</tr>
<tr>
<td>Priestley, Seible, &amp; Calvi (1996)</td>
<td>0.08L + 0.15f,d,</td>
</tr>
<tr>
<td>Sheikh &amp; Khoury (1993)</td>
<td>0.1h</td>
</tr>
<tr>
<td>Bayrak &amp; Sheikh (1998)</td>
<td>1.0h</td>
</tr>
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</table>
### Table 2 - Deformation Capacities used for Designing of a Column ($p_1 = 1$ and $2\%$)

<table>
<thead>
<tr>
<th>Concrete Strength</th>
<th>Section Size (mm x mm)</th>
<th>Axial Load</th>
<th>Aspect Ratio</th>
<th>Comment</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>40 MPa</strong></td>
<td>300 x 300 ($\gamma = 0.8$)</td>
<td>0.3$P_o$</td>
<td>3</td>
<td>Case 1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.5$P_o$</td>
<td>6</td>
<td>Case 2</td>
</tr>
<tr>
<td></td>
<td>500 x 500 ($\gamma = 0.9$)</td>
<td>0.3$P_o$</td>
<td>3</td>
<td>Case 3</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.5$P_o$</td>
<td>6</td>
<td>Case 4</td>
</tr>
<tr>
<td><strong>80 MPa</strong></td>
<td>300 x 300 ($\gamma = 0.8$)</td>
<td>0.3$P_o$</td>
<td>3</td>
<td>Case 9</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.5$P_o$</td>
<td>6</td>
<td>Case 10</td>
</tr>
<tr>
<td></td>
<td>500 x 500 ($\gamma = 0.9$)</td>
<td>0.3$P_o$</td>
<td>3</td>
<td>Case 13</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.5$P_o$</td>
<td>6</td>
<td>Case 14</td>
</tr>
</tbody>
</table>
### Table 3(a) – Required Amount of Confinement and Predicted Drift ($\rho_{f} = 1\%$)

<table>
<thead>
<tr>
<th></th>
<th>ACI 318-02</th>
<th>ATC-32</th>
<th>NZS 3101:1995</th>
<th>Sheikh &amp; Khoury</th>
<th>Wehbe et al.</th>
<th>Saatcioglu</th>
</tr>
</thead>
<tbody>
<tr>
<td>CASE 1</td>
<td>0.87</td>
<td>3.5</td>
<td>0.99</td>
<td>3.8</td>
<td>0.87</td>
<td>3.5</td>
</tr>
<tr>
<td>CASE 2</td>
<td>0.87</td>
<td>2.4</td>
<td>0.99</td>
<td>2.5</td>
<td>0.87</td>
<td>2.4</td>
</tr>
<tr>
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<td>1.26</td>
<td>2.1</td>
<td>1.79</td>
<td>3.2</td>
</tr>
<tr>
<td>CASE 4</td>
<td>0.87</td>
<td>1.5</td>
<td>1.26</td>
<td>1.6</td>
<td>1.79</td>
<td>1.9</td>
</tr>
<tr>
<td>CASE 5</td>
<td>0.87</td>
<td>4.7</td>
<td>0.99</td>
<td>4.4</td>
<td>0.79</td>
<td>4.9</td>
</tr>
<tr>
<td>CASE 6</td>
<td>0.87</td>
<td>3.2</td>
<td>0.99</td>
<td>3.3</td>
<td>0.79</td>
<td>3.1</td>
</tr>
<tr>
<td>CASE 7</td>
<td>0.87</td>
<td>3.6</td>
<td>1.26</td>
<td>4.2</td>
<td>1.77</td>
<td>4.6</td>
</tr>
<tr>
<td>CASE 8</td>
<td>0.87</td>
<td>2.5</td>
<td>1.26</td>
<td>2.7</td>
<td>1.77</td>
<td>2.8</td>
</tr>
<tr>
<td>CASE 9</td>
<td>1.74</td>
<td>2.5</td>
<td>1.93</td>
<td>2.7</td>
<td>2.25</td>
<td>3.0</td>
</tr>
<tr>
<td>CASE 10</td>
<td>1.74</td>
<td>1.6</td>
<td>1.93</td>
<td>1.6</td>
<td>2.25</td>
<td>1.6</td>
</tr>
<tr>
<td>CASE 11</td>
<td>1.74</td>
<td>1.9</td>
<td>2.45</td>
<td>2.9</td>
<td>4.32</td>
<td>4.0</td>
</tr>
<tr>
<td>CASE 12</td>
<td>1.74</td>
<td>1.2</td>
<td>2.45</td>
<td>1.5</td>
<td>4.32</td>
<td>2.3</td>
</tr>
<tr>
<td>CASE 13</td>
<td>1.74</td>
<td>4.5</td>
<td>1.93</td>
<td>4.4</td>
<td>2.23</td>
<td>4.4</td>
</tr>
<tr>
<td>CASE 14</td>
<td>1.74</td>
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<td>1.93</td>
<td>2.9</td>
<td>2.23</td>
<td>3.0</td>
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<td>CASE 15</td>
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<td>4.0</td>
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<tr>
<td>CASE 16</td>
<td>1.74</td>
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<td>2.45</td>
<td>2.8</td>
<td>4.25</td>
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</tr>
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</table>

Table 3(b) — Required Amount of Confinement and Predicted Drift ($\rho_f = 2\%$)

<table>
<thead>
<tr>
<th></th>
<th>ACI 318-02</th>
<th>ATC-32</th>
<th>NZS 3101:1995</th>
<th>Sheikh &amp; Khoury</th>
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<th>Saatcioglu</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\rho_s,\text{req} (%)$</td>
<td>$\delta,\text{achieved} (%)$</td>
<td>$\rho_s,\text{req} (%)$</td>
<td>$\delta,\text{achieved} (%)$</td>
<td>$\rho_s,\text{req} (%)$</td>
<td>$\delta,\text{achieved} (%)$</td>
</tr>
<tr>
<td>CASE 1</td>
<td>0.87</td>
<td>3.7</td>
<td>1.03</td>
<td>4.2</td>
<td>0.78</td>
<td>3.4</td>
</tr>
<tr>
<td>CASE 2</td>
<td>0.87</td>
<td>2.7</td>
<td>1.03</td>
<td>2.8</td>
<td>0.78</td>
<td>2.6</td>
</tr>
<tr>
<td>CASE 3</td>
<td>0.87</td>
<td>1.9</td>
<td>1.33</td>
<td>2.4</td>
<td>1.77</td>
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</tr>
<tr>
<td>CASE 4</td>
<td>0.87</td>
<td>1.7</td>
<td>1.33</td>
<td>1.9</td>
<td>1.77</td>
<td>2.2</td>
</tr>
<tr>
<td>CASE 5</td>
<td>0.87</td>
<td>5.4</td>
<td>1.03</td>
<td>5.3</td>
<td>0.78</td>
<td>5.6</td>
</tr>
<tr>
<td>CASE 6</td>
<td>0.87</td>
<td>3.6</td>
<td>1.03</td>
<td>3.8</td>
<td>0.78</td>
<td>3.5</td>
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<tr>
<td>CASE 7</td>
<td>0.87</td>
<td>3.3</td>
<td>1.33</td>
<td>4.8</td>
<td>1.74</td>
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<td>CASE 8</td>
<td>0.87</td>
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<td>CASE 9</td>
<td>1.74</td>
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<td>1.97</td>
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<td>4.1</td>
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<td>1.97</td>
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<td>2.51</td>
<td>3.1</td>
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<td>1.74</td>
<td>1.7</td>
<td>2.51</td>
<td>2.1</td>
<td>4.37</td>
<td>2.9</td>
</tr>
<tr>
<td>CASE 13</td>
<td>1.74</td>
<td>4.8</td>
<td>1.97</td>
<td>4.6</td>
<td>2.22</td>
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</tr>
<tr>
<td>CASE 14</td>
<td>1.74</td>
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<td>1.97</td>
<td>3.5</td>
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<tr>
<td>CASE 15</td>
<td>1.74</td>
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<td>2.51</td>
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<tr>
<td>CASE 16</td>
<td>1.74</td>
<td>2.8</td>
<td>2.51</td>
<td>3.1</td>
<td>4.22</td>
<td>3.3</td>
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</tbody>
</table>
Figure 1 — Definitions of ductility parameters

Figure 2 — Relationship between curvature and displacement ductilities (Park and Pauley (1975))
Figure 3 — Relationship of moment and lateral load capacities

Figure 4 — Model verification

(a) Displacement Ductility ($\mu_\Delta$)  
(b) Drift (%)
Figure 5 — Relationship between various ductility parameters

(a) Concrete Model

P-Δ effect was not considered.

(b) Axial Load

(c) Concrete Strength
Figure 5 (cont.) — Relationship between various ductility parameters
Figure 6 — Comparison of confinement requirements
Research Activities on Confined Concrete in Japan

by F. Watanabe

Synopsis: A key of seismic design of ductile frame is to provide the adequate flexural ductility to potential plastic hinge regions. This is realized by limiting the amount of tension reinforcement index, providing transverse reinforcement and others. For columns, the application of transverse reinforcement to potential plastic hinge region is essential, that is, the compressive ductility of concrete is improved and results in larger flexural ductility. In the 1980s, a new RC project was carried out as a Japanese National Project to establish the design and construction guidelines for high-rise buildings up to 200 meters high. For columns at the lower part of high-rise buildings, the use of high-strength concrete (HSC) is required. However, HSC fails in brittle manner and results in small flexural ductility of potential plastic hinges. Therefore the new RC project gave an opportunity to re-recognize the importance of lateral confinement to concrete. This paper presents the recent research works on confined concrete in Japan, mainly for HSC. Some experimental works and idealizations of stress-strain curve of confined concrete are introduced. Maximum compressive strength covered in this paper is 176 MPa.

Keywords: concrete; confinement; high strength; idealization; stress-strain
Watanabe

Fumio Watanabe. He is a professor of structural engineering at the Graduate School of Engineering, Kyoto University Japan. His research interests include ductility improvement of concrete by lateral confinement, strength and ductility design of concrete members for combined bending and shear, seismic design of concrete building structures and seismic strengthening of existing buildings. He serves on several committees at AIJ (Architectural Institute of Japan), JCI (Japan Concrete Institute) and fib (International Federation for Structural Concrete) and others.

INTRODUCTION

A key of seismic design of reinforced concrete ductile frames is the assurance of flexural ductility at column potential plastic hinge region. Therefore the application of transverse reinforcement to potential plastic hinge region is essential, that is, the compressive ductility of concrete is improved and results in larger flexural ductility.

During the Tokachi-oki earthquake in 1968, several reinforced concrete buildings suffered serious structural damage to columns. Main damage was the brittle shear failure of short columns due to poor transverse reinforcement detailing. Therefore the main research target was placed on the prevention of shear failure in Japan. On the other hand, the improvement of flexural ductility of column was also discussed by several researchers for the future development of ductility design method of reinforced concrete buildings. Then several researchers conducted the research on confined concrete. However the output from these researches was not directly reflected to 1981 revision of the Building Standard Law, where the checking procedure for ultimate lateral strength based on structural ductility was adopted.

In the 1980s, New RC Project was carried out as a Japanese National Project to establish the design and construction guidelines for high-rise buildings up to 200 meters high. Columns at the lower stories of high-rise building are constructed with HSC due to high axial load. However HSC generally shows brittle failure in compression and results in small flexural ductility of column potential plastic hinges. That is, the ductility improvement is inevitable for the effective utilization of HSC. Therefore the ductility improvement of HSC by lateral confinement was taken up as one of the research programs in the project.

In 1993, an idealization of stress strain curve of confined HSC was proposed as a project output. The results of New RC Project and the succeeding progress of structural analyses and construction technology have made the realization of reinforced concrete high-rise buildings more possible. However covered range of the compressive strength of concrete of New RC guidelines are less than 60 MPa. The requirement for the use of HSC (more than 100 MPa) arose for the construction of reinforced concrete skyscrapers (150 meters high or more). Recently the construction of high-rise building in Japan is very active using HSC more than 100 MPa in compressive strength. To use such ultra high strength concrete, loading tests and idealization of stress strain behavior of confined ultra high strength concrete are being conducted in Japan.
The stress strain curve of concrete has a descending branch after peak load. Generally the negative slope of descending branch is getting steep as the increase of compressive strength. Some examples of stress strain curve of HSC \(2\) are shown in Fig. 1. As indicated in the figure HSC has a sharp peak and the steep descending branch. These behaviors may result in the brittle flexural failure of RC columns without any ductility. In this figure the idealized stress strain curves are also indicated with the available limit strain, which give the maximum value of stress block factor \(k_3\) (ratio of average stress to peak stress). This available limit strain is the measure to evaluate the flexural ductility of RC sections. Therefore a key of ductility improvement of concrete is how to increase this available limit strain by lateral confinement.

Research on confined HSC

Muguruma et al \(^3\) conducted the axial loading tests on 14 laterally confined columns without longitudinal reinforcement to investigate the confining efficiency of transverse steel to HSC. Each column had 140 x 140 mm square confined core section (distance between centers of transverse steel) without cover concrete and 400 mm height. Twelve columns were confined by single peripheral spirals (deformed bars of 7.4 mm in diameter) with 50 mm spacing (volumetric ratio was 2.13 \%). Remaining two columns were confined by double peripheral spirals with 5 cm spacing (volumetric ratio was 4.26 \%). Concrete strength ranged from 34.2 to 87.5 MPa and yield strength of spiral hoops was 1360 MPa. Axial strain was measured by two displacement transducers at the central part of columns with gage length of 200 mm. Some examples of the measured stress strain curves were indicated in Fig. 2. Figure 2 indicates that the compressive ductility of high strength concrete can be improved by lateral confinement. From these test results and past experiments by other researchers, an idealization of stress strain curve for confined concrete with square section was proposed (see Fig. 3). Stress strain curve for plain concrete are given by a combination of a parabola and a straight line as equations (1) and (2) (see Fig. 3).

\[
\begin{align*}
\varepsilon_c & \leq \varepsilon_m & \sigma_c &= E_c\varepsilon_c^2 + \left(\frac{\varepsilon_c}{\varepsilon_m}\right)^2 (f_c' - E_c\varepsilon_m) \quad (1) \\
\varepsilon_c & > \varepsilon_m & \sigma_c &= (\sigma_u - f_c')\frac{\varepsilon_c - \varepsilon_u}{\varepsilon_u - \varepsilon_m} + f_c' \quad (2) \\
E_c &= 20600\sqrt{f_c' / 19.6} \quad (3) \\
k_m & \leq 1.5 & \varepsilon_m &= 0.000814k_m + 0.00167 \quad (4) \\
& & \varepsilon_u &= (-0.265k_m + 1.71)\varepsilon_m \quad (5) \\
k_m & > 1.5 & \varepsilon_m &= 0.002871 \quad (6) \\
& & \varepsilon_u &= 1.31\varepsilon_m \quad (7)
\end{align*}
\]
\[ k_m = \left( \frac{f_c}{54} \right) \left( \frac{C}{500} \right)^2 \left( \frac{W}{200} \right) \]  \hspace{1cm} (8)

Where \( \sigma_c \) and \( \varepsilon_c \): stress and strain, \( f'_c \) and \( \varepsilon_m \): compressive strength of plain concrete (\( N/mm^2 \)) and strain at \( f'_c \), \( E_c \): initial stiffness of plain concrete (\( N/mm^2 \)), \( \sigma_u \) and \( \varepsilon_u \): stress at \( \varepsilon_u \) and strain at which average stress reaches maximum, \( k_m \): coefficient of concrete property, \( C \): unit cement content (\( kg/m^3 \)), \( W \): unit water content (\( kg/m^3 \)),

Stress \( \sigma_u \) at \( \varepsilon_u \) can be obtained according to the definition of \( \varepsilon_u \). The average stress after peak is given by

\[ \sigma_{av} = \left( \frac{1}{\varepsilon_c} \right) \left[ S + \frac{\sigma_u - f'_c (\varepsilon_c - \varepsilon_m)^2}{\varepsilon_u - \varepsilon_m} + f'_c (\varepsilon_c - \varepsilon_m) \right] \]  \hspace{1cm} (9)

Where \( S \) is the area below the stress strain curve up to the peak. At \( \varepsilon_u \), average stress takes the maximum value. Therefore we obtain the stress \( \sigma_u \) as

\[ \frac{\partial \sigma_{av}}{\partial \varepsilon_c} = 0 \] \hspace{1cm} \[ \sigma_u = f'_c + \frac{2(S - f'_c \varepsilon_m)}{\varepsilon_u + \varepsilon_m} \]  \hspace{1cm} (10)

Similarly stress strain curve of confined concrete beyond \( \varepsilon_m \) is given by a combination of a parabola and a straight line as equations (11) and (12).

\[ \varepsilon_m < \varepsilon_c \leq \varepsilon_{cm} \] \hspace{1cm} \[ \sigma_c = \frac{f'_c - f'_{cc}}{(\varepsilon_m - \varepsilon_{cm})^2} (\varepsilon_c - \varepsilon_{cm})^2 + f'_{cc} \]  \hspace{1cm} (11)

\[ \varepsilon_{cm} < \varepsilon_c \] \hspace{1cm} \[ \sigma_c = \frac{\sigma_{cu} - f'_{cc}}{\varepsilon_{cu} - \varepsilon_{cm}} (\varepsilon_c - \varepsilon_{cm}) + f'_{cc} \]  \hspace{1cm} (12)

Where \( \sigma_c \) and \( \varepsilon_c \): stress and strain, \( f'_{cc} \) and \( \varepsilon_{cm} \): compressive strength of confined concrete (\( N/mm^2 \)) and strain at \( f'_{cc} \), \( \sigma_{cu} \) and \( \varepsilon_{cu} \): stress at \( \varepsilon_{cu} \) and strain at which average stress reaches maximum.

From the definition of \( \varepsilon_{cu} \), \( \sigma_{cu} \) is obtained by similar procedure for \( \sigma_u \).
Values of \(f'_{cc}, \epsilon_{cm}\) and \(\epsilon_{cu}\) were given by empirical equations using a confining coefficient \(C_c\).

\[
C_c = 0.313\rho_s \frac{\sqrt{f_y}}{f_c} \left( 1 - 0.5 \frac{s}{w} \right)
\]

\[
f'_{cc} = (1 + 49C_c)f'_{c}
\]

\[
\epsilon_{cm} = (1 + 341C_c)\epsilon_m
\]

\[
\epsilon_{cu} = (1 + 509C_c)\epsilon_u
\]

Where \(S_c\): area below the stress strain curve up to the peak \(f'_{cc}\), \(f_y\): yield strength of lateral reinforcement \((N/mm^2)\), \(s\): pacing of lateral reinforcement, \(w\): sectional dimension, \(\rho_s\): volumetric ratio of transverse reinforcement.

Sun and Sakino \(^5\) conducted the axial loading tests on 36 confined square columns with longitudinal reinforcement. Tested columns had 335 by 335 mm section size and 285 by 285 mm confined core. Concrete strength ranged from 24.0 to 128 MPa and the yield strength of hoops (6 mm in diameter deformed bar) was 1107 MPa. Volumetric ratio of transverse reinforcement ranged from 1.39 to 4.44 %. Test results indicated that the ductility of high strength concrete can be improved by lateral confinement. However for very high strength concrete (128 MPa) at least the volumetric ratio of 4.44 % is necessary to compensate for the loss of load carrying capacity due to the spalling of cover concrete.

Nishiyama et al \(^6\) conducted the axial loading tests on 14 confined square columns with longitudinal reinforcement. Tested columns had 250 by 250 mm sectional dimensions and 214 by 214 mm confined core. Average concrete strength was 111 MPa. Each column had peripheral hoops and two legs of internal cross tie in both directions. Volumetric ratio of hoop reinforcement ranged from 1.79 to 4.06 %. Grade 80 (yield strength was 813-840 MPa) and Grade 40 (yield strength was 462-481 MPa) bars were used as the
transverse reinforcement. Obtained results are, 1) transverse reinforcement with 800 MPa yield strength and more than 4 % volumetric ratio is needed to prevent the brittle failure of high strength concrete, 2) spalling of cover concrete does not affect the stress strain curve of confined concrete and 3) internal cross ties first yield near the peak load while the peripheral hoop is still in elastic for specimens confined by high strength steel.

Similar tests were conducted by Yagenji et al \(^7\) and Tanaka et al \(^8\). These Japanese tests and others indicated that 1) large amount of high strength transverse reinforcement is necessary to obtain the ductile compressive behavior of high strength concrete: the volumetric ratio of at least 2 % for circular section and 4 % for square section, 2) internal cross tie should be used for square section and 3) sufficient amount of transverse reinforcement should be provided to compensate for the loss of load carrying capacity due to spalling of cover concrete.

New RC idealization proposed by Sun and Sakino \(^9\)

Several stress strain idealizations for confined concrete were proposed in Japan. As one of the output of the New RC Project, Sun and Sakino \(^9\) proposed the stress strain idealization for square column section. Stress strain curve of confined concrete is given by equation 18 as a continuous curve. The feature of this idealization is that the effect of a ratio of transverse-bar diameter to unsupported length is introduced. Numerical examples of stress strain curve of confined concrete (for confined core) are indicated in Fig. 4, where an unsupported length of transverse reinforcement is assumed to be 200 mm.

\[
\frac{\sigma_c}{f_{cc}} = \frac{AX + (D-1)X^2}{1+(A-2)X + DX^2} \quad (18)
\]

\[
f_{cc}' = f_c' + \kappa \rho_h f_{hy} \quad (19)
\]

\[
\kappa = 11.5 \left( \frac{d^*}{C} \right) \left( 1 - \frac{s}{2D_{core}} \right) \quad (20)
\]

\[
X = \frac{\epsilon_c}{\epsilon_{cm}} \quad (21)
\]

\[
A = E_c \epsilon_{cm} / f_{cc}' \quad (22)
\]

\[
K = f_{cc}' / f_c' \quad (23)
\]

\[
K \leq 1.5 \quad \epsilon_{cm} = \epsilon_m \{1 + 4.7(K - 1)\} \quad (24)
\]

\[
K > 1.5 \quad \epsilon_{cm} = \epsilon_m \{3.35 + 20(K - 1.5)\} \quad (25)
\]

\[
\epsilon_m = 0.00093 \left( f_c' \right)^{0.25} \quad (26)
\]
\[ E_c = 41000 \left( \frac{f_c'}{100} \right)^{0.333} (\gamma / 2.4)^2 \]  
(27)

\[ D = 1.50 - 0.0171 f_c' + 1.6 \sqrt{(K - 1) f_c'} / 23 \]  
(28)

Where \( d'' \): diameter of transverse reinforcement, \( s \): spacing of transverse reinforcement, \( C \): unsupported length of transverse reinforcement, \( D_{core} \): width of confined core, \( \sigma_c, \varepsilon_c \): stress and strain of confined concrete, \( \varepsilon_m \): strain of plain concrete at peak, \( \varepsilon_{cm} \): strain of confined concrete at peak, \( \rho_{hi} \): volumetric ratio of transverse reinforcement to confined core, \( f_c' \) and \( f_{cc}' \): compressive strengths of plain and confined concrete \((N/mm^2)\), \( \gamma \): specific gravity of plain concrete, \( f_{hy} \): yield strength of transverse steel \((N/mm^2)\), \( m \): 1.0 for normal aggregate, 1.2 for hard aggregate and 0.9 for soft aggregate.

New Approach by Beni Assa et al

Recently Beni Assa et al \(^{10, 11}\) proposed the steel concrete interaction model to predict the strength and stress strain curve of confined concrete. This model is based on the lateral confining stiffness of transverse reinforcement. A total of thirty-two 145 by 300 mm concrete cylinders were tested under monotonic concentric compression. No cover concrete was provided in all specimens. The target strength of concrete ranged from 20 MPa to 90 MPa. As the transverse reinforcement 6.25 mm helical spirals and welded circular hoops were used. Their nominal yield strengths were 1300 MPa and 800 MPa, respectively. The spacing of spirals or circular hoops was varied from 19 mm to 75 mm. The axial strain of concrete cylinder was measured at the central part of specimens with the gage length of 145 mm as indicated in Fig. 5. The strain of the lateral steel was measured using nine wire strain gages placed approximately along three loops of hoops within the central region. From strain measurements, average lateral strain and lateral stress to concrete was calculated. Stress and strain of confined concrete at each characteristic point in stress strain curve were experimentally obtained as

\[ f_{cc}' = f_c' + 3.36 f_{rp} \]  
(29)

\[ \varepsilon_{cm} = (1 + 21.5 f_{rp} / f_c') \varepsilon_m \]  
(30)

\[ \varepsilon_{80} = (2.74 + 32.8 f_{rp} / f_c') \varepsilon_m \]  
(31).

The relationship between the lateral strain and the lateral stress at peak was also obtained as a unique equation as equation 32. This equation is the best fit to the test results. This equation is a key equation in their study and named the peak load condition line.

\[ \varepsilon_{rp} = 0.0021 + 0.016 f_{rp} / f_c' \]  
(32)
Where \( f'_{cc} \): strength of confined concrete (N/mm²), \( f'_c \): strength of plain concrete (N/mm²), \( f_{rp} \): lateral pressure at peak (N/mm), \( \varepsilon_{cm} \): strain at peak of confined concrete, \( \varepsilon_{80} \): strain at 0.80 \( f'_{cc} \) after peak, \( \varepsilon_m \): strain at peak of plain concrete, \( \varepsilon_{rp} \): lateral strain at peak.

These test results can be used to predict the stress strain behavior of concrete confined by several types of transverse reinforcement configurations. If the lateral pressure \( f_r \) for any lateral expansion (lateral strain) \( \varepsilon_r \) is theoretically obtained for any transverse reinforcement configurations as indicated by \( f_r - \varepsilon_r \) line in Fig. 6, the lateral pressure at peak load, \( f_{rp} \), can be easily obtained at the intersection point between the peak load condition line and \( f_r - \varepsilon_r \) line. Then stress-strain coordinate at peak load is obtained. For circular transverse reinforcement, \( f_r - \varepsilon_r \) curve has similar shape to that of stress strain curve of transverse reinforcement. For other types of transverse reinforcement details, \( f_r - \varepsilon_r \) can be predicted by finite element analysis. An example of the transverse-steel concrete interaction model is indicated in Fig. 7. Perimeter hoop is divided into finite beam-column elements, which is interconnected at nodal point. Integration point and layering of section of perimeter hoop are indicated in Fig. 7. Internal cross tie is modeled as truss element. This transverse steel system is pushed out by concrete bar element due to its lateral expansion. Compressive stiffness of concrete bar element is given to have the same stiffness of corresponding triangular concrete element (see Fig. 7) where tensile stiffness of concrete element is assumed to be zero. This is to express the separation between concrete surface and perimeter hoop due to bowing out of it. By giving free elongation of concrete bar element corresponding to lateral strain of concrete, \( \varepsilon_r \), average lateral pressure to concrete section \( f_{rp} \) can be obtained.

Fig. 8 indicates the typical examples of numerical calculation, where the volumetric ratio of transverse reinforcement, concrete strength and yield strength of hoop are assumed to be 2.15%, 34.1MPa and 1300MPa, respectively. This figure indicates that the confining efficiency of transverse reinforcement strongly depends on the detailing of it.

Figure 9 indicates the process to obtain the stress strain coordinate at peak load and the predicted stress strain curve for Unit-2 specimen (Scott et al. 1982) with their observed one. The numerical expression of stress strain curve of confined concrete is given by Popovics proposal.

\[
\sigma_c = f'_{cc} \frac{x\beta}{\beta - 1 + x\beta} \quad (33)
\]
\[
x = \varepsilon_c / \varepsilon_{cm} \quad (34)
\]
\[
\beta = E_c (E_c - f'_{cc} / \varepsilon_{cm}) \quad (35)
\]
Where $E_c$: initial stiffness of concrete, $\varepsilon_{cm}$: strain at peak of confined concrete and given by Eq. 30.

**BEHAVIOR OF CONFINED ULTRA HIGH STRENGTH CONCRETE**

In Japan, application of HSC to high-rise buildings has rapidly increased after the publication of design and construction guidelines prepared by the New RC Project. The New RC guidelines, the succeeding progresses of structural analyses and construction technology have made the realization of reinforced concrete high-rise buildings more possible. However covered range of the compressive strength of concrete in New RC guidelines are less than 60 MPa. The requirement of higher strength of concrete (more than 100 MPa) arose for the construction of reinforced concrete skyscrapers (150 meters high or more). For the use of such ultra high strength concrete the ductility improvement by lateral confinement becomes much more important.

**Recent work on confined ultra high strength concrete**

Komuro et al \(^{12}\) conducted strain controlled axial tests on confined ultra high strength concrete from 100 to 176 MPa in compressive strength. Tested column specimens are indicated in Fig. 10 and Tab. 1. As longitudinal reinforcement 12 D-16 deformed bars with yield strength of 685 MPa were provided to all specimens. In addition four high strength steel rods (22 mm in diameter and yield strength of 1219 MPa) were placed in a central part of section to obtain the stable axial response. The contribution of these longitudinal bars to axial resistance was observed by wire strain gages and was deducted from the total axial force. For columns with square section perimeter hoops and internal ties were arranged. In the experiments high strength transverse reinforcement was used to improve the compressive ductility effectively. The yield strengths of transverse reinforcement were 1515 MPa (5.1 mm in diameter bar) and 1440 MPa 6.1 mm in diameter bar). Volumetric ratios of transverse reinforcement ranged from 2 to 4.4 % for square columns and from 0.8 to 1.8 % for circular columns, respectively. Some of columns were tested under cyclic loading to investigate the effect of load cycling to the envelope of stress strain curve. Test results indicated that the envelope curve of cyclic loading is almost identical to the virgin curve. Some examples of stress strain curve obtained from monotonic loadings were indicated in Fig. 11, where the stress of concrete was calculated for confined core concrete section. From this experiments Komuro et al modified the stress strain idealization proposed by Muguruma et al (Equations 1 to 17). For such high strength concrete, $k_m$ value of equation 8 is generally larger than 1.5. Therefore $\varepsilon_m$ and $\varepsilon_u$ for plain concrete become 0.002871 and 0.003772 from equations 6 and 7, respectively.

\[
\varepsilon_m = 0.002871 \tag{36}
\]

\[
\varepsilon_u = 0.003772 \tag{37}
\]

For square section

\[
f'_{cc} = (1 + 49C_c)f'_c \tag{38}
\]
\[ \varepsilon_{cm} = (1 + 179C_c)\varepsilon_m \quad (39) \]
\[ \varepsilon_{cu} = \varepsilon_u \quad C_c < 0.0013 \quad (40) \]
\[ \varepsilon_{cu} = (-1.44 + 1890C_c)\varepsilon_u \quad C_c \geq 0.0013 \quad (41) \]

For circular section
\[ f'_{cc} = (1 + 75C_c)f'_{c} \quad (42) \]
\[ \varepsilon_{cm} = (1 + 250C_c)\varepsilon_m \quad (43) \]
\[ \varepsilon_{cu} = \varepsilon_u \quad C_c < 0.00041 \quad (44) \]
\[ \varepsilon_{cu} = (0.401 + 1460C_c)\varepsilon_u \quad C_c \geq 0.00041 \quad (45) \]

Some examples of theoretically predicted stress strain curves are indicated in Fig. 11 by thick dotted line. This idealization for stress strain curve of confined ultra high strength concrete was used for the practical design of a high-rise building with 130 MPa concrete.

CONCLUSIONS

The use of HSC is spreading worldwide to construct the large-scale structures such as high-rise buildings. For the effective utilization of HSC to ductile frames, ductility improvement of HSC by lateral confinement is inevitable. The reason is that HSC generally fails in brittle manner in compression and results in small flexural ductility of column potential plastic hinges.

This paper presented the recent research works on confined concrete in Japan, especially for HSC. Some experimental works and idealizations of stress strain curve of confined concrete were introduced. Maximum compressive strength covered in this paper was 176 MPa. These research results have been successfully applied to the construction of high-rise buildings in Japan. Examples of actual constructions are shown in figures 12 and 13. Fig. 12 indicates a condominium building constructed in Tokyo in 2002. Building height and number of stories are 157.4 meters and 47 stories, respectively. Fig. 13 is also a condominium building in Tokyo. In these buildings high strength concrete of nominal compressive strength of 100 MPa were used for lower two stories. This year the construction of a building with 130 MP concrete has started in Tokyo.

At this moment some design trials of buildings with 150 MPa concrete is going on. The author tried to introduce Japanese research as much as possible, but could only review a limited number of papers. However it is quite sure that HSC has increasingly come into wide use worldwide. We need more mutual exchange of information about the confined HSC for the effective utilization of it.

ACKNOWLEDGEMENT

Author would like to express hearty thanks to Mr. Tsutomu Komuro at Taisei Construction Company who kindly provided test data on confined HSC and photos.
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Table 1 List of columns tested by Komuro et al.12)

<table>
<thead>
<tr>
<th>Spec. No.</th>
<th>Section (mm)</th>
<th>Concrete (MPa)</th>
<th>Transverse reinforcement</th>
<th>Volumetric ratio</th>
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<tr>
<td>1</td>
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<td>Confined Core 250x250</td>
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* Cyclic loading.

Yield strength of transverse reinforcement
5.1 mm bar: 1515 N/mm²
6.1 mm bar: 1440 N/mm²

Figure 1 – S-S curve of HSC.2)
Figure 2 – S-S curve of confined HSC³)

Figure 3 – S-S model by Muguruma et al ³)

Figure 4 – Numerical examples of New RC equation proposed by Sun and Sakino⁹)
Figure 5 – Tests by Beni Assa et al$^{10, 11}$

Figure 6 – Peak load condition line

Figure 7 – Transverse steel concrete interaction model by Beni Assa et al$^{10, 11}$
Figure 8 – Predicted lateral strain lateral pressure curves for various types of transverse steel configuration

Figure 9 – Comparison between observed stress strain curve and predicted one (Unit-2, Scott et al, [1982])
Figure 10 – Column specimens tested by Komuro et al\textsuperscript{12)}

Figure 11 – Experimentally observed stress strain curves of confined ultra high-strength concrete (Komuro et al\textsuperscript{12}).
Figure 12 – Siodome D-South-Block Building, 47 stories, 157.4 meters high, 2002, Tokyo, Minatoku

Figure 13 – Kawada-cho Residence C-Tower, 41 stories, 131.8 meters high, 2002, Tokyo, Shinjyuku-ku
80  Watanabe
Influence of Confinement Modeling on Cyclic Response of Reinforced Concrete Columns

by S.K. Kunnath

Synopsis: Concepts in ductile design have led to an increased interest in understanding the role of confinement in improving the seismic performance of reinforced concrete members. While transverse reinforcement is regarded as a form of passive confinement in RC members, the observed increase in the strength of confined concrete is typically a function of the axial strain levels. Confinement models have been developed by numerous researchers to describe the stress-strain behavior of concrete as a function of certain key parameters that are related to the amount and type of transverse reinforcement. Accurate constitutive models of confined concrete are necessary for direct use in fiber-model based discretization of RC components or for indirect use in hysteresis based phenomenological models. This paper examines the relevance and importance of accurate confinement modeling in predicting the inelastic behavior of well-confined concrete columns. In particular, the influence of incorporating confinement effects in predicting the monotonic and cyclic response of RC columns is investigated. It is analytically demonstrated that the role of the longitudinal reinforcing bars play a more significant role in determining the overall force-deformation behavior of RC components. Detailed fiber-based discretizations that rely entirely on constitutive models are incapable of reproducing post-yield softening and deterioration because of their inability to incorporate complex large deformation behavior of both the longitudinal and the confining reinforcement. Approximate phenomenological models will continue to see widespread use in inelastic analysis of RC structures until these limitations of constitutive-based element models are overcome.

Keywords: analytical modeling; confined concrete; cyclic response; ductility; seismic
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INTRODUCTION

The significance of transverse reinforcement in enhancing the strength and ductility of reinforced concrete (RC) columns is well recognized, however, the role of confinement modeling in analytical simulation of the cyclic response of RC elements has not received as much attention. This is primarily because most of the analytical effort has been directed towards the development of uniaxial constitutive relationships for confined concrete. Accurate constitutive models of confined concrete are needed in two computer-based modeling applications: (i) for direct use in fiber-model based discretization of RC components; and (ii) for indirect use in phenomenological models that rely on predictive hysteretic behavior of sections. This paper examines the relevance and importance of accurate confinement modeling in predicting the inelastic behavior of reinforced concrete columns. Since the role of confinement models is to predict the behavior of well-confined concrete, this study focuses primarily on well-confined columns.

Of the many issues that have concerned researchers in developing confinement models is the role of axial strain in mobilizing the lateral pressure provided by the transverse reinforcement. The effect of axial strain on the strength of confined concrete can be more clearly understood by examining tests on concrete confined by fiber-reinforced plastics (FRP) or steel tubes. While concrete confined by transverse reinforcement exhibits softening behavior after attaining its peak stress, FRP-confined concrete displays a monotonically increasing stress-strain behavior because the confining pressure provided by the continuous jacket increases with increasing axial strain.

A systematic analytical study is underway at the University of California at Davis examining the role of confinement in cyclic behavior of reinforced concrete columns. This paper presents a preliminary summary of the findings from this ongoing work. The following issues are the overall objectives of the project:

1. The effectiveness of confinement models to predict the strength of confined concrete for varying levels of axial strain
2. The significance of predicting the confined concrete strength in estimating seismic deformation demands in RC elements
3. Issues in predicting the degrading response of RC members and its importance in performance-based seismic engineering

This paper highlights findings related to the second and third of the above objectives.
CONFINED STRENGTH MODELS

The strength enhancement provided by confining steel is generally modeled using simple assumptions on the state of stress in the transverse steel and the distribution of lateral stress on the core concrete. Some attempts to incorporate additional considerations are also evident in the literature. For the purpose of this study, models that predict the strength of confined concrete are classified into two categories.

Simple Models based on Modification Factors

The literature on confined concrete is abundant with models characterizing the increase in strength and ductility resulting from passive confinement. The general forms of expressions relating the effect of lateral confining pressure on uniaxial strength and corresponding strain, attributed to the early work of Richart et al. (1928), are given by:

\[
\begin{align*}
E_{cc} &= E_{co} \left(1 + q_{ce} \frac{f_{L}}{f_{co}} \right) \\
f_{cc}' &= f_{cc} + q_{c}f_{L} \\
\end{align*}
\]

where \(f_{cc}', E_{cc}\) are the maximum strength and strain at maximum strength of the confined concrete, \(f_{cc}, E_{co}\) are the corresponding unconfined properties, \(f_{L}\) is the effective lateral confining pressure and \(q_{c}, q_{ce}\) are material parameters that depend on numerous factors. A widely used and often referenced model for predicting the strength of confined concrete is the following expression proposed by Mander et al. (1988):

\[
\frac{f_{cc}'}{f_{co}'} = 2.254 \sqrt{1 + 7.94 \frac{f_{L}}{f_{co}'} - 2 \frac{f_{L}}{f_{co}'} - 1.254} \\
\]

For sections with circular spirals:

\[
f_{L} = 0.5\rho_{s}f_{yt} \left(\frac{1-0.5s'/d_{s}}{1-\rho_{cc}}\right)
\]

where \(\rho_{s}\) is the volumetric confinement ratio, \(f_{yt}\) is the yield strength of the confining steel, \(s'\) is the clear distance between the hoops, \(d_{s}\) is the center-to-center spiral diameter and \(\rho_{cc}\) is the ratio of the longitudinal steel area to the area of the core concrete. The constants that appear in the equation (3) were obtained from empirical calibration of experimental data to a five-parameter multiaxial failure surface proposed by William and Warnke (1974). The key assumption in deriving effective confinement pressure is that a uniform hoop tension is developed in the transverse steel and that the confining steel is
yielding when the equilibrium of forces is considered to equate hoop stress and the average lateral stress in the concrete core. Other approaches for estimating the confined strength are also loosely based on what can be referred to as strength enhancement factors. The formulations by Sheikh and Uzumeri (1982) recognized the arching action resulting from confining stresses—a concept further refined by Mander et al. (1988). Saatcioglu and Razvi (1992) developed an expression for confinement effectiveness by assuming an average distribution of lateral pressure and equating the total force to the sum of forces in each transverse bar. Assumptions similar to the Mander model are evident in the formulation.

Alternative Strength Models

This class of models accounts for the state of stress in the transverse reinforcement at the expected peak strength of the confined concrete. In actuality this requires careful consideration of the three-dimensional triaxial stress state in concrete. The model of Sheikh and Uzumeri (1982) referenced earlier suggests using the actual stress state in the confining reinforcement but does not offer a procedure to estimate this parameter. Madas and Elnashai (1992) propose an iterative procedure to estimate the instantaneous lateral pressure during an incremental analysis. Cusson and Paultre (1995) suggest a relatively simpler iterative technique to consider the state of stress in the confining steel as follows.

1. Assuming that the confining steel is yielding, compute the effective lateral confining pressure (for sections confined by spirals, Equation 4 can be used; similar expressions can be derived for rectangular sections), and estimate the corresponding peak uniaxial strength and corresponding strain (Cusson and Paultre, 1995):

\[
f_{cc}' = f_{co}^c[1.0 + 2.1(f_L / f_{co})^{0.7}]
\]

\[
\varepsilon_{cc} = \varepsilon_{co} + 0.21(f_L / f_{co})^{1.7}
\]

2. Estimate the strain in the transverse reinforcement. In the method presented by Cusson and Paultre (1995), this is accomplished by assuming isotropic elasticity and evaluating lateral strains induced by compressive strains in the concrete, as follows:

\[
\varepsilon_L = 0.5\varepsilon_{cc}[1 - f_L / f_{cc}']
\]

Based on observed experimental evidence, the Poisson ratio at maximum concrete strength is taken as 0.5 in deriving the above expression.

3. Determine the resulting stress in the transverse reinforcement using the stress-strain relationship for the confining steel. The computed stress should be limited to the yield stress. Use this stress to re-compute the effective lateral confining pressure and repeat the above steps till convergence is achieved.
Finally, it should be pointed out that efforts in three-dimensional finite element continuum modeling of confined concrete do exist but are beyond the scope of this paper. The reader is referred to the work of Ahmad et al. (1986), Vecchio (1993) and Kwon and Spacone (2002).

CONFINED CONCRETE CONSTITUTIVE MODELING

The complete stress-strain curve of concrete, confined or unconfined, is represented by two segments: an ascending branch and a descending branch, as displayed in Figure 1. The following form of the ascending branch of the stress-strain curve was proposed by Sargin (1971):

\[
f'_{cc} = \frac{k_a \varepsilon_c}{\varepsilon_{co}} + (k_b - 1) \left( \frac{\varepsilon_c}{\varepsilon_{co}} \right)^2
\]

Note that the above model reduces to the well-known Hognestad (1951) model for the case \(k_a = 2\) and \(k_b = 0\). The initial model of Ahmad and Shah (1982) and the later version by Ahmad and Abdel-Fattah (1991) represent extensions of the above relationship.

A polynomial form of the ascending branch was recently proposed by Hoshikuma et al. (1997), as follows:

\[
f_c = C_1 \varepsilon_c^n + C_2 \varepsilon_c + C_3
\]

Four boundary conditions relating the strength and stiffness limits at \(\varepsilon_c = 0\) and \(\varepsilon_c = \varepsilon_{cc}\) provides the stress-strain behavior for the ascending branch:

\[
f_c = E_c \varepsilon_c \left[ 1 - \frac{1}{n} \left( \frac{\varepsilon_c}{\varepsilon_{cc}} \right)^{n-1} \right]
\]

\[
n = \frac{E_c \varepsilon_{cc}}{E_c \varepsilon_{cc} - f'_{cc}}
\]

Though several equations have been proposed for the descending portion of the stress-strain curve, a linear approximation is generally adequate. This requires knowledge of the failure strain and the residual strength.
The primary purpose of developing an accurate confinement model is to facilitate its use in nonlinear analysis of structures where the constitutive modeling of concrete plays a role in the response analysis. To evaluate the significance of confinement modeling in inelastic analysis of RC structures, a series of analytical studies were carried out using different confinement models. Simulation models of an RC column, for which experimentally recorded data is available, were developed. The column represents a bridge pier that was tested by El-Bahy et al. (1999). Figure 2 displays the cross-sectional details of the model column that was subjected to monotonic and cyclic loading histories and constitutes the benchmark problem for the analytical studies reported in this paper.

Simulation Models

Two simulation models were considered in the study: the first model employs a force-based nonlinear beam-column element based on a detailed fiber section discretization while the second model comprised a generalized beam element with inelastic rotational springs whose properties were derived from a sectional moment-curvature analysis. A schematic description of the two models is shown in Figure 3. Model SA refers to the fiber section model that is implemented in the open source finite element software OpenSees (2004). In the present study, a mesh consisting of 8 divisions in the radial direction and 20 segments in the circumferential direction were used. The outermost ring is modeled as unconfined concrete while the inner seven rings are modeled as confined concrete. The reinforcing steel is modeled using a bilinear stress-strain relationship. Model SB is based on a macro nonlinear element with inelastic rotational springs at the ends of the element (Kunnath, S.K., 2004). The moment-curvature input for the element is derived from a detailed fiber section analysis of the column cross-section. Model SB is representative of numerous current approaches to analyzing reinforced concrete structures. Model SB was used only for the cyclic response analysis and details of the hysteretic modeling are presented in a subsequent section.

Confined Concrete Models Used in Evaluation Study

Three models representing different approaches to confinement modeling are evaluated in this investigation. In the first extreme approximation, the entire section is treated as unconfined. This means that confinement effects are practically ignored. The second model is based on the popular Mander model described earlier in the paper. The third model is derived from the formulation of Cusson and Paultre which accounts for the likely stress in the transverse steel at peak strength of the confined concrete. The resulting stress-strain curves for the three models are shown in Figure 4. As is evident from the figure each model predicts different peak strength, residual strength and corresponding strain limits. The following properties, derived from model formulations reported earlier and based on experimentally reported values of unconfined concrete, are used in the ensuing simulations:
Evaluation of Monotonic Response

The first case to be examined is the monotonic response of the column. Initially, the moment-curvature response of the column section subjected to combined axial force and bending will be evaluated. This is followed by an examination of the force-deformation response of the model under monotonically increasing lateral load. In both cases, two aspects of modeling will be studied: (a) influence of confinement modeling, and (b) influence of longitudinal steel modeling.

Influence of Confinement Modeling

The moment-curvature response of the column cross-section under a constant axial force of approximately 200 kN (which corresponds to $0.1f'_c A_g$) is illustrated in Figure 5 for the three confinement models shown previously in Figure 4. The resulting curves indicate that the response is not sensitive to the type of confinement model used in the analysis. Figure 6 shows the resulting force-deformation response of the column. There is evidence of spalling of the cover concrete and is more pronounced when confinement effects are completely ignored. The post-yield response is also not influenced by the effects of confinement.

Influence of Longitudinal Steel

However, as Figure 7 demonstrates, the behavior of the section is generally controlled by the response of the longitudinal steel. It was observed that both the yield strength and post-yield stiffness of the longitudinal steel played a more significant role in controlling the behavior of the section. This observation can be carried forward to the estimation of the force-displacement response which is displayed in Figure 8. A greater deal of flexibility in controlling the peak and post-peak behavior is possible with variations in the longitudinal steel modeling than the modeling of confinement effects. The check on strain-hardening is facilitated by P-delta effects, which tend to soften and compensate for strain hardening effects. A low post-yield stiffness can lead to P-delta collapse.

Evaluation of Cyclic Response

Since the different confinement models did not produce significant changes in the sectional moment-curvature response, it was not surprising that the cyclic response using the three confinement models produced essentially the same cyclic response. Model I in Figure 9 was obtained with Mander’s confinement model. The force-based fiber section model overestimates energy dissipation and is unable to capture any degradation in strength. The only change introduced in Model II and Model III is the post-yield stiffness.
of the longitudinal bar. This has the effect of limiting the overall strength but does not overcome the issues raised with respect to Model I.

Fiber Section Models vs. Global Macromodels

While it may be argued that the results presented in Figure 9 are acceptable simulations of observed behavior, it is clear that such models cannot be used in predicting damage and failure in RC components. To demonstrate the fact that reasonable results can be obtained with equivalent macromodels, Model SB is utilized to simulate the cyclic response of the column model. The input to the macromodel consists of the cracking moment, the yield moment, the initial flexural rigidity, the yield curvature and some control parameters to model the cyclic response. In this study these basic parameters were taken from the moment-curvature simulations presented in Figure 5 and 7. Program IDASS with standard default parameters for degrading behavior of well-confined columns were used in the simulations. The resulting analytical response is compared to the experimentally observed behavior in Figure 11. The overall response is generally better than the fiber-based model that relies on accurate constitutive modeling of confined concrete and longitudinal steel.

SUMMARY OF FINDINGS

This work summarizes preliminary findings from an ongoing project that is concerned with the sensitivity of modeling parameters in nonlinear cyclic response of RC structures. In particular, the issue of confinement modeling was addressed. The literature is abundant with models that express uniaxial stress-strain relationships as a function of confinement effectiveness. Commonly used models are based on assuming the transverse reinforcement to have yielded when the concrete attains its maximum strength. Other researchers have correctly pointed out that the actual stress in the transverse reinforcement is an important factor in determining the confined behavior of concrete. However, as indicated in Figure 12, the axial strain levels in the section are non-uniform and vary from extreme compression to extreme tension on opposite faces. The strain levels are also sensitive to the assumed stress-strain behavior of the unconfined concrete and the longitudinal steel. The initial axial strains induced by gravity loading are typically insignificant compared to the non-uniform strains induced by lateral loading. Hence the treatment of the axial strain profile in estimating lateral confinement pressure needs to be addressed. The axial strain level may become a non-issue if the peak strains under lateral loading cause the confining steel to yield.

While the significance of confinement in concrete cannot be overlooked from a post-elastic and collapse perspective, findings from this study indicate that the properties of confined concrete by itself does not play a significant role in the inelastic response of RC structures. The behavior of the longitudinal reinforcement is more critical and controls the overall response of the element. If confinement effects are to be incorporated, it is more important to consider the consequences of confinement failure rather than focusing on stress-strain behavior of the confined concrete region. Approximate phenomenological models are effective in capturing overall degrading response of RC
elements; however the calibration of hysteretic models may pose a formidable challenge. Detailed fiber model representations are necessary to model local failure but the associated challenges in modeling complex large-deformation effects such as buckling and hoop fracture are equally daunting.

REFERENCES


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Figure 1. General stress-strain model for concrete identifying control parameters for unconfined and confined concrete
Figure 2. Details of model column used in simulations

Figure 3. Simulation models – Model SA: detailed fiber element model with five integration points; Model SB: simplified macromodel with moment-rotation inelastic springs

Figure 4. Stress-strain models used in analytical study
Figure 5. Effect of confinement modeling on moment-curvature response

Figure 6. Effect of confinement modeling on monotonic response

Figure 7. Effect of longitudinal steel modeling on moment-curvature response – Model L1: post-yield stiffness 1%; Model L2: post-yield stiffness 2%; Model L3: post-yield stiffness 0.5% and 5% increase in yield strength
Figure 8. Effect of longitudinal steel modeling on monotonic response

Figure 9. Influence of confinement modeling and longitudinal bar modeling on cyclic response of RC column
Figure 10. Hysteretic model implemented in IDASS and utilized in simulation

Figure 11. Simulation of cyclic response using approximate macromodel

Figure 12. Variation of axial strain at hoop locations during monotonic loading
Analytical Performance of Reinforced Concrete Columns using Various Confinement Models

by A. Esmaeily and K. Lucio

Synopsis: Accuracy of various stress-strain relationship models for concrete confined by lateral steel reinforcement in prediction of the moment-curvature response of a section under various loading patterns was investigated. Fiber model was implemented in a computer program developed for flexural analysis of reinforced concrete columns. Both monotonic and hysteretic analyses were performed. Each confined concrete model was used in the analysis for several loading cases on a circular, and a loading case on a rectangular section keeping all other parameters fixed. Results were compared to each other and also validated against experimental data from six large-scale reinforced concrete circular columns and a rectangular section tested under the analyzed loading cases. In general, analysis underestimated the moment and overestimated the curvature capacities under a high level of axial load regardless of the model used and there was a better agreement between analysis and test for low level or no axial load. Results from various models were close to each other, though different from test results, for cases with a constant or proportionally variable axial load. For a monotonic curvature with non-proportionally variable axial load, predictions by some recent models had a better agreement with experimental data. Evidently the level of axial load, and in turn the depth of compression zone, is related to the degree of confinement-utilization, and affects the confined concrete behavior. This is rarely addressed by the models.

Keywords: axial load; confined concrete; material models; moment-curvature
INTRODUCTION

Prediction of the performance of a structure during its life-time and its behavior under various loading conditions is a key element in “performance-based design” as the future design methodology, especially for reinforced concrete structures. Sophisticated methods such as a detailed finite element analysis along with advanced material models and constitutive laws are an option but probably not a favorable choice for an engineer in a design office, where a simple analytical approach along with commonly used models, are preferred. Investigation of the applicability of conventional material models and analytical methods to predict the performance of reinforced concrete structures with an acceptable accuracy is therefore a necessity. This covers a wide range of the analytical methods, such as plastic hinge assumption, and material models for steel, concrete and confined concrete. The goal of this study is limited to exploring various analytical models for concrete confined conventionally by lateral steel reinforcement in simulating the performance of a reinforced concrete member, and in particular a column in a building or bridge. The analytical work in this study is based on a moment curvature analysis of the section under various loading conditions using different confined concrete models. This will prevent other factors, such as assumptions on curvature distribution on a member in a force deflection analysis, to obscure the comparative studies on material models. A window-based application was developed to carry out the monotonic or hysteretic moment curvature analysis for various cross-sections under a desired loading pattern. Different confined concrete models were used in the analysis for the section and each loading case, and the results were compared to each other and also with the experimental data available for the section under the analyzed loading case.

The experimental data for the circular section column was limited to various loading cases on a single section, and therefore, all other parameters such as geometry, lateral reinforcement, and arrangement of longitudinal steel were fixed. As a result, this study highlights the role of loading and in particular axial load and loading pattern and the accuracy of a model in prediction of the moment-curvature response for a certain loading case.
ANALYTICAL PROGRAM

A computer program developed by the author for non-linear monotonic and hysteretic moment curvature and force deflection analysis of reinforced concrete columns under any kind of loading or displacement history was used to predict the moment curvature response of the section used in this study.

Analysis is based on fiber model which divides the section into small elements, assuming that the column consists of uniaxially stressed fibers along the longitudinal axis. This model has been used effectively in analysis of reinforced concrete columns subjected to reversed loading paths. The effect of confinement is considered in the monotonic and hysteretic stress-strain relationship of concrete. Strain hardening of the steel was implemented in the material model used in the program.

Since the only information required to analyze a section is the monotonic or hysteretic stress-strain behavior of the confined and plain concrete and steel, and there is no predetermined rule enforced to define the general behavior, the method used in the analysis is versatile in terms of the ability to predict the behavior of a reinforced concrete section under any loading history.

The circular section used in this study was 16 inch (406 mm) in diameter, reinforced by 12 #4 (13) and confined by a W2.5 spiral with 1.25 inch (32 mm) pitch.

All of the confined concrete models studied here, were used in the program to predict the moment-curvature response of the section for each of the experimental loading cases.

The four loading patterns included two cases with a cyclic curvature, one under a constant axial load equal to 30% of section capacity, $A_g f'_c$ and the other under an axial load proportional to the moment with a ratio of 66 for moment (kip-in) to axial load (kip); a case with a full cycle of curvature without axial load and; a case with a monotonically increasing curvature under a non-proportionally variable axial load within $30\% A_g f'_c$ and $-10\% A_g f'_c$.

MATERIAL MODELS

For a consistent comparison between the confined concrete material models for a certain type of loading and to investigate the degree of the accuracy of each model for various loading cases, the only variables were confined concrete monotonic stress-strain response model and loading pattern, and all other parameters, including section geometry, reinforcement ratio and arrangement and, material models for steel and plain concrete and the hysteretic rules for steel and concrete were fixed. The fixed material models used in this analysis are as follows (Esmaeily, A., Xiao, Y.; 2002).
Monotonic Stress-Strain Model for Steel: A flexible model was developed for monotonic stress-strain relationship of steel with 4 main parameters. These parameters, as follow, can be adjusted to simulate the behavior of different types of steel.

\( K_1 \) is the ratio of the strain at start of the strain hardening to the yield strain.

\( K_2 \) is the ratio of strain at maximum stress to yield strain.

\( K_3 \) is the ratio of ultimate strain to yield strain.

\( K_4 \) is the ratio of the maximum stress to yield stress.

As shown in Figure 3, the curve is linear up to the yield point and has a pure plastic deformation from the yield point up to a strain of \( K_1 \) times the yield strain. The maximum stress is achieved at a strain of \( K_2 \) times the yield strain, and is equal to \( K_4 \) times yield stress. Steel ruptures at a strain of \( K_3 \) times the yield strain. A quadratic curve joins the start of strain hardening point, the maximum stress point and the rupture point.

The values used as the input data for the monotonic stress-strain curve of steel in the analysis were as follows:

\[ f_y = 68 \text{ ksi} \quad (469 \text{ MPa}) , \quad E = 290.000 \text{ ksi} \quad (2000.000 \text{ MPa}) \]

\[ K_1 = 4.5 , \quad K_2 = 25.7 , \quad K_3 = 40.0 , \quad K_4 = 1.3 \]

Hysteresis Rules for Steel Stress-Strain Model: The model developed and used for the hysteretic behavior of steel has the three major parts as is common for any hysteretic model. Before any strain reversal, the stress and strain follow the monotonic stress-strain curve of steel as described in the monotonic stress-strain curve for steel. At the turning point (strain reversal) the modulus of elasticity is assumed to be the same as initial modulus of elasticity of steel. Bauschinger effect has been considered by decreasing the stiffness of steel to a portion of the initial value. Figure 4 shows an instance of the hysteretic stress-strain model, as implemented in the analysis.

Hysteresis Rules for Concrete Stress-Strain Model: The monotonic stress-strain curve for the confined concrete as proposed by different researchers serves as the envelope for hysteretic stress-strain response of confined concrete developed by the author. The model considers the tensile strength of concrete. The history of the stress and strain is tracked for each concrete element so that it will not have any tensile strength after the first crack and no compressive strength after the first crash in compression.

At a strain reversal the curve follows a parabolic path that is concave upward. The initial slope of the reversal curve is taken equal to the initial stiffness of confined concrete. The stress decreases to zero when the tensile strength is ignored, or will decrease to the tensile strength, with a predefined slope after the sign change of the stress. At the second reversal of strain, the stress remains zero up to a strain where the stress had vanished or changed sign in the first reversal, and then it grows with a slope equal to initial stiffness of confined concrete, in the beginning. The slope decreases as the strain and corresponding stress increase. The stress increases up to the envelope curve and then follows that curve. It should be added that for ascending and descending paths of the hysteretic curve, we may apply different initial stiffness that in turn may be different from the confined concrete initial stiffness. In present analysis, these values have been chosen to be identical. Figure 5 shows the concrete hysteretic behavior curves for curved return paths.
While a large number of confined concrete material models have been developed and proposed by different researchers, this study is limited to five representative models from the very early and primitive models to the more recent models with a more realistic and in turn more parameters. A custom option was developed in the analytical program to provide more flexibility for simulating a material model not implemented as a built-in component in the program. A brief description of the models used is as follows:

**Richart’s Model** (1928): The pioneer work on the effect of transverse reinforcement on concrete compression behavior was conducted by Richart (Richart et. al., 1928, 1929). In this model, the strength and corresponding strain of confined concrete is directly proportional to the increase in transverse pressure.

\[
f_{cc} = f'_{c} + Kf_{r} \tag{1}
\]

\[
\varepsilon_{cc} = \varepsilon_{c} \left[ 1 + 5 \left( \frac{f_{cc}}{f_{c}} - 1 \right) \right] \tag{2}
\]

where \( \varepsilon_{c} \) is the strain at the peak stress of plain concrete cylinders. Richart did not propose a stress-strain curve for confined concrete. For analysis in this study it was assumed that the stress-strain curve is a parabola up to the peak point and a straight line afterwards connecting the peak point to a point with a stress of \( 0.75f'_{cc} \) and a strain of \( 2\varepsilon_{cc} \).

**Sheikh and Uzumeri** (1982): This model was created based on the concept of effectively confined concrete area. The stress-strain curve consists of three sections. The first section is a parabola with vertex at \( (f_{cc}, \varepsilon_{s1}) \). The term \( f_{cc} \) represents the compressive strength of confined concrete and is equal to \( Ks \cdot f'_{c} \), in which \( f'_{c} \) is the compressive strength of plain concrete and \( Ks \) is the strength gain factor. The second section of the model consists of a horizontal line that extends from \( \varepsilon_{s1} \) up to \( \varepsilon_{s2} \). Finally the last section consists of a straight line with slope \( Z \) that extends to a stress about 30% of the maximum value. The \( \varepsilon_{s85} \) is the value of strain corresponding to 85% of \( f_{cc} \) on the third section of the curve. The increase in strength of confined concrete is calculated based on the effectively confined area. This area is less than the core area and defined by the distribution of longitudinal steel fully supported by the bend of a tie. \( Ks \) is evaluated by the following equation, in which \( A_{co} \) is the core area enclosed by the centerline of outer tie; \( n \) is the number of arcs; \( C \) is the center-to-center distance between longitudinal bars; \( s \) is the tie spacing; \( B \) is the width of the section; and \( H \) is the height of the section.

\[
Ks = 1.0 + \frac{1}{P_{occ}} \left[ \sum_{i=s}^{n} C_{i} \right]^{2} \left[ 1 - \frac{n}{\alpha A_{co}} \right] \left[ 1 - 0.5s \tan \theta \right] \left[ 1 - 0.5s \tan \theta \right] \tag{3}
\]

\( \varepsilon_{s1} \) is the minimum strain corresponding to the maximum concrete stress and is defined by the following equation where \( f'_{c} \) is in psi.
\[ \varepsilon_{s1} = 0.55K_s f'_c \times 10^{-6} \]  

(4)

\[ \varepsilon_{s2} \] is the maximum strain corresponding to the maximum concrete stress and is defined by the following equation.

\[ \varepsilon_{s2} = \varepsilon_c \left( 1 + \frac{0.81}{C} \left( 1 - 5 \left( \frac{s}{B} \right)^2 \right) \rho_s f'_s \sqrt{f_c} \right) \]  

(5)

The slope of the third section of the stress-strain curve is defined as:

\[ Z = \frac{-2 f_{co}}{3 \rho_s \sqrt{B/S}} \]  

(6)

According to the experimental tests conducted by Sheikh and Uzumeri (Sheikh et. al., 1982) most of the specimens reached a maximum strain corresponding to 0.85 times the maximum confined concrete stress.

\[ \varepsilon_{s85} = 0.225 \rho_s \sqrt{B/S} + \varepsilon_{s2} \]  

(7)

**Mander, Priestly, and Park** (1988): This model is applicable to both circular and rectangular transverse reinforcement. The main equation for the monotonic stress-strain curve for confined concrete is defined as:

\[ f_c = \frac{f_{cc} x_r}{r - 1 + x^r} \]  

(8)

where, \( x \) is the ratio of strain, \( \varepsilon_c \), to the strain at peak stress; \( \varepsilon_{cc} \), \( f_{cc} \) is the peak stress for confined concrete, and \( r \) is the ratio of concrete’s initial modulus of elasticity to the difference of initial and secant moduli of elasticity. \( \varepsilon_{cc} \) and \( r \) are defined as follows.

\[ \varepsilon_{cc} = \varepsilon_c \left[ 1 + 5 \left( \frac{f_{cc}}{f'_c} \right) \right] \]  

(9)

\[ r = \frac{E_c}{E_c - E_{sec}} \]  

(10)

where \( f'_c \) and \( \varepsilon_c \) represent the plain concrete strength and corresponding strain. \( \varepsilon_c \) is usually assumed as 0.002. \( E_c \) is the tangent modulus of elasticity of the concrete and can be evaluated as:

\[ E_c = 5000 \sqrt{f'_c} \text{ MPa} \]  

(11)
The effective lateral confining pressure $f_l$ is determined based on the same principle of effectively confined area that Sheikh used to develop his stress-strain model. Therefore, the arches produced between the levels of transverse steel are considered as ineffectively confined core. Midway between the levels of transverse reinforcement, the area of ineffectively confined concrete will be the largest. The arching action is assumed to form a parabola with an initial tangent-slope of 45°. Therefore each different configuration of transverse steel reinforcement will have a different confinement effectiveness coefficient given by:

$$K_e = \frac{A_e}{A_{cc}}$$

(13)

where $A_e$ is the area of effectively confined concrete core and

$$A_{cc} = A_c (1 - \rho_{cc})$$

(14)

where $\rho_{cc}$ is the ratio of longitudinal reinforcement to the area of core section and $A_c$ is the area of concrete within the centerlines of the perimeter spiral or hoop.

The effective lateral confining pressure is then defined as:

$$f_l' = f_l K_e$$

where $f_l = \frac{1}{2} \rho_s f_{yh}$

(15)

and $\rho_s$ is the ratio of the volume of transverse confining steel to the volume of confined concrete core.

Confined compressive stress is formulated based on a multi-axial failure criterion and is given as:

$$f_{cc} = f_c' \left( -1.254 + 2.254 \sqrt{1 + \frac{7.94 f_l'}{f_c} - 2 f_l' \frac{f_l'}{f_c}} \right)$$

(16)

The ultimate concrete strain, $\varepsilon_{cu}$ is assumed to occur after the first hoop fracture. An energy balance approach was used to predict when the first hoop fracture occurs. Complete description of the model can be found elsewhere (Mander et. al, 1988).

**Cusson and Paultre** (1995): This model does not assume that the stirrup yields at the peak strength of confined concrete, and the actual stress in the stirrups at the peak concrete stress is estimated through an iterative process (Cusson et. al., 1995). The ascending branch of stress-strain curve, originally proposed by Popovics, is described as:
\[ f_c = f_{cc} \left( \frac{k \left( \frac{\varepsilon}{\varepsilon_{cc}} \right)}{k+1+\left( \frac{\varepsilon}{\varepsilon_{cc}} \right)} \right), \quad \varepsilon \leq \varepsilon_{cc} \]  

(17)

where:

\[ k = \frac{E_c}{E_c - \left( \frac{f_{cc}}{\varepsilon_{cc}} \right)} \]  

(18)

\[ f_{cc} \] is the peak stress of confined concrete corresponding to a strain \( \varepsilon_{cc} \). \( K \) is a parameter which controls the initial slope and the curvature of the ascending branch, and \( E_c \) is the modulus of elasticity of concrete.

The curve for the ascending branch was originally proposed by Fafitis and Shah but modified by Cusson and Paultre.

\[ f_c = f_{cc} \exp \left( k_1 \left( \varepsilon_{c50c} - \varepsilon_{cc} \right)^{k_2} \right), \quad \varepsilon \leq \varepsilon_{cc} \]  

(19)

where

\[ k_1 = \frac{\ln 0.5}{(\varepsilon_{c50c} - \varepsilon_{cc})^{k_2}} \quad \text{and} \quad k_2 = 0.58 + 16 \left( \frac{f_{le}}{f_{co}} \right)^{1.4} \]  

(20)

The coefficient \( k_1 \) controls the slope of the descending branch, whereas the coefficient \( k_2 \) controls its curvature.

The strength and ductility of the confined concrete is based on the effective confinement pressure. Cusson and Paultre followed the same approach used by Mander to find the effective confinement pressure.

**Fafitis and Shah** (1985): The governing equations to define the stress-strain relationship are:

\[ f_c = f_{cc} \left( 1 - \left( 1 - \frac{\varepsilon}{\varepsilon_{cc}} \right)^A \right) \quad \text{for} \quad 0 \leq \varepsilon \leq \varepsilon_{cc} \]  

(21)

\[ f_c = f_{cc} \exp \left[ -k (\varepsilon - \varepsilon_{cc})^{1.15} \right] \quad \text{for} \quad \varepsilon > \varepsilon_{cc} \]

where \( f_{cc} \) is the peak stress of confined concrete and \( \varepsilon_{cc} \) is the corresponding strain. \( A \) and \( K \) are the parameters that control the ascending and descending branches of the stress-strain curve respectively and are given by the following equations

\[ A = \frac{E_c \varepsilon_{cc}}{f_{cc}} \quad \text{and} \quad k = 0.17 f_{cc} \exp \left( -0.01 \frac{f_{le}}{\lambda_1} \right) \]  

(22)
where $E_c$ is the initial modulus of elasticity of the concrete and $\lambda_1$ is a parameter that depends on the strength of the concrete and the degree of confinement

$$\lambda_1 = 1 - 2.5 \frac{f_r}{f_c} \left[ 1 - \exp \left( - \frac{f_c}{6500} \right) \right]^{25} \tag{23}$$

The peak stress $f_{cc}$ and corresponding strain $\varepsilon_{cc}$ are given in the following equations:

$$f_{cc} = \lambda_2 \left[ f'_c + \left( 1.15 + \frac{3048}{f'_c} \right) f_r \right] \tag{24}$$

$$\varepsilon_{cc} = 1.027 \times 10^{-7} f'_c + 0.0296 \lambda_2 \frac{f_r}{f'_c} + 0.00195 \tag{25}$$

where $\lambda_2$ is a parameter that depends on the relative confinement ($f_r/f'_c$) and $f_r$ is the confinement pressure based on the assumption that the lateral steel has yielded.

$$\lambda_2 = 1 + 15 \left( \frac{f_r}{f'_c} \right)^{3} \text{ and } f_r = \frac{2 A_s f_y}{s D} \tag{26}$$

where $A_s$ is the cross sectional area of the spiral or hoop, $f_y$ is the yielding stress of the lateral reinforcement, $s$ is the spacing of the lateral reinforcement, and $D$ is the core diameter of the column. For square columns the lateral confinement pressure is calculated assuming an equivalent circular column with an effective diameter equal to the side of the confined square core (Fafitis et. al. 1985).

**COMPARISON OF RESULTS AND DISCUSSIONS**

As mentioned earlier analytical predictions for the moment curvature response of the section, using various models were very close to each other, for most of the load cases. To investigate the relative effect of the peak strength of concrete on the peak flexural strength and the corresponding curvature, for the sections studied here, a range of plain-concrete compressive strengths starting from 3.65 ksi (25 MPa) and growing up to 21.9 ksi (151 MPa) were used and a monotonic moment-curvature analysis was performed for several axial load cases. Results are summarized in Figure 1. When no axial load is applied on the circular section, increasing the compressive strength of concrete 100% (double the strength) leads to only 5.5% increase in flexural strength while all other parameters such as section geometry, reinforcement arrangement and properties remain fixed. For the same situation under an axial load equal to 400 kips (1780 kN), corresponding to approximately 30% $\frac{f'_c}{f'_c}$, the increase in flexural strength is 26%, and when the axial load is 800 kips (3559 kN) this increase will jump to 90%. Increasing the plain concrete strength to 6 times as much as the original strength (500% increase), increases the flexural capacity for 15% under no axial load and around 200% under a very high axial load. In any case, the increase rate in flexural strength is much slower
compared to the increase in concrete strength. A similar trend was observed for a rectangular section.

In Table 1, the maximum confined concrete strengths evaluated by the models are summarized. The enhancement in compressive strength is at most 45% and especially the differences between various models are far less than this value. This will not lead to a considerable difference between moment-curvature analysis results for the section using various confined concrete models even at a high level of axial load. However, the ultimate strain of confined concrete in the model plays a considerable role in cases with a non-proportionally variable axial load.

It should be noted that for cases with a low level or no axial load, the model used for steel is more critical in general response of the section, and even for relatively high axial load levels, any change in steel strength has a more pronounced effect on flexural response of the section. This issue is not discussed here considering the scope of the report.

As a representative case, for a cyclic curvature under a constant axial load of 30% $\frac{A_g f_c}{c}$ analytical moment curvature results using Cusson and Paultre's model are compared with experimental data in Figure 7. Predictions by this and all other models for this loading case were very conservative.

It has been observed that for a larger compression zone due to a higher axial load level, analytical results are more conservative. As is evident from the models studied, level of axial load, or depth of the compression zone on the section does not play any role in the models in terms of its strength and ductility. In other words, for the dark fiber shown in Figure 2, all of the models will give the same stress for cases (a), (b) and (c) if the strain is the same, while it is more realistic to relate the strength and ductility of the model to the degree of confinement utilization. This is conceptually illustrated by the curves in terms of the change in peak stress and strain. When a larger portion of the section is in compression, the confinement is utilized more. The assumption of yielded lateral reinforcement in most of the models implies that the confinement steel is fully utilized which is not a realistic assumption. So, if the model with this assumption is scaled against experimental data in which the axial load level has been low or moderately high, predictions using such a model for a high axial load will be conservative. Even in some models like Cusson where the real stress in confining steel corresponding to the peak confined concrete strength is found through an iterative process and used in the model, the aforesaid fact is not addressed. So, it is important to implement this effect in the model for a more realistic stress-strain curve for confined concrete both in terms of the peak strength and its corresponding strain.

For the loading case with a cyclic curvature and a proportionally variable axial load analytical moment curvature results using Sheikh and Uzmeri's model are compared with experimental data in Figure 8. Axial load was positive for negative curvatures and negative for positive curvatures. The peak absolute values of axial load were approximately within 1% $\frac{A_g f_c}{c}$. For this low level of compressive axial load, predictions were close to the experimental results, and when the axial load was negative (tensile axial load), analysis slightly overestimated the real values.
Figure 9 compares analytical moment-curvature results for one curvature cycle without any axial load, using Mander’s model for confined concrete with test results. The maximum experimental curvature was dictated by the testing facilities and the section may have achieved more curvature and in turn moment before failure. It is evident from the figure that predictions get more conservative for higher curvatures. This may be due to the fact that a larger portion of the section will experience compression to compensate for the tensile forces developed by steel for higher curvatures, leading to a more conservative analytical prediction.

Analytical predictions using models by Sheikh and Cusson are compared with test results in Figure 10 and Figure 11. Cusson and Paultré’s model leads to a better agreement with experimental data. Using the actual stress in confining steel corresponding to the peak compressive strength of confined core which is found through an iterative process and the relatively higher strain at the peak strength may be a major factor for this agreement.

CONCLUSIONS

Several models for confined concrete stress-strain relationship were used to analytically predict the moment-curvature response of a circular reinforced concrete section under certain loading patterns. These predictions were then compared with the test results for the corresponding loading cases.

Analytical predictions for the moment curvature response of the section using various models were very close to each other, for most of the loading cases. This is due to the fact that under low to moderate axial loads, the change in the concrete compressive strength has a very small effect on the flexural strength of the section. The difference between the peak compressive strength of the models studied here was within 50% which resulted in similar analytical responses by all models for most of the loading cases. The effect of steel properties is pronounced more for cases for low level of axial load and is considerable for other cases. This is not within the scope of this paper and is not addressed. For a monotonically increasing curvature and non-proportionally variable axial load, predictions by models with a higher strain corresponding to peak strength of confined concrete, such as Cusson’s model, or models with a higher strain capacity were more realistic.

For a high level of axial load with a monotonic or cyclic curvature, all of the models underestimated the flexural strength and overestimated the ductility. For cases with low or no axial load, predicted flexural strength was close to experimental results, but for a tensile axial load, the strength was slightly overestimated. The displacement capacity of the section was underestimated by all of the models for no or very low axial load cases. As discussed earlier, this behavior is related to the fact that axial load level, or in general depth of compression zone, as a factor directly related to the degree of utilization of the confining material, is not addressed and considered by the models studied so far.
REFERENCES


Table 1 – Properties of confined concrete per various models.

<table>
<thead>
<tr>
<th>Concrete Model</th>
<th>Peak Stress ksi (MPa)</th>
<th>Strain at Peak Stress</th>
</tr>
</thead>
<tbody>
<tr>
<td>Plain Concrete</td>
<td>7.3 (50.3)</td>
<td>0.003</td>
</tr>
<tr>
<td>Richart</td>
<td>10.7 (73.8)</td>
<td>0.0066</td>
</tr>
<tr>
<td>Mander</td>
<td>9.44 (65.1)</td>
<td>0.007</td>
</tr>
<tr>
<td>Sheikh</td>
<td>7.4 (51.0)</td>
<td>0.004</td>
</tr>
<tr>
<td>Cusson</td>
<td>12 (82.7)</td>
<td>0.015</td>
</tr>
<tr>
<td>Fafitis</td>
<td>7.4 (51.0)</td>
<td>0.0046</td>
</tr>
</tbody>
</table>

Figure 1 – Relative effect of concrete strength on flexural response for a circular (left) and rectangular (right) section.

Figure 2 – A fiber under the same compressive strain in three different cases.
Figure 3 – Steel monotonic stress-strain model, developed and used in the analytical program.

