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ACI Shear and Torsion Provisions for Prestressed Hollow Girders

by Thomas T. C. Hsu

New torsion design provisions have been proposed for the 1995 ACI Building Code. As compared to the 1989 provisions, these generalized 1995 provisions have three advantages: First, they are applicable to closed cross sections of arbitrary shapes. Second, they are applicable to prestressed concrete. Third, they are considerably simplified by deleting the "torsional concrete contribution" and its interaction with shear. These new provisions are suitable for application to concrete guideways and bridges, because these large structures are always prestressed and are often chosen to have hollow box sections of various shapes. This paper discusses the background of the new code provisions, suggests modifications to code formulas, and illustrates the application of the code provisions to prestressed hollow girders by way of a guideway example.

Keywords: code provisions; concrete structures; design; guideways; hollow sections; prestressed concrete; reinforced concrete; shear; torsion.

INTRODUCTION

New design provisions for torsion have been proposed for the 1995 ACI Building Code. These provisions are the product of more than three decades of research in torsion. This research includes the establishment of the equilibrium truss model (Nielsen, 1967; Lampert and Thurlimann, 1968; Elfgren, 1972; CEB-FIP, 1978; Thurlimann, 1979), the compression field theory (Collins, 1973; Collins and Mitchell, 1980), and the softened truss model. (Hsu and Mo, 1985a, 1985b, 1985c; Hsu, 1988).

The equilibrium truss model provides a complete set of equilibrium equations, thus furnishing torsion design with a clear concept. The compression field theory, which considers also Mohr compatibility condition, allows the torque-twist relationship to be predicted. The softened truss model takes into account the softened biaxial constitutive laws in addition to equilibrium and compatibility. As a result, it clarifies the stress and strain conditions in the shear flow zone. This understanding of the shear flow zone (Hsu, 1990, 1993) allows us to define the thickness of the shear flow zone and the lever arm area.

Because of the rationality of these theories, the 1995 torsion provisions are considerably more generalized as compared to the 1989 version (ACI-318, 1989). They are now applicable to closed cross sections of arbitrary shapes and to prestressed concrete. The new torsion design process is also considerably simplified by deleting the "torsional concrete contribution" and its interaction with shear (MacGregor and Ghoneim, 1995).

RESEARCH SIGNIFICANCE

This paper provides background information for the shear and torsion provisions in the 1995 ACI Code. This information is presented in a systematic and logical manner. The presentation starts out with the derivation of basic equilibrium equations, from which the design equations are developed. The design procedures are clearly summarized in a flow chart, and are illustrated by a guideway design example.

Concrete guideways and bridges are usually made of large prestressed hollow girders. Such structures can now be designed by the new ACI shear and torsion provisions. Special problems pertaining to hollow box girders are discussed. Three code provisions, which are at present applicable only to solid sections, are generalized to include hollow sections.

BASIC EQUILIBRIUM EQUATIONS

Equilibrium in Element Shear

A membrane element subjected to a shear flow q is shown in Fig. 1 (a). The element has a thickness of h and a square shape with a unit length in both directions. The longitudinal bars are arranged in the ℓ-direction (horizontal axis) with a uniform spacing of s_ℓ. The transverse bars are arranged in the t-direction (vertical axis) with a uniform spacing of s. After cracking, the concrete is separated by diagonal cracks into a series of concrete struts as shown in Fig. 1(b). The cracks are oriented at an angle θ with respect to the ℓ-axis. The diagonal

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concrete struts, the longitudinal bars, and the transverse bars form a truss which is capable of resisting the shear flow q.