Figure 4 – Steel hysteretic stress-strain model developed and used in the analytical program.

Figure 5 – Concrete hysteretic behavior curve based on the rules in the analysis.
Figure 6 – Simulating a model by the custom-model option.

Figure 7 – Analytical moment curvature results using Cusson and Paultre’s model compared to the experimental data, for a cyclic curvature under a constant axial load of 30% $A_{g}f'_{c}$.

Figure 8 – Analytical moment curvature results using Sheikh and Uzmeri’s model compared to the experimental data, for a cyclic curvature under a proportionally variable axial load (with proportionality ratio of 66 in the US customary system).
Figure 9 – Analytical moment curvature results using Mander’s model compared to the experimental data, for one curvature cycle without any axial load.

Figure 10 – Analytical moment curvature results using Sheikh and Uzumeri’s model compared to the experimental data, for a monotonic curvature under a non-proportional axial load varying between $+30\% A_{g}f_{c}'$ and $-10\% A_{g}f_{c}'$.

Figure 11 – Analytical moment curvature results using Cusson and Paultre’s model compared to the experimental data, for a monotonic curvature under a non-proportional axial load varying between $+30\% A_{g}f_{c}'$ and $-10\% A_{g}f_{c}'$. 
Modeling of Stress-Strain Relationship and Deformability for Confined High-Strength Concrete

by H. Kinugasa, Y. Xiao, and A. Martirosyan

Synopsis: Some of the most widely referenced models for stress-strain relationship of confined concrete columns with square cross section are reviewed, and the performance under monotonic compressive loading are evaluated based on a database in which compressive strength ranges from 40 to 140 MPa and yield strength of transverse reinforcement ranges from 400 to 1400 MPa. It is found that majority of the models examined can predict the strength of confined concrete reasonably well, however, the predictability to the deformation is rather low. A model for predicting deformability of confined concrete column subjected to monotonic compressive loading is proposed in terms of strain energy provided by confining reinforcement up to the peak point and the point of 85% of the maximum stress beyond the peak point. Predictions based on the strain energy are found to be in good agreement with experimental results.

Keywords: confinement; deformability; high-strength concrete; performance; strain energy; stress-strain models
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INTRODUCTION

Large number of studies have been made on the modelling of stress-strain behavior of confined concrete. A model is required to accurately describe the ascending branch, the peak strength and strain and the descending branch. However, the discussion of the accuracy of models is a subjective matter. Very often a model is judged better by its developer using limited set of data. The authors attempt to propose a rational method based on “Four parameters” to compare stress-strain models with test results, which are taken to represent the whole stress-strain curve.

In this study, a large database of HSC columns with square cross section tested under monotonic and concentric axial loading was formed. Capabilities of several existing models are examined based on the four parameters using the database.

SUMMARY OF VARIOUS STRESS – STRAIN MODELS

This paper provides a review of some of the most widely referenced models. In this paper, only stress-strain relationship for columns having square cross section subjected to monotonic loading are considered.
Legeron and Paultre's model (2003)

This model was proposed based on strain compatibility and transverse force equilibrium. The model is capable of predicting the effectiveness of transverse reinforcement, which is the key in modelling the behaviour of HSC confined with high-yield-strength steel. The model is validated on test results from more than 200 circular and square large-scale columns tested under slow and fast concentric loading.

\[
f_{cc} = f'_{cc}\left[\frac{k(e_{cc}/e'_{cc})}{k-1+(e_{cc}/e'_{cc})^k}\right] \quad \varepsilon_{cc} \leq \varepsilon'_{cc} \quad f_{cc} = f'_{cc} \exp\left[k_1(e_{cc}-e'_{cc})^{k_2}\right] \quad \varepsilon_{cc} \geq \varepsilon'_{cc}
\]

\[
f'_{cc} = \left[1 + 2.4(I'_{c})^{0.7}\right]f'_{c} \quad \varepsilon'_{cc} = \left[1 + 35(I'_{c})^{1.2}\right]e_{c}
\]

\[
f'_{cc} = \frac{0.25f'_{c}}{\rho_{oy}(\kappa-10)} \geq 0.433e_cE_{c} \quad (\kappa > 10) \quad \varepsilon_{cc} = 0.5(\varepsilon_{c:50} - \varepsilon'_{cc})^2 \quad k_1 = \frac{\ln 0.5}{(\varepsilon_{c:50} - \varepsilon'_{cc})^2} \quad k_2 = 1 + 25(I_{c:50})^2
\]

\[
f'_{cc} = \rho_{oy}f'_{c} \quad I_{cc} = \frac{f'_{cc}}{f'_{c}} \quad \kappa = \frac{f'_{cc}}{\rho_{oy}E_{c}e_{c}} \quad \rho_{oy} = K_e \frac{A_{hy}}{s}\quad I_{c:50} = \frac{\rho_{oy}f'_{c}}{f'_{c}} \quad k = \frac{E_{ct}}{E_{ct} - (f'_{cc}/\varepsilon'_{cc})}
\]

\[K_e :\text{geometrical effectiveness coefficient introduced by Sheikh and Uzumeri (1982) and by Mander et al (1984)}\]

Li's model (1994)

Li conducted numerous tests on circular and square high-strength reinforced concrete columns. Based on the results, he modified Mander et al.'s model for predicting the performance of HSC columns with various types of reinforcement configurations.

\[
f_{cc} = E_c e_{cc} + f'_{cc} - E_c e'_{cc} \left(\varepsilon'_{cc}\right)^2 \quad 0 \leq \varepsilon_{cc} \leq \varepsilon'_{cc}
\]

\[
f_{cc} = f'_{cc} + \frac{f'_{cc} - f^o_{cc}}{f'_{cc} - e'_{cc}} (e_{cc} - e'_{cc})^2 \quad \varepsilon_{cc} \leq \varepsilon_{cc} \leq \varepsilon'_{cc}
\]

\[
f_{cc} = f'_{cc} - \beta f'_{cc} (e_{cc} - e'_{cc}) \quad \varepsilon'_{cc} \leq \varepsilon_{cc}
\]

\[
f'_{cc} = f'_{c'} \left[-0.413 + \frac{2f'_{c'}}{f'_{cc}} - 1.413 \left[1 + \frac{11.4f'_{c'}}{f'_{cc}}\right]\right]
\]

\[
\varepsilon'_{cc} = \varepsilon'_{c'} \left[1 + 11.3 \left(\frac{f'_{c'}}{f'_{cc}}\right)^{0.7}\right] \quad 60 \leq f'_{c'} \leq 80 \quad \varepsilon_{cc} = \varepsilon'_{cc} \left[-8.1 + 9.1 \exp\left(\frac{f'_{c'}}{f'_{cc}}\right)\right] \quad f'_{c'} \geq 80
\]
\[
\beta = (0.048f' - 2.14) - (0.098f' - 4.57)\sqrt{\frac{f''}{f'}} \quad f'' = \frac{1}{2}K_c\rho_s f_y
\]

Unit: MPa, mm

\(K_c\): confinement effectiveness coefficient.

**Mander, Priestley and Park’s model (1988)**

This model has been widely used in analyzing columns with both circular and rectangular cross section. The transverse reinforcement can be of different types, circular or spiral, rectangular hoops together with cross ties or without cross ties. For development of this model, tests on full scale confined reinforced concrete columns, with concrete strength of 30 MPa and steel yield strength of about 300 MPa were conducted.

\[
f_{cc} = \frac{f_{cc} \cdot x}{r + 1 + x'}
\]

\[
f''_{cc} = f_c \left(1 - 1.254 - \frac{2f''}{f_c} + 2.254 \sqrt{1 + \frac{7.94f''}{f_c}}\right) \quad \varepsilon''_{cc} = \left[R\left(\frac{f''_{cc}}{f_c} - 1\right) + 1\right]\varepsilon_c
\]

Unit: MPa, mm

\[
f'' = \frac{1}{2}K_c\rho_s f_y
\]

\[
K_c = \frac{1 - \sum w_i (6/BH) \left(1 - \frac{s'}{2B}\right) \left(1 - \frac{s}{2H}\right)}{(1 - \rho_c)}
\]

\(w_i\): \(i\)th clear transverse spacing between adjacent longitudinal bars, \(R\): empirically determined coefficient (\(R\) varies from 3 to 6, depending on the concrete strength).

**Saatcioglu and Razvi’s model (1992)**

The stress-strain curve consists of two segments. The first segment is parabolic ascending branch; the second part is a linear descending branch. Lateral reinforcement in the form of equivalent uniform lateral pressure was used to develop the model characteristics for strength and ductility. The model has been compared with different types of column tests, including circular, square and rectangular.

\[
f_{cc} = f_{cc}' \left(2\left(\frac{\varepsilon_{cc}}{\varepsilon_{cc}'} - \left(\frac{\varepsilon_{cc}}{\varepsilon_{cc}'}\right)^2\right)^{\frac{1}{21 + 2k}}\right) \leq f_{cc}'
\]

\[
\varepsilon_{85} = 260\rho_c \varepsilon_c' + \varepsilon_{085}
\]

\[
f_{cc}' = f_c' + k_2f_c
\]

\[
\varepsilon_{cc}' = \varepsilon_c'(1 + 5K)
\]

\[
K = \frac{k_1f_{cc}}{f_c}
\]

\[
k_1 = 6.7(f_c)^{0.37}
\]

\[
f_{cc} = k_2f_c
\]

\[
k_2 = 0.26 \left[\sqrt{\frac{B}{s}} \left(\frac{H}{s} - \frac{1}{f_c}\right)\right] \leq 1.0
\]

\[
\rho_c = \frac{\sum A_i}{s(B + H)}
\]

\[
f_c = \sum A_i f_{yw} \sin \alpha
\]

Unit: MPa, mm

\(\varepsilon_{085}\): strain corresponding to 85% strength level beyond the peak stress of unconfined concrete (if no test data is available, \(\varepsilon_{085} = 0.0038\)), \(s\): spacing of laterally supported longitudinal reinforcement.
Sun and Sakino’s model(1990)

The concrete model confined by square hoop reinforcement or circular reinforcement was proposed based on many tests of circular and rectangular confined concrete columns. The coefficients in evaluating equations of maximum strength were determined to match with a variety of experimental data in which several types of transverse reinforcement were used. The maximum strengths of concrete and transverse reinforcement used in examined specimens were 132 and 1109MPa, respectively.

\[ f'_{cc} = f'_c + \kappa \rho f_{hy} \quad \text{if } f_{hy} \leq 687 \text{MPa} \]
\[ \varepsilon'_{cc} = \varepsilon'_c \left(1 + 4.7(K - 1)\right) \quad (K < 1.5) \]
\[ \varepsilon'_{cc} = \varepsilon'_c \left(3.35 + 20(K - 1.5)\right) \quad (K \geq 1.5) \]
\[ Y = \frac{AX + (D - 1)X^2}{1 + (A - 2)X + DX^2} \]

Unit : MPa, mm

\[ K = 11.5(\phi / C')(1 - 0.5s / B) \]
\[ Y = f'_{cc} / f'_c \]
\[ X = \frac{\varepsilon_{cc}}{\varepsilon'_c} \]
\[ A = E_{ct} / f'_{cc} \]
\[ \varepsilon'_c = 0.93(f'_{cc})^{1/4} \times 10^{-3} \]
\[ \gamma = 16 \]
\[ D = 1.5 - 1.71 \times 10^{-2} f'_c + \gamma \sqrt{(K - 1)f'_c / 2.3} \]
\[ E_{ct} = 4.1k(f'_c / 100)^{1/3} \times 10^4 \]

\[ C': \text{ length between effective supports of transverse reinforcement (mm), } \phi : \text{ diameter of hoop (mm), } k \text{ varies from 0.9 to 1.0, depending on the quality of aggregate.} \]

Sheikh and Uzumeri’s model(1982)

This model is one of the earliest ones developed for prediction of stress-strain relationship for confined concrete. The model was developed based on experimental results of 24 tests, as well as number of tests conducted by other researchers. The proposed stress-strain curve consists of three segments. The first segment is parabolic ascending branch with its center coordinates \((f'_{cc}, \varepsilon'_{cc1})\); the second part is horizontal line from \((f'_{cc}, \varepsilon'_{cc1})\) to \((f'_{cc}, \varepsilon'_{cc2})\); and the third section is an inclined line with a slope \(Z\). It continues up to the point of \(0.3f'_{cc}\), after which it again continues horizontally.

\[ f'_{cc} = K_s f'_c \]
\[ \varepsilon'_{cc1} = 80K_s f'_c \times 10^{-6} \]
\[ \varepsilon'_{cc2} = \varepsilon'_c \left[1 + \frac{248}{C} \left(1 - 5.0 \left(\frac{\rho s}{B} f_{hy} / f'_c\right) \right)^2 \right] \]
\[ Z = \frac{0.5}{3 - 4 \rho s} \sqrt{\frac{B}{s}} \]
\[ K_s = 1.0 + \frac{B^2}{140f_{occ}} \left\{1 - \frac{nC^2}{5.5B^2} \left(1 - \frac{s}{2B}\right)^2 \right\} \sqrt{\rho_s f_{hy}} \]
\[ f_{occ} = f'_c (A_0 - At) / 1000 \]
\[ A_0 = B \times H \]

Unit : MPa, mm

\[ n: \text{ number of curvatures between the longitudinal bars, } At: \text{ total cross section of longitudinal bars.} \]

Watanabe and Muguguma’s model(1993)

The concrete model confined by square or circular hoop reinforcement have been studied by Watanabe and Muguguma et al. for a long time. The concrete strength of examined specimens ranged from 26 MPa to 130 MPa and that of transverse reinforcement ranged from 191 MPa to 1353MPa for square specimens.
\[ f'_{cc} = (1 + 49C_c)f'_c \quad \varepsilon'_{cc} = (1 + 341C_c)\varepsilon'_c \]

\[ f'_{cc} = E_c\varepsilon'_{cc} + \left((f'_c - E_c\varepsilon'_c)/\varepsilon'_c\right)^2 \quad 0 < \varepsilon'_{cc} < \varepsilon'_c \]

\[ f'_{cc} = \frac{(f'_c - f'_{cc})}{(\varepsilon'_c - f'_{cc})^2} (\varepsilon'_{cc} - \varepsilon'_c)^2 + f'_c \quad \varepsilon'_c < \varepsilon'_{cc} < \varepsilon'_c \]

\[ f'_{cc} = \frac{(f'_{up} - f'_{cc})}{(\varepsilon'_{cc} - \varepsilon'_{up})} (\varepsilon'_{cc} - \varepsilon'_{up}) + f'_c \quad \varepsilon'_{cc} < \varepsilon'_{up} \]

Unit: MPa, mm

\[ \varepsilon_{up} = (1 + 611C_c)\varepsilon_e \quad f_{up} = \frac{2(A_2 - f'_{cc}\varepsilon'_{cc})}{\varepsilon_{cc} + \varepsilon_{up}} + f'_{cc} \quad \varepsilon'_e = 0.0013(1 + f'_c/98.6) \]

\[ \varepsilon'_{cc} = \sqrt{(0.008 - \varepsilon'_c)\varepsilon'_c - (0.004 - \varepsilon'_c)2A_1/\varepsilon'_c} \quad A_1 = \varepsilon'_e (E_e\varepsilon'_e + 2f'_c)/6 \quad E_{ct} = 22700\sqrt{f'_c/19.6} \]

\[ C_c = 0.31312 f_{hy} / f'_c (1 - 0.5S / B) \]

\[ A_2 : \text{area between the idealized s-s curve and X-axis to the peak point} \]

**Yong, Nour and Nawy’s model (1988)**

This model was developed based on experimental results of HSC columns subjected to monotonic axial loading, in which the effects of rectilinear confinement were investigated. Twenty-four columns of HSC were tested. The concrete strength ranged from 84 to 94 MPa. All specimens failed with a single shear failure plane.

\[ f'_{cc} = Kf'_c \quad Y = \frac{AX + DX^2}{1 + (A - 2)X + (D + 1)X^2} \quad \varepsilon'_{cc} \leq \varepsilon'_{cc} \quad \varepsilon'_e = 0.00265 + \frac{0.008(1 - 0.734S / B)}{\sqrt{f'_c}} (\rho_s f_{hy})^{2/3} \]

\[ Y = f_{cc} / f'_c \quad X = \varepsilon_{cc} / \varepsilon'_e \quad E_{ct} = 36.78w^{15} \sqrt{f'_{cc}} \quad \text{Unit: MPa, mm} \]

\[ D = \left[ \frac{(A - 1)^2}{0.55} \right] - 1 \quad A = E_{ct}\varepsilon_{cc} / f'_c \quad K = 1 + 0.11 \left( 1 - \frac{0.245S}{B} \right) \left( \rho_s + \frac{n\phi_s}{0.31\phi_f \rho_l} \right) \sqrt{f'_c} \]

Similar polynomial equation was used for descending curve beyond peak. \( \rho_l \): Volumetric ratio of longitudinal bars, \( \phi_v \) and \( \phi_l \): Nominal diameter of transverse and longitudinal reinforcement, \( w \): Specific weight of concrete in kN/m³, \( n \): Number of longitudinal bars.
COMPARATIVE STUDIES

Definition of Characteristic Parameters

The modelling for confined concrete is required to precisely describe its ascending branch, peak value, and descending branch. The authors propose to establish a method based on four parameters, \( E_{50}, f'_{cc}, \varepsilon'_{cc}, \) and \( \varepsilon_{85} \), which are taken to represent the whole stress-strain curve as shown in Fig.1 (Martirossyan 1998). The four parameters consist of peak stress \( f'_{cc} \), peak strain \( \varepsilon'_{cc} \), modulus of elasticity at 50% of strength, \( E_{50} \), and strain at 15% stress degradation beyond peak point, \( \varepsilon_{85} \). Parameters \( E_{50} \) and \( \varepsilon_{85} \) represent the behavior of ascending and descending branch respectively. Ultimate strain is an important parameter for ductility predictions in seismic design. However, there is no universally accepted definition for the ultimate strain of confined concrete. Sheikh et al. (1982) suggested to use the point where the compression stress has degraded to 85% of the peak strength as the ultimate state for confined concrete. It varies from 85 to 50%, depending on researchers. Since descending part of stress-strain curve beyond peak point tends to be unstable especially when the transverse reinforcement is not sufficient or the concrete strength is very high, it tends to be difficult to obtain the true characteristics of deformability. In order to avoid the decrease in reliability, strain at 15% stress degradation which is relatively small degradation point is used as the ultimate strain, similar to Sheikh et al.'s suggestion.

Data Base of HSC

A database (Martirossyan 1998) of HSC columns with square cross section tested under monotonic and concentric axial loading was used. The data base includes tests conducted by six different researcher groups; Cusson and Paultre (1994), Li (1994), Nagashima et al. (1992), Sakino et al. (1990), Yong et al. (1988), and Martirossyan and Xiao (1998). The range of material strength is shown in Fig.2. The compressive strength of unconfined concrete ranges from about 40 to 140 MPa and yield strength of transverse reinforcement ranges from about 400 to 1400 MPa. The range of volumetric ratio of transverse reinforcement is shown in Fig.3. The volumetric ratio ranges from about 0.002 to 0.055. The database includes five configurations of transverse reinforcement shown in Fig.4.

Evaluation of Existing Models

The models reviewed in the previous section have each range of applicability concerning material strength. Since in the following evaluation of the models, some models are applied beyond their range of applicability, this section is not to show which model is more excellent. The purpose of this section is to investigate the applicability and the present stage of the modelling.
Fig. 5 illustrates the performance of the models for peak stress $f_{cc}$, in which hollow symbols indicate specimens which yield strength of confinement is larger than 1000 MPa. Li’s and Mander et al.’s model have a tendency to overestimate the strength of specimens with high-strength confining steel. This is considered to result from the assumption that confining steel reaches yield point at maximum stress. This may be approximately correct for normal-strength confining steel, however for high-strength confining steel it is not accurate. High performance is obtained from Sun & Sakino, Watanabe & Muguruma, and Legeron & Paulitre’s model which took into account the effect of high-strength steel on peak stress.

Fig. 6 and Fig. 7 illustrate the performance of models for strain at maximum stress, $\varepsilon_{cc}$, and strain at 85% of peak stress, $\varepsilon_{85}$, respectively. It should be noted that all the models gave scattered predictions for them. This is remarkable when compared with the performance for maximum stress $f_{cc}$ shown in Fig. 5. Confining concrete provides an increase in deformability as well as an increase in strength. Although many studies have been made, focusing on the peak stress rather than the deformability, it is clear that the significance of confinement is an increase in plastic energy and plastic deformation, and the performance of deformability greatly affects the results of RC member analysis. In the next chapter, a prediction model for strains $\varepsilon_{cc}$ and $\varepsilon_{85}$ will be proposed in terms of strain energy which is provided by confinement.

DEFORMABILITY MODEL FOR CONFINED CONCRETE

Energy Provided by Confinement

Mander et al. (1988) proposed an energy balance approach to estimate ultimate strain. The idea that the effect of confinement can be analyzed in terms of energy balance should be highly evaluated. Stress-strain relationships for confined and unconfined concrete are illustrated in Fig. 8. The area between these curves (shown shaded in Fig. 8) indicates the strain energy provided by confining reinforcement, and the value is considered to be affected by amount and its effectiveness of confinement. Many researchers have investigated deformability of confined concrete based on the gain in ultimate strain due to confinement. However, since the gain in ductility results from the increase in strain energy due to confinement, it is essential to directly investigate the strain energy and calculate the corresponding strain. Moreover, considering that the significance of confining concrete in hinge region is to increase energy dissipation as well as ductility, it is meaningful to examine the amount of strain energy provided by confinement.

The strain energy provided by confinement up to the peak point, $E_{Gcc}$ (the area A in Fig. 8(1)), and the strain energy up to the strain at 85% of peak stress, $E_{G85}$ (the area A+B in Fig. 8(1)), were calculated and examined. However, since it is difficult to precisely calculate the areas, they are simplified as shown in Fig. 8(2). Although the simplification
decreases the accuracy a little especially in EGcc, it is considered that the essential
tendency will never be lost.

EGcc and EG85 can be calculated as follows.

\[
EG_{cc} = \frac{1}{2} \left[ \bar{\varepsilon}_{cc} (f'_c + f'_e) - \bar{\varepsilon}_{cc} (2f'_e - \gamma \bar{\varepsilon}_{cc}) \right] \quad \varepsilon'_{cc} \leq \varepsilon_{cu}
\]
\[
= \frac{1}{2} \left[ \bar{\varepsilon}_{cc} (f'_c + f'_e) - \bar{\varepsilon}_{cu} f'_c \right] \quad \varepsilon_{cu} \leq \varepsilon'_{cc} \quad --- (1)
\]

\[
EG_{85} = \frac{1}{2} \left[ \bar{\varepsilon}_{cc} (f'_c + f'_e) + 1.85 f'_e (\bar{\varepsilon}_{85} - \bar{\varepsilon}_{cc}) - \bar{\varepsilon}_{85} (2f'_e - \gamma \bar{\varepsilon}_{85}) \right] \quad \varepsilon_{85} \leq \varepsilon_{cu}
\]
\[
= \frac{1}{2} \left[ \bar{\varepsilon}_{cc} (f'_c + f'_e) + 1.85 f'_e (\bar{\varepsilon}_{85} - \bar{\varepsilon}_{cc}) - \bar{\varepsilon}_{cu} f'_c \right] \quad \varepsilon_{cu} < \varepsilon_{85} \quad --- (2)
\]

where \( \bar{\varepsilon}_{cc} = \varepsilon_{cc} - \varepsilon_c \), \( \bar{\varepsilon}_{85} = \varepsilon_{85} - \varepsilon_c \), \( \bar{\varepsilon}_{cu} = \varepsilon_{cu} - \varepsilon_c \), \( \gamma = f'_c / \varepsilon_{cu} \), \( \varepsilon_{cu} \) is assumed to be 0.01. The other notations were defined in Fig.8.

In Fig.9(1) and 9(2), the vertical axes show the experimental strain energy \( EG_{cc} \) and \( EG_{85} \) respectively, and horizontal axes show a ratio, \( \rho h f_{hy} / f'_c \), where \( \rho h = A_{hy} / (B \cdot S) \); \( A_{hy} \) = total cross section of transverse reinforcement in x or y direction within \( S \) (= spacing of transverse reinforcement); \( B \): concrete core dimension. The ratio \( \rho h f_{hy} / f'_c \) is similar to the so-called confinement index which has been used by many researchers to represent the effectiveness of confinement. As can be seen from these figures, it is clear that the strain energy \( EG_{cc} \) and \( EG_{85} \) have correlation with the ratio \( \rho h f_{hy} / f'_c \) and have an increasing tendency with that. The solid lines in these figures which describe the increasing tendency can be given by the following equations.

\[
EG_{cc} = 0.44 \times (\rho h f_{hy} / f'_c) \quad 0 < \rho h f_{hy} / f'_c \leq 0.14
\]
\[
= 5.8 \times (\rho h f_{hy} / f'_c) - 0.75 \quad 0.14 < \rho h f_{hy} / f'_c \quad --- (3)
\]

\[
EG_{85} = 2.18 \times (\rho h f_{hy} / f'_c) \quad 0 < \rho h f_{hy} / f'_c \leq 0.1
\]
\[
= 10.9 \times (\rho h f_{hy} / f'_c) - 0.87 \quad 0.1 < \rho h f_{hy} / f'_c \quad --- (4)
\]

**Evaluation of Proposed Model**

Solving Eqs.(1) and (2) for \( \varepsilon'_{cc} \) and \( \varepsilon_{85} \) gives

\[
\varepsilon'_{cc} = \frac{1}{2} \left( f'_c - f'_e + \sqrt{(f'_c - f'_e)^2 + 8\gamma \cdot EG_{cc}} \right)/2\gamma + \varepsilon'_c \quad \varepsilon'_{cc} \leq \varepsilon_{cu}
\]
\[
= \left( 2EG_{cc} + \bar{\varepsilon}_{cu} f'_c \right) \left( f'_e + f'_e \right) + \varepsilon'_c \quad \varepsilon_{cu} < \varepsilon'_{cc} \quad --- (5)
\]
\[ \varepsilon_{\text{AS}} = \left\{ (1.85 f'_c - 2 f'_c) + \sqrt{(1.85 f'_c - 2 f'_c)^2 - 4\gamma(2EG\varepsilon' + 0.85 f'_c) - 2EG\varepsilon'} \right\}/2\gamma + \varepsilon' \]

\[ = \left\{ 2EG\varepsilon' + \varepsilon'_c - \varepsilon'_c (f'_c + f'_c) \right\}/[1.85 f'_c] + \varepsilon'_c + \varepsilon' \]

\[ \begin{align*}
\varepsilon_{\text{AS}} & \leq \varepsilon_{\text{cu}} \\
\varepsilon_{\text{cu}} & < \varepsilon_{\text{AS}}
\end{align*} \quad \quad \text{Eq.}(6) \]

Fig.10(1) and Fig.11(2) compare the prediction of \( \varepsilon'_c \) and \( \varepsilon_{\text{AS}} \) calculated by Eqs.(5) \&(6) to the experimental results respectively. The predicted \( \varepsilon'_c \) and \( \varepsilon_{\text{AS}} \) show good performance compared with that of existing models which were shown in Fig.6 and Fig.7, indicating that the formulation based on strain energy is effective.

CONCLUSIONS

Several widely referenced models for stress-strain relationship of confined concrete columns with square section were reviewed in this study, and the performance under monotonic compressive loading was evaluated based on a database of columns with high-strength transverse reinforcement and concrete. As a result of the evaluation, it was found that the predictability for deformability is considerably lower than that for maximum stress. In this paper, a model for predicting deformability of confined concrete column subjected to monotonic compressive loading was proposed in terms of strain energy provided by confining reinforcement up to the maximum stress and 85% of the maximum stress beyond the peak point. Predictions based on the strain energy were found to be in good agreement with the experimental results.

NOTATIONS

The following notations are used in common in this paper.

- \( f'_c \): compressive strength of confined concrete, \( \varepsilon'_c \): axial strain corresponding to compressive strength of confined concrete, \( f'_u \): compressive strength of unconfined concrete, \( \varepsilon'_u \): axial strain corresponding to compressive strength of unconfined concrete, \( E_{ct} \): tangent modulus of elasticity of unconfined concrete, \( \rho_s \): volumetric ratio of transverse reinforcement, \( f_{hy} \): yield strength of unconfined concrete, \( E_s \): modulus of elasticity of transverse reinforcement, \( C \): center-to-center distance between longitudinal bars, \( s \): spacing of the transverse reinforcement, \( A_s \): the area of the transverse reinforcement, \( B \) and \( H \) are center-to-center distance of the perimeter hoop of the rectangular concrete core, \( s_c \): clear spacing between transverse reinforcement, \( \rho_{cc} \): ratio of area of longitudinal steel to the area of the core.

REFERENCES


Li, B., 1994, “Strength and Ductility of Reinforced Concrete Members and Frames Constructed Using High Strength Concrete,” Research Report ISSN 0110-3326, Department of Civil Engineering, University of Canterbury, Christchurch, New Zealand.


Figure 1 – Four parameters for confined concrete

Figure 2 – Range of material strength in database

Figure 3 – Range of volumetric ratio of transverse reinforcement in database
Figure 4 – Configurations of transverse reinforcement in database

Figure 5 – Compressive strength of confined concrete
Figure 6 – Strain at compressive strength of confined concrete
Figure 7 – Strain corresponding to 85% of compressive strength beyond peak point
Figure 8 – Strain energy up to maximum stress and 85% of maximum stress beyond peak

Figure 9 – Increase in strain energy $E_{Gcc}$ and $E_{G85}$ with confinement index $\rho_{fb}f_{fb}/f_c$

Figure 10 – Strain calculated by modeling of strain energy $E_{Gcc}$ and $E_{G85}$
Flexural Stress-Strain Curves of Confined High-Strength Concrete (ISCC-2004)

by T.-H. Tan and N.-B. Nguyen

Synopsis: This paper presents an analytical procedure for determining the stress-strain curves of concrete under flexure. The procedure is used to derive the stress-strain curves of confined concrete from tests of thirteen HSC columns confined by transverse reinforcement and subjected to different strain gradients. The effect of strain gradient to the stress-strain curves of confined concrete is observed and an analytical model for confined concrete that account for this effect is proposed.

Keywords: concrete; confinement; flexure; high strength; strain; stress
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INTRODUCTION

The confinement effect of transverse reinforcement in structural members subjected to compression has been well recognized. Under axial compression the core concrete expands laterally thus creating tension stress in the transverse reinforcement. As a result there is a passive confining from the reinforcement applied to the core through the contact area between the core and the reinforcement. Since this contact area is very small, only a part of core concrete is effectively confined as shown in Figure 1.

Experimental studies on confined concrete columns subjected to concentric loading have shown that the confinement from either circular or rectilinear hoops improves both strength and ductility of the core concrete. From these studies a number of confinement models have also been developed. For simplicity most of existing models assume the core concrete as confined concrete although the stress in the core concrete is not uniform as it consists both effectively and ineffectively confined concretes. Based on this assumption the stress of confined concrete can be computed from tests and the experimentally derived stress-strain curves of core concrete are used to develop confinement models. However there are still controversial opinions on the effect of strain gradient to the stress-strain curves of both plain and confined concrete. Therefore the applicability of those models developed from concentric tests still needs to be examined.

There are limited experimental studies on the confinement effect in confined concrete columns subjected to flexure. Unlike concrete subjected to concentric loading, it is very difficult to obtain the flexural stress-strain curves of plain and confined concretes. Alternatively there are some methods for determining the flexural stress-strain curves which are based on the strain compatibility and force and moment equilibriums. Based on the numerical differential method modified from the one used by Hoggestad et al., Ibrahim and MacGregor obtained the flexural stress-strain curves of core concrete in high-strength reinforced concrete columns. However this method is very sensitive to any small experimental errors that are unavoidable in most tests. Moreover the spalling of concrete cover is usually brittle and it can significantly affect the derivation of stress-strain curves of core concrete using such a method.

In this paper a procedure for determining the stress-strain curves of concrete from tests of specimens having T-shape cross-sections is presented. This procedure is used in combination with the procedure developed by the authors for rectangular cross-sections to derive the stress-strain curves of core concrete from the authors' tests on
square columns subjected to combination of compression and bending. Based on these stress-strain curves as well as the ones from concentric tests, a model for confined concrete is proposed.

**DETERMINATION OF FLEXURAL STRESS-STRAIN CURVES OF CONCRETE FROM T-SHAPE SECTIONS**

Figure 2 shows the assumption of stress and strain distributions in a T-shape section subjected to flexure where the most compressed fiber is at the flange. The stress-strain curve of concrete is assumed to follow a polynomial expressed by Eq. (1), of which degree $n$ and coefficients $a_i$ are to be determined.

$$F(\varepsilon_y) = a_1\varepsilon_y + a_2\varepsilon_y^2 + \cdots + a_{n-1}\varepsilon_y^{n-1} + a_n\varepsilon_y^n = \sum_{i=1}^{n} a_i\varepsilon_y^i$$  \hspace{1cm} (1)

The strain compatibility can be expressed as:

$$\frac{y}{c} = \frac{\varepsilon_y}{\varepsilon_{cm}}$$  \hspace{1cm} (2)

From Figure 2 the force equilibrium is written as:

$$P = \int_{y_0}^{y_w} b_w F(\varepsilon_y)dy + \int_{y_0}^{y_f} b_f F(\varepsilon_y)dy$$  \hspace{1cm} (3)

where $b_w$ and $b_f$ are respectively the width of the web and the width of the flange; $y_0$ and $y_w$ are distances as shown in the figure; $c$ is the neutral axis depth; and $P$ is the force carried by the concrete. Assuming the tensile stress of concrete is zero thus $y_0 = 0$ if $c \leq h$ where $h$ is the total height of the section.

Changing variables in Eq. (3) using Eq. (2) gives:

$$P = \frac{b_w}{\varepsilon_{cm}} \int_{y_0}^{\varepsilon_y} F(\varepsilon_y)d\varepsilon_y + \frac{b_f}{\varepsilon_{cm}} \int_{\varepsilon_y}^{y_f} F(\varepsilon_y)d\varepsilon_y$$  \hspace{1cm} (4)

Substituting $F(\varepsilon_y)$ from Eq. (1) into Eq. (4) and taking $f_0 = \frac{P}{b_w}\varepsilon_{cm}$ yields:

$$f_0 = \frac{b_w}{b_f\varepsilon_{cm}} \int_{y_0}^{\varepsilon_y} \left(\sum_{i=1}^{n} a_i\varepsilon_y^i\right)d\varepsilon_y + \frac{1}{\varepsilon_{cm}} \int_{\varepsilon_y}^{y_f} \left(\sum_{i=1}^{n} a_i\varepsilon_y^i\right)d\varepsilon_y$$  \hspace{1cm} (5)

Noting that for $\varepsilon_y = 0$ then $f_0 = 0$ and evaluating the integrations in Eq. (5) yields:

$$f_0 = \frac{b_w}{b_f\varepsilon_{cm}} \sum_{i=1}^{n} a_i \left(\varepsilon_y^{i+1} - \varepsilon_0^{i+1}\right) + \frac{1}{\varepsilon_{cm}} \sum_{i=1}^{n} a_i \left(\varepsilon_y^{i+1} - \varepsilon_0^{i+1}\right)$$  \hspace{1cm} (6)

Let $r_y^{i+1} = \varepsilon_y^{i+1}/\varepsilon_{cm} = y_0/c$; $r_w^{i+1} = \varepsilon_w/\varepsilon_{cm} = y_w/c$; rewrite Eq. (6) we have:

$$f_0 = \sum_{i=1}^{n} \frac{a_i}{1 + 1} \left(1 + \frac{b_w}{b_f} r_y^{i+1} - r_y^{i+1} - \frac{b_w}{b_f} r_w^{i+1}\right)$$  \hspace{1cm} (7)

The coefficients $a_i$ can be found in different ways depending on the testing condition. In case of columns tested under constant neutral axis depth then $r_y$ and $r_w$ are
constant and the relationship between $f_0$ and $\varepsilon_{cm}$ for the entire range of test data can be approximated by a function $f_0(\varepsilon_{cm})$ as follows:

$$f_0(\varepsilon_{cm}) = \sum_{i=1}^{n} A_i \varepsilon_{cm}^i$$  \hspace{2cm} (8)

$$A_i = \frac{a_i}{i+1} \left(1 + \frac{b_w r_y^{i+1}}{b_f} - r_y^{i+1} - \frac{b_w r_y^{i+1}}{b_f} r_y^{i+1}\right); \hspace{1cm} i = 1 \text{ to } n$$  \hspace{2cm} (9)

From Eq. (8) the coefficients $A_i$ and the degree $n$ can be found by curve-fitting the $f_0$ and $\varepsilon_{cm}$ data, which are solely computed from test data without using any assumption. Once $A_i$ are determined, the coefficients $a_i$ can be computed from Eq. (9).

If $r_y$ and $r_w$ are not constant as in case of columns subjected to constant end eccentricities then the coefficients $a_i$ can be found by performing linear regression analysis of the entire test data of which each set of test data must satisfy Eq. (7).

The flexural stress-strain curve of concrete can also be derived from moment equilibrium in the same manner. The final expression for the equilibrium condition for each set of test data is:

$$m_0 = \sum_{i=1}^{n} \frac{a_i}{i+2} \varepsilon_{cm}^i \left(1 + \frac{b_w r^{i+2}_y}{b_f} - r^{i+2}_y - \frac{b_w r^{i+2}_y}{b_f} r^{i+2}_y\right)$$  \hspace{2cm} (10)

where $$A_i = \frac{a_i}{i+2} \left(1 + \frac{b_w r^{i+2}_y}{b_f} - r^{i+2}_y - \frac{b_w r^{i+2}_y}{b_f} r^{i+2}_y\right); \hspace{1cm} i = 1 \text{ to } n$$  \hspace{2cm} (11)

The coefficients $a_i$ can be found by either fitting the $m_0 - \varepsilon_{cm}$ data to find coefficients $A_i$, and calculate $a_i$ from Eq. (11), or performing linear regression analysis of the test data, depending on the position of the neutral axis as described earlier.

DETERMINATION OF STRESS-STRAIN CURVES OF CORE CONCRETE
FROM FLEXURE TESTS OF REINFORCED CONCRETE COLUMNS

Data of thirteen RC columns tested under varying strain gradient and three columns tested under concentric loading are used to derive the stress-strain curves of core concrete. The columns were of the same size of 200 x 200 x 800 mm and reinforced with eight longitudinal steel bars having 10mm diameter. Three concretes with target cylinder strengths of 40, 70, and 90 MPa were used which gave the actual strengths at the days of the tests ranging from 45.5 MPa to 100.6 MPa. Transverse reinforcement with different configurations and yield strengths were used. Details of the tests are tabulated in Table 1. Figure 3 shows the test setup and the configuration of transverse reinforcement. The column was tested under two loading points. The primary load $P_1$ was applied by a 5 MN testing machine under a constant stroke of 0.05 mm/min. The secondary load $P_2$ was applied by a manually operated hydraulic jack in such a way that the strain monitor at the
back face (the left hand side of the specimen in Figure 3) was always kept at zero. The behaviors of the columns and test results are given elsewhere.

It is assumed that the concrete stress in the core, defined as the prism bounded by the centerlines of the perimeter hoops, follows one and only stress-strain curve for confined concrete and the concrete stress in the cover follows the stress-strain curve for unconfined concrete. It is further assumed that the stress-strain curve of the core concrete is not affected by the spalling of concrete cover. This assumption is based on the observation from the stress-strain curves of confined concrete obtained from concentric tests of specimens having no cover concrete. For simplicity the stress in a part of the concrete cover strip in the least compressed side is assumed to follow the confined concrete stress-strain curve as shown in Figure 4. The strain is assumed to be linearly distributed across the entire section.

The derivation of stress-strain curves of core concrete and cover concrete of columns tested under varying strain gradient is divided into two stages. In the first stage, test data up to spalling of concrete cover were used to obtain the stress-strain curve of gross concrete using the procedure for rectangular sections. In the second stage, the section was divided into two distinct portions; the cover concrete and the core concrete. Their individual stress-strain curves were obtained by using an iterative procedure as follows:

1- Initially, for the first stage, the stress in the core and cover concrete is assumed to follow the stress-strain curve of gross concrete obtained. Load and moment applied to the core concrete were found by subtracting the load and moment carried by concrete cover and longitudinal reinforcement. To account for the concrete area displaced by the longitudinal reinforcement, the force of confined concrete corresponding to the strain at the center of the steel bars was subtracted from the force in the steel bars.

2- The modified force term of the core concrete was calculated from the load applied to the confined concrete area as shown in Figure 4. The proposed procedure was applied to the entire range of test data to derive the first approximation of the stress-strain curve of the core concrete using both force equilibrium and moment equilibrium.

3- The stress-strain curve of the core concrete obtained from step 2 is used to back-calculate the load and moment applied to the cover concrete. The stress-strain curve of the cover concrete was derived using the procedure applied to the T-shape section. The stress in the longitudinal reinforcement was also recalculated to account for the area of core concrete displaced by the area of the steel bars in the cross-section.

4- Steps 2 and 3 were repeated until the results converge to a desired level.

Figure 5 shows the stress-strain curves of gross concrete, cover concrete, and core concrete obtained from column S40-B-N3. There are two curves for the core concrete. One is obtained from force equilibrium and the other obtained from moment equilibrium. Both the curves are almost the same in the descending part but there is some discrepancy near and after the peak stress. This discrepancy may result from the assumption that the core concrete follows one and only stress-strain curve while it consists of both effectively and ineffectively concretes which should behave differently.
The spalling of ineffectively confined concrete that follows the spalling of cover concrete should have different impact on the force and moment equilibriums. To satisfy both equilibriums, it is decided that the stress-strain curve of the core concrete be taken as the average of the curves obtained from each equilibrium condition.

Figure 6 to Figure 8 show the average stress-strain curves of core concrete obtained from columns with 2.4% volumetric ratio of hoops (configuration B). The stress-strain curves of core concrete obtained from columns under eccentric loading show a more ductile behavior than those obtained from similar columns under concentric loading. This is probably because for columns tested under concentric loading the spalling of ineffectively confined concrete around the perimeter of the cross-section midway between two hoop sets takes place at the same time. Consequently the loss of the load carried by the ineffectively confined concrete could not be compensated for by the increase in the stress of the effectively confined core and the average stress of the entire core concrete, which is considered as confined concrete stress, would reduce after the spalling of ineffectively confined concrete. In columns tested under eccentric loading, the spalling of ineffectively confined concrete in the core occurs gradually from the most compressed side to the other extreme, thus the confined concrete stress calculated based on the entire core section would not be as strongly affected as in the case of concentric loading. Moreover, for columns tested under eccentric loading the stress in the transverse reinforcement increases gradually from the most compressed side of the cross section to the other extreme. As a result, the yielding of the transverse reinforcement would progress slowly from most compressed side to the less compressed side thus maintaining a considerable lateral pressure to the core through out the tests.

ANALYTICAL MODEL FOR THE STRESS-STRAIN CURVE OF CONFINED CONCRETE

Figure 10 shows the proposed stress-strain relationship for confined concrete. For confined concrete subjected to strain gradient there is a horizontal part at the maximum stress of confined concrete to account for the differences between confined concrete under axial compression and under strain gradient. The stress-strain curve of confined concrete consists of four parts described by the following equations:

- For $\varepsilon_c \leq \varepsilon_{c1}$:
  $$f_c = \frac{f_{cc} \left( \frac{\varepsilon_c}{\varepsilon_{c1}} \right) r_c}{r_c - 1 + \left( \frac{\varepsilon_c}{\varepsilon_{c1}} \right)^{1.5}}$$  (12)

- For $\varepsilon_{c1} \leq \varepsilon_c \leq \varepsilon_{c2}$:
  $$f_c = f_{cc}'$$  (13)

- For $\varepsilon_{c2} \leq \varepsilon_c \leq \varepsilon_{20}$:
  $$f_c = f_{cc}' - \frac{0.15 f_{cc}' (\varepsilon_c - \varepsilon_{c2})}{(\varepsilon_{20} - \varepsilon_{c2})}$$  (14)

- For $\varepsilon_c > \varepsilon_{20}$:
  $$f_c = 0.2 f_{cc}'$$  (15)
where $f_{cc}'$ is the maximum stress of confined concrete; $\varepsilon_{c1}$ is the lowest strain at maximum stress of confined concrete; $\varepsilon_{c2}$ is the highest strain at maximum stress of confined concrete; $\varepsilon_{85}$ is the strain at 85 percent of maximum stress of confined concrete in the descending part; $\varepsilon_{20}$ is the strain at 20 percent of maximum stress of confined concrete in the descending part; and $r_c$ is the coefficient to control the slope of the ascending part.

The maximum stress of confined concrete is computed by Eq.(16), which was proposed by Li \(^3\) and reexamined by the authors against a large number of test data on confined concrete columns subjected to concentric tests with yield strength of transverse steel up to 1000 MPa.

$$f_{cc}' = f_{co}' \left[ -0.413 + 1.413 \sqrt{1 + 11.4 \frac{f_{le}}{f_{co}}} \right] - 2 \frac{f_{le}}{f_{co}} \quad (16)$$

where $f_{co}'$ is the strength of unconfined concrete; $f_{le}$ is the effective lateral confining pressure, which is computed as:

$$f_{le} = 0.5 K_e \rho_{sh} f_{yh} \quad (17)$$

where $K_e$ is the confinement effectiveness coefficient; $\rho_{sh}$ and $f_{yh}$ are the volumetric ratio and the yield strength of transverse reinforcement, respectively.

The confinement effectiveness coefficient is the ratio of the critical area of the effectively confined core area to the concrete core area.

$$K_e = \frac{A_{eff}}{(A_c - A_t)} \quad (18)$$

where $A_c$ is the core area measured center-to-center of perimeter tie; $A_t$ is total cross sectional area of longitudinal bars; and $A_{eff}$ is critical area of the effectively confined core.

Based on the assumption that the effectively confined core is created from series of second-degree parabolas with an initial angle of 45°, the coefficient $K_e$ can be calculated for any cross sectional area and configuration of reinforcement in reinforced concrete columns. Mander et al. \(^5\) have established the formulas for the most commonly used configurations in which the formula for concrete confined by rectilinear hoops is as follows:

$$K_e = \frac{1 - \sum_{i=1}^{n} \left( \frac{c_i^2}{6 b_{cx} b_{cy}} \right) \left( 1 - 0.5 \frac{s_i'}{b_{cx}} \right) \left( 1 - 0.5 \frac{s_i'}{b_{cy}} \right)}{1 - \rho_l} \quad (19)$$

where $b_{cx}$, $b_{cy}$ are core dimensions measured centre-to-centre of confining reinforcement parallel to the x- and y-axes, respectively; $c_i$ is $i^{th}$ clear spacing between two adjacent longitudinal bars that are laterally supported; $n$ is number of longitudinal bars that are laterally supported; $\rho_l$ is volumetric ratio of longitudinal steel to volume of core, which is calculated as:
\[ \rho_i = \frac{A_i}{A_c} \]  

(20)

The strains at maximum stress and strain at 85 percent of maximum stress in the descending branch are computed as follows:

\[ \varepsilon_{c1} = \varepsilon_{co} + 0.06 \frac{f_{lc}}{f_{co}} \]  

(21)

\[ \varepsilon_{c2} = \varepsilon_{c1} + 0.18 \sqrt{\frac{f_{lc}}{f_{co}}} \]  

(22)

\[ \varepsilon_{85} = \varepsilon_{085} + \frac{7.0f_{lc}}{f_{co}^2} + (\varepsilon_{c2} - \varepsilon_{c1}) \]  

(23)

where \( \varepsilon_{co} \) is the strain at maximum stress of unconfined concrete and \( \varepsilon_{085} \) is the strain at 85 percent of maximum stress of unconfined concrete in the descending branch.

\[ \varepsilon_{co} = 0.002 + \frac{(f_{co}' - 20)}{80 \times 10^{-3}} \]  

(24)

\[ \varepsilon_{085} = \varepsilon_{co} + \frac{0.023}{f_{co}} \]  

(25)

In case of confined concrete subjected to concentric loading \( \varepsilon_{c2} \) is taken equal to \( \varepsilon_{c1} \).

The coefficient to control the slope of the ascending part is calculated as follows:

\[ r_c = \frac{E_c}{E_c - \frac{f_{ce}'}{\varepsilon_{c1}}} \]  

(26)

\[ E_c = 3320 \sqrt{f_{c}'} + 6900 \]  

(27)

CONCLUDING REMARKS

An analytical procedure for determining the stress-strain curves of concrete for a T-shape cross-sections has been proposed. This procedure was used in combination with the one proposed for rectangular members to derive the stress-strain curves of confined concrete from test data of thirteen HSC columns confined by transverse reinforcement and subjected to varying strain gradient. The flexural stress-strain curves have a higher ductility compared with the stress-strain curves of confined concrete obtained from similar columns tested under concentric loading. This is contrary to the common assumption that strain gradient has no effect on the stress-strain relationship. An analytical model for confined concrete that account for this difference has been proposed.
REFERENCES


Table 1. Details of columns

<table>
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<tr>
<th>Column</th>
<th>Cylinder strength, MPa</th>
<th>Yield strength of longitudinal reinforcement, MPa</th>
<th>Configuration</th>
<th>Yield strength, (MPa)</th>
<th>Spacing (mm)</th>
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*) Columns tested under concentric loading.

Figure 1 – Different parts of concrete in circular and square reinforced concrete columns
Figure 2 – Assumption of stress and strain distributions in a T-shape section under flexure tests

Figure 3 – Test setup and configurations of transverse reinforcement

Figure 4 – Assumption for derivation of stress-strain curve of confined concrete
Figure 5 – Stress-strain curves of columns S40-B-N3

Figure 6 – Average stress-strain curves of core concrete of NSC columns having hoop configuration B

Figure 7 – Average stress-strain curves of core concrete of HSC columns having hoop configuration B
Figure 8 – Average stress-strain curves of core concrete of very high-strength concrete columns having hoop configuration B

Figure 9 – Average stress-strain curves of core concrete of columns having hoop configurations C, D, and E

Figure 10 – Proposed stress-strain relationship for confined concrete
Empirical Models for Confined Concrete under Uniaxial Loading

by B. Oh and R. Sause

Synopsis: Theoretically sound empirical models for the axial stress-strain behavior as well as the transverse deformation behavior of concrete with constant confinement under uniaxial compression loading are proposed based on plasticity theory using existing empirical models and test data in the literature. The proposed axial stress-strain model provides strength and ductility increases with increasing confining pressure. The confining pressure is applied in the model as an initial hydrostatic compression loading region. The proposed empirical transverse deformation model uses the plastic strain rate as a function of the axial strain to provide a basis for quantifying the compaction and dilation behavior of confined concrete under uniaxial loading conditions. Concretes with various compressive strengths are considered in combination with confining pressures up to 50% of the unconfined concrete strength. Parameters needed to describe the axial and transverse deformation behaviors are identified and their recommended values are provided.

Keywords: confined concrete; empirical models; stress-strain models; transverse deformation
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INTRODUCTION

The paper proposes a new empirical axial stress-strain model for both unconfined and confined concrete under uniaxial compression loading. The model satisfies theoretical conditions and is, in certain aspects, more accurate than existing models. For unconfined concrete the model is composed of three regions: linear elastic, nonlinear ascending, and nonlinear descending regions. It has the following advantages compared to existing empirical models. First, a distinct linear elastic region is provided. The lack of a linear elastic region implies inelastic behavior occurs as soon as load is applied, which is not the expected behavior of concrete. Second, separate parameters are used to control the ascending and descending regions. This approach enables the initial stiffness of the ascending region to match the stiffness of the elastic region without affecting the descending region. Third, the slope of the descending region at the peak stress is equal to that of the ascending region, which is zero.

For confined concrete, the proposed model also includes the three (linear elastic, nonlinear ascending, and nonlinear descending) regions. As expected, the linear elastic region increases in size as the confining pressure increases, and the strength and ductility increase with increasing confining pressure. In the proposed model, however, unlike most previous models, the conditions under which the confining pressure is applied are clearly shown, by including an initial triaxial compression region. In this region, the confining pressure is applied by hydrostatic compression. The complete confined concrete model with initial hydrostatic compression is, therefore, composed of four regions: an initial hydrostatic compression loading region, and the linear elastic, nonlinear ascending, and nonlinear descending regions.

The paper also proposes a new empirical transverse deformation. The empirical model provides reasonable behavior when studied in terms of several different transverse deformation variables. Among five different transverse deformation parameters, the plastic strain rate is chosen as the primary variable. The model satisfies theoretical conditions and is, in certain aspects, more accurate than the existing models. It has the following advantages compared to existing empirical models. First, a distinct linear elastic region is provided. Behavior in this region is described using the theory of elasticity. Second, the secant strain ratio at the peak stress corresponds to volumetric...
compaction at the peak stress not volumetric dilation. Third, the maximum transverse
deformation permitted by the model limits the plastic volume dilation according to
theoretical conditions.

**EMPIRICAL AXIAL STRESS-STRAIN MODEL**

As noted earlier, for confined concrete the proposed model is composed of four
regions as shown in Figure 1: initial hydrostatic compression loading, linear elastic,
nonlinear ascending, and nonlinear descending regions. For unconfined concrete, the
initial hydrostatic compression loading region and the effects of confinement on the size
of the elastic region, the strength, and the ductility are eliminated. Note that for the model
compression stresses and strains are treated as negative quantities.

**Initial hydrostatic compression loading region**

The initial hydrostatic compression loading region is defined for axial strain, $\varepsilon_1$, in
the range $0 \leq |\varepsilon_1| \leq |\varepsilon_{cp}|$ where $\varepsilon_{cp}$ is defined below. Under hydrostatic loading, the
behavior is assumed linear elastic and from the theory of elasticity,

$$1 1 2 3 \frac{1}{K V V P} \frac{1}{V_1}$$

where $K$ is the ratio of the transverse strain to the axial strain in the elastic region, $\sigma_1$ is
the axial stress, $\sigma_2$ equals $\sigma_3$, which is the confining pressure, and $E_c$ is the elastic
modulus. Since $\sigma_1 = \sigma_2 = \sigma_3$, from the above equation,

$$\sigma_1 = 3K^* \varepsilon_1$$

where $K^*$ is bulk modulus of elasticity. Therefore, the initial slope of the stress-strain
curve equals $3K^*$, not $E_c$. The strain corresponding to the upper limit of this region is

$$\varepsilon_{cp} = \frac{\sigma_{cp}}{3K^*}$$

where $\sigma_{cp}$ is the predetermined value of confining pressure.

**Elastic region**

The linear elastic region is defined for axial strain in the range $|\varepsilon_{cp}| \leq |\varepsilon_1| \leq |\varepsilon_{1ic}|$
where $\varepsilon_{1ic}$ is axial strain corresponding to $\sigma_{1ic}$ which is defined below. The stress-strain
relationship is

$$\sigma_1 = E_c \left( \varepsilon_1 - \varepsilon_{cp} \right) + \sigma_{cp}$$

The stress at the linear elastic limit for confined concrete is

$$\sigma_{1ic} = \left( c_i + c_{ic} \phi_c \right) \sigma_{10}$$

where $c_i$ is the elastic limit parameter for unconfined concrete, $c_{ic}$ is the confining
pressure parameter for the elastic limit of confined concrete, $\phi_c = \sigma_{cp} / \sigma_{10}$ is the confining
pressure ratio, and $\sigma_{10}$ is the peak compressive stress for unconfined concrete. In
developing the model, $c_i$ and $c_{ic}$ are assumed to be 0.3 and 2, respectively; other values
are possible. The strain corresponding to the upper limit of the linear elastic region is
\[ \varepsilon_{\text{ic}} = \left( \sigma_{\text{ic}} + 2\eta \sigma_{\text{cp}} \right) / E_c \]

Note that, for confined concrete without the initial hydrostatic compression loading region, \(\sigma_1 = E_c \varepsilon_1, \varepsilon_{\text{cp}} = 0\), and \(\varepsilon_{\text{ic}} = \sigma_{\text{ic}} / E_c\). Differences between \(\varepsilon_{\text{ic}}\) for confined concrete model with the initial hydrostatic compression loading region and that of the simpler confined concrete model are shown in Figure 2.

**Ascending region**

The nonlinear inelastic ascending region starts from \(\sigma_{\text{ic}}\) and reaches \(\sigma_{\text{io}}\), which is the peak value of the \(\sigma_1\) for confined concrete. This region is defined for axial strain in the range \(|\varepsilon_{\text{ic}}| \leq |\varepsilon| \leq |\varepsilon_{\text{io}}|\), where and \(\varepsilon_{\text{io}}\) is the strain when the peak stress \(\sigma_{\text{io}}\) is reached. For the model, \(\sigma_{\text{io}} = \sigma_{\text{io}} + 4.1 \sigma_{\text{cp}}\) and \(\varepsilon_{\text{io}} = \varepsilon_{\text{i0}} \left[1 + 5 \left( \sigma_{\text{io}} / \sigma_{\text{i0}} - 1 \right) \right]\) have been adopted from Richart et al. [1928], where \(\varepsilon_{\text{i0}}\) is the strain at the peak stress for unconfined concrete.

For the stress-strain relationship in this region, the stress-strain function proposed by Popovics [1973] (hereafter called the Popovics model, for simplicity) is adopted and modified as follows

\[ \sigma_1 = (\sigma_{\text{io}} - \sigma_{\text{i0}}) \omega_u \frac{r_a}{r_a - 1 + \omega_u r_u^0} + \sigma_{\text{i0}} \]

where \(\omega_u = (\varepsilon_1 - \varepsilon_{\text{i0}}) / (\varepsilon_{\text{io}} - \varepsilon_{\text{i0}})\), \(r_a = E_c / (E_c - E_{\text{ac}})\), and \(E_{\text{ac}} = (\sigma_{\text{io}} - \sigma_{\text{i0}}) / (\varepsilon_{\text{io}} - \varepsilon_{\text{i0}})\).

**Descending region**

The nonlinear inelastic descending region is defined for axial strain in the range \(|\varepsilon_{\text{i0}}| \leq |\varepsilon| \leq |\varepsilon_{\text{i0}}|\) where \(\varepsilon_{\text{i0}}\) is the maximum value of axial strain that is considered in the model. \(\varepsilon_{\text{i0}}\) is determined from test data provided from previous research, for example as listed in Table 1 and Table 2. Judgment is used to determine \(\varepsilon_{\text{i0}}\). A simple formula is proposed for \(\varepsilon_{\text{i0}}\) as a function of \(\phi_c\)

\[ \varepsilon_{\text{i0}} = -0.008 - 0.1\phi_c \]

The stress-strain relationship in this region adopts the Popovics model but uses a different parameter \(r_d\).

\[ \sigma_1 = \sigma_{\text{io}} \left( \frac{\varepsilon_1}{\varepsilon_{\text{io}}} \right) \frac{r_d}{r_d - 1 + \left( \frac{\varepsilon_1}{\varepsilon_{\text{io}}} \right)^{r_d}} \]

where the parameter \(r_d\) is a function of the concrete strength as well as a function of \(\phi_c\). From studies [Oh 2002] of the model proposed by Mander et al. [1988] (hereafter called the Mander model, for simplicity), it is observed that the parameter ‘r’ of the Mander model provides a reasonable descending region behavior when \(\phi_c\) is moderate, but
provides a descending region slope that is too flat when \( \phi_c \) is small. Thus, the proposed value of \( r_d \) is

\[
r_d = \frac{E_c}{E_c - E_sc} g(\phi_c)
\]

where \( E_{sc} = \sigma_{10c} / \varepsilon_{10c} \) and \( g(\phi_c) \) reflects the effect of confining pressure on ductility. \( g(\phi_c) \) is defined so that when \( \phi_c \geq 0.5 \), \( r_d \) equals ‘\( r \)’ of the Mander model, and when \( \phi_c = 0 \), \( r_d \) equals \( r_{du} \) which is given by

\[
r_{du} = 0.58 - 0.0464\sigma_{10} + 0.00162\sigma_{10}^2
\]

in which \( \sigma_{10} \) in MPa units. This formula for \( r_{du} \) was determined from regression analysis [Oh 2002] over a range of strains up to \( \varepsilon_{iu} \) given by the previous formula.

When \( \phi_c \geq 0.5 \), in \( g(\phi_c) = 1.0 \) and \( r_d = \langle r \rangle \) of the Mander model, as shown in Figure 3 schematically. The value of 0.5 is chosen by assuming that the descending region of the Mander model is reasonable for \( \phi_c = 0.5 \) and beyond. For smaller values of \( \phi_c \), the linear function for \( g(\phi_c) \) shown in Figure 3 is derived as follows. When \( \phi_c = 0 \),

\[
\frac{E_c}{E_c - E_{sc}} g(\phi_c) = r_{du}
\]

By using \( \phi_c = \sigma_{cp} / \sigma_{10c} \) along with \( \sigma_{10c} \) and \( \varepsilon_{10c} \) introduced above, \( E_{sc} \) can be expressed as

\[
E_{sc} = \frac{E_{su} (1 + 4.1\phi_c)}{1 + 20.5\phi_c}
\]

where \( E_{su} = \sigma_{10} / \varepsilon_{10c} \). Thus, when \( \phi_c = 0 \), \( E_{sc} = E_{su} \) and

\[
g(\phi_c = 0) = \left(1 - \frac{E_{su}}{E_c}\right) r_{du}
\]

With \( g(\phi_c = 0.5) = 1.0 \), the resulting linear function for the range \( 0 \leq \phi_c \leq 0.5 \) is

\[
g(\phi_c) = \left(1 - \frac{E_{su}}{E_c}\right) r_{du} (1 - 2\phi_c) + 2\phi_c
\]

The function \( g(\phi_c) \) is shown in Figure 4 for different concrete strengths. The corresponding \( r_d \) is shown in Figure 5. Note that to generate the results shown in Figures 4 and 5, \( \varepsilon_{10} = \varepsilon_{10H} = -0.00027 \sqrt[4]{\sigma_{10}} \) which is an empirical expression for \( \varepsilon_{10} \) based on experimental data from Hognestaed et al. [1955], where \( \sigma_{10} \) is in psi units, and \( E_c \) is from the ACI-318 formula [ACI Committee 318 1999].

**EMPIRICAL TRANSVERSE DEFORMATION MODEL**

Development of an empirical plastic strain rate-axial strain model is presented. The model provides reasonable behavior in terms of five different transverse deformation variables (\( \eta_1^p \), \( \eta_2 \), \( \eta_3 \), \( \varepsilon_1 \), \( \varepsilon_2 \)) which are explained below. The model satisfies theoretical conditions and is, in certain aspects, more accurate than the existing models. Note that for the model, compression strains are treated as negative quantities.
Transverse deformation variables

**Transverse strain rate:** The transverse strain rate (also called the tangent strain ratio) is \( \eta_t = \frac{d \varepsilon_3}{d \varepsilon_1} \). In the linear elastic region, \( \eta_t = \eta_i \), since the plastic deformations are zero.

**Transverse strain:** The transverse strain, \( \varepsilon_3 \), is obtained from \( \eta_t \) by integration

\[
\varepsilon_3 = \int_0^\eta_t d \varepsilon_1
\]

Under linear elastic hydrostatic compression loading, the transverse strain is the same as the axial strain, and the transverse strain increases at the same rate as the axial strain until the confining pressure reaches the predetermined value, \( \sigma_{cp} \). The maximum strain under hydrostatic loading is \( \varepsilon_{cp} \) for both the axial direction and transverse direction, as shown in Figure 6(a). Under this loading condition, both the secant strain ratio (\( \eta_s = \varepsilon_3 / \varepsilon_1 \)) and the tangent strain ratio (\( \eta_t = d \varepsilon_3 / d \varepsilon_1 \)) equal 1 as shown by line II in Figure 6(a), (b), and (c). At the end of the hydrostatic compression loading region, the concrete is assumed to be loaded under uniaxial compression with constant confining pressure. Under this loading condition, \( \varepsilon_3 \) increases linear elastically until reaching \( \varepsilon_{3ic} \), which is the transverse strain at the linear elastic limit for confined concrete. Under linear elastic uniaxial loading, the constant tangent strain ratio is \( \eta_t \) as shown by line III in Figure 6(a).

**Volumetric strain:** The volumetric strain, \( \varepsilon_v \), is the sum of axial and transverse strains (i.e., \( \varepsilon_1 + 2\varepsilon_3 \)). Since \( d\varepsilon_3 = d\varepsilon_1 \) under linear elastic hydrostatic compression loading

\[
\frac{d \sigma_1}{d \varepsilon_v} = \frac{3K' d \varepsilon_3}{d \varepsilon_1 + 2d \varepsilon_3} = K'
\]

as shown in Figure 6(d). To illustrate volume changes in concrete under uniaxial loading and other loading conditions, axial stress is often plotted against volumetric strain as shown in Figure 7. The slope of the axial stress-volumetric strain curve is

\[
\frac{d \sigma_1}{d \varepsilon_v} = \frac{d \sigma_1/d \varepsilon_1}{d \varepsilon_1 + 2d \varepsilon_3/d \varepsilon_1} = \frac{d \sigma_1/d \varepsilon_1}{1 + 2\eta_t}
\]

Two points of concern in Figure 7 are the point of peak stress and the point where the slope of the curve is infinity, which occurs when \( \eta_t = -0.5 \) according to the above equation, indicating no volume change with stress. Figure 7(a), (b), and (c) show conditions where \( \eta_t \) reaches \(-0.5\) before the peak stress is reached, when the peak stress is reached, and after the peak stress is reached, respectively. In Figure 7(d), \( \eta_t \) does not reach \(-0.5\). These conditions will be considered in evaluating possible transverse deformation models.

**Secant strain ratio:** The secant strain ratio is defined as \( \eta_s = \varepsilon_3 / \varepsilon_1 \), and has been used in previous research on transverse deformation of concrete under uniaxial loading by, for example, Darwin and Pecknold [1977] and Elwi and Murray [1979]. The secant strain ratio can be calculated as
The secant strain ratio in the linear elastic uniaxial loading region after hydrostatic compression loading, denoted as $\eta^E_1$, is neither constant nor linear as shown by curve III in Figure 6(b). $\epsilon_3$ in this region equals $\epsilon_{cp} + \eta_1^1(\epsilon_1 - \epsilon_{cp})$, and

$$\eta^E_1 = \epsilon_3 / \epsilon_1$$

By substituting $\epsilon_{cp} = \sigma_{cp} / 3K = \phi_c \sigma_{10}(1 + 2\eta_1^1) / E_c$ into the above equation, $\eta^E_1$ is expressed as a function of $\phi_c$.

$$\eta^E_1 = \epsilon_1 + (1 - \eta_1^1) \frac{\epsilon_{cp}}{E_c}$$

Transverse plastic strain rate: The transverse plastic strain rate (also called the tangent plastic strain ratio) is $\eta^p = d\epsilon^p_t / d\epsilon^t$. Just as the total strains, $\epsilon_1$ and $\epsilon_3$, can be decomposed into elastic parts and plastic parts, the infinitesimal increments of strain, $d\epsilon_1$ and $d\epsilon_3$, can be decomposed into elastic parts, $d\epsilon^e_1$ and $d\epsilon^e_3$, and plastic parts, $d\epsilon^p_1$ and $d\epsilon^p_3$. The ratio of the infinitesimal increments of plastic strain define $\eta^p$. Using $\Omega$ equal to the ratio of the infinitesimal increment of elastic axial strain, $d\epsilon^e_1$ to the infinitesimal increment of total axial strain, $d\epsilon_1$, $\eta^p_1$ can be expressed in terms of $\eta_1$, $\eta_t$ as follows

$$\eta^p_1 = \eta_1 - \eta_t \frac{\Omega}{1 - \Omega}$$

where $\Omega = \frac{1}{E_c} \frac{d\sigma_{10}}{d\epsilon^t}$.

Transverse deformation model based on plastic strain rate

Theoretical conditions that are satisfied by the transverse deformation model based on the plastic strain rate $\eta^p$ are as follows: (1) setting the initial value of $\eta^p$, $\eta^p_{10}$, equal to $\eta_1$ gives appropriate transverse strain conditions at the end of the elastic region; (2) at the peak axial stress ($\sigma_{10}$ or $\sigma_{10c}$), the infinitesimal increments of elastic strain become zero instantaneously (i.e., $d\epsilon^e_{10c} = 0$ and $d\epsilon^e_{10c} = 0$) and thus the value of $\eta^p_1$ at the peak axial stress, $\eta^p_{10}$ is set equal to the value of $\eta_1$ at the peak stress, $\eta_{10}$; and (3) a limiting value for $\eta^p_1$ is imposed [Oh 2002], which is denoted $\eta^p_{tu}$.

The following hyperbolic function is used to express $\eta^p$ as a function of the axial strain (i.e., $\eta^p_1 = \eta^p_1(\epsilon_1)$)

$$\eta^p_1 = \frac{1}{a \epsilon_1 + b} + \eta^p_{tu}$$

where $a = \frac{-(\eta_0 - \eta_1)}{(\epsilon_{10c} - \epsilon_{10c}^p)(\eta_1 - \eta^p_{tu})(\eta_0 - \eta^p_{tu})}$ and $b = \frac{\epsilon_{10c}(\eta_1 - \eta_{tu}^p) - \epsilon_{10c}^p(\eta_1 - \eta^p_{tu})}{(\epsilon_{10c} - \epsilon_{10c}^p)(\eta_1 - \eta^p_{tu})(\eta_0 - \eta^p_{tu})}$.
Derivations of $a$ and $b$ are found in Oh [2002]. Some values for $a$ and $b$ are given in Table 3. $\eta$ is a continuous function of $\varepsilon_1$, but the value of $\eta$ at the peak axial stress, $\eta_{0\theta}$, can be determined using a set of expressions derived by Oh [2002]. As mentioned earlier, the limiting value of $\eta^p$ is $\eta^q_w$. Oh [2002] discusses the possible values of $\eta^q_w$, and shows that if $\eta^q_w$, which is negative to produce dilation after the peak stress, is less than $-2$ (say $-3$), the transverse plastic strain rate can exceed that implied by an associated flow rule from plasticity theory and produce quite large values of $\varepsilon_3$.

RESULTS

Axial stress-strain

Results from the proposed model for unconfined 4 ksi (27.6 MPa) concrete are compared to those from the Popovics model in Figure 8. The results from the Popovics model with $\varepsilon_{10} = \varepsilon_{10W}$ (labeled PW), where $\varepsilon_{10W}$ is the strain at peak stress from the test data of Watanabe [1972], agree well with the results from the proposed model (labeled OS) in the elastic region and in early part of the ascending region. However, $\varepsilon_{10W}$ is smaller than $\varepsilon_{10H}$ that was used for the OS results. For the descending region, the PW results do not agree well with the OS results. The results from the Popovics model with $\varepsilon_{10} = \varepsilon_{10H}$ (labeled PH) do not agree well with the OS results in the elastic and in the ascending regions. For the descending region, the PH results are flatter than the OS results. The PH results match the OS results only at $V_{10}$. The results demonstrate that the rather soft initial stiffness and flat slope of the descending region of the Popovics model have been improved in the proposed model (OS results), which has a controlled initial stiffness (equal to the specified $E_c$), and a descending region controlled independently of the ascending region.

A comparison with the Mander model is provided in Figure 9. The results from the original Mander model (labeled MO) agree well with the OS results in the early part of the ascending region. The original Mander model uses a slightly larger $E_c$ than used for the OS results. $\varepsilon_{10}$ of the original Mander model is fixed at $-0.002$, and thus the results do not agree well with the OS results as $\varepsilon_{10}$ increases with the strength of concrete as suggested by Hognestaed et al. [1955]. For the descending region, the MO results are much flatter than the OS results. The results from the Mander model using $\varepsilon_{10H}$ and the same $E_c$ as the OS results (labeled MM) agree with the OS results in the elastic and the ascending regions, but the descending region is even flatter than the MO results.

Results from the proposed model for confined 4 ksi (27.6 MPa) concrete with $\phi_c = 0.2$ are compared to the results from the Mander model in Figure 10. The proposed model results are labeled OS. The Mander model results using $\varepsilon_{10}, E_c,$ and $\sigma_{10c}$ recommended by Mander et al. [1988] are labeled MO. The Mander model results using the same $\varepsilon_{10}, E_c,$ and $\sigma_{10c}$ as the OS results are labeled MM. The MO results are initially stiffer than the OS results, but deviate early from linear behavior. The MO peak stress is larger in magnitude than the OS peak stress since the value of $\sigma_{10c}$...
calculated by the Mander model is greater than that from the formula of Richart et al. [1928] when $c_1$ is smaller than 0.35. For the descending region, the MO results are much flatter than the OS results. The MM results start with the same stiffness as the OS results but deviate early from linear behavior and reach the same peak stress as the OS results. For the descending region, the MM results are much flatter than the OS results and almost parallel to the MO results.

The comparisons with previous empirical models in Figures 8, 9, and 10 show that using separate parameters for the ascending and descending regions enables the initial stiffness and post-peak behavior to be modeled independently, and, therefore, the proposed model offers the potential for greater accuracy compared to test data.

Transverse deformation

The proposed transverse deformation model, based on the $\eta^p_i$ function given above, is compared to transverse deformation models that use constant $\eta^p_i$ in Figure 11. Three different values of $c_1$ are considered. The values of $\eta^p_i$ for both the proposed model with the $\eta^p_i$ function and the constant $\eta^p_i$ models are in the range $[-0.15] \leq |\eta^p_i| \leq [-2]$. For the proposed model, $\eta^p_i = -0.15$, and $\eta^p_{ui} = -2$. These lower and upper limits, and an intermediate value of $\eta^p_i$, -0.7, are chosen for the constant $\eta^p_i$ models.

The results from the transverse deformation models based on $\eta^p_i$ are compared with respect to $\eta_i$ in Figure 12. $c_1$ equals 0.2. For the model with constant $\eta^p_i = -0.15$, $\eta_i$ does not vary. However, the other models with constant $\eta^p_i$ have $\eta_i$ that varies much like the proposed model (labeled OS in Figure 12). However, $\eta_i$ for the constant $\eta^p_i$ models becomes insensitive to changes in $\varepsilon_i$ whereas the proposed model results show changes in $\eta_i$ with increasing $\varepsilon_i$. Note that the results for the constant $\eta^p_i$ models with $\eta^p_i = -0.7$ and -2 have a discontinuity in slope at the beginning of the inelastic region. The proposed model does not have this discontinuity.

Figure 13 compares $\eta^p_i$ results from the four models. The behavior in the elastic hydrostatic compression loading region is clearly demonstrated for all the models. The model with constant $\eta^p_i = -0.15$ exhibits linear behavior in the inelastic region while the model with constant $\eta^p_i = -2$ exhibits highly nonlinear behavior with rapidly growing $\eta^p_i$.

Figure 14 compares $\eta_i$ results from the four models. The models with constant $\eta^p_i$ seem to converge to the constant values of $\eta^p_i$ as $\varepsilon_i$ increases. However, the proposed model (labeled OS) yields increasing $\eta_i$ over the range of $\varepsilon_i$ considered.

Figure 15 compares $\varepsilon_v$ results from the four models. The results from the model with constant $\eta^p_i = -0.15$ exhibit only volumetric compaction. For the models with constant $\eta^p_i = -0.7$ and -2, the peak stress occurs with volumetric dilation. For the proposed model (labeled OS), the peak stress occurs with volumetric compaction, but
dilation occurs after the peak stress is reached.

CONCLUSIONS

The proposed empirical axial stress-strain model can be used to predict the behavior of unconfined concrete and concrete with constant confinement under uniaxial compression loading. The axial strain corresponding to the peak value of the axial stress in unconfined concrete, $\varepsilon_{10}$, and the modulus of elasticity of concrete, $E_c$, have significant influence on the ascending region behavior of the proposed model. The descending region parameter unconfined concrete, $r_{du}$, and the descending region parameter for confined concrete, $r_d$, have significant influence on the descending region behavior of the proposed model. Comparisons with previous empirical models show that using separate parameters for the ascending and descending regions enables the initial stiffness and post-peak behavior to be modeled independently and therefore with potentially greater accuracy compared to test data.

The proposed empirical transverse deformation model can be used to predict the behavior of unconfined concrete and concrete with constant confinement under uniaxial compression loading. The proposed model used the plastic strain rate, $\eta_p^p$, as a function of the axial strain, $\varepsilon_p^p$, to provide a basis for calculating the transverse deformations. The transverse deformation behavior using different variables, such as tangent ratio of transverse strain to axial strain, $\eta_t$, transverse total strain, $\varepsilon_t$, secant ratio of transverse strain to axial strain, $\eta_s$, and volumetric strain, $\varepsilon_v$, can be predicted based on the proposed model. Results for these variables were shown, and can be compared with measured data when such data is available.

REFERENCES

ACI Committee 318 (1999), “Building code requirements for structural concrete (318-99) and commentary (318R-99),” American Concrete Institute.


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Table 1 -- Maximum $\varepsilon_1$ recorded (unconfined concrete)

Note: K97 stands for Kestner et al. [1997]
AS86 stands for Ahmad and Shah [1986]
XW00 stands for Xiao and Wu [2000]
IP96 stands for Imran and Pantazopoulou [1996]
Table 2 -- Maximum $\varepsilon_r$ recorded (confined concrete)

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Note: K97 stands for Kestner et al. [1997]
MS97 stands for Mirmiran and Shahawy [1997]
XW00 stands for Xiao and Wu [2000]
IP96 stands for Imran and Pantazopoulou [1996]

Table 3 -- Parameters $a$ and $b$ as a function of $\phi_c$

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Figure 1 – Confined concrete model (schematic)
Figure 2 – Effect of initial hydrostatic compression

Figure 3 – Function $g(\phi_c)$ (schematic)

Figure 4 – Function $g(\phi_c)$

Figure 5 – $r_d$ for different concrete strengths
Figure 6 – Elastic behavior including hydrostatic compression

(a) Transverse strain versus axial strain
(b) Secant strain ratio versus axial strain
(c) Tangent strain ratio versus axial strain
(d) Axial stress versus volumetric strain

Figure 7 – Schematic representation of axial stress versus volumetric strain

Figure 8 – Comparison of proposed unconfined concrete model with Popovics model
(4 ksi [27.6 MPa] concrete)
Figure 9 – Comparison of proposed unconfined concrete model with Mander model (4 ksi [27.6 MPa] concrete)

Figure 10 – Comparison of proposed confined concrete model with Mander model: $\phi_c = 0.2$ (4 ksi [27.6 MPa] concrete)

Figure 11 – Effect of confining pressure on $\eta^p_f$
Figure 12 – Effect of confining pressure on $\eta_t (\phi_c = 0.2)$

Figure 13 – Effect of confining pressure on $\varepsilon_3 (\phi_c = 0.2)$

Figure 14 – Effect of confining pressure on $\eta_s (\phi_c = 0.2)$

Figure 15 – Effect of confining pressure on $\varepsilon_v (\phi_c = 0.2)$
Accuracy and Improvements for Variable and Constant Confinement Concrete Models

by C.J. Naito and F. Cetisli

Synopsis: A series of experiments were conducted on the performance of concrete under varying levels of passive confinement. Glass and carbon fiber reinforced polymers, and steel plate were used to provide confinement. The research presented compares the measured performance of the jacketed systems with current predictor models and finite element estimations. Two categories of predictor models are examined: Constant Confinement (CC) and Variable Confinement (VC). The CC model assumes that the confining pressure provided by the jacket is active and constant over the load history. VC model assumes that the confinement passively increases as lateral dilation occurs. The research shows that VC models provide improved accuracy over CC models in predicting the stress-strain response of confined concrete. Use of multi-linear relationships between axial and transverse strain combined with stress-strain relationships previously developed provide a comprehensive method of estimation. The limitations of these methods are clearly illustrated through examples.

Keywords: constitutive models; experimental tests; FEM analysis; FRP; steel jacketed
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Fatih Cetisli, Graduate Student Researcher at Civil & Environmental Engineering Department of the Lehigh University. He has earned his M.Sc. degree from Lehigh University [2003] and B.Sc. degree from Istanbul Technical University [1999]. His research interests are concrete design and earthquake engineering. Contact: fac5@lehigh.edu

INTRODUCTION

Lateral confinement has been shown to provide an effective means of strengthening concrete under axial compression. Lateral confinement is achieved through internal transverse reinforcement, in the form of closely spaced stirrups or spirals, or external jacketing. Concrete filled steel tubes (CFT) have been used for many years for externally increasing the lateral confinement to concrete. With the development of fiber polymer materials new strengthening materials are being used and implemented. Externally bonded glass fiber reinforced polymer (GFRP) and carbon fiber reinforced polymer (CFRP) sheets have been successfully used for rehabilitation and hardening of existing buildings and bridge systems. Strength-based design recommendations have recently been established by the American Concrete Institute [ACI 2002a]. However, accurate prediction of deformation response under these conditions has not been fully validated. To assess the displacement-based performance of a structure, the stress-strain response of the materials must be accurately predicted.

Under axial loading, concrete dilates laterally and is passively restrained by the confining jacket or tube system. This restraint increases the concrete longitudinal strength beyond its unconfined strength. Characterization of concrete subject to passive confinement initiated with the studies conducted by Richart [1928]. This research defined a relationship between confined concrete strength, \( f_{cc} \), the unconfined strength, \( f_{co} \), and the lateral confining stress, \( f_l \) (Eq.1).

\[
f_{cc} = f_{co} + 4.1 \times f_l \tag{1}
\]

This work has continued with renewed effort in the last 30 years and has resulted in a variety of empirical and theoretical approaches. The majority of concrete axial stress-strain models developed are based on methods developed by Popovics et al. [1973] and the popular design-oriented model of Mander [1988].

CONSTANT AND VARYING CONFINEMENT MODELS

One of two basic assumptions is used to predict the material response of confined concrete: the assumption of constant confinement (CC) [Mander 1988] and the assumption of varying confinement (VC) [Madas and Elnashai 1992]. Under the constant confinement approach the external jacket or transverse reinforcement is assumed to
produce a constant confining stress to the concrete. This stress is active at all axial strain levels (Figure 1). The variable confinement model is based on a theoretical response. When concrete is subjected to axial strain, the system dilates laterally due to Poisson’s effect (Figure 2a). Lateral dilation of the concrete is shared by the confining material due to compatibility (Figure 2b). From this strain, \( a_i \), and the jacket constitutive relationship a tensile jacket stress, \( f_j \), is generated (Figure 2c). Due to force equilibrium a confining stress on the dilating concrete, \( f_l \), is produced (Figure 2d). Thus for the VC model the lateral confining stress acting on the concrete varies as a function of the axial strain (Figure 1).

The application of CC models by Mander [1988] is based on the use of steel transverse stirrups or spiral reinforcement. The assumption of constant confinement under these conditions is reasonable due to the comparative constitutive behaviors of concrete and steel. Confining pressure is calculated from the free body diagram shown in Figure 2. For cylindrical members this can be shown to equal to \((2t/D) \ f_j(a_i)\), where \( t \) is the jacket thickness, \( D \) is the cylinder diameter, and \( f_j(a_i) \) is the jacket constitutive relationship.

The majority of concrete dilation occurs after the peak compressive strength is reached. At this point extensive cracking and crushing of the concrete material occurs, thus producing a rapid increase in dilation. Concrete lateral strain at peak stress is on the order of 0.1%. A36 steel yields after 0.12% strain is reached. Thus plastic deformation of the jacket and plastic lateral dilation of the concrete occur at the same deformation levels. Consequently, the assumption of constant confinement, while not theoretically correct, produces good predictions for the peak stress generated by steel confinement. At low strains, however, the confining action is over predicted while at higher strains when the jacket strain hardens, the response is under-predicted.

The use of constant confinement models for FRP materials produces poor results. The constitutive relationships for GFRP and CFRP are typically elastic-brittle. Previous researchers [Samaan and Mirmiran 1998 and Lam and Teng 2002] recommended the use of the ultimate stress level for the value of constant confinement. FRP jackets typically rupture after 1% axial elongation of the fibers (1% lateral dilation of concrete). Lateral dilation of concrete, however, only reaches approximately 0.1% at the peak stress. At this value of lateral dilation the confining pressure produced by the FRP jacket is less than 10% of its rupture strength. The use of rupture stress significantly over estimates the available confining stresses at peak. To provide a better fit ACI committee 440 [2002b] recommends that a strain of 0.4% be used. This assumption over-predicts the confining pressure by a factor of four at the peak concrete stress. To accurately predict the peak compressive strength of concrete confined with FRP materials, alternate predictive methods must be utilized.

The variable and constant confinement methodologies for computing the stress-strain response of confined concrete are illustrated in the flowcharts of Figure 3. The CC methodology assumes a constant confining stress equal to the yield or rupture stress of the jacket. For steel jackets the yield stress is used. For elastic-brittle materials such as GFRP and CRFP a stress of 0.004 \times\) jacket elastic modulus, but not exceeding 75% of the rupture stress is recommended [ACI 2002b]. From this value the confined compressive
strength and corresponding axial strain is predicted through the use of methods described in Table 2. The stress-strain response can thus be predicted with the models discussed in Table 1.

The VC model builds on the methods used for the CC methodology, with the difference that a new constant confining stress is computed for each increment of axial strain. This iterative method starts with an axial strain value which is converted to a lateral strain using the dilation ratio relationships presented in Table 3. From the lateral concrete strain the jacket strain is computed using compatibility. The jacket stress can be computed from the jacket’s constitutive relationship. The resulting confining pressure is found by force equilibrium. From this point the VC method follows the steps of CC; confined concrete strength and corresponding strain are calculated and used in a stress-strain relationship. The stress at the current level of strain provides one data point. The method is repeated for increasing strain levels until the lateral strain of concrete reaches the failure strain of jacket. The final response is developed from a family of stress-strain responses at different confinement levels (Figure 4).

**Empirical Prediction Methods**

To use the VC and CC methods, the peak compressive stress and strain and the loading and unloading stress strain relationships must be defined. A number of predictive models have been developed based on empirical data fit [Richart et al. 1928; Mander et al. 1988; Cusson and Paultre 1995; Saatcioglu et al. 1995; Diniz and Frangopol 1997; Samaan et al. 1998; Razvi and Saatcioglu 1999; Xiao and Wu 2000; Assa et al. 2001; Susantha et al. 2001; Imran and Pantazopoulou 2001; Lam and Teng 2002; Harries and Kharel 2002]. The confined compressive strength, $f_{cc}$, and corresponding strain, $\varepsilon_{cc}$, are typically presented in the form of $f_{cc} = f_{c0} + k_1 \times f_l$ and $\varepsilon_{cc} = \varepsilon_{c0} \times \phi_1$. Where $f_{c0}$ is unconfined compressive strength of concrete, $\varepsilon_{c0}$ is the axial strain at unconfined peak strength, $f_l$ is the confining pressure. The modification factors $k_1$ and $\phi_1$ are used to provide an adjustment in the peak compressive stress and corresponding strain from the unconfined response. Recommendations for these values are presented in Table 2.

The inelastic stress-strain model for unconfined concrete by Popovics [1973] has been adopted for predicting confined concrete behavior [Mander et al. 1988]. Empirically based modifications to descending branch have been recommended [Cusson and Paultre 1995; Razvi and Saatcioglu 1999; Susantha et al. 2001]. New CC analytical models have also been developed [Saatcioglu et al. 1995; Diniz and Frangopol 1997; Samaan et al. 1998; Assa et al. 2001]. Table 1 present the axial stress strain models based on the previously defined values of $f_{cc}$ and $\varepsilon_{cc}$.

VC method requires the definition of a lateral strain to axial strain relationship for concrete, also referred to as the dilation relationship, $\gamma(\varepsilon)$. For elastic response of concrete this value is defined as a constant equal to a Poisson’s ratio, $\nu$, of 0.15. Due to nonlinear response of concrete, the dilation relationship varies as axial deformation increases. Two models have been developed: Elwi and Murray [1979] developed a dilation relationship for unconfined concrete and Harries and Kharel [2002] who defined...
a tri-linear empirical dilation relationship for FRP jacketed concrete. Table 3 summarizes these two models.

**Modified CC Method for Elastic-Brittle Materials**

To accurately predict the peak compressive strength and strain for elastic-brittle jackets an iteration method is proposed. Oh [2002] has shown that the dilation ratio of both confined and unconfined concretes reaches a value of 0.4 at the peak axial stress. From this value and any of the existing models (Table 2) an iteration-based solution can be made as shown in Figure 5. In the iteration shown, the model by Mander et al [1988] is used for illustration.

**EXPERIMENTAL PROGRAM**

To examine the accuracy of the current design and analysis procedures, an experimental study was conducted with 150 by 300mm concrete cylinders. Steel jackets, CFRP, and GFRP sheets were used for confinement. Three specimens of each type were tested. Details of their material characteristics are given in Table 4. FRP material properties were experimentally determined during previous studies [Kestner et al. 1997], and steel properties were from the mill certification. Unidirectional CFRP carbon-Aramid and GFRP sheets were used. The sheets were bonded with a two-part epoxy resin/hardener. A 102mm lap was used on both FRP specimens. The concrete filled steel tube was fabricated from bent steel plate butt-welded along one line. Ready-mix concrete with a design unconfined compressive strength of 27.6 MPa was used for this study. The mix design has a weight ratio of water/cement/coarse aggregate/fine aggregate slag cement of 1.00/1.48/6.09/4.79/0.37. Concrete was wet cured according to ASTM C192-00.

FRP specimens failed either by jacket fracture or delamination. CFT specimens failed due to fracture near the welding zone. Strains measured on the jacket however exceeded the rupture strain specified by mill certification. The FRP materials failed due to delamination and fiber rupture. On average the FRP jackets reached at least 70% of their ultimate strain capacity prior to failure.

**Experimental Results**

External FRP and CFT jacketing result in an increase of magnitude and maintenance of compressive strength with increasing strain over that of unconfined concrete. The average measured axial stress-strain and dilation ratio (lateral/axial strain) to axial strain responses are presented for FRP in Figure 6. Axial strains corresponding to the peak compressive strengths are presented as vertical lines. The dilation response of the unconfined and confined concretes can be divided into three regions. In the first zone, which occurs up to the peak compressive stress, the dilation ratio remains approximately constant at a value of 0.15. After the peak stress is reached lateral dilation increases rapidly. This is followed by a decrease in the rate of change of the dilation ratio to either a horizontal or marginally increasing rate. This behavior is consistent with the observed behavior. Up to peak strength only marginal damage is observed. Once the strength is exceeded, cracking rapidly propagates throughout the cylinder. For unconfined concrete, this progression slows as the load carrying capacity drops; for the confined specimens the jackets become active and slow the progression.
ANALYTICAL PREDICTION OF RESPONSE

The relationship between axial stress (normalized to peak stress) and dilation ratio illustrates a clear trend for both confined and unconfined concretes. At the peak stress the value of dilation ratio can be taken as 0.4 (Figure 7). This supports previous observations by others [Oh 2002]. This value can be used to accurately predict the response of FRP jacketed concrete using the iterative method previously mentioned (Figure 5). The prediction defined by Cusson and Paultre [1995] is used to compare the effect of three assumptions for lateral stresses: rupture strength of jacket [Samaan and Mirmiran 1998], ACI [2002b] recommendation, and the iterative approach. These assumptions result in three different levels of confinement (Figure 8). The ACI approach provides the lowest level of effective confinement and thus underestimates the ductility. The use of the rupture stress greatly over-predicts the strength and response. The iterative method produces a marginal under-prediction of the peak compressive stress and strain but accurately matches the ductility of the model. The under prediction provides a conservative estimate of behavior for design.

The quasi-static experimental results and predictive models are compared for unconfined concrete. To provide a direct correlation, the models use the measured values of $E_c$, $f_{c0}$, and $H_{c0}$. Under these conditions the unconfined response is accurately predicted by the majority of the models. The ascending branch matches the experimental response for all but the Diniz model which marginally underestimates the response. Post peak response is over-predicted by the Paultre and Diniz models and under-predicted by the others. The Army model provides the closest prediction of response.

Appropriate use of the CC and VC methods require the accurate prediction of the peak confined stress and strain, $f_{cc}$ and $\varepsilon_{cc}$. The accuracy of the models defined in Table 2 is compared to the experimental results for CFT, GFRP, and CFRP (Figure 14). The peak confined stress for steel jacketed concrete is closely predicted by the majority of models. The prediction of the corresponding strain is captured by a number of the models; however, the Paultre and Mirmiran models do not accurately predict the measured response. For FRP jackets the peak strength is typically over-predicted. The models of Xiao and Mirmiran provide good correlation for both GFRP and CFRP. The best prediction both of peak confined stress and strain, however, is provided by the iterative method previously discussed using the stress-strain relationship of Paultre. The method provides a conservative under prediction of strength while accurately predicting the corresponding strain.

The variable confinement method relies on the accurate prediction of the dilation ratio. Previous models have provided a close estimate of the dilation of unconfined concrete up to the peak compressive stress [Elwi and Murray 1979] (Figure 9). Unfortunately this model overestimates the dilation post peak and underestimates the dilation when applied to FRP confined concretes. New models [Harries and Kharel 2002] provide an improved estimate of dilation pre- and post-peak for FRP jacketed columns. For steel jacketed systems, dilation relationships have not been developed. A multi-linear approach, similar to the multi-linear approximation for FRP materials, shows promise for steel jacketed systems. A tri-linear relationship was fit to the experimental results (Figure 10). Two VC models (based on Mander) are compared with the CFT experimental
results. The first method (VC) uses the dilation relation of Elwi, and the second (VC-modified) uses the relationship defined in Figure 10. The modified method provides an accurate estimate of response and failure strain (Figure 11).

The estimated stress-strain response of CC and VC models are presented in Figure 11 to Figure 13. Due to the inaccurate prediction of peak stress and corresponding strain, a number of the models over-estimate the response of steel jacketed concrete. As shown, Watanabe and Paultre under-estimate the ductility of steel jacketed systems, while Mander and Saatcioglu over-predict the strength. All CC models used the ACI [2002b] recommendations for estimating the lateral stress generated by the FRP jacket. These results are compared to the VC method using Paultre constitutive relationship and the dilation ratio relationship of Harries. The VC models consistently provide greater accuracy in predicting the response.

Modified CC model using the iterative approach can be used with any stress–strain model to predict the response of confined concrete. With the exception of the Mander model the iterative approach provides a good correlation with the measured behavior (Figure 15). Diniz, Paultre and Watanabe showing the most promise for application of this method.

**Finite Element Analysis**

To further examine the performance of the concrete a continuum approach using three-dimensional finite element methods was conducted using ABAQUS 6.3 [Hibbit et al. 2002]. The cylinders were modeled with 20 nodded iso-parametric brick elements and 8-noded shell elements for the jackets. The concrete was modeled with damaged plasticity, compression hardening, and tension with nonlinear softening. The damaged plasticity model provides an approximate method for estimating the damage due to cracking. The model does not discretely form cracks. Concrete materials were modeled to match the unconfined experimental results. FRP Jackets were modeled as elastic and the steel jacket was modeled as elastic-plastic.

The FEM models provide a good prediction of the measured response. The models provide a reasonable estimation of both the peak confined stresses (Figure 16) and the dilation behavior (Figure 17). Using these models a parametric study is conducted to determine a relationship for peak confined stress and strain.

**Parametric FEM Study**

Prediction of confined concrete stress-strain response is dependent on the accurate determination of the peak confined strength and strain. Typically models for estimating these values are dependent on the strength of the jacket and unconfined concrete. An improved correlation can be determined from the effective stiffness of the confinement system. This effective stiffness can be computed from the relative stiffness of the confining material to that of the axial stiffness of the concrete. To examine this effect a FEM parametric study is used to develop a new predictive model for $f_{cc}$ and $\varepsilon_{cc}$.

The modulus of the confining pressure is calculated from free body diagram of the confined concrete ($E_{\text{confining}} = 2(t/D)E_i$). For the same jacket material an increase in the thickness results in an increase in the modulus of confining pressure. The parametric study varies the thickness of the steel jacket to evaluate the effect of confining pressure.
The results are calibrated to a lateral/axial stiffness ratio, \( RE \) (Eq.-2). The jacket thickness is varied from 0.4mm to 4.0mm and the peak stress and strain are noted. Using the relationships below a linear fit is obtained. From the relationship (Eq.-3) the peak compressive strength, \( f_{cc} \), can be found with respect to the ratio \( RE \). The corresponding strain, \( \varepsilon_{cc} \), can be calculated using Eq.-4. These equations are compared with the experimental results in Figure 14. In all cases (CFT, CFRP and GFRP), the proposed model provides an accurate estimate of the peak confined stress and corresponding strain.

\[
RE = \frac{E_{confining}}{A \cdot \frac{E_c}{L} \cdot \pi \cdot D^2 \cdot \left( \frac{E_c}{E_c} \right) \cdot \frac{t}{L}}
\]

\[
f_{cc} = f_{cd} \cdot (226.96 \cdot RE + 1.058) [R^2 = 0.9876]
\]

\[
\varepsilon_{cc} = \varepsilon_{cd} \cdot (0.9508 \cdot (f_{cc}/f_{cd}) + 0.068) [R^2 = 0.9979]
\]

**SUMMARY AND CONCLUSIONS**

External confinement of concrete enhances the strength and ductility of concrete under axial demand. Two methods of estimating the stress-strain response of concrete are presented: constant confinement and variable confinement. Limitations of these models and recommendations for improving the accuracy are made with respect to experimental results on steel and, carbon fiber and glass fiber reinforced polymer jackets. From the discussion presented the following conclusions can be made:

- Accurate prediction of stress-strain response of the confined concrete is strongly dependent on the estimation of peak confined strength and corresponding strain.
- Measured peak confined strength is typically over-estimated by the discussed models for jacketed concrete. The corresponding strain can be accurately predicted for steel jacketed concrete, however, most models over predict the strain for FRP confinement.
- Constant confinement models provide a reasonable estimate of axial stress-strain response for steel jacketed systems due to the coinciding occurrence of steel yield and concrete crushing.
- CC models poorly predict the response of FRP jacketed concrete due to the elastic response of the jacketed material and the over prediction of confining stress. ACI recommendations for confining stress over predict the confined strength and under predict the ductility. Use of the jacket’s rupture strength significantly over predicts the response and should not be recommended.
- To accurately determine the peak confined compressive strength and corresponding strain of the FRP jacketed concrete, an iterative solution for \( f_{cc} \) and \( \varepsilon_{cc} \) can be used with the CC method. This model can be accurately used for confinement by elastic materials.
- With a proper dilation relationship, VC models provide the most accurate prediction of confined stress-strain behavior for all types of jacketed concrete. Existing dilation
models by Harries and Kharel [2002] provide an accurate model for FRP jacketed concrete. A tri-linear dilation relationship is shown to provide a comparable accuracy for steel jacketed systems.

- FEM can be used to study the confinement effects of jacketing on concrete. Parametric study resulted in an accurate model for prediction of both peak confined stress and strain for jacketed concrete.

ACKNOWLEDGEMENT

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<th>Researcher</th>
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<td>US Army (1986)</td>
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<td>Cusson and Paultre (1995)</td>
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<td>[ f(\varepsilon) = f_{cc} \times \exp(-k_{c} \times (\varepsilon - \varepsilon_{cc})^{5}) ]</td>
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<td>[ f(\varepsilon) = f_{cc} \times (1 - (\frac{\varepsilon}{f_{cc}})^{5}) ]</td>
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<td>Assa et al. (2001)</td>
<td>[ f(\varepsilon) = f_{cc} \times \frac{(\alpha \times \frac{f_{cc}}{f_{c}}) + (\beta_{r} - 1) \times \left( \frac{f_{cc}}{f_{c}} \right)^{2}}{\alpha + (\alpha - 2) \times \frac{f_{cc}}{f_{c}} + \beta_{r} \times \left( \frac{f_{cc}}{f_{c}} \right)^{2}} ]</td>
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### Table 2 — Modification factors for compressive strength and corresponding strain

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<td>Cusson-Paultre ‘95</td>
<td>(2.1 \times \left(\frac{f_f}{f_c}\right)^{-0.16})</td>
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<td>Samaan-Mirmiran ‘98</td>
<td>(6.0 \times f_f^{-0.33})</td>
<td>(</td>
</tr>
<tr>
<td>Mirmiran ‘99</td>
<td>2.98</td>
<td>N.A.</td>
</tr>
<tr>
<td>Saafi ‘99</td>
<td>(2.2 \times \left(\frac{f_f}{f_c}\right)^{-0.16})</td>
<td>N.A.</td>
</tr>
<tr>
<td>Saatcioglu-Razvi ‘99</td>
<td>(3.5 \times (f_f/f_c)^{0.45})</td>
<td>N.A.</td>
</tr>
<tr>
<td>Toutanji ‘99</td>
<td>(4.1 - 0.75 \times \frac{f_f^2 \times D}{2 \times t \times E_f})</td>
<td>N.A.</td>
</tr>
<tr>
<td>Xiao ‘00</td>
<td>3.36</td>
<td>N.A.</td>
</tr>
<tr>
<td>Asse-Watanabe ‘01</td>
<td>(4.0)</td>
<td>(1 + 21.5 \times \left(\frac{f_f}{f_c}\right))</td>
</tr>
<tr>
<td>Susantha ‘01</td>
<td>(4.0)</td>
<td>(1 + 5 \times \left(\frac{f_f}{f_c}\right) - 1)</td>
</tr>
<tr>
<td>Harries ‘02</td>
<td>(4.264 \times f_f^{-0.443})</td>
<td>N.A.</td>
</tr>
<tr>
<td>Lam ‘02</td>
<td>2.15</td>
<td>N.A.</td>
</tr>
<tr>
<td>Mander ‘88</td>
<td>(f_c = f_c \times (-1.254 + 2.254 \times (1 + 7.94 \times \left(\frac{f_f}{f_c}\right)^{0.83} - 2 \times \left(\frac{f_f}{f_c}\right))))</td>
<td>(1 + 5 \times \left(\frac{f_f}{f_c}\right) - 1)</td>
</tr>
<tr>
<td>Imran-Pantazopoulou ‘01</td>
<td>(f_c = f_c \times \left[(\frac{f_f^2}{f_c} - 0.021) + (1.043 + 10.571 \times \left(\frac{f_f}{f_c}\right)^{0.83})\right])</td>
<td>N.A.</td>
</tr>
</tbody>
</table>

### Table 3 — Dilation relationships

<table>
<thead>
<tr>
<th>Researcher</th>
<th>Dilation Relationship</th>
</tr>
</thead>
<tbody>
<tr>
<td>Elwi-Murray 1979</td>
<td>(\eta(\varepsilon) = \nu \times [1 + 1.3763 \times \frac{\varepsilon}{\varepsilon_{\varepsilon}} - 5.36 \times \left(\frac{\varepsilon}{\varepsilon_{\varepsilon}}\right)^2 + 8.586 \times \left(\frac{\varepsilon}{\varepsilon_{\varepsilon}}\right)^3])</td>
</tr>
<tr>
<td>Harries-Kharel 2002</td>
<td>(\eta(\varepsilon) = \nu \times [1 - 0.99 \ln(\varepsilon_{\varepsilon}) + 12\ln(\varepsilon_{\varepsilon}) - 0.66 \ln(\varepsilon_{\varepsilon}) + 8])</td>
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\[\text{MPa}\]
Table 4 — Material characteristics of jackets

<table>
<thead>
<tr>
<th>Property</th>
<th>CFRP</th>
<th>GFRP</th>
<th>Steel Tube</th>
<th>Concrete</th>
</tr>
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<tbody>
<tr>
<td>Elastic Modulus $E_i$ [MPa]</td>
<td>233,700</td>
<td>23,300</td>
<td>200,000</td>
<td>32,060</td>
</tr>
<tr>
<td>Thickness $t$ [mm]</td>
<td>0.165</td>
<td>0.864</td>
<td>2.46</td>
<td>-</td>
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<tr>
<td>Yield Strength $f_y$ [MPa]</td>
<td>N/A</td>
<td>N/A</td>
<td>248</td>
<td>-</td>
</tr>
<tr>
<td>Yield Strain $e_y$</td>
<td>N/A</td>
<td>N/A</td>
<td>0.0012</td>
<td>-</td>
</tr>
<tr>
<td>Ultimate Strength $f_u$ [MPa]</td>
<td>3,510</td>
<td>496</td>
<td>310 $(f_{cd}) 38.4$</td>
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</tr>
<tr>
<td>Ultimate Strain $e_u$</td>
<td>0.015</td>
<td>0.019</td>
<td>0.025 $(e_{cd}) 0.00164$</td>
<td></td>
</tr>
</tbody>
</table>

Figure 1 — Confining pressure variation assumptions for CC and VC models

Figure 2 — Confined stress-strain response
Figure 3 — Methodology for computation of stress-strain response

Figure 4 — VC family of responses
Figure 5 — Recommended iteration scheme for the peak confined compressive strength and strain of confined concrete.

Figure 6 — FRP jacketed concrete response.

Figure 7 — Normalized axial strength-dilation ratio relationship.
Figure 8 — Measured and predicted response of CFRP jacketed concrete

Figure 9 — Prediction of dilation relationship

Figure 10 — Tri-linear data fit for dilation of CFT

Figure 11 — Axial stress-strain behavior of CFT
Figure 12 — Axial stress-strain behavior of CFRP jacketed concrete

Figure 13 — Axial stress-strain behavior of GFRP jacketed concrete
Figure 14 — Comparison of the peak strength and strain of jacketed concrete.

Figure 15 — CC predictor models with modification of iterative prediction-GFRP.
Figure 16 — Stress-strain behavior prediction by using FEM analysis

Figure 17 — Dilation relationship prediction with FEM analysis
Modeling the Stress-Strain Behavior of Confined Concrete Columns

by K.M. El-Dash and O.O. El-Mahdy

Synopsis: In this paper, an analytical stress-strain model of confined concrete columns is developed and presented. The model is based on the extensively obtained data from tests of column specimens subjected to concentric compression loading. The tests included a wide range of varieties including both normal and high-strength concretes. The cross sections of the columns were of circular, rectangular, or elliptical shapes. The model incorporates the effective relevant parameters of confinement like concrete strength, yield strength of transverse reinforcement, spacing between lateral confining element, and dimensional configuration of column specimen and its transverse reinforcement. The model can be used for concrete confined by spirals, rectilinear hoops, crossties, and combinations of these reinforcements. The model demonstrates good predictive capability for concrete columns of compressive strength ranging from 20 MPa to 120 MPa. In addition, the model is shown to be applicable for a wide range of quantity and configuration of lateral reinforcement with volumetric ratio to concrete from 0.2% to 4%.

Keywords: columns; concrete; confined; ductility; high strength; model; stress-strain
INTRODUCTION

Behavior of confined concrete members has been studied extensively in the last two decades. The effect of lateral reinforcement is not considered up to 40-50% of the concrete maximum capacity, which is the actual working range. The real contribution of confinement takes place at higher range of loading when the lateral strains of concrete become high. The lateral dilation of concrete forces the lateral confining elements to stretch outside producing excessive internal strains and stresses. This behavior from the concrete towards the confining elements attracts passive pressure that enhances the strength and ductility of concrete (Van Mier, 1986, Mander et al., 1988, Muguruma et al., 1990, Karabinis and Kiousis, 1994, Priestley et al., 1994, Cusson and Paultre, 1995, and Mei et al., 2001).

Behavior of concrete under concentric compressive load is governed by bond stresses between the paste and aggregates. When the applied load approaches the ultimate capacity of concrete, slippage between paste and aggregates occurs. This slippage is accompanied by crack initiation that propagates with the incremental increase of loading. If excessive lateral pressure is applied to the concrete, the contact bond will be stronger and the slippage between paste and aggregates will be delayed to a higher range of loading. When the confining reinforcement is sufficient to resume tolerable confining stress, the propagation of cracks will be slower than propagation of cracks for unconfined concrete. The slower rate of propagation causes the better ductility behavior obtained for confined concrete under compressive loading (Van Mier, 1986).

RESEARCH SIGNIFICANCE

In the current study, it is targeted to establish a comprehensive stress-strain relationship of confined concrete columns subjected to concentric compressive load. The model considers columns confined by spirals or ties with/without cross ties. The presented analytical relationship considers columns made by concrete with compressive strength starting with conventional strength of 20 MPa up to high compressive strength of 124 MPa. In addition, the model is shown to be applicable for a wide range of quantity
and configuration of lateral reinforcement with volumetric ratio to concrete from 0.2% to 4%. The cross sections of the columns employed were of circular, rectangular, or elliptical shapes. The peak strength and peak strain of the stress-strain relationship are presented in a comparative study for different confinement situations. The ascending and descending branches of the stress-strain curve are incorporated in a single equation to produce a single expression for the overall needed relationship.

ANALYTICAL MODEL

In the presented model, a fractional equation is used to predict the stress-strain relationship of laterally confined concrete. This equation has been used by Sargin et al., 1971, Ahmad and Shah, 1982, Martinez et al., 1984, and El-Dash, 1995, to predict the stress-strain curve for different types of columns. The equation is given by;

\[ y = \frac{Ax + (B - 1)x^2}{1 + (A - 2)x + Bx^2} \]  

(1)

where; \( y = \frac{f_c}{f_{cc}} \), \( x = \varepsilon_c / \varepsilon_{cc} \), \( A = E_c / E_p \), \( f_c \) is the confined concrete stress at strain of \( \varepsilon_c \), and \( f_{cc} \) and \( \varepsilon_{cc} \) are the peak stress and the corresponding strain.

The parameter \( A \) controls the slope of the ascending branch of the curve depending on the modulus of elasticity, \( E_c \) that is calculated as per ACI-318, 2002, equation;

\[ E_c = 0.036 w_c^{1.5} \sqrt{f_c'} \]  

(2)

where \( w_c \) is the unit weight of concrete in kg/m\(^3\) and \( f_c' \) is in MPa.

The ascending branch is finished at the peak point that provides the model with plastic modulus, \( E_p \), expressed as;

\[ E_p = f_{cc} / \varepsilon_{cc} \]  

(3)

The parameter \( B \), which primarily controls the shape of the stress-strain curve in the post-peak portion, is determined by establishing the strain of one representative point in the post-peak portion of the curve. This point used to be at 85% of the peak stress on the descending branch with the corresponding strain denoted as; \( \varepsilon_{85} \). For high-strength concrete, the stress-strain relationship is very sensitive in the post-peak portion. The representative point chosen for strain in the post-peak portion is at 50% of the maximum stress, \( \varepsilon_{50} \). The 50% strength post-peak point gives a good representation for the shape of post-peak portion of the curve. It is an intermediate location on the descending portion, far from the sensitivity zone near the peak point, and ahead of the tail end of the curve. This point was utilized by Cusson and Paultre, 1994, in the test measurements and in their analytical model, 1995. In Equation (1), when \( x > 1 \), the value of \( y \) should not be less than 0.2.
To predict the peak stress and the corresponding strain, the following formulations are utilized:

\[ f_{cc} = f_{co} + \Delta f_c \]

and

\[ \varepsilon_{cc} = \varepsilon_{co} + \Delta \varepsilon_c \]

where; \( f_{co} \), and, \( \varepsilon_{co} \), are the peak stress and strain of the unconfined concrete, and, \( \Delta f_c \), and, \( \Delta \varepsilon_c \), are the enhancements in concrete strength and the corresponding strain due to lateral confinement, respectively.

The parameters primarily influencing the stress-strain relationship of confined concrete include the strength of concrete, yield strength of the confining reinforcement, volumetric ratio of the confining reinforcement to the concrete core as well as spacing between confining reinforcement, dimensions of the column, and the configuration of the lateral confining reinforcement. All these parameters are considered in the presented model. Table (1) presents the experimental work used in the analysis including 157 concrete specimens with different cross sections, heights, compressive strength, and transverse reinforcements.

**Lateral Pressure**

The strength capacity of concrete columns varies considerably with the amount and spacing of lateral confining reinforcement, and the strength of unconfined concrete. The lateral confining pressure in the case of confined circular columns can be easily quantified because the lateral pressure is almost uniform. For rectangular or elliptical column cross-sections, the distribution of the lateral confining pressure is not uniform. The configuration of the transverse reinforcement plays a big role in the behavior of such columns. Besides the variability in the confining pressure due to the shape of the cross section of the column, there is a variability of the pressure in the longitudinal direction due to the spacing between the hoops or the pitch of the spiral.

It is common to assume that when the concrete reaches its maximum resistance, the confining pressure can be computed by assuming that the lateral confining reinforcement yields as it was assumed by Cusson and Paultre, 1995, Hoshikuma et al., 1997, and Razvi and Saatcioglu, 1999-a. Saatcioglu and Razvi, 1998, validated this assumption experimentally for the heavily confined high-strength concrete specimens with volumetric transverse reinforcement ratio of 1.3\% for circular cross sections and 2\% for rectangular cross sections. For lightly confined columns, the lateral reinforcement may not reach the yield strength but the assumption resulted in acceptable analytical results. In both cases, the consideration of the ratio of the concrete unconfined strength to the yield strength of lateral reinforcement needs to be included in the mathematical expression. A fine measurement for the effective lateral pressure at the maximum resistance can be calculated exploiting the following equation:

\[ f_l = k_s k_f \rho_{st} f_{yt} \]
where, $\rho_{st}$, is the volumetric ratio of the transverse reinforcement to the confined concrete core and $f_y$, is the yield strength of the transverse reinforcement. Figure (1) shows the relationship between the lateral reinforcement ratio, $\rho_{st}$, and the enhancement in confined concrete strength. The relationship is almost directly linear proportional with steady enhancement of concrete strength as the transverse reinforcement increases. The trend lines obtained from linear regression proved that the circular columns gain more strength than rectangular columns with the same lateral confinement. Figure (2) presents the relationship between spacing between transverse reinforcement to column breadth, $s/b$, and the enhancement in concrete compressive strength, $f_{cc}/f_{co}$. It is noticed from the figure that when, $s/b$, is less than 0.3 the enhancement gained could be considerably higher considering all other affecting parameters. The coefficient, $k_s$, is induced to consider the effect of lateral pressure variability in the vertical direction. It was deduced in the analysis that columns with rectangular cross section are much more sensitive to the spacing of the lateral reinforcement than the columns with circular cross section. Hence, two different mathematical expressions are presented for this coefficient in the model;

$$k_s = \left(1 - \frac{s}{b}\right)^{0.5}$$

for circular cross sections and

$$k_s = \left(1 - \frac{s}{b}\right)^2$$

for rectangular cross sections.

The coefficient, $k_f$, accounts for the change in confining pressure with the change in the ratio of unconfined concrete strength to yield strength of the lateral reinforcement. The factor applies the well-known phenomena that the higher concrete strength columns demand higher lateral reinforcement to obtain same properties enhancements. Figure (3) presents this effect on the enhancement in concrete strength, $f_{cc}/f_{co}$. It is observed clearly that when, $f_{co}/f_y$, is less than 0.10 the enhancement in concrete strength could be impressive when other effective parameters are constant. The coefficient is calculated employing the following equation for both rectangular and circular cross sections;

$$k_f = 1 - \left(\frac{f_{co}}{f_y}\right)^{0.5}.$$  \hspace{1cm} (9)

**Peak Stress**

It is shown in the previous sections and in Figures (1), (2), and (3) that peak stress of confined concrete columns depends primarily upon the strength of unconfined concrete, dimensions of confined core, and amount and configuration of the lateral reinforcement. For the computation of peak stress of normal and high-strength concretes, knowledge of unconfined concrete strength, $f_{co}$, and the effective lateral pressure, $f_l$, is needed. The response of confined concrete columns to the lateral pressure varies drastically by the change in the cross section. Columns with circular cross sections
experience strength enhancement about double that experienced by columns with rectangular cross section when subjected to same lateral confining pressure. This is referred due the irregularity in the distribution of pressure on the cross section of the rectangular columns. Hence, the following relationships are derived after excessive mathematical calibrations for different types of expressions to represent the confined concrete strength in terms of its unconfined strength and the applied lateral confining pressure.

The following relationships are deduced from regression analysis to predict the compressive strength of confined concrete columns;

\[ f_{cc} = f_{co} + 3.8f_l \]  

(10)

for circular cross sections and

\[ f_{cc} = f_{co} + 1.8f_l \]  

(11)

for rectangular cross sections. The results of the elliptical concrete columns included in the analysis show mechanical response to the lateral confinement that is similar to that of the rectangular columns.

Figure (4) shows the relationship between the experimentally recorded results for the confined concrete strength versus the values derived analytically from the proposed model. The correlation between the two sets of values is terrific for most of the specimens. The calculated, \( R^2 \), value is 0.942 that displays the fine matching between experimental and analytical results.

Peak Strain

The results obtained experimentally for confined concrete specimens proved that high-strength concrete columns require a considerably higher level of lateral confining pressure to simulate the same ductility enhancements of normal strength concrete columns. Records of Razvi and Saatcioglu, 1999-b, and Saatcioglu and Razvi, 1998, shows considerably lower peak strains than those recorded by, Cusson and Paultre, 1994, Hoshikuma et al., 1997, and Lin et al., 2004, for the same concrete strength, dimensions, and transverse reinforcement quantity and configuration. Results obtained by Liu et al., 2000, showed very high values for the peak strain of the confined columns so that it is excluded from the analytical derivation for the peak strain expression. The variation in the recorded peak strain values is dramatic from one research to another that can be referred to the mix proportions, age of concrete at testing, additives used in the mix, and type of aggregate utilized in the specimen. These parameters are not included neither in this study nor in any previous study because of difficulties of quantifying of these influences.

Figure (5) shows general relationship between the confining pressure ratio to the unconfined strength, \( f_l / f_{co} \), and the peak strain of the experimented specimens. The figure illustrates the proportionality of the confining pressure with the peak strain value in general but a wide scatter can be easily noticed in the figure due to the reasons mentioned earlier. After considerable trials, the following mathematical relationship is
found to best fit the relationship between the strain at peak stress value and the effective lateral pressure, $f_l/f_{co}$. The relationship is:

$$
\varepsilon_{cc} = \varepsilon_{co} + 0.57 \left( \frac{f_l}{f_{co}} \right)^3
$$

(12)

where the peak strain of the unconfined concrete, $\varepsilon_{co}$, is expressed as per the recommendation of Shah and Ahmad, 1994:

$$
\varepsilon_{co} = 0.00165 + 0.0000165 f'_c
$$

(13)

The peak strain mathematical expression has an, $R^2$, value of 0.718 with respect to the recorded values for the experimental results.

**Descending Branch**

The shape of the post-peak portion of the stress-strain curves is primarily governed by the parameter, $B$, in Equation (1). To calculate the value of this parameter, a post-peak point is needed to be established. The point at 85% or 50% of the confined concrete strength in the post-peak portion of the curve can be utilized as this reference point. The 85% strength post-peak point is preferred to be used for low strength concrete and well-confined specimens since the descending portion should not have steep descending curve. On the contrary, columns with high-strength concrete and low to medium confinement are sensitive to the strain beyond the peak point. Hence, the point at 50% strength has a good representation for the post-peak portion of the later type. It has an intermediate location on the descending portion, far from the sensitivity zone near the peak point, and ahead of the collapse of the specimen.

A detailed investigation was carried out to find an appropriate mathematical equation that would represent the strain of concrete at 85% and 50% of the peak strength, $\varepsilon_{85}$ and $\varepsilon_{50}$, respectively. The strains of concrete in the post-peak portion of the response, $\varepsilon_{85}$ and $\varepsilon_{50}$, are found to be correlated to the strength of confined concrete more than to the strength of unconfined strength. Figure (6) presents the relationship between the ratio of lateral confining pressure to peak strength and the measured strain at 85% of the peak strength on the descending branch. Figure (7) presents a similar relationship utilizing the strain at 50% of the peak strength. The usage of 50% and 85% was based on the available data. Nevertheless, the strain at 50% of the peak strength is used for brittle specimens like high strength concrete types. Based on the experimental records, the following expressions are found to be most representative of the test data:

$$
\varepsilon_{50} = \varepsilon_{cc} + 0.033 \left( \frac{f_l}{f_{cc}} \right)^{0.5}
$$

(14)

and

$$
\varepsilon_{85} = \varepsilon_{cc} + 0.021 \left( \frac{f_l}{f_{cc}} \right)^{0.5}
$$

(15)
The above-mentioned mathematical expressions have, $R^2$, values of 0.94 and 0.73, respectively, with respect to the experimental results included in the study. Once the reference point, $\varepsilon_{85}$ or $\varepsilon_{50}$, is established, the parameter, $B$, can be obtained by back substitution in equation (1). It shows as:

$$B = \frac{1 - Ax - 2x + 2x^2}{x^2}$$  \hspace{1cm} (16)

where $x$, is used as for $\varepsilon_{85}$ or $\varepsilon_{50}$.

**VERIFICATION OF MODEL**

Comparisons between the complete stress-strain curves predicted by the proposed model and the experimental results are shown in Figures (8) and (9). Figure (8) shows the results for a square column of 500 mm side length and 1,000 mm height. The specimen was confined by 13 mm diameter welded hoops spaced at 40 mm intervals. The volumetric ratio of the transverse reinforcement was 2.6%. The unconfined compressive strength of concrete was 24.3 MPa and the yield strength of the hoops was 295 MPa. Figure (9) presents the results for a square column of 235 mm side length and 1,400 mm total height. The specimen was confined by 9.5 mm diameter hooked hoops spaced at 50 mm intervals. The volumetric ratio of the transverse reinforcement was 2.8%. The unconfined compressive strength of concrete was 99.9 MPa and the yield strength of the hoops was 705 MPa.

The figures show the effect of lateral confinement on the behavior of concrete columns with respect to the unconfined concrete strength. The lateral confining pressure for the specimen shown in Figure (8) is 4.63 MPa versus 7.99 MPa for the specimen shown in Figure (9). Despite that the lateral pressure for the first specimen is less than the second one, it experienced higher strength and ductility enhancements because its unconfined strength is much less than the second one. In both cases, the proposed model demonstrated high predictive capacity for different values of unconfined concrete strength, cross sectional dimensions, yield strength of transverse reinforcement, and spacing of hoops.

It could be noticed by comparison for the behavior of normal strength and high strength concrete columns that for comparable specimens, the higher strength concrete specimens have lower deformability and energy absorption and dissipation capacities initially. During the latter part of the displacement excursions, these properties improve rapidly and the total values are comparable to those of lower strength concrete specimens. The same conclusion was reported by Bayrak and Sheikh, 1998.

**PRACTICAL APPLICATION**

The confinement of concrete columns has a neglected effect on strength and ductility up to 40-50% of the compressive strength. The real benefit from confinement arises when the applicable load is close to the concrete strength or beyond this limit. The
The parameter, $\rho_d f_{st} / f_{co}$, was used in ACI-ASCE Committee 441, 1997, as a guidance measure for the confinement. Saatcioglu and Razvi, 1998, proposed a minimum value of 0.18 for the term $\rho_d f_{st} / f_{co}$ for columns with rectangular cross section. In addition, Razvi and Saatcioglu, 1999-b, proposed a minimum value of 0.09 for the same term for concrete columns with circular cross sections. Based on the extensive study carried out in the research, it was reached that a value of 0.10 for $\rho_d f_{st} / f_{co}$ for circular columns most probably results in 20% strength enhancement and ductility index ($\varepsilon_{50}/\varepsilon_{cc}$) of 3. In addition, a value of 0.15 for the same term with rectangular columns may result in 10% strength enhancement and ductility index of 3. The lately mentioned values are recommended for columns severely exposed to quakes or similar cases.

CONCLUSIONS

A numerical model is presented to predict the stress-strain relationship for normal and high strength concrete columns of rectangular and circular sections confined with spirals, ties, and/or cross ties. The model is based on the experimental results of 157 concrete specimens subjected to different types and amounts of transverse reinforcement and tested under concentric loading. Comparisons are made between the predictions of the model and the available experimental results. It can be concluded from the study that:

- The model demonstrates good predictive capability and is applicable for a wide range of variables that include range of unconfined concrete strength from 20 MPa to 120 MPa and transverse reinforcement ratio from 0.2% to 4.9% by volume.
- The strength enhancement, $f_{cc} / f_{co}$, of the confined concrete decreases with the increase of concrete strength but the total absorbed energy by the column increases with the same strength and lateral confinement configuration.
- The ductility of the confined columns decreases drastically with the increase of concrete strength for the same confinement pattern. Equation (12) shows the exponential relationship between the peak strain and unconfined concrete strength.
- It is realized that a value of 0.10 for $\rho_d f_{st} / f_{co}$, for circular columns most probably results in 20% strength enhancement and ductility index ($\varepsilon_{50}/\varepsilon_{cc}$) of 3. In addition, a value of 0.15 for the same term with rectangular columns may result in 10% strength enhancement and ductility index of 3. The lately mentioned values are recommended for columns severely exposed to quakes or similar cases.

NOTATIONS

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_c$</td>
<td>Modulus of elasticity</td>
</tr>
<tr>
<td>$E_p$</td>
<td>Plastic modulus</td>
</tr>
<tr>
<td>$E_{des}$</td>
<td>Deterioration rate</td>
</tr>
<tr>
<td>$b$</td>
<td>Smaller side of column or diameter</td>
</tr>
<tr>
<td>$f_c$</td>
<td>Concrete stress</td>
</tr>
<tr>
<td>$f'_c$</td>
<td>Specified concrete strength</td>
</tr>
<tr>
<td>$f_{cc}$</td>
<td>Confined concrete strength</td>
</tr>
<tr>
<td>$f_{co}$</td>
<td>Unconfined concrete strength</td>
</tr>
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</table>
Lateral pressure
Yield strength of lateral reinforcement
Constants
Spacing between hoops
Unit weight of concrete
Concrete strain
Confined concrete strain at peak point
Unconfined concrete strain at peak point
Concrete strain at 50% of the confined strength on the post-peak branch
Concrete strain at 85% of the confined strength on the post-peak branch
Volumetric ratio of lateral reinforcement to concrete core

REFERENCES

ACI Committee 318, 2002, “Building Code Requirements for Reinforced Concrete and Commentary (ACI 318-02/ACI 318R-02)”, American Concrete Institute, Detroit.


Table 1 — Experimental work included in the analysis

<table>
<thead>
<tr>
<th>Author</th>
<th>Number of specimens</th>
<th>Shape</th>
<th>Cross section (mm)</th>
<th>Height (mm)</th>
<th>Compressive strength (MPa)</th>
<th>Transverse reinforcement %</th>
</tr>
</thead>
<tbody>
<tr>
<td>Liu (2000)</td>
<td>12</td>
<td>Circular</td>
<td>D=250</td>
<td>1600</td>
<td>60 - 96</td>
<td>0.58 – 3.18</td>
</tr>
<tr>
<td>Razvi (1999-b)</td>
<td>20</td>
<td>Circular</td>
<td>D=250</td>
<td>1500</td>
<td>60 - 124</td>
<td>0.41 – 3.05</td>
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<tr>
<td>Pessiki (1997)</td>
<td>8</td>
<td>Circular</td>
<td>D=559</td>
<td>2235</td>
<td>37.9 – 84.7</td>
<td>1.32 – 2.61</td>
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<tr>
<td>Hoshikuma (1997)</td>
<td>11, 13</td>
<td>Circular, Rectangular</td>
<td>D=200-500, D=200-1000</td>
<td>600-1500, 600-1000</td>
<td>18.5 – 28.8, 23.2 – 24.3</td>
<td>0.19 – 4.66, 0.39 – 4.66</td>
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<tr>
<td>Sevcioglu (1998)</td>
<td>24</td>
<td>Square</td>
<td>250</td>
<td>1500</td>
<td>60 - 124</td>
<td>0.99 – 4.59</td>
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<tr>
<td>Cusson (1994)</td>
<td>27</td>
<td>Square</td>
<td>235</td>
<td>1400</td>
<td>52.6 – 115.9</td>
<td>1.40 – 4.80</td>
</tr>
<tr>
<td>Lin (2004)</td>
<td>24</td>
<td>Square</td>
<td>300</td>
<td>1400</td>
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Figure 1 – Relationship between volumetric ratio of lateral reinforcement, $\rho_{st}$ %, and compressive strength enhancement, $\frac{f_{cc}}{f_{co}}$

Figure 2 – Relationship between spacing of transverse reinforcement to column breadth and the compressive strength enhancement
Figure 3 – Relationship between concrete unconfined strength to yield strength of transverse reinforcement and the compressive strength enhancement

Figure 4 – Correlation between experimental and analytical concrete confined strength

Figure 5 – Relationship between ratio of lateral confining pressure to concrete compressive strength and peak strain
Figure 6 – Effect of lateral confining pressure on the strain at 85% of the peak strength

Figure 7 – Effect of lateral confining pressure on the strain at 50% of the peak strength

Figure 8 – Analytical vs. experimental stress-strain relationship for LS-3 specimen tested by Hoshikuma et al. (1997)
Figure 9 – Analytical vs. experimental stress-strain relationship for A-5 specimen tested by Cusson and Paultre (1994)
El-Dash and El-Mahdy
Experimental Study on Stress-Strain Relationship of Confined Concrete with GFRP Jackets

by C.D. Zhou and X.L. Lu

Synopsis: A two-stage analytical model for evaluating the stress-strain behavior of square concrete columns confined with glass fiber-reinforced polymer (GFRP) is proposed. Fifty-one square columns (divided into seventeen groups) were tested under static axial compression up to failure. The main factors considered in the test are as follows: grades of concrete, bonding shape, amounts of GFRP jackets, and space of fiber strips. The results clearly showed the efficiency of the GFRP jackets in enhancing the ultimate strain and the strength of the columns. The analytical model was calibrated using data from the tests, and good agreement between test and computation results was obtained.

Keywords: analytical model; confined concrete; GFRP; stress-strain relationship
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Xi Lin Lu, holds the Cheung Kong Scholarship and serves as a professor of civil and structural engineering at the Tongji University. His research interests include confined concrete, nonlinear analysis of concrete structures and seismic behavior of high-rise structures.

INTRODUCTION

Fiber reinforced polymer (FRP) materials have been recognized as new innovative material for construction industry. They are being increasingly used as an alternative to steel for reinforcing and strengthening of concrete structures. The common way to strengthen a concrete column using FRP is to wrap the column with FRP jackets. Under the increased axial compressive loading, the core concrete is confined with the surrounding wrapped FRP jackets. As a result, the compressive strength and ultimate strain of concrete is improved (H. Saadatmanesh et al. 1994, Amir Mimiran et al. 1997, Chris P. Pantelides 1999, Houssam A. 1999, ACI Committee 440 2000, Yung C. Wang et al. 2001), thus the bearing-capacity and the ductility of concrete columns are raised, and the seismic resistance is enhanced.

When square concrete columns are strengthened with bonding glass-fiber reinforced polymer (GFRP) jackets, lateral pressure of core concrete is not uniform. Premature failure of GFRP jackets often occurs because of stress concentration on the corner of columns (Chris P. Pantelides 1999, Yung C. Wang et al. 2001). So the strength and ductility is lower than that of confined circular columns, and its calculation and analysis are more troublesome than that of circular ones.

In this paper, experimental results on 51 square columns were presented to study confined effect of concrete with GFRP jackets. Based on reported test results, a two-stage analytical model is provided.

TEST PROGRAM

Description of test

Experimental results (Z. H. Guo 1999) indicate that the axial compressive strength of concrete prism or cylinder is close to fixed value while the ratio of length to width is more than 3, that is \( h/b \geq 3 \) (see figure 1). Accordingly the ratio of length to width of specimens for this experiment is also three. The cross sections of columns are shown in figure 2.
Figure 3 depicts reinforcement details of the test units. 17 groups of 450 mm high columns were built and tested. In order to assure the reliability of test results, each group was comprised of three columns. The main factors that affect the confinement of concrete are as follows: grade of concrete, bonding shape (strip spacing wrapped and completely wrapped), amount of GFRP jackets, and space of FRP strips.

All specimens were divided into three series. The first series, group SL0, SM0 and SH0, were left unwrapped to act as control specimens; the second series, group SL1, SM1-4 and SH1 were tested after completely wrapped with different layers of GFRP sheets; the third series, group SM5-12, were tested after bonded GFRP strips.

The 0.169mm nominal thickness GFRP wraps were applied when the columns were cured one month later. The reciprocal radius of columns was 20 mm. The surface of column was smoothed before an epoxy jacket was bonded. The method of application consisted of applying the continuous epoxy-saturated fabric until the specified numbers of wraps were achieved. The final wrap overlapped 100 mm the beginning of first wrap. The jackets were left curing at least 1 week before testing. The extent of the wraps is shown in figure 4. The specimen grouping was listed in table 1.

Material properties

Table 1 presents the main mechanical properties of the concrete. The concrete strength $f'_c$ was transferred from its standard cube compression strength. Table 2 provides the GFRP jacket and the epoxy material properties for the tensile specimens tested as part of this study. Both GFRP jacket specimens exhibited a linear response to sudden rupture failure. The stress-strain relationship of GFRP was shown in figure 4.

Test setup

Concentric compression loading was applied with a 2000kN universal testing machine (see figure 5). The axial load was applied in small increments about 100kN per minute. The axial load was monitored using a load cell.

Figure 6 illustrates the instruments of the columns for determining longitudinal and transverse strain. The longitudinal and transverse strain was monitored by two 25 mm strain gauges bonded on the GFRP surface, respectively. The longitudinal displacement was automatically measured using two electric displacement gauges.

TEST RESULTS

Failure modes

Phenomena in the test and failure modes of the specimens (see figure 7) showed that the ductility and strength of concrete square columns was significantly improved after boned GFRP jackets. Failure modes in this test were consistent with other test results (J. B. Mander et al. 1988, Chris P. Pantelides 1999, Yung C. Wang et al. 2001, Beni Assa et al. 2001, Murat Saatcioglu et al. 2002).
The unwrapped columns under axial compression loads initially developed some mini-cracks that gradually propagated with the increase of the loads. The lateral strain in the middle of the columns rapidly increased and expanded at the 80% of $f_c'$, ultimate strength of concrete; finally the columns were abruptly crushed at $f_c'$. For completely wrapped square columns, the concrete had little lateral expansion at the initial stages of loading, so the GFRP sheets played a little the confinement role owing to the slow development of the lateral strain. When compression stress reached to $f_c'$, the middle of specimens expanded outside and part of the sheets began to rupture with incidental sounds, and some lateral white stripes appeared. Then the lateral strain of specimens developed so rapidly as the increase of loads, the GFRP jackets appeared apparent folds and started to expand due to too large axial stress. When compression stress reached to $f_{cc}'$, ultimate strength of confined concrete, the jackets on the middle of columns suddenly ruptured and rapidly expanded outside, then the whole columns went to failure.

Although some smoothing treatment with the edges and corners of square columns has been done, there is still a certain extent stress concentration that resulted in the first fracture of the GFRP spirals in these places.

The core concrete was crushed after failure accompanying with some falling concrete grains. The diameter of grain depended on the number of layers of the GFRP sheets: the more the number of plies were, the smaller the diameter was. This indicated that increasing number of GFRP jackets could provide more confinement.

**Confined concrete strength**

Table 3 shows that the ultimate strengths of unwrapped columns of the three different grade strengths are remarkably increased while wrapped with GFRP jackets. Where $f_{cc}'$, is the ultimate strength of confined concrete; $\varepsilon_{cc}'$ is the strain corresponding to $f_{cc}'$; $f_{cc1}'$ and $\varepsilon_{cc1}'$ represent the test results; $f_{cc2}'$ and $\varepsilon_{cc2}'$ represent the computation results.

Figure 8 shows some deregulation that the ultimate strength of the unwrapped concrete columns of the largest grade concrete is less than that of the moderate grade concrete. The columns of the largest grade concrete behaved more brittle than the moderate grade concrete columns, and they went to failure as soon as the crack appeared, namely the crack load is ultimate load.

As shown in Figure 9, the ultimate strength is increased as the number of the GFRP jacket plies increased. When the interface of columns were wrapped with GFRP from 1 to 4 plies, the ultimate strength was significantly enhanced by 20%, 20%, 24% and 24.5%, respectively. The increase of ultimate strength is significantly less than that of the GFRP jackets plies. This is because that the confinement provided by GFRP jackets is uneven along the columns interface; moreover, the vicinity of the middle of the
cross section is not in three-dimension compression.

Figure 10 presents the effect of gap between GFRP strips on the axial stress. The gaps are 0, 50mm, 100mm and 150mm in test, respectively, and the corresponding ultimate strength was reduced from 37.4MPa to 36.9, 34.9 and 34.1MPa, respectively. Compared with the unwrapped columns, the improvement of the ultimate strengths was 20%, 18%, 12% and 9%, respectively.

Figure 11 shows the effect of GFRP strip space. According to intervals 0, 25mm, 50mm, 75mm and 100mm, the ultimate strength was 37.4, 35.0, 36.4, 33.8 and 31.1 MPa, respectively. Compared with the unwrapped columns, the ultimate strength was increased by 20%, 12%, 16%, 8% and -0.5%, respectively.

Figure 10 and 11 indicate that ultimate strength decreased as the increase of the intervals and gaps between GFRP jacket strips. The confinement effect of GFRP maybe completely disappeared beyond a certain space.

**Axial and lateral strains**

Unwrapped columns behaved brittle failure without obvious descent in the stress-strain curves with the peak value of axial strain of 0.0015 and lateral strain from 0.0004 to 0.00055. The ductility of unwrapped concrete columns decreased as the concrete strength increasing.

The ductility of the square columns completely wrapped with GFRP jackets was significantly improved and obvious descent in the stress-strain curves could be observed with the peak value of axial strain from 0.0023 to 0.0105 and lateral strain from 0.0025 to 0.006. The descent curve was more slowly as the number of plies increasing, namely the ductility was improved. Though the increase of the concrete grade could decrease the ductility of the columns with the same number of layers of GFRP jackets, the confined columns still have excellent ductility corresponding to the unwrapped concrete columns.

The ductility of the square columns wrapped with GFRP strips was also improved as the number of plies of strips increasing. However, the effect of confinement by strips was not better than that completely wrapped columns. The axial strain of columns in failure varied from 0.0015 to 0.0028, and the lateral strain was from 0.001 to 0.003. The ductility lied on the effective confinement of the GFRP strips, namely larger intervals provided less effective confinement. So the descent part of the stress-strain curves changes to be steep as the space increasing.

The test results listed in table 3 showed that the ultimate strain of GFRP jackets was much less than their rupture strain for all the wrapped square columns. This is because the premature failure of GFRP jackets often occurs for stress concentration on the corner of columns.
ANALYTICAL MODEL

The area confined by GFRP jackets

When a GFRP-confined concrete specimen is subjected to axial compression, the concrete expands and this expansion is resisted by the GFRP. The GFRP jacket is subjected to tension in the hoop direction, and the core concrete is compressed by off-lying GFRP jackets. The area of the concrete core confined by the GFRP jackets may be approximated by considering the parabolic arching action (J. B. Mander et al. 1988, Yung C. Wang et al. 2001) (see figure 12) that takes place between two GFRP strips in the vertical direction and in the transverse direction. If in figure 12 the arching action is assumed to occur in the form of a second-degree parabola with an initial tangent slope of 45°, the area of an effectively confined concrete core is

\[ A_e = \left( b^2 - \frac{2x^2}{3} \right) \left( 1 - \frac{s}{2b} \right)^2 \]  

(1)

\[ f_i' = f_i k_e \]  

(2)

\[ k_e = \frac{A_e}{A_{cc}} \]  

(3)

\[ A_{cc} = b^2 \]  

(4)

Where \( b \) is the width of specimen; \( s \) is clear vertical spacing between GFRP strips; \( A_{cc} \) is the sectional area of specimens; and \( f_i' \) is the effective lateral confining stress. The confining stress is not uniformly distributed over the surface of the concrete core. To simplify analysis, it may be assumed that the confining stress is uniformly distributed over the surface of specimens. This nominal stress \( f_i' \) can be got from force equilibrium illustrated in figure 13.

\[ f_i (s + s')b = 2f_{frp}ts' \]  

(5)

Where \( f_{frp} \) is the stress in GFRP jackets, and it is a variable value before GFRP rupture. \( t \) is the nominal thickness of GFRP jackets; and \( s' \) is the width of GFRP strips. Taking these parameters into equation (2), the effective lateral stress \( f_i' \) can be calculated by the following equation:

\[ f_i' = \left( \left( 1 - \frac{2x^2}{3b^2} \right) \left( 1 - \frac{s}{2b} \right)^2 \right) \frac{2f_{frp}ts'}{(s + s')b} = \left( 1 - \frac{2(x^2 + s^2)}{3b^2} \right) \frac{2E_{frp}\varepsilon_{frp}ts'}{(s + s')b} \]  

(6)

Where \( E_{frp} \) is the elastic modulus of GFRP jackets; \( \varepsilon_{frp} \) is the strain of GFRP.
jackets; $\varepsilon_r$ is the effective strain of GFRP jackets, and it can be equal to 0.004 in this paper (ACI Committee 440, 2000). Taking $\varepsilon_r$ into equation (6), the value of $f'_{cl}$ can be uniquely defined.

**Stress-strain relationship of confined concrete**

Early investigations (J. B. Mander et al. 1988, Houssam A. 1999) showed that the confined strength $f'_{cc}$ and the corresponding longitudinal strain $\varepsilon'_{cc}$ can be calculated by equation (7) and (8). The test results were listed in table 4. And good agreement between test and calculation results is achieved.

$$f'_{cc} = f'_{c} \left[ 1 + 3.5 \left( \frac{f'_{cl}}{f'_{c}} \right)^{0.85} \right] \text{ (MPa)} \quad (7)$$

$$\varepsilon'_{cc} = \varepsilon'_{c} \left( \frac{5f'_{cc}}{f'_{c}} - 4 \right) \quad (8)$$

According to the concrete stress-strain relationship model (Z. H. GUO 1999), this paper proposed the concrete stress-strain relationship model confined with GFRP jackets. The concrete stress-strain relationship is shown in figure 14 and represented by equation (9) and (10), and the concrete stress-strain relationship is shown in figure 15 and represented by equation (11) and (12). The axial compressive stress $f'_{c}$ can be calculated by the following equations:

$$f'_{c} = f'_{c} \left[ 2 \left( \frac{\varepsilon'_{c}}{\varepsilon'_{c}} \right) - \left( \frac{\varepsilon'_{c}}{\varepsilon'_{c}} \right)^2 \right] \quad (9a)$$

$$f'_{c} = f'_{c} \left[ 1 - 0.15 \left( \frac{\varepsilon'_{c} - \varepsilon'_{cc}}{\varepsilon'_{u} - \varepsilon'_{cc}} \right) \right] \quad \varepsilon'_{c} < \varepsilon'_{cc} < \varepsilon'_{u} \quad (9b)$$

$$f'_{c} = f'_{cc} \left[ 2 \left( \frac{\varepsilon'_{c}}{\varepsilon'_{cc}} \right) - \left( \frac{\varepsilon'_{c}}{\varepsilon'_{cc}} \right)^2 \right] \quad \varepsilon'_{c} \leq \varepsilon'_{cc} \quad (10a)$$

$$f'_{c} = f'_{cc} \left[ 1 - 0.15 \left( \frac{\varepsilon'_{c} - \varepsilon'_{cc}}{\varepsilon'_{cc}} \right) \right] \quad \varepsilon'_{c} < \varepsilon'_{cc} < \varepsilon'_{u} \quad (10b)$$

where $f'_{cc}$ and $\varepsilon'_{cc}$ were calculated by equation (7) and (8); and $\varepsilon'_{u} = 1.5 \varepsilon'_{cc}$.

The stress-strain curves of the calculation results based on the proposed model are shown in figure 16. The computation results can be probably agreement with the test results. The reliability of the proposed model is proved by the test results.
CONCLUSIONS

Experimental work was conducted on 51 square concrete columns. GFRP jackets were applied to 42 columns. The other 9 columns were tested in their as-built conditions. The columns were tested under concentric loading. The test results indicate that GFRP wrapping is an effective means of confinement, as they significantly increase both the strength and ductility of concrete.

The test results showed that the ultimate strain of GFRP jackets was much smaller than their rupture strain for all the wrapped square columns. This is because the premature failure of the GFRP jackets often occurs due to stress concentration on the corner of square columns.

An analytical model is introduced to predict the stress-strain relationship of square columns confined with GFRP. The model is based on concrete stress-strain relationship model, the arching action, and the variable confinement provided by GFRP jackets. The key parameters in this model include the ultimate strength $f_{cc}'$, the corresponding longitudinal strain $\varepsilon_{cc}'$ and the ultimate strain $\varepsilon_u$ of the concrete square columns confined with GFRP jackets. The validation of the model presented herein is proved by the test results.

REFERENCES

ACI Committee 440. Guide for the design and construction of externally bonded FRP systems for strengthening concrete structures. 24 May 2000


Houssam A. Toutanji. Stress-strain characteristics of columns externally confined with advanced fiber composite sheets. ACI Materials Journal, V.96, No.3, May-June 1999 397-404


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<th>Reinforcement method</th>
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Table-2. Material properties

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Table 3. Test results

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<th>$f'_{\text{oo2}}$, MPa</th>
<th>$f'<em>{\text{oo1}} / f'</em>{\text{oo2}}$</th>
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Figure 1 - Stress zone of specimen.

Figure 2 - Section of specimen.
Figure 3 - Confined schemes of specimens; (a) unwrapped column; (b) completely wrapped column; (c) strip space 25mm; (d) strip space 50mm; (e) strip space 75mm; (f) strip space 100mm; (g) gap 50 mm; (g) gap 100mm; (i) gap 150mm.

Figure 4 - Stress-strain relationship of GFRP.

Figure 5 - Test setup.
Figure 6 - Strain gauges.

Figure 7 - Failure modes: (a) unwrapped column; (b) completely wrapped column; (c) strip spacing wrapped column; (d) gap 50 mm; (e) gap 100 mm; (f) gap 150 mm.

Figure 8 - Effect of concrete strength.

Figure 9 - Effect of GFRP jacket plies.
Figure 10 - Effect of gap between GFRP strip.

Figure 11 - Effect of GFRP strip space.

Figure 12 - Arching action.
Figure 13 - Confined stress distribution.

Figure 14 - Stress-strain relationship of concrete.

Figure 15 - Stress-strain relationship of confined concrete.
Figure 16 - Stress-strain relationship of specimen.
Energy-Based CDM Model for Nonlinear Analysis of Confined Concrete

by J. Li and J.-Y. Wu

Synopsis: In this paper, a continuum damage mechanics (CDM) based constitutive model appropriate for confined concrete is presented. Basically, the tensile and shear damage variables are adopted to describe the degrading of macro-mechanical properties of concrete, and the corresponding damage criteria are established based on elastoplastic damage energy release rates, which can predict the enhancement of strength and ductility of concrete under biaxial compression. To describe the compressive consolidation mechanism under triaxial compressive confinement, the above Drucker-Prager type shear damage criterion is modified to take the third invariant of effective stress into account. The irreversible plastic strains are determined empirically in this paper, though the effective stress space plasticity method is also introduced here. Comparing to experimental tests of concrete under biaxial and triaxial compressive stress states, the predictive results of proposed model show good agreement with test data, which validate the capability of present model for reproducing the nonlinearity of confined concrete.

Keywords: confined concrete; constitutive model; continuum damage mechanics; nonlinear analysis
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INTRODUCTION

Obviously there are many advantages in using the high-strength concrete over the normal-strength concrete, especially the enhancement of compressive strength. However, the high-strength concrete is still not widely used in engineering structures yet, mainly due to the brittleness and less ductility accompanied with high strength, which can be greatly reduced or even overcome by imposing confinement.

As is generally accepted, lightly-confined concrete does not improve the properties of material too much, where well-confined one implies high ductility. To study the effects of confinement in both high-strength concrete and normal-strength concrete, much effort has been made, and many material models that take the effects of confinement into consideration have been proposed, for example, Park et al. (1982), Sheikh and Uzumeri (1982), Bjerkeli et al. (1990), Cusson and Paultre (1994), etc. However, most of the proposed models are in fact an empirically axial constitutive model, whose basis philosophy still remains the same as Richart et al. (1928) and ACI Code (1995).

Within the framework of continuum damage mechanics (CDM), an energy-based concrete constitutive model appropriate for numerical analysis of confined concrete, is presented in this paper. Consistent with thermodynamics theory, the plastic Helmholtz free energy is accounted for the damage growth, and the damage criteria are based on the elastoplastic damage energy release rates. Such a material model is indeed a 3D one, which can provide more detailed information about the stress states in the structures.

DAMAGE VARIABLES AND CONSTITUTIVE RELATION

To clearly distinguish the behaviours of concrete material under tension and compression, and produce the proposed tension-shear nonlinear mechanisms of degradation, the concept of classical effective stress $\bar{\sigma}$ in CDM can be generally defined and decomposed into positive and negative components ($\bar{\sigma}^+, \bar{\sigma}^-$) as following (Ju, 1989; Faria et al., 1998; Wu and Li, 2004)

$$\bar{\sigma} = C_\sigma : \varepsilon' = C_\sigma : \left( \varepsilon - \varepsilon^p \right)$$  \hspace{0.5cm} (1)

$$\bar{\sigma}^+ = P^+ : \bar{\sigma} ; \quad \bar{\sigma}^- = \bar{\sigma} - \bar{\sigma}^+ = P^- : \bar{\sigma}$$  \hspace{0.5cm} (2)
where $C_0$ denotes the usual fourth-order isotropic linear-elastic constitutive tensor; $\varepsilon$, $\varepsilon'$ and $\varepsilon''$ are all rank two tensors, which correspond to strain tensor, elastic strain tensor and plastic strain tensor, respectively; $P^+$ and $P^-$ are both fourth-order symmetric tensor, introduced as the positive and negative projection tensor of $\sigma$, respectively, with (Faria, et al., 2000; Wu et al., 2005a, b)

$$P^+ = \sum_i H(\hat{\sigma}_i)(P'' \otimes P'') + P^- = I - P^+$$

(3)

here, $\otimes$ is tensor product; $I$ is the fourth-order identity tensor; $H(\hat{\sigma}_i)$ denotes the Heaviside function computed for the $i$th principal effective stress $\hat{\sigma}_i$; and with $i$th corresponding unit principal direction vector $p_i$, second-order symmetric tensor $P''$ is expressed as

$$P'' = p_i \otimes p_i$$

(4)

Correspondingly, introducing the elastic free energy potential as function of the above free and internal variables, combining with the Clausius-Duheim inequality, and making use of standard arguments (Coleman and Gurtin, 1967), along with the additional assumption that damage and plastic unloading are elastic process, one can obtain the constitutive law (Wu and Li, 2004; Wu et al., 2005a, b)

$$\sigma = (I - D) : \bar{\sigma}$$

(5)

with the fourth-order damage tensor $D$ expressed as

$$D = d^+ P^+ + d^- P^-$$

(6)

Here the tensile damage scalar $d^+$ and the shear damage scalar $d^-$, namely tensile damage under tensile stress and shear damage under compressive stress, are adopted to describe the degradation of the macro-mechanical elastic and plastic properties.

**DAMAGE CRITERIA AND EVOLUTION LAWS OF INTERNAL VARIABLES**

**Damage criteria and damage evolution laws**

To complete the entire material model, the evolution laws of two types of internal variables, i.e., two damage variables and plastic strains tensor, which are the keys to a damage model, must be introduced first.

From the basic thermodynamics theory, the evolution laws of damage variables should be established based on the tensile and shear elastoplastic damage release rates,
which correspond to their conjugated thermodynamics general forces. In present model, the plastic Helmholtz free energy of concrete material is considered to obtain the following elastoplastic damage energy release rates (Wu et al., 2005a, b)

\[ Y^+ = \sqrt{E_0 \left( \sigma^+ : \Lambda_0 : \sigma \right)} ; \quad Y^- = a \bar{T}_1 + \sqrt{3 \bar{T}_2} \]  

(7)

where \( E_0 \) is the initial elastic modulus of concrete; \( \Lambda_0 = C_0^{-1} \) is the initial compliance tensor; \( \bar{T}_1 \) and \( \bar{T}_2 \) are the first invariant of effective stress tensor and the second invariant of effective deviator stress tensor, respectively.

Letting \( f_0^+ \), \( f_0^- \) and \( f_{b0}^- \) denote the corresponding strength beyond which nonlinearity becomes visible under axial tension, axial compression and equibiaxial compression, respectively, the damage parameter \( \alpha \) in Eqn. (7), can be expressed as (Wu et al., 2005a, b)

\[ \alpha = \frac{(f_{b0}^- / f_0^-) - 1}{2(f_{b0}^- / f_0^-) - 1} \]  

(8)

In this paper the typical value of 1.16 for \( f_{b0}^- / f_0^- \) is taken, and then \( \alpha \) takes the value of 0.1212.

Applying Eqns (7) and (8) the thresholds of tensile and shear damage energy release rates, can be established as

\[ r_0^+ = f_0^+ ; \quad r_0^- = (1 - \alpha) f_0^- \]  

(9)

With the already referred definitions for the damage energy release rates, the state of damage can then be characterized by means of the damage criteria \( g^\pm \), with the following functional form (Wu and Li, 2004; Wu et al., 2005a, b)

\[ g^\pm (Y_{a, n}^\pm, r_{n}^\pm) = Y_{a, n}^\pm - r_{n}^\pm \leq 0 \]  

(10)

\[ r_{n}^\pm = \max \left\{ r_{0}^\pm, \max_{r \in [0, \alpha]} Y_{r}^\pm \right\} \]  

(11)

where, the subscript \( n \) refers to the value at current time \( n \); variables \( r_{n}^\pm \) are current damage thresholds (energy barriers), i.e. the radiiuses of the damage surfaces, which control the size of the expanding damage surfaces. The physical meaning of Eqns. (10) and (11) can be referred to Simo and Ju (1987).
The above damage criteria can predict the typical nonlinear behaviors under biaxial stress states fairly well, e.g., tension-compression softening effect, the enhancement of strength and ductility under biaxial compression (Wu and Li, 2004; Wu et al., 2005a, b). However, due to the inherent shortcomings of the Drucker-Prager type criteria, it will be shown later that in triaxial compression, the predictive results seem too much conservative.