Equilibrium in the longitudinal direction is shown by the force triangle on the left face of the shear element, Fig. 1(b). The shear flow q pointing upward is resisted jointly by a longitudinal steel force n_ℓ and a diagonal concrete force, $(\sigma_d h)\cos\theta$. The steel force n_ℓ is defined as the longitudinal steel force per unit length, $A_\ell f_\ell / s_\ell$, where A_ℓ is the cross sectional area of one longitudinal bar and f_l is the stress in the longitudinal bars. The concrete force, $(\sigma_d h)\cos\theta$, represents the diagonal concrete stress, σ_d, acting on a thickness of h and a width of $\cos\theta$. The $\cos\theta$ relationship is shown by the geometry in Fig. 1(a). From this force triangle the shear flow q can be related to the longitudinal steel force nl by the geometry:

$$q = n_\ell \tan\theta \tag{1}$$

Similarly, equilibrium in the transverse direction is shown by the force triangle on the top face of the shear element, Fig. 1(b). The shear flow q pointing leftward is resisted jointly by a transverse steel force n_t and a diagonal concrete force, $(\sigma_d h)\sin\theta$. The steel force n_t is defined as the transverse steel force per unit length, $A_t f_v / s$, where A_t is the cross sectional area of one transverse bar and f_v is the stress in the transverse bars. The concrete force, $(\sigma_d h)\sin\theta$, represents the diagonal concrete stress, σ_d, acting on a thickness of h and a width of $\sin\theta$. The $\sin\theta$ relationship is also shown by the geometry in Fig. 1(a). From this force triangle the shear flow q can be related to the transverse steel force n_t by the geometry:

$$q = n_t \cot\theta \tag{2}$$

The shear flow q can be related to the diagonal concrete stress σ_d using either the force triangle in the longitudinal direction or the force triangle in the transverse direction. From geometry of the triangles we obtain:

$$q = (\sigma_d h)\sin\theta\cos\theta \tag{3}$$

Equilibrium in beam shear

A beam subjected to a concentrated load $2V$ at midspan is shown in Fig. 2 (a). Since the reaction is V, the shear force is a constant V throughout one-half of the beam, and the moment diagram is a straight line. When a beam element of length d_v is isolated and the moment on the left face is defined as M, then the moment on the right face is $M + Vd_v$. The shear forces on both the left and right faces are, of course, equal to V.

A model of the isolated beam element is shown in Fig. 2 (b). The top and bottom stringers are separated from the web element, so that the two different mechanisms operating to resist bending and shear can be clearly illustrated. The stringers are resisting the bending moment and the web element is carrying the shear force.

Assuming that the shear flow q is distributed uniformly over the depth and along the length of the web element, then the web element can now be treated as a large shear element with a depth of d_v, where d_v is the vertical distance between the two stringers. This large shear element in Fig. 2 may be slightly different from the unit element shear in Fig. 1 in that the longitudinal steel may not be uniformly distributed. To take care of this non-uniform distribution of longitudinal steel, we define $\overline{N}_\ell = n_\ell d_v$, where $\overline{N}_\ell$ is the total force of the longitudinal steel over the depth d_v to resist the shear force $V = q_{dv}$. Multiplying Eqs. (1) to (3) by the depth d_v gives the following three equations:

$$V = \overline{N}l \tan\theta \tag{4}$$

$$V = n_t d_v \cot\theta \tag{5}$$

$$V = (\sigma_d h) d_v \sin\theta\cos\theta \tag{6}$$

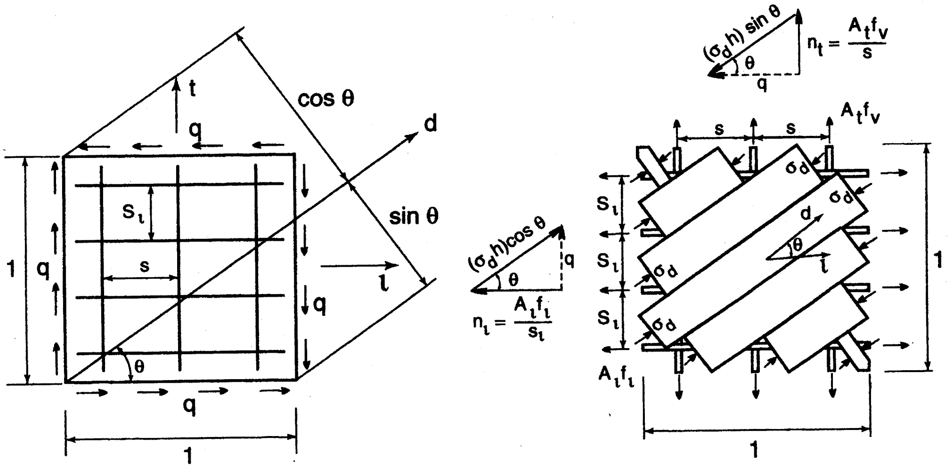
Equilibrium in torsion

A hollow prismatic member of arbitrary bulky cross section and variable thickness is subjected to torsion as shown in Fig. 3 (a). According to St. Venant's theory, the twisting deformation will have two characteristics. First, the cross sectional shape will remain unchanged after twisting; and second, the warping deformation perpendicular to the cross section will be identical throughout the length of the member. Such deformations imply that the in-plane normal stresses in the wall of the tube member should vanish. The only stress component in the wall is the in-plane shear stress, which forms a circulating shear flow q on the cross section. The shear flow q is the resultant of the shear stresses in the wall thickness and is located on the dotted loop shown in Fig. 3 (a). This dotted loop is defined as the center line of shear flow.

A membrane wall element ABCD is isolated and shown in Fig. 3 (b). It is subjected to pure shear on all four faces. Let us denote the shear stress on face AD as τ_1 and that on face BC as τ_2. The thicknesses at faces AD and BC are designated h_1 and h_2, respectively. Taking equilibrium of forces on the element in the longitudinal ℓ-direction we have

$$\tau_1 h_1 = \tau_2 h_2 \tag{7}$$

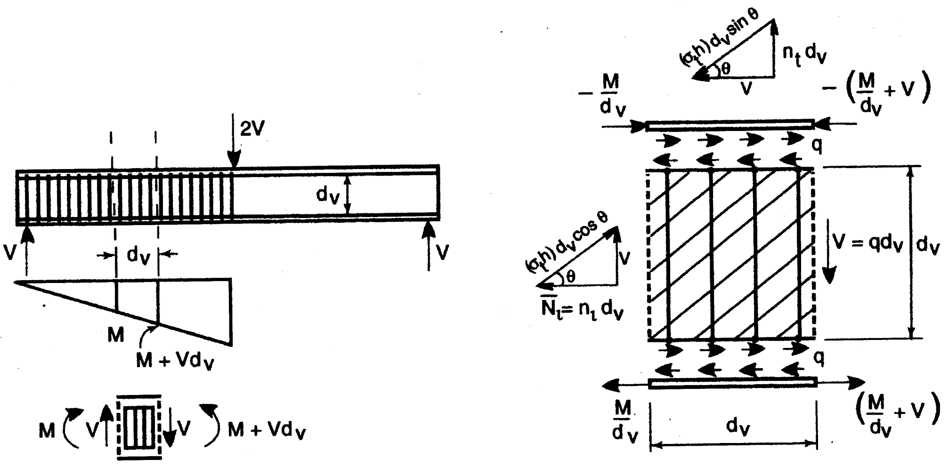
Since shear stresses on mutually perpendicular planes must be equal, the shear stresses on face AB must be τ_1 at point A and τ_2 at point B. Eq. (7), therefore, means that τh on face AB must be equal at points A and B. Since we define $q = \tau h$ as the shear flow, q must be equal at points A and B. Notice also that the two faces AD and BC of the element can be selected at an arbitrary distance apart without violating the equilibrium condition in the longitudinal direction. It follows that the shear flow q must be constant throughout the cross section.