Since present constitutive model is directed towards failure analysis, and therefore no attempt will be made in the region in which failure does not occur. If the high-pressure region in which the failure surface remains open in the direction of hydrostatic compression, leading to the hardening goes on indefinitely, is excluded (should this become necessary, the cap model might be resorted to), the shear damage energy release rate in Eqn. (7)_2 can be modified to take the third invariant of effective stress into account as

\[ Y^- = \alpha T_1 + \sqrt{3J_2} - \gamma (-\hat{\sigma}_{i,\text{max}}) \]  

where the McAuley bracket is defined as \[ \langle x \rangle = \frac{(x + |x|)}{2} \].

Note that when \( \hat{\sigma}_{i,\text{max}} = 0 \), i.e. in biaxial compression, Eqn. (12) is just the same as the Drucker-Prager criterion in Eqn. (7)_2 and the coefficient \( \gamma \) appears only in triaxial compression, that is, in stress states with \( \hat{\sigma}_{i,\text{max}} < 0 \). This coefficient can be determined by comparing the damage conditions along the tensile and compressive meridians.

By definition, the tensile meridian (TM) is the locus of stress states satisfying the condition \( \hat{\sigma}_{i,\text{max}} = \hat{\sigma}_1 > \hat{\sigma}_2 = \hat{\sigma}_3 \), and the compressive meridian (CM) is the one such that \( \hat{\sigma}_{i,\text{max}} = \hat{\sigma}_1 = \hat{\sigma}_2 > \hat{\sigma}_3 \). It can be obtained that \( \hat{\sigma}_{i,\text{max}}^\text{TM} = \left( T_1 + 2\sqrt{3J_2} \right)/3 \) and \( \hat{\sigma}_{i,\text{max}}^\text{CM} = \left( T_1 + \sqrt{3J_2} \right)/3 \) along the tensile and compressive meridians, respectively. With \( \hat{\sigma}_{i,\text{max}} < 0 \), the corresponding shear damage criterion can be expressed as

\[ (\gamma + 3\alpha) T_1 + 3(1-\alpha) f_0^- (\text{TM}) \]  
\[ (\gamma + 3\alpha) T_1 + (\gamma + 3)(1-\alpha) f_0^- (\text{CM}) \]  

Let \( K_c = \sqrt{J_2^\text{TM}}/\sqrt{J_2^\text{CM}} \) for any given value of the hydrostatic pressure \( T_1 \) with \( \hat{\sigma}_{i,\text{max}} < 0 \), then \( K_c = (\gamma + 3)/(2\gamma + 3) \), which is assumed a constant according to experimental evidence (Lubliner et al., 1989). The coefficient \( \gamma \) is, therefore, evaluated as
\[\gamma = \frac{3(1 - K_c)}{2K_c - 1}\]  

A value of \( K_c = 2/3 \), which is typical for concrete, gives \( \gamma = 3.0 \), is adopted in present model. Typical damage surfaces defined by Eqns. (7) and (12) are shown in Figure 1 in the deviator plane and in Figure 2 for plane-stress conditions.

Once the damage criteria are obtained, the evolution laws for the damage variables can be established according to the normal rules, which imply the Kuhn-Tucker optimality conditions of a principle of maximum damage dissipation, as follows

\[\dot{d}^\pm = \dot{\lambda}^d \frac{\partial g^\pm(Y_n^\pm, r_n^\pm)}{\partial Y_n^\pm}; \quad \dot{r}_n^\pm = \dot{\lambda}^d \]

\[\dot{\lambda}^d \geq 0; \quad g^+(Y_n^\pm, r_n^\pm) \leq 0; \quad \dot{\lambda}^d g^+(Y_n^\pm, r_n^\pm) = 0\]  

where \( \dot{\lambda}^d \) are damage consistency parameters.

With some standard derivations (Simo and Ju, 1987; Faria et al., 1998; Wu et al., 2005a, b) and the initial conditions \( \dot{d}_0 = 0 \) are considered, following expressions for the damage variables can be obtained

\[d^\pm = G^\pm(r_n^\pm)\]  

Obviously, the functions \( G^\pm(r_n^\pm) \) should satisfy the following requirements

\[0 \leq G^+(r_n^\pm) \leq 1; \quad \dot{G}^+(r_n^\pm) \geq 0; \quad G^+(r_0^\pm) = 0\]  

In present model, different functions \( G^\pm(r_n^\pm) \) are adopted for tensile damage and shear damage

\[d^+ = G^+(r_n^+) = 1 - \frac{r_0^+}{r_n^+} \exp \left[ A^+ \left( 1 - \frac{r_n^+}{r_0^+} \right) \right] \quad (r_n^+ \geq r_0^+)\]  

\[d^- = G^-(r_n^-) = 1 - \frac{r_0^-}{r_n^-} (1 - A^-) - A^- \exp \left[ B^- \left( 1 - \frac{r_n^-}{r_0^-} \right) \right] \quad (r_n^- \geq r_0^-)\]  

where parameters \( A^+ \) involved in Eqn. (20) can be computed by equating the concrete
fracture energy $G_f$ per unit of the characteristic length $l_{ch}$ to the time integral of dissipation on a one-dimensional tensile test, rendering (Oliver et al., 1990)

$$A^* = \left[ \frac{G_f E_o}{l_{ch} (f_{ch}^0)^2} - \frac{1}{2} \right]^{-1} \geq 0 \tag{22}$$

$A^*$ and $B^*$ may be determined by imposing the one-dimensional numerical curve to fit the curve extracted from the one-dimensional test. Noted that Eqn. (20) ignores the hardening under tension and the concrete is assumed to behave linear-elastic before the tensile stress arrives at the tensile strength, which seems rational, while Eqn. (21) allows reproducing the hardening in concrete, as well as the softening which characterizes the post-peak behaviors.

**Plastic strains**

The irreversible plastic strains can be accounted for through the effective stress space plasticity (Ju, 1989), which can be coupled with the proposed model as

$$\dot{\varepsilon}^p = \dot{\lambda}^p \partial_{\sigma} F^p; \quad \dot{\kappa} = \dot{\lambda}^p h^p; \quad F(\sigma, \kappa) \leq 0, \quad \dot{\lambda}^p \geq 0, \quad \dot{\lambda}^p F(\sigma, \kappa) \leq 0 \tag{23}$$

Many plastic yield function $F$ and plastic potential function for $F^p$ appropriate for concrete material can be adopted in present model, e.g. Lee and Fenves (1998) for $F$ and Drucker-Prager type function for $F^p$. The hardening parameters $\kappa$ can be referred to the equivalent plastic stains under tension and compression (Wu et al., 2005a, b), respectively.

However, due to the fact that the present model is mainly intended for the nonlinear analysis of confined concrete structures, high algorithm efficiency has to be ensured, which is contradictory to the complexity of the effective stress space plasticity. Within the above context, in the present paper, the plastic strains are accounted for only as an “overall effect” (Faria et al., 1998), introducing some drastic simplifications based on the effective stress space plasticity as (Wu, 2004)

$$\dot{\varepsilon}^p = b^p \sigma; \quad b^p = \xi^p E_o H \left( \dot{\varepsilon} \right) \frac{\varepsilon : \dot{\varepsilon}}{\sigma : \sigma} \geq 0 \tag{24}$$

with $\xi^p$ being a material parameter controlling the rate intensity of plastic strains estimated as (Wu et al., 2005a, b)

$$\xi^p = \left( 1 + \alpha \right)^{-1} = E_o / \left( E_0 + E^p \right) \tag{25}$$
where \( \alpha_E \) is the ratio between the plastic modulus \( E^p \) under uniaxial compression and the elastic modulus \( E_0 \), i.e., \( \alpha_E = E^p / E_0 \).

Numerical simulation results (Wu, 2004) of experimental tests of concrete material and structures have shown little difference between the above two methods, while the empirical method of Eqn. (24) is much more computational effective.

### NUMERICAL SIMULATIONS AND MODEL VERIFICATIONS

If the effective stress space plasticity is used to determine the irreversible strains, the plastic flows and damage evolutions can be decoupled, and three steps numerical system (Wu et al., 2005a), i.e., the elastic-trial part, the plastic-corrector part and the damage-corrector part, can be established according to the operator-split method (Simo and Hughes, 1998). For the empirically determined plastic strains, “backward-Euler” based integration algorithm (Wu, 2004) is adopted in present model due to its unconditional stability. The detailed algorithms involved will not be given here [should it be necessary, one can refer to Wu (2004) and Wu et al. (2005a) where the tangent matrix are also derived].

With the above damage model and its corresponding algorithm, a finite element program was coded to simulate the experimental results of confined concrete and validate the model’s capability.

### Biaxial compression

A set of experimental tests reported in Kupfer et al. (1969), performed with concrete specimens under biaxial compression \( (\sigma_2 = 0) \), according to the following load conditions: (1) \( \sigma_3 / \sigma_1 = -1/0 \); (2) \( \sigma_3 / \sigma_1 = -1/-1 \); and (3) \( \sigma_3 / \sigma_1 = -1/-0.52 \), is referred to Figure 3. The material properties adopted in the simulation were: \( E_0 = 3.1 \times 10^4 \text{MPa} \), \( v_0 = 0.20 \), \( f_0^+ = 3.0 \text{MPa} \), \( f_0^- = 15.0 \text{MPa} \), \( G_f = 70 \text{N/m} \) \( (A^+ = 0.683 \text{N/m}) \), \( A^- = 1.0 \), \( B^- = 0.213 \), \( \xi^p = 0.10 \). As can be shown in Figure 3, the predicted stress-stress curves exhibit fairly well agreement with the test ones, capturing rather satisfactorily the overall experimental behaviours.

To illustrate the capability of present constitutive model for predicting the nonlinear behaviours under other load conditions, using the same material properties as above, the obtained biaxial strength envelop curves are also shown in Figure 3, demonstrating close results with the series of experimental tests of Kupfer et al. (1969).

As clearly perceptible in Figure 3, an important attribute of the present constitutive model, is its ability to predict not only the softening effect under tension-compression, but
also the enhancement of strength and ductility under biaxial compression, which is essential for analysis of confined concrete structures.

Inferred from the state-of-art included in Mazars and Pijaudier-Cabot (1989), these two features, with an evident relevance for nonlinear behaviors of concrete, were not captured by their elastic damage model, whose results are also included in Figure 3.

Triaxial compression

To further illustrate the capability of present damage model for confined concrete, Figure 4 compares the numerical predictions with the experimental results tested by Green and Swanson (1973) for concrete under 3D compression. Tests were driven with an increasing normal stress along the specimen vertical axial, and three different sets of confining stresses along the horizontal directions: (1) $\sigma_1 = \sigma_2 = 0.0\text{MPa}$; (2) $\sigma_1 = \sigma_2 = -6.895\text{MPa}$; and (3) $\sigma_1 = \sigma_2 = -13.79\text{MPa}$. The material properties adopted in the simulations were: $E_0 = 3.7 \times 10^4\text{MPa}$, $\nu = 0.20$, $f_p = 15.0\text{MPa}$, $A = 1.0$, $B = 0.13$, $\xi = 0.10$. The above parameters were calibrated in order to fit the uniaxial stress-strain curve, whose result is referred to Figure 4. In the numerical simulations, two shear damage criteria, i.e. Eqn. (7) and (12) were used to show their discrepancy.

It can be clearly seen that under 3D compressive confinement the predictive results from Eqn. (7) are far more conservative since the Drucker-Prager type function does not take the third invariant of effective stress into consideration. Meanwhile, if the shear damage criterion of Eqn. (12) is used to determine the evolution of shear damage, the overall response curves, as well as the enhancement of strength and ductility produced by the 3D confinement, can be satisfactorily predicted by present damage model as illustrated in Figure 4.

CONCLUSIONS AND FUTURE WORKS

Within the framework of continuum damage mechanics, this paper presents an energy based elastoplastic damage constitutive model, mainly intended for the nonlinear analysis of confined concrete structures. To describe the degradation of macro-mechanical properties of concrete, tensile and shear damage variables are adopted, whose damage criteria are established based on the elastoplastic damage energy release rates upon considering the plastic free energy. To model the compressive consolidation mechanism under triaxial compressive confinement, the Drucker-Prager type shear damage criterion is modified to take the third invariant of effective stress into account. The irreversible plastic strains can be determined according to effective stress-space plasticity or empirical evolution law, and the later is adopted for the intended purpose.

The model has been implemented into a general finite element program capable of predicting the typical nonlinear behaviors of concrete under different stress states, whose results demonstrate its adequate accuracy for the biaxial stress states and triaxial...
Along with the lateral reinforcement discretized via truss elements, whose stress can be usually determined with the yield assumption or strain estimation (Claeson, 1995), the present model can be applied to nonlinear analysis of confined concrete structures, and this part of jobs will be forthcoming.

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Figure 1 – Shear damage surface in the deviator plane for different values of $K_c$
Figure 2 – Damage surface in biaxial stress states

Figure 3 – Comparisons of numerical results with experimental tests (Kupfer et al., 1969)
a) Shear damage criterion: Eqn. (7)   

b) Shear damage criterion: Eqn. (12)

Figure 4 – Comparisons of numerical results with experimental tests  
(Green and Swanson, 1973)
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Finite Element Study of Confined Concrete

by W.-F. Chen and Y.-M. Lan

Synopsis: Recent worldwide applications of fiber reinforced polymers (FRP) wraps and tubes for existing and new structural members have continued to emphasize the significance of confined concrete. This study presents an overview of the related finite element (FE) studies for, but not limited to, the FRP-confined concrete applications of commercial FE programs such as ABAQUS and ANSYS. The capabilities, limitations, and remarks of the concrete models available in the programs are addressed. When the built-in options cannot be satisfied, several recommendations are also given for the user-defined concrete material models. The needs for future research and developments are also indicated.

Keywords: analysis; computation; concrete; confinement; plasticity
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INTRODUCTION

Recent worldwide applications of Fiber Reinforced Polymers (FRP) wraps and tubes for existing and new structural members have continued to emphasize the significance of confined concrete. Despite the differences in confining materials (such as steel and FRP) and configurations (for instance, stirrups, spirals, wraps, and tubes), the characteristics of concrete under triaxial compressive conditions remain the same - concrete plasticity. This plasticity behavior can be best simulated and understood numerically using finite element (FE) methods. FE analysis has frequently been used to predict and compare the results of experiments and other analysis. However, the FE studies for the FRP-confined concrete applications are still sparse.

In general, a FE model built on commercially available software such as ABAQUS, ANSYS, and MARC, is often easily accessible, well maintained, and possibly extendable. Thus, these programs may be the first choice when FE analysis is considered. Proper applications of the programs require a good understanding of their capabilities and limitations. Since these codes are programmed for general purposes, they may not necessarily have all the capabilities to accurately model the application of interest.

The current study was aimed to provide an overview for, but not limited to, the FRP-confined concrete applications of commercially available FE software. To this goal, the related FE studies were reviewed to examine the concrete models available in the programs. To satisfy diverse needs, issues for the user-defined material options were also addressed. Based on the present overview, the needs for future research were identified.

COMMERCIALY AVAILABLE CONCRETE MODELS AND THEIR CONFINED APPLICATIONS

This section focuses the studies utilizing the concrete models available in the popular commercial FE software, particularly ABAQUS and ANSYS. Tables 1a and 1b show an overview of the related studies in terms of the yielding criteria, hardening rules, element types, confining materials and shapes, and subjects investigated.
ABAQUS

The CONCRETE option in ABAQUS is capable of simulating reinforced concrete (RC) using smear and discrete approaches. The smear model is often used with the REBAR option to define reinforcements in concrete. The interaction between rebars and concrete, that is, the stress degradation after cracking, can be described by the TENSION STIFFENING option. Among others, Mahfouz et al. (2001) adopted the smear concrete to obtain the uniaxial failure loads of the upgraded and repaired RC columns using an innovative FRP confining system, Sandwich Wrapping Confining System (SWCS). The analytical predictions are very close to the experimental results by about 10%. For FRP-confined applications, Fang (1999) also employed the smear model to study the load-carrying capacity, stiffness, and failure mechanism of the RC beams.

On the other hand for the discrete approach, the CONCRETE option can also be used for plain concrete, whereas steel reinforcements can be represented by one-dimensional truss or beam elements. Such discrete configuration has also been used in literature. For instance, Mullen et al. (1999) studied the deformed shapes and internal stresses of FRP-retrofitted shear columns under double bending.

The yielding surfaces of the CONCRETE option have linear compressive meridian planes and linear tensile “crack detection” surfaces in stress space. This is well defined for tensile cracking applications. However, for compressive confined concrete applications, Lan (1998) has found that the definition limitations of the friction angle of the compressive yielding planes along with the associate flow restrict the use of this option for low confining pressure.

In the ABAQUS concrete model, the isotropic hardening rule is defined by an unconfined uniaxial compressive stress-strain relation, without involving any hydrostatic stresses or strains. This leads the concrete model to underestimate the ductility of confined concrete. Lan (1998) proposed a modification to account for the hydrostatic effect using the Solution-Dependent Field Variable (SDFV) option. With the modification, he obtained good agreement between the numerical and the experimental results for both active and passive confined concrete.

In summary, when the CONCRETE material option is employed for confined concrete applications, several issues to be considered are noted:

1. **Applications.** It is expected to perform better for low confining pressure applications under monotonic loading. In general, inelastic volume strain can be overestimated due to the associated flow assumed in the model. Also, under reverse loading, one may anticipate elastic response alone without any stiffness degradation (ABAQUS User’s Manual 2001).

2. **Inputs.** For the uniaxial compression input, assume concrete to exhibit a linear post-peak softening branch, in order to better capture the ductility increase (Schneider, 1998; Johansson and Gylltoft 2001, 2002; Johansson 2002).
3. User-defined options. Using SDFV to make up the hydrostatic sensitivity missing in the hardening rule may help to generate better results.

ANSYS

Similar to ABAQUS, ANSYS offers the SOLID 65 option to model RC with smear and discrete approaches. Like the REBAR option in ABAQUS, the SOLID 65 option is capable of describing single and distributed reinforcements in three-dimensional (3D) space. The input data required for this option include Real Constants, Material Models, and Data Tables. Real Constants specify the locations, angles and distribution ratios of the embedded reinforcements. Material Models define the elastic modulus, poison ratios, and densities of concrete and reinforcements. Data Tables determine the inelastic constitutive relations. For concrete, it requires two sets of these tables, one for the yielding criterion with the hardening rule, and the other for the uniaxial or multiaxial strength.

Among other yielding criteria, the William-Wranke model with 5 parameters has been employed and simulated well the structural behavior of the FRP-wrapped square RC columns under uniaxial compression (Feng et al. 2002; Lu et al. 2002; Lu and Jiang 2003). The Drucker-Prager model on the other hand, was adopted by Mirmiran et al. (2000) and Shahawy et al. (2000). They concluded that the perfect-plastic assumption has failed to capture the dilation characteristics and over-predicted the strength and stiffness of the FRP-concrete tubes.

Numbers of remarks have been made for using the SOLID 65 option (Lu and Jiang 2003):

1. Supports. Avoid placing a support directly on a single node, because this may cause stress to concentrate and fail at that location. Applying a stiff plate or enlarging the nearby mesh can practically resolve this problem.
2. Mesh size. If the mesh size is too fine, it may again introduce stress concentration and result in early failure of the concrete (as compared with experimental data). Such a problem can be prevented effectively with elements greater than 2 inches (50 mm).
3. Elements. For concrete applications in general, hexahedral elements are more stable and efficient in convergence than tetrahedral elements. Hence, hexahedral elements are recommended wherever possible.
4. Convergence. Like in the experiments, nodal displacement control rather than force control is preferred to trace the pre- and post-peak curves. Also, when cracking starts and failure is about to occur, proper release of the tolerance will help the convergence.

USER-DEFINED CONCRETE MODELS
AND THEIR CONFINED APPLICATIONS

When the built-in options cannot be satisfied, all above-mentioned FE programs offer the user-defined material options. When these options are considered, one may re-
evaluate the important characteristics of the concrete plasticity model such as strain-space formulation, yielding criterion, and computational stability and efficiency.

The strain-space plasticity formulation for compressive concrete is different from the stress-space formulation currently available in commercial FE codes. The strain-space formulation can clearly identify loading conditions (for example, strain-softening or unloading, perfect plastic or neutral loading), whereas the stress-space formulation has difficulty distinguishing. In addition, the strain-space formulation is more natural for FE implementation because it is strain-driven. The superiority of strain-space plasticity has been well recognized by numerous researchers (Il’yushin 1961; Yoder and Iwan 1981; Casey and Naghdi 1981, 1983; Kiousis 1985; Han and Chen 1986; Mizuno and Hatanaka 1992; Hidaka et al. 1994; Farahat et al. 1995; Lan 1998).

To better represent confined concrete, the hardening function with confinement-sensitivity is recommended (Farahat et al. 1995; Johansson and Akesson, 2002). As to the yielding and plastic potential surfaces, they should be hydrostatic-dependent. Simple functions such as the von Mises, Drucker-Prager and Mohr-Coulomb criteria, or complicated functions such as the Willam-Warnke, Ottosen, and Hsieh-Ting-Chen criteria, are often used.

Computational stability and efficiency are frequently concerned when the integration of the rate constitutive equations is performed numerically. In computational plasticity, the inconsistency of the numerical integration algorithm and the tangent stiffness operator employed is the main cause of computational inefficiency and instability. The incorporation of the Return-Mapping integration algorithms with the Consistent Tangent operators has been found to provide optimal convergence, accuracy and stability (Nyssen 1981; Yoder and Whirley 1984; Simo and Taylor 1985; Ortiz and Popov 1985; Dodds 1987). However, these optimal algorithms and operators are not currently available in all above-mentioned programs. Instead, the tangent-based schemes and the elastoplastic tangent operators have been used. For the user-defined material model, one may take this computational improvement into consideration.

In summary, it is an ultimate goal for the user-defined concrete model to capture important features such as loading-condition detection and confinement-sensitivity, and yet remain reasonably simple (for input, computation, implementation, and maintenance).

Among others, some researchers have worked with ABAQUS on its user-defined material subroutine UMAT for confined concrete applications (see Table 1a). In strain space, Lan (1998) has formulated and implemented the Drucker-Prager plasticity with nonlinear-bounding surfaces, isotropic hardening and nonassociated flow using the Return-Mapping algorithm and the Consistent Tangent operator. The comparisons with the experimental data have validated his model for concrete under both active (low and high confining pressure) and passive confinements (FRP). For steel-confined applications, Johansson and Akesson (2002) have again adopted the Drucker-Prager model but have done so in stress space with isotropic hardening and associated flow, and
implemented it using C++ language. They reported that except the post-peak behavior, the pre-peak behavior of concrete-filled steel tubes was well predicted.

**RESEARCH NEEDS**

From Tables 1a, 1b and the above overview, it can be seen that as far as FRP-confined concrete is concerned, there are still some aspects to be thoroughly studied or explored numerically in order to arrive full understanding of the structural behavior for future design and analysis. These aspects, including stress interactions, force-moment interaction diagrams, cyclic loadings, Impacts, configurations, short-term and long-term effects, are discussed as follows.

Deeper insights of the stress and strain interactions need to be examined for various configurations of the retrofitted and new members and structures subjected to monotonic and cyclic loadings. These interactions could be, for example, the bond stresses between existing and fresh concrete, the interfacial stresses between confinements and concrete, and the steel stresses of the confined connections at a particular loading stage. Full understanding of these interactions can help to assess and predict the structural behavior including the failure modes and mechanisms.

The force-moment interaction diagrams are essential for design. Accordingly, parametric studies are required in terms of configurations, reinforcement and confinement ratios, and slenderness. These design curves can be developed numerically at relatively low costs.

Additional advancements (such as stiffness reduction) are needed to extend the monotonic case to the cyclic case for FRP-confined applications. These developments are especially valuable for post-strengthening assessments of RC structures in earthquake regions. It will greatly assist the validation of the monotonic design under seismic conditions.

Loads applied at different strain rates affect concrete ductility. The applications could be automobile or vessel impacts to piers. This strain-rate effect may require not only the concrete model to be modified, but also the FRP model.

The optimal configurations of FRP-confined systems for different applications still undergo explorations. New confining systems are expected to provide not only greater strength and ductility, but also more warnings and safety features prior to total collapse. Numerical explorations in this area can provide a safe, economical and educational alternative.

The short-term (such as creep and shrinkage) and long-term effects (such as fatigue and environmental effects) have continued to be a very important issue. With few numerical investigations available in this area, the need is apparent.
CONCLUSIONS

Based on the related FE studies, an overview of the capabilities, limitations and remarks of the commercially available concrete models is presented for their confined concrete applications. Some recommendations for the user-defined concrete models are also given for the extended uses. The needs for future research and developments are indicated accordingly. Whether it is an existing problem or innovative design, proper models of commercial FE programs with a good understanding of the capabilities and limitations can predict well the structural behavior of confined concrete members and structures.

REFERENCES


Table 1a. Confined-concrete studies with commercial FE software

<table>
<thead>
<tr>
<th>Software/Publication</th>
<th>Concrete Material Option</th>
<th>Hardening Rule</th>
<th>Element</th>
<th>Confining Material</th>
<th>Jacket or Tube Shape</th>
<th>Studied Subject</th>
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<tr>
<td>ABAQUS</td>
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<td></td>
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<td>Johansson and Gylltoft 2001, 2002; Johansson and Akesson 2002; Johansson 2002</td>
<td>CONCRETE, User-defined concrete</td>
<td>Hardening-softening</td>
<td>3D solid (9 &amp; 6 nodes)</td>
<td>Steel</td>
<td>Circular, Square</td>
<td>Load-deflection curves under uniaxial monotonic loading, confining stresses, stress distributions</td>
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<tr>
<td>Mahfouz, Rizk and Sarkani 2001</td>
<td>CONCRETE (Smear)</td>
<td>Hardening</td>
<td>3D solid (8 nodes)</td>
<td>GFRP, CFRP</td>
<td>Rectangular, Square RC</td>
<td>Failure loads under static axial loading, stress distributions</td>
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<td>Mullen, Rice, Hackett and Ma 1999</td>
<td>CONCRETE (Discrete)</td>
<td>Not known</td>
<td>3D solid</td>
<td>CFRP</td>
<td>Rectangular, Circular RC</td>
<td>Deformed shapes, internal stresses of shear columns under double bending</td>
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<td>FRP</td>
<td>Rectangular RC beams</td>
<td>Load-carrying capacity, stiffness, failure mechanisms</td>
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<td>Schneider 1998</td>
<td>CONCRETE</td>
<td>Perfect-plastic-softening</td>
<td>3D solid (20 nodes)</td>
<td>Steel</td>
<td>Circular</td>
<td>Uniaxial load-deflection curves, stress distributions, diameter/thickness ratios</td>
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<td>Lan, Sotelino and Chen 2003; Lan 1998</td>
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<td>3D solid (8 nodes)</td>
<td>AFRP, CFRP</td>
<td>Circular</td>
<td>Uniaxial load-deflection curves under active and passive confinements, stress interactions</td>
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Table I b. Confined-concrete studies with commercial FE software (continued)

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<th>Studied Subject</th>
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<td>Lu and Jiang 2003; Lu, Feng and Ye 2002; Feng, Lu and Ye 2002</td>
<td>William-Warnke (Smear &amp; Discrete)</td>
<td>Hardening-softening</td>
<td>3D SOLID 65</td>
<td>GFRP, CFRP</td>
<td>Square RC</td>
<td>Uniaxial load-deflection curves, stress and strain distributions, failure mechanisms</td>
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<td>Drucker-Prager</td>
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<td>3D solid (8 nodes)</td>
<td>GFRP</td>
<td>Circular, Square</td>
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<td>Parvin and Wang 2002</td>
<td>Mohr-Coulomb</td>
<td>Hardening</td>
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<td>FRP</td>
<td>Circular</td>
<td>Load-deflection curves under lateral monotonic and cyclic loadings</td>
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<td>Parvin and Wang 2001</td>
<td>Mohr-Coulomb</td>
<td>Hardening</td>
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<td>CFRP</td>
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<td>Load-deflection curves under eccentric loadings, stress distributions</td>
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Failure Criteria of Concrete under Triaxial Compression

by T.-H. Tan and X. Sun

Synopsis: This paper presents an experimental investigation to determine the ultimate strength of concrete under triaxial compression. Concrete of 4 different strength levels were employed and triaxal tests were performed on 100 x 300 mm cylindrical specimens to establish the failure criteria for low, normal and high-strength concrete. The effects of confining pressure and stress path on different grades of concrete were also studied and test results were used to verify the failure criteria proposed by other researchers.

Keywords: compressive strength; concrete; confining pressure; deviatoric stress; failure envelope; hydrostatic stress; triaxial compression
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INTRODUCTION

Since the failure of concrete in a structure can occur differently under complex stress states, the understanding of the behaviour of concrete under multiaxial stress states is needed to develop the failure criteria for concrete. The compressive strength of concrete is the principal property employed in the design of reinforced concrete structures. After the pioneering investigation in 1920s by Richart et al on triaxial behaviour of concrete, many other researchers have also conducted studies on the concrete behaviour in multiaxial compression and several failure criteria of concrete have been proposed. More recently, the investigation has also been extended to include high strength concrete. However, most of the triaxial tests were conducted on equipment developed for rock testing and therefore the specimens were smaller (50 mm diameter) and the aspect ratio was kept at 2. With the advancement in servo-hydraulic test equipment and digital technology, more complicated and sophisticated triaxial tests can now be performed.

In this study, an experimental programme was undertaken to study the behaviour of concrete for a wide range of strength under various confining pressure. The equipment used can accommodate a bigger specimen and also a larger aspect ratio. Beside the usual active confining stress environment, passive confining stress condition can also be produced. The test results provided a better understanding of the ultimate compressive strength of concrete under lateral confining pressures and the effect of stress path. A failure envelope is proposed and it is compared with those put forward by other researchers.

EXPERIMENTAL PROGRAMME

Specimen preparation

It is well known that the friction between the steel platen and the ends of concrete specimen will give rise to tri-axial stress state in the concrete near the ends. This frictional restraint may prevent the specimen from expanding in the direction of specimen-platen interface and results in an overestimation of concrete compressive strength. Kotsovos reported that using a height to width ratio of 2.5 can minimize the complex and indefinable stress state of the concrete ends caused by this frictional restraint. In this study, specimens with height to diameter ratio of 3 were chosen and the dimension of concrete cylinder was 100 x 300 mm. The mix proportions for the four
strength groups namely G10, G25, G50 and G80 is shown in Table 1. In order to ensure statistical homogeneity of the specimens, all the concrete cylinders were extracted from larger existing concrete blocks measuring 2000 x 300 x 350 mm using a coring machine. The cores were then cut to lengths of 300 mm and both ends of each core were ground using a special surface grinder, ensuring that the ends are flat and perpendicular to the long axis of the specimen. In order to ensure the consistency of concrete specimens, prior to testing, all specimens were moist-cured for at least one week to ensure that they are fully saturated before testing.

**Test equipment**

Triaxial test apparatus comprise mainly of the following components: a) Rockcell Model 10 Triaxial Cell; b) Instron Load Frame; c) Confining Pressure System and d) Axial and Circumferential Deformation Device. The triaxial cell is designed to withstand a maximum lateral confining pressure of 70 MPa and the maximum loading capacity of the Instron loading frame is 2000 kN. The confining pressure system is servo-hydraulic and close-loop control. It is totally independent of the axial loading system and consists of a hydraulic pressure supply and a hydraulic pressure intensifier. The pressure intensifier is used to provide hydraulic pressure in the triaxial cell. The close-loop command and feedback control for the pressure system is managed by Instron digital controller.

The concrete specimen was first aligned with the lower and upper platen and then jacketed with a heat-shrink membrane to prevent hydraulic oil from penetrating the concrete during the test. Two LVDTs with 2.5 mm maximum stroke were used to measure axial deformation of the specimen and a circumferential LVDT, with 2.5 mm maximum stroke, was used to measure lateral deformation of the specimen. The axial LVDTs with gauge length of 100 mm were positioned in the central portion of the specimen. The circumferential LVDT was placed around the specimen at the midheight of the specimen. (Fig. 2)

**Test procedure**

Prior to testing, a load of about 10% of the uniaxial failure load was applied to the specimen to minimize the initial take-up reading due to the presence of any minute gaps at the interfaces. The load was gradually reduced to zero and similarly the confining pressure was adjusted to zero. For safety purpose, the maximum limit for the failure load was pre-set to prevent any sudden changes to occur during the loading. All the test readings such as load, pressure, position, time as well as axial and radial displacement were automatically logged by an Instron test programme named MAX. For triaxial tests with active confining stress, the axial compressive stress, $\sigma_1$, and the confining stress ($\sigma_2 = \sigma_3$) were initially increased up to a predetermined value ($\sigma_1 = \sigma_2 = \sigma_3$), and thereafter the confining stress was kept constant and the axial load was further increase until the specimen fails ($\sigma_1 > \sigma_2 = \sigma_3$).
Before concrete reaches the unstable fracture stage i.e. at about 70% to 80% of the peak stress, the circumferential deformation is low and the loading can be executed using the axial strain for control. Beyond that stage, particularly for unconfined concrete but less so for confined one, the circumferential deformation increases rapidly and uncontrollably. Shown in Fig. 3 is a relationship of $\varepsilon_1$ (axial strain) and $\varepsilon_2$ (circumferential strain) versus time under single channel control of $\varepsilon_1$. It is to be noted that $\varepsilon_2$ increases rapidly when the axial stress is close to the peak stress if the axial loading rate is held constant. This phenomenon makes the use of axial strain an unstable control parameter which will result in a sudden brittle failure in high strength concrete. Moreover, the response of the confining pressure may not be fast enough to keep up with the rate of circumferential deformation.

Cross compensation control - In order to have a more controllable failure, the loading of the specimen has to be changed to one based on the circumferential strain. Therefore, a cross compensation control method is used in this study. In conventional testing, the servo feedback comes from single channel which can be either the axial strain or the circumferential strain. Cross compensation control is based on the feedback from the combination of two signals. Fig. 4 reveals the strains versus time relationships under cross compensation control. The feedback consists of two signals in the combination of $(a\varepsilon_1 + b\varepsilon_2)$, where $a$, $b$ are compensation factors and $\varepsilon_1$, $\varepsilon_2$ are feedback signals. It should be noted that at the beginning of loading, the feedback is axial strain dominant and the loading rate is high. When concrete is approaching its peak stress, the loading rate is slowed down when the feedback is gradually transferred to circumferential strain dominant. Thus, it is possible to achieve a less brittle failure in order to obtain a complete stress-strain curve when the feedback signal is gradually dominated by the circumferential strain.

In this investigation, four groups of concrete specimens termed as G10, G25, G50 and G80 were tested. The uniaxial compressive strength of these groups was 10.35, 27.2, 51.8 and 77.46 MPa respectively. At least 3 specimens were tested to determine the uniaxial compressive strength for each group. Stress-strain relationships of G10 and G80 concrete under active confinement of various confining stresses are shown in Fig. 5 and 6 respectively.

EXPERIMENTAL RESULTS AND DISCUSSIONS

As expected, the results show that the peak stress level is dependent on the confinement level. The higher the confinement, the higher the peak stress and the corresponding strain the concrete can achieve. The test results are summarised in Table 2.

Proposed failure envelope for concrete

The general shape of failure envelope of concrete is usually described as open-ended and has a convex polar figure which has threefold symmetry with respect to the hydrostatic axis. The failure curve is nearly triangular for tensile and small compressive stresses, and becomes more circular corresponding to the increasing value of hydrostatic
pressure. Among the failure criteria proposed in the past, William and Warnke’s “five-parameter” model\(^4\) reflects the principal features of the triaxial failure surface of concrete. In this study, their model was adopted to define the failure envelope for concrete under triaxial stress state. The ultimate failure condition for concrete can be defined in terms of compressive and tensile surfaces, which are functions of the octahedral normal stress. The tensile and compressive meridians are expressed as follows:

\[
\frac{\tau_0}{f_c} = a_0 + a_1 \frac{\sigma_0}{f_c} + a_2 \left( \frac{\sigma_0}{f_c} \right)^2 \quad \text{at } \theta = 0^\circ \quad \text{Tensile meridian}
\]

\[
\frac{\tau_0}{f_c} = b_0 + b_1 \frac{\sigma_0}{f_c} + b_2 \left( \frac{\sigma_0}{f_c} \right)^2 \quad \text{at } \theta = 60^\circ \quad \text{Compressive meridian}
\]

The coefficients of the equations are chosen so that the surfaces pass through a set of control points given by the “five parameters”, namely the unconfined concrete compressive strength, the tensile strength, the equal biaxial compressive strength and a defined point on each of the two surface meridians. For the compressive meridian, the number of parameters is reduced to three. These three parameters can be derived by regression of the experimental results. For the triaxial test condition, the equation of compressive meridian can be written as follows:

\[
\frac{\sigma_o}{f_c} = \frac{\sigma_1 + \sigma_2 + \sigma_3}{3f_c} = -\frac{f_o}{3f_c} - \frac{2f_r}{3f_c}
\]

\[
\tau_o = \frac{1}{3f_c} \sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2} = -\sqrt{2} \left( \frac{\sigma_o + \frac{f_r}{f_c}}{f_c} \right)
\]

For triaxial case \(\sigma_1 = \sigma_2 = -f_r, \theta = 60^\circ\):

\[
\tau_o = b_0 + b_1 (\sigma_o) + b_2 (\sigma_o)^2
\]

Substituting Eq.2 into Eq.3, produces

\[
b_2 (\sigma_o)^2 + (b_1 + \sqrt{2})(\sigma_o) + \left( b_0 + \frac{\sqrt{2}f_r}{f_c} \right) = 0
\]

Solving Eq.4 and incorporating Eq. 1, the result is

\[
\frac{f_o}{f_c} = \frac{3(b_1 + \sqrt{2})}{2b_2} + \sqrt{\left( \frac{3(b_1 + \sqrt{2})}{2b_2} \right)^2 - \frac{9b_0}{b_2} - \frac{9\sqrt{2}f_r}{b_2f_c} - \frac{2f_r}{f_c}}
\]
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By regression of experimental data, for concrete uniaxial strength within 10.35~77.46 MPa, the three parameters can be defined as: $b_0 = 0.19$, $b_1 = -0.8725$ and $b_2 = -0.087$. Therefore, Eq.5 can be transformed into:

$$f_0 = f'_c \left( -9.338 + 10.338 \sqrt{1 + 1.368 \frac{f_r}{f_c} - 2 \frac{f_r}{f_c}} \right)$$

(6)

where $f'_c$ = peak strength of confined concrete; $f_r$ = lateral confining pressure on concrete; $f_c$ = Standard concrete cylinder strength.

The proposed failure envelope was compared with experimental results as shown in Fig. 7. Based on experimental observation, it should be noted that for concrete under active confinement, concrete with different uniaxial compressive strength will result in different failure envelopes. However, the differences between these envelopes are insignificant especially for normal and high strength concrete. In this study, the proposed failure envelope is suitable for low, normal and high-strength concrete. Comparisons were also made between active and passive confinement with different lateral stiffness. It was found that the failure envelope under passive confinement is slightly different from that of active confinement for the same grade of concrete (Figure 8); the higher the stiffness of lateral confinement, the closer the failure envelope to that of active confinement. This suggests that different stress paths produce different failure envelopes but the difference is minor. Hence the proposed failure envelope can still represent all the data with reasonable accuracy. It was observed that the proposed failure envelope also has a close fit with the test results tested by other researchers (Figures 9 and 10) for a wide range of concrete uniaxial compressive strength of up to 119 MPa.

In Fig.11, proposed failure envelope is compared with failure envelopes proposed by other researchers. It is interesting to find that the proposed failure envelope is close to failure surfaces proposed by Hobbs and Kotsovos. Up to about lateral stress ratio of 0.4, the linear relationship is more conservative but beyond that, Mander’s model produces a lower strength boundary and Setunge’s model gives an upper boundary for all lateral stress ratio. Moreover, all of these models proposed are assumed to be stress path independent. The differences between these models may be caused by different experimental results for different aspect ratio of specimen and various test conditions.

CONCLUDING REMARKS

In this paper, the results of experimental work on the strength properties of low, normal and high strength concrete were presented. A cross compensation control method was adopted to avoid sudden brittle failure of concrete especially for high-strength concrete during the test. The failure surface of concrete under lateral confinement was determined through regression analysis of the experimental data. The following conclusions are drawn based on the findings of this study:
1. The strength and ductility of concrete under lateral confinement are influenced by the lateral confining stress. The higher the confining stress, the higher the peak stress and peak strain concrete can achieve.

2. For concrete under active confinement, different concrete uniaxial strength will result in different failure envelopes. However, the differences between these envelopes are insignificant especially for normal and high-strength concrete. The differences caused by stress-path on failure envelope were also found to be small. It is reasonable to establish a single failure criterion to describe the strength property for concrete under lateral confinement.

3. The proposed failure envelope is suitable for low, normal and high-strength concrete, even for very high-strength concrete with uniaxial compressive strength up to about 120 MPa.

REFERENCES


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Table 1. Mix design of concrete

<table>
<thead>
<tr>
<th>Grade of concrete</th>
<th>G10</th>
<th>G25</th>
<th>G50</th>
<th>G80</th>
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<tr>
<td>Compressive strength at 3 months (MPa)</td>
<td>10.35</td>
<td>27.2</td>
<td>51.8</td>
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<td>Slump (mm)</td>
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<td>Cement (kg)</td>
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<td>540</td>
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<td>Silica fume (kg)</td>
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<td>0</td>
<td>0</td>
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<tr>
<td>Water (kg)</td>
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<td>190</td>
<td>160</td>
<td>145</td>
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<tr>
<td>Aggregate (kg)</td>
<td>1020</td>
<td>1000</td>
<td>1000</td>
<td>1050</td>
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<tr>
<td>Sand (kg)</td>
<td>870</td>
<td>810</td>
<td>760</td>
<td>595</td>
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<td>Water/cement ratio</td>
<td>0.82</td>
<td>0.55</td>
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Table 2. Test results of concrete under active confinement

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<tr>
<th>Spec. Number</th>
<th>$f_c$ (MPa)</th>
<th>D (mm)</th>
<th>H (mm)</th>
<th>Max $\sigma_2$ (MPa)</th>
<th>Failure load (KN)</th>
<th>Peak $\sigma_1$ (MPa)</th>
<th>Peak $\sigma_1$ (mm/m)</th>
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<tr>
<td>G10A-1</td>
<td>10.35</td>
<td>100.9</td>
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Figure 1 – Rockcell Model 10 triaxial cell

Figure 2 – Setup of specimen
Figure 3 – Single channel control

Figure 4 – Cross compensation control

Figure 5 – Axial stress-strain relationship for low-strength concrete (G10) under lateral confinement
Figure 6 – Axial stress-strain relationship for high-strength concrete (G80) under lateral confinement

Figure 7 – Effect of compressive strength on the proposed failure envelope

Figure 8 – Effect of stress path on the proposed failure envelope
Figure 9 – Test data (Xie, 1995)\(^6\) compared with proposed failure envelope confinement

Figure 10 – Test data (Imran, 1996)\(^7\) compared with proposed failure envelope confinement

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Three-Dimensional Finite Element Modeling of Confined High-Strength Concrete Columns

by P. Bhargava, R. Bhowmick, U. Sharma, and S.K. Kaushik

Synopsis: The use of high-strength concrete (HSC) tied columns is becoming increasingly popular in engineering practice. Researchers are working to obtain the proper post-peak behavior of tied columns with concrete strength greater than 60 MPa. Many empirical confinement models have been reported in the literature for the prediction of stress-strain behavior under concentric loading. However, nothing significant has been said about the numerical modeling of the problems wherein the nonlinear response of HSC tied columns may be reasonably predicted. In the present study, the nonlinear behavior of concrete material has been idealized by William-Warnke five-parameter model, which, to date, is the most widely accepted and sophisticated criterion. Within the framework of rate independent associative elasto- plasticity, a full backward Euler integration algorithm for stress updating has been implemented in the present work. A fixed crack smeared approach based upon fracture energy concept and non-local material softening law has been employed for the tensile modeling of concrete material. The computational model also involves the provision for cover spalling. A couple of examples have also been presented for validation of the numerical methodology proposed in this work.

Keywords: confinement; hardening; model; softening; spalling
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INTRODUCTION

High-strength concrete (compressive strength greater than 60 MPa) is a relatively new construction material with enormous potential. Structures using High Strength Concrete (HSC) are becoming increasingly common. Higher compressive strength, greater modulus of elasticity and substantial savings are all properties of HSC that appeal to the designers. One of the main applications of HSC is in the lower-storey columns of tall buildings. The use of HSC allows a significant reduction of column sizes, which increases the floor area, leading to economic advantages. But HSC when subjected to short term or sustained loads tends to be brittle when loaded to failure, lacking the plastic deformation typical of normal strength concrete. The ductility performance of HSC is very poor when unconfined or under low confinement. This type of concrete behavior raises questions about the application of the material to the structures, especially in the seismic regions.

Over the last two decades experiments have been carried out on high strength concrete tied columns by researchers to observe the behavior under concentric loading. Numerous models exist in the literature to predict stress-strain law of normal and high strength concrete confined columns [Muguruma et al. 1993, Yong et al. 1988, Nagashima et al. 1992, Cusson and Paultre 1995, Razvi et al. 1999, Legeron et al. 2003]. Most of these models have limited validity in terms of concrete strength, column geometry, transverse reinforcement yield strength and loading condition. A comparative study of confinement models undertaken by the authors earlier indicated that the capability of Legeron and
Paultr (2003) model to predict the actual test behavior is best among all these empirical models [Sharma et al. 2004].

In the last three decades, tremendous progress has been made in the modeling of concrete and the numerical analysis of reinforced concrete structures. The application of finite element method to tied concrete columns is still at the initial stage [Chen and M au 1989, Abdel-Halim et al. 1989, Xie et al 1996, Foster and Liu 1998].

RESEARCH SIGNIFICANCE

Nonlinear analysis of reinforced concrete structures has become increasingly important in recent years. It is only by carrying out a complete progressive failure analysis of the structure up to collapse that it is possible to assess all safety aspects of a structure and find its deformational characteristics. Post-peak response (both with regards to compression and the tension) is a complex phenomenon for numerical modeling as it involves problems of mesh / load step sensitivity, uniqueness, stability, and detection of localization and mesh ability to capture localization. For pressure sensitive quasi-brittle materials, additional problems like pressure dependent nonlinear hardening (softening) moduli, choice of damage parameters, elastic degradation, non-associative plastic flow etc. further complicates the mathematical modeling.

The key issues identified in the context of non-linear finite element analysis of confined concrete columns are: Modeling of concrete in compression including the computational aspects of elasto-plasticity, hardening hypothesis and the consistent iterative path independent solution algorithms. Development of a conceptual model for concrete cracking using fixed smeared elastic approach and integrating the added features like fracture energy, non local material softening law and the tension-softening/tension-stiffening phenomena. To this end, a software has been developed for the analysis purpose.

MATERIAL MODELING

Modeling in compression

The actual behaviour of concrete in three-dimensional state of stress is extremely complicated. It is well established that under tensile and low compressive stresses, concrete fails by brittle fracture. On the other hand, it can yield and flow like a ductile material under high hydrostatic pressure. Several models have been developed for brittle and ductile behaviour of concrete [Chen and Saleeb 1982; Chen 1982]. The most commonly used failure criteria are defined in stress spaces by a number of material constants varying from one to five independent control parameters. These models can be classified as one-, two-, three-, four-, and five-parameter models [Chen and Saleeb 1982; Chen 1982; Buyukozturk and Shareef 1985]. Though all these models have certain inherent advantages and disadvantages, yet the William-Warnke five-parameter model, to date, is most versatile and sophisticated criterion for the elasto-plastic modeling of concrete [Chen and Saleeb 1982]. Modeling of concrete in compression includes:
Elasto-plastic five parameter model -- It is widely accepted that the failure surface may conveniently be represented in Haigh-Westerguard space as \( \phi(\xi, \rho, \theta) = 0 \), where the set of variables \( \xi, \rho \) and \( \theta \) may be defined as

\[
\xi = \frac{I_1}{\sqrt{3}} \quad ; \quad \rho = \sqrt{2J_2} \quad ; \quad \theta = \frac{1}{3} \cos^{-1}\left(\frac{3\sqrt{3}J_3}{2J_2^{3/2}}\right) \tag{1}
\]

Note that \( I_1 \) is the first invariant of stress tensor, \( J_2 \) and \( J_3 \) are second and the third invariant of the deviatoric stress tensor respectively.

The geometry construction of such surface in the principal stress space is best described in by its cross-sectional shape in deviatoric plane and its meridians in the meridian plane. The cross-section of the failure surface is the intersection curve between the failure surface and a deviatoric plane (parallel to the \( \pi \)-plane) which is perpendicular to the hydrostatic axis with \( \xi = \) constant. The meridians of the failure surface are the intersection curves between the failure surface and a plane (meridian plane) containing the hydrostatic axis with \( \theta = \) constant. For the assumed five parameter model, the deviatoric section has a three-fold symmetry and is approximated by an elliptic curve for each sector, \( 0 \leq \theta \leq 60^0 \), with smooth transition because of isotropy. The hydrostatic section comprises of two second-order parabolas along the meridians \( \theta = 0^0 \) and \( \theta = 60^0 \), called the tensile and compressive meridians, respectively. Moreover, both parabolas have a common apex at the hydrostatic axis (corresponding to hydrostatic tensile yield).

The tensile and compressive curved meridians are expressed as

\[
\frac{\rho_t}{\sqrt{5}f'_c} = r_t = a_0 + a_1 \left( \frac{\sigma_m}{f'_c} \right) + a_2 \left( \frac{\sigma_m}{f'_c} \right)^2 \quad \text{at} \ \theta = 0^0 \tag{2a}
\]

\[
\frac{\rho_c}{\sqrt{5}f'_c} = r_c = b_0 + b_1 \left( \frac{\sigma_m}{f'_c} \right) + b_2 \left( \frac{\sigma_m}{f'_c} \right)^2 \quad \text{at} \ \theta = 60^0 \tag{2b}
\]

Where \( f'_c \) is the uniaxial compressive cylinder strength, \( \sigma_m \) is the hydrostatic stress and \( a_0, a_1, a_2 \) and \( b_0, b_1, b_2 \) are the unknown parameters. Taking into account the common apex at the equisectrix, the number of parameters is reduced to five and they are determined from experimental data. The elliptic section on \( \sigma_m = \xi \) deviatoric plane is given by

\[
r(\sigma_m, \theta) = \frac{2r_c(r_c^2 - r_t^2) \cos \theta + r_c^2 (2r_t - r_c)}{4(r_c^2 - r_t^2) \cos^2 \theta + (r_c - 2r_t)^2} \tag{3}
\]

Five-parameter model predicts the failure if the stress-state satisfies the condition

\[
\phi(\sigma_m, \tau_m, \theta) = \frac{\tau_m}{r(\sigma_m, \theta)f'_c} - 1 = 0 \quad ; \quad \tau_m = \sqrt{2/5J_2} \tag{4}
\]
The classical elasto-plastic constitutive equations pertaining to rate independent plasticity with associative flow rule are adopted [Crisfield 1991, 1997],

\[
\dot{\varepsilon} = \dot{\varepsilon}_e + \dot{\varepsilon}_p \tag{5a}
\]
\[
\dot{\sigma} = D_e \cdot \dot{\varepsilon}_e = D_e \left( \dot{\varepsilon} - \dot{\varepsilon}_p \right) \tag{5b}
\]
\[
\dot{\varepsilon}_p = \dot{\lambda} \frac{\partial \phi}{\partial \sigma} = \dot{\lambda} a \tag{5c}
\]

Where, the total strain rate, \( \dot{\varepsilon} \) is decoupled into an elastic component \( \dot{\varepsilon}_e \) and a plastic component \( \dot{\varepsilon}_p \); the stress rate \( \dot{\sigma} \) is related to elastic strain rate through elastic constitutive matrix \( D_e \) and \( \phi \) is the yield function given by Eq. (4). \( \dot{\lambda} \), the plastic multiplier, is to be determined with the aid of loading-unloading criterion which is expressed in Kuhn-Tucker form as

\[
\phi(\sigma) \leq 0 \tag{6a}
\]
\[
\dot{\lambda} \geq 0 \tag{6b}
\]
\[
\dot{\lambda} \phi = 0 \tag{6c}
\]

The above equations may be viewed as a constraint requiring that the stress trajectories be confined to the elastic domain i.e. plastic consistency condition.

Integration algorithm and consistent tangent modulus: In any numerical scheme employed for the analysis of elasto-plastic problems, it is essential to integrate the constitutive equations governing the material behavior. It is well known that the precision with which these relations are being integrated has a direct impact on the overall accuracy of the analysis. A number of integration algorithms like forward Euler (explicit), semi-backward Euler (cutting plane) and full implicit or backward Euler (closest point projection) are being popularized in finite element literature. It is quite established, by now, that in order to enhance the robustness of the elasto-plastic integration algorithms, evaluation of consistent tangent modulus (rather than the continuum tangent modulus) is of paramount importance, as it allows for the exact linearization in the closed form and, thus ensures the quadratic rate of convergence of the Newton-Raphson iterative procedure [Simo and Taylor 1985; Crisfield 1991; Ramm and Matzenmiller 1988]. The task of evaluating the tangent modulus is exceedingly laborious as it involves computation of first and second order derivatives of yield criterion at the end of each iteration.

In the present study a full implicit Backward Euler integration algorithm along with the consistent tangent modular has been implemented in the context of William-Warnke five-parameter model [Ortiz et al. 1986; Simo 1985]. As per this class of algorithms, for a typical step \([ t_n, t_{n+1} \] \) the set of constitutive parameters is defined with the aid of following equations

\[
\Delta \sigma_{n+1} = D_e \cdot \left( \Delta \varepsilon_{n+1} - \Delta \varepsilon_{p,n+1} \right) \tag{7a}
\]
The value of stress tensor at $t_{n+1}$ can be expressed in terms of elastic predictor as

$$\sigma_{n+1} = \sigma_{n+1}^{trial} - \Delta \lambda_{n+1} D_e \frac{\partial \phi_{n+1}}{\partial \sigma_{n+1}}$$

A starting estimate for $\sigma_{n+1}$ is conveniently obtained by using the cutting plane algorithm [Crisfield 1997]. Generally this starting estimate fails to satisfy the yield function, as the normal at trial position $\sigma_{n+1}^{trial}$ does not coincide with the final normal at $\sigma_{n+1}$. It requires the setting up of an iterative loop by defining the following residual vectors at iteration $k$:

$$\hat{R}^{(k)}_{\sigma, n+1} = \sigma^{(k)}_{n+1} - \sigma_{n+1}^{trial} + \Delta \lambda_{n+1}^{(k)} D_e \frac{\partial \phi_{n+1}^{(k)}}{\partial \sigma_{n+1}^{(k)}}$$

Obviously, the two residuals must tend to zero in order to obtain an admissible solution. This is achieved by adopting the normal practice of writing the first order Taylor series expansion and making the new residuals

$$0 = R^{(k)}_{\sigma, n+1} + \frac{\partial \phi_{n+1}^{(k)}}{\partial \sigma_{n+1}^{(k)}} \delta \sigma_{n+1}^{(k)} - A^{(k)}_{n+1} \delta \lambda_{n+1}^{(k)}$$

Where $\delta \sigma_{n+1}^{(k)}$ and $\delta \lambda_{n+1}^{(k)}$ are the changes in the stress and the consistency parameter during the iteration loop $[k, k+1]$. Solution of equations (10a-b) enables to have the value of iterative plastic multiplier $\delta \lambda_{n+1}^{(k)}$ as

$$\delta \lambda_{n+1}^{(k)} = \frac{R^{(k)}_{\phi, n+1} - \frac{\partial \phi_{n+1}}{\partial \sigma_{n+1}^{(k)}} \hat{D}^{(k)}_{n+1} \cdot R^{(k)}_{\sigma, n+1}}{A^{(k)}_{n+1} + \frac{\partial \phi_{n+1}}{\partial \sigma_{n+1}^{(k)}} \hat{D}^{(k)}_{n+1} \cdot \frac{\partial \phi_{n+1}^{(k)}}{\partial \sigma_{n+1}^{(k)}}}$$

Where $\hat{D}^{(k)}_{n+1}$, algorithmic modulus, is defined as

$$\hat{D}^{(k)}_{n+1} = \left[ D_e^{-1} + \Delta \lambda_{n+1}^{(k)} \left( \frac{\partial^2 \phi}{\partial \sigma^2} \right)_{n+1} \right]^{-1}$$

Differentiating equation (10) and combining it with equation (14) for algorithmic
tangent modulus, it is possible to write
\[ \dot{\sigma}_{n+1} = \dot{D}_{n+1} : \left[ \dot{\sigma}_{n+1} - \delta n + \frac{\partial \phi_{n+1}}{\partial \sigma_{n+1}} \right] \]  
(13)

Subsequently, enforcement of the consistency condition results in the following incremental elasto-plastic constitutive law
\[ \delta \sigma_{n+1} = D_{ep,n+1} : \dot{\sigma}_{n+1} \]  
(14)

Where the consistent tangent modulus \( D_{T,n+1} \) is given as
\[ D_{ep,n+1} = \dot{D}_{n+1} - \frac{\partial \phi_{n+1}}{\partial \sigma_{n+1}} \frac{\partial \sigma_{n+1}}{\partial \sigma_{n+1}} \]  
(15)

In addition, an expression for consistent hardening parameter based upon strain hardening hypothesis has also been employed.

**Modeling in Tension**

Smeared cracking is a continuum approach for the numerical solution of fracture mechanics problems in which local discontinuities are distributed (i.e. smeared). The literature suggests that the classical smeared cracking models are unobjective, exhibiting spurious mesh sensitivity and converge to an incorrect failure mode with zero energy dissipation [Bazant and Oh 1983; and Borst 1984]. It is known that the choice of the element size affects the finite element solutions of strain softening problems, if no provision for non-local material model is made. Therefore, more sophisticated crack simulation theories, enriched by the introduction of additional terms such as fracture energy and characteristic length, have been proposed to overcome this problem. Significant improvement of this concept has been provided by the fictitious crack model [Hillerborg et al. 1976] and the crack band theory [Bazant and Oh 1983]. In the fictitious crack approach, the tensile strength \( f'_t \) and the fracture energy \( G_f \) are the two model parameters. The crack band theory, on the other hand, includes a third parameter, which is the crack bandwidth \( h \) in addition to two parameters \( (f'_t \) and \( G_f \)).

In the original formulation of the smeared crack approach, the strain vector represented the overall strain of the cracked material. In this way no distinction was made between the cracks and the solid material between them. However, the modern approach for smeared cracking is based on the idea of strain decomposition into strain in the cracked material and also in the uncracked concrete as
\[ \Delta \varepsilon = \Delta \varepsilon_{co} + \Delta \varepsilon_{cr} \]  
(16)

In order to incorporate crack traction- crack strain laws, it is convenient to adopt a local \((n, s, t)\) coordinate system, which is aligned with the crack orientation, i.e. with the principal directions of orthotropy. In fracture mechanics terminology, the local
incremental crack strain vector $\Delta \varepsilon_{cr}$ consists of mode-I crack normal strains and mode-II crack shear strains. The transformation between the global and local crack strain/stress components is given as

$$
\Delta \varepsilon_{cr} = N \Delta \varepsilon_{cr} \quad ; \quad \Delta t = N^T \Delta \sigma
$$

(17)

Where $N$ denotes the orthogonal transformation (rotation) matrix, which is a function of the current orientation of the crack with respect to the global coordinate axes. In the present smeared crack approach, $N$ is assumed to remain fixed upon crack formation (fixed crack concept). To relate the incremental stress vector to incremental strain vector, the constitutive relations for the intact continuum between the cracks and the smeared cracks are assumed as

$$
\Delta \sigma = D_{co} \Delta \varepsilon_{co} \quad ; \quad \Delta t = D_{cr} \Delta \varepsilon_{cr}
$$

(18)

Where $D_{co}$ is the constitutive matrix of instantaneous properties of solid material between the cracks and $D_{cr}$ is the matrix incorporating the mode –I and mode-II and mixed-mode properties of the crack. Finally the overall relation between global stress and global strain may be derived as

$$
\Delta \sigma = \left[ D_{co} - D_{co} N \left[ D_{cr} + N^T D_{cr} N \right]^{-1} N^T D_{co} \right] \Delta \varepsilon
$$

(19)

Note that $D_{co}$ for an elastic isotropic material is a function of elastic modulus $E$ and the Poisson’s ratio $\nu$. The cracked constitutive modulus, $D_{cr}$, comprises of mode –I ($D^I_{cr}$) and mode –II stiffness moduli ($D^{II}_{cr}$) for a single smeared crack. The mode-I stiffness modulus, $D^I_{cr}$, is the most important factor in the present material model because it governs the crack propagation in mode- I. This modulus depends on selected shape of descending branch of stress strain relation and the tensile strength of concrete.

A family of crack normal stress versus crack normal strain ($p - \varepsilon_{cr}$) relations have been proposed in the literature based on curve fitting of a large number of experimental data [Leibengood et al. 1986; Reinhardt, et al. 1986]. The nonlinear, exponential ($p - \varepsilon_{cr}$) diagram employed in this study is as follows

$$
\frac{p}{f_t} = 1 - \left( \frac{\varepsilon_{cr}}{\varepsilon_{cr}^u} \right)^q
$$

(20)

Where $\varepsilon_{cr}^u$ is the ultimate strain corresponding to stress free crack opening (i.e. opening at total loss of load carrying capacity in the normal direction of the crack). The above softening relations enable to obtain the modulus $D^I_{cr}$ as

$$
D^I_{cr} = \frac{\partial p}{\partial \varepsilon_{cr}} = -\frac{q f_t'}{\varepsilon_{cr}^u} \left( 1 - \frac{\varepsilon_{cr}}{\varepsilon_{cr}^u} \right)^{q-1}
$$

(21)
For smeared cracks, the crack shear modulus $D_{cr}$ has in past been assigned a constant value, which corresponds to linear ascending relation between shear stress and shear strain across the crack. An improvement is obtained by making the shear stiffness after cracking a decreasing function of crack normal strain. The model accounts for the fact that the interlock of aggregate particles diminishes with increasing crack opening. The mode-II stiffness modulus can be related with the elastic shear modulus $G$ via the shear retention factor as $[\text{Rots and Blauwendraad 1989}]$

$$D_{cr}^I = \frac{\beta}{1 - \beta} G ; \quad \beta = \frac{1}{1 + 4447 \varepsilon_{cr}} \tag{22}$$

Fracture energy and strain softening -- Material softening and strain localization have been the focus of significant computational and experimental research efforts directed to elucidate problems such as stability, uniqueness and localization. The concept of constant fracture energy release rate provides a sound and well-accepted base within the framework of smeared crack approach. For the smeared crack the fracture is assumed to be distributed over the crack bandwidth $h$, which is related to the particular finite element configuration. Assuming constant strain distribution over the cracked bandwidth $h$, the fracture energy $G_f$ may be defined as

$$G_f = h g_f = h \int_0^{\varepsilon_{cr}} p \, d \varepsilon_{cr} \tag{23}$$

Where $g_f$, the fracture energy density indicates the total area under stress versus crack strain curve. $p$, the normal stress component corresponding to crack normal strain $\varepsilon_{cr}$.

From the equations (20 and 23), the parameter $\varepsilon_{cr}^u$ may be conveniently obtained as

$$\varepsilon_{cr}^u = \frac{(q + 1) g_f}{q f'_t} \tag{24}$$

Note that the stress free strain $\varepsilon_{cr}^u$ at softening completion is not conceived as a separate material property. It is a consequence of the crack bandwidth $h$ and the three parameters of the model i.e. tensile strength, fracture energy and softening diagram.

In order to avoid snap back instability [Crisfield 1986] at local integration point level, a condition for maximum limit for crack band-width $h$ may be stated as

$$h \leq \frac{(q + 1) G_f}{q f'_t} \left( E \frac{1}{q f'_t} \right)^{1/q} \varepsilon_{cr}^{(1-q)/q} \tag{25}$$

Tension softening and tension stiffening -- After cracking concrete can continue to carry tensile stresses as a result of two independent mechanisms: tension softening and tension stiffening. Tension softening refers to the fracture associated mechanism particularly significant in concrete structures containing little or no reinforcement. The concrete post cracking tensile stress, $p_{soft}$, associated with tension softening is derived from crack band theory as discussed above.
On the other hand, tension stiffening involves post-cracking tensile stress in the concrete as a result of interaction between reinforcement and concrete. Concrete-related models for consideration of tension stiffening are more popular than steel-related models and, therefore, concrete based model has been used in the present study. In areas between cracks, load is transferred from reinforcement to concrete via bond stresses producing significant level of average tensile stresses in the concrete. The concrete tension stiffening stresses are modeled as

\[ p_{stf} = \frac{f'_t}{1 + c_t\varepsilon_{cr}} \]  

(26)

Where, the parameter \( c_t \) indicates the degree of tension stiffening and depends on various factors like the reinforcement ratio \( \bar{\rho} \), bar diameter \( d_b \), the orientation of the reinforcement \( \theta \), (the angle between the reinforcement and normal to the crack surface). The parameter \( c_t \) is evaluated as

\[ c_t = 2.2 \left[ \sum_{i=1}^{n} \frac{4\bar{\rho}_i d_b \cos \theta_i}{d_b \cos \theta_i} \right]^{-1} \]  

(27)

Where, \( n \) is the number of steel layers. The resulting average principal tensile stress in concrete is larger of two values determined by tension softening and tension stiffening approaches as \( p = \max(p_{solf}, p_{stf}) \). The concrete structure involving the complexities associated with tension softening, spurious modes and snap-back behavior cannot be analyzed by simply using the Newton-Raphson technique. It is recommended that N-R methodology must be augmented with the well-known arc-length technique in order to achieve an acceptable and accurate solution [Crisfield 1982; Crisfield 1991]

NUMERICAL EXAMPLES

In this section an attempt is being made to validate the computational methods presented in the earlier sections by comparing the numerical predictions with experimental results for a couple of square column sections. Two test specimens CS17 [Razvi and Saatcioglu 1999] and 2B [Cusson and Paultre 1994] have been chosen for this purpose.

Specimen CS17 (Razvi and Saatcioglu 1999)

The geometry and reinforcement arrangement of the column is shown in Fig.1a and Fig.1b. Finite element modeling has been carried out for a test zone length of 935 mm. The concrete has been modeled using 8-noded isoparametric solid elements, allowing translations in three directions, with a numerical integration over a 2x2x2 Gaussian quadrature. The longitudinal and transverse reinforcements have been modeled using 2-noded truss elements, allowing translations in axial direction, with numerical integration over 2 point Gauss quadrature. Complete strain compatibility has been assumed between the reinforcement and the concrete elements. The finite element mesh as shown in the
Fig. 1c, consists of 1127 nodes, 792 concrete elements and 464 steel elements. The material properties considered for the present analysis are as follows:

- Compressive strength of concrete = 68.9 MPa
- Tensile strength of concrete = 5.2 MPa
- Yield strength of longitudinal reinforcement = 450 MPa
- Yield strength of lateral reinforcement = 400 MPa
- Modulus of elasticity of concrete = 42037 MPa
- Modulus of elasticity of steel = 200000 MPa
- Poisson’s ratio of concrete = 0.2
- Poisson’s ratio of steel = 0.3

The boundary conditions have been applied at the ends such that, translations in two directions in the horizontal plane are restrained and the translation in the vertical direction is permitted. The load has been applied by imposing varying incremental vertical displacements at the ends. The value of the displacement increment is on higher side before the commencement of spalling process and gradually it dies down as one approaches toward the end of the solution.

To study the behavior of concrete columns with the core confined by tie reinforcement, cover spalling has been included in the analysis by setting the cover elements to a low stiffness once a threshold tensile strain value in the range of 0.0007 to 0.00075 is attained [Foster 1998]. Non-linear finite element analysis has been carried out for this test example. The stress-strain curve for the confined concrete column using the FE analysis and the experimentally observed one is illustrated in Fig 2a. It is apparent that the two curves match reasonably well in respect to peak stress value of 74.6 MPa (computed)/75.2 MPa (experimental) and the corresponding strain level of 0.0028 (computed)/0.0033 (experimental). Also, the analysis indicates the stress level of around 0.3 times the specified tensile strength of concrete at the instant the spalling of the cover elements initiates. The elements at the edges develop triaxial stress state with two tensile stresses and one compressive stress, whereas elements at other areas develop triaxial stress state with one tensile stress and two compressive stresses.

The confining and the axial stresses are plotted in XZ plane [Fig 3] for the complete domain at four stages of analysis i.e. before cover spalling, after cover spalling but before the peak load is attained, at the peak load and at the end of the descending branch (or the solution). Before the cover spalls, the confining stress is observed to be low, at around 0.7 – 0.75 MPa and the axial stress level is more or less uniform over the test region, having a stress level of around 67 – 68 MPa. The levels of both confining and axial stresses increase thereafter. At the peak stress the confining stresses at the region where failure occurred, is observed to be around 4.5 – 5.5 MPa, while the axial stress level at the similar region is observed to be around 72 – 74 MPa. At the post-peak region and near the end of the solution, the confining stress level is found to have increased considerably to the order of 20 MPa, whereas the axial stress level decreases to around 48-52 MPa. At the boundary regions the stress levels are high due to the restraint provided. In respect to the reinforcement, it is being observed that the longitudinal steel yields at peak stress, but ties do not yield, though the experimental report indicates yielding of ties at peak stress [Table 1]. Many studies (Cusson & Paultre 1994 and Li et al. 1994) in the past have concluded that yield strength of ties may not be attained at peak of confined stress-strain curve for high strength concrete, especially with low confinement. Even the fact is well illustrated in the context of normal strength concrete.
(Sheikh & Uzmeri 1980). Legeron & Paultre (2003) confinement model, which estimates the behavior of confined concrete better than most of the other models (Sharma et al. 2004), also predicts the value of stress in lateral ties at peak close to the value given by the proposed FE model i.e. less than the yield value (Table 1). Apart from this there is a high level of uncertainty in the experimental measurement of strains in ties, especially near the peak when there is a sudden change in load and strain values.

**Specimen 2B (Cusson and Paultre 1994)**

The geometric configuration [Fig 4] and the material properties for the concrete column specimen are as follows:

- X-sectional dimensions = 235 x 235 mm; length of column = 1400 mm.
- Compressive strength of concrete = 85 MPa; Tensile strength of concrete = 6.35 MPa; Yield strength of longitudinal reinforcement = 450 MPa; Yield strength of lateral reinforcement = 415 MPa; Modulus of elasticity of concrete = 45000 MPa.
- Modulus of elasticity of steel = 200000 MPa.
- A test gauge length of 900 mm has been discretized using 8-noded brick and 2-noded bar elements for concrete and reinforcement respectively. The finite element mesh consists of 1008 concrete elements and 920 steel elements.

Once again, the finite element results have been summarized in the form of stress-strain curve for concrete, level of stress in longitudinal and the transverse reinforcement and the triaxial state of stress corresponding to various stages of loading. The stress-strain plot as given in Fig. 5 suggests an excellent agreement between finite element prediction and experimental observation. As far as the stress in the reinforcement is concerned, it is less than the yield value though the lateral reinforcement is comparatively more stressed. The plots of confining and axial stresses in the XZ plane for the complete model before cover spalling and at the peak load is shown in Fig. 6. Before the cover spills, the confining and axial stress level are in the range of 0.9-1.1 MPa and 80 – 83 MPa respectively. However, these values increase marginally as the peak confined stress level is attained.

**SUMMARY AND CONCLUSIONS**

Finite element simulation to predict the behavior of the structures that are made up of quasi-brittle material like concrete requires sophisticated constitutive models for concrete in tension as well as in compression. The non-linear behavior of concrete in compression has been idealized by employing an elasto-plastic strain-hardening model. The William-Warnke five-parameter model has been used to define the initial and the subsequent yield surfaces. Modeling of concrete in compression has been undertaken considering the computational aspects of elasto-plastic integration schemes, consistent tangent modular along with the Newton-Raphson iterative technique. A fixed crack smeared approach based upon fracture energy concept and nonlocal material softening law has been employed for the modeling of the tensile behavior of the concrete material. The resulting model is robust in the sense that finite element analysis do not diverge prematurely, and is capable of providing accurate predictions of reinforced concrete behavior as has been shown by a couple of examples.


### Table 1. Comparison of Stresses in Ties at Peak Load obtained from FEA, Legeron & Paultre (2003) Model and Experimental Results for CS17

<table>
<thead>
<tr>
<th>Figure</th>
<th>Tie Locations</th>
<th>Stresses at Peak Load from FEA</th>
<th>Stresses at Peak Load from Legeron’s Model</th>
<th>Stresses at Peak Load from Experimental Report</th>
</tr>
</thead>
<tbody>
<tr>
<td>A, D, E, H, I, L, M, P</td>
<td>100.2</td>
<td>200</td>
<td>400</td>
<td></td>
</tr>
<tr>
<td>Q, T, U, X</td>
<td>93.87</td>
<td>200</td>
<td>400</td>
<td></td>
</tr>
<tr>
<td>R, S, V, W</td>
<td>82.18</td>
<td>200</td>
<td>400</td>
<td></td>
</tr>
</tbody>
</table>
Table—2. Comparison of stresses in ties before first cover spalling, at the peak load and at the end of the solution for 2B

<table>
<thead>
<tr>
<th>Figure</th>
<th>Tie Locations</th>
<th>Stresses before cover spalling</th>
<th>Stresses at Peak Load</th>
<th>Stresses at the end of the solution</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>A,D,E,H, I,L,M,P</td>
<td>110</td>
<td>202</td>
<td>355</td>
</tr>
<tr>
<td></td>
<td>Q,T,U,X</td>
<td>256</td>
<td>333</td>
<td>400</td>
</tr>
<tr>
<td></td>
<td>R,S,V,W</td>
<td>256</td>
<td>333</td>
<td>400</td>
</tr>
</tbody>
</table>

Fig 1-- (a) geometry, (b) reinforcement arrangement, (c) FE mesh and BC’s for CS17

Fig 2 -- (a) Comparison of stress-strain values from FE analysis with experimental results for concrete, (b) stress-strain plot for longitudinal steel
Fig 3-- Principal stress surfaces: (a) confining stresses and (b) axial stresses before cover spalling; (c) confining stresses and (d) axial stresses at the peak load; (e) confining stresses and (f) axial stresses at the end of solution for Column CS17.

Fig 4 -- (a) geometry and (b) reinforcement arrangement for Column 2B.
Fig 5-- Comparison of stress-strain values from FE analysis with experimental results for Column 2B

Fig 6--Principal stress surfaces: (a) confining stresses and (b) axial stresses before cover spalling; (c) confining stresses and (d) axial stresses at the peak load
Confined Concrete in Concrete-Filled Steel Tubular Columns

by K. Sakino

Synopsis: It has been widely known that concrete filled steel tubular (CFT) columns have much higher strength and deformation capacities than common reinforced concrete (RC) columns because of beneficent interactive confinement effect between the filled concrete and the steel tube. The confinement effect by steel tube furthermore contributes to improving ductility of high-strength concrete, and enables application of the CFT columns in high-rise buildings located on seismic regions. This paper describes the axial and the flexural behaviors of CFT columns with circular and square sections based on many experimental researches conducted in Japan. The emphasis of this paper will be placed on the stress-strain curve models for concrete in CFT columns, which play the fundamental role in assessing both of the axial and the flexural behaviors of the CFT columns.