(a) SHEAR ELEMENT (THICKNESS OF h)

(b) TRUSS MODEL

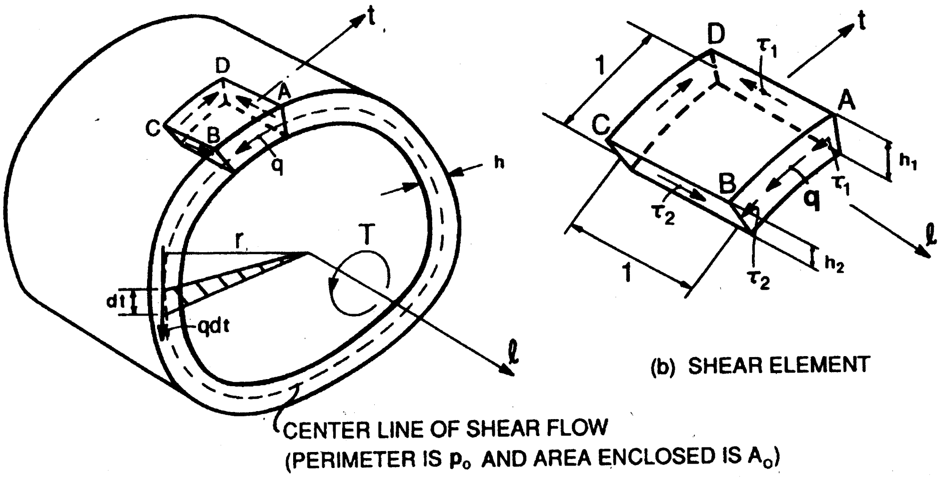
Fig. 1—Equilibrium in element shear



(a) ISOLATION OF BEAM ELEMENT

(b) MODEL FOR BEAM SHEAR

Fig. 2—Equilibrium in beam shear



(b) SHEAR ELEMENT

(a) TUBE IN TORSION

Fig. 3—Equilibrium in torsion

The relationship between T and q can be derived directly from the equilibrium of moments about the l-axis. As shown in Fig. 3 (a), the shear force along a length of wall element dt is qdt. The contribution of this element to the torsional resistance is $q(dt)(r)$, where r is the distance from the center of twist (ℓ-axis) to the shear force qdt. Since q is a constant, integration along the whole loop of the center line of shear flow gives the total torsional resistance:

$$T = q\oint rdt \tag{8}$$

From Fig. 3 (a) it can be seen that rdt in the integral is equal to twice the area of the shaded triangle formed by r and dt. Summing these areas around the whole cross section results in:

$$\oint rdt = 2A_O \tag{9}$$

where A_o is the gross area enclosed by the center line of shear flow. This parameter A_o is a measure of the lever arm of the circulating shear flow and will be called the lever arm area. Substituting $2A_o$ from Eq. (9) into Eq. (8) gives:

$$q = \frac{T}{2A_O} \tag{10}$$

Eq. (10) was first derived by Bredt (1896).

A shear element isolated from the wall of a tube of bulky cross section, Fig. 3 (b), may be subjected to a warping action in addition to the pure shear action discussed above. If the warping action is neglected, then this shear element becomes identical to the shear element in Fig. 1 which is subjected to pure shear only. As a result, the three equilibrium Eqs. (1) to (3) derived for the element shear in Fig. 1 become valid. Substituting q from Eq. (10) into Eqs. (1), (2) and (3), we obtain the three equilibrium equations for torsion:

$$T = \frac{\overline{N}_\ell}{p_O}(2A_O)\tan\theta \tag{11}$$

$$T = n_t(2A_o)\cot\theta \tag{12}$$

$$T = (\sigma_d h)(2A_O)\sin\theta\cos\theta \tag{13}$$

Notice in Eq. (11) that $\overline{N}_\ell = n_\ell p_O$. This is because n_ℓ, which is the longitudinal force per unit length, must be multiplied by the whole perimeter of the center-line of the shear flow p_o to arrive at the total longitudinal force due to torsion, $\overline{N}_\ell$.