Keywords: concrete-filled steel tubular column; confined concrete; moment-curvature analysis; ultimate moment
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INTRODUCTION

Historical background of concrete filled steel tubular system in Japan

Concrete filled steel tubular (CFT) columns have been used for columns carrying large axial force since it was first used in Great Britain for the construction of road bridges in the late 1870's. In Japan, the first design recommendations for CFT columns were established in 1967 by the Architectural Institute of Japan (AIJ). Then it was revised in 1981. In these recommendations, CFT columns were considered as a kind of Steel Reinforced Concrete (SRC) columns because of their similarity in usage and mechanical characteristics. The recommendations made it possible to utilize CFT columns in practice. However, the limit of the width (diameter)-to-thickness ratio for structural steel tubes was not relaxed in comparison with the ratio for bare structural steel tubes. Also, the filled concrete strength was not increased in the estimation of actual strength even though it was confined by a steel tube and was under tri-axial stress conditions. Moreover, the CFT columns without concrete cover could not be practically used without a special approval. The Building Standard Law in Japan in force at that time said that covering concrete for a CFT column was an essential assumption to be considered to be an SRC column. Thus, the diffusion of CFT column constructions was quite slow. This situation has not changed even after the design method for CFT was included in the AIJ-SRC standards revised in 1987, in which the width (diameter) -to-thickness ratio was improved by further studies.

In the “New-Urban Housing Project” organized in 1985 by the Ministry of Construction, very broad research and development (R&D) were conducted. In practice, quantitative estimation of the strength and ductility of CFT beam-columns brought by the synergistic action of structural steel and filled concrete was studied and formulated into design formulas. The mixture of filling concrete and the casting methods were also investigated. The fire resistance of CFT columns was also extensively examined to study the possibility of the omission of covering concrete considering thermal capacity of filled concrete. However, the research results were monopolized only by the private companies involved in this R&D.

Based on that status of CFT column system, the committee for CFT structures was organized in AIJ to make the guidelines for engineers to design the CFT column system in rational manner based on many researches conducted by the year 1995, and published Recommendations for Design and Construction of Concrete Filled Steel Tubular Structures (referred to as AIJ Recommendations for CFT Structures, hereafter) in 1997. On the other hand, a five-year research program on Composite and Hybrid Structures (CHS) was started in fiscal year of 1993 as the fifth phase of the US-Japan Cooperative Earthquake Research Program. A research program on CFT system (referred to as US-Japan Research Program, hereafter) was carried out as a part of CHS
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project. The main objective of the research program was to conduct an experimental research on the behavior of CFT columns with much larger width (diameter)-to-thickness ratio and high strength of materials

**STRESS-STRAIN CURVE MODEL FOR CONCRETE CONFINED BY RECTILINEAR TRANSVERSE REINFORCEMENT OR SQUARE STEEL TUBE JACKET (ORIGINAL MODEL)**

Sakino and Sun (1994) have investigated the axial behavior of the concrete confined by rectilinear transverse reinforcement and proposed a stress-strain curve model for confined concrete based on the experimental data of RC columns tested by the author and other investigators. The test columns were confined by rectilinear hoops as well as square steel tubes, and tested under concentric axial load only. The author and Sun also conducted a comparative study of various stress-strain curve models available in literature for the concrete confined by rectilinear transverse reinforcement, and reached conclusion that the proposed model was able to predict the behavior of concentrically loaded columns better than the other models studied (Yoshioka, Sun and Sakino, 1994). The proposed model has two features: the first one is of a wide application from the viewpoint of material strengths, in other words, the model is applicable to columns with high strength concrete up to \( f'_c = 100 \text{N/mm}^2 \), and high strength bars with yield strength as high as \( V_{sy} = 700 \text{ N/mm}^2 \). This feature can be of a great advantage, since the high strength materials are being increasingly used in RC high-rise buildings as well as buildings with large spans. The second feature is of a wide application from the viewpoint of configurations of transverse reinforcement. The proposed model is applicable to many types of configurations of the transverse reinforcement as shown in Figure 1.

The proposed stress-strain model for confined concrete is given by Equation (1) (see Figure 2).

\[
Y = \frac{VX + (W-1)X^2}{1 + (V-2)X + WX^2}
\]

where \( X \) and \( Y \) are concrete stress \( (\sigma_c) \) and strain \( (\varepsilon_c) \) normalized by corresponding peak values \( \sigma_{ccB} \) and \( \varepsilon_{cco} \); \( V = E_c / E_{sec} \), \( E_c = (0.69 + 0.33\sqrt{f'_c}) \times 10^4 \) is the Young’s modulus of elasticity of concrete in \( \text{N/mm}^2 \); \( E_{sec}(=\sigma_{ccB}/\varepsilon_{cco}) \) is the secant modulus at the point of peak stress, and \( W \) is the parameter governing the slope of descending portion of the stress-strain model.

In order to predict the stress-strain curve, it is necessary to determine the values of three parameters in Equation (1): (1) the strength of confined concrete, \( \sigma_{ccB} \), (2) the strain corresponding to the maximum stress, \( \varepsilon_{cco} \), and (3) parameter \( W \). They are given as follows;
in which $\varepsilon_{co} (= 0.94 (f_c')^{1/4} \times 10^{-3})$ is the strain at maximum stress of unconfined (plain) concrete; $K (= \sigma_{cyc}/f_c')$ is the strength enhancement ratio of confined concrete; $f_c'$ is the compressive strength of concrete cylinder; $\rho_h$ is the volumetric ratio of rectilinear transverse reinforcement to the confined core measured center-to-center of perimeter transverse reinforcement; $\sigma_{yh}$ is the yield strength of transverse reinforcement with an upper limitation of 700 N/mm$^2$; $d''$ and $C$ are the nominal diameter and laterally unsupported length of transverse reinforcement (see Figure 1), respectively; $S$ is the spacing of transverse reinforcement; and $D_c$ is the distance between center of perimeter transverse reinforcement. Units of length and stress are mm and N/mm$^2$, respectively.

The proposed model given by Equation (1) can also be applied to define the stress-strain relationship for unconfined concrete by substituting $\rho_h = 0$ into Equation (2). The descending portion of the stress-strain curve given by Equation (1), however, cannot be used for unconfined high strength concrete with $f_c' > 80$ N/mm$^2$, since the parameter $D$ for such a high strength concrete takes zero or negative value, which results in discontinuity of the stress-strain curve near the peak point. Then a straight line connecting the peak point and point ($\varepsilon_{spall}$, 0) is assumed for the descending portion of stress-strain for unconfined high-strength concrete with $f_c' > 80$ N/mm$^2$. The value of $\varepsilon_{spall}$ is taken as 0.004 following Park et al’s assumption (Park and Kent, 1972).

Comparisons between the proposed model and experimental results of concentrically loaded columns tested by Adachi et al (1993) are shown in Figure 3, where modified Kent and Park model (Scott and Park et al., 1982) and other models for confined concrete proposed by Shiekh and Uzumeri (1982), Mander et al. (1988), and Muguruma and Watanabe (1983) are also shown for comparison. The test columns were large-size specimens of 1300 mm height, and 500mm×500mm cross section made of high-strength concrete with $f_c' = 77$ N/mm$^2$. High-strength transverse deformed bars with $\sigma_{yh} = 920$ N/mm$^2$ were used to confine the core concrete of these specimens. The experimental results are most appropriately predicted up to large strains by the model given by Equation (1) as shown in Figure 3. One of reasons for other models’ overestimating the stress-strain curve at large strains may be that, in these models, parameters governing the slope of stress-strain curve after peak point were determined from the experimental results of normal-strength concrete columns.
Ultimate strength of axially loaded CFT short columns

Circular columns—In the initial stages of loading of the circular CFT columns subjected to concentric axial load, Poisson’s ratio for the concrete is lower than that for steel, therefore a separation between steel tube wall and concrete core takes place provided that the adhesive bond between the steel and concrete does not work. As the load increases, the longitudinal strain reaches a certain critical strain, and the lateral deformation of the concrete catches up with that of the steel tube. When the load increases further, a tensile hoop stress is developed in the steel tube, and the concrete core is subjected to triaxial compression. This phenomenon results in an increase of axial compressive load capacity.

The equation for axial load capacity is obtained by the following procedure: First, the strength formula for concrete is assumed to be given by Equation (5).

\[ \sigma_{ccB} = \gamma_U \cdot f'_c + k \cdot \sigma_t \]  
(5)

where \( \sigma_{ccB} \) = strength of confined concrete; \( f'_c \) = compressive strength of 10cm by 20cm concrete cylinder; \( \gamma_U \) = strength reduction factor for concrete introduced to take scale effect into consideration \( = 1.67D_c^{-0.112} \) (Sakino and Nakahara et al., 2004); \( D_c \) = diameter of concrete core (in mm); \( k \) = confinement coefficient = 4.1 (Richart, Brandzaeg and Brown, 1929); \( \sigma_t \) = confining stress (lateral pressure).

The stresses induced in the steel tube at ultimate load are assumed to be given by Equation (6).

\[ \sigma_{s\theta} = \alpha_u \cdot \sigma_{sy}, \quad \sigma_{sz} = \beta_{uc} \cdot \sigma_{sy} \]  
(6)

where \( \alpha_u, \beta_{uc} \) = coefficients determined based on experimental results, assumed to be independent of material properties and dimensions of columns. Referring to Figure 4, the relation between the hoop stress \( \sigma_{s0} \) and the lateral pressure is given by Equation (7).

\[ \sigma_t = -\frac{2t}{D - 2t} \sigma_{s\theta} \]  
(7)

In the course of the evaluation of the confining effect on concrete strength, it is assumed that the difference between the ultimate strength \( N_u \) and the nominal squash load \( N_o \) given by Equation (8) is provided by the confining effect on concrete strength, and this gain depends upon the tube strength \( N_{so} \), and thus.

\[ N_o = N_{so} + N_{co} = A_s \sigma_{sy} + A_c \gamma_U \cdot f'_c \]  
(8)
\[ N_u - N_o = \lambda N_{so} \quad ; \quad \frac{N_u}{N_o} = 1.0 + \lambda \left( \frac{N_{so}}{N_o} \right) \]  \hspace{1cm} (9)

where \( A_s \) and \( A_c \) = cross sectional area of steel tube and filled concrete, respectively; \( N_{so} \) = axial yield strength of steel tube (=\( A_s \cdot \sigma_{sy} \)); and \( \lambda \) = augmentation factor determined based on experimental results. On the other hand, \( N_u \) is given by Equation (10)

\[ N_u = A_s \cdot \sigma_{sz} + A_c \cdot \sigma_{ccB} \]  \hspace{1cm} (10)

Substituting Equations (5), (6) and (7), and using Equation (8), Equation (11) is developed.

\[ N_u - N_o = A_s \cdot \beta_{uc} \cdot \sigma_{sy} + A_c \cdot (\gamma_U \cdot f_c' + k\sigma_T) - A_s \cdot \sigma_{sy} - A_c \cdot \gamma_U \cdot f_c' \]
\[ = A_s \cdot \sigma_{sy} (\beta_{uc} - 1) - A_c \cdot k \cdot \frac{2t}{D - 2t} \cdot \sigma_{so} \]
\[ = A_s \cdot \sigma_{sy} (\beta_{uc} - 1 - \frac{A_c}{A_s} \kappa \cdot \frac{2t}{D - 2t} \cdot \alpha_u) \]

\hspace{1cm} (11)

From Equation (9), the factor \( \lambda \) is given by Equation (12).

\[ \lambda = \frac{N_u - N_o}{N_{so}} = \beta_{uc} - 1 - \frac{(D - 2t)}{2(D - t)} k \cdot \alpha_u \]  \hspace{1cm} (12)

Equation (12) shows that the value of \( \lambda \) becomes constant for the steel tube with the same D/t ratio if the values of coefficients \( k \), \( \alpha_u \), and \( \beta_{uc} \) are constant. A constant value of \( \lambda \) defines the normalized axial compressive load capacity as a linear function of the parameter \( N_{so}/N_o \) as given by Equation (9). The value of \( \lambda \) can be determined by a regression analysis based on available experimental data as described later. The relation between stress coefficients \( \alpha_u \) and \( \beta_{uc} \) is obtained from the assumption that steel stresses at the ultimate state given by Equation (6) satisfies the von Mises yield criterion given by Equation (13).

\[ \sigma_{sy}^2 = \sigma_{so}^2 \cdot \sigma_{sz} + \sigma_{sz}^2 = \sigma_{sy}^2 \]  \hspace{1cm} (13)

where \( \sigma_{so} \) = hoop stress of steel tube in yield condition; \( \sigma_{sz} \) = axial stress of steel tube in yield condition; and thus

\[ \alpha_u^2 - \alpha_u \beta_{uc} + \beta_{uc}^2 = 1.0 \]  \hspace{1cm} (14)

Once the value of \( \lambda \) is fixed, the values of \( \alpha_u \) and \( \beta_{uc} \) are determined by solving Equations (12) and (14), where \( k=4.1 \) as described before, and D/t=50 as a representative
value to avoid the dependency of $\alpha_u$ and $\beta_{uc}$ on the $D/t$ ratio.

Figure 5 shows the relationships between experimental axial load capacity $N_{\text{exp}}$ of CFT columns and yield load of the steel tube $N_{so}$, both divided by nominal squash load $N_0$. The open circles show the existing experimental results obtained elsewhere in Japan. The design formula in AIJ Recommendations for CFT Structures was proposed based on the open circle data, where the slope of the dotted line defines $\lambda$ in Equation (9), and the value of $\lambda$ was determined as 0.27. The solid circles show the results of the data from US-Japan Research Program. The slope of the solid line based on the US-Japan data is slightly lower than that of dotted line. However, it is not necessary to revise the existing design formula. The value of $\lambda$ equal to 0.27 gives values of coefficients $\alpha_u$ and $\beta_{uc}$ as $-0.19$ and $0.89$, respectively, from Equations (12) and (14). Figure 6 shows comparisons between experimental results on the axial load capacity of circular CFT stub columns tested in the US-Japan Research Program and calculated capacities using Equation (15) which is obtained from the above procedure.

$$N_u = N_0 + \lambda \cdot N_{so} = N_0 + 0.27N_{so} \quad (15)$$

Square Columns --In the case of square columns, it is necessary to take into consideration a capacity reduction due to local buckling of the steel tube wall of the column with large $D/t$ ratio rather than the confinement effect of the steel tube. Figure 7 shows relationships between the axial load capacity factor of the steel tube $S$ and the normalized width-to-thickness ratio $(B/t)\sqrt{\sigma_{sy}/E_s}$, where $S$ denotes the ultimate compressive strength divided by the yield axial strength of steel tube. The axial load capacity factor of the hollow steel tube stub columns and the steel tube in CFT stub columns shown in Figure 7 are given by Equations (16), (17) and (18).

$$N_{su} = A_s \cdot \sigma_{scr}, \quad \sigma_{scr} = \min(\sigma_{sy}, S \cdot \sigma_{sy}) \quad (16)$$

$$\frac{1}{S} = 0.698 + 0.128 \left( \frac{B}{t} \right)^2 \frac{\sigma_{sy}}{E_s} \quad (17)$$

for hollow steel tube stub columns

$$\frac{1}{S} = 0.698 + 0.128 \left( \frac{B}{t} \right)^2 \frac{\sigma_{sy}}{E_s} \times \frac{4.00}{6.97} \quad (18)$$

for steel tube in CFT stub columns.

Equation (17) was obtained by a regression analysis using the experimental results of the hollow steel tube stub columns tested in the US-Japan Research Program, and modified into Equation (18) by multiplying $4.0/6.97$. This modification is based on an elastic buckling theory by considering the difference in boundary conditions or buckling modes between the hollow steel tube (simply-supported plate) and steel tube in CFT columns (clamped plate) shown in Figure 7.

The axial load capacity of CFT columns can be estimated by Equation (19).
\[ N_u = N_{su} + N_{co} = A_s \cdot \sigma_{scr} + A_c \cdot \gamma_u \cdot f'_c \]  

(19)

Figure 8 shows the comparisons between experimental results on axial load capacity of the square CFT stub columns tested in the US-Japan Research Program and calculated capacities obtained by Equation (19) which gives a slightly more conservative value for columns with small B/t ratio. The reason for this is considered to be a strain hardening effect of steel tubes rather than the confinement effect.

**Stress-strain models for filled concrete**

To predict the load-deformation relationships of centrally-loaded CFT columns, a stress-strain model for confined concrete is necessary. A method to apply the stress-strain curve model given by Equation (1) to the confined concrete in CFT columns with circular or square section is discussed in this section. As given by Equation (1) through Equation (4), the model is actually a two-parameter model for which the stress-strain curve can be determined if the strengths of unconfined (plain) and confined concrete are given. In the case of circular CFT columns, the strength enhancement factor \( K \) defined as \( \frac{V_{ccB}}{V_{cp}} \) (\( V_{cp} \) is the strength of unconfined concrete, and assumed to be \( \gamma_u \cdot f'_c \)) is given by Equation (20) which is obtained from Equations (5) and (7).

\[
K = 1.0 - k \frac{2t}{(D - 2t)} \frac{\sigma_{s0}}{\sigma_{cp}}
\]

(20)

Once the values of \( k \) and \( \sigma_{s0} \) are fixed as 4.1 and \(-0.19\sigma_{sy}\) as described before, Equation (20) can be transformed into Equation (21) where \( D/t=50 \) as a representative.

\[
K = 1.0 + 0.032(\sigma_{sy} / \sigma_{cp})
\]

(21)

In the case of square CFT columns, the value of \( K \) factor should be 1.0 from the view point of axial load capacity of centrally-loaded short columns; in other words the confinement effect on ultimate axial load cannot be expected as described before. It is expected, however, that the axial deformation capacity of filled concrete after reaching ultimate axial load can be improved by the confinement effect of square steel tubes as observed in many experimental results. The slope of the falling branch of the stress-strain curve for the confined concrete is governed by the constant \( W \) given by Equation (4), which is a function of \( \sigma_{cp} \) and effective lateral pressure index \( \sigma_{re} \). In the case of the concrete confined by the square steel tube acting only as the transverse reinforcement (referred to as the steel jacket), \( \sigma_{re} \) is given by Equation (22) instead of \( \sigma_{re} \) defined in Equation (2) (Sakino and Sun, 1994).

\[
\sigma_{re} = \frac{2t^2 (B - t)}{(B - 2t)^3} \sigma_{sy}
\]

(22)

In this paper, it is assumed that the value of \( W \) for the square steel tube in CFT column is
equal to that for the square steel jacket. Table 1 gives all the information to obtain stress-strain curves for concrete confined by square steel jackets (original model) and steel tubes in circular and square CFT columns. Figure 9 shows these stress-strain curve models along with the curve for unconfined concrete. The unconfined concrete strength and the yield strength of circular (D/t=60) and square (B/t=60) steel tubes shown in Figure 9 are 20MPa and 300MPa, respectively.

**Stress-strain models for steel tube**

For the circular CFT columns, the stress-strain relationship of the steel tube is developed as elastic-perfectly plastic relation model as shown in Figure 10. The maximum stress of the steel tube is 0.89 \( \sigma_{sy} \) as described before. In the case of square CFT columns, the stress-strain models for steel tubes are proposed as shown in Figure 11. The three types of multi-linear model are described in the figure, where Type-1 is the model for steel tubes with small B/t ratio and the maximum stress is expected to be larger than the yield stress due to the strain hardening effect. The maximum stress of steel tubes with large B/t ratio (Type-3) does not reach the yield stress due to local buckling. In the case of the steel tube with the medium B/t ratio (Type-2), the maximum stress of the steel tube is defined as the yield stress. The classification for modeling is according to the value of the generalized B/t ratio, as shown in Figure 7. The specific values of \( \sigma_{SB}, \epsilon_{SB}, \epsilon_{SE}, \sigma_{ST}, \) and \( \epsilon_{ST} \), are calculated by using the equations summarized in Table 2 for each type. In the table, \( \sigma_{SB} \) and \( \epsilon_{SB} \) are the stress and strain at the local buckling, respectively, and \( \epsilon_{SE} \) is strain at elastic limit. In the case of Type-1 and Type-2, \( \epsilon_{SE} \) is the same as \( \epsilon_{sy} \), and \( \sigma_{ST} \) and \( \epsilon_{ST} \) are the stress and strain at the terminal point of the falling branch.

**Comparisons of the experiment and analytical models**

The proposed analytical load-deformation curves are compared with the six experimental results for each shape of CFT columns in Figures 12 and 13. The properties of test specimens of circular and square columns are given in Table 3. These figures show the relations between experimental or calculated axial load of CFT columns divided by the nominal squash load \( N_o \) and longitudinal strain. The thick solid lines show experimental results and the thin solid lines show the analytical curves. The dashed line and chained line show calculated loads of the filled concrete and steel tube, respectively. In each figure, specimens with different D/t or B/t ratio are shown for comparisons. The agreement observed between the predicted and experimental behavior is satisfactory for these columns.

**FLEXURAL BEHAVIOR OF CFT COLUMNS**

**Moment-curvature analysis**

Theoretical moment-curvature relationships for CFT column sections under combined flexural and axial load can be derived on the following assumptions: (1) plain sections remain plain after bending, (2) the stress-strain curves for concrete in circular and square tube are given by Curve 1 and Curve 2 shown in Figure 9, respectively, (3) the stress-strain curves for steel tubes in circular and square sections are given by Figures 14 and 15, respectively. It is noteworthy that strain gradient effect on the
stress-strain curves is ignored as described in second assumption. It is also noteworthy that the effect of hoop stress on the tensile yield strength of the steel tube in tension is considered for both of the circular and square sections as shown in Figures 14 and 15. The moment-curvature relationships for a given column can be obtained by the laminas method in which the section is divided into a number of discrete laminas.

Comparisons between theoretical and experimental moment-curvature relationships of the columns subjected to uniform bending moment under constant axial load (see Figure 16) are shown in Figures 17 and 18, for circular and square sections, respectively. The properties of test specimens of circular and square columns are given in Table 4. It is seen that reasonable agreements exist between the theoretical and experimental moment-curvature relationships.

**Ultimate moment of CFT columns**

Circular columns—The AIJ Recommendations for CFT Structures prescribes that the ultimate moment of circular CFT section can be estimated as a full plastic moment calculated by using stress blocks for concrete and steel tube shown in Figure 19. It is noteworthy that a magnitude of the hoop stress induced in steel tube at ultimate state is assumed to be \(-0.19\sigma_y\) which is the same value as assumed in obtaining the design formula for axial load capacity of CFT short columns. The hoop stress of \(-0.19\sigma_y\) gives the magnitudes of tensile and compressive stresses for stress block of steel tube as \(-1.08\sigma_y\) and \(0.89\sigma_y\), respectively, as shown in Figure 19, and also gives confining stress which results in the maximum stress of confined concrete, \(\sigma_{ccB}\), given by Equation (20). The stress in concrete at ultimate state is assumed to be \(\sigma_{ccB}\) in magnitude, and to uniformly distributed as shown in Figure 19.

When the circular CFT columns using high strength concrete is subjected to bending moment under high axial load, the CFT columns behave in rather brittle manner as shown in Figure 17. Then, it is necessary to introduce an ultimate strain and shape factors for concrete stress block as the case of ordinary reinforced concrete columns. The detailed information on the concrete stress block for CFT columns shown in Figure 20, which is obtained based on an analytical study using a stress-strain model for confined concrete shown in Figure 9, is given in Reference (Sun and Sakino, 1998). Figure 21 shows comparisons between experimental results on ultimate moment of the circular CFT columns tested in US-Japan Research Program and calculated ones obtained by using stress blocks for steel and concrete shown in Figures 19 and 20, respectively.

Square columns—The AIJ Recommendations for CFT Structures prescribes that the ultimate moment of square CFT section can be estimated as a full plastic moment calculated by using stress blocks for concrete and steel tube. The magnitude of stress blocks of concrete and steel are recommended to be \(\rho_0 \cdot f'_c\) and \(\sigma_{sy}\), respectively. In order to widen application limits of AIJ Recommendations for CFT Structures, the stress block for high-strength concrete should be taken as one shown in Figure 22 for which the values of \(k_1\) and \(k_2\) are given by Equation (23).
\[
k_1 = 0.831 - 0.076(\gamma_U \cdot f_c' / 41.2) \quad \text{(in N/mm}^2) \]
\[
k_2 = 0.429 - 0.010(\gamma_U \cdot f_c' / 41.2) \quad \text{(in N/mm}^2) \]

The magnitude of stress for steel tube with large \(D/t\) ratio, the reduced compressive stress, \(\sigma_{scr}\), given by Equation (16) and (18) is introduced to take an effect of local buckling of steel tube walls into consideration to the steel stress block as shown in Figure 22. Figure 23 shows comparisons between experimental results on ultimate moment of the square CFT columns tested in US-Japan Research Program and calculated ones obtained by using stress blocks for concrete and steel tube shown in Figure 22. This method, however, does not estimate the experimental results very well, and gives overestimation of the ultimate moment to many specimens as shown in Figure 23. The main reason for this is attributed to an elastic region of steel tube web.

**CONCLUDING REMARKS**

The mechanical behavior of CFT columns with circular or square section subjected to concentric axial load or to monotonic bending moment under constant axial load is discussed in this paper on a basis of synergistic action of steel tube and filled concrete. The confinement effect in CFT columns had been ignored for a long time in Japanese Standards as well as ACI Code until Recommendations for Design and Construction of Concrete Filled Steel Tubular Structures published by AIJ in 1977, which has limitations of application concerning material strengths and thickness of steel tube wall though. The emphasis of this paper is placed on the behavior of CFT columns with high strength materials and thin steel tube wall. It is concluded that the mechanical behavior of CFT columns can be predicted by the stress-strain curve models for steel and concrete described in this paper. It is also shown that the axial load capacity of CFT short columns and the ultimate moment of CFT column sections can be predicted by the simplified methods considering synergistic action between steel tube and filled concrete.

**ACKNOWLEDGEMENTS**

The author started an investigation on the stress-strain curve for concrete confined by circular steel tube jacket in 1984 with a graduate student Yan Xiao, who is a professor at the University of Southern California and one of the organizers of ISCC-2004, under a supervision of Professor Tomii at Kyushu University. This research has been further extended as reported in this paper by author and many collaborators. The stress-strain curve model for concrete confined by a square steel tube jacket or rectilinear transverse reinforcement was proposed by Dr. Sun, Professor at Kyushu University, and the author. Dr. Nakahara, Research Associate at Kagoshima University, made a great contribution to the investigation on the mechanical behavior of CFT columns. He also helped the author to complete this paper. Finally, the research described in this paper could not have been completed without the resources of the laboratory and staff of the Department of Architecture at Kyushu University.
REFERENCES


Richart, F. E. et al. (1929), “The Failure of Plain and Spirally Reinforced Concrete in Compression,” Bulletin No. 190, Univ. Illinois, Engineering Experimental Station, Urbana, Ill.


Table 1-- Specific values for stress-strain curve models for concrete

<table>
<thead>
<tr>
<th></th>
<th>Original</th>
<th>Circular CFT</th>
<th>Square CFT</th>
</tr>
</thead>
<tbody>
<tr>
<td>( X = )</td>
<td>( \varepsilon_c )</td>
<td>( \varepsilon_c )</td>
<td>( \varepsilon_c )</td>
</tr>
<tr>
<td>( Y = )</td>
<td>( \varepsilon_{ce0} )</td>
<td>( \varepsilon_{ce0} )</td>
<td>( \varepsilon_{ce0} )</td>
</tr>
<tr>
<td>( Z = )</td>
<td>( \sigma_c )</td>
<td>( \sigma_c )</td>
<td>( \sigma_c )</td>
</tr>
<tr>
<td>( \nu = )</td>
<td>( E_c \cdot \varepsilon_{ce0} )</td>
<td>( E_c \cdot \varepsilon_{ce0} )</td>
<td>( E_c \cdot \varepsilon_{ce0} )</td>
</tr>
<tr>
<td>( \sigma_{re} = )</td>
<td>( \frac{2t^2(B-t)\sigma_{sy}}{(B-2t)^3} )</td>
<td>( \frac{k e \sigma_r}{(B-2t)^3} )</td>
<td>( \frac{2t^2(B-t)\sigma_{sy}}{(B-2t)^3} )</td>
</tr>
</tbody>
</table>

\[
\sigma_{ce0} = \frac{1}{k_c} \frac{\sigma_{re}}{\sigma_{cp}} = 1 + k_c \frac{\sigma_r}{\sigma_p} \quad k_c = 4.1,
\]

\[
W = 1.50-171 \times 10^{-3} \sigma_{cp} + 2.39 \sqrt{\sigma_{re}}
\]

\[
E_c = (6.90 + 3.32 \sqrt{\sigma_{cp}}) \times 10^3
\]

\[
\varepsilon_{ce0} = \begin{cases} 1.0 + 4.7(K - 1) & K \leq 1.5 \\ 3.4 + 20(K - 1.5) & K > 1.5 \end{cases}
\]

\[
\varepsilon_c = 0.94(\sigma_{cp})^{1/3} \times 10^{-3}
\]

\[
k = 4.1, \quad k_c = 23, \quad \sigma_u = -0.19, \quad \sigma_r = \frac{2td_u \cdot \sigma_{sy}}{D - 2t}
\]

Table 2-- Specific values for stress-strain models for steel tubes in square CFT columns

<table>
<thead>
<tr>
<th>Type-1</th>
<th>Type-2</th>
<th>Type-3</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \sqrt{\alpha_s} \leq 1.54 )</td>
<td>1.54 &lt; ( \sqrt{\alpha_s} &lt; 2.03 )</td>
<td>2.03 &lt; ( \sqrt{\alpha_s} )</td>
</tr>
<tr>
<td>( \sigma_{sb} = \sigma_{sy} \cdot S_{eq.13} )</td>
<td>( \sigma_{sy} )</td>
<td>( \sigma_{sy} \cdot S_{eq.14} )</td>
</tr>
<tr>
<td>( \varepsilon_{sb} = \left( 6.06 - \frac{1}{\alpha_s} - 0.801 \frac{1}{\alpha_s} + 1.10 \right) \cdot \varepsilon_{sy} )</td>
<td>( \varepsilon_{sy} )</td>
<td>( \sigma_{sb} / E_s )</td>
</tr>
<tr>
<td>( \varepsilon_{se} = \varepsilon_{sy} )</td>
<td>( \varepsilon_{sy} )</td>
<td>( \sigma_{sb} / E_s )</td>
</tr>
</tbody>
</table>

\[
(\varepsilon_{cf} - \varepsilon_{sb}) = 3.59, \quad \sigma_{sy} = 1.19 - 0.207 \sqrt{\alpha_s}
\]
Table 3—Properties of specimens of short CFT columns subjected to concentric loading

(a) Circular CFT columns

<table>
<thead>
<tr>
<th>Specimen</th>
<th>D(mm)</th>
<th>t(mm)</th>
<th>(\sigma_{sy}(\text{MPa}))</th>
<th>(f'(\text{MPa}))</th>
<th>(D/t)</th>
<th>(\gamma_0)</th>
<th>(N_{exp}(\text{kN}))</th>
<th>(N_{exp}/N_0)</th>
</tr>
</thead>
<tbody>
<tr>
<td>CC4-A-4-1</td>
<td>149</td>
<td>2.96</td>
<td>308</td>
<td>40.5</td>
<td>50.3</td>
<td>0.075</td>
<td>0.96</td>
<td>1,064</td>
</tr>
<tr>
<td>CC4-C-4-1</td>
<td>300</td>
<td>2.96</td>
<td>279</td>
<td>41.1</td>
<td>101.4</td>
<td>0.137</td>
<td>0.88</td>
<td>3,277</td>
</tr>
<tr>
<td>CC4-D-4-1</td>
<td>450</td>
<td>2.96</td>
<td>279</td>
<td>41.1</td>
<td>152.0</td>
<td>0.206</td>
<td>0.84</td>
<td>6,870</td>
</tr>
<tr>
<td>CC8-A-8</td>
<td>108</td>
<td>6.47</td>
<td>853</td>
<td>77.0</td>
<td>16.7</td>
<td>0.069</td>
<td>1.00</td>
<td>2,713</td>
</tr>
<tr>
<td>CC8-C-8</td>
<td>222</td>
<td>6.47</td>
<td>843</td>
<td>77.0</td>
<td>34.4</td>
<td>0.141</td>
<td>0.92</td>
<td>7,304</td>
</tr>
<tr>
<td>CC8-D-8</td>
<td>337</td>
<td>6.47</td>
<td>823</td>
<td>85.1</td>
<td>52.0</td>
<td>0.208</td>
<td>0.87</td>
<td>13,776</td>
</tr>
</tbody>
</table>

(b) Square CFT columns

<table>
<thead>
<tr>
<th>Specimen</th>
<th>B(mm)</th>
<th>t(mm)</th>
<th>(\sigma_{sy}(\text{MPa}))</th>
<th>(f'(\text{MPa}))</th>
<th>(B/t)</th>
<th>(\sqrt{\alpha_a})</th>
<th>(\gamma_0)</th>
<th>(N_{exp}(\text{kN}))</th>
<th>(N_{exp}/N_0)</th>
</tr>
</thead>
<tbody>
<tr>
<td>CR4-A-4-1</td>
<td>148</td>
<td>4.38</td>
<td>262</td>
<td>40.5</td>
<td>33.8</td>
<td>1.21</td>
<td>0.95</td>
<td>1,414</td>
<td>1.03</td>
</tr>
<tr>
<td>CR4-C-4-1</td>
<td>215</td>
<td>4.38</td>
<td>262</td>
<td>41.1</td>
<td>49.1</td>
<td>1.75</td>
<td>0.91</td>
<td>2,424</td>
<td>0.95</td>
</tr>
<tr>
<td>CR4-D-4-1</td>
<td>323</td>
<td>4.38</td>
<td>262</td>
<td>41.1</td>
<td>73.7</td>
<td>2.63</td>
<td>0.85</td>
<td>4,950</td>
<td>1.04</td>
</tr>
<tr>
<td>CR8-A-8</td>
<td>119</td>
<td>6.47</td>
<td>835</td>
<td>77.0</td>
<td>18.4</td>
<td>1.17</td>
<td>0.97</td>
<td>3,318</td>
<td>1.00</td>
</tr>
<tr>
<td>CR8-C-8</td>
<td>175</td>
<td>6.47</td>
<td>835</td>
<td>77.0</td>
<td>27.0</td>
<td>1.72</td>
<td>0.94</td>
<td>5,366</td>
<td>1.00</td>
</tr>
<tr>
<td>CR8-D-8</td>
<td>265</td>
<td>6.47</td>
<td>835</td>
<td>80.3</td>
<td>40.9</td>
<td>2.61</td>
<td>0.88</td>
<td>8,990</td>
<td>0.91</td>
</tr>
</tbody>
</table>

Table 4—Properties of specimens of CFT columns subjected to bending moment under constant axial load

(a) Circular CFT columns

<table>
<thead>
<tr>
<th>Specimen</th>
<th>D(mm)</th>
<th>Dlt</th>
<th>(\sigma_{sy}(\text{MPa}))</th>
<th>(f'(\text{MPa}))</th>
<th>(\gamma_0)</th>
<th>(N(kN))</th>
<th>(M_{exp}(\text{kNm}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>EC4C8045</td>
<td>300</td>
<td>101</td>
<td>77.6</td>
<td>0.88</td>
<td>279</td>
<td>2720</td>
<td>194.0</td>
</tr>
<tr>
<td>EC4C8060</td>
<td>300</td>
<td>101</td>
<td>77.6</td>
<td>0.88</td>
<td>279</td>
<td>3630</td>
<td>160.8</td>
</tr>
<tr>
<td>EC4C4040</td>
<td>450</td>
<td>152</td>
<td>39.9</td>
<td>0.84</td>
<td>279</td>
<td>2970</td>
<td>409.8</td>
</tr>
<tr>
<td>EC4C4060</td>
<td>450</td>
<td>152</td>
<td>39.9</td>
<td>0.84</td>
<td>279</td>
<td>4450</td>
<td>346.9</td>
</tr>
<tr>
<td>EC6C8030</td>
<td>239</td>
<td>52.6</td>
<td>77.6</td>
<td>0.91</td>
<td>507</td>
<td>1560</td>
<td>217.4</td>
</tr>
<tr>
<td>EC6C8060</td>
<td>239</td>
<td>52.6</td>
<td>77.6</td>
<td>0.91</td>
<td>507</td>
<td>3110</td>
<td>177.8</td>
</tr>
<tr>
<td>EC6C4030</td>
<td>360</td>
<td>79.3</td>
<td>39.9</td>
<td>0.87</td>
<td>525</td>
<td>2050</td>
<td>461.0</td>
</tr>
<tr>
<td>EC6C4060</td>
<td>360</td>
<td>79.3</td>
<td>39.9</td>
<td>0.87</td>
<td>525</td>
<td>4100</td>
<td>396.4</td>
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</table>

(b) Japanese CFT columns

<table>
<thead>
<tr>
<th>Specimen</th>
<th>B(mm)</th>
<th>t(mm)</th>
<th>(\sqrt{\alpha_a})</th>
<th>(\sigma_{sy}(\text{MPa}))</th>
<th>(\sigma_{cm}(\text{MPa}))</th>
<th>(N(kN))</th>
<th>(N_{exp}(\text{kN}))</th>
<th>(N_{exp}/N_0)</th>
<th>(M_{exp}(\text{kNm}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>BRA4-6-5-02</td>
<td>200</td>
<td>5.93</td>
<td>33.7</td>
<td>1.57</td>
<td>320</td>
<td>47.6</td>
<td>570</td>
<td>3158</td>
<td>0.180</td>
</tr>
<tr>
<td>BRA4-6-5-04</td>
<td>200</td>
<td>5.93</td>
<td>33.7</td>
<td>1.53</td>
<td>320</td>
<td>47.6</td>
<td>1140</td>
<td>3158</td>
<td>0.361</td>
</tr>
<tr>
<td>BRA4-4-5-02</td>
<td>200</td>
<td>4.25</td>
<td>47.1</td>
<td>1.51</td>
<td>211</td>
<td>47.6</td>
<td>426</td>
<td>2448</td>
<td>0.174</td>
</tr>
<tr>
<td>BRA4-4-5-04</td>
<td>200</td>
<td>4.25</td>
<td>47.1</td>
<td>1.51</td>
<td>211</td>
<td>47.6</td>
<td>851</td>
<td>2448</td>
<td>0.348</td>
</tr>
<tr>
<td>BRA4-2-5-02</td>
<td>200</td>
<td>2.04</td>
<td>98.0</td>
<td>3.44</td>
<td>253</td>
<td>47.6</td>
<td>380</td>
<td>2236</td>
<td>0.170</td>
</tr>
<tr>
<td>BRA4-2-5-04</td>
<td>200</td>
<td>2.04</td>
<td>98.0</td>
<td>3.44</td>
<td>253</td>
<td>47.6</td>
<td>761</td>
<td>2236</td>
<td>0.340</td>
</tr>
</tbody>
</table>

Figure 1 – Typical configuration of transverse reinforcement
Figure 2 – Stress-strain curve for concrete

Figure 3 – Comparison between experimental and theoretical stress-strain curves of confined concrete

Figure 4 – Calculation of lateral pressure

Figure 5 – Experimental axial compressive load capacities of circular CFT columns
Figure 6 – Comparisons between experimental and calculated results (circular CFT columns)

Figure 7 – Axial compressive load capacities of hollow square steel tubes

Figure 8—Comparisons between experimental and calculated results (square CFT columns)
Figure 9 – Stress-strain models for concrete in CFT columns

Figure 10 – Stress-strain model for steel tubes in circular CFT columns

Figure 11 – Stress-strain models for steel tubes in square CFT columns
Figure 12 – Comparisons between experimental results and proposed models (circular CFT columns)

Figure 13 – Comparisons between experimental results and proposed models (square CFT columns)
Figure 14 – Stress-strain models for circular steel tube

Figure 15 – Stress-strain models for square steel

Figure 16 – Loading condition
Figure 17 – Comparisons between experimental and theoretical moment-curvature relationships of circular CFT columns
Figure 18 – Comparisons between experimental and theoretical moment-curvature relationships of square CFT columns

Figure 19 – Stress blocks for concrete and steel in circular CFT section

\[ \sigma_{cb} = \gamma_b / \sigma_b + 4.1 \cdot \sigma_c / \sigma_b + 4.1 / D_c \]
\[ \beta_1 = 0.89, \quad \beta_2 = 1.08, \quad \alpha_c = 0.19 \]

Figure 20 – Stress block for high-strength concrete in circular CFT section
Figure 21 – Comparisons between experimental ultimate moments and calculated ones for circular CFT columns

Figure 22 – Stress blocks for concrete and steel in square CFT section

Figure 23 – Comparisons between experimental ultimate moments and calculated ones for square CFT columns
Hysteretic Behavior of Concrete-Filled Steel Tubular Columns under Uniform Bending

by H. Nakahara and K. Sakino

Synopsis: In order to evaluate the load carrying capacity and ductility of hinging zone of the concrete filled steel tubular (CFT) columns, tests are carried out on 12 circular specimens and 18 square specimens subjected to uniform bending under a constant axial load. The experimental parameters are: 1) depth to thickness ratio ($D/t$ ratio) and width to thickness ratio ($B/t$ ratio) of steel tube; 2) axial load ratio; 3) material strength; 4) deformation history and 5) annealing. One of the features of the test is the wide range of $D/t$ and $B/t$ ratio. The range of $D/t$ ratio of the circular CFT columns are 40.5-160 and the range of $B/t$ ratio of the square CFT columns are 32.8-98.0, respectively. The experimental load-deformation relations are compared with those of the elasto-plastic analysis based on the proposed stress-strain relationships established for the filled concrete and for the steel tube. The analytical results show good agreement with the test results for all specimens. This implies that the proposed stress-strain relationships for CFT columns are useful to predict the characteristics of the filled concrete and the steel tube.

Keywords: confined concrete; cyclic loading; stress-strain relations
INTRODUCTION

In seismic design against large earthquakes, structural members are allowed to perform inelastic behavior in order to dissipate large energy. The ductile members are required to dissipate the input energy efficiently. It is well known that the concrete steel tubular (CFT) columns possess the stable inelastic behaviors according to extensive amount of experimental works. Therefore, the structural system with CFT columns is useful for moment resisting frame and used widely in Japan. To control and predict the post-elastic behavior of CFT column under seismic excitation, it is important that the cyclic load-deformation relationships of the columns are investigated.

Experimental and analytical studies are carried out on 12 circular specimens and 18 square specimens to investigate the cyclic bending behavior of CFT columns. Most of the beam-column tests in Japan are conducted in the manner that specimens are subjected to shearing force under a constant axial load as shown in Figure 1 (a). To estimate the flexural behavior from this double curvature column, however, there are following uncertain factors.

1) The effect of shearing force is not clear.
2) The extra confinement caused by the rigid loading beam is not clear.

From the viewpoint of the basic study of CFT columns, the above uncertain effects should be removed to clarify the moment-curvature characteristics. Therefore, all tests are conducted under uniform bending moment and a constant axial load as shown in Figure 1 (b). In this paper, we deal with the details and results of the test and investigate the flexural behavior of CFT columns based on the elasto-plastic analysis.

EXPERIMENT AND SPECIMEN

The outline of the experiments of CFT columns is shown in Table 1. The experimental program is divided by four series where Series 1 and 2 are for circular CFT columns and Series 3 and 4 are for square CFT columns. Important test parameters are depth (width) to wall-thickness ratio and axial load ratio. The two types of deformation histories are selected to conduct the cyclic tests as shown in Figure 2. For the loading pattern of Type 1, the amplitude of $\phi D$ increases stepwise by 0.05 rad, where the $\phi$ denotes the curvature of the specimen and $D$ denotes the depth of the specimen, after three successive cycles up to 0.2. Type 2 is one cycle loading and intended to include the behavior of
pushover test. The peak of $\phi D$ of Type 2 is 0.35 which is limited by the stroke of the hydraulic jacks of the loading apparatus shown in Figure 3. The annealing is also selected as one of the test parameters.

The test specimens are shown in Table 2. The specimens are named by the designations shown in Figure 4. The height $L$ of all specimen is three times of its depth $D$ or width $B$. The depth of the specimens in Series 1 is 165 mm. The depth and width of specimens in the other series are 200 mm. The notation $t$ shows wall-thickness of steel tube. In the table, $\sigma_y$ is the yield stress of steel tube and $\sigma_p$ is the compressive strength of concrete. The value of $\sigma_y$ is taken in standard tensile test coupons known as type A-1 in Japan. The value of $\sigma_p$ is taken by testing cylinders of which diameter are 100 mm and height are 200 mm. The steel tubes with normal strengths which are from 211 to 412 MPa are used. The concrete of the tests of Series 1 and 3 is normal compressive strength of 34 or 48 MPa. For the other series of the tests, the high strength of concrete of 69.1 MPa is used. In the table, notations $N$ and $N_0$ stand for the applied axial force and nominal squash load, respectively. The axial load ratio is shown as $N/N_0$.

ANALYTICAL MODEL

To compare with the experimental results, moment-curvature relationships are obtained by the numerical analysis using the proposed stress-strain relationships. The method of the analysis is based on the iterating calculation to find the distribution of strain satisfying the equilibrium condition of forces at the critical section. The axial force and bending moment are obtained by integrating the product of the stress and the area of each fiber. The following assumptions are used to generate the moment-curvature curve of layered section of the CFT columns;

1) the section remains in the same shape,
2) a linear strain distribution is assumed,
3) tensile stress of concrete is neglected,
4) an envelop curve of stress-strain shown in Figure 5 is used for concrete fiber,
5) an envelop curve of stress-strain shown in Figure 6 is used for steel tube fiber,
6) a hysteretic stress-strain model shown in Figure 7 is used for concrete fiber,
7) a hysteretic stress-strain model shown in Figure 8 is used for steel tube fiber.

In Figure 5, the three solid curves show the compressive stress-strain relationships in CFT columns or steel jacketed column with 300 MPa of $\sigma_y$ and 20 MPa of $\sigma_p$ where $\sigma_p$ is the compressive strength of plain concrete. Curve-1 is the confined concrete in circular CFT column with $D/t$ ratio of 60 and the Curve-2 shows the concrete in square CFT column with $B/t$ ratio of 60. In order to identify the characteristic of concrete in CFT, the concrete confined by steel jacket with $B/t$ ratio of 60 is shown as Curve-3. And Curve-4 expressed by break line is the behavior of unconfined concrete. The difference between Curve-2 and Curve-3 is only the existence of axial stress in steel tube. The confining effect from the steel tube wall of square CFT decreases by the axial stress in steel tube. The peak value of stress of concrete in square CFT is less than that of concrete in steel jacket and is assumed to the maximum stress of plain concrete.

All curves in Figure 5 are obtained by the simple equation as follows;
where all variables are summarized in Table 3. Curve-3 and 4 are developed for normal reinforced concrete by Sakino and Sun (Sakino and Sun 1994). The stress-strain relationships for concrete in CFT columns are modeled by authors (Sakino et al. 2004) based on the test results of the fifth phase of U.S.-Japan Cooperative Earthquake Research Program.

The circumferential stress of the circular steel tube of CFT is tensile and assumed to be 0.19 $\sigma_y$. Therefore, strength enhance ratio $K$ is calculated as the value of 1.4 by using the sample specimen shown in Figure 5 and equation shown in Table 3. On the other hand, $K$ for the concrete in square CFT is assumed 1. The confining effect by steel tube of square CFT is neglected for strength enhancement but contributes to improve the deformation capacity as shown in Figure 5.

On the above assumption of the circumferential stress of circular CFT, the axial stress-strain relation shown in Figure 6 (a) is obtained for the steel tube of circular CFT by using the two dimensional yield criterion by von Mises. The stress-stain relations shown in Figure 6 (b) are already proposed by the authors (Nakahara, Sakino, and Inal 1998) for the locally-buckling steel tube of the square CFT. Sun’s stress-strain relationship (Sun 1991) is used for the unloading and reloading branches of concrete. Meng-Ohi-Takanashi stress-strain relationship (Meng et al. 1992) is used for hysteretic rule of steel tube. They are shown in Figure 7 and Figure 8, respectively. In Figure 8, compressive and tensile skeleton parts are defined separately. The feature of the Meng model is to make the opposite skeleton adjustable to fit the test results. According to the plastic strain $\varepsilon_p$, the target point is moved by multiplying a coefficient $\Psi$. In this case, the values of $\Psi$ are 0 and 0.6 for the analysis of circular columns and square columns, respectively. The Bauschinger part of the curve is expressed by the Ramberg-Osgood function; the coefficients defining the round of the curves are 7 and 4.5 for circular and square column to fit the experimental results.

**COMPARISON BETWEEN EXPERIMENT AND ANALYSIS**

The characteristics of the cyclic behaviors of circular and square CFT columns are addressed in this section by comparing the experimental results with the analytical results. The results of the test and analysis of Series 1 for circular CFT and Series 4 for square CFT are selected as the samples of explaining the major conclusions. Relationships between the bending moment $M$ and the non-dimensional curvature $\phi D$ are shown in Figure 9, and relationships between the strain at the centroid of the section $\varepsilon_g$ and $\phi D$ are shown in Figure 10. In these figures, upper side figures show the experimental results and lower side figures show the analytical results corresponding to the above ones. An average curvature $\phi$ through the main gauge at the middle of the specimen is measured by a couple of the transducers shown in Figure 3. The main gauge length is twice of the depth of specimen. An axial strain of the section $\varepsilon_g$ is also measured by these transducers. In the
M-ϕD relations, the calculated full plastic moments $M_{cal}$ are shown to compare with the maximum bending moments obtained by experiments and analyses. The mark "X" shows fracture of the specimen due to breaking at the welded joint or failing in sustaining the axial load. Tests are stopped at the fracture of the specimens. The main reason of presenting the $ε_{ϕD}$ relations is to observe the convergence-divergence of the shrinkages due to the axial load levels. The typical behavior of CFT columns is that the load-deformation curve shows stable manner while the shrinkages is increasing continuously. In spite of obtaining the stable manner of $M$-ϕD relations, it is necessary to check the axial deformations to estimate the damage level especially for CFT columns as indicated by Sakino and Tomii (Sakino and Tomii 1981).

The Figure 9 and 10 show the results of non-annealed specimens in Series 1. The axial load level of the left side specimen is 0.20 and that of right side is 0.60. The experimental maximum moment of specimen under low axial load is almost the same as the full plastic moment $M_{cal}$. On the other hand, the capacity of right side specimens is over $M_{cal}$ due to the confining effect of concrete. The $M$-ϕD and $ε_{ϕD}$ relationships are significantly affected by the axial load ratio. The analytical results are able to predict the different behaviors of the strength deterioration and cumulative shrinkage according to the axial load level. The results of annealed specimens in Series 1 are shown in Figure 11 and 12. The experimental results of right side specimen under height axial load show typical $M$-ϕD and $ε_{ϕD}$ characteristics. The strength deterioration due to the cyclic loading is not observed but the shrinkage is divergent. The analytical results do not trace the $M$-ϕD relations but can estimate the increasing manner of the axial deformation. The influence of the annealing is not observed clearly from the comparisons of the results of Series 1.

The results of the square CFT columns with small $B/t$ ratio of 32.8 in Series 4 are shown Figure 13 and 14. In the Figure 15 and 16, the results of the square columns with large $B/t$ ratio of 88.1 are shown. From the observation from these relationships, it can be seen that the flexural behavior of square CFT columns is dominated by the axial load ratio and $B/t$ ratio. According to the increases of axial load ratio, $B/t$ ratio or both, the cyclic behaviors of the columns become brittle. In Figure 13, the right side specimen "BR4-6-6-047-C" with $B/t$ ratio of 32.8 decreases the resisting capacity after peak moment, but strength deteriorations seem to converge after $ϕD$ of 1.0 x 10^{-2} (rad). While for the specimens with $B/t$ ratio of 88.1 in Figure 15, the moment resistant capacities fall down continuously. The analytical result of the specimen "BR4-6-6-047-C" slightly overestimate in the $M$-ϕD and the $ε_{ϕD}$ relations. On the other hand, the analytical result of $ϕD$ relation of the specimen "BR4-2-6-02-C" predicts the larger shrinkages than that of experiment. But the other analytical results show good agreement with the test results.

The tests and analyses of Series 2 and Series 3 are already discussed in our previous papers (Nakahara and Sakino 2000; Nakahara, Ninakawa and Sakino 2003). The details of the almost all specimens shown in Table 2 can be completed by being referred to these above papers and this paper.

**CONCLUSIONS**

Twelve circular CFT columns and eighteen square CFT columns were tested under uniform bending moment and a constant axial load and an numerical analysis was con-
ducted to predict the experimental behaviors. The following conclusions can be reached from the tests and analyses of hysteretic behavior of the single curvature CFT columns.

1) The flexural behavior of circular CFT columns was dominated by the axial load ratio, but the effect of annealing was not observed clearly.
2) The ductility of square CFT columns was significantly affected by the axial load ratio and width to thickness ratio.
3) The cyclic behavior of the CFT columns were predicted by the elasto-plastic analysis using the proposed stress-strain curve models for the filled concrete and the steel tube.

The main reason that the analysis well traced the test results of the relations between bending moment and the non-dimensional curvature and of the relations between the strain at the centroid of the section and the non-dimensional curvature was attributed to the confined concrete models for CFT columns. Because the concrete models were easy to express the different type of confined concrete and were accurate to predict the behaviors, the utilities and availabilities of the models were shown in this paper.

REFERENCES


Nakahara, H. and Sakino, K., 2000, "Flexural Behavior of Concrete Filled Square Steel Tubular Beam-Columns," Proceedings of 12th WCEE, Auckland, New Zealand, CD-ROM.


# Table 1 -- Outline of experiment

<table>
<thead>
<tr>
<th>Section</th>
<th>Experimental Series</th>
<th>Number of Specimens</th>
<th>Axial load ratio</th>
<th>Depth (Width) to wall-thickness ratio</th>
<th>Loading Pattern</th>
<th>Anoaling</th>
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<td>Circular</td>
<td>Series 1</td>
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<td>0.2, 0.4, 0.6</td>
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<td>Anoaling, Non-Anealing</td>
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<td>Series 2</td>
<td>6</td>
<td>0.2, 0.4</td>
<td>88, 160</td>
<td>Type 1, Type 2</td>
<td>Non-Anealing</td>
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<tr>
<td>Square</td>
<td>Series 3</td>
<td>11</td>
<td>0.2, 0.4</td>
<td>34, 47, 98</td>
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<td>Anoaling</td>
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<td>Series 4</td>
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<td>0.15 ~ 0.45</td>
<td>33, 88</td>
<td>Type 1</td>
<td>Non-Anealing</td>
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# Table 2 -- Properties of specimens

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<th>Specimen</th>
<th>D,B(mm)</th>
<th>t (mm)</th>
<th>D/t, B/t</th>
<th>$\sigma_y$ (MPa)</th>
<th>$\sigma_s$ (MPa)</th>
<th>N/Nc</th>
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<td>BC4-2-6-02-C</td>
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<td>88.1</td>
<td>265</td>
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<td>BC4-1-6-02-C</td>
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<td>160</td>
<td>241</td>
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<td>Series 4</td>
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</table>

B: breadth of steel tube, D: depth of steel tube, t: wall thickness of steel tube, $\sigma_y$: yield stress of steel tube, $\sigma_s$: strength of concrete cylinder, N/Nc: axial load ratio
Table 3 -- Parameters for proposed stress-strain models for concrete

<table>
<thead>
<tr>
<th></th>
<th>Original (Curve-3,4)</th>
<th>Circular CFT (Curve-1)</th>
<th>Square CFT (Curve-2)</th>
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<td>$X$</td>
<td>$\frac{E}{e} \left(\frac{e}{e_c}\right)$</td>
<td>$\frac{E}{e} \left(\frac{e}{e_c}\right)$</td>
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<tr>
<td>$Y$</td>
<td>$\frac{\sigma}{\sigma_c} \left(\frac{e}{e_c}\right)$</td>
<td>$\frac{\sigma}{\sigma_c} \left(\frac{e}{e_c}\right)$</td>
<td>$\frac{\sigma}{\sigma_c} \left(\frac{e}{e_c}\right)$</td>
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<tr>
<td>$V$</td>
<td>$\frac{E \cdot e}{\sigma} \left(\frac{e}{e_c}\right)$</td>
<td>$\frac{E \cdot e}{\sigma} \left(\frac{e}{e_c}\right)$</td>
<td>$\frac{E \cdot e}{\sigma} \left(\frac{e}{e_c}\right)$</td>
</tr>
<tr>
<td>$\sigma_{\infty}$</td>
<td>$\frac{2E(B-t)\sigma}{\sigma_c} + \frac{k \sigma}{\sigma_c} \left(\frac{e}{e_c}\right)$</td>
<td>$\frac{k \sigma}{\sigma_c} \left(\frac{e}{e_c}\right)$</td>
<td>$\frac{k \sigma}{\sigma_c} \left(\frac{e}{e_c}\right)$</td>
</tr>
<tr>
<td>$\sigma_{\infty}$</td>
<td>$1 + \frac{k \sigma}{\sigma_c} \left(\frac{e}{e_c}\right)$</td>
<td>$1 + \frac{k \sigma}{\sigma_c} \left(\frac{e}{e_c}\right)$</td>
<td>$1$</td>
</tr>
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</table>

$W = 1.50 - 17.1 \times 10^{-3} \frac{\sigma}{\sigma_c} + 2.39 \sqrt{\sigma_{\infty}}$

$E = (6.90 + 3.32 \sqrt{\frac{\sigma}{\sigma_c}}) \times 10^{1}$

$\varepsilon = 0.94 (\frac{\sigma}{\sigma_c})^{1/2} \times 10^{-2}$

$\varepsilon_{\infty} = \begin{cases} 1.0 + 4.7(K-1), & K \leq 1.5 \\ 3.4 + 20(K-1), & K > 1.5 \end{cases}$

$k = 4.1, \ a = -0.19, \ \sigma = -\frac{2t\alpha \cdot \sigma}{D-2t}$

Notations:

- $B$ : Outside width of steel tube
- $b$ : Inner width of steel tube (b = B - 2t)
- $D$ : Outside diameter of steel tube
- $t$ : Thickness of steel tube wall
- $E$ : Young's modulus of elasticity concrete (in MPa)
- $\varepsilon_c$ : Strain at maximum stress of plain concrete
- $\varepsilon_{\infty}$ : Strain at maximum stress of confined concrete
- $\sigma_{\infty}$ : Compressive strength of concrete (in MPa)
- $\sigma_{tb}$ : Strength of confined concrete (in MPa)
- $\sigma_y$ : Yield stress of steel tube (in MPa)
Figure 1 -- Loading conditions

Figure 2 -- Deformation histories
Figure 3 -- Loading apparatus and measuring frames (Unit : mm)

Figure 4 -- Designations of the test specimens
Figure 5 -- Stress-strain relations for concrete

Figure 6 (a) -- Stress-strain relations for steel tube of circular CFT

Figure 6 (b) -- Stress-strain relations for steel tube of square CFT
Figure 7 -- Hysteresis curve model for concrete

Figure 8 -- Hysteresis curve model for steel tube

Figure 9 -- M-φD relations of non-anealed circular CFT columns (Series 1)
Figure 10 -- $\varepsilon_g$-$\phi D$ relations of non-anealed circular CFT columns (Series 1)

Figure 11 -- $M$-$\phi D$ relations of anealed circular CFT columns (Series 1)
Figure 12 -- $\varepsilon - \phi D$ relations of anealed circular CFT columns (Series 1)

Figure 13 -- $M - \phi D$ relations of square CFT columns with small $B/t$ ratio (Series 4)
Figure 14 -- $\varepsilon_g$-$\phi D$ relations of square CFT columns with small $B/t$ ratio (Series 4)

Figure 15 -- $M$-$\phi D$ relations of square CFT columns with large $B/t$ ratio (Series 4)
Figure 16 -- $\epsilon - \phi D$ relations of square CFT columns with large $B/t$ ratio (Series 4)
Seismic Retrofit for R/C Rectangular Columns by Circular Steel Jackets

by J.H. Wang, K. Yoshimura, K. Kikuchi, and M. Kuroki

Synopsis: In order to investigate the seismic behavior of existing reinforced concrete (R/C) rectangular columns which are strengthened by circular steel jackets, a total of twenty column specimens with and without strengthening were designed, constructed, and were tested under three different constant axial-stresses of 5.6, 13.2 and 19.4 MPa, and alternately repeated lateral forces. Test results of retrofitted columns with circular steel jackets are compared with those obtained from the column specimens with and without rectangular steel- and CF sheet-jackets, and circular CF sheet jackets. One of the main conclusions is that the ultimate flexural strengths and deformation capacity of the R/C rectangular columns which are confined laterally by the circular steel- and CF sheet-jacketing are quite effective, especially in case of the columns under high axial-compression.

Keywords: circular steel jackets; high axial compression; RC moment-resisting frames; rectangular columns; seismic strength
INTRODUCTION

Background

Since a number of reinforced concrete (R/C) short columns in ordinary school buildings failed in brittle shear failure mode during the 1968 Tokachi-oki earthquake, the structural design provisions of the Building Standard Law (BSL) of Japan was slightly revised in 1971. This is to prevent the R/C short columns from brittle shear failure. After this, R/C building columns have been designed so as to provide a sufficient amount of transverse reinforcement in their column section.

During the 1971 San Fernando earthquake in the United States and 1975 Oita earthquake in Japan, however, R/C moment resisting frames with soft first-stories such as Olive View Hospital and K. L. Hotel buildings collapsed drastically, and the extensive structural damage was concentrated into the soft first-stories of those buildings. In addition, R/C buildings with large eccentricity in plan were also damaged severely during the 1978 Miyagi-ken-oki earthquake. As the results, the seismic structural design provisions of the BSL were completely revised in 1981, which has been so called “the New Seismic Design Method”. According to the New Seismic Design Method, ordinary building structures shall be designed so as to be safe within the allowable stresses against small- or medium-size earthquakes with peak ground acceleration of 0.08 to 0.1g first, and then lateral stiffness distribution along the height of the building, as well as mass- and stiffness-eccentricity in plan, must be investigated so as to be within the limited values. In addition, in case of the buildings with high irregularity in lateral stiffness distribution along the building height or large eccentricity in plan, or in case of the buildings higher than 31 meters, seismic safety of the buildings must be investigated against severe earthquakes with peak ground acceleration of 0.3 to 0.4g as the second
After the New Seismic Design Method was published in 1981, the 1995 Hyogo-ken Nanbu (Kobe) earthquake was the biggest earthquake in Japan both in human and dwelling damage, where approximately 6,500 people were dead and more than 100,000 buildings and houses were completely collapsed. It was noted that most of the severely damaged buildings by this earthquake had been designed and constructed in accordance with the design provisions of the old building codes and/or design standards published before 1971 and/or 1981 as mentioned above.

In addition to those facts that the old R/C framed buildings designed before 1981 were damaged severely during the 1995 Kobe earthquake, however, it is worthy of note that some of the relatively new R/C buildings with soft lower stories which had been designed in accordance with the New Seismic Design Method after 1981 were also damaged severely and failed in flexural failure mode in their soft lower stories. In order to prevent this kind of flexural failure of the R/C independent columns located in the soft lower stories in R/C moment resisting frames, the design values for the Stiffness Ratio ($R_s$) and Shape Factor ($F_{es}$) in the BSL were also revised after the occurrence of this earthquake. At the same time, the buildings shall be designed so as not to fail in weak-column failure mode at their ultimate states in case when being subjected to strong motion earthquakes.

Based on those facts, the new “Code for Promotion of the Seismic Strengthening of Existing Buildings” was established on December in 1995. In accordance with this new Code, “the Specified Buildings” with three stories or more, and having more than 1,000m$^2$ in total floor area, are recommended to conduct their seismic safety evaluations, and if necessary, it is also recommended to retrofit the buildings against future big earthquakes. After this new Code was published in 1995, seismic capacity of a large number of existing building structures has been investigated using the very popular seismic evaluation method in Japan (JBDPA 1977).

During the period starting from October in 1995 to the end of December in 2003 seismic safety evaluations of more than five hundred existing buildings have been conducted in Oita Prefecture, Japan, and more than one hundred R/C buildings have been already retrofitted. Among those retrofitted existing buildings with poor seismic capacity whose R/C independent columns are expected to subject the quite high axial-compression during coming big earthquakes, some of the R/C rectangular columns located in the soft first-stories were retrofitted by adopting the circular steel jackets filled with concrete inside the circular steel jacket as shown in Figure 1. Herein, seismic performance of those R/C rectangular columns, which were strengthened by the circular steel jackets, is investigated by using a total of twenty column specimens with and without strengthening.

Since the 1989 Loma Prieta earthquake attacked many of the R/C highway bridges located at around the San Francisco Bay Area in the United States, a large number of experimental and theoretical studies were conducted for retrofitting the R/C highway
bridges located in the California State, and based on those excellent research works, more than seven-hundred R/C bridges with poor seismic capacity have been retrofitted by circular and/or elliptic steel jackets filled with concrete materials. During the 1994 Northridge earthquake, it is well known that more than 50 bridges with those steel jacketed columns were subjected to peak ground acceleration of 0.3g or higher, however, none of those bridges suffered any structural damage to their columns (Priestley, Seible, and Calvi 1996). On the contrary, there are quite few systematic studies on the effectiveness of those seismic strengthening for existing R/C rectangular building columns by using the circular or elliptical steel- and/or CF sheet-jackets (Takiguchi and Abdullah 2001; Kitano et al. 1997; Utsunomiya et al. 1998).

Prototype RC columns

In many of the earthquake countries, a number of extensive earthquake damage to the R/C rectangular building columns have been reported, and even in recent years, many of the R/C rectangular short- and long-columns collapsed in brittle shear and flexural failure modes, and/or failed in combined shear, flexure and high axial compression forces during the 1999 Turkey (Kocaeli) and Taiwan (Chi-chi) earthquakes. This implies the importance of further effort for developing the different types or much more effective seismic strengthening methods for the R/C rectangular building columns which are expected to fail under high axial compression and/or alternately repeated earthquake forces. Herein, in order to prevent the R/C rectangular building columns with poor seismic resistance from the extensive earthquake damage, the effectiveness of the circular steel jackets is investigated by using twenty model column specimens with and without seismic strengthening.

The circular columns shown in Figure 1 are one of the prototype columns adopted in the present study, which is the first application example of the seismic strengthening using circular steel jacketing to the first-story independent R/C rectangular columns of an actual nine-story existing office building in Oita City, Japan. This building was designed in 1970 in accordance with the old Japanese Building Standard Law and related Structural Design Standards of the building structures, and was completed in 1973. Since this building did not have sufficient seismic resistance against future big earthquakes, seismic strengthening was conducted in 2000 by adopting the three different seismic strengthening methods, that is, steel reinforced concrete (SRC) composite shear walls and circular steel jacketing both in the first-story as shown in Figures 1 and 2, and steel braced frames in the second through eighth-story of this building. Figure 2 shows the first (or ground) floor plan of this building after retrofitting in 2000. Before strengthening, there were eleven independent R/C rectangular columns located respectively along the extreme north-side and south-side column-lines in the longitudinal (or east-west) direction. While in the transverse (or north-south) direction, two R/C bearing walls (or coupling-walls) were located in the extreme east- and west-side wall-lines, respectively. In addition to those independent columns and coupling walls, there were many cast-in-place R/C boxed bearing walls provided at around the central part of this floor as being understood in Figure 2. In retrofitting the first-story of this
building, both of the extreme two-bays located in the north-side and south-side column-lines were strengthened by providing the eight SRC composite shear walls as shown in Figure 2. In addition to these cast-in-place SRC shear walls, every two independent R/C rectangular columns located in the same column-lines with the above SRC walls were also retrofitted by using the 16mm thickness circular steel-plates filled with concrete materials. These two different retrofitting techniques adopted were not only for seismic strengthening against lateral loadings but also intending to prevent the complete crushing of the first-story during the severe earthquake. Any seismic strengthening was not provided into the remaining eight independent columns. As being understood later, those of two different R/C rectangular columns with and without retrofit by circular steel jackets are the prototypes of the tested columns adopted in the present study. In addition to the retrofitted R/C columns by the circular steel jackets as mentioned above, three other different seismic strengthening techniques are also adopted herein to the original non-retrofitted R/C rectangular columns, which are retrofitted by using the square steel jacket, and circular and square CF sheet jackets.

In the experimental phase of the present investigation, a total of twenty one-third-scale model columns are tested under three different constant axial compression stresses of 5.6, 13.2 and 19.4MPa, and alternately repeated lateral forces, respectively. Herein, the axial-compression of 5.6MPa (or low axial-load) adopted is the corresponding axial-stress to which the retrofitted rectangular columns shown in Figure 1 are subjected as the long-term gravity-loads. On the contrary, those of two different high axial-compression stresses of 13.2MPa (medium axial-load) and 19.4MPa (high axial-load) are the expected values to which the retrofitted R/C columns are subjected in case when the adjacent non-retrofitted rectangular R/C columns have lost their axial load-carrying capacities first, and then (or furthermore), in case when the prototype nine-story building as mentioned above has finally reached its ultimate failure mechanism in the transverse direction, respectively.

In determining the thickness of the circular steel jackets with tube-thickness of 16mm which is used for retrofitting the actual existing R/C rectangular shown in Figure 1, the highest value of the axial-compression of 19.4MPa as mentioned above was adopted as the column axial-compression. Besides the horizontal component of the earthquake forces, there is also the vertical component of earthquake ground motion that cannot be neglected, and this vertical component will induce an additional effect on the axial-loads in the strengthened columns. Since it is impossible to determine the magnitude of the effect of the vertical component of earthquake ground motion correctly, a factor of safety in axial-stress of column was considered while determining the thickness of the circular steel-plate. The axial-stress of column is designated by (\( \sigma_c \)) and assumed as (\( \sigma_c = 3F_c \)) after considering various factors such as the factor of safety, workmanship required during the circular steel-tube welding and so on. Based on this axial-stress the diameter (\( d \)) and thickness (\( t \)) of circular steel-plate having a circumferential yield stress (\( \sigma_i \)) of 300MPa was adopted as the strengthening material. The compressive strength of concrete confined by the circular steel jacket was determined from the following equation proposed by Richard (Tomii, Yoshimura, and Morishita 1977).
Where \( F_c \) is the specified design compressive strength of concrete, \( \sigma_r \) is the radial pressure that balances the hoop tension \( \sigma_h \) under equilibrium condition given by the following equation.

\[
\sigma_r = \frac{t}{(d/2) - t} \sigma_h
\]

Although the required thickness of the steel-tube obtained from the above equation is about 16.5mm, \( t = 16 \) mm was adopted as the nearest size available from the manufacturer.

During the retrofitting construction, two faced semi-circular steel jackets with tube-thickness of 16mm were welded each other to form a circular steel jackets having diameter \( d \) is equal to 1.0 meter. The volumetric ratio of strengthening steel jacket for circular column \( \rho_s \) is equal to 6.4\% \( (\rho_s = 4t/d) \). While in the non-retrofitted original R/C rectangular column considered as the prototype column, the cross-sectional size and shape of the RC rectangular columns were \( b \times D = 600 \times 700 \) mm with longitudinal steel reinforcement ratio, \( p_{gr} = 1.1\% \), shear reinforcement ratio, \( p_{sw} = 0.24\% \) and supporting long-term vertical axial-load \( (L/N) \) of 2,373kN, where the corresponding axial-stress \( \sigma_0 = L/N/bD \) is 5.6M Pa as mentioned before.

TEST SPECIMENS

Details of twenty specimens and material properties used for the specimen are shown in Tables 1 and 2, and Figure 3. Among these, eight of the twenty specimens are the main test specimens for the present study, where four specimens are non-strengthened R/C rectangular specimens (R-H) and other four are the strengthened specimens with circular steel jacket (C-SP). The remaining twelve specimens are constructed and tested for the purpose of comparative study and these include four each with rectangular steel jacket (R-SP), and rectangular and circular CF sheet jackets (R-CF) and (C-CF). All the twenty specimens are tested under the three different axial compressions of 5.6, 13.2 and 19.4M Pa, and the repeated lateral forces, respectively.

Test specimens are approximately one-third scale model of the first-story columns of the nine-story RC office building as presented before, and their cross-sectional dimensions are 210 mm by 245 mm. Eight D10 (or #3) bars are provided for the longitudinal (or main) reinforcement, in which the ratio of the total cross-sectional area of the main Re-bars to the R/C column section is \( p_{gr} = 1.1\% \). Transverse (or hoop) reinforcement is D6 (#2) bars provided with a spacing of 130 mm and shear reinforcement ratio \( p_{sw} \) is 0.23\%. Here, it is noted that the volumetric ratio of \( \rho_s = 5.1\% \) in the retrofitted specimens with circular steel jacket is slightly less than that of the prototype-strengthened columns having the value of \( \rho_s = 6.4\% \), because of using the circular steel jacket with tube-thickness \( t = 4.5 \) mm and diameter \( d = 350 \) mm,
respectively.

The specimens shown in Figures 3(b) through (d) are classified into two groups, such as “half-length” and “full-length” columns, depending on their column sizes. Assuming that the inflection point of the flexural deformation of the specimen columns are located at the mid-height of the column, heights of the half-length column specimens, which are subjected to low and medium axial-loads, become 435mm and 345mm, respectively. While in the full-length column specimens subjected to medium and high axial-loads, clear height of the columns becomes to be 1,170mm.

The typical size and shape, and bar arrangement of all the test specimens with half-length and full-length column specimens are shown in Figures 3(b) through (d), respectively. Each of the test specimens is designated by four symbol codes, where the first letters “R” or “C”, represent the rectangular or circular cross-sectional shapes, second letters “SP” or “CF” represent that the original rectangular columns “H” with only hoop reinforcement are strengthened by steel plates or carbon fiber sheets as their transverse reinforcement. The third numerals are the applied constant axial-compression in MPa, and the last letters “H” or “F” represent the specimens are half-length columns or full-length columns, respectively.

The amount of seismic strengthening provided by the rectangular and circular steel- or CF sheet-jackets is represented by the values of $p_w$ and $p_x \sigma_{wy}$ (or $\rho_x$ and $\rho_x \sigma_{wy}$), where $p_w$ is the shear reinforcement ratio, $\rho_x$ is the volumetric ratio of the circular steel jacket, and $\sigma_{wy}$ is the yield strength of steel-plate and design yield strength of CF sheet (=2300MPa), respectively. In case of the circular steel jacket (C-SP) specimens, non-shrinkage mortar was filled inside the circular steel jackets after circular steel jackets were welded, while in case of the circular CF sheet (C-CF) specimens, non-shrinkage mortar was cast within the circular form around the original rectangular (R-H) column sections first, and then CF sheets were bonded on the perimeter surface of the circular columns after curing the mortar and removing the circular forms. Between upper- and lower-stubs, and, top- and bottom-edges of the steel- and CF sheet-jackets in the strengthened specimens, 10 mm clearance was provided as shown in Figure 3. This is to avoid the direct touch of the stubs to the top- and bottom-edges of the steel- and CF sheet-jackets filled with mortar during the application of lateral loading reversals to the column specimens.

**TEST SETUP**

Test setups adopted in the present study are shown in Figures 4(a) and (b) together with one of the test specimens respectively. The test setup adopted for the half-length column specimens with low axial-stress of 5.6 MPa is shown in Figure 4(a), and the test setup used for all the full-length column specimens with medium and high axial-stresses of 13.2 and 19.4MPa is shown in Figure 4(b). Lower-stub of each specimen was fixed to the laboratory test floor by using high strength steel tendons. Two different oil jacks installed in vertical and horizontal directions, respectively, applied a constant
vertical axial-load and alternately repeated lateral forces. All the specimens were tested under constant low, medium and high axial-loads of 287, 678 and 1000kN (or 5.6, 13.2 and 19.4M Pa in column-section), respectively, and alternately repeated lateral forces. Height of the application point of the repeated lateral forces is the location of the inflection point of flexural deformation of the column specimens, and was kept to be located at the mid-height of the whole column height measured from the top-surface of the lower-stub. According to this applied lateral loading position, shear span ratio of the tested columns becomes to be 2.7. Important displacements and strains in reinforcing bars and jacketing materials were measured by using the displacement transducers and strain gages, and all the measured information was processed simultaneously by a personal computer.

TEST RESULTS AND DISCUSSIONS

$Q$–$R$ relations

Relations between lateral forces ($Q$) and story-drifts ($R$) of all the specimens are shown in Figure 5. Solid circles and squares, and open circles, squares and triangles shown in the figures represent the initial yielding of the longitudinal Re-bars in tension and compression respectively, which were measured by the strain gages located at 30mm, 585mm and 1,140mm above the bottom of each column. ($Q_{mu1}$) represented by solid lines are the ultimate flexural strengths when the tested columns developed their ultimate flexural strengths ($M_{u1}$), which are calculated by the stress block of an unconfined concrete based on the popular A CI criteria. ($Q_{mu2}$) shown by solid lines are the ultimate lateral strengths corresponding to the ultimate flexural strengths obtained from the moment-curvature (or $M$-$\phi$) relationship, which was derived from the stress ($\sigma$) -strain ($\varepsilon$) curves of the concrete for respective specimens (Komure, Tagaki, and Nakatsuka 1997; Morita and Sakino 1993; Park, Priestley, and Gill 1982). ($Q_{su1}$) shown by dashed lines are the ultimate lateral shear strengths calculated by modified Ohno and Arakawa equation (AIJ 1990). ($Q_{su2}$) shown by dotted lines are determined from the existing shear-strength evaluation formula (AIJ 1999).

In all of the non-strengthened (R-H) specimens shown in Figures 5(a) through (d), yielding of the longitudinal Re-bars in the column-sections was initiated when the story-drifts were from $R$=+0.5x10$^{-2}$rad to $R$=+1.0x10$^{-2}$rad, and immediately after developing their ultimate lateral strengths, deterioration in lateral load-carrying capacity took place due to the extensive shear cracks. Especially in the non-strengthened (R-H) specimens with medium and high axial-loads, rapid strength deterioration in the lateral load-carrying capacity took place when the story-drifts ($R$) were between $R$=0.4 and 1.0x10$^{-2}$rad. This was due to the brittle shear failure of the tested columns after developing their ultimate flexural strengths, and all the columns lost their axial load-carrying capacity. On the contrary, all of the specimens retrofitted by the steel- and CF sheet-jackets showed the excellent lateral load-carrying capacity and deformability until the story-drift ($R$) reached $R$=5.0x10$^{-2}$rad. Note that the lateral strengths of the rectangular steel jacketing specimens (R-SP-19.4-F) were increased rapidly after
$R = +2.0 \times 10^{-2}$ rad. This was caused by the direct touch of the top of the steel jacket to the lower surface of the upper-stub of the column specimen. Also it can be noted that the lateral strengths of the (C-SP-19.4 and C-CF-19.4) specimens with circular steel- and CF sheet-jackets which were subjected to quite high axial-compression of 19.4 MPa increased little by little until the experiments finished at $R = 5.0 \times 10^{-2}$ rad. All of those facts mean that the seismic performance of the original non-retrofitted R/C rectangular columns were well improved by adopting the seismic strengthening techniques presented in this investigation.

**Q-R envelope curves**

Figures 6(a) through (d) show the envelope curves obtained from the average of the positive and negative $Q-R$ hysteresis loops shown in Figures 5(a) through (d). Values of the Ductility index ($F$), which is one of the most important indexes to evaluate the seismic safety of the existing R/C rectangular columns (JBDPA 1977), is also presented in the figures. As being observed from the figures, all the non-strengthened (R-H) specimens except for (R-H-5.6-H) with low axial-load failed in brittle shear failure mode within the small story-drift ($R$) less than 1.0 $\times 10^{-2}$ rad after developing their ultimate flexural strengths, while in all of the strengthened specimens, their lateral load carrying-capacity and deformation behavior are well improved from the non-strengthened of (R-H) specimens.

Herein, in order to investigate the ultimate lateral-strength deterioration in a large inter-story displacement area, the lateral strengths at the story-drift of $R = 4.0 \times 10^{-2}$ rad (or $R = 4\%$ rad) were calculated for all the specimens, and the calculation results are given by the ratio of $(Q_{R=4\%})$ to the maximum lateral strengths, $(Q_{max})$, in Table 3, where $(Q_{R=4\%})$ is the lateral strengths at the story-drift of $R = 4.0 \times 10^{-2}$ rad and $(Q_{max})$ is the maximum lateral strengths observed within the story-drifts less than $R = 2.0 \times 10^{-2}$ rad, respectively. It can be understood from Table 3 that the value of $(Q_{R=4\%}/Q_{max})$ of the rectangular CF sheet jacket specimens decreases from 0.95 to 0.78 with the increase of the axial compression load in the retrofitted columns, while in columns with rectangular steel jackets, the ultimate lateral strength deterioration is less than 10%. On the contrary, the values $(Q_{R=4\%}/Q_{max})$ of circular CF sheet- and steel-jackets specimens increase gradually from 0.95 to 1.04 with the increase of the axial column compression loads. This fact means the ultimate lateral strengths and deformability of the circular jacketing (C-SP and C-CF) specimens are little bit higher and much more excellent than the rectangular jacketing (R-SP and R-CF) specimens.

**$\delta$-$R$ relations**

Axial column deformation ($\delta$) versus story-drift ($R$) relations observed during the positive loadings is shown in Figure 7 for all the specimens. Here, elongation of the axial deformation ($\delta$) is presented in the positive direction.

In the case of the specimens with low axial-stress of 5.6 MPa (Figure 7(a)),
followings are obtained as the test results: In the non-strengthened specimen (R-H-5.6-H) which failed in brittle failure mode at $R=+3.2 \times 10^{-2}$ rad, rapid shortening initiated in the longitudinal direction of the column specimen. On the contrary, longitudinal elongation of axial deformation was observed in both of the circular CF sheet- and steel-jackets specimens (C-CF-5.6-H and C-SP-5.6-H).

In the case of the column specimens with medium axial-stress of 13.2 MPa (Figures 7(b), (c)), obtained test results are as follows: In the non-strengthened specimens (R-H-13.2-H, R-H-13.2-F) which failed finally in brittle shear failure mode, rapid shortening initiated just after the occurrence of brittle shear failure in their column sections. Also the gradual shortening in axial deformation was observed in the rectangular half-length retrofitted specimens, (R-SP-13.2-H), while in the rectangular full-length steel jacket specimen, (R-SP-13.2-F), almost no longitudinal shortening or elongation in the axial deformation were observed. On the contrary, longitudinal elongation of the axial deformation was observed in all the circular steel- and CF sheet-jackets specimens with the increase of their inter-story displacements. This is due to the higher confining effect that is expected in circular jacketing specimens than the rectangular column specimens.

In the case of the column specimens with high axial-stress of 19.4 MPa (Figure 7(d)), remarkable test results were obtained as follows: The non-strengthened specimen (R-H-19.4-F) failed finally in brittle shear failure mode at $R=+0.44 \times 10^{-2}$ rad, and just after the occurrence of this brittle shear failure in its column section, rapid shortening of the column initiated in the longitudinal direction of the column specimen. In addition, shortening in axial deformation was also observed in both of the rectangular steel- and CF sheet-jacketed specimens, (R-SP-19.4-F, R-CF-19.4-F). With the increase of the inter-story displacements, however, it is worthy of note that there was almost no axial shortening observed in both of the circular steel- and CF sheet-jacketing specimens, (C-SP-19.4-F, C-CF-19.4-F) even in those columns under quite high axial-compression and alternately repeated flexural and lateral shear forces, and in the very large story-drifts of $R=+4.0 \times 10^{-2}$ rad or more. This is due to the considerable high confining effect, which is given by the circular steel- and CF sheet-jackets. From the measurements by the tri-axial strain gauges on the surface of the steel jackets, it can be noted that the maximum stress induced in the steel jacket of the (C-SP-19.4-F) specimen was observed at the story-drift ($R$) of $+4.53 \times 10^{-2}$ rad, and the corresponding equivalent stress at this moment was within yield stress given by the Von Mises yield criterion.

**Theoretical prediction of the ultimate lateral strengths**

The ultimate lateral strengths obtained from all the test specimens ($Q_{max}$) were determined by using the $Q-R$ hysteresis loops shown in Figure 5, and are presented in Table 3 together with the predicted ultimate flexural strengths, ($Q_{mu1}$) and ($Q_{mu2}$), and lateral shear strengths, ($Q_{su}$), respectively. The observed ultimate lateral strengths, ($Q_{max}$), in Table 3 are defined as the average of the positive and negative ultimate lateral strengths which were observed less than the story-drifts of $R=\pm 2.0 \times 10^{-2}$ rad.
The values of \( Q_{mu1} \) in Table 3 are the corresponding ultimate flexural strengths when the tested columns developed their ultimate flexural strengths, \( (M_u) \), which are calculated by assuming the stress blocks of the unconfined concrete based on the popular ACI criteria. On the contrary, values of \( Q_{mu2} \) in Table 3 are the corresponding ultimate flexural strengths when the tested columns developed their ultimate flexural strengths \( (M_u) \), which are determined by considering the confining effects of concrete. Herein, the flexural strengths, \( (M_u) \), of the rectangular CF sheet specimens, and circular steel- and CF sheet-jacket specimens are defined as the flexural moment capacity of the corresponding columns when the longitudinal reinforcing bars in the column sections reached the strains when the strain-hardening initiated during the material testing given in Table 2. Solid circles shown in Figure 8 are the stress-strains of the extreme fiber concrete at their ultimate flexural strengths for all the specimens under high axial compression stress of 19.4MPa.

The values \( Q_{su} \) of the non-strengthened specimens are the corresponding lateral shear strengths in shear failure mode, and were determined based on the existing proposed equations. The values \( Q_{su} \) of the rectangular CF sheet- and steel -jacket specimens are the corresponding lateral shear strengths, which were determined based on the existing proposed equations (AIJ 2001; Yoshimura et al. 2000).

Since the ultimate lateral strengths of all the specimens were determined by the flexural failure mode, test results of the ultimate lateral strengths \( (Q_{max}) \), were compared with the calculated flexural strength, \( (Q_{mu1}) \) and \( (Q_{mu2}) \), respectively, and the results are given in Table 3. In case of the half-length column specimens with low and medium axial-stresses and full-length column specimens with medium and high axial-stresses, the ratios of \( (Q_{max}) \) to \( (Q_{mu1}) \) are 1.08 to 1.19, 1.28 to 1.64, 1.41 to 2.07 and 1.45 to 3.42 respectively, while the ratios of \( (Q_{max}) \) to \( (Q_{mu2}) \) become 1.01 to 1.10, 0.86 to 1.12, 0.83 to 1.36 and 0.72 to 1.28 respectively.

In case of the specimens with low axial- stress of 5.6MPa, the ultimate lateral strengths, \( (Q_{max}) \), can be well predicted by the value of \( (Q_{mu2}) \), because the values of \( (Q_{max}/Q_{mu2}) \) in Table 3 fall between 1.01 and 1.10. Also in the specimens with medium and high axial-stresses of 13.2 and 19.4MPa, the values of \( (Q_{max}/Q_{mu2}) \) of the circular jacket specimens become to be 0.89 to 1.08, and thus the observed ultimate lateral strengths, \( (Q_{max}) \), can be also well predicted by \( (Q_{mu2}) \).

On the contrary, since the values of \( (Q_{max}/Q_{mu2}) \) in the rectangular CF sheet jacket specimens with medium and high axial- stresses of 13.2 and 19.4MPa are between 0.72 and 0.86, the ultimate lateral strengths of the CF sheet jacket specimens are conservatively evaluated by the existing proposed equations, \( (Q_{mu2}) \).

While in the rectangular non-retrofitted specimens and rectangular steel jacket specimens with medium and high axial-stresses of 13.2 and 19.4MPa, the values of \( (Q_{max}/Q_{mu2}) \) in Table 3 become to be between 1.12 and 1.36, and 1.12 and 1.31, respectively. This shows that the experimental ultimate lateral strength, \( (Q_{max}) \), seems to
be evaluated too small. One of the main reasons to this cause is coming from the location of the flexural yield hinges, which assumed to be formed at the top and bottom of the tested columns. Regarding to this point, further discussions are made in the subsequent Section.

**Interaction between axial-compression (N) and ultimate flexural strength (Mu2)**

Interaction between the ultimate flexural strength and axial load-carrying capacity can be evaluated by conducting a simple analysis. Results are given in Figure 9, where interaction diagrams between the ultimate flexural moment (Mu2) and column axial load-carrying capacity (N) are given for all the columns of the tested specimens. (Mu2) – (N) interaction curves in Figure 9 are obtained from the bending moment (M) versus curvature (ϕ) relations which were determined by using the (Vc) - (Vc) relations shown in Figure 8. Open squares and circles in Figure 9 represent the experimental ultimate flexural moments (Mue), which were determined by multiplying the ultimate lateral strengths (Qmax) by the half height of the tested columns. Of course, P-Δ moment caused by the axial compression and lateral story-drift is also included in those values shown by the open squares and circles in Figure 9. Remarkable differences in the ultimate flexural strengths are observed between the non-strengthened specimens (R-H-13.2-F, R-H-19.4-F) and other jacketed specimens. The reason to this is caused by the confining effect of the concrete given by the jacketing materials of the retrofitted specimens especially under medium and high axial compression loads.

The experimental and predicted values of the ultimate flexural strengths for all the specimens are shown in Table 4. In the circular jacket specimens, it can be understood that the experimental values are well evaluated by the predicted values, because the ratios of the experimental values, (Mue), to the predicted values, (Mu2), become to be from 1.00 to 1.19. On the contrary, the ratios, (Mue/Mu2), of the non-strengthened specimens and rectangular steel jacket specimens in Table 4 become to be (1.43 and 1.34), and (1.45 and 1.30), respectively, in which the experimental values are conservatively predicted by the calculation. One of the main reasons to this cause is coming from the location of the flexural yield hinges, which assumed to be formed at the top and bottom of the tested columns, or on the bottom- and top-surfaces of the upper- and lower-stubs, respectively. In the actual cases, however, the flexural yield hinges of the rectangular R/C long columns as being tested here are not formed just at these locations as mentioned above, but seems to be formed at the inner locations toward the mid-height of the columns, which are little bit apart from the upper-most or lower-most ends of the tested columns. Due to this reason, it is possible to occur that the predicted values of ultimate lateral strengths, (Mu2), of the rectangular column specimens except for the CF sheet jacketed columns became to be higher than the experimental values, (Mue). And thus, the predicted values of the ultimate flexural strengths of the rectangular specimens modified by using the following equation (Sun, Sakino, and Aklan 1996).
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\[ n = \frac{N}{A_c \cdot f_c} \quad \frac{M_{u3}}{M_{u2}} = \begin{cases} 1.1 & n \leq 0.3 \\ 1.1 + 0.8 (n - 0.3)^2 & n > 0.3 \end{cases} \] (3)

Where \( N \) is axial compression, \( A_c \) is cross-section area of column, \( f_c \) is compressive strengthening of column concrete, \( M_{u3} \) is modified value of the ultimate flexural strength, \( M_{u2} \) is predicted value of the ultimate flexural strength, and \( n \) is ratio of axial compression.

In Table 4, the modified values of the ultimate flexural moment, \( (M_{u3}) \), are given together with the values of the ratios, \( (M_{ue}/M_{u3}) \), for all the rectangular specimens. As being observed, experimental values, \( (M_{ue}) \), can be predicted well by using the \( (M_{u3}) \) better than \( (M_{u2}) \), because the ratios, \( (M_{ue}/M_{u3}) \), of the non-strengthened specimens (R-H-13.2 and R-H-19.4) and rectangular steel jacket specimens, (R-SP-13.2-F and R-SP-19.4-F) in Table 4 become to be (1.22 and 1.05), and (1.18 and 0.97), respectively.

Finally, the values of \( (M_{ue}/M_{u3}) \) in the rectangular CF sheet jacket specimens (R-CF-13.2-F and R-CF-19.4-F) are 0.78 and 0.62, these indicate that the predicted values are quite higher than the experimental ones. At this point, further discussion and investigation will be needed including the compressive stress \( (\sigma_c) \) versus strain \( (\varepsilon_c) \) relations of the concrete, which is confined by the rectangular CF sheet jacketing.

CONCLUSIONS

In order to investigate the effectiveness of the seismic strengthening methods by providing the circular steel jackets for existing R/C rectangular columns, twenty specimens with and without jacketing were tested under three different constant axial compression stresses of 5.6, 13.2 and 19.4 MPa, and alternately repeated lateral forces. Results of the experiments obtained are summarized in the followings:

1. Seismic retrofit with steel- and CF sheet-jackets are quite effective even under quite high axial compression loads.
2. The ultimate lateral strengths of the circular jacket specimens are much higher than the rectangular jackets specimens.
3. In case when the columns are subjected to low axial-compression of 5.6 MPa, there were quite few difference observed in the ultimate lateral strengths and deformation capacity between circular and rectangular jacketed column specimens.
4. In case when the columns are subjected to much higher axial compression such as 13.2 and 19.4 MPa, difference in ultimate lateral strength and deformation capacity became prominent between circular and rectangular jacket specimens.
5. The strengthening effect given by the rectangular CF sheet-jackets decreases with the increase of the axial-compression. In case of the rectangular steel jacketed columns, however, more than 10% deterioration in lateral load-carrying capacity were not observed at least until the story-drift reached \( R = \pm 4\% \).
6. In case when the R/C rectangular column is strengthened by the circular steel jacket, almost no extensive lateral strength deterioration of 5% or more was not observed
until the story-drift reached $R = \pm 4\%$, even under quite high axial compression of 19.4 MPa.

7. In the rectangular jacket specimens, shortening of columns in longitudinal direction was observed with the increase of the axial compression load and inter-story displacement, however, there was no or only slight shortening was observed in the circular jacket specimens even under quite high axial compression of 19.4 MPa.

8. In all the column specimens retrofitted by the rectangular and circular steel- and CF sheet-jackets, any deterioration in axial load-carrying capacity did not take place even under quite high axial-compression of 19.4 MPa.

9. The ultimate lateral strengths of the circular jacketing specimens can be well predicted by the existing proposed equations by considering the confining effects given by the circular steel- and CF sheet-jackets.

10. The ultimate lateral strengths of the rectangular steel jacket specimens can be also well predicted by the existing proposed equations by considering the location of the flexural yield hinges.

11. Further investigation of the compressive stress-strain relation of the concrete confined by the rectangular CF sheet jacketing will be needed furthermore.

REFERENCES


Table 1 -- List of test specimens

<table>
<thead>
<tr>
<th>Specimen</th>
<th>Cross-section of column</th>
<th>Seismic strengthening</th>
<th>$f'_{c}$ (MPa)</th>
<th>$f'_{s}$ (MPa)</th>
<th>$\sigma_s = N/An$ (MPa)</th>
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<tbody>
<tr>
<td>R-14.5-6-H</td>
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<td></td>
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<td>59.8</td>
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<td>25.9</td>
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<td>R-19.4-3-H</td>
<td>Rectangular</td>
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<td>22.3</td>
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<td>13.2</td>
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<td>R-CF-19.4-H</td>
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<td>57.5</td>
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<tr>
<td>C-SSP-19.4-H</td>
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<td>$t = 4.5$</td>
<td>14.30</td>
<td>22.9</td>
<td>55.1</td>
</tr>
</tbody>
</table>

Remarks:
- $b$: breadth of R/C rectangular columns
- $D$: depth of R/C rectangular columns
- $t$: thickness of strengthening material
- $d$: column diameter
- $\rho_s$: shear reinforcement ratio
- $\rho_p$: volumetric ratio of strengthening plate (circular column)
- $(\rho_s + 2t/d)$
- $\sigma_y$: yield strength of strengthening material
- $\rho_p \sigma_y$: amount of shear reinforcement index (circular column)
- $\rho_s \sigma_y$: amount of shear reinforcement index
- $f'_{cc}$: compressive strength of non-shrinkage mortar
- $\sigma_s$: applied constant axial stress in original column section
Table 2 -- Material properties

<table>
<thead>
<tr>
<th>$d_e$ (MPa)</th>
<th>Reinforcing bar</th>
<th>Tensile strength (MPa)</th>
<th>Elongation (%)</th>
<th>Steel plate</th>
<th>Tensile strength (MPa)</th>
<th>Elongation (%)</th>
<th>Unit weight of CF sheets (g/m²)</th>
<th>Thickness of CF sheet (mm)</th>
<th>Tensile strength (MPa)</th>
<th>Elastic modulus (GPa)</th>
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<td>5.6</td>
<td>D10 357 502 25</td>
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<td>344 44</td>
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<td>0.222 4220 267</td>
<td>200</td>
<td>0.111 4110 246</td>
<td>400</td>
<td>0.222 4220 267</td>
</tr>
<tr>
<td>13.2</td>
<td>D5 466* 541 10</td>
<td>347</td>
<td>472 36</td>
<td>400 228</td>
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<td>400</td>
<td>0.222 4110 246</td>
<td>400</td>
<td>0.222 4110 246</td>
</tr>
<tr>
<td>19.4</td>
<td>D6 328* 499 24</td>
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<td>472 36</td>
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Note: * The yield strength of 0.2% offset

Table 3 -- Evaluation of ultimate lateral strengths

<table>
<thead>
<tr>
<th>Specimen</th>
<th>Strength of concrete (MPa)</th>
<th>Experimental value</th>
<th>Predicted value</th>
<th>Ratio</th>
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<td>$Q_{\text{ex}}$</td>
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<td>(kN)</td>
<td>(kN)</td>
<td>(kN)</td>
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<tr>
<td>R-CF-13.2-H</td>
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<td>110 0.89 72 129</td>
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<td>345</td>
<td>0.99 0.99</td>
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Note: * The top of the steel contacted the lower surface of the upper stub of the specimen.

Table 4 -- Experimental value and predicted value of the ultimate flexural strengths

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<tr>
<th>Specimen</th>
<th>Strength of concrete (MPa)</th>
<th>Ratio of axial compression $\alpha$</th>
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<th>Predicted value</th>
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<th>Ratio</th>
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<td>$M_{\text{pr}}$</td>
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<td>(kN m x m²)</td>
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<td>0.59 84.4</td>
<td>10.0</td>
<td>1.22</td>
<td>0.89 78</td>
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<td>1.00</td>
<td>0.93 90</td>
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<td>93.1</td>
<td>93.1</td>
<td>1.11</td>
<td>0.97 97</td>
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Figure 1 -- Seismic retrofit for R/C rectangular column by circular steel jacket (2000 Oita, Japan)

Figure 2 -- First floor plan
Figure 3 -- Details of test specimens

(a) Section of column

(b) Rebar arrangement of column

(c) Rebar arrangement of column

(d) Rebar arrangement of column

Figure 4 -- Test setup

(a) Low axial-load

(b) Medium and high axial-load
Figure 5 -- Lateral force ($Q$) – story drift ($R$) relations

(a) $\sigma_{0}=5.6\text{MPa}$ (Half-length column specimen)

(b) $\sigma_{0}=13.2\text{MPa}$ (Half-length column specimen)

(c) $\sigma_{0}=13.2\text{MPa}$ (Full-length column specimen)

(d) $\sigma_{0}=19.4\text{MPa}$ (Full-length column specimen)
Figure 6 -- $Q - R$ envelope curves

Figure 7 -- Axial deformation ($\delta$) – story drift ($R$) relations

Figure 8 -- ($\sigma_c$) – ($\varepsilon_c$) relations of concrete
Figure 9 -- Axial compression ($N$) versus ultimate flexural strength ($M_{u2}$) interaction
Behavior and Modeling of FRP-Confined Concrete: A State-of-the-Art Review

by J.G. Teng and L. Lam

Synopsis: Over the past decade, fibre-reinforced polymer (FRP) composites have found wide applications in civil engineering, particularly in the retrofit of structures. One important application of FRP composites in the retrofit of reinforced concrete (RC) structures is to provide confinement to columns for enhanced strength and ductility. As a result, a large number of studies have been carried out on the compressive behaviour of FRP-confined concrete. This paper provides a state-of-the-art review of existing studies on this subject, with the emphasis being on the revelation of the fundamental behaviour of FRP-confined concrete and the modelling of this behaviour. Both monotonic loading and cyclic loading are covered, although only a limited amount of work is available on the latter. The paper is explicitly limited to concrete confined with FRP jackets, in which the fibres are oriented only or predominantly in the hoop direction, but many of the observations made in this paper are also relevant to concrete confined with FRP jackets with a significant axial stiffness, as found in concrete-filled FRP tubes as new columns.

Keywords: behavior; concrete; confinement; FRP; modeling; stress-strain curve
INTRODUCTION

Fibre reinforced polymer (FRP) composites have found increasingly wide applications in civil engineering due to their high strength-to-weight ratio and high corrosion resistance. One important application of FRP composites is as wraps or jackets for the confinement of reinforced concrete (RC) columns for enhanced strength and ductility. In FRP-confined concrete subject to axial compression, the FRP is principally loaded in hoop tension while the concrete is loaded in tri-axial compression, so that both materials are used to their best advantages. Both the strength and the ultimate strain of concrete can be greatly enhanced as a result of FRP confinement, while the high tensile strength of FRP can be fully utilised. Instead of the brittle behaviour exhibited by both materials, FRP-confined concrete possesses greatly enhanced ductility.

This paper provides a state-of-the art review of existing studies, with the emphasis being on the revelation of the fundamental behaviour of FRP-confined concrete and the modelling of this behaviour. The paper is limited to concrete confined with FRP jackets in which the fibres are oriented only or predominantly in the hoop direction as such jackets are commonly used in column retrofit. Nevertheless, many of the observations made in this paper are also applicable to concrete confined with FRP jackets with a significant axial stiffness, as found in concrete-filled FRP tubes as new columns.

BASIC BEHAVIOUR OF FRP-CONFINED CONCRETE

Confining action of FRP jackets

When a concrete cylinder confined with an FRP jacket is subject to an axial compressive stress $\sigma_c$, it expands laterally. This expansion is confined by the FRP jacket which is loaded in tension in the hoop direction. The confining pressure provided by the FRP jacket increases continuously with the lateral strain of concrete because of the linear elastic stress-strain behaviour of FRP, in contrast to steel-confined concrete in which the confining pressure remains constant when the steel is in plastic flow. Failure of FRP-confined concrete generally occurs when the hoop rupture strength of the FRP jacket is reached. For FRP jackets with fibres oriented only or predominantly in the hoop direction, the lateral (radial) confining pressure acting on the concrete core $\sigma_l$ is given by

$$\sigma_l = \frac{E_{frp} \varepsilon_h}{R}$$  \hspace{1cm} (1)
where $E_{frp}$ is the elastic modulus of FRP in the hoop direction, $t$ is the thickness of the FRP jacket, $\varepsilon_h$ is the hoop tensile strain in the FRP jacket, and $R$ is the radius of the confined concrete core, respectively. The lateral confining pressure reaches its maximum value $f_l$ at the rupture of FRP, given by Eq. 1 with $\varepsilon_h$ being the FRP hoop rupture strain $\varepsilon_{h,rup}$. The FRP hoop rupture strain is generally not the same as the ultimate tensile strain of the FRP from tensile tests of flat coupons as discussed in some detail later in the paper. The ratio between the confining pressure $f_l'$ when the jacket ruptures (i.e. the maximum confining pressure) and the compressive strength of unconfined concrete $f_{co}'$ is commonly referred to as the confinement ratio.

Axial stress-strain behaviour of confined concrete

Fig. 1 shows typical stress-strain curves of carbon FRP (CFRP)-confined concrete and unconfined concrete, obtained by Lam et al.\textsuperscript{1} from compression tests on 152 mm x 305 concrete cylinders. Also shown in this figure are those from compression tests conducted by Sfer et al.\textsuperscript{2} on a 150 mm x 300 mm concrete cylinder with active confinement at a constant lateral pressure of 1.5 MPa, and by Candappa et al.\textsuperscript{3} on 100 mm x 200 mm concrete cylinders with active confinement at lateral pressures of 4 and 12 MPa respectively. The unconfined concrete cylinder strengths of these specimens are in the range of 32-42 MPa and all these tests were conducted under displacement control. For the purpose of comparison, the axial stress $\sigma_c$ is normalized by the compressive strength of the unconfined concrete $f_{co}'$ while the axial strain $\varepsilon_c$ or lateral strain $\varepsilon_l$ is normalized by the axial strain of the unconfined concrete at its peak stress $\varepsilon_{co}$. The axial strains of concrete are defined as positive and the lateral strains negative. The CFRP had a nominal thickness of 0.165 mm per ply, an elastic modulus around 250 GPa based on the nominal jacket thickness and an ultimate tensile strain of 1.52 %.

The two axial stress-axial strain curves of CFRP-confined concrete shown in Fig 1 feature a monotonically ascending bi-linear shape. By contrast, the axial stress-axial strain curves of actively confined concrete feature a descending branch. This is because in the case of FRP-confined concrete, as the axial stress increases, the confining pressure provided by the jacket also increases instead of remaining constant. If the amount of FRP provided exceeds a certain threshold value, this confining pressure increases fast enough to ensure that the stress-strain curve is monotonically ascending. This bilinear phenomenon was also previously observed by Xiao et al.\textsuperscript{4} for concrete stub columns confined with steel tubes before the yielding of steel. His steel tubes were primarily used as transverse reinforcement and were not directly loaded in the axial direction. Monotonically ascending stress-strain curves have been observed in the majority of existing tests on FRP-confined concrete. Naturally, if the amount of FRP is small, a descending branch is possible and has been observed in tests\textsuperscript{5-7}.

Dilation properties

The dilation properties of unconfined concrete and actively confined concrete have been well established\textsuperscript{8,9}. Under axial compression, unconfined concrete has an initial
Poison's ratio (the lateral-to-axial strain ratio at \( c_H = 0 \), given as absolute value) between 0.15 and 0.22 and experiences a volumetric reduction or compaction up to 90% of the peak stress. Thereafter the concrete shows volumetric expansion or dilation as a result of the rapidly increasing lateral-to-axial strain ratio. Unstable dilation after the initial compaction has also been observed in actively confined concrete in tri-axial compression tests, although at a higher confining pressure, the dilation is less pronounced as described by Pantazopoulou.

A number of studies have been concerned with the dilation properties of FRP-confined concrete. Mirmiran and his co-workers compared the volumetric responses of FRP-confined concrete with those of plain concrete and steel-confined concrete, which behaves similarly to actively confined concrete after the yielding of the confining steel. They demonstrated that for steel-confined concrete, unstable dilation occurs when steel yields, but for FRP-confined concrete, the linearly increasing hoop stress of the FRP jacket can eventually curtail the dilation if the amount of FRP is sufficiently large.

In Fig. 2, the normalized lateral strain is plotted against the normalized axial strain for all the specimens examined in Fig. 1. It can be seen from this figure that the normalized lateral-to-axial strain curve of the FRP-confined specimen initially follows that of the unconfined specimen but gradually deviates from it as the axial strain increases, and then sequentially intersects each of the corresponding curves of the actively confined specimens. Huang et al. observed that at these points of intersection, the current confinement ratio of the FRP-confined specimen is almost equal to the confinement ratio of the corresponding actively confined specimen. Here, the current confinement ratio for a given state of deformation is defined as the ratio between the confining pressure provided by the FRP jacket as defined by Eq. 1 and the unconfined concrete strength. Moreover, they found that the lateral strain-axial strain responses of unconfined, actively confined and FRP-confined concrete can all be approximated by the following unified equation:

\[
\frac{\varepsilon_c}{\varepsilon_{co}} = 0.85 \left[ 1 + 0.75 \left( \frac{-\varepsilon_l}{\varepsilon_{co}} \right) \right]^{0.7} \exp \left[ -7 \left( \frac{-\varepsilon_l}{\varepsilon_{co}} \right) \right].
\]

Eq. 2 is based on the same sign convention for concrete strains as that adopted in plotting Fig. 1. The hoop strain in the FRP jacket is assumed to have the same magnitude as the lateral strain of the concrete but the opposite sign. Fig. 3 shows the performance of Eq. 2 for the specimens examined in Figs 1 and 2.

**Ultimate condition**

As eventual failure of FRP-confined concrete is by the rupture of the FRP jacket, the ultimate condition of the confined concrete, often characterized by its compressive strength and ultimate axial strain, is intimately related to the ultimate tensile strain or tensile strength of the confining FRP jacket in the hoop direction. In most existing theoretical models for FRP-confined concrete, it has been assumed that tensile rupture of FRP occurs when the hoop stress in the FRP reaches its tensile strength from material...
tests, either flat coupon tests\textsuperscript{17} or ring splitting tests\textsuperscript{18}. However, extensive experimental results have shown that the material tensile strength of FRP cannot be reached in FRP-confined concrete as the hoop rupture strains of FRP measured in FRP-confined cylinder tests have been found to be considerably smaller than the ultimate tensile strains obtained from material tensile tests\textsuperscript{6,19,20}.

This uncertainty with FRP hoop rupture strains has led to difficulties in predicting the ultimate condition of FRP-confined concrete, particularly the ultimate axial strain. This is because the ratio between the FRP hoop rupture strain and the material ultimate tensile strain varies with the type of FRP\textsuperscript{21}. De Lorenzis and Tepfers\textsuperscript{22} showed that of the models they reviewed and assessed\textsuperscript{6,12, 23-29}, none was able to predict the ultimate axial strain with reasonable accuracy if the hoop strain in the FRP jacket at rupture is taken to be equal to the material ultimate tensile strain. Xiao and Wu\textsuperscript{6}, Jin\textsuperscript{30} and Moran and Pantelides\textsuperscript{31} suggested the use of a reduced FRP strength. Lam and Teng\textsuperscript{21} recently suggested that in developing confinement models, the actual hoop rupture strain $\varepsilon_{h, rup}$ measured in the FRP jacket should be used to evaluate the stress in the FRP rather than simply using the FRP material tensile strength $\sigma_{frp}$.

Several causes have been suggested for the difference in the ultimate tensile strain between FRP tensile test specimens and FRP jackets confining concrete\textsuperscript{6, 19-22,29}. These suggestions are generally speculative without sound experimental evidence. Lam and Teng\textsuperscript{32,33} recently conducted the first carefully planned study involving comparative experiments in an attempt to clarify the causes for the reduced strain capacity of FRP when used to confine concrete. The experimental program covered flat coupon tensile tests\textsuperscript{17} and ring splitting tests\textsuperscript{18} on CFRP and GFRP specimens, and compression tests on concrete cylinders wrapped with one to three plies of CFRP and GFRP. Based on the test observations, Lam and Teng\textsuperscript{32} concluded that the average hoop rupture strains of FRP measured in FRP-confined concrete cylinders are affected by at least three factors: (a) the curvature of the FRP jacket; (b) the deformation non-uniformity of cracked concrete; and (c) the existence of an overlapping zone, in which the measured strains are much lower than strains measured elsewhere. While the effect of curvature is material dependent, which means that the curvature of the FRP jacket has a stronger detrimental effect on CFRP than on GFRP in the context of their study, the non-uniformity of strain distribution is independent of the type of FRP.

In addition, Harries and Carey\textsuperscript{34} recently investigated the effect of bond between the FRP jacket and the concrete core on the hoop rupture strain $\varepsilon_{h, rup}$. Their test results showed that on the unbonded specimens which contained a 0.08 mm thick plastic wrap between the FRP jacket and the concrete, the hoop strains were nearly uniform around the circumference. However, the average hoop rupture strains of these specimens did not appear to be higher than those measured on the specimens with the FRP jacket bonded to the concrete. Thus, the role played by adhesive bonding in reducing the hoop rupture strain has not been clearly established by these tests.
From the large number of studies on FRP-confined concrete, many stress-strain models have resulted. These models can be classified into two categories: (a) design-oriented models, and (b) analysis-oriented models. In the first category, stress-strain models are presented in closed-form expressions, while in the second category, stress-strain curves of FRP-confined concrete are predicted using an incremental iterative numerical procedure in which the interaction between the concrete core and the confining FRP is explicitly accounted for. Design-oriented models are more suitable for direct use in design, but analysis-oriented models are more versatile and powerful and are particularly suitable for use in nonlinear computer analysis.

**Design-oriented models**

Apart from models representing the stress-strain curve of FRP-confined concrete with two or three strictly linear segments\(^\text{6,35,36}\), two expressions have frequently been exploited in describing the stress-strain behaviour of FRP-confined concrete: an equation proposed by Sargin\(^\text{37}\) and a four-parameter stress-strain curve proposed by Richard and Abbot\(^\text{38}\).

The equation proposed by Sargin\(^\text{36}\) has the following form:

\[
\sigma_c = \frac{A \varepsilon_c + (D - 1) \left( \frac{\varepsilon_c}{\varepsilon_{co}} \right)^2}{1 + (A - 2) \frac{\varepsilon_c}{\varepsilon_{co}} + D \left( \frac{\varepsilon_c}{\varepsilon_{co}} \right)^2}
\]

where \(A\) and \(D\) are constants controlling the initial slope and the descending path of the stress-strain curve respectively. Eq. 3 has been used in stress-strain models for steel-confined concrete\(^\text{39,40}\) and unconfined concrete\(^\text{41}\). The well-known Hognestad\(^\text{42}\) parabola is a special case of Equation 3 with \(A = 2\) and \(D = 0\). Ahmad et al.\(^\text{43}\) used Eq. 3 to represent the whole stress-strain curve of FRP-confined concrete, with \(f_{co}^\prime\) and \(\varepsilon_{co}\) being replaced by the stress and strain at the peak stress of FRP-confined concrete. Their model does not feature a bilinear shape. In all other models\(^\text{25, 27,28,44-46}\) which exploit Eq. 3 or a modified version, the stress-strain curve consists of two segments with only the first segment being represented by Eq. 3 or the Hognestad parabola while the second segment is defined using a separate expression.

The four-parameter curve of Richard and Abbot\(^\text{38}\), which was proposed to describe the elastic-plastic behaviour of structural systems, is given by

\[
\sigma = \frac{(E_1 - E_2)\varepsilon}{1 + \left( \frac{(E_1 - E_2)\varepsilon}{f_{co}} \right)^n + E_2\varepsilon}
\]

where \(E_1\) and \(E_2\) are the elastic moduli of the concrete and FRP, respectively, \(\varepsilon\) is the strain, \(\varepsilon_{co}\) is the peak strain, and \(n\) is the strain hardening exponent.
where $\sigma$ and $\varepsilon$ are the stress and the strain respectively, $f_o$ is a reference stress, $E_1$ is the initial modulus, $E_2$ is the plastic modulus, and $n$ is the shape parameter controlling the transition from the first portion to the second portion of the stress-strain curve. An advantage of Eq. 4 is that the bilinear shape of a stress-strain curve can be described by a single equation. Consequently, Eq. 4 has been used in a number of stress-strain models for FRP-confined concrete\textsuperscript{12,31,47-49} and a model by Toutanji and Saafi\textsuperscript{50} for concrete confined with FRP-reinforced PVC tubes. A model proposed by Campione and Miragia\textsuperscript{51} uses a modified form of Eq. 4.

The present authors recently developed a design-oriented model\textsuperscript{21} based on a careful interpretation of a large test database assembled by them from the open literature. This model consists of a parabolic first portion with its initial slope being the elastic modulus of unconfined concrete and a linear second portion which intercepts the stress axis at the strength of unconfined concrete. The parabolic first portion meets the linear second portion with a smooth transition. This model allows the use of test values or values suggested by design codes for the elastic modulus of unconfined concrete and accounts for the effect of FRP confinement on the non-linear response of concrete before the transition point. It reduces to the design stress-strain curve for unconfined concrete recommended by Eurocode 2\textsuperscript{41} provided the same initial elastic modulus is used, which is an important advantage of the model in practical applications.

Analysis-oriented models

A number of analysis-oriented models for FRP-confined concrete\textsuperscript{14,29,52-54} have been developed on the basis of the assumption that the axial stress and axial strain of concrete confined with FRP at a given lateral strain are the same as those of the same concrete actively confined with a constant confining pressure equal to that supplied by the FRP jacket. This assumption is equivalent to assuming that the stress path of the confined concrete does not affect its stress-strain behaviour. The validity of this assumption has recently demonstrated by Huang et al.\textsuperscript{15,16}. Based on this assumption, an active confinement model for concrete can be used to evaluate the axial stress and axial strain of FRP-confined concrete at a given confining pressure and the interaction between the concrete and the FRP jacket can be explicitly accounted for by equilibrium and radial displacement compatibility considerations. As a result, the stress-strain curve of FRP-confined concrete can be generated as a curve that crosses a series of stress-strain curves for the same concrete confined with a range of lateral pressures. A model proposed by Mander et al.\textsuperscript{55} or a modified version has been used as the active confinement model in the above five models for FRP-confined concrete. The lateral strain at a given axial strain, which is the key for an accurate analysis-oriented model, is found by different means in these models.

Apart from the above five models based on an active confinement model, other approaches have also been used for modelling FRP-confined concrete. Harmon et al.\textsuperscript{13} developed a model for FRP-confined concrete based on the concept of crack slip and separation in the concrete. Becque et al.\textsuperscript{56} proposed a model based on Gerstle's octahedral stress-strain models\textsuperscript{57,58} with some modifications. In addition, the plasticity model
proposed by Karabinis and Rousakis for FRP-confined concrete may also be classified as an analysis-oriented model, although the numerical integration required by the plasticity approach makes it more complicated than other analysis-oriented models.

A recent model developed at The Hong Kong Polytechnic University followed an approach similar to that adopted in the five models based on an active confinement model. In this model, Eq. 2 is used to determine the lateral strain-axial strain relationship, and a modified version of Mander et al.’s model is used to determine the stress-strain curve under current confining pressures, as demonstrated in Fig. 4.

PERFORMANCE OF STRESS-STRAIN MODELS

To assess the performance of existing stress-strain models, their predictions are compared here with results from tests on three CFRP- and two GFRP-wrapped concrete cylinders (152 mm × 305 mm) recently conducted by the authors. These tests were carefully conducted with hoop strains measured at 8 points around a circumference. The hoop strains used for comparison here were averaged from strains measured outside a 150 mm overlapping zone. It should be noted that the conclusions reached in this section are in agreement with those reached through similar comparisons undertaken by the authors using the test data of Xiao and Wu, which are not reported here due to space limitation.

The comparisons between the test results and the predictions of the design-oriented models and analysis-oriented models are shown in Figs 5 and 6 respectively. The details of the specimens are also shown in these figures. The elastic moduli and material ultimate tensile strains of the FRPs were obtained from flat coupon tests according to ASTM D3039 and calculated based on the nominal thicknesses, which were 0.165 mm per ply for the CFRP and 1.27 mm ply for the GFRP. In predicting the stress-strain curves, the elastic modulus and initial Poisson’s ratio of unconfined concrete were either those specified in an individual model or taken to be $E_c = 4730 \sqrt{f'_{cc}}$ (MPa) and $\nu_c = 0.18$ if they are not specified in the model. The axial strain at the compressive strength of unconfined concrete $\varepsilon_{cc}$ was assumed a constant value of 0.002 for all models for a more direct comparison even though Huang et al. originally suggested $\varepsilon_{cc} = 0.0022$ for use with their model.

Fig. 5 shows comparisons between stress-strain curves from tests on CFRP-confined concrete and those from the more accurate design-oriented models as identified in a preliminary comparison. In Fig. 5a, comparisons are made for the CFRP-confined cylinders where the hoop strain or stress in the FRP at rupture as defined in the original model is used. For the models of Xiao and Wu and Jin, reduction factors of 0.50 and 0.96 are used for the ultimate FRP tensile strain respectively, according to their definitions. For Moran and Pantelides’ model, the CFRP hoop rupture strain is taken as 0.0085 based on their own suggestion. For Lam and Teng’s model, the CFRP is assumed to reach 58.6% of the material tensile strength, which is the average value from a large test database. For other models, the FRP tensile
strength from flat coupon tests is used. In Fig. 5b, the actual average hoop rupture strains obtained from the confined concrete tests are used for all models. It can be observed from Figure 5a that if the hoop strain at the rupture of the FRP jacket is assumed the value specified in the original model, the three models using a substantially reduced value for this rupture strain\(^{21,31,49}\) perform better than the other models. However, if the actual hoop rupture strain is used instead, then the performance of the other models becomes better. Among these models, the models of Saafi et al.\(^{28}\), Jin\(^{30}\), Moran and Pantelides\(^{31}\), Xiao and Wu\(^{49}\), and Lam and Teng\(^{21}\) give closer predictions of the shape of the stress-strain curve and the ultimate condition.

Fig. 6 shows comparisons between stress-strain curves obtained from both tests conducted on CFRP and GFRP-confined cylinders and analysis-oriented models. The predicted stress-strain curves terminate at the point where the average FRP hoop rupture strain from the tests is reached. For the CFRP-confined concrete cylinders (Fig. 6a), the models of Spoestra and Monti\(^{29}\), Fam and Rizkalla\(^{53}\), Chun and Par\(^{54}\) and Huang et al.\(^{15}\) perform well and are superior to other models. However, for the GFRP-confined concrete cylinders (Fig. 6b), only the model by Huang et al.\(^{15}\) provides close predictions.

The above comparisons suggest that central to a design-oriented model is an accurate definition of the ultimate condition. Provided that the ultimate condition of FRP-confined concrete is accurately defined, the stress-strain curve can be closely matched using different forms of equations. For analysis-oriented models, the key for accurate predictions is the lateral-to-axial strain relationship for FRP-confined concrete.

**FRP-CONFINED CONCRETE UNDER CYCLIC COMPRESSION**

While the behaviour of FRP-confined concrete under monotonic compression has been extensively studied, only a very limited amount of work has been conducted on the behaviour of FRP-confined concrete subject to cyclic compression\(^{1,10,60-65}\). To the best knowledge of the authors, only one stress-strain model has been proposed for FRP-confined concrete under cyclic compression\(^{62}\).

A recent experimental study conducted by Lam et al.\(^{1}\) at The Hong Kong Polytechnic University included tests of CFRP-confined standard concrete cylinders of 152 mm by 305 mm subjected to monotonic compression and cyclic compression respectively. Both the loading schemes and the instrumentation were carefully planned. The study led to a better understanding of several aspects of the behaviour of FRP-confined concrete under cyclic compression. Details of the study are available elsewhere\(^{65}\). Fig. 7 shows typical stress-strain curves of FRP-confined concrete under monotonic or cyclic compression, obtained from tests of concrete cylinders with an unconfined concrete strength of 38.9 MPa and a two-ply CFRP jacket. The stress-strain curves of unconfined concrete under monotonic loading are also shown for comparison.

A basic hypothesis in studies on the cyclic stress-strain behaviour of unconfined and steel-confined concrete is that an envelope curve exists for the stress-strain responses and this envelope curve is approximately the same as the stress-strain curve of the same
concrete under monotonic loading. Lam et al. showed that this hypothesis is also valid for FRP-confined concrete\textsuperscript{1,65}, as can be seen from Fig. 7, where the envelope curve of the cyclic stress-strain response (the heavy dashed line) almost coincides with the stress-strain curve obtained from the monotonic loading test. Lam et al. also showed that for the same concrete, the plastic strain, defined as the residual axial strain when concrete is unloaded to the zero stress, is linearly related to the axial strain at the starting point of unloading from the monotonic loading curve (or the envelope curve), but seems to be independent of the amount of FRP-confinement\textsuperscript{1,65}. Furthermore, Lam et al. pointed out that the cyclic response of FRP-confined concrete in the stress-strain domain is not unique in that the loading history has a cumulative effect on the plastic strain and stress deterioration of concrete\textsuperscript{1,65}, as previously found by Karson and Jirsa\textsuperscript{66} for unconfined and steel-confined concrete. This non-uniqueness can be demonstrated by examining the cyclic stress-strain response of a specimen subjected to repeated unloading/reloading cycles at a prescribed displacement level, as shown in Fig. 7. It is clear that an increase in the number of cycles is accompanied by an increase in the plastic strain and a decrease in the new stress at the unloading strain (Fig. 7). The ultimate condition of FRP-confined concrete under cyclic loading was also examined in detail by Lam et al.\textsuperscript{1}. They noticed that although all their FRP-confined specimens subjected to cyclic loading failed by FRP hoop rupture, the hoop rupture strains in these specimens are higher than those observed in corresponding specimens subjected to monotonic loading\textsuperscript{1,65}. A similar observation has also been made by Theodoros\textsuperscript{62}. As a result of this higher FRP hoop rupture strain, the compressive strength and ultimate axial strain of FRP-confined concrete subject to cyclic loading are higher than those of FRP-confined concrete subject to monotonic loading. A detailed discussion of the ultimate condition of FRP-confined concrete under cyclic compression is available in Reference\textsuperscript{1}.

Lam and Teng's design-oriented stress-strain model\textsuperscript{21} for FRP-confined concrete has been shown to accurately predict the envelope curve, compressive strength and ultimate strain of FRP-confined concrete under cyclic compression\textsuperscript{1,65}. However, the only cyclic stress-strain model by Shao\textsuperscript{62} for FRP-confined concrete does not appear to be adequate, based on the comparisons given in References\textsuperscript{1,65} with Lam et al.'s test data\textsuperscript{1,65}. In Shao's model, the unloading path is represented using a polynomial and the reloading path is represented using straight lines. The envelope curve is predicted using Samaan et al.'s stress-strain model\textsuperscript{12}. A comparison between Shao's model and Lam et al.'s test result (Fig. 8) shows that the predictions of Shao's model for the unloading curves deviate significantly from the test curves, although it gives reasonably close predictions for the reloading curves. Another deficiency of Shao's model is the uniqueness assumption. As a result, this model is incapable of predicting the cumulative effect of loading history on the permanent strain and stress deterioration of FRP-confined concrete.

CONCLUDING REMARKS

This paper has provided a critical review of existing research on normal strength concrete uniformly confined with FRP jackets, with which most existing studies have been concerned. For such FRP-confined concrete, it may be concluded that the large number
of studies on FRP-confined concrete have led to a good understanding of its behaviour and many stress-strain models, the latest of which provide close predictions of test results. While there will inevitably be further refinement of models for such FRP-confined concrete, much more attention should be directed to several other issues of FRP-confined concrete which have received limited attention. These include (a) FRP-confined high strength concrete; (b) interaction between steel confinement and FRP confinement; (c) concrete with non-uniform confinement as found in FRP-confined rectangular and other non-circular sections and sections under eccentric compression; (d) FRP-confined concrete subject to cyclic loading; and (e) concrete confined by hybrid FRP composites. The limited existing research on FRP-confined concrete subject to cyclic compression has been noted in the paper, while a detailed survey of existing work on FRP-confined concrete in rectangular columns has been presented by Lam and Teng67.

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Figure 1 – Axial stress vs axial and lateral strains of FRP-confined and actively confined concrete
Figure 2 – Lateral strain vs axial strain of FRP-confined and actively confined concrete

Figure 3 – Performance of Equation 2

Figure 4 – Generation of a stress-strain curve in an analysis-oriented model of FRP-confined concrete based on an active confinement model
Figure 5 – Performance of design-oriented models

(a) Using the original definition of FRP hoop rupture strain

(b) Using the test FRP hoop rupture strain
Figure 6 – Performance of analysis-oriented models

(a) Specimens confined with 2 plies of CFRP

(b) Specimens confined with 2 plies of GFRP

Figure 7 – Cyclic stress-strain curves of CFRP-confined concrete in comparison with monotonic stress-strain curves of confined and unconfined concrete
Figure 8 – Prediction of unloading and reloading paths using Shao’s model
The Influence of Concrete Strength and Confinement Type on the Response of FRP-Confined Concrete Cylinders

by J.F. Stanton and L.M. Owen

Synopsis: This paper describes a series of tests on concrete cylinders confined by carbon fiber jackets. The primary variables in the investigation were the thickness of the jackets (and therefore the lateral confinement stress), the size of the cylinders, the loading speed, and the loading type (monotonic vs. cyclic), and the jacket type (individual layers or continuous wrap). Of these parameters, the lateral confining stress was found to have the greatest influence, but the coefficient for the concrete used was found to be slightly lower than the 4.1 that is implicit in ACI318-02. The cylinder size, the loading speed and the cyclic loading regimes were found to have essentially no influence on stress and strain at failure. The continuously wound jackets were found to initiate failure by delamination, rather than fracture of the fiber, and to lower the stress and strain at failure.

Keywords: concrete; confinement; cylinders; failure; fiber; jacket
INTRODUCTION

Responses of Interest

The response of concrete to loads is improved by confinement. However, analysis of confined concrete is relatively complex, because it must account for several media (concrete, the confining medium, and in some cases the bond between them), because the concrete cracks and so is a discontinuous medium with both frictional and cohesive properties, and because the confining stress may not be uniform throughout the concrete. Thus, in most cases simplifications are necessary and there is a need to focus on the response quantities of greatest interest. The primary ones are the stress and strain at failure. Secondary properties include the stiffness at different stages of loading, especially if a finite element model is to be formulated. Most researchers have concentrated on establishing values for the stress and strain at failure.

Modeling Approaches

The modeling approach adopted generally follows the intended end use. Many structural elements are essentially one-dimensional and are confined bi-axially, in which case a relatively simple model, suitable for hand calculation, is appropriate. Many of such models (e.g. Mirmiran et al. 1997) are based on one or more empirical assumptions, and are mainly suitable for low levels of confinement, where the confined strength is no greater than, say, three times the unconfined strength. In the vast majority of cases, this limitation proves not to be critical.

More comprehensive models account for additional behaviors, such as the nonlinear relationship between the octahedral normal and shear stresses at failure, and are of necessity more complex. Examples are given by the failure criteria developed by Willam and Warnke (1975), and Ottosen (1977), which are described in some detail by Chen (1982). Because of their complexity, these models are best suited for implementation in Finite Element codes. The economics of the profession dictate that such models are used on only a small number of projects, in which the problem is complex, the confinement is high or the response must be known with great reliability.
Important variables.

The most important variables are:

- Confinement stress (at failure), $f'_l$. This has the strongest influence of any parameter on the confined strength.
- Constitutive law of the confining material. FRP is approximately elastic up to failure, whereas steel yields. The two display very different stress-strain curves for the confined concrete.
- Uniformity of confinement stress. In circular columns subjected to concentric axial load, the confining stress is uniform. However, in rectangular cross-sections, when the confinement is provided by an external jacket or by perimeter ties alone, the confinement stress is high at the corners and negligible at the mid-points of the sides.
- Stress gradient. High concrete stresses are most commonly induced by bending, so that is the circumstance under which confinement is most needed. However, the mechanics and benefits of confinement are much more difficult to establish for cross-sections that experience a stress gradient than for members loaded by concentric compression. Even the compression strains are difficult to measure, because they are usually obtained indirectly from curvature, which requires the assumption of a plastic hinge length.
- Concrete properties. Confined concrete possesses strength by virtue of cohesive strength ($C$) and internal friction (angle $\phi$). The values of these parameters are not necessarily the same for all concrete mixes, and do not necessarily remain constant through the load history. In addition, stiffness parameters (e.g. an effective bulk modulus, $K_{eff}$, explained later in this paper) are needed if axial strain is to be related to lateral strain, which controls failure of the confining medium.

Given the uncertainties in the foregoing parameters, a complex model is often not justified in practice, and use of a simple failure criterion, such as the Mohr-Coulomb surface, is appropriate. This is particularly true when the confinement stress is relatively low.

Objectives and Scope

The purpose of the work described here was to explore the influence on confined concrete strength of some fundamental properties, such as concrete strength, confinement strength and stiffness, jacket construction, specimen size and loading type. The experimental work was conducted using concentrically loaded concrete cylinders.

THEORETICAL BACKGROUND

FRPs typically display behavior that is linearly elastic up to a stress very close to failure. This means that concrete confined by FRP represents a statically indeterminate system in which the axial stress at any time depends on the radial stress, which in turn depends on the hoop strain in the jacket. Thus stress and strain are linked during the load path. Only if the system fails when the fiber fails, and the stress at which it does so is
known, is the system statically determinate. By contrast, in a steel-confined system, the jacket has a stress-strain curve that is closer to being elasto-plastic, so the axial stress in the concrete can be more easily determined. Furthermore, even if the jacket hoop strain at failure is known, the compressibility of the concrete must be known in order to determine the axial strain in the concrete at system failure.

In this study a simple model was sought to predict the behavior displayed in the tests. The model was based on the concept of a Mohr-Coulomb failure envelope, with a constant internal friction angle in the compressive region. A detailed description of the behavior when at least one principle stress is positive (tensile) is not necessary for the present purposes, because all three principal stresses in a confined cylinder are compressive.

The Mohr-Coulomb failure envelope is shown in Figure 1. When the Mohr’s circle for the state of stress in the concrete touches the envelope, failure occurs by shear along an internal plane. As the hydrostatic stress increases, so does the normal stress on all internal planes, and so does the shear stress required to cause slip. When the confining stress is constant, and independent of the radial strain, slip constitutes failure of the system. However, in FRP-confined cylinders, the slip may lead to an increase in radial strain and therefore an increase in confining stress. In that case, first slip is not necessarily synonymous with failure.

When the concrete is experiencing internal shear slip, the axial stress, \( \sigma_z \), and the confining stress, \( \sigma_r \), are linked by

\[
\sigma_r = \sigma_z \left( \frac{1 - \sin \phi}{1 - 2 \sin \phi} \right) + 2C \left( \frac{\cos \phi}{1 - \sin \phi} \right)
\]

(1)

where

\( C \) = the cohesive strength of the concrete, given by the intercept on the vertical (shear stress) axis.
\( \phi \) = angle of internal friction

Here \( \phi \) is assumed to remain constant at all confining stresses, in the interests of simplicity. Numerous experiments have shown that in fact it drops slightly with increasing confining stress, probably due to crushing of the concrete particles, but for confining stresses of the level likely to be seen in practice, little error is introduced.

Differentiation of Equation 1, and the assumption that \( \phi \) is constant, lead to the incremental form

\[
d\sigma_r = d\sigma_z \left( \frac{1 - \sin \phi}{1 - 2 \sin \phi} \right)
\]

(2)
However, the radial and hoop strains in a circular cylinder are identical. If the jacket is elastic, the hoop strain in the jacket and the radial stress in the concrete are then related by

$$\sigma_r = -K_j \varepsilon_r$$

where the jacket stiffness, $K_j$ is given by

$$K_j = \frac{t_j E_j}{R}$$

Combining Equations 2, 3 and 4 leads to

$$d \sigma_z = -K_j \left( \frac{1+ \sin \phi}{1- \sin \phi} \right) d \varepsilon_r$$

If the concrete volumetric strain and hydrostatic stress are related through an incremental relationship of the form

$$d \sigma_{hyd} = K_{c,eff} d \varepsilon_{vol}$$

where $K_{c,eff}$ is an effective tangent bulk modulus that relates the incremental volumetric stress and strain, then the incremental axial stress and strain on the cylinder can be shown to be related by

$$\frac{d \sigma_z}{d \varepsilon_z} = K_j \frac{1+ \sin \phi}{2(1- \sin \phi)+ \frac{K_j}{K_{c,eff}} \left(1 - \frac{\sin \phi}{3}\right)}$$

Equation 7 gives the slope of the stress-strain curve of a confined cylinder. $K_{c,eff}$ is not necessarily constant with respect to hydrostatic stress, especially since the concrete in the cylinder is subjected simultaneously to shear and direct compression. However, $\phi$ is known to remain nearly constant provided that the confining stress does not exceed about $2f'_c$. Then, if the secondary slope of the stress-strain curve is approximately constant, $K_{c,eff}$ must also be approximately constant in that range. Experimental stress-strain data for a number of $K_j$ values would allow identification of the values of $K_{c,eff}$ and $\phi$.

A relationship between the axial and radial strains can be obtained by combining Equations 5 and 7 to give
If \( K_{c,\text{eff}} \) is constant, Equation 8 may be integrated and added to the unconfined concrete strain capacity to give, at failure of the jacket,

\[
\varepsilon_z = \varepsilon_{\text{ccu}} = \varepsilon_{\text{cou}} - \varepsilon_{\text{ju}} \left(2 + K_j C_c \right) + C_1
\]

(9)

where:

- \( \varepsilon_{\text{cou}} \) is the unconfined concrete strain at failure = -0.004
- \( \varepsilon_{\text{ju}} \) is the ultimate tensile jacket strain
- \( C_c \) is a constant for the concrete being used.

\[
C_1 = \frac{1 - \sin \phi}{3 K_{c,\text{eff}} (1 - \sin \phi)}
\]

If Equation 9 is to be consistent with values for unconfined cylinders, \( C_1 \) must be equal to \( 2 \varepsilon_{\text{ju}} \), so

\[
\varepsilon_{\text{ccu}} = \varepsilon_{\text{cou}} - \varepsilon_{\text{ju}} \left( K_j C_c \right)
\]

(10)

Note that, in Equation 10, tension strain is positive. The equation provides a rational basis for establishing the axial strain in a cylinder when the fiber bursts.

**EXPERIMENTAL PROGRAM**

**Test Variables**

Experiments were conducted on concrete cylinders confined by carbon fiber jackets. In all cases, reference cylinders without jackets were also tested. The variables investigated were:

- Concrete strength (4 – 7 ksi, 28 – 49 MPa)
- Jacket thickness (number of layers)
- Specimen size (4", 6" 12" diameter cylinders, 100, 150, 300 mm))
- Loading type (monotonic, cyclic)
- Loading speed
- Jacket wrapping style (continuous wrap vs. individual sheets for each layer)
Specimen Preparation and Test Set up

The concrete specimens were cast in steel molds. They were cured in a fog room, and were then removed, allowed to dry, and wrapped with a jacket using a wet lay-up procedure. In most cases the jacket was applied using one sheet of fiber per layer, lapped by 6 inches (150 mm) at the break. In multi-layer specimens, all the laps were placed over one another so that the hoop strain was constant around most of the circumference.

In two of the specimens, the jacket was made by continuous wrapping, using a single tow sheet. This is not the method specified by the fiber manufacturer (separate sheets for each layer), but, since application of several individual layers with the specified 30 minute wait between layers was found to be time-consuming, the continuous wrapping method was investigated to explore whether there was a performance penalty to be paid for the gain in application speed.

The carbon fiber was supplied by the Tonen Corporation, and was designated as type C1-20 or C1-30. The two types were essentially identical except for their effective thicknesses, of 0.00433 and 0.0065 inches (0.11 and 0.165 mm) respectively. Coupon tests showed that the mean modulus was 38,000 ksi (262 GPa) and the strain at failure was 1.6%, which leads to a stress at failure of 610 ksi (4200 MPa). The manufacturer gives mean values of modulus of 34,000 ksi (238 GPa) and strain at failure of 1.73%, and essentially the same stress at failure. In the cylinder tests, the C1-20 and C1-30 materials were used interchangeably as needed. However, all results were expressed in equivalent layers of C1-20.

The tests were conducted using a manually controlled 2400 kip (1100 tonne) Baldwin Universal Test Machine. Cyclic loading was therefore limited to a small number of cycles conducted at a speed that was feasible using the manual control system.

The test setup is shown in Figure 2. The cylinders were bedded on a thin layer of high-strength plaster, and seated on a series of plates on the lower platen of the test machine. An upper plate rested on a second layer of plaster on top of the cylinder and a spherical head was used to transmit the load from the head of the test machine.

The instrumentation consisted of two vertical potentiometers to measure axial displacement, three strain gages around the circumference of the cylinder at mid-height to measure the local hoop strain, and a Circumferential Displacement Device (CED) to measure the increase in circumference. Two strain gages were placed symmetrically, 120° apart, on the unlapped region of the jacket, and a third gage was placed in the lap region.

The CED consisted of a thin, greased cable surrounding the cylinder, fixed at one end and attached to a potentiometer at the free end. The cable was horizontal around the specimen, it then passed over a pulley and was tied to a smooth steel rod, which slid vertically in a heavy steel guide cylinder. The weight of the rod maintained a constant
tension in the cable. The potentiometer was attached to the steel cylinder and measured
the movement of the weight.

The strain gages measured local hoop strain, and the CED readings were
converted to average hoop strains.

EXPERIMENTAL RESULTS

General.

The average axial stress-strain curves for a subset of the specimens from are
shown in Figure 3. The cylinders in that group were 4” x 8” (100 x 200 mm), and were
made from concrete with an unconfined strength of 7 ksi (49 MPa). They were tested at
an age between 167 and 221 days, so the strength varied little between specimens due to
additional aging between tests. The results show characteristics that are typical of many
of the other tests, and are used here to illustrate general trends in behavior.

The shape of the curve may be characterized by three regions. In the first, up to
approximately the unconfined strength, the response is essentially the same as that of the
unconfined cylinder. In the third, the response is essentially linear at a slope that depends
on the jacket thickness, and therefore, stiffness. This region resembles the strain-
hardening portion of the stress-strain curve for a metal. The second region represents a
curved transition between the first and third.

The failure conditions for confined concrete show that both the stress and strain
at failure are higher than for the unconfined concrete by an amount that depends on the
jacket thickness. These findings are in agreement with those of many other researchers.

The behavior may also be characterized by plotting the hydrostatic stress against
the volumetric strain, as shown in Figure 4 for the same set of cylinders. The hoop
strains were taken from the CED and the volumetric strain was computed as

$$\varepsilon_{vol} = \varepsilon_z + 2\varepsilon_r$$

where $\varepsilon_z$ and $\varepsilon_r$ are the axial and radial strains (positive in tension). The axial stress was
obtained from the measured axial load, and the confining stress was computed from the
hoop strains. The hydrostatic stress was obtained from them.

All the curves show an initial region in which the volumetric strain is
proportional to the hydrostatic stress. Theoretically the two are linked by $K$, the bulk
modulus of the concrete, where

$$K = \frac{E}{3(1-2\nu)}$$
However, the accuracy with which $K$ can be calculated from the test data is not good, largely because the strain field is not constant over the length of the specimen. The friction on the loading plates provides confining stress at the cylinder ends, so the hoop strain is likely to be largest at mid-height, where it was measured, whereas the axial strain is the average value. Thus the individual values used in computing the volumetric strain are not truly consistent with each other.

At approximately $0.8f'_c$, the curves turn towards volumetric expansion. This occurs at a stress slightly higher than the critical stress at which, in unconfined concrete, micro-cracks start to cause volumetric expansion. In cylinders with low confinement, (1 or 2 layers, corresponding to $\rho_j = 0.43$ and 0.87% in Figure 4), the expansion continues monotonically until the jacket bursts. This behavior may be viewed as the concrete determining the kinematics of the system. With higher $\rho_j$ values, the curve turns back again to the direction of reduction of volume. At moderate $\rho_j$ values (4 layers, or 1.73%), the jacket bursts before the volumetric strain becomes negative (reduction in volume); at high $\rho_j$ values (8 layers or 3.46%) the volumetric strain is always negative. At such high confinement values, the deformations are determined by the interaction of the jacket hoop stiffness and the bulk compressibility of the concrete, with the jacket exerting the greater influence.

As already explained, the values shown should be regarded as trends, rather than exact values. However, Figures 3 and 4 show that, for each set of parameters, the two nominally identical specimens displayed behavior that was almost the same. This indicates that the scatter in the instrument readings was small enough not to influence the findings significantly.

**Effect of Jacket Thickness on Failure Stress and Strain**

The effect of the jacket thickness on the complete stress-strain curve of some of the specimens can be seen in Figure 3. The effect on the failure stress is illustrated in Figure 5, in which the normalized failure stress, $f'_{cc}/f'_c$, for all specimens is plotted against the normalized confinement stress, $f'_L/f'_c$. $f'_{cc}$ is the axial stress, $\sigma_z$, at failure, while $f'_L$ is the radial stress, $\sigma_r$, at failure. The figure includes all three cylinder sizes, and two concrete batches (B1 and B2, with $f'_c = 7$ and 4 ksi, or 49 and 28 MPa, respectively). The best fit straight line is given by

$$f'_{cc} = 1 + 3.52f'_L$$

with a correlation coefficient of 0.96. The widely-used equation developed by Mander et al. (1988), based on steel-confined specimens but expressed in terms of the confining stress at failure,

$$f'_{cc} = f'_c \left( -1254 + 2254 \sqrt{1 + 7.94 \frac{f'_L}{f'_c} - 2 \frac{f'_L}{f'_c}} \right)$$

(14)
was found to give a less satisfactory fit. At the higher confining stresses, it predicted a value for \( f'_{cc} \) that was too low.

The effect on axial strain at failure is shown in Figure 7. The best fit linear equation is

\[
\varepsilon_{ccu} = \varepsilon_{c0u} + 0.0034 \frac{f'_{L}}{f'_{c}}
\]

(15)

where \( \varepsilon_{c0u} \), the failure strain for unconfined concrete, is taken as 0.004 in/in (mm/mm). The correlation (0.88) is less good than for axial stress, partly because the strain in the jacket at failure showed considerable scatter, which resulted in scatter in the axial strain at failure, as predicted by Equation 10.

**Effect of Cylinder Size.**

The size of the cylinders (4” to 12”, or 100 mm to 300 mm, dia) was found to have almost no effect on the results, in terms of the axial stress, axial strain or secondary slope of the stress-strain curve. The same concrete was used for all specimens (maximum aggregate size 3/8” or 10 mm) and the effects were evaluated for different numbers of FRP layers. Even for the unconfined cylinders, in which a slight reduction in strength was expected for the larger cylinders, no clear trend was apparent.

**Effect of Loading Speed.**

Cylinders were loaded at 1, 4 and 15 times the mean speed dictated by ASTM C39, of 35 psi/sec (0.24 MPa/sec). The highest speed used was the fastest that could be achieved reliably using the available equipment. At the highest speed, each unconfined test was completed in approximately ten seconds. The loading speed was found to have a negligible effect on the axial stress and strain at failure.

**Effect of Cyclic Loading.**

Cylinders from Batch 2 concrete (\( f'_{c} = 4 \) ksi, or 28 MPa) were subjected to two different cyclic loading regimes. In Cyclic Loading Type I, the axial strain was increased in constant increments up to failure. In Cyclic Loading Type II, the specimen was loaded to 75% of the axial strain achieved by the comparable monotonic specimen, cycled 16 times between that strain and zero load, then loaded monotonically to failure. The result for Cyclic Loading Type I is illustrated in Figure 7 (using 6”, or 150 mm, diameter cylinder wrapped with 3 layers of C1-20 tow sheet). The figure also contains the curve for the two comparable monotonically loaded specimens, and shows that the monotonic curve is a reliable envelope for the cyclic one. Furthermore, the failure stress and strain were almost the same in both cases.

Similar curves for Cyclic Loading Type II are shown in Figure 8. The specimen was wrapped with 6 layers of C1-20 tow sheet. Due to logistical constraints, the cyclically loaded cylinders were tested at 78 days, whereas the comparable monotonic specimens were tested at 33 days. Thus the figure contains five curves: two control
specimens, the two control specimens with projected strength increases to simulate the effects of the 45 days of additional aging, and the cyclic specimen. The cyclic specimen failed at an axial stress and strain very close to those of the projected monotonic specimens. However, during cycling, the peak stress dropped slightly with each cycle, suggesting a modest progressive plastic compaction of the concrete. The overall conclusion was that cyclic loading had a negligible effect on the failure properties.

**Effect of Fiber Wrapping Technique.**

Two cylinders were tested using a continuously wrapped jacket rather than the standard jacket that consisted of separate sheets for each layer. The stress-stain curves for the two continuously-wrapped jackets and their discrete counterparts are shown in Figure 9. All four curves lie essentially on top of one another up to about 1.8% strain. At that strain, failure initiated in the continuously wrapped cylinders, but the control cylinders with individual layers continued to support load without any sign of failure up to about 2.3% strain.

Inspection of the specimens showed that, in both the continuously wrapped specimens, the failure had been started by inter-laminar shear failure in the resin. The outer two of the four layers of C1-30 material (equivalent to six layers of C1-20) had delaminated, while the inner two had fractured. This was true in both specimens. The cause appeared to be inadequate penetration of the resin into the fibers. Failure initiated by delamination had also been observed in a small number of individually wrapped specimens during the program, but the delamination had never been found in any but the outermost layer. All the jackets were installed by the same personnel, so some reason other than operator differences must be sought for the difference.

Two explanations conclusions appear plausible. The first is that the continuous wrapping approach makes good preparation and resin penetration inherently more difficult, so that delamination becomes more likely unless particularly skilled operators apply the jackets. The second is that the bond stresses in the overlap region are highly non-uniform, being higher near the free ends of the tow sheet, and that the effect is more significant in the continuously wrapped specimens because, once bond failure has started, it propagates around the cylinder with no discontinuities to act as crack arrestors. The fiber manufacturer specifies that the layers should be applied using individual sheets, but offers no reasons for so doing. It should be noted that here the jackets were applied with the cylinder axes horizontal, whereas column retrofit jackets are necessarily applied with the axis vertical. This raises the possibility of the jacket sagging if too many layers of wet resin and fiber are applied at once, and provides a reason for using separate layers.

The limited evidence from the two tests conducted here suggests that Tonen’s specifications have a valid basis. Further examination of the issue is necessary for firm conclusions to be drawn, but the preliminary finding is that the use of continuous wrapping presents a greater risk of premature failure. However, selection of the procedure should take into account the costs of both labor and materials. Applying one additional layer to compensate for the lower strength, and saving time by using
continuous wrapping, may be more cost-effective than using a smaller number of separate layers.

CONCLUSIONS

The following conclusions may be drawn from the study:

- Response can be divided into three regions: elastic (I), transition (II), and “strain-hardening” (III).
- In Region I, while the concrete is still elastic, the jacket provides no useful confinement.
- In Region III, at high strains, the Mohr-Coulomb model provides an acceptable description of failure.
- At high confining stresses, the concrete suffers bulk compressive strain. This must be taken into account if the axial strain at failure is to be computed correctly from the lateral strain capacity of the jacket.
- In Region II, the response is more difficult to quantify rationally. An empirical transition curve may be appropriate.
- The absolute size of the specimen has a negligible effect on response.
- The loading speed has no effect on response, for speeds up to 15 times the mean speed specified by ASTM C39.
- Cyclic loading has a negligible effect on the failure stress and strain. Many cycles to the same compressive strain cause a gradual degradation in stress at that strain, but subsequent monotonic loading to failure follows a path that soon rejoins the virgin monotonic curve.
- Use of a jacket made from a single continuously wrapped sheet, rather than layers made from separate sheets, leads to a slight reduction in the stress and strain at failure.

REFERENCES


Figure 1 -- Mohr-Coulomb failure envelope.

Figure 2 -- Test setup.
Figure 3 -- Axial stress-strain curves for batch 1 specimens.

Figure 4 -- Hydrostatic stress vs. volumetric strain for batch 1 specimens.

Figure 5 -- Effect of lateral confining stress on axial stress at failure.
Figure 6 -- Effect of lateral confining stress on axial strain at failure.

Figure 7 -- Cyclic loading Type I.

Figure 8 -- Cyclic loading Type II.
Figure 9 – Continuously wrapped jackets.
External Confinement of Low-Strength Brittle Reinforced Concrete Short Columns

by A. Ilki, O. Peker, E. Karamuk, and N. Kumbasar

Synopsis: In this study, FRP jacketed short reinforced concrete columns with low concrete strength and inadequate transverse reinforcement were tested under uniaxial compression. The diameter of the longitudinal bars and spacing of the transverse bars were designed to allow buckling of longitudinal bars. The effects of the jacket thickness, pre-damage, cross-section shape and the usage of FRP jackets either continuous or as straps like hoops or spirals were investigated. The test results showed that external confinement of this type of columns was very effective in terms of deformability and strength enhancement. The buckling of the longitudinal bars was delayed significantly by the FRP jackets. The pre-damage did not have an adverse effect on the performance of the jacketed members. It was also observed that for equivalent amount of FRP, continuous jackets and straps provided similar performances. The compressive strengths and the corresponding axial deformations of the columns were also predicted by the empirical equations proposed by the authors before. It was seen that these predictions were in reasonable agreement with experimental results.

Keywords: columns (supports); confinement; ductility; reinforced concrete; strength
INTRODUCTION

Many existing buildings suffer from various design and construction deficiencies all over the world. These deficiencies may be either because of the insufficiency of relatively older codes, or due to inadequate construction practice. Among these deficiencies, low concrete quality and inadequate transverse reinforcement are quite common, particularly in developing countries. The seismic performance of these types of structures may be poor due to low concrete strength, and particularly insufficient ductility. For preventing severe seismic damage, and obtaining satisfactory seismic performance, the deformability and the axial load capacity of columns with these deficiencies may need to be enhanced. External confinement of these structural members with high strength fiber reinforced polymer (FRP) composite sheets can enhance the deformability and axial strength significantly.

Various studies proved that significant enhancement in strength and deformability of concrete is possible by external confinement with FRP composite sheets. According to Fukuyama and Sugano (2000) the repair and seismic strengthening by continuous fiber sheet wrapping method was first developed in Japan, whose research was first carried out in 1979. The studies of Fardis and Khalili (1982), Harmon and Slattery (1992), and Nanni and Bradford (1995) are among the initial research works on confinement of concrete members with FRP composites. Karbhari and Gao (1997), Toutanji (1999), Saafi et al. (1999), Fam and Rizkalla (2001) and Becque et al. (2003) developed extensive experimental data for cylinder specimens, for a variety of fiber types, orientations and jacket thickness, either for FRP jacketed concrete or concrete filled FRP tubes. Demers and Neale (1999) tested reinforced concrete columns of circular cross-section with FRP jackets. Rochette and Labossiere (2000), and Wang and Restrepo (2001), tested square and rectangular concrete columns confined by FRP composites. Xiao and Wu (2000) investigated the effect of concrete compressive strength and thickness of CFRP jacket and proposed a simple bilinear stress-strain model for CFRP jacketed concrete. Tan (2002) tested half scale reinforced concrete rectangular columns with a section aspect ratio of 3.65 under axial loads and investigated the effects of fiber type and configuration and fiber anchors on the strength enhancement of the columns. Ilki and Kumbasar (2002) tested both damaged and undamaged cylinder specimens, which were externally confined...
with different thickness of CFRP jackets, under monotonic increasing and repeated compressive stresses. Based on experimental results they proposed simple expressions for ultimate strength and corresponding axial strain of CFRP wrapped concrete. Lam and Teng (2002) carried out an extensive survey of existing studies on FRP confined concrete and proposed a simple model based on the linear relationship between confined concrete strength and lateral confining pressure provided by FRP composites, which was quite similar to the model proposed by Ilki and Kumbasar (2002) before. Ilki and Kumbasar (2003), after testing CFRP wrapped concrete specimens with square and rectangular cross-sections, modified the expressions that they have proposed before to cover non-circular cross-sections. Lam and Teng (2003a, 2003b) proposed design oriented stress-strain models for both uniformly and non-uniformly confined concrete members. Ilki et al. (2004) tested FRP jacketed low strength concrete members with circular and rectangular cross-sections, and stated that when the unconfined concrete quality was lower, the efficiency of the FRP jackets was higher. De Lorenzis and Tepfers (2003), stated that none of the available models could predict the strain at peak stress with reasonable accuracy.

In this study, 13 reinforced concrete short columns with circular, square and rectangular cross-section and retrofitted with CFRP jackets were tested under uniaxial compression. For representing the columns of relatively older existing buildings, the concrete mix-proportion was specially designed for obtaining low concrete strength. For the same purpose, the transverse bars were not sufficient for a ductile behavior either. Two pre-damaged specimens were also included in the test program. After the tests of unconfined and CFRP jacketed specimens, it was observed that the efficiency of the CFRP jackets on deformability and strength enhancement of low strength concrete members was very remarkable for specimens with circular and non-circular cross-sections. The FRP jackets delayed the buckling of the longitudinal bars significantly, even for the specimens jacketed by 1 ply of CFRP sheets. The pre-damage did not have any adverse effect on the performance of the jacketed members. It was also observed that for equivalent amount of CFRP, continuous jackets and straps provided similar performances. One specimen with relatively higher concrete strength was also included in the test program for a direct comparison of effect of unconfined concrete strength on the behavior of CFRP jacketed reinforced concrete members. This comparison showed that the contribution of CFRP jacket was more pronounced in the case of lower unconfined concrete strength. The compressive strengths and the corresponding axial deformations of the columns with low strength concrete were also predicted by the empirical equations proposed by the authors before, (Ilki and Kumbasar, 2003). It was seen that these predictions were in reasonable agreement with experimental results.

RESEARCH SIGNIFICANCE

Although extensive experimental data is available for FRP confined concrete, most of these studies are on small size cylinders without longitudinal and transverse reinforcement. Almost all of these studies were also for medium to high concrete strengths. However, many existing structures, in need of retrofit, suffer from low quality of concrete, as well as other deficiencies like inadequate transverse reinforcement. In this
study, the behavior of relatively larger size specimens with longitudinal and transverse reinforcement, cast with relatively lower strength concrete, was investigated.

EXPERIMENTAL WORK

General characteristics of the specimens

In this study, 13 reinforced concrete short columns with circular, square and rectangular cross-section and jacketed with different thickness of CFRP jackets were tested under uniaxial compression, as well as specimens without external confinement. The test program included 11 cylinder specimens with the diameter of 250 mm and height of 500 mm, 1 specimen with the cross-sectional dimensions of 150 mm × 300 mm, and 1 specimen with the cross-sectional dimensions of 250 mm × 250 mm. The heights of the specimens with rectangular cross-section were also 500 mm. The concrete cover was 25 mm for all specimens and the corners of the rectangular specimens were rounded with the radius of 40 mm. For representing the columns of relatively older existing buildings, the concrete mix-proportion was specially designed for obtaining low concrete strength, except one specimen (NS-C-3-a) with the unconfined member concrete strength of 28.5 MPa. For all different cross-sections, the spacing of the transverse reinforcement, s, was chosen as approximately 14.5 times of the diameter of the longitudinal bars, \( d \), to allow buckling of longitudinal reinforcing bars under axial stresses, and for representing frequently met transverse bar spacing in relatively older structures. The geometric ratio of the longitudinal reinforcement was around 1%. The specimens were tested after being jacketed with 1, 3 or 5 plies of CFRP sheets. Two pre-damaged specimens were also included in the test program. General characteristics of the specimens are given in Table 1 and Fig. 1. In Table 1, \( f_{\text{c}} \), \( n \), \( A_{\text{s}} \) and \( A_{\text{sh}} \) represent unconfined concrete strength of the member, number of CFRP plies, longitudinal reinforcement and transverse reinforcement, respectively.

Materials

Concrete: Specially designed ready mixed low strength concrete was used for all of the specimens, (except NS-C-3-a). The concrete mix-proportion and fresh concrete characteristics are given in Table 2. \([\text{Water/Cement}]\) and \([\text{Water/(Cement+Pozzolan)}]\) ratios were 1.4 and 1.1, respectively. Ordinary Portland cement with the 28 days strength of 42.5 MPa and a mid-range superplastisizer were used in the mixture, where the maximum aggregate size was around 10 mm. The 28 days standard cylinder strength, \( f_{\text{c}} \), was around 11 MPa. The cylinder strength-concrete age relationship is given in Fig. 2. Since the specimens were tested between the ages of 103 and 184 days, the corresponding standard cylinder strengths were between 14.2 and 15.9 MPa.

Reinforcement: For longitudinal reinforcement \( \phi 10 \), \( \phi 14 \) and \( \phi 12 \) bars were used for specimens with circular, square and rectangular cross-sections, respectively. For transverse reinforcement \( \phi 8 \) bars were used for all specimens. Only plain bars were used both for longitudinal and transverse reinforcement for reflecting the columns of existing
relatively older structures in developing countries. The mechanical properties of reinforcing bars are given in Table 3.

CFRP: The mechanical properties of the CFRP composite sheets as given by the manufacturer are presented in Table 4. In this table $f_\text{fu}$, $E_f$, $\varepsilon_\text{fu}$, and $t_f$ are the tensile strength, elasticity modulus, ultimate rupture strain and nominal thickness, respectively.

**Preparation of the specimens**

Preparation steps of the specimens are shown in Fig. 3. As seen in this figure, the spacing between the transverse bars was reduced outside the middle zone of the specimens for obtaining the damage in the middle portion, where most of the measuring devices were located. A clear cover of 20 mm was formed for longitudinal reinforcement at the bottom and top faces of the specimens for preventing direct loading of reinforcing bars. Before CFRP sheet wrapping in transverse direction with 0-degree orientation, surface preparation procedure was carried out, which included sanding, cleaning, forming one layer of epoxy-polyamine primer and one layer of epoxy putty. Then epoxy adhesive was used for bonding CFRP jacket on the specimens. Additional layers of epoxy adhesive were applied between the CFRP jacket plies and on the outer ply of CFRP jacket when more than one ply was wrapped. The compressive and tensile strengths of the epoxy system were around 80 and 50 MPa, respectively. Tensile elasticity modulus of the epoxy system was around 3000 MPa and its ultimate elongation was 0.025. For obtaining satisfactory bonding, 150 mm overlap length was formed at the end of the wrap. In the cases of wrapped sheets to be more than one ply, the sheet was wrapped continuously and 150 mm overlap was formed at the end of the wrap.

**Loading and measuring systems**

A 5000 kN capacity Amsler loading machine was used for loading. The average vertical displacements through all height of the specimens (gage length: 500 mm) and in 270 mm gage length around mid-height were measured by displacement transducers. The deformations of the internal longitudinal and transverse reinforcement were measured by post-yield type strain gages, as well as the longitudinal and transverse strains of the FRP jackets. Strain gages with 3 and 5 mm gage lengths were used on internal reinforcement, while 60 mm gage length strain gages were used on FRP sheets. For data acquisition a 50 channel TML switch box and a TML-TDS-303 data logger were used.

**EXPERIMENTAL RESULTS**

The test results for low strength concrete specimens are outlined in Table 5. As seen in this table, with respect to unconfined concrete, confined concrete compressive strengths, $f'_c$, increased about 2.4, 4.5 and 6.8 times for CFRP jackets of 1, 3 and 5 plies, respectively. Note that unconfined member concrete strength, $f_c$, was assumed to be 85% of the standard cylinder strength. The average increments in the axial strains corresponding to CFRP jacketed concrete strengths, $\varepsilon_c$, were around 11.5, 24 and 32 times for 1, 3 and 5 plies of CFRP jackets. The axial strain corresponding to unconfined
concrete strength, $e_{co}$, was assumed to be equal to 0.002. The transverse strains on CFRP jackets at failure, $e_{ct}$, were between 0.010 and 0.016 independent of the thickness of the jacket, with an average value as 0.013. Note that only the strain gages that could work until failure were taken into consideration. The appearances of some of the damaged specimens are presented in Fig. 4. Generally, the failures were sudden at the mid-heights of the specimens. While rupture of CFRP sheets was only one vertical cut with the height of 200-250 mm for the jackets of 1 ply, 3-4 vertical cuts with the heights of 40-70 mm for the jackets of three plies and more vertical cuts of lower heights for jackets of 5 plies were observed. For rectangular specimens with CFRP jackets of 3 plies, the rupture of sheets was around the corners just after the rounded portion. Similar to circular specimens jacketed with 3 plies of CFRP, there were 3-4 vertical cuts with the heights of 40-70 mm in the square and rectangular specimens.

The axial stress-axial strain relationships in the gage length of 500 mm, and the axial stress-transverse strain relationships in the gage length of 60 mm for the specimens with circular cross-section are given in Fig. 5 and 6, respectively. In Fig. 5, the gage length for the vertical deformation measurements of unconfined standard cylinders was 150 mm. As seen in Fig. 5, significant enhancement in strength and deformability was provided by CFRP jacketing. As the thickness of the jacket increased, the enhancement in the axial stress-axial strain behavior became more remarkable. However, as seen in Fig. 6, ultimate transverse strains of CFRP jackets were around 0.012-0.013, independent of the jacket thickness. The initial axial stiffness of the specimens did not show a tendency to increase with increasing jacket thickness. This is due to inefficiency of CFRP jackets before a certain amount of transverse deformation occurs. Note that all stress-strain relationships are given with the non-dimensional vertical axis, and while determining the stress-strain relationship of confined concrete, the contribution of longitudinal reinforcing bars was subtracted by considering the actual stress-strain relationships of these bars including the strain-hardening region. It should also be noted that although the major contribution of the enhancement in strength and deformability was provided by the external CFRP jacket, the internal transverse reinforcement had also some influence. The contribution of internal transverse reinforcement is minimal for the specimens with square and rectangular cross-section.

For demonstrating the effects of gage lengths for vertical measurement, the axial stress-axial strain relationships obtained by the displacement transducers in 270 and 500 mm gage lengths and by strain gages in 60 mm gage length for specimens LS-C-1-a and LS-C-5-a are presented in Figs. 7 and 8, respectively. As seen in these figures, the average axial deformations measured in the gage lengths of 270 and 500 mm were almost same, while the deformations measured in the gage length of 60 mm by strain gages were slightly less than those. However, it should be noted that since the deformation capacity of strain gages is limited, it is not possible to carry out measuring of the axial strains by strain gages until the end of the tests. The axial stress-axial strain relationships of the undamaged and pre-damaged equivalent specimens are shown in Fig. 9. No practical difference in terms of strength, deformability and axial stiffness was observed between the performances of pre-damaged (LS-C-3-a-PD, LS-C-3-b-PD) and undamaged specimens (LS-C-3-a, LS-C-3-b). Note that during the loading for pre-damage, the
specimens were loaded until the axial strain level of 0.004 and the specimens have lost a small portion of their strengths. Similar results for different damaged specimens were obtained by Demers and Neale (1999) and Ilki and Kumbasar (2002) before. For equivalent amount of FRP, the installation of CFRP jackets continuous or in the form of straps (either like individual hoops or continuous spirals) did not have a remarkable effect on the behavior, (Fig. 10). Note that, both for specimen LS-C-3-a-H (with hoop type straps) and LS-C-3-a-S (with spiral type straps), the straps were 50 mm wide and installed on the specimens with 50 mm clear spacing. In Fig. 11, the axial stress-axial deformation and axial stress-transverse deformation relationships for specimens LS-C-3-a, LS-R-1-3-a and LS-R-2-3-a are presented. As seen for same thickness of CFRP jacket, all the specimens with circular, square and rectangular cross-section experienced a remarkable enhancement in strength and deformability. While the strength enhancement was more remarkable for the specimen with circular cross-section, specimens with square and rectangular cross-sections exhibited higher axial deformations. In Fig. 12, the stress-strain relationships of the specimens LS-C-3-a ($f_{cc}^{\text{co}}=12.8$ MPa) and NS-C-3-a ($f_{cc}^{\text{co}}=28.5$ MPa) are presented. As seen in this figure, the strength and ductility enhancements were more pronounced for the specimen with relatively lower unconfined concrete strength.

**PREDICTION OF CONFINED STRENGTH AND DEFORMABILITY**

For prediction of ultimate strengths and corresponding axial strains of CFRP jacketed low strength concrete specimens, the empirical equations proposed by Ilki and Kumbasar (2003) were used, Eqs. (1) and (2). These equations were proposed based on experimental results on the specimens with circular and rectangular cross-section. In these equations, $f_{cc}$ and $\varepsilon_{cc}$ are the confined concrete strength and corresponding axial strain, $f_{max}$ is the effective lateral confinement stress and $\varepsilon_{co}$ is the strain corresponding to unconfined concrete strength and it is assumed as 0.002 in this study. $\alpha$ coefficient is introduced for considering the shape effect of rectangular cross-sections and it is determined as a function of lengths of short side ($b$) and long side ($h$) of the cross-section and assumed as 1 for circular cross-sections, Eq. (3).

$$\left[ \frac{f_{cc}'}{f_{co}'} \right]_{\text{FRP}} = \alpha \left[ 1 + 2.23 \frac{f_{max}}{f_{co}'} \right]$$  \hspace{1cm} (1)

$$\left[ \frac{\varepsilon_{cc}}{\varepsilon_{co}} \right]_{\text{FRP}} = \left[ 1 + 15 \left( \frac{f_{max}}{f_{co}'} \right)^{0.75} \right]$$  \hspace{1cm} (2)

$$\alpha = 0.6 + 0.2 \frac{b}{h}$$  \hspace{1cm} (3)

$f_{max}$, the effective lateral confinement stress provided by CFRP jacket can be determined by Eq. (4). In Eq. (4), $\kappa_a$ is the efficiency factor that is to be determined based on the section geometry as the ratio of effectively confined cross-sectional area to the gross
cross-sectional area, $\kappa_a$, can be assumed as 1 for circular cross-sections. For rectangular cross-sections, $\kappa_a$ can be determined by Eqs. (5), (6) and (7), as also proposed by Wang and Restrepo (2001). In Eq. (4), $f_t$ and $\rho_f$ are the tensile strength and ratio of wrapping material to the concrete cross-section, respectively. In Eqs. (5), (6) and (7) $A_1$ and $A_2$ are the areas of uneffectively confined concrete on the sides and at the corners, respectively. The ratio of the wrapping material to the concrete cross-section can be determined by Eq. (8) and (9) for circular and rectangular cross-sections, respectively. In these equations, $\rho$ is the ratio of cross-sectional area of the longitudinal reinforcement to the cross-sectional area of wrapped member, $\theta$ is the arching angle and $r$ is the radius of the member corner. Wang and Restrepo (2001) reported that $\theta$ varied between 42 and 47 degrees. In this study, based on the observations on the damaged specimens, $\theta$ is assumed as 45 degrees. In Eqs. (8) and (9), $t_f$ and $n_f$ are the effective thickness and the number of plies of wrapping material, $D$ is the diameter of the circular cross-section, and $b$ and $h$ are lengths of the short and long sides of the rectangular member to be wrapped. The effective lateral confinement stress provided by CFRP jacket was effected by the cross-sectional shape and dimensions. For example, $f_{\text{max}}$ values for specimens LS-C-3-a, LS-R-1-3-a and LS-R-2-3-a were 13.6, 9.0 and 9.6 MPa, respectively.

$$f_{\text{max}} = \frac{\kappa_a \rho_f f_t}{2}$$

$$\kappa_a = 1 - A_1 - A_2 - \rho$$

$$A_1 = \frac{(b - 2r)^2 + (h - 2r)^2}{3bh} \tan \theta$$

$$A_2 = \frac{4r^2 - \pi r^2}{bh}$$

$$\rho_f = \frac{4n_f t_f}{D}$$

$$\rho_f = \frac{2n_f t_f (b + h)}{bh}$$

While predicting the confined concrete strength and corresponding axial strain, the contribution of internal transverse reinforcement (ITR) was also taken into account. For this purpose, the empirical equations proposed by Ilki et al. (2003) and Mander et al. (1988) were used for strength and ultimate axial strain, respectively, Eqs. (10) and (11). After the strength and deformability enhancements provided by external CFRP jacket and internal transverse reinforcement were determined separately, the total enhancement in strength and corresponding strain was calculated by using Eqs. (12) and (13). The predictions for the enhancements provided by CFRP jacket and ITR are presented in Figs.
13 and 14 for compressive strength and corresponding axial strain, respectively. In the same graphs, comparisons of the predicted total enhancements provided by ITR and CFRP jacket, and experimentally determined strength and deformability enhancements are also presented. Note that the effectively confined cross-sectional areas by CFRP jacket and internal transverse reinforcement were determined as shown in Fig. 15.

\[
\frac{f'_cc}{f'_co} = 1 + 4.54 \frac{f_{l,\text{max}}}{f'_cco} \quad (10)
\]

\[
\frac{\varepsilon_{cc}}{\varepsilon_{cco}} = 1 + 5 \left( \frac{f'_cc}{f'_cco} - 1 \right) \quad (11)
\]

\[
\frac{f'_cco - f'_cco}{f'_cco}_{\text{TOTAL}} = \left[ \frac{f'_cco}{f'_cco} - 1 \right]_{\text{CFRP}} + \left[ \frac{f'_cco}{f'_cco} - 1 \right]_{\text{ITR}} \quad (12)
\]

\[
\frac{\varepsilon_{cc} - \varepsilon_{cco}}{\varepsilon_{cco}}_{\text{TOTAL}} = \left[ \frac{\varepsilon_{cc}}{\varepsilon_{cco}} - 1 \right]_{\text{CFRP}} + \left[ \frac{\varepsilon_{cc}}{\varepsilon_{cco}} - 1 \right]_{\text{ITR}} \quad (13)
\]

For combined contribution of FRP jacket and internal transverse reinforcement, Wang and Restrepo (2001) have determined the effective lateral confinement stresses provided by FRP jacket and internal transverse reinforcement separately and then used the model proposed by Mander et al. (1988) for concrete confined by internal steel reinforcement by replacing the transverse confinement stress provided by internal reinforcement with the total transverse confinement stress provided by internal reinforcement and external FRP jacket.

**CONCLUDING REMARKS**

After the uniaxial compression tests on low strength reinforced concrete columns without sufficient transverse reinforcement, following conclusions are derived. The CFRP jackets significantly increased the compressive strength and ultimate axial strain of the columns with circular, square and rectangular cross-sections. While the strength enhancement was more pronounced for circular cross-sections, deformability enhancement was more for rectangular cross-sections. Although the spacing of transverse bars and the diameter of longitudinal bars were chosen for allowing buckling of the longitudinal bars, as well as the premature buckling of the longitudinal reinforcement was prevented, the contribution of longitudinal reinforcing bars to the axial resistance and ductility was maintained until very large axial strains. Although the enhancement provided with one ply of CFRP jacket was very remarkable, further increase in jacket thickness resulted with significantly higher strengths and axial deformation capabilities. Independent of the jacket thickness, the measured maximum transverse deformations of CFRP jackets were between 0.010 and 0.016, with the average of 0.013. Measurements of axial deformation were carried out for three different gage lengths, namely mid 60 mm, mid 270 mm and 500 mm,
which was entire height of the specimens. It was seen that the measured axial deformations in different gage lengths were quite close to each other, particularly in 270 and 500 mm gage lengths. The performance of CFRP jacketed pre-damaged specimens was similar as the CFRP jacketed undamaged specimens. For equivalent amount of FRP, the application of CFRP jacketing as straps in the form of hoops or spirals instead of continuous wrapping did not have a significant positive or negative effect on the behavior. However, considering the difficulty of application of CFRP jackets as hoops or spirals, continuous application may be more preferable. The contribution of CFRP jacket was less pronounced for the specimen with relatively higher unconfined concrete strength. The empirical equations, proposed by the authors before, predicted the compressive strength and corresponding axial strains of the specimens tested in this study with a reasonable accuracy. For the 12 low strength concrete specimens tested, the average of the ratio of experimental to predicted confined concrete strengths is 1.26, with a standard deviation of 0.12, and the average of the ratio of experimental to predicted confined concrete ultimate axial deformations is 1.81, with a standard deviation of 0.41.

ACKNOWLEDGMENTS

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REFERENCES


Table 1 – Specimen characteristics

<table>
<thead>
<tr>
<th>Specimens</th>
<th>Section shape</th>
<th>$f'_{co}$ MPa</th>
<th>n</th>
<th>Jacket</th>
<th>Pre-damage</th>
<th>$A_{sj}$</th>
<th>$A_{st}$</th>
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<tr>
<td>LS-C-1-a</td>
<td>Circular</td>
<td>12.8</td>
<td>1</td>
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<td>13.1</td>
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<td>3</td>
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<td>No</td>
<td>6φ10</td>
<td>φ8/145</td>
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<td>3</td>
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<td>6φ10</td>
<td>φ8/145</td>
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<tr>
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<td>12.8</td>
<td>5</td>
<td>Continuous</td>
<td>No</td>
<td>6φ10</td>
<td>φ8/145</td>
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<tr>
<td>LS-C-5-b</td>
<td>Circular</td>
<td>13.5</td>
<td>5</td>
<td>Continuous</td>
<td>No</td>
<td>6φ10</td>
<td>φ8/145</td>
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<tr>
<td>NS-C-3-a</td>
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<td>28.5</td>
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<tr>
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<tr>
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<td>13.5</td>
<td>3</td>
<td>Continuous</td>
<td>Yes</td>
<td>6φ10</td>
<td>φ8/145</td>
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<tr>
<td>LS-C-3-a-H</td>
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Table 2 – Mix-proportion (kg/m³), Fresh concrete properties (cm)

<table>
<thead>
<tr>
<th>Cement</th>
<th>Water</th>
<th>Sand</th>
<th>Gravel</th>
<th>SP</th>
<th>Fly ash</th>
<th>Admixture</th>
<th>Slump</th>
<th>F/DAST</th>
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<tr>
<td>150</td>
<td>210</td>
<td>638</td>
<td>982</td>
<td>286</td>
<td>40</td>
<td>1.14</td>
<td>20-24</td>
<td>30-31</td>
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SP: stone powder, F/DAST: flow diameter after slump test

Table 3 – Mechanical properties of reinforcing bars

<table>
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<tr>
<th>Bar</th>
<th>$\phi$ mm</th>
<th>$f_y$ MPa</th>
<th>$\epsilon_y$</th>
<th>$f_{u,\text{max}}$</th>
<th>$\epsilon_{u,\text{max}}$</th>
<th>$f_{u}$ MPa</th>
<th>$\epsilon_{u}$</th>
<th>Type</th>
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<tbody>
<tr>
<td>$\phi8$</td>
<td>8.0</td>
<td>476</td>
<td>0.0024</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>Plain</td>
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<tr>
<td>$\phi10$</td>
<td>10.1</td>
<td>367</td>
<td>0.0018</td>
<td>523</td>
<td>0.19</td>
<td>377</td>
<td>0.27</td>
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<tr>
<td>$\phi12$</td>
<td>12.2</td>
<td>330</td>
<td>0.0017</td>
<td>471</td>
<td>0.23</td>
<td>335</td>
<td>0.30</td>
<td>Plain</td>
</tr>
<tr>
<td>$\phi14$</td>
<td>13.8</td>
<td>345</td>
<td>0.0017</td>
<td>477</td>
<td>0.23</td>
<td>294</td>
<td>0.31</td>
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</table>

$\phi$: bar diameter, $f_y$: yield strength, $\epsilon_y$: yield strain, $f_{u,\text{max}}$: tensile strength, $\epsilon_{u,\text{max}}$: strain corresponding to $f_{u,\text{max}}$, $f_u$: ultimate stress, $\epsilon_u$: ultimate elongation
Table 4 – Mechanical properties of CFRP sheets

<table>
<thead>
<tr>
<th>$f'_{cu}$ MPa</th>
<th>$E_t$ GPa</th>
<th>$\varepsilon'_f$</th>
<th>$t_0$ mm</th>
<th>Unit weight, kg/m$^3$</th>
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<tr>
<td>3430</td>
<td>230</td>
<td>1.5%</td>
<td>0.165</td>
<td>1820</td>
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Table 5 – Test results for externally confined low strength concrete columns

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<tr>
<th>Specimen</th>
<th>$f_{ec}$ MPa</th>
<th>$e_{ec}$ 500mm</th>
<th>$e_{ec}$ 270mm</th>
<th>$e_{ch}$</th>
<th>$f_{ec} / f_{co}$</th>
<th>$e_{ec} / e_{co}$ 500mm</th>
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<tr>
<td>LS-C-1-a</td>
<td>29.0</td>
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<td>0.023</td>
<td>0.010</td>
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<tr>
<td>LS-C-1-b</td>
<td>30.8</td>
<td>0.024</td>
<td>-</td>
<td>0.015</td>
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<td>2.2</td>
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<tr>
<td>LS-C-3-a</td>
<td>56.1</td>
<td>0.043</td>
<td>0.043</td>
<td>0.007*</td>
<td>4.4</td>
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<td>LS-C-3-b</td>
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<td>-</td>
<td>0.012</td>
<td>4.6</td>
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<td>LS-C-5-a</td>
<td>84.9</td>
<td>0.066</td>
<td>0.068</td>
<td>0.015*</td>
<td>6.6</td>
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<tr>
<td>LS-C-5-b</td>
<td>92.6</td>
<td>0.062</td>
<td>-</td>
<td>0.004*</td>
<td>6.9</td>
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<td>LS-C-3-a-PD</td>
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<td>-</td>
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<td>5.4</td>
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<td>LS-C-3-a-H</td>
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<td>0.023</td>
<td>0.012</td>
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<td>LS-C-3-a-S</td>
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<td>0.032</td>
<td>0.035</td>
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<td>LS-R-1-3-a</td>
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<td>0.055</td>
<td>0.045</td>
<td>0.016</td>
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<td>LS-R-2-3-a</td>
<td>34.9</td>
<td>0.062</td>
<td>0.064</td>
<td>0.009*</td>
<td>2.6</td>
<td>3.0</td>
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</table>

* out of order before reaching peak stress  ** on the longer side of the cross-section

Figure 1 – Details of the specimens

Figure 2 – Standard cylinder strength-concrete age relationship
Figure 3 – Preparation of the specimens

Figure 4 – Specimens after test (a) LS-C-1-a, (b) LS-C-3-a, (c) LS-C-5-a, (d) LS-C-3-a-H, (e) LS-C-3-a-S, (f) LS-R-1-3-a, (g) LS-R-2-3-a
Figure 5 – Axial stress-axial strain relationships for specimens with circular cross-section in 500 mm gage length

Figure 6 – Axial stress-transverse strain relationships for specimens with circular cross-section in 60 mm gage length

Figure 7 – Axial stress-axial strain relationships – different gage lengths (LS-C-1-a)
Figure 8 – Axial stress-axial strain relationships – different gage lengths (LS-C-5-a)

Figure 9 – Axial stress-axial strain relationships for pre-damaged and undamaged specimens

Figure 10 – Axial stress-axial strain relationships for continuous and strap type jackets
Figure 11 – Axial stress-axial strain relationships for circular, square and rectangular cross-sections

Figure 12 – Axial stress-axial strain relationships for specimens with low and normal unconfined concrete strengths (LS-C-3-a and NS-C-3-a)

Figure 13 – Contributions of ITR and CFRP jacket on strength enhancement and comparison of predicted and measured enhancement in strength
Figure 14 – Contributions of ITR and CFRP jacket on deformability enhancement and comparison of predicted and measured enhancement in ultimate deformation

Figure 15 – Effectively confined cross-sectional areas for CFRP jacket and internal transverse reinforcement
Experimental Study on Concrete Cylinders Confined with Various FRP under Uniaxial Compression


Synopsis: External confinement of concrete by means of FRP composites can significantly enhance its strength and ductility as well as result in large energy absorption capacity. The present paper focuses on studying the behavior of concrete cylinders confined with hybrid FRP composites. A total of thirty 150 mm by 300mm cylinders were tested, which included 3 plain concrete specimens, 12 concrete cylinders confined with one kind of FRP, and 20 specimens confined with hybrid FRP sheets. Experimental parameters included the different types of FRP sheets, different hybrid ratio of FRP. The relationships of axial stress-axial strain of FRP-confined concrete are studied, as well as the variation of Poisson’s ratio of FRP confined concrete is discussed. Behaviors of concrete cylinders confined with hybrid FRP and counterpart specimens confined with one kind of FRP sheets are compared. Finally, some equations are proposed to evaluate the ultimate strength and strain of concrete cylinders confined with hybrid FRP. This paper provides a framework for better understanding of confining effects of concrete cylinders confined with different FRP composites.

Keywords: concrete cylinders; confinement; fiber-reinforced polymer (FRP); hybrid; strain; stress
INTRODUCTION

The use of fiber reinforced polymer (FRP) composites for strengthening or rehabilitation of concrete structures is gaining increasing popularity in the civil engineering community. One of the most attractive applications of FRP materials is their use as confining devices for columns, which may result in remarkable increases of strength and ductility. In the last decades, a big pool of experiments has been performed to investigate the behavior of concrete columns confined with FRP composites (e.g. Karabinis and Rousakis 2001; Mirmiran and Shahawy 1997; Pesski et al. 2001; Saafi et al. 2000; Seible et al. 1997; Xiao and Wu 2000). And also, various models were calibrated for FRP-confined concrete. (e.g. Hosotani et al. 1998; Karbhari et al. 1997; Lam and Teng 2002; Spoelstra and Monti et al.; Toutanji 1999; Wu et al. 2003).

The FRP technology develops fast in the last couple of years, and more and more FRPs were used in civil engineering, such as AFRP, PBO (short for Polypara-phenylene-Benzo-bis-Oxazole), DFRP (Dyneema FRP). Existing experiments were mainly focused on the behavior of CFRP or GFRP-confined concrete cylinders. So it is necessary for a systematic assessment of the effectiveness of different types of FRP or combination of FRP. Based on this point of view, two types of FRP are combined (named as hybrid FRP) to confine the concrete by using the advantage of each FRP. It is expected to get higher strength and ductility of cylinders confined with hybrid FRP.

In this paper, the performances of concrete cylinders confined with various FRP and hybrid FRP composites are studied. Equations to predict the ultimate strength and strain of hybrid FRP-confined concrete cylinders are suggested. The objective of this work is to get a better understanding of the performance of concrete columns confined with various FRP and hybrid FRP composites.
Properties of FRP composites

Five different types of FRP composites were used in this paper. These were high strength CFRP (CF1), high modulus CFRP (CF7), aramid FRP (AF), glass FRP (GF), and PBO FRP (PF). The mechanical properties of FRP composites are listed in Table 1. In Table 1, \(f_{fm}, E_{fm}, \) and \(e_{fm}\) are the ultimate strength, elasticity modulus of FRP and the ultimate strain of FRP provided by manufactures. \(f_{fu}, E_{f}, \) and \(e_{fu}\) are the test results.

TEST SPECIMENS

A total of 35 concrete cylinders having dimension 152mm in diameter and 300mm in height were tested in the laboratory. These specimens included 3 unconfined cylinders, 12 cylinders confined with one kind of FRP sheets and 20 cylinders confined with hybrid FRP sheets. Details of the test specimens are shown in Table 2. The FRP jackets were formed in a wet lay-up process by impregnating a continuous fiber sheet with matching epoxy resin. For the concrete cylinders confined with one kind of FRP, regardless of the number of FRP layers, a single continuous sheet with the main fibers oriented in the hoop direction was wrapped around cylinders, the finishing end of the sheet overlapping the starting end by a 100mm length as shown in Fig.1. For hybrid FRP-confined cylinders, it was continuous for the same FRP, and the location of the overlap for different FRP was distributed. Eight unidirectional resistance gauges were used to measure the strain of FRP, and four among of them were distributed at mid-height to measure the hoop strain of the FRP jacket. In addition, axial displacement of FRP-confined concrete cylinders was determined using two linear variable displacement transducers (LVTDs). The speed is 10kN/min.

EXPERIMENTAL RESULTS

Table 2 also summarizes the results of all specimens, where \(f_{cc,e}\) is the peak (maximum) strength of concrete cylinders, and \(e_{cc,e}\) is the strain at the maximum strength. The average maximum strength and the corresponding strain of three unconfined concrete cylinders are 23.1MPa and 0.00267. The ultimate strength and strain can be obvious increased after the cylinders confined with FRP, and which is related to the type of FRP, thickness of FRP and the hybrid ratio. Failure of the cylinders confined with FRP marked by fracture of the sheet with bursting along the mid-height of the specimens.

Axial stress-axial strain relationship of FRP-confined concrete

The axial stress-axial strain curves of concrete cylinders are shown in Fig.1. Fig.1 and Table 2 show that the confinement of FRP can considerably enhance the structural performance of concrete columns, especially with regard to ductility. The stress-strain responses of present specimens have no strain-softening component because the
confinement from FRP is strong (Wu et al. 2003).

Fig. 1 (a) shows the axial stress-axial strain diagrams of concrete cylinders confined with one layer CF1 (abbreviated to CF1), and one layer CF7 together with one layer CF1 (abbreviated to CF7+CF1). For the concrete cylinder confined with CF1, the initial stress-strain behavior was similar to that of plain concrete, since the lateral expansion of the core was insignificant. When the stress level was close to the maximum strength of unconfined concrete, a transition zone occurred with the increase in microcracks. And the stress-strain curve was generally linear until FRP ruptures since FRP was fully activated. The stress-strain curves of concrete cylinders confined with CF7+CF1 could be divided into three stages. In the first region, the curve was similar to that of plain concrete. In the second region, the concrete cracked and the FRP was fully activated. The curve in this section was often not linear, because two different types of FRP together confined the core concrete and the fiber gradually ruptured for the ultimate tensile strain of CF7 was low. When the axial stress of FRP-confined concrete reached 47.3 MPa, most of CF7 would ruptured, the confinement stress originally beard by CF7 was transferred to CF1. In the third region, the core concrete was only confined by CF1, the slope of stress-strain curve was smaller than that in the second region. The stress-strain response was linear, and the axial stress and strain would still be improved until CF1 ruptured.

Fig. 1 (b) shows the comparison of the stress-strain curves confined by PF, CF7+PF, and CF7+2PF. For the concrete cylinders confined with PF, the ultimate strength and strain were 45.0 MPa and 0.0237 respectively. For the concrete cylinder confined with CF7+PF, when the axial stress reached about 44.5 MPa, CF7 ruptures. Because one layer PF was not strong enough to absorb all the energy originally absorbed by CF7+PF, so the PF would rupture right after the CF7, and the concrete cylinder was damaged. The axial ultimate strain of concrete cylinder confined by CF7+PF was even smaller than that of cylinder confined with PF. here we comes to the conclusion that if the confinement strength of high stain FRP is too low, hybrid has no effect, and the ductility of confined concrete is even worse than that of counterpart specimens confined with one fold of FRP. However, the concrete cylinder confined with CF7+2PF hybrid was effective.

Fig. 1 (d) shows the stress-strain of concrete confined with 2C1, CF7+2CF1 and 2CF1. It showed that the stress-strain curves of cylinders confined with CF7+2CF1 and 2CF1 were almost the same, which means one more CF7 sheet will not obviously improved ultimate strength. There were two reasons: (1) the confinement stress of CF7+2CF1 was very high, so the effectiveness coefficient was lower than that when the confinement stress is low, for example 2CF1; (2) the ratio of confinement stress of 2CF1+CF7 was 3.9, so the effect of CF7 on the concrete cylinder was not so obvious like 2CF1. It means if the ratio of the confinement stress of high ductility FRP to low ductility FRP is too big, the effect of high ductility FRP on the hybrid is small and hybrid is not economic.

Effect of hybrid ratio on the behavior of hybrid FRP-confined concrete

The theoretic strain $\varepsilon_{f_{2,c}}$ of high ductility FRP after the low ductility FRP
ruptures can be calculated according to energy balance.

\[ \varepsilon_{f2,c} = \frac{f_{f1}t_{f1} + E_{f2}\varepsilon_{fu1}t_{f2}}{E_{f2}t_{f2}} \]  

(1)

Where \( f_{f1}, t_{f1} \) and \( \varepsilon_{fu1} \) are the strength, thickness and ultimate strain of low ductility FRP; \( E_{f2}, t_{f2} \) are the modulus and thickness of high ductility FRP.

Because the inner FRP with low ductility is confined by the outer FRP, when low ductility FRP ruptures, the actual value of the strain \( \varepsilon_{f2,c} \) of high-ductility FRP is smaller than the value of \( \varepsilon_{f2,c} \). And this is why the lateral confinement strength of concrete cylinder confined by CF7+CF1 is the same as that of the cylinder confined by CF1 at the ultimate stage, while the ultimate strength is higher than that of cylinders confined by CF1. The actual strain of high ductility FRP after low ductility FRP ruptures can be calculated by reducing \( \varepsilon_{f2,c} \) with a factor of \( \eta \).

\[ \varepsilon_{f2,c} = \eta \varepsilon_{f2,c} \]  

(2)

The factor of \( \eta \) is related to the confinement strength ratio \( \beta \) of the two FRPs, and which can be determined by Eq.(3) according to the regression of the present test results. The confinement strength ratio \( \beta \) can be calculated by Eq.(4).

\[ \eta = 0.195 \beta + 0.1689 \quad 1.45 < \beta \leq 4.26, \]  

(3)

\[ \beta = \frac{f_{f2}t_{f2}}{f_{f1}t_{f1}} \]  

(4)

Where \( f_{f2}, t_{f2} \) are the ultimate strength and thickness of high ductility FRP; \( f_{f1}, t_{f1} \) are the ultimate strength and thickness of low ductility FRP;

Based on the test results of the present experiment results, \( \beta \) is suggested to be between 1.45 and 4.26. If \( \beta \leq 1.45 \), the high ductility FRP cannot absorb the energy released by low ductility FRP when it ruptures, so the hybrid has no effect. If \( \beta > 4.26 \) \( \eta \) will be 1.0 according to Eq.(3), that means the confinement strength of the high ductility FRP is much stronger than that of low ductility FRP, and high ductility FRP has a little effect on the confined concrete, so the hybrid is not economic.

**ULTIMATE STRENGTH OF FRP-CONFINED CONCRETE**

**Ultimate strength of concrete cylinders confined with one kind of FRP**

The compressive strength \( f_{cc} \) of FRP-confined concrete has been discussed in detail in recent study. Eq.(5) has been used by most researchers to estimate the ultimate stress of confined concrete, assuming that the failure of the system occurs when the
confined pressures researches its maximum.

\[ f_{cc} = f_{co} + kf_1 \]  \hspace{1cm} (5)

Where \( f_{co} \) is the compressive strength of unconfined concrete; \( f_1 \) is the lateral confinement strength of FRP; \( k \) is the strength enhance coefficient.

Wu et al. (2003) presented a comprehensive review of existing compressive strength models, together with an assessment of their accuracy using a large test database. They suggested \( k \) could be taken as 2.4 for high modulus CFRP confined concrete and 2.0 for other FRP confined concrete. Strength enhance coefficient \( k \) calculated based on the present experiment are between 2.4 and 3.1, and the average value is 2.75, which is larger than that suggested by Wu et al. (2003). That may due to the unconfined concrete strength of present specimens is lower than that of specimens discussed by Wu et al. (2003). There is often a better confinement effect for low strength concrete and \( k \) will be bigger.

**Ultimate strength of concrete cylinders confined with hybrid FRP**

Fig. 3 shows the curves of axial stress and \( f_1 \) of FRP-confined concrete cylinders. For concrete cylinder confined with one kind of FRP, in the first stage, the confinement of FRP is very small. In the second region, there is a linear portion between the increase in axial stress from the intercept of the stress axis \( f_0 \) and the confinement stress. For hybrid FRP confined concrete, the relationship of axial stress and \( f_1 \) is similar to the concrete cylinders confined with one kind of FRP before low ductility FRP ruptures. The axial stress of FRP-confined concrete will keep at the stress level same to that before low ductility FRP ruptures, the strain of high ductility FRP will jump from \( \varepsilon_{f1} \) to \( \varepsilon_{f2,\sigma} \), and the confinement stress of FRP will reduce from \( \sigma_{l1} \) to \( \sigma_{l2} \). \( \sigma_{l1} \) can be calulated by

\[ \sigma_{l1} = \frac{E_f e f_2 t f_2}{R} \]  \hspace{1cm} (6)

Where \( R \) is the radius of concrete cylinders.

After that, the stress level in FRP-confined concrete continues to increase with increasing strain, because concrete still be effectively confined with high ductility FRP. And it can be noticed that the axial stress from \( f_{cc1} \) increases linearly to the confinement stress of high ductility FRP until to FRP ruptures.

The axial stress of FRP-confined concrete when low ductility FRP ruptures can be calculated by

\[ f_{cc1} = f_{co} \left( 1 + k_1 \frac{f_{l1}}{f_{co}} \right) \]  \hspace{1cm} (7)

Where \( k_1 \) is the stress enhancement coefficient in the first region, \( f_{l1} \) is the confinement strength of hybrid FRP.
The strength of hybrid FRP-confined cylinders when high ductility FRP ruptures can be calculated by

\[ f_{cc}'' = f_{cc1}'' \left( 1 + k_2 \frac{f_{i2} - \sigma_{i1}}{f_{cc1}''} \right) \]  

(8)

where \( f_{i2} \) is the confinement strength of high ductility FRP, and can be calculated by \( f_{i2} t_{f2} / R \); \( k_2 \) is the strength enhancement coefficient in the second region.

The stress enhancement coefficient \( k_2 \) is usually greater than \( k_1 \), because the strength of concrete in the first stage is low, and the confinement effect is high. Here \( k_1, k_2 \) are supposed to be equal to \( k \) for simplicity.

Comparison of presented and experimental strength

The experimental results obtained in this study are compared with the analytical results obtained by proposed model as shown in Table 2. In table 2, \( f_{cc, a}'' \) is the ultimate strength calculated by Eq.(7) or Eq.(8) and supposing \( k = 2.75 \) based on the present results. Error between presented values and experimental values are between -11.7\% and 9.1\%. \( f_{cc, b}'' \) is the ultimate strength calculated by Eq.(7) or Eq.(8) and supposing \( k = 2.4 \) in the first region and 2.0 in the second region (Wu et al. 2003). Error between presented values and experimental values are between -5.2\% and -31.7\%.

ULTIMATE STRAIN OF FRP-CONFINED CONCRETE

Influence parameters on the axial strain of FRP-confined concrete

The stress-strain curves of concrete cylinders confined with CF1 and 2CF1 are shown in Fig.(4). The ultimate strain increases with the layers of FRP. The stress-strain curves of concrete cylinders confined with 2CF7, CF1+3GF are also shown in Fig.(4). The confinement strengths of them are 9.70MPa, 9.43MPa, and 8.47MPa, and the strains of them are 0.0124, 0.0208, and 0.0243 respectively. That shows the fact that the ultimate strain of FRP-confined concrete is not only related to the \( f_{i1} / f_{co}'' \), but also to the ultimate strain of FRP, which is different from that of the ultimate strength of FRP-confined concrete.

Poisson’s ratio of FRP-confined concrete

One approach to examine the behavior of FRP-confined concrete is to study its Poisson’s ratio \( \nu \) of FRP-confined concrete.

\[ \nu = \varepsilon_r / \varepsilon_c \]  

(9)

Where \( \varepsilon_r \) is the lateral strain, and it is supposed to be equal to the strain of FRP according to strain compatibility; \( \varepsilon_c \) is the axial strain of FRP-confined concrete.
Fig. 5 shows the Poisson’s ratio of concrete cylinders confined with FRP. The Poisson’s ratio of unconfined concrete increased dramatically with the growth of micro cracks, and became unstable near the maximum strength. For the cylinders confined with FRP, when axial stress was low, the ultimate Poisson’s ratio of concrete was a constant value. When the stress was near to the strength of unconfined concrete, micocracks increased rapidly and lateral expansion was significant. The Poisson’s ratio of FRP-confined stabilized at an asymptotic value after FRP was fully activated. The confinement strength of the concrete cylinders confined with CF1, 2GF, PF, and AF were 9.43MPa, 8.47MPa, 7.10MPa and 8.86MPa, and the ultimate Poisson’s ratio of them were between 0.78 and 0.84. It seems that the ultimate Poisson’s ratio is mainly related to the ratio of confinement strength to unconfined concrete strength. The confinement strength of concrete cylinder confined with 2CF7 was 9.70MPa, which was similar to common modulus FRP, but the Poisson’s ratio was 0.31, less than that of other FRP-confined concrete cylinders. So the ultimate Poisson’s ratio is not only related to the value of $f_u/f_{co}$, but also related to the modulus of FRP when the modulus of FRP is greater than 250GPa (Wu et al. 2003). For the concrete confined with CF7+AF, the Poisson’s ratio of concrete rapidly increased near to stress of the strength of unconfined concrete, and because the concrete was confined with CF7+CF1, the Poisson’s ratio was stabilized while axial stress was still increasing. The rupture of CF7 resulted in a rapid expansion, so the Poisson’s ratio increased by from 0.38 to about 0.8. After the high ductility FRP was fully activated, the Poisson’s ratio trend to a stable value again until FRP ruptures. And it also can be found that the ultimate Poisson’s ratio of hybrid FRP-confined concrete is mainly related to the confinement strength of high ductility FRP and which is approximately equal to that of concrete cylinders confined with the same quantity of one kind of FRP.

**Ultimate strain of FRP-confined concrete**

Based on the test results shown in Fig.1 and Fig.(5), we can conclude that the ultimate strain of concrete cylinders confined with hybrid FRP is similar to that of counterpart specimens confined with high ductility FRP only. We can determine the ultimate strain of hybrid FRP-confined concrete with the method same to that of concrete cylinders confined with one kind of FRP. According to the model suggested by Wu et al. (2003), the ultimate Poisson’s ratio $v_u$ of concrete cylinders confined with common modulus CFRP sheet, GFRP sheet and AFRP sheet can be approximated by Eq.(10).

$$v_u = 0.56 \left( \frac{f_{u2}}{f_{co}} \right)^{0.66}$$

After $v_u$ is determined, the axial ultimate strain of FRP-confined concrete can be calculated from Eq.(11) according to strain compatibility.

$$\varepsilon_{cc} = \varepsilon_{fu2}/v_u$$

**Comparison of presented and experimental strain**

The experimental results obtained in this study are compared with the analytical results obtained by proposed model as shown in Table 2. Error of presented values and
experimental values are between -10.3% and -46.0%. Presented strain of FRP-confined concrete is less than that of experimental values for the low compressive strength of concrete specimens in this paper.

CONCLUSIONS

The following conclusions can be drawn from the results of this study:

1. The ultimate strength of FRP-confined concrete is mainly related to the $f_i/f_{co}$. The ultimate strain of FRP-confined concrete is not only related to the $f_i/f_{co}$ but also the ultimate strain of FRP.

2. For concrete cylinders confined with hybrid FRP, the ratio of hybrid is very important. If the ratio of confinement strength of high ductility FRP to low ductility FRP is too low, the hybrid may be ineffective, and if the ratio is too high, the hybrid will not be economic.

3. Model of Wu et al. (2003) can be used to predict the ultimate strength and strain of concrete confined with one kind of FRP. Based on that, models for predicting the ultimate strength and strain of hybrid FRP-confined concrete are suggested, and they are in agreement with the experimental results.

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REFERENCES


Wu et al.


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<th>$f_{fu}$ (MPa)</th>
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### Table 2: Description and test results of FRP-confined cylinders

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Where $f'_{cc,e}$ and $f'_{cc,a}$ are the presented ultimate strength by two different models, and $\varepsilon'_{cc,e}$ is the presented axial ultimate strain by the model of Wu et al. 2003.
Figure 1-Wrapping types of FRPs
Figure 2-Axial stress-axial strain curves of FRP-confined concrete
Figure 3-Model curves of axial stress and $f_i$.

Figure 4-Comparison of the stress-strain curves.

Figure 5-Poisson’s ratio of FRP-confined concrete.
Experimental Study of RC Columns Strengthened with CFRP Sheets Under Eccentric Compression


Synopsis: The objective of this research project is to conduct an experimental study of the behavior of carbon fiber-reinforced polymer (CFRP) materials for strengthening reinforced concrete structures. The test program consisted of 23 reinforced concrete columns strengthened with CFRP sheets under static eccentric compressive loading. It is shown that under large eccentric compression loading, ultimate capacity of the columns can be effectively increased with the longitudinal CFRP sheets, and the ductility factors of the columns can be increased with the transverse CFRP sheets. It was observed that the material strength utilization of transverse CFRP sheets was more sufficient than that of longitudinal CFRP. The transverse CFRP sheets were broken at failure of a column, but the maximum measured strain in the longitudinal CFRP was only about $5000\mu\varepsilon$ at peak load.

Keywords: CFRP; columns; eccentric loading; experiment; strengthening
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INTRODUCTION

The strengthening and retrofit of existing concrete structures with CFRP (Carbon Fiber Reinforced Polymer) has become common in China. There are many engineering applications in which CFRP was bonded onto the surface of reinforced concrete elements to improve their strength and ductility. One of the most important applications is the confinement of concrete columns with CFRP sheets. It is an effective method proved by many engineering applications and experiments (Mirmiran, 1997, Alex, 2001, Shahaway, 1999). Most of the concrete column research programs were focused on the investigation of seismic behavior of flexural columns strengthened with CFRP sheets, and tested under reverse cyclic bending loading with constant axial compression load (Sheikh, 2002, Sasdatmanesh, 1994, Chaallal, 2000, Nanni, 1995). Another investigation was uniaxial compression testing of concrete cylinder or square columns wrapped with FRP sheets (Toutanji, 1999, Mirmiran, 1997, Wang, 2001). In the two kinds of research program the enhancement action of the transverse CFRP sheets was results from confinement of concrete in the columns. It was shown in testing that though ductility of the columns was increased with the confinement, the enhancement of ultimate flexural capacity of concrete columns strengthened with transverse CFRP was very limited. In the test program presented in this paper, an experimental investigation was conducted to study the performance of reinforced concrete columns strengthened with longitudinal and/or transverse CFRP sheets under monotonically applied eccentric compression. Twenty-three reinforced concrete columns were tested in this research. The purpose of the study was to examine the ultimate capacity enhancement by the CFRP. In addition, this study also investigated the deformation behavior or ductility of reinforced concrete columns retrofitted with CFRP under eccentric static loading.

EXPERIMENT PROGRAM

Design of test specimen
The column specimens have rectangular sections with the dimensions of 250mm x
350mm, and a length of 2200mm. Fig. 1 shows the column dimension and reinforcement details. The yielding stress of longitudinal and transverse reinforcement is 365MPa and 272MPa, respectively. In column specimen C00L-1, C10L-1 and C01L-1 three longitudinal steel bars of 18mm diameter were arranged in each side of the section, and in other columns two steel bars of 18mm diameter were arranged.

Properties of CFRP and adhesive

High tensile strength carbon fiber tow sheets (Type Hex-3R) were used and impregnated using three-part epoxy polymer adhesive. The tensile strength and felted strength of the resin adhesive is 20MPa and 3.5MPa, respectively. The low bound mechanical properties of the resulting CFRP laminate according to manufacturer’s literature are listed in Table 1.

The CFRP laminate was applied by the manual lay-up procedure. First the column specimen was burnished so that the radius of the corners was about 10mm. Subsequently a layer of primer glue was padded on the concrete surface. Then a two-part saturant was thoroughly mixed and a thin layer was applied on carbon fiber sheet using a sponge brush. With careful handling, the carbon fiber ply was wrapped around the specimen or longitudinally along the specimen. A plastic roller was used to remove the air entrapped between fiber ply and saturant. After approximately 30 minutes, a third layer of epoxy glue was applied and the plastic roller was used again to work the resin into the fibers. The system is cured for 4-5 days at room temperature to reach full strength before strain gages were attached. Three identical specimens but with different bonding mode were manufactured for each set of columns.

There were five bonded modes of CFRP sheets as shown in Fig. 2 and listed in Table 2.

Test program

A 5000 kN four-posts compression testing machine was used to load the columns. An incremental loading program was used in testing. Before cracking of concrete the loading increment was 20kN then changed to 50kN after tension cracking of concrete. Beyond the maximum load the hydraulic pressure oil was continuously supplied to the cylinder of the testing machine until the load was dropped to the 80 percent of maximum load.

The lateral displacement data were collected from five locations using dial gauges as shown in Figure 3. In order to record the displacement beyond the maximum load a strain displacement transducers was placed in the mid-height of the columns. Resistance strain gauges with a 100 mm gauge length were placed at selected sites throughout the columns. Strain data were collected at the mid-height section as shown in Figure 3. Other important strains were collected as needed. Gauges were placed on the concrete surface, on the FRP surface, or on the steel inside the column. Cracking was recorded during the testing.

TEST RESULTS AND DISCUSSIONS

The capacity and lateral displacement at the mid-height of a column are summarized in Table 2. In this table the capacity enhancement is calculated as follows:
\[ \Delta = \frac{\tilde{N}}{N_c}, \quad \tilde{N} = \frac{N}{f_c bh_0} \]

in which \( \Delta \) is the capacity enhancement, \( N \) is the measured maximum load, \( f_c \) is the axial compression strength of concrete, \( b \) is the width of the section and \( h_0 \) is the effective depth of the section. The subscript “c” denotes the control column.

It is shown that longitudinal CFRP sheets enhanced the ultimate capacity of columns, with maximum enhancement of 22\% (C20L-3) greater than the unstrengthened control column. For the columns loaded with large eccentricity \( (e_0 = 0.5h, h = \text{the depth of columns}) \) the enhancement action of the CFRP sheets longitudinally bonded on the tensile concrete surface was in the range of 8\%–22\%. From table 2, some important observations include the followings:

1. The maximum load of the columns was controlled by the ultimate compressive resultant stress of concrete and the maximum stress of CFRP. As the longitudinal steel ratio was increased, maximum curvature of section of the column was decreased at the maximum load, so the stress of CFRP would be at a low level and the enhancement action was small, such as column C10L-1 which had steel ratio 50\% greater than other columns but only 8 percent greater maximum load than the control column.

2. The concrete strength had an important effect on the enhancement action of ultimate capacity of columns with CFRP strengthening. As shown in Table 2, the column C11L-2 and C11L-3 had the same parameters but the concrete strength, the enhancement extent of maximum load was 16.3\% and 21.1\%, respectively. The same result was observed in the other group, C20L-2 (C20L-2a) and C20L-3, in which the enhancement extent was 13.7\% (16.7\%) and 22\%, respectively. According to the experimental results it appears that the concrete strength grade should not be lower than C35–C40 if one expects significant capacity enhancement in the columns wrapped with longitudinal CFRP sheets.

3. The enhancement in ultimate capacity of the columns strengthened with only transverse CFRP sheets was small. In the column C01L-1, C01L-2 and C01L-3, the enhancement of 6.7\%, 1.7\% and 4.6\% was recorded, respectively. It is noted that the column C01L-1 had the greatest capacity enhancement by the transverse CFRP sheets but C10L-1 was the lowest one of the columns strengthened with longitudinal CFRP sheets. It was shown that the transverse confinement action was more effective for a column with higher steel ratio.

4. Another comparisons were the two sets of column, C11L-2 to C10L-2a and C11L-3 to C10L-3. The columns in each set had the nearly same parameters other than the bonding mode. The columns C11L-2 and C11L-3 were strengthened with transverse CFRP sheets besides longitudinal CFRP sheets. As shown in Table 2, the ratios of capacity enhancement of the two sets were 16.3 / 12.4 = 1.31 and 21 / 13.4 = 1.57. Obviously the transverse CFRP not only confined concrete but also enhanced the anchoring effect of longitudinal CFRP sheets, which increased the maximum tensile stress in longitudinal CFRP at failure of the columns.
To the columns loaded with smaller eccentricity only transverse CFRP sheets were wrapped. It was shown in Table 2 that the ultimate capacity of columns increased with CFRP sheet layers. Contrary to the columns strengthened with CFRP sheets longitudinally bonded under large eccentric compression, the enhancement of maximum load is greater for the columns with lower concrete strength. For the set of columns C01S-1 and C02S-1 which had the concrete strength of 28.4MPa and 30.1MPa, the test results in Table 2 showed 7.7% and 13.3% increase in maximum load over the control column, respectively. In the columns C01S-2 and C02S-2, which had concrete strength of 49.4MPa and 38.4MPa, there were only 3.6% and 5.1% increase in maximum load over the control column, respectively.

Displacement ductility of a reinforced concrete member is defined as the ratio of the ultimate displacement to the yielding displacement. In this study the lateral displacement at the mid-height of a column under maximum load was taken as the yield displacement, and the displacement at which the load was dropped to 85% of the maximum load was taken as the ultimate displacement. According to this definition, the ductility factors of the test columns are listed in Table 2. The load vs. lateral displacement curves of columns are shown in Figure 4.

As shown in Table 2, the columns strengthened only with longitudinal CFRP sheets had lower ductility, and the ductility factors varied between 1.24 ~ 1.65. But the ductility factors of the columns wrapped with transverse CFRP sheets varied between 1.82 ~ 2.25, despite existence of longitudinal CFRP sheets. It was seen that the ductility of reinforced concrete columns could be significantly improved by the confinement of transverse CFRP sheets. It was also shown that the number of CFRP sheet layer had a great effect on the ductility of the columns under small eccentricity compressive loading. For example, the ductility factor of column C02S-1 was increased to 3.07 with two layers of CFRP sheets wrapped.

As indicated in Figure 4, the load-displacement curves of the columns wrapped with CFRP could be divided into three stages. At the first stage the load-displacement curves behaved like a straight line, and the load at this stage was about 80% ~90% of the maximum load for columns under small eccentric compression. And the straight line was ended as cracking of concrete occurred for the columns under large eccentric compression. At this stage, the stresses in reinforcement and CFRP sheets were low.

At the second stage, for the columns under large eccentric compression the strain in longitudinal CFRP sheets and reinforcement increased synchronously until maximum load, as shown in Figure 5. The strain in longitudinal CFRP sheets was about 2500 $\mu$ε for columns with lower concrete strength and 4500 ~ 5000 $\mu$ε for columns with higher concrete strength (see Figure 6). And the strain in transverse CFRP sheets, obviously lagging behind the strain in longitudinal CFRP, reached only about 1000 $\mu$ε (see Figure 7). For columns under small eccentricity load, with only transverse CFRP sheets applied, the strain in CFRP also increased to 2000 $\mu$ε. From experimental observation and data analysis it was found that the confinement action of CFRP didn’t begin to increase until the vertical cracks appeared at the maximum load.
Beyond the peak point, the load began to decrease but the strain in CFRP sheets increased rapidly. The strain in longitudinal CFRP finally reached 8000 ~ 12000 \( \mu \varepsilon \) when the load had dropped below the 85% of maximum load and the maximum concrete compression strain became greater than 0.003. At failure of columns the longitudinal CFRP sheets were peeled from concrete surface but didn’t fracture. The strain of transverse CFRP sheets in the columns loaded in small eccentricity rose rapidly with the decrease of load. Finally, the transverse CFRP sheets were broken at the failure section though the measured strain was still less than 10000 \( \mu \varepsilon \). The transverse CFRP sheets in the columns loaded with large eccentricity had a small effect on the capacity but enhanced the ductility of columns. The longitudinal CFRP sheets were also broken at failure.

### CALCULATION OF ULTIMATE CAPACITY OF TEST COLUMNS

According to the experimental observations, the ultimate capacity of the columns strengthened with longitudinal CFRP sheets can be calculated based on the similar assumptions used in design of reinforced concrete columns, except using the CFRP stress at the maximum load. It was shown that the longitudinal CFRP strain reached about 4500 ~ 5000 \( \mu \varepsilon \) at maximum load for columns under large eccentric compression. Because of measurement errors, the actual CFRP strain was always larger than the measured strain in CFRP sheets. Therefore taking 5000 \( \mu \varepsilon \) and corresponding stress as the CFRP stress at maximum load of the columns should result in conservative estimate of the stress in the longitudinal CFRP sheets. Using the CFRP stress at 5000 \( \mu \varepsilon \), the calculated results of ultimate capacity of the columns by the Chinese Code formulas (GB50010-2002) are listed in Table 3. As shown in this table, the difference between experimental and calculation results is not beyond 5%.

As confirmed by experimental results, the ultimate capacity of columns under eccentric loading was enhanced by transverse wrapping CFRP sheets. But the compressive concrete stress-strain relationship under the non-uniform confinement condition could not be directly obtained from the testing of concrete columns under eccentric loading. Considering confinement action applied by the transverse CFRP sheets, the enhancement factors of compressive concrete are introduced as follows (Wu, 2002)

\[
\begin{align*}
 f_{cc} &= k_{p1} f_c, \quad \varepsilon_{cc} = k_{p2} \varepsilon_0 \\
 k_{p1} &= 1 + 0.0025 \alpha \lambda, \quad k_{p2} = 1 + 0.0045 \alpha \lambda \\
 \lambda &= \frac{\rho_f E_{cf}}{\sqrt{f_c}}, \quad \alpha = \frac{25}{f_c} \quad (f_c \text{ in MPa})
\end{align*}
\]

where \( f_{cc} \) and \( f_c \) = peak stress of confined and unconfined concrete, respectively; \( \varepsilon_{cc} \) and \( \varepsilon_c \) = the strain of confined and unconfined concrete at peak stress, respectively; \( k_{p1} \) and \( k_{p2} \) = the enhancement factors of peak stress and strain of confined concrete, respectively; \( \rho_f \) = the volume ratio of the transverse CFRP sheets to the confined concrete core, and \( E_{cf} \) = the elastic modulus of CFRP.

In calculation of ultimate strength of columns, the rectangular stress block parameters are accordingly adjusted based on the equations (2)~(4), then ultimate strength of the
columns under small eccentricity and central loading can be calculated and the results are also listed in Table 3. It can be seen from table 3 that the calculation results is agree well with the test results.

CONCLUSIONS

Based on the experimental study on reinforced concrete columns strengthened with CFRP sheets under eccentric loading, the following observations and conclusions are drawn:

1. It was shown in the tests that the ultimate capacity of the columns strengthened with longitudinal CFRP sheets under large eccentric compression was enhanced. The steel ratio, concrete strength, number of CFRP layers and CFRP bonding mode had different effects on the ultimate strength of the columns. Under large eccentricity, the ultimate capacity enhancement of the columns could reach to 20% if the steel ratio was less than 1.5%, the concrete strength grade was greater than C35 and the CFRP sheets was well bonded on the surface of concrete.

2. For the columns strengthened with only transverse CFRP sheets under large eccentric compression, the ultimate strength enhancement of the columns was smaller compared with the columns strengthened with longitudinal CFRP sheets. But the ductility factors of the columns confined by transverse CFRP sheets were obviously greater than those of columns strengthened with only longitudinal CFRP sheets.

3. When the eccentricity was small, the ultimate strength and ductility of columns could be obviously improved by the transverse CFRP sheets. The transverse CFRP sheets were ruptured at failure of columns, and it was shown that the material strength of transverse CFRP sheets could be fully utilized at the ultimate limit state.

4. The measured strains of longitudinal CFRP sheets were about 4500~5000 $\mu\varepsilon$ in the columns under large eccentric compression. Because of non-uniformly distribution of the strain along the columns after cracking, the actual maximum strain should be greater than 5000 $\mu\varepsilon$. Considering the action of tensile CFRP sheets and based on the Code (GB50010-2002) formulas, the ultimate strength of columns under large eccentric compression were calculated. The calculated results agreed well with the test results. For the columns under small eccentric or axial loading, confinement action enhanced the peak stress of compressive concrete, and hence the ultimate strength of columns. Introducing a peak stress enhancement factor, the ultimate strength of columns wrapped with transverse CFRP sheets under small eccentric compression can also be accurately calculated by the Code formulas.

ACKNOWLEDGMENTS

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REFERENCES


<table>
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<tr>
<th>Table 1 -- Manufacturer provided CFRP properties</th>
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</thead>
<tbody>
<tr>
<td>Ultimate strength (MPa)</td>
</tr>
<tr>
<td>Tensile modulus (GPa)</td>
</tr>
<tr>
<td>Ultimate strain (mm/mm)</td>
</tr>
<tr>
<td>Thickness (mm)</td>
</tr>
<tr>
<td>Density (g/cm³)</td>
</tr>
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</table>
### Table 2 -- The basic parameters of columns and main test results

<table>
<thead>
<tr>
<th>Specimen No.</th>
<th>( f_{cu} ) (MPa)</th>
<th>( e_0 ) (mm)</th>
<th>Max. Load (kN)</th>
<th>Disp. at Max. Load (mm)</th>
<th>Enhanced in Max. Load (%)</th>
<th>Ductility factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>C00L-1</td>
<td>29.0</td>
<td>175</td>
<td>820</td>
<td>9.65</td>
<td>0</td>
<td>1.43</td>
</tr>
<tr>
<td>C10L-1</td>
<td>30.3</td>
<td>175</td>
<td>900</td>
<td>10.67</td>
<td>5.0</td>
<td>1.24</td>
</tr>
<tr>
<td>C01L-1</td>
<td>29.0</td>
<td>175</td>
<td>875</td>
<td>9.37</td>
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<td>1.82</td>
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<td>650</td>
<td>8.76</td>
<td>0</td>
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</tr>
<tr>
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<td>580</td>
<td>8.54</td>
<td>-10.5</td>
<td>1.42</td>
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<tr>
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<td>24.4</td>
<td>175</td>
<td>678</td>
<td>10.0</td>
<td>12.4</td>
<td>1.26</td>
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<td>C01L-2</td>
<td>29.8</td>
<td>175</td>
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<td>10.78</td>
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<td>9.19</td>
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<td>16.3</td>
<td>2.20</td>
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<td>10.97</td>
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<td>13.4</td>
<td>1.35</td>
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<td>C01L-3</td>
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<td>175</td>
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<td>9.26</td>
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<td>C20L-3</td>
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<td>1500</td>
<td>2.15</td>
<td>7.4</td>
<td>7.19</td>
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</tbody>
</table>

**Note:**
* First letter ‘C’ means Column, first number represents the number of layers of longitudinal CFRP sheets, second number represents the number of layers of transverse CFRP sheets, the letter 'L' and 'S' means larger and smaller eccentricity respectively, and ‘A’ means axial compressive loading.

**Note:**
* \( e_0 \) = eccentricity.

† 150mm cube compressive strength.

‡ Failed in an end of the column.
Table 3 -- Experimental and Calculation Results of the Ultimate Capacity of Columns

<table>
<thead>
<tr>
<th>Specimen No.</th>
<th>$f_c$ (MPa)</th>
<th>Experiment (kN)</th>
<th>Calculation (kN)</th>
<th>Difference (%)</th>
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<tr>
<td>C00L-1</td>
<td>22.0</td>
<td>820</td>
<td>855</td>
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<td>23.0</td>
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<td>890</td>
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Figure 1-- Dimensions and reinforcement details of test specimen.
Figure 2 -- Bonded modes of CFRP sheets.

Figure 3 -- Test setup.
Figure 4 -- Load-displacement curves of test specimen.
Figure 5 -- Load versus strains in CFRP sheets and steel: (a) C11L-2, (b) C20L-3, and (c) C11L-3.
Figure 6 -- Load versus longitudinal strain in CFRP sheets.
Figure 7 -- Load versus strain in transverse CFRP sheets: (a) Eccentricity = 0.5h and (b) Eccentricity = 0.1h.
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Flexural Confinement of Plastic Hinges in Circular Bridge Columns Using Unstressed Prestressing Strand as Transverse Reinforcement

by A.M. Budek, M.J.N. Priestley, and C.O. Lee

Synopsis: Four concrete columns using prestressing strand as transverse reinforcement were tested to establish design parameters for the use of high-strength transverse reinforcement under seismic loading. Two tests were dynamic. The confinement of flexural hinges was satisfactory at reinforcement levels below that called for by code, provided that the spiral pitch was small enough to prevent buckling of the longitudinal reinforcement. Dynamic loading did not have any unanticipated effect on flexural performance.

Keywords: bridge; column; confinement; experimental; flexure; prestressing; reinforcement; seismic
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**RESEARCH SIGNIFICANCE**

Current code requirements for transverse reinforcement in spirally confined columns subject to seismic loading, such as those adopted in ATC32 [1], can result in severe congestion of reinforcing steel. This is exacerbated by the need to keep transverse reinforcement at or below a stress level of 413 MPa, to control crack width. The present study shows that the use of high-strength reinforcing steel can significantly reduce the volumetric ratio of transverse reinforcement if a higher tensile stress is permitted, with performance equivalent or superior to that offered by conventional reinforcement.

Current seismic bridge design standards, as outlined in ATC32[1] and Caltrans BDS [2] call for a limitation on stress on transverse reinforcement of 413 MPa. This is based largely on the perceived requirement to control inelastic crack widths in shear-critical columns. If high-yield-strength reinforcement is used, however, cracking will be elastic, and the transient nature of seismic loading will make the lower crack-width limit unnecessarily conservative, as the cracks will close on reversal of load.

Growing interest in the use of high-strength concrete (HSC) has led to investigations of high-strength confinement to fully exploit HSC’s full capacity [3, 4, 5, 6, and 7]. This has generally used either high-strength wire, or ‘Ulbons’, a high-strength deformed bar.

The use of multi-wire prestressing strand offers several advantages. It is available in a number of diameters, manufactured in long lengths, and is flexible enough to be wrapped around a column cage. It is also less expensive, on a strength basis, than Grade 60 reinforcement.
Research into the behavior of members confined with high-strength transverse reinforcement has been driven by increasing interest in the use of high-strength concrete (HSC), which requires high confining pressures to achieve adequate ductility for seismic design. These can be achieved either through increasing the volumetric transverse reinforcement ratio, or using higher-strength transverse reinforcement. The bulk of research in the area (and thus in the area of high-strength transverse reinforcement) has been in building column configurations, rather than bridge columns.

**Axial load tests**

Li, Park, and Tanaka [5] -- Li, Park, and Tanaka tested a large number of square and circular columns under axial load, with normal (f_{yt}=445 MPa) and high-strength (f_{yt}=1318) transverse reinforcement (trade name ‘Ulbon’), over a range of concrete strengths (35-82 MPa). High-strength reinforcement enhanced the axial strength and ultimate compression strain of the confined concrete and provided satisfactory antibuckling performance.

Razvi and Saatcioglu [6] -- Twenty-two circular cylinders were tested under monotonic axial compression. High-strength concrete (60-120 MPA) was used. Yield strength of transverse reinforcement ranged from 400 to 1000 MPa, and its pitch was varied. It was observed that the proclivity of HSC to brittle failure could be ameliorated by providing sufficient confining pressure (proportional to the product $f_{yt}P_{yt}$). Thus the volumetric ratio of transverse reinforcement required could be reduced through the use of higher-strength material, though full mobilization of the transverse reinforcement would require greater dilation of the core.

**Flexural Tests**

Li, Park, and Tanaka [5] -- Cyclic lateral loading was carried out on five square-section columns. Three of the tests used high-strength (f_{yt}=1318 MPa) transverse reinforcement. All used high strength concrete. Ductile behavior was not achieved in the plastic hinge regions. It was suggested that the lower dilation of high-strength concrete prevented the full mobilization of high-strength confinement.

Muguruma, Nishiyama, Watanabe, and Tanaka [3] -- This series of cyclic lateral-load tests was performed to address the confinement of high-strength concrete (80-130 MPa) with high strength (f_{y}=873 MPa) hoop reinforcement. The use of high-strength reinforcement did not increase flexural ductility at lower axial load; in both cases tested, transverse reinforcement was found to have fractured. The failure mechanism, which was longitudinal bar buckling, was delayed in the test unit using high-strength transverse reinforcement.

Sato, Tanaka, and Park [4] -- The researchers tested two 400 mm square columns using high-strength transverse reinforcement with f_{y}=1368 MPa, and normal-
strength longitudinal reinforcement. The columns had an aspect ratio of 2.5, and underwent reversed cyclic loading. Moderate-to-high levels of axial load were tested. The columns were reinforced under the assumption that the design strength of 1275 MPa for the transverse reinforcement could be used. The transverse reinforcement remained well within its elastic range (<0.7% strain) into high displacement ductility levels. Buckling of the longitudinal bars was forestalled by the use of high-strength transverse reinforcement.

Aziznamini and Saatcioglu [7] -- Nine 305 mm square columns with rectilinear ties were tested in reversed cyclic loading under moderate axial load; seven used HSC, and of these, two had high-strength (f_{yt} = 827 MPa) transverse reinforcement. The use of high-strength confinement was not found to play a significant role in the columns’ flexural response. The researchers concluded that the dilation of the HSC under the loading conditions examined was not sufficient to fully mobilize the reinforcement.

FLEXURAL TEST INSTALLATION, TEST UNIT DESIGN, PROCEDURE

The test configuration used a moderate aspect ratio of 5.28 (column diameter 457 mm, length 2.415 m, in single bending). The test setup is shown in fig. 1. Axial load was applied through two high-strength steel post-tensioning rods whose line of action was concurrent with the column centerline. Target axial load ratio applied was 0.15 f’cA_g.

The volumetric ratio of transverse steel in reference test unit HS4 was determined using the value adopted by ATC32 [1] to ensure adequate ductile performance:

\[
\rho_t = 0.16 \frac{f'_c}{f_{yt}} \left( 0.5 + \frac{1.25 P_{axial}}{f'_c} \right) + 0.13(\rho_l - 0.01) \tag{1}
\]

in which

- f’c = unconfined concrete compressive strength
- f_{yt} = yield strength of transverse reinforcement
- P_{axial} = applied axial load
- \rho_l = longitudinal reinforcement ratio

Direct use of the equation would have required a transverse reinforcement ratio of 0.009; however, a lower value of 0.008 was chosen for construction to ensure that a flexural failure would occur within the stroke capacity of the actuator used. This resulted in the use of #3 Grade 60 spirals (413 MPa nominal yield) pitched at 86 mm (s_p = 5.4 d_bf).

Design of the transverse reinforcement of HS5 and HS7 (dynamic) was based on an equivalent strain energy approach, in which it was assumed that all of the strain energy absorbed by the transverse reinforcement to an ultimate strain of 0.07 would be
available to balance the compressive strain energy of the concrete at maximum concrete compressive strain. The modified Mander [10] expression for ultimate concrete compression strain is [9]

$$
\varepsilon_{cu} = 0.004 + \frac{1.4 f'_st \varepsilon_{ut}}{f'_{cc}}
$$

(2)

in which $f'_{cc}$, the strength of confined concrete, may be approximated as 1.5 $f'_c$. The transverse steel strain energy in this equation is proportional to the product $f'_{yt} \varepsilon_{ut}$. Thus, for a given value of $\varepsilon_{cu}$ obtained through the use of Grade 60 transverse reinforcement, one can replace $f'_{yt} \varepsilon_{ut}$ with an equivalent strain energy term reflecting the properties of high-strength steel. Accordingly, the stress-strain curves of both the high-strength and Grade 60 reinforcement were integrated (see fig. 2), to achieve the following relationship:

$$
f_{pu} \varepsilon_{pu} = \frac{k_2}{k_1} f_{yt} \varepsilon_{ut}
$$

(3)

in which, $f_{pu} =$ yield strength of high-strength reinforcement (1723 MPa)

$\varepsilon_{pu} =$ ultimate strain of high-strength reinforcement (0.07)

$f_{yt} =$ expected yield strength of Grade 60 reinforcement (455 MPa)

$\varepsilon_{ut} =$ ultimate strain of Grade 60 reinforcement (0.12)

$k_1, k_2 =$ constants (see fig. 2-10; $k_1$=1.34, $k_2$=0.82)

The model for Grade 60 stress-strain behavior used was that of the Mander model for confined concrete [10], assuming yield strain of 0.002276 at 455 MPa, a yield plateau to a strain of 0.008, and a parabolic strain hardening path of an ultimate strain of 0.12 at 689 MPa. Design transverse reinforcement ratio for HS5 was $\rho_t=0.00564$.

The design of HS6 and HS8 (dynamic) was specified to achieve an equivalent inelastic confining pressure to HS4. Confining pressure provided by spiral reinforcement is given by

$$
f_i = \frac{2 f_u A_{sp}}{D s'}
$$

(4)

in which

$A_{sp} =$ area of spiral steel

$f_u =$ spiral steel ultimate stress

$D' =$ spiral diameter to center of spiral

$s =$ spiral pitch

This may be related to the volumetric transverse reinforcement ratio,

$$
\rho_t = \frac{4 A_{sp}}{D' s}
$$

(5)
to give

\[ f_i = 0.5\rho_i f_u \]  

(6)

The inelastic confining pressures for HS4 and HS6 were equated to determine the required volumetric reinforcement ratio for HS6.

Reinforcement details for HS4-8 are shown in table 1. Material properties are shown in table 2.

The testing of HS4-6 was quasi-static and cyclic; the early stages of loading were carried out in load control, to predetermined force targets. This was done in four single-cycle steps with the shear force for the fourth step corresponding to first yield of the flexural reinforcement. Displacement ductility (\( \mu \)) was a multiple of yield displacement normalized to ideal flexural strength. Three full displacement cycles were performed at each level of displacement ductility tested beyond yield. The testing of HS7 and HS8 followed the same displacement pattern as the quasi-static tests. Rate of cycling was at 2Hz through \( \mu = 1.5 \); thereafter, as the structural period increased with the formation of a plastic hinge, the frequency was progressively reduced to a final value of 0.3 Hz at \( \mu = 8 \). A ‘hard-start’ was used for HS7; that is, the actuator was commanded to move to full speed as quickly as possible. This resulted in large-magnitude force spikes immediately after movement commenced (which had no discernable effect on HS7’s performance). In the case of HS8, a ‘soft-start’ option was chosen, which brought the actuator more gradually up to full speed through the addition of an extra cycle at a level of displacement below the target displacement (this did affect HS8’s performance, as will be discussed below). The input signal to the actuator controller was a sinusoid. Dynamic testing of HS7 was stopped after three cycles at \( \mu = 8 \), because the limits of actuator travel (+/- 217 mm) had been reached. The column showed no overt signs of incipient failure.

**RESULTS**

**HS4 (reference test – conventional reinforcement)**

Designed to current seismic standards (with the above proviso), HS4 showed excellent performance through cyclic loading through \( \mu = 6 \). The hysteresis loops shown in fig. 3 are wide and stable, indicating a high degree of energy absorption. Failure was caused by buckling of the longitudinal bars at \( \mu = 8 \). Buckling failure may not be a completely accurate representation of failure of a conventionally reinforced column for two reasons. First, the material chosen for the spiral was #3 bar, and to obtain the requisite reinforcement ratio the pitch was over 5 longitudinal bar diameters. Also, the transverse reinforcement ratio was 11% lower than specified by eq. 1 (to keep maximum stroke within actuator capacity), with commensurately greater bar spacing. Confining steel strains are shown in fig. 4, and are consistent with observed damage (yielding of the
spirals at \( \mu=6 \) in the lower part of the column. The development of high strains at a height of about one column diameter may be the result of strain penetration from the shear faces, permitted by bond slip in the spirals as the integrity of the cover concrete was compromised by cracking. Curvature profiles, shown in fig. 5, are consistent with flexural action, and exceed the predicted maximum curvature. Curvature at the column base at \( \mu=8 \) was not recorded because the potentiometers were removed to prevent damage.

**HS5 (equivalent strain energy)**

HS5 (and its structural twin HS7) was the most heavily reinforced of the specimens. The hysteresis loops of the force-displacement response (shown in fig. 6) show a great deal of energy absorption; the column retained its integrity and competence through cycling at \( \mu=8 \). Indeed, its strength forced a considerable amount of damage into the footing. Uplift in the footing cover concrete (exposing the footing steel) was noted at \( \mu=8 \), indicating significant strain penetration from the column longitudinal bars into the footing. No incipient buckling in the column longitudinal bars was noted at this point. Failure occurred at \( \mu=10 \), through buckling of longitudinal bars followed by confinement rupture. Curvature profiles for HS5 are shown in fig. 8. Maximum recorded curvatures were somewhat in excess of those predicted. Curvature at high levels of ductility is not reliable due to uplift in the footing cover concrete (from which measurements were referenced); uplift would have tended to reduce recorded curvature in a section. Fig. 9 shows HS5 at \( \mu=6 \); extensive spalling is visible, but the column steel is showing no distress. Fig. 10 shows the plastic hinge of HS5 after testing. The buckled longitudinal bars and damaged footing are very evident.

**HS6 (equivalent confining pressure)**

HS6 was the most lightly reinforced of all of the columns tested. Its force-displacement response (fig. 11) shows stable hysteretic behavior through \( \mu=6 \), but failure during the first negative excursion to \( \mu=8 \). It paralleled the reference specimen HS4 fairly closely, though its failure slightly preceded that of HS4. Failure was directly attributable to longitudinal bar buckling. The wide spacing of the spiral, coupled with the reduced stiffness of the strand, and a slightly lower elastic modulus of prestressing material, combined to allow bar buckling that was more widespread and serious than seen at commensurate ductility levels in HS4. Confining steel strains for HS6 are shown in fig. 12. While the recorded data (taken at first-cycle displacement peaks) does not indicate strains in the inelastic range, it should be noted that a significant amount of data was lost through strain gauge failure at high damage levels. Also, critical gauges were monitored in real time during the test, and these (lost before displacement peaks were reached) indicated inelastic strains in the plastic hinge region. Curvature profiles are shown in fig. 13, and reflect the effect of local buckling causing very high localized curvature at failure (\( \mu=8 \)). The effect of incipient buckling can be seen in push at \( \mu=6 \).
HS7 (equivalent strain energy - dynamic)

HS7 was the dynamically-tested twin to HS5, and its performance was similar. The force-displacement response (fig. 14) shows wide and stable hysteresis loops through $\mu=8$ (this was the limit of the available stroke on the dynamic actuator). HS7 carried slightly higher shear than did HS5; this is attributable to dynamic effects [10]. At the end of cycling at $\mu=8$, there was no visible damage to the column steel. Substantial damage had been forced into the footing. Testing of HS7 was completed with three extra quasi-static cycles at $\mu=8$, which resulted in failure of the spirals in confinement. To preserve the clarity of the comparison with the quasi-statically tested HS5, this is not shown in fig. 14. Fig. 15 shows confining steel strains, and these generally remained within the elastic range, as in the case of the similarly reinforced HS5. Curvature profiles are shown in fig. 16. Aside from one obvious artifact (at a height of 200 mm in push), the profiles are consistent and exceed that which was predicted as a maximum. Again, some uplift in the footing cover may have reduced the measured curvature at high levels of ductility.

HS8 (equivalent confining pressure - dynamic)

The response of HS8 was probably dominated by the choice of the ‘soft-start’ option for dynamic loading. As seen in the force-displacement response (fig. 17), failure occurred during cycling at $\mu=6$, earlier than in its quasi-statically tested structural mate, HS6. ‘Soft-start’ used an extra half-cycle, at a lower displacement level, to bring the actuator up to full speed without the force spikes caused by hard-start. This extra half-cycle put enough energy into the system to precipitate an earlier failure than HS6. Confining steel strains for HS8 are shown in fig. 18. They go into the inelastic range at the end of the loading history, and their peak location correlates with the location of longitudinal bar buckling and spiral damage. Curvature profiles are shown in fig. 19, and indicate maximum curvatures somewhat lower than those predicted by the Mander model [10]. This may be due to the column being ‘over-exercised’ by the use of soft-start for loading; considerably more energy (on the order of 20%, as estimated by displacement) was put into the column at each load step than in the other tests. Fig. 20 shows HS8 after cycling at $\mu=6$. A ruptured spiral is visible, as are several buckled longitudinal bars.

Comparisons of HS4 – HS8

Comparing first the static tests, HS5 and 6, we can see that the closer spiral spacing and resulting larger transverse reinforcement ratio resulted in considerably better performance. That this would be the case is obvious, but the nature of HS6’s failure is not. HS6 failed through buckling of the longitudinal bars, which began between spirals, but the spiral stiffness was not sufficient to provide the same restraint as would be given by conventional reinforcement. The seven-wire strand was sufficiently flexible to be bent away from the developing buckled region in the vertical place (up and down the column face), and horizontally (pushed outward from the column) to a lesser degree. This clearly allowed the buckled region to expand. HS4, with an even wider pitch than HS6, actually provided better antibuckling restraint, and though both columns failed at $\mu=8$, HS4’s failure was slightly later; it did survive a full cycle. The enhanced performance of HS5
and HS7 may therefore be attributed to a combination of transverse reinforcement properties, and the closer spacing used.

Use of the equivalent strain energy approach in designing HS5 resulted in an underestimation of its capacity; this may have come about as a result of the column’s ability to shed excess energy into the footing.

Dynamic effects played only a small role in column performance, and this could be easily predicted using current analytical tools [10]. The dynamically tested columns carried a slightly higher overall shear. The performance and observed damage in HS5 and HS7 (through μ=8) was nearly identical. The use of the soft-start, as mentioned above, subjected HS8 to a higher energy input (in the form of the extra half-displacement-cycle) than HS6, and probably caused its early demise.

In conclusion, while other parameters need to be examined (such as axial load variation, effect of longitudinal bar diameter, and concrete quality and strength), the results from this test program indicate significant potential for the enhancement of flexural performance when high-strength transverse reinforcement is used.

**CONCLUSIONS**

1. The high elastic capacity of high strength transverse reinforcement was advantageous in resisting core dilation, provided spiral pitch was sufficiently close to prevent buckling of the longitudinal reinforcement.

2. Designing for an equivalent strain energy capacity (when compared to Grade 60) in the transverse reinforcement provides a higher rotation capacity than does conventional reinforcement.

3. Designing for equivalent inelastic confining pressure will probably provide equivalent performance to conventional reinforcement, providing antibuckling criteria are met.

4. The nominal spiral pitch of $s \leq 6d_b$ was inadequate to forestall buckling of the longitudinal reinforcement, due to: 1) the reduced section stiffness of seven-wire prestressing strand when compared to a conventional reinforcing bar, and 2) the reduced elastic modulus of prestressing material. A maximum spacing of $4d_b$ is suggested pending future research.

5. Strain rate enhancement of confined concrete strength was not affected by the use of high-strength transverse reinforcement, and could be predicted through standard methods.
ACKNOWLEDGMENTS

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International Symposium on Confined Concrete

Table 1: Test Unit Reinforcement Details for HS5-7 and HS4

<table>
<thead>
<tr>
<th>Test Unit</th>
<th>Type</th>
<th>Longitudinal Reinforcement (Gr. 60)</th>
<th>Transverse Reinforcement</th>
<th>( s )</th>
<th>( \rho_t )</th>
</tr>
</thead>
<tbody>
<tr>
<td>HS4</td>
<td>Flexure Reference test</td>
<td>20 #5 (D15.9)</td>
<td>#3 (D9.5) Gr. 60</td>
<td>86 mm</td>
<td>0.008</td>
</tr>
<tr>
<td>HS5,7</td>
<td>Flexure (HS7 dynamic)</td>
<td>20 #5 (D15.9)</td>
<td>6.2 mm strand (Gr. 250)</td>
<td>36.5 mm</td>
<td>0.0057</td>
</tr>
<tr>
<td>HS6,8</td>
<td>Flexure (HS8 dynamic)</td>
<td>20 #5 (D15.9)</td>
<td>6.2 mm strand (Gr. 250)</td>
<td>69 mm</td>
<td>0.0032</td>
</tr>
</tbody>
</table>

Table 2: Day of Test Material Properties and Axial Load for HS5-8, and HS4

<table>
<thead>
<tr>
<th>Test Unit</th>
<th>Column Strength ( f_c ) (MPa)</th>
<th>Longitudinal Steel Strength (MPa)</th>
<th>Transverse Steel Strength (MPa)</th>
<th>Axial Load (kN)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( f_c ) yield ultimate</td>
<td>( f_y ) ultimate</td>
<td>( f_y ) ultimate</td>
<td></td>
</tr>
<tr>
<td>HS4</td>
<td>32.0</td>
<td>429.2</td>
<td>720</td>
<td>385.8</td>
</tr>
<tr>
<td>HS5</td>
<td>32.5</td>
<td>429.2</td>
<td>720.0</td>
<td>1569</td>
</tr>
<tr>
<td>HS6</td>
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</tr>
<tr>
<td>HS8</td>
<td>35.9</td>
<td>429.2</td>
<td>720.0</td>
<td>1569</td>
</tr>
</tbody>
</table>

Fig. 1-- Flexural column test setup
Fig. 2: Strain energy of Gr. 250 prestressing strand compared with that of Gr. 60 reinforcement.

Fig. 3: HS4 force-displacement (Gr. 60 transverse, $P_{\text{ax}} = 0.15 f'c_{\text{Ag}}$)

Fig. 4: HS4 confining steel strains (Gr. 60 transverse, $P_{\text{ax}} = 0.15 f'c_{\text{Ag}}$)
Fig. 5: HS4 curvature profiles (Gr. 60 transverse)

Fig. 6: HS5 force-displacement (Gr. 250 7-wire, equivalent strain energy)

Fig. 7: HS5 confining steel strains (Gr. 250 7-wire, equivalent strain energy)
Fig. 8: HS5 curvature profiles (Gr. 250 7-wire, equivalent strain energy)

Fig. 9: HS5 at $\mu=6$

Fig. 10: HS5 after test; footing steel exposed, buckled column bars
Fig. 11: HS6 force-displacement (Gr. 250 7-wire, equivalent confining pressure)

Fig. 12: HS6 confining steel strains (Gr. 250 7-wire, equivalent confining pressure)

Fig. 13: HS6 curvature profiles (Gr. 250 7-wire, equivalent confining pressure)
Fig. 14: HS7 force-displacement (Gr. 250 7-wire, equivalent strain energy, dynamic)

Fig. 15: HS7 confining steel strains (Gr. 250 7-wire, equivalent strain energy, dynamic)

Fig. 16: HS7 curvature profiles (Gr. 250 7-wire, equivalent strain energy, dynamic)
Fig. 17: HS8 force-displacement (Gr. 250 7-wire, equiv. confining pressure, dynamic)

Fig. 18: HS8 confining steel strains (Gr. 250 7-wire, equiv. confining pressure, dynamic)

Fig. 19: HS8 curvature profiles (Gr. 250 7-wire, equiv. confining pressure, dynamic)
Fig. 20: HS8 after cycling at $\mu=6$, showing ruptured spirals and buckled longitudinal bars (Gr. 250 7-wire, equivalent confining pressure, dynamic)
Compressive and Flexural Behavior of RC Columns Laterally Prestressed by High-strength Steel Bars

by T. Hitaka, K. Sakino, T. Yamakawa, and A. Furukawa

Synopsis: A dry column confining method using high-strength steel bars proposed by one of the authors is applied to a total of 24 reinforced concrete columns. Two series of tests were conducted on these columns. Hoop steel ratio of the columns was 0.1%, typical of RC columns designed before the Japanese Building Code in 1981. These RC columns were laterally reinforced using 19 sets of four high-strength steel bars and L-shaped steel blocks that were attached around the column by tightening the high-strength steel bars. Five specimens were exposed to concentric compression. Uniform bending test was conducted on 19 specimens under constant axial load. High-strength steel bars were pre-tensioned before testing. The amount of pre-tension was controlled by strain in the bars. It was found that the new confining method is effective to increase ductility of the RC columns. Strength of a RC column increases in a proportional manner to the amount of pre-stress in high-strength steel bars. Strength estimation method is proposed. The measured strength varied between 93% to 116% of the calculated strength.

Keywords: confined concrete; high-strength steel bar; moment-curvature analysis; prestress confinement; retrofit; ultimate moment
INTRODUCTION

A dry column confining method using high-strength steel bars, as shown in Figure 1, has been proposed by Yamakawa (2004). This technology is used to retrofit reinforced concrete (RC) columns designed under old design codes. It may also be applied to enhance axial strength and ductility of RC columns right after being severely damaged by an earthquake. Topic of this paper is compressive and flexural behavior of such confined RC columns.

Compressive strength of concrete is known to increase by confining RC columns. This effect of confinement is expressed by the following equation (Richart, Brandzaeg and Brown, 1929),

$$\sigma_{ccB} = \gamma_U \cdot f'_c + k \cdot \sigma_r$$  

(1)

where $\sigma_{ccB}$ is the strength of confined concrete, $\gamma_U$ is the scale effect coefficient, $f'_c$ is the compressive strength of concrete cylinder and $\sigma_r$ is the confining stress in the concrete, which occurs when the concrete bares out against the transverse reinforcement. The notation, $k$, is the coefficient that reflects shape of section or configuration of transverse reinforcement.

Behavior of confined concrete in RC columns was actively studied to develop methods to estimate the compressive strength and ultimate strain of the confined concrete. In the method proposed by Sakino and Sun (1994), calculation of $k \cdot \sigma_r$ in equation (1) is given as the following.

$$k \cdot \sigma_r = 11.5 \rho_h \sigma_{yh} \left( \frac{d''}{C} \right) \left( 1 - \frac{S}{2D_c} \right)$$

(2)
where $\rho_h$ is the volumetric ratio of rectilinear transverse reinforcement to the confined core measured center-to-center of perimeter transverse reinforcement; $\sigma_{yh}$ is the yield strength of transverse reinforcement with an upper limitation of 700 N/mm$^2$; $d''$ and $C$ are the nominal diameter and laterally unsupported length of transverse, respectively; $S$ is the spacing of transverse reinforcement; and $D_c$ is the distance between center of perimeter transverse reinforcement. Units of length and stress are mm and N/mm$^2$, respectively. More detail is found in the paper written by Sakino included in the proceedings.

In most researches referenced in deriving this method, yield strength of the transverse reinforcement is about 300 N/mm$^2$, and the reinforcement elements act passively, i.e. the reinforcing steel strain increases in response to the concrete bearing out against the transverse steel. Yield strength of the transverse reinforcement substitutes for $\sigma_{yh}$ in equation (2). This is based on the observation during the tests providing data for the method, that the transverse steel yields at a compressive stress of the column approaching its unconfined strength. This is not the case, however, if yield strain of the transverse steel is too large. As yield strain of the reinforcement is proportional to its yield strength, yield strength should not be applied in such cases. Yield strength of the transverse steel applicable for this method is therefore limited not to exceed 700N/mm$^2$. This limitation is prescribed based on a few test data suggesting that such high-strength steel does not yield at the maximum strength stress level.

High-strength steel used for the transverse reinforcement in this presented study is 1041 N/mm$^2$, three-fold of typical reinforcing bars used in Japan. The yield ratio is 0.9. Although the transverse reinforcing method employed for the RC columns studied in this paper is different from the typical methods, the tests provide data on general behavior of elastic transverse reinforcement and its effect on RC columns' behavior. Moreover, the notation, $\sigma_{r}$, in equation (1) is confining stress active in RC columns. The equation (1) suggests the possibility that the strength of a RC column may be increased by actively confining, i.e. pre-tensioning, the reinforcement. Supposing that this is the case, pre-tensioning is an effective method to use high-strength steel reinforcement to its full capacity. The objective of this study is to verify the compressive and flexural performance of RC columns confined by non/pre-tensioned high-strength steel bars. First, this paper describes two series of tests on RC columns, confined or non-confined. It is followed by discussion of test results. In the discussion, the strain behavior of confining bars is closely studied.

**TEST**

**Specimens**

Two series of tests were conducted with a total of twenty-four specimens, as summarized in Table 1. Figure 2 shows detail of the column and the jacket. Series A specimens are concentrically compressed, while Series B specimens are subjected to universal bending under constant axial load. Test parameters are (1) the amount of pre-stress in high-strength steel bars and (2) axial load ratio ($N/N_o = N/(bD_c f_{c}')$, $N$: applied axial load, $b$ and $D_c$: width and depth of column section, $f_{c}'$: cylinder strength of concrete)
and (3) loading protocol (monotonic/cyclic), as summarized in Table 1.

The specimens are designated as follows: RC indicates that the specimen is a RC specimen (without high-strength steel bar confining). NPS and PS specimens are those covered with high-strength steel bar sets along the full length. In the high-strength steel bars of NPS minimum tension required to prevent the steel blocks from slipping down the surface. PSL, PSM and PSH specimens are substantially pre-stressed, as summarized in Table 1. The amount of pre-tension in high-strength steel bars were monitored in terms of strain, measured by gages installed on the surface of each high-strength steel bar at the mid-length.

All specimens are 250x250x750mm RC columns of relatively low-strength concrete (design strength, $f_c = 18$ N/mm$^2$). Mechanical properties of the used concrete and steel are shown in Table 1 and Table 2, respectively. Each specimen is reinforced by twelve 10 mm-diameter deformed longitudinal bars, hence the reinforcement ratio is 1.5%. Four millimeter-diameter deformed bars are used as hoops, located at 105 mm intervals. Transverse reinforcement is thus sparse, similar to the RC columns designed before the revision of Japanese Building Codes in 1981. Top and bottom surface of the series A specimens was smoothed using plaster. Four high-strength steel bars of 5.4mm-diameter and L-shaped steel blocks are coupled as shown in Figure 2 and one set is placed around the column at each level (numbered from 1 through 18) at an interval of 41mm. Steel corner blocks are attached to the column by friction between the block and the column surface. Steel block has a threaded pit and a hole. The high-strength steel bars are threaded on both ends. One end is fixed into the threaded pit of the steel block, while the other penetrates the hole and bolted. Twenty-one RC columns are covered with eighteen high-strength steel bar-steel block sets along all its height. Middle portion of four specimens (RC), 451mm-long, were left bare.

### Loading and Measurement

Specimens are concentrically loaded by 5MN testing machine as schematically shown in Figure 3. Bending moment is applied to the pin-supported Series B specimens by a hydraulic jack, which is fixed at the tip of rigid girders that extend from the ends of the specimen. For Series B tests, displacements measured by transducers are used to calculate $\phi$, nominal curvature of the specimen. For the Series B tests, the loading protocol is designated in terms of $\phi D_c$, which is increased by 0.005rad. One deformation cycle is repeated three times at each deformation level until the flexural deformation, $\phi D_c$ is increased to more than 3%. The specimens were axially loaded first. Pre-tension was induced into the high-strength steel bars after the axial load was augmented to the level designated for each specimen.

Two strain gages were installed on all high-strength steel bars. For those located between No.7 to No.12 sections in Figure 2a, two gages were mounted per each, to eliminate strain caused by flexural deformation.
RESULTS

Series A: Concentric Compression Test

Figure 4a shows compressive force - compressive strain relation. When the high-strength steel bars of PS specimens (pre-tensioned specimens, which are PSL, PSM and PSH) are tensioned before vertical loading, the columns theoretically elongate a little due to Poisson’s effect. Noticeable elongation was observed only for PSH specimen, which was 0.03%. All specimens behaved in a similar manner up to an axial strain ($\varepsilon_2$) of 0.2%. Stiffness of the specimens was roughly equal. Loading was continued until axial resistance was lost for RC specimen, while for other specimens, loading was terminated when any one of high-strength steel bars broke. Behavior of each specimen is summarized as follows,

RC: Compressive force reached its peak at a compressive strain ($\varepsilon_2$) of 0.2 to 0.3%. Behavior beyond this deformation level was brittle. Strength was reduced rapidly.

NPS: Similar to RC specimen, strength increased close to its maximum strength at around $\varepsilon_2 =0.2$ to 0.3%. Maximum strength was almost the same as that of RC specimen. While behavior before this level is similar to RC specimen, strength scarcely deteriorated until axial strain, $\varepsilon_2$, exceeded 1.7%, when a high-strength steel bar broke in the threaded portion, which is different from RC specimen. The cause of fracture was bending of the bars caused by concrete popping out of the section, rather than elongation.

PS: Compressive force reached close to the maximum strength at around a compressive strain of 0.3%. Strength of PSL started to deteriorate after $\varepsilon_2$ exceeded 1.4%. Compressive strength decreased at a smaller deformation level for PSM and PSH, which was roughly $\varepsilon_2 =0.5%$. While maximum strength was larger relative to the amount of pre-tension for PSL and PSM, maximum strength was the same between PSM and PSH. Loading continued until compressive strain was increased to about 1.5%, when one high-strength steel bar broke in a similar manner as in the case of NPS specimen.

Figure 4b shows relations between tensile strain ($\varepsilon_p$) in high-strength steel bars and axial compressive strain of the RC columns. Two things are noted in observing Figure 4a and b. Tensile strain increases by only as much as 0.02% while $\varepsilon_2$ is increased to 0.2%. At $\varepsilon_2 =2\%$, compressive stress is larger than 95% of the maximum strength. Compressive stiffness beyond this level of the RC specimen decreases rapidly. At $\varepsilon_2 =0.3\%$, the curve is almost flat for NPS and PS specimens. Tensile strain in high-strength steel bars increases mainly after compressive strain exceeds 0.2%. Ultimate tensile strain of high-strength steel bars is about 0.5%, a close value to yield strain ($\sigma_y/E$). It is also noticed that ultimate compressive strain of columns is about 1.5 to 1.7%. While the ultimate strain is somewhat larger for specimens with smaller pre-tension in high-strength steel bars, the strain is roughly the same regardless of the amount of pretension. Taking the margin of strain glued on opposite sides of one high-strength steel bar, it is observed that flexural deformation of high-strength steel bars increases at a compressive strain of 0.7 to 0.8%. This strain is close to the compressive strain, at which RC specimen collapsed. Axial strain at which substantial failure of the concrete occurs
appears the same for confined and non-confined columns.

Series B: Uniform Bending Test under Constant Axial Load

Six to eight cracks were observed on the tension side of all Series B specimens at a nominal curvature of $\phi D_c = 0.5\%$. Intervals of cracks were relatively equal along height. Crashing of concrete started sometime during the first cycle of $\phi D_c = 1\%$. One high-strength steel bar broke during the first cycle of $\phi D_c = 2\%$ for specimen NPS6C. Similar event occurred for PSM6C during the first cycle of $\phi D_c = 3\%$. Strength deteriorated rapidly after this event, which led to termination of the testing. Fracture of the bar occurred in the thread section close to the corner block, where flexural deformation of the bars caused by bearing of the concrete popping out of the column section is the most severe. Studying the specimens after the testing, buckling of longitudinal reinforcing bars was observed in most specimens. Photographs of three specimens (RC6C, NPS6C, and PSM6C) are shown in Figure 5.

Figure 6 shows moment - $\phi D_c$ relation of the first half excursion of the monotonically loaded specimens (the dotted lines in Figures 7 through 9 show the full hysteresis). P-$\Delta$ moment is included in the moment (vertical axis) of Figures 6 and 7 through 9. While the flexural strength rapidly deteriorated for RC specimen at $\phi D_c = 0.5\%$, strength deterioration was not admitted for NPS and PS (i.e. confined) specimens. Comparing RC and NPS specimens, maximum strength was larger for the confined specimen. Maximum strength was larger for specimens with higher active confinement (pre-tension), although strength increase was not as large as observed for Series A tests. Figure 10a shows relations between strain of high-strength steel bars and $\phi D_c$. Strain in high-strength steel bars increased from the initial strain by 0.1% during the first half excursion of loading. Although the strain decreased by 0.045% when unloaded, it started to increase again when the deformation was reversed. Maximum strain during the latter half of the excursion was larger than the strain during the first half by 0.04%.

Cyclic hysteresis of the Series B specimens is shown in Figures 7 through 9. The envelop curve of the hysteresis of specimens tested under axial load of $N_N = 0.6$ is similar to the monotonic test results. Effect of high-strength steel bar confining is larger for specimens under larger axial load. Like the monotonic test, strength of RC specimen decreased sharply at a flexural deformation of $\phi D_c = 0.5\%$. Strength of all specimens was more than 95% of the maximum strength at this deformation. Strength was larger for specimens with higher pre-tension in the steel bars, which is also similar to the monotonic test results. Deformation affordable for specimens was larger for the confined specimens. The RC specimen under 0.2$N_N$ axial load resisted to flexural moment without strength deterioration until the first excursion of $\phi D_c = 1.5\%$ cycle. The confined specimens behaved in a far more ductile manner. Enhancement of ductility (compared with RC specimens) was the same for all the confined specimens regardless of the amount of pre-tension. Test was terminated for NPS6C and PSM6C, when a high-strength steel bar broke, which was caused by bearing of the concrete. Axial strain of the column at this event was 4% for both. During the loading of PSM6C, which is the first specimen tested among the Series B specimens, concrete debris that came off the
column was not removed from the cage of steel bars. This may have caused steel bars of PSM6C to break prematurely.

Figures 10b and c show relations between strain of high-strength steel bars and $\phi D_c$ of the cyclically loaded specimens. Similar to the monotonic test results, strain of high-strength steel bars of the specimens tested under axial load of 0.6$N_c$ remained the same as the initial strain level until $\phi D_c$ exceeded 0.5%, the deformation at which RC specimen collapsed. Strain accumulated as loading cycle of $\phi D_c >1\%$ was repeated. Comparing the maximum strain during the first cycle of $\phi D_c =2\%$, the increase of strain from the initial strain is 1.4 times larger for cyclic tests than the monotonic test results. Rate of strain increase was larger for specimens tested under larger axial load.

STRENGTH ESTIMATION

The methodology to estimate compressive and flexural strength is developed applying the method used to formulate equation (1). Additionally, based on the findings in the experiment, the following are assumed.

1) Confining stress, $\sigma_r$, is estimated as the nominal compression stress in RC columns, which reacts to the force transferred from corner blocks.
2) Confining stress, $\sigma_r$, does not exceed 2.2 N/mm².
3) Compressive stress-strain relation of the confined concrete is described by rigid-plastic model. Concrete does not resist to tension.
4) Stress-strain relation of steel is perfect elastic-plastic.
5) Confinement by hoops is negligible.

Calculating equations (1) and (2), confinement of concrete by hoops is estimated to increase the concrete strength only by 1%. The assumption 5) is based on this finding.

In order to use estimation equation (1) to predict strength of confined concrete of the specimens, the confining stress, $\sigma_r$, scale effect coefficient, $\varepsilon r_U$, and coefficient, $k$, must be determined. An estimation method suggested in the Reference may be used to calculate the scale effect coefficient, $\varepsilon r_U$, which is,

$$\varepsilon r_U = 1.67d^{0.112}$$

where $d$ is equivalent diameter ($=D_c(4/\pi)^{1/2}$). The value obtained by calculating (3) is 0.89

Subtracting compressive force carried by longitudinal bars from the compressive strength of Series A specimens, we get compressive strength of the confined concrete. Yield strength is used to calculate the compressive force carried by the longitudinal bars. The compressive strength varies with the confining stress transferred from corner blocks. Figure 11 shows increase of concrete’s compressive strength divided by the strength of concrete not confined by the high-strength steel bars ($\sigma_{c_b}/\sigma_{RC}$). Equation (1) indicates
that the coefficient $k$ is the incremental ratio relative to $\sigma_{ccB}/\sigma_{RC} (=0.13$, the angle shown in Figure 11) multiplied with plain concrete strength (about 20N/mm$^2$), which is about 2.6.

Flexural strength of RC specimens is obtained employing the method proposed by Sun and Sakino. Assumption 3) is used in calculating flexural strength of NPS and PS specimens. Calculated and measured strength, $M_{cal}$ and $M_{ex}$, respectively, are compared in Figure 12. Vertical axis is the ratio of $M_{ex}$ to $M_{cal}$. Horizontal axis is the axial load ratio. Accuracy of the estimation method is confirmed by comparison to test data shown in Figure 12.

CONCLUDING REMARKS

Compressive and Flexural Behavior of such RC columns employing a retrofit method using high-strength steel bars and steel corner block as confining elements, was investigated. Steel bars of some confined specimens were pre-tensioned. Two series of tests suggested that the lateral pre-stressing of RC columns is effective to enhance compressive strength of concrete. It was found that strength of concrete does not deteriorate if the column is confined with the steel bars, thus ductility is enhanced to two to four fold, depending on the axial load level. A method to predict compressive and flexural strength of the confined column was proposed. Confining stress is calculated using pre-stress transferred from the steel bars, which is then used to obtain the confined concrete strength. Rigid-plastic model is used to calculate the ultimate flexural strength. Measured strength of the confined RC columns showed good agreement with the strength calculated using the proposed method.

ACKNOWLEDGMENTS

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REFERENCES


Richart, F. E. et al. (1929), “The Failure of Plain and Spirally Reinforced Concrete in Compression,” Bulletin No. 190, Univ. Illinois, Engineering Experimental Station, Urbana, Ill.

## Table 1. Test Specimens

### (a) Series A: Concentric Compression Test

<table>
<thead>
<tr>
<th>Specimen</th>
<th>Initial Strain (x 10^-6)</th>
<th>Pre-tension Cylinder (N/mm²)</th>
<th>P_max (kN)</th>
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<tr>
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### (b) Series B: Uniform Bending Test – Monotonic Loading Test

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<th>Specimen</th>
<th>ε_p (x10^-6)</th>
<th>σ_y (N/mm²)</th>
<th>σ_y/N (N/mm²)</th>
<th>σ_y/N (N/mm²)</th>
<th>M_max (kN m)</th>
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<tr>
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<tr>
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<td>20.2</td>
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### (c) Series B: Uniform Bending Test – Cyclic Loading Test

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<th>σ_y/N (N/mm²)</th>
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## Table 2. Material Properties of Steel

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<td>High-Strength Steel Bar</td>
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<tr>
<td>Hoop</td>
<td>442</td>
<td>0.85</td>
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Figure 1 – RC column retrofitted using high-strength steel bars and steel corner block

Figure 2 – Specimen

(a) Installation
(b) Finish

(a) Specimen

(b) Corner Block

Bearing area = 120mm²

Unit: mm
Figure 3 – Loading

Figure 4 – Series A Test Results

(a) Axial Force – Axial Strain Relation

(b) Strain of High-strength Steel Bars Deformation
Figure 5 – Specimens of Series B (monotonic) after testing

Figure 6 – Moment-Flexural deformation
Figure 7 - Moment-Flexural deformation ($\phi_D$) relation (RC and PSL, Cyclic)
Figure 8 - Moment-Flexural deformation ($\phi_D$) relation (PSL and PSM, Cyclic)
Figure 9 - Moment-Flexural deformation ($\phi D_c$) relation (PSH, Cyclic)
Figure 10 – Strain of high-strength steel bars – (Series B)
Figure 11 – Prestress and confined concrete strength

Figure 12 – Comparison of measured and calculated flexural strength
Experimental Investigation into Strengthening of RC Columns using Composite Mortar Laminates Reinforced with Mesh Reinforcements

by S.P. Shang, L.M. Jiang, M.X. Zhang, and L.H. Zeng

Synopsis: Eleven approximate full-size specimens including nine eccentrically compressed columns of monotonic loading and two axially compressed columns of laterally cyclic loading were tested. By a series of comparison experiment of specimens strengthened using a kind of composite mortar laminates reinforced by mesh reinforcements (CMMR laminates) and no strengthened specimens, it was found that the RC columns strengthened with attached CMMR laminates demonstrated greater degree of improving in load-bearing capacity, in which the carrying capacity increment of the strengthened eccentrically compressed columns with lesser eccentricity was greater than that of the same type of columns with bigger eccentricity under the same strengthening conditions; the strengthening effects of the specimens with lower concrete grade are better than that of those ones with higher concrete grade; the ductility and the deformation capacity and energy dissipation ability of the strengthened columns were remarkably increased. In this paper, the test results is described, the principle and regularity that this category of strengthening laminate improved the ultimate load-bearing capacity, ductility, cracking behavior and mode of failure etc. of the RC columns are analyzed. The studying results proved that this strengthening measure for RC columns is superior to make the strengthening effect notable, working behavior of strengthened column excellent, strengthening construction easy and economical.

Keywords: composite mortar laminate; ductility; mesh reinforcement; RC column; strengthening; ultimate capacity
INTRODUCTION

Ferrocement or composite mortar reinforced with relatively small diameter mesh reinforcements is a typical inorganic composite material applied to the thin-wall structures. Also it is ideally suited as alternative strengthening material for RC structures or masonry structures for it possesses a higher ratio of tensile strength to weight and a considerably greater toughness, ductility, durability and cracking resistance than other conventional cement-based materials. Furthermore, it can easily be cast into any shape to fit the contour of the elements being strengthened. Examples of these were reported by P. Paramasimam et al. (1998) and P.J.Nedwell et al. (1994), respectively.

The above-mentioned strengthening measure has the advantages of superior fire resistance, ageing resistance, ductility, durability, economy and easy construction over other kinds of strengthening methods with epoxy resin adhesive applied on the interface for the strengthened structure. Furthermore, the compatibility, the working harmony, and the permeability between the original structural concrete and ferrocement or CMMR laminate are much better than that of the latter. This kind of strengthening laminate has generally a thickness of about 20mm, among which the ideal thickness of protective coating of reinforcement mesh or wire mesh is 5mm. This conclusion was presented by G.J.Xiong et al. (1998). Hence, this strengthening laminate don’t obviously enlarge cross section and weight of the original structure. Composite mortar is one of two main component of strengthening laminate, which can be used as adhesive on the original structural concrete/strengthening laminate interface and also as the protective coating material of reinforcement mesh or wire mesh. Therefore, this strengthening measure is freed from manufacturing extra protective coating for strengthened structures like in other strengthening methods such as carbon fiber bonding, steel plate bonding etc.

Even though there was a little research and application in strengthening RC beam or slab, but the research achievement on strengthening of RC column with ferrocement or CMMR laminate is seldom found. The use of CMMR in strengthening of RC columns is relatively new technique. Among this experimental investigation of approximate full-size specimens, nine eccentrically compressed columns of monotonic loading and two axially compressed columns of laterally cyclic loading were tested. The study and analysis focused on the ultimate load-bearing capacity, load versus mid-span deflection curves or ductility and failure characteristics of the columns strengthened with CMMR laminates.
Preparation of experiment

Description of basic specimens and choice of strengthening program -- The specimens were divided into four groups, A, B, C, and D. The details of nine specimens are shown in Figure 1 and summarized in Table 1. In order to diminish the interference of the dimension and effect of additional eccentricity and take into account of measuring range of test machine, all of the basic columns were designed as approximate full-size short columns which had the 300mm × 300mm square cross section and overall length of 1200mm. The main longitudinal reinforcements of basic columns were hot-rolled screw-threaded steel bars (f_y = 358 MPa), in which the tensile longitudinal bars were bent into corbel as oblique reinforcements. The horizontal hoop reinforcements of basic columns were smooth mild steel bars (f_y = 281 MPa). Two kinds of concrete nominal strength grades, that is, 25 MPa and 40 MPa are chosen. Two set of three companion 150mm × 150mm × 150mm cubes were cast with each basic column to test the realistic strength. Both of the basic columns and cubes were cured under damped hessian for more than 28 days. Furthermore, in order to examine the effect of a little increasing of cross-sectional areas due to attaching CMMR laminates for series specimens with an eccentricity of 60mm, one of the specimens was strengthened only with plain composite mortar without reinforcement mesh as the control specimen.

Choice of components of strengthening laminate -- If wire meshes with relatively small size of diameter and grid were chosen as reinforced component of mortar like ferrocement, the requirement of reinforcement ratio of strengthening layer or strengthening effect could not be met. Furthermore, if the wire mesh amount of layers was increased too much to get higher reinforcement ratio, the cross sections of specimens are increased, and it is difficult to vibrate and compact the mixture of mortar. Cold-rolled steel bar with ribs is a versatile material which has been found various and extensive applications in the last few years. Its strength is higher than that of ordinary hot-rolled steel bar; its behavior of anchoring bond and ductility is better than that of cold-drawn low-carbon wire; also it possess the good weld ability and cheap price. Welded meshes of cold-rolled steel bars with ribs with 60mm × 60mm square openings and single reinforcement diameter of 6.45mm were used to the strengthening laminates of these specimens. The average yield tensile strength and ultimate tensile strength of its single reinforcement were 551.2 MPa and 627.7 MPa, respectively.

The mortar as one of two main components of strengthening laminate is required to possess high strength, good toughness, a degree of percentage of elongation and excellent cohesiveness. One hand, performance of a kind of composite mortar reinforced with polypropylene fiber was tested. The test results showed that its compressive strength and tensile strength were higher than that of identical conditional ordinary cement mortar for more than 2.5 times and 1.2 times, respectively. Its obvious plastic deformation could be observed too. On the other hand, bending tensile test and shear test of specimens which were made of basic concrete and the composite mortar and had the joining interface brushed using inorganic cement based adhesive between the two parts were
conducted, respectively. The test results indicated that the cohesiveness of the basic concrete/composite mortar interface was superior. A summary of the components and the mixture ratio of the composite mortar and the interfacial adhesive is shown in Table 2.

In Table 2, the extra component of the composite mortar is the powder consisted of the polypropylene fiber, expanding agent of type of entingite, silica ash and pulverized fuel ash etc. The A component of interfacial adhesive is the water-reducing agent of type of resin, its B component is the cement based inorganic powdered mixture containing 18% other extra components.

**Strengthening method and procedure** -- Firstly, in order to increase the composite action between the basic column and CMMR laminate, four-side surfaces of the basic column except for the range of corbels were roughened by mechanical chipping prior to the installation of the CMMR laminate. Namely, mortar coating of surface of the basic column was chipped until the coarse aggregates were exposed. Secondly, the new concrete surface was washed by pressure water and was brushed with steel wire brush simultaneously till dust was totally removed. Thirdly, the transverse steel bars shaped in advance according to \[ \Box \] shape and the longitudinal straight steel bars precut off according to the exact size of the column were tied with plastic cable ties in terms of the size \( 60\text{mm} \times 60\text{mm} \) grid, while the longitudinal bars were placed on the internal layer, the transverse steel bars with splicing length \( 100\text{mm} \) were placed on the external layer, the lap joints of transverse steel bars were avoided to put at the corners of the column, and adjoining lap joints were avoided to lap to each other on the same side surface of the column. All of the lap joints and intersecting points of the mesh were electric welded so that the cage enclosing the column as close to its surface as possible was installed. Fourthly, when the wet specimen was air dried until the color of its surface was dark gray, the interfacial adhesive which had been stirred into the paste in advance was brushed onto the rough surface of the column by slightly exerting power. Lastly, machine stirred composite mortar were carefully hand plastered to it to ensure full penetration and compaction. The specimen was cured for more than 30 days under wet covers. The thickness of the CMMR laminate was controlled within 25 mm.

**Test setup, program of applied load and tested contents** -- Tests were carried out on a long column test machine with 5000KN(500T) capacity. The loading setup and the test scheme are shown in Fig. 2. Pre-loading was conducted prior to being formal to check if every part of test setup had been in a normal work state. All of the specimens were test at a steady loading ramp of 20KN until the initial crack emerged. Then applied load in loading ramp of 50KN until achieved 80–90% maximum load and again applied load in 20KN until maximum load. Under every grade of load, the load was sustained steadily for 5 minutes before test data were recorded, while the cracks were monitored and traced visually. The load was successively applied after exceeding the maximum load until the ultimate failure occurred.

Test contents for each column included the strains of the tensile and compressive main reinforcements, the longitudinal strains of concrete surfaces of two sides which were vertical to the moment plane of the column, longitudinal strains of mortar surfaces
and longitudinal steel bars of mesh of the same two sides, the tensile strains of transverse steel bars of the mesh at the plane position and the corner position of the compressive side, the mid-span lateral deflection of the column, tensile or compressive strains of mortar surfaces of two sides which were parallel to moment plane of the column.

**Results and Discussion**

Main results of nine test specimens are summarized in Table 3. The experimentally tested axial load-lateral deflection responses are given in Fig. 3, and the axial load-strain responses of mesh reinforcements of part strengthened specimens are presented in Fig. 4.

**Improved mechanism of load-bearing capacity and ductility** -- From Table 3, it can be seen that CMMR laminate contributes greatly to increase of ultimate load-bearing capacity and ductile capacity.

As to strengthened big eccentrically compressed columns with 150mm eccentricity, in each CMMR laminate, the longitudinal steel bars of mesh on the tensile side shared the tensile stress, the longitudinal steel bars of mesh and the mortar on the compressive side shared the compressive stress. The transverse steel bars of mesh could effectively confined the concrete of the compressive zone and anchored the longitudinal steel bars of mesh thus prevented relative slippage between the basic column and the CMMR laminate.

As shown in Fig. 4(a), it can be seen to specimen Col-2 (Col-4 and Col-2 had the similar test curves) that the strains of longitudinal steel bars of mesh on the compressive side increased rapidly and almost had the same increase rate with the longitudinal main reinforcements of the same side under relatively low load, then the strain development slowed down with load increase, after reaching ultimate strength of the column the strains decreased. It was indicated that there was a good interfacial cementation and the CMMR laminate of compressive side harmoniously worked together with compressive concrete well until longitudinal main reinforcements of the same side reached yield strength. The other reason for a rapid increase of strains of compressive side longitudinal steel bars of mesh during loading ascending period was that this side reinforcement mesh was close to the axial load acting line.

The strains of longitudinal steel bars of mesh on the tensile side of specimen Col-2 also increased at almost the same rate with the longitudinal main reinforcements of the same side when load was low. But after the load reached the 30% ultimate strength, the strain increase of the former was gradually slower than that of the latter. It was showed to shell out between the CMMR laminate and the base column. But the former kept continuous increase of strain before the column reached ultimate failure due to the transverse steel bars of mesh provided the anchor action to longitudinal steel bars of mesh especially at its upper and lower ends. Above results show that longitudinal steel bars of mesh have effect to some extent on raising the load-bearing capacity, on preventing development of horizontal cracks, and on improving the ductility.
On compressive side of specimen Col-2, only after the longitudinal main reinforcements reached yield strength, the strain increase of the transverse steel bars of mesh became quick. In Fig. 4(a), it can be seen that the strain increase of transverse steel bars of mesh at corner position was faster than at the plane position when load was low, for example, the strain at corner position was 420 με, and at the plane position was 51 με when the load reached the 50% ultimate strength. But when the ultimate failure occurred, the strain at corner position was 1734 με and the strain at plane position was 1838 με. The result show that the attached strengthening laminate at corner position had more effective effect on improving the load-bearing capacity than it did at plane position, but their confined action tended to be equal when the column got close to ultimate failure.

As shown in Fig. 5 (a), it can be seen as to Col-2 that the compressive side mortar and the same side concrete almost had same increase rate of strain before the load reached the 2/3 ultimate strength. After that, the strain development of mortar was obviously slower than that of the concrete. The main reason was that the C M M R laminate was not directly compressed, and the compressive stress of mortar was gained only by interfacial shear stress between C M M R laminate and the base column. Under the prerequisite that special measures of mechanical anchorage such as use of shear connectors wasn’t taken, the relative slippage on the interface during the loading latter period was the direct reason of resulting in this lagging phenomenon. Although the high compressive strength of mortar wasn’t fully utilized due to shear slippage on the interface, its compressive strain increase still indicated the contribution to improve the load-bearing capacity of column. When horizontal cracks appeared on tensile side, the mortar of tensile side had stopped to work, so it had little strengthening effect. Moreover, the good cementation of the mortar to the concrete of basic columns and mesh reinforcements promoted the integral work behavior of C M M R laminate and it also played a good role of protective coating for mesh reinforcements.

As to strengthened small eccentrically compressed columns with 60mm centricity, from the test results, it is shown that almost whole cross section of each column supported the compressive stress and only a little part of cross section supported the tensile stress thus the distributional confined effect on the cross section was obviously nonuniform and was gradually diminished from the compressive zone to the tensile zone.

Yu et al. (2000) reported that the confined effect of every category of strengthening material is passive, the confined mechanism is related to the ring directional stiffness of the strengthening coating. The confined action of C M M R laminate to basic column may be divided into two stages. During the first stage, concrete was in the linear elastic range. The axial compressive strain and the transverse tensile strain of the basic columns were small, thus the passive confinement effect of transverse steel bars of mesh could not be brought into play. During the second stage, the stiffness of basic columns was reduced, the longitudinal cracks on compressive side were fast developed, the concrete swelled along transverse direction, and the tensile strains of transverse steel bars of mesh distinctly increased on compressive side, thus the passive confined mechanism of C M M R laminate was made the best use. The axial load-mesh reinforcements strain curve of
strengthened small eccentrically compressed column Col-6 is shown in Fig. 4 (b) (Col-6 and Col-9 had the similar test curves). After the column reached ultimate strength, some of the transverse steel bars of mesh were broken with the load-bearing capacity decreasing slowly. The tensile strains of transverse steel bars of mesh versus several Special grades of load is given in Table 4.

As to the improved strength, from Table 4, it can be seen that the confined effect of the transverse steel bars of mesh at plane position and at corner position was obvious different during the loading rising phase, but after that the effect of two sides tended to be the same.

Furthermore, the tensile strains of small eccentrically compressed columns was 1.5~3 times as that of big eccentrically compressed columns at the same position of transverse steel bars of mesh on the compressive side when the applied load reached ultimate strength of specimens. It was proved that the confined effect of attached CMMR laminate to small eccentrically compressed columns is better than to big eccentrically compressed columns. Therefore, it can be inferred that this confined effect should be best to axial compressive columns.

The compressive strains of longitudinal steel bars of mesh on the compressive side of specimen Col-6 always kept slow increase during the whole loading phase, which indicated that the compressive longitudinal steel bars of mesh only played finite roles to improve the ultimate load-bearing capacity and the ductility of the column. The tensile longitudinal steel bars of mesh almost had little of effect because the tensile cross-sectional area of specimen Col-6 was much small.

As shown in Fig. 5(b), it was similar with strengthened big eccentrically compressed columns that the high compressive strength of mortar of specimen Col-6 couldn’t be still made the best use even though strengthened axial compressive columns so long as the strengthening laminate didn’t directly supported the pressure and the special measures of shear anchorage was not token to prevent the shear slippage of interface during the loading later period.

Effect of concrete grades and cross-section area increase--To both of the strengthened big eccentrically compressed columns and the small eccentrically compressed columns, the confined effect provided by CMMR laminates to C25 series specimens was better than to the C40 series specimens, especially the small eccentrically compressed columns, this phenomenon seem to be more obvious.

From Table 3 and Fig. 3 (d), it is indicated that specimen Col-8 strengthened only by plain composite mortar and no strengthened specimen Col-7 almost exhibited the same test results on the load-bearing capacity, cracking behavior, the stiffness and the ductility. The one reason was that the tensile strength of plain composite mortar was very low, so its transverse confined effect to basic column was little. The other reason was that the mortar of compressive side was not directly compressed, the strain increase of mortar was similarly slower than that of the same side concrete during the loading later period.
thus shared pressure by it was very little. It was proved that the strengthening effect of a little increasing of cross-sectional area due to attached CMMR laminate for the series small eccentrically compressed columns was not obvious.

Comparison of stiffness and ductility -- As shown in Fig. 3 (a) and Fig 3 (b), the attached CMMR laminate played a great role in improving ductility of the strengthened big eccentrically compressed column. But the improved stiffness is relatively little, since most of steel bars of mesh gained a quick strain increase until the longitudinal main reinforcements reached yield.

From Fig 3 (c) and Fig 3(d), as to the strengthened small eccentrically compressed column, the passive confined mechanism of attached CMMR laminate had a great effect on improving the ductility, yet the improved stiffness still was relatively little, since the tensile strain and compressive strain of longitudinal steel bars of mesh always kept a slow increase and the strain of transverse steel bars of mesh obtained rapid increase until the longitudinal main reinforcements were yielded.

The displacement ductility ratio is used in this paper, it is the ratio of the mid-span lateral deflection as the load was 85% ultimate strength during loading descendent phase to the corresponding displacement as the load was ultimate strength. From Table 3 it is can be seen that this strengthening measure is in favor of improving ductile capacity of either the big or small eccentrically compressed columns.

The coordinating work between the basic column and attached CMMR laminate due to the interfacial composite action not only raised the peak point of axial load-lateral deflection curves but also made its descendent section became relatively level, namely load-bearing capacity withdrawn slowly and the energy dissipation capacity of columns was improved significantly.

Comparison of modes of cracking and failure -- From Table 3, it is shown that the first cracking load of strengthened eccentrically compressed columns seem to be not increased as compare to corresponding control columns. It may be explained as follows: The mesh reinforcement of CMMR had bigger grid size and diameter of steel bars than the wire mesh of ferrocement and the prestress technology wasn’t used in installing of the reinforcement mesh. The strain of steel bars of mesh is very little during loading initial stage, which resulted in contributing little to the crack-bearing capacity. But, with the continuous increase of load, the tensile side longitudinal steel bars of mesh could prevent development of transverse cracks and the compressive side transverse steel bars of mesh could prevent the development of longitudinal cracks. Thereby, comparing to the control columns, the strengthened eccentrically compressed columns had the little average crack spacing and the crack width , and had big amount of cracks.

As shown in Fig.6 (a), the control specimen Col-3 exhibited the typical failure model of RC big eccentrically compressed column, four horizontal cracks with the average interval 250 mm were formed at the mid-span on tensile side. With increasing of the load, the cracks developed into the compressive zone, and after longitudinal main
reinforcements reached yield strength, the concrete of compressive side abruptly crushed and came off, the failure omen was not obvious. In contrast, as shown in Fig. 6 (b), the average crack spacing and the crack width of strengthened big eccentrically compressed column Co1-4 was less than that of control column Co1-3 in each identical grade of load. After exceeding ultimate strength, its compressive side surface gradually swelled and expanded especially at the upper and lower ends of the CMMR laminate since the inclined planes of corbels pushed outwardly the CMMR laminate when this specimen obtained fairly big lateral deflection. The CMMR laminate of upper and lower ends emerged debond at the tensile side due to the anchorage effect lost due to transverse steel bars of mesh breaking. The strengthened columns including Col-2 and Col-4 showed good structural ductility.

As shown in Fig. 6 (c), control specimens Col-5 had the typical failure model of RC small eccentrically compressed column. The longitudinal cracks firstly appeared on compressive side. When the load was close to ultimate bearing capacity, the concrete of compressive side was abruptly crushed and came off, and the same side longitudinal main reinforcements reached yield strength. The columns exhibited relatively brittle failure mode. In contrast, as shown in Fig. 6 (d), the longitudinal average crack spacing of strengthened column Col-6 was closer and its average crack width was less under each same grade of load. After exceeding the ultimate load-bearing capacity, the compressive side surface fast swelled and expended outwardly and was divided into many longitudinal strips by more and more longitudinal cracks. The scattered abrupt voice sent from the tensile transverse steel bars of mesh could be heard; the horizontal cracks of tensile side also gradually emerged. The ductile failure mode was exhibited by the specimens Col-6 and Col-9.

Verification of plane deformation assumption -- In order to test and verify if the eccentrically compressed columns strengthened with CMMR laminates still accord with the plane deformation assumption, five dial gauges were installed at mid-span of one flank of each specimen (Fig. 2) and the gauge length is 200 mm, so that measured the tensile and compressive strain. In Fig. 7, the longitudinal strain distribution at mid span for strengthened columns Col-2, Col-6 under several typical grades of load are given. It can be seen that when the load was relatively low, the strain distribution showed a good agreement with the plane deformation assumption; when the load was close to the ultimate load-bearing capacity, the growth of strains deviated from linear distribution, but it may be approximately considered that strengthened eccentrically compressed columns still accord with the plane deformation assumption.

**EXPERIMENTAL INVESTIGATION OF AXIALLY COMPRESSED COLUMNS UNDER LATERALLY CYCLIC LOADING**

Two specimens including one control column without strengthening and one column strengthened with CMMR laminate have been tested under constant axial load and lateral cyclic load to investigate seismic performance of RC columns strengthened with this strengthening measure during present experimental investigation.
The specimen was a vertical cantilever column fixed to a bottom base beam which was fixed to the test platform and was strong enough to provide a fixed end for the column. Height of each column was $H=1500\text{mm}$ from the point of lateral loading to the top end of the base beam. The basic Column Section Size was $b \times h=300\text{mm} \times 300\text{mm}$ with round corners of $200\text{mm}$ radius and center of circle to be the section centroid to avoid stress concentration of CMMR laminate, where $b$ and $h$ are the section width and height, respectively. The transverse steel hoops were $6\text{mm}$ in diameter with a spacing of $200\text{mm}$ for the specimens. Eight longitudinal reinforcement bars with a diameter of $16\text{mm}$ were placed around the perimeter of the section.

The strengthening materials, method and procedure basically were the same as previous statement in this paper. But the reinforcement mesh with $50\text{mm} \times 50\text{mm}$ square openings was employed in CMMR laminate of enveloping entire column, and in the potential plastic hinge zone, approximately $1.5$ times of section height, the spacing between transverse steel bars of mesh was decreased as $25\text{mm}$. Moreover, fifteen millimeter gap was provided between the bottom end of CMMR laminate and the top end of base beam to prevent the strengthening coating from carrying any direct axial stress for an extremely high lateral displacement used in this experimental investigation.

Each specimen was tested under a lateral reversal cyclic load $P$ acting at $1500\text{mm}$ from the bottom of the column with a pseudo controlled push-pull hydraulic actuator and a simultaneous constant axial load $N$ on the top of the column. This expression $n=N/f_{c}bh$ is called axial load ratio and $n=0.38$ was adopted for two columns, where, $f_{c}$ is compressive strength of concrete. A detailed explanation of the test setup designed by Y.Xiao et al. (2004) is available from the corresponding reference.

During testing, the applied lateral load was controlled by displacement increment. Single cycles of lateral load were applied to the specimen in terms of lateral displacement $\Delta=4\text{mm}, 8\text{mm}, 12\text{mm}$ etc. respectively, before the yield strain of longitudinal main reinforcements in tensile zone being reached. The lateral displacement corresponding to this yielding strain is termed yielding displacement $\Delta_{y}$. Then, three repetitive loading cycles were applied for each of peak lateral displacement as order: $\Delta_{y}, 2\Delta_{y}, 3\Delta_{y}, 4\Delta_{y}$ and etc. until the decreased $85\%$ maximum lateral load-bearing capacity and again single loading cycles were applied in continually increased lateral displacement as above order until the stage where the specimen was judged as unsuitable for further loading.

The hysteretic responses of the lateral load and displacement for the columns are shown in Fig.8. Compared with the control specimen, the lateral load-bearing capacity of strengthened column was enhanced by $41\%$, its significant ductility improvement was achieved too. As can be founded from Fig.8(b) that up to drift ratio $\Delta/H=9.5\%$, the strengthened column exhibited extremely stable lateral load-displacement response due to the strong confinement provided by the strengthening coating. It was observed that both of the control specimen and the strengthened column exhibited flexural failure. As to the control specimen, flexural cracks perpendicular to the column axis developed first in the region close to the bottom end of the column at a load of about $67\text{KN}$. The final failure was due to widening of three major flexural cracks and crash then disintegration of
concrete of lacked confinement in compressive zone at the lower part of the control column. For the strengthened column, the first flexural cracks was observed at the bottom end of the column at a lateral load of about 67KN which is identical to the first crack load recorded for the control column. As the load was increased, several flexural cracks formed and widened within the gap. Except for a few fine cracks mostly parallel to the column axis, no physical damage was observed on the strengthening coating throughout the test, and it remained in good condition even after the completion of the test. It was observed that when the lateral displacement increased beyond a drift ratio of approximately 9%, concrete within the gap started to crush. The test was terminated after the lateral displacement 145mm being reached.

CONCLUSIONS

(1) Compared with the control specimens, the columns strengthened with attached C M M R laminate demonstrate various degrees of enhancing in load-bearing capacity, In which the load-bearing capacity increment of the small eccentrically compressed columns was greater than that of the big eccentrically compressed columns under the same strengthening conditions. The strengthening effects of the eccentrically compressed columns with lower grade of concrete are better than that of the columns with higher grade of concrete.

(2) By comparing to the control specimens, the stiffness of the strengthened eccentrically compressed columns were enhanced to a certain extent after the yield load of longitudinal main reinforcements being reached. The ductility of the columns was notably improved, and the deformation ability and energy dissipation ability were remarkably increased.

(3) For the strengthened big eccentrically compressed column, the longitudinal steel bars of mesh on the tensile side shared the tensile stress, the longitudinal steel bars of mesh and the mortar on the compressive side shared the compressive stress. The transverse steel bars of mesh could effectively confined the concrete of the compressive zone and anchored the longitudinal steel bars of mesh thus prevented or diminished slippage between the basic column and the C M M R laminate. For the strengthened small eccentrically compressed column, during the middle and latter periods of loading, due to the expanding and swelling of concrete was speeded up and its longitudinal cracks was developed rapidly on compressive side, the passive confined mechanism of the strengthening coating was brought into play fully.

(4) By providing external confinement using C M M R laminate to the basic column, the seismic performance could be improved significantly.

(5) The composite mortar with strong cohesiveness, high strength, good ductility and the function of protective coating, provided the essential condition for the excellent work behavior of C M M R laminate.
(6) The attached strengthening laminate remarkably improved the mode of cracking and effectively prevented crack developing. The slight increase of the cross-sectional dimension due to strengthening was not evident to affect the strengthening effect of the columns so long as the strengthening layer was not directly compressed. It can be approximately considered that the plane deformation assumption is still suitable to the columns strengthened with this strengthening method.

(7) It can prevent or diminish the relative slippage and the debond between the basic column and the strengthening coating to decrease the spacing of transverse steel bars of mesh at upper and lower ends.

ACKNOWLEDGMENT

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REFERENCES

### Table-1. Classified details of specimens

<table>
<thead>
<tr>
<th>Group</th>
<th>Column NO.</th>
<th>Eccentricity (mm)</th>
<th>Concrete grade(MPa)</th>
<th>Main reinforcement ratio (%)</th>
<th>Strengthening program</th>
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<tbody>
<tr>
<td>A</td>
<td>Col-1</td>
<td>150</td>
<td>C25</td>
<td>1.69</td>
<td>No strengthened</td>
</tr>
<tr>
<td></td>
<td>Col-2</td>
<td>150</td>
<td>C25</td>
<td>1.69</td>
<td>Strengthened with CMMR</td>
</tr>
<tr>
<td>B</td>
<td>Col-3</td>
<td>150</td>
<td>C40</td>
<td>1.69</td>
<td>No strengthened</td>
</tr>
<tr>
<td></td>
<td>Col-4</td>
<td>150</td>
<td>C40</td>
<td>1.69</td>
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<tr>
<td>C</td>
<td>Col-5</td>
<td>60</td>
<td>C25</td>
<td>1.69</td>
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<tr>
<td></td>
<td>Col-6</td>
<td>60</td>
<td>C25</td>
<td>1.69</td>
<td>Strengthened with CMMR</td>
</tr>
<tr>
<td>D</td>
<td>Col-7</td>
<td>60</td>
<td>C40</td>
<td>1.69</td>
<td>No strengthened</td>
</tr>
<tr>
<td></td>
<td>Col-8</td>
<td>60</td>
<td>C40</td>
<td>1.69</td>
<td>Strengthened with plain mortar</td>
</tr>
<tr>
<td></td>
<td>Col-9</td>
<td>60</td>
<td>C40</td>
<td>1.69</td>
<td>Strengthened with CMMR</td>
</tr>
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### Table-2. The components and the mixture ratio of composite mortar and interfacial adhesive

<table>
<thead>
<tr>
<th>Composite mortar</th>
<th>Ordinary cement : Natural sand : Extra component : Water (P.O.42.5) (d≤0.25mm)</th>
<th>1.00 : 1.50 : 0.16 : 0.44</th>
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<tbody>
<tr>
<td>Interfacial adhesive</td>
<td>A component : B component : water</td>
<td>1.00 : 33.30 : 9.00</td>
</tr>
</tbody>
</table>

### Table-3. Main testing results of all specimens

<table>
<thead>
<tr>
<th>Group</th>
<th>Column</th>
<th>Cracking load (kN)</th>
<th>Ultimate bearing capacity (kN)</th>
<th>Increased ratio of bearing capacity (%)</th>
<th>Lateral deflection of maximum load (mm)</th>
<th>Ductility factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>Col-1</td>
<td>600</td>
<td>1420</td>
<td>-</td>
<td>3.877</td>
<td>1.272</td>
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<tr>
<td></td>
<td>Col-2</td>
<td>600</td>
<td>1600</td>
<td>12.68</td>
<td>2.615</td>
<td>2.702</td>
</tr>
<tr>
<td>B</td>
<td>Col-3</td>
<td>600</td>
<td>1380</td>
<td>-</td>
<td>4.861</td>
<td>1.347</td>
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<tr>
<td></td>
<td>Col-4</td>
<td>600</td>
<td>1550</td>
<td>12.31</td>
<td>3.133</td>
<td>2.696</td>
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<tr>
<td>C</td>
<td>Col-5</td>
<td>2180</td>
<td>2450</td>
<td>-</td>
<td>1.075</td>
<td>0</td>
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<tr>
<td></td>
<td>Col-6</td>
<td>2100</td>
<td>2900</td>
<td>18.36</td>
<td>2.105</td>
<td>2.993</td>
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<tr>
<td>D</td>
<td>Col-7</td>
<td>2300</td>
<td>2700</td>
<td>-</td>
<td>2.775</td>
<td>0</td>
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<tr>
<td></td>
<td>Col-8</td>
<td>2260</td>
<td>2680</td>
<td>-0.74</td>
<td>3.25</td>
<td>2.427</td>
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<tr>
<td></td>
<td>Col-9</td>
<td>2100</td>
<td>3100</td>
<td>14.81</td>
<td>1.724</td>
<td>4.812</td>
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Table 4. Tensile strain of horizontal steel bar of mesh on compressive sides

<table>
<thead>
<tr>
<th>Column NO.</th>
<th>Position</th>
<th>Raising phase of loading</th>
<th>Descendent phase of loading</th>
</tr>
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<tbody>
<tr>
<td></td>
<td></td>
<td>70%N* max</td>
<td>N max</td>
</tr>
<tr>
<td>Col-6</td>
<td>Plane position</td>
<td>$302 \times 10^{-6}$</td>
<td>$875 \times 10^{-6}$</td>
</tr>
<tr>
<td></td>
<td>Corner position</td>
<td>$802 \times 10^{-6}$</td>
<td>$1600 \times 10^{-6}$</td>
</tr>
<tr>
<td>Col-9</td>
<td>Plane position</td>
<td>$155 \times 10^{-6}$</td>
<td>$716 \times 10^{-6}$</td>
</tr>
<tr>
<td></td>
<td>Corner position</td>
<td>$711 \times 10^{-6}$</td>
<td>$1790 \times 10^{-6}$</td>
</tr>
</tbody>
</table>

Note: *Maximum axial load

Figure 1- The details of basic specimens

(a) C and D groups  
(b) A and B groups
Figure 2- Test setup

Figure 3- Axial load-lateral deflection curves of all specimens
Figure 4-Axial load-mesh reinforcement’s strain curves of strengthened columns

Figure 5- Axial load-compressive strain curve of compressive side concrete and mortar for strengthened columns
Figure 6- Failure patterns

(a) Col-3

(b) Col-4

(c) Col-5

(d) Col-6
Figure 7 - Longitudinal strain distribution at midspan

Figure 8 - Lateral load-displacement responses of the columns
Seismic Behavior of Reinforced Concrete Frame with Split Columns

by Z.-X. Li

Synopsis: Based on the success of authors' previous study, the seismic behavior of a 1/3 scale model of two-bay and three-story reinforced concrete frame with short columns being replaced by split columns at lower two stories is experimentally investigated under cyclic loads to present the seismic behavior of reinforced concrete frames by using the technology of split column. At first, a test model of two-bay and three-story reinforced concrete frame scaled to 1/3 of actual frame is designed, in which the original columns at lower two stories of the model frame are short columns and are replaced by the split columns, and is applied to constant vertical loads and cyclic horizontal loads at the top of the frame. The hysteresis curves between the cyclic horizontal load and the lateral displacement at the top of the model frame is obtained, from which it is seen that under the excitation of cyclic load, the model frame underwent the process of cracking, yielding, and maximum loading, and was destroyed under the ultimate load finally. It is also seen that the model frame with split columns represents better ductility, and the ductility factor, defined as the ratio of ultimate displacement by yielding displacement, of the model frame reaches 6.0. The yielding process of the model frame is obtained from the strain values of the longitudinal bars of beams and columns, from which it is seen that the frame with split columns can realize that the plastic hinges are generated at the ends of beams at first and then the columns begin yielding while the frame has still the load and deformation capacity. When the cyclic load reaches the maximum load, the columns begin yielding, but the deformation of frame may increase continually. It is demonstrated that splitting the short columns can change the failure mode from shear to flexure, thus enabling a frame to have much better ductility.

Keywords: cyclic load; ductility; frame; model experiment; plastic hinge; reinforced concrete; seismic behavior; short column; split column; yielding
Introduction

With the increment of the height of reinforced concrete tall buildings, there are many reinforced concrete short columns with shear-span ratio less than 2.0 on the equipment floors, transition floors, refuge floors and parking floors in the buildings. Inferior behavior of reinforced concrete short columns had been proved by past earthquake lessons, in which the brittle damage of the short columns may result in the collapse of the buildings during earthquakes. Therefore, how to improve the seismic behavior of short columns is the key problem to increase the seismic performance of tall buildings in civil engineering.

Some measures had been suggested to improve its behavior, such as special detailing of reinforcements and the adoption of SRC (Coffman et al., 1993; Tang et al., 1992; Wong et al., 1993; Zerbe et al., 1990), but these measures show complexity in construction and ineffectiveness in case of ultra short column with shear-span ratio less than 1.5. To improve the seismic behavior of reinforced concrete short columns especially with shear-span ratio less than 1.5, a technology of split column is proposed recently (Li et al., 2003), in which a rectangular short column is split to 2 or 4 long elemental columns reinforced individually by the partitions as shown in Figure 1. The series studies on the seismic behavior of the split columns and the beam-column joints with the split columns show the significant effectiveness to improve the seismic behavior of reinforced concrete short columns.

In this paper, based on the success of authors’ previous studies on the technology of split column, an experimental study on the seismic behavior of a one-third scale model of two-bay and three-story reinforced concrete frame with split columns at lower two stories is performed under cyclic loads to prove the integral seismic behavior of reinforced concrete frames employing the technology of split column.

Test Model and Setup

A test model of two-bay and three-story reinforced concrete frame scaled to 1/3 of actual frame is design as shown in Figure 2, according to the requirements of the ductility design of seismic category 1 of Chinese Code GB50011-2001. The original columns at lower two stories are short columns with shear-span ratio, \( \lambda = \frac{M_c}{Vh_0} \), of 1.5 and 1.9, and the split columns are employed for all the columns at lower two stories. The left split columns are designed with 50mm length of transition zone in which the integral section is kept to constrain the split cracks to extend to the joints of beam-split column, but the middle and right split columns are designed without the transition zone. The main designed parameters of the model frame are given in Table 1.
The test setup is shown in Figure 3, in which the model frame is fixed on the ground, the axial loads of the columns are applied through the vertical jacks against the reaction frame, and the cyclic loads on the model frame is applied through the horizontal jack at the top of the frame against the reaction wall.

The hybrid loading control of force and displacement is adopted in the loading process, in which the force control is adopted at first before the yielding of the frame and then the displacement control is adopted according to the requirement of ductility after the yielding of the frame.

TEST RESULTS

The hysteresis curve of the horizontal cyclic load and the lateral displacement at the top of the model frame is obtained as shown in Fig.4, from which it is seen that under the excitation of cyclic load, the model frame underwent the process of cracking, yielding, and maximum loading, and was destroyed under the ultimate load finally. From Figure 4, it is also seen that the model frame with split columns represents better ductility, and the ductility factor, defined as the ratio of ultimate displacement by yielding displacement, of the model frame reaches 6.0, which is higher than ordinary reinforced concrete frames whose ductility factor is about 4.0.

Table-2 gives the loads and displacements at the top of model frame at different loading steps, in which the integral displacement angle, defined as the ratio of the top displacement by the total height of the frame is also illustrated.

Figure 5 gives the photo of the destroyed frame, from which the integral failure pattern can be observed, and Figure 6 gives the more detail drawings of the cracks at all the beam-column joints of the destroyed frame after, in which the figures (a), (b) and (c) shows the cracks on the joints of 3rd floor, the figures (d), (e) and (f) shows the cracks on the joints of 2nd floor, and the figures (g), (h) and (i) shows the cracks on the joints of 1st floor.

The axial strains of concrete and longitudinal bar of the split columns at the bottom of the columns are measured, and the measuring points of the strains of concrete on the surface of column is shown in Figure 7(a). Figure 7(b) gives the changing graph of strain in the bottom cross section of a split column in the loading process, from which some important phenomenon can be observed. When the load is enough smaller, the split column works as an integral column, and the strain in the integral section of column is continuous and linear. With the increasing of the load, the partition may break and the column is split to the split columns, while the strain in the integral section is broken to two individual parts in which each part represents the strain of an elemental column and is continuous and linear.

TEST ANALYSIS

It is observed from the test process that there is no inclined shearing crack on any split
column as shown in Figure 5. The split column avoids the brittle damage and the frame with split columns develops significant ductility. Therefore, it can be said that a poor ductility frame with short columns has been changed to an exceedingly ductile frame with split columns.

By comparing the figures (d), (e) and (f) in Figure 6, also the figures (g), (h) and (i) in Figure 6, it is seen that the damage of the cores of the left joints of beam-split columns is clearly less than that of the cores of the middle and right joints of beam-split columns, because the transition zones are set on the top and bottom of the left split columns. From which, it can be said that the transition zones at the ends of the split columns constrain the split crack to extend to the cores of the joints effectively.

The yielding and plastic hinge developing process of the model frame can be obtained from the strain values of the longitudinal bars of beams and columns, and the occurrence sequence of the plastic hinges of the model frame is shown in Figure 8.

From the yielding process of the frame shown in Figure 8, it can be seen that the frame with split columns can realize that the plastic hinges are generated at the ends of beams at first and then the columns begin yielding while the frame has still the load and deformation capacity. When the cyclic load reaches the maximum load, the columns of the 1st and 2nd stories begin yielding, but the deformation of frame may increase continually. Until the frame is destroyed, the split columns still express better load and deformation capacity, which demonstrates that the frame replaced the short columns with the split columns may obtain better ductility and may meet the design requirement of ductility for an ordinary frame.

CONCLUSIONS

Through the model experiment on the seismic behavior of a reinforced concrete frame with split columns, some conclusions are given as:

After the short column is designed as the split column in the reinforced concrete frame, the design concept of changing the short column to long column directly can be realized, and the failure pattern of the column may change from the shear type of short column to bending type of split column, from which it is seen that the split column is effective measure to improve the seismic behavior of the reinforced concrete short column.

The transition zone set at the ends of split column may constrain the split crack of the split column to extend to the joints of beam-split column. To prevent the double shearing cores occurring in the joint of beam-split column, the transition zone must be set at the top and bottom end of the split column.

The sequence of plastic hinges of a reinforced concrete frame with split columns may be similar to that of an ordinary reinforced concrete frame, even the ductility of the frame with split columns is higher than that of the ordinary frame.
Therefore, the seismic behavior of the reinforced concrete frame may be improved significantly by replacing the short columns with split columns, even may exceed the seismic behavior of ordinary reinforced concrete frames. Because of the simplicity and effectiveness to improve the seismic behavior of reinforced concrete tall buildings, the technology of split column explodes the prospects of wide application in actual engineering.

ACKNOWLEDGMENTS

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REFERENCES


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<th>Column</th>
<th>Size (mm)</th>
<th>Strength (MPa)</th>
<th>Shear-span ratio</th>
<th>Stirrup ratio (%)</th>
<th>Axial compression ratio</th>
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<td>Middle column</td>
<td>Side column</td>
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<td>1st story</td>
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<td>2nd story</td>
<td>250 × 250</td>
<td>35.4</td>
<td>1.9</td>
<td>2.0</td>
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<td>37.2</td>
<td>3.5</td>
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Table 2—Load and top displacement at different loading steps

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<tr>
<th>Step</th>
<th>Direction</th>
<th>Load P(kN)</th>
<th>Displacement Δ(mm)</th>
<th>Displacement angle Δ/H</th>
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<tr>
<td></td>
<td>Pull</td>
<td>30</td>
<td>2.3</td>
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<td></td>
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<td>150</td>
<td>24.3</td>
<td>1/183</td>
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<tr>
<td>Maximum</td>
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<td>87.1</td>
<td>1/51</td>
</tr>
<tr>
<td>Ultimate</td>
<td>Push</td>
<td>203</td>
<td>87.1</td>
<td>1/51</td>
</tr>
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<td></td>
<td>Pull</td>
<td>147</td>
<td>136.8</td>
<td>1/31</td>
</tr>
<tr>
<td></td>
<td>Push</td>
<td>173</td>
<td>136.8</td>
<td>1/31</td>
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</table>

Figure 1--Structural figure of split column

Figure 2--Model frame with split columns at lower two stories
Figure 3--Test setup

Figure 4--Hysteresis curve of cyclic load and top displacement
Figure 5--Photo of the destroyed frame

Figure 6--Detailed cracks on the joints of model frame
Figure 7--Measuring of strain at the end of split column

Figure 8--Occurrence sequence of plastic hinges of the model frame
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