Comparison

Comparison of the three sets of equations for element shear, beam shear and torsion shows that they are basically the same. The three equations for beam shear, Eqs. (4) to (6), are simply the three equations for element shear, Eqs. (1) to (3), multiplied by a length of d_v. The three equations for torsion, Eqs. (11) to (13), are simply those for element shear, Eqs. (1) to (3), multiplied by the area, $2A_o$. Hence it is only necessary to understand the geometric and algebraic relationships of one set of three equations for element shear.

The six equations for beam shear and torsion are derived in a consistent and logical manner. Such clarity in concept is one of the main advantages of the equilibrium truss model. With these six equations serving as the basis of design, the new ACI shear and torsion provisions become clear and rational.

SHEAR AND TORSIONAL STEEL DESIGN

Torsional steel

Transverse torsional steel—Assuming the yielding of steel, $f_v = f_{yv}$, where f_{yv} is the yield strength of closed stirrups provided for torsion, then the symbols n_t in Eq. (12) becomes $n_t = A_t f_{yv}/s$, and the symbol T becomes $T_n = T_u/\phi$. The torsional transverse steel can be directly designed according to Eq. (12):

$$\frac{A_t}{s} = \frac{T_u}{\phi 2A_O f_{yv}\cot\theta} \tag{14}$$

In Eq. (14) the angle θ is limited to a range of 30 deg < θ < 60 deg in order to control cracking. It has been shown by Thurlimann (1979) that crack width increases very rapidly when θ moves away from this range. An angle of $\theta = 45$ deg is recommended for reinforced concrete because this angle represents the best crack control (Hsu, 1993). For prestressed concrete, however, the angle for best crack control should be less than 45 deg because of the longitudinal prestress. ACI code provision suggests an angle of 37.5 deg for prestressed concrete (MacGregor and Ghoneim, 1993).

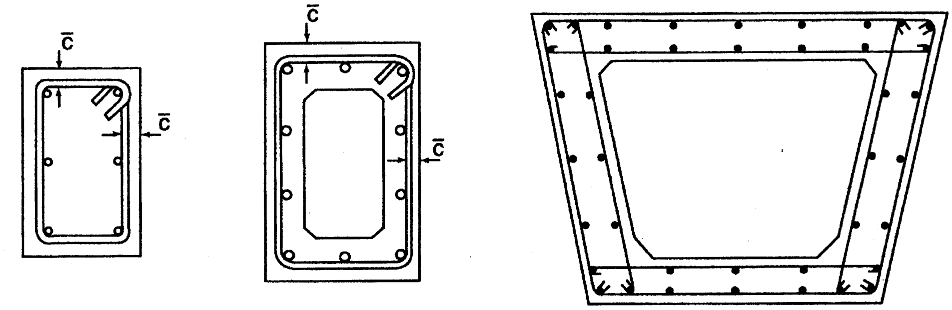
The lever arm area A_o depends on the thickness of the shear flow zone t_d, which, in turn, is a function of the applied torsional moment, T_n. The larger the torsional moment T_n, the larger the shear flow zone t_d and the smaller the lever arm area A_o. These relationships can be derived theoretically from the warping compatibility condition of the wall. For design practice, however, simplified expressions are given for t_d and A_o as follows (Hsu, 1990 and 1993):

$$t_d = \frac{4T_u}{\phi f'_c A_{cp}} \tag{15}$$

$$A_o = A_{cp} - \frac{t_d}{2}p_{cp} \tag{16}$$

where A_{cp} is the area enclosed by the outside perimeter of concrete cross section, and p_{cp} is the outside perimeter of the concrete cross section. Substituting t_d from Eq. (15) into Eq. (16), A_o becomes

$$A_o = A_{cp} - \frac{2T_u p_{cp}}{\phi f'_c A_{cp}} \tag{17}$$



(a) SMALL SOLID SECTIONS (b) SMALL HOLLOW SECTIONS (c) LARGE HOLLOW SECTIONS

Fig. 4—Various cross sections

For structural members commonly employed in buildings, Fig. 4 (a), ACI Code suggests a simpler but less accurate expression:

$$A_o = 0.85A_{oh} \tag{18}$$

where A_{oh} is the area enclosed by the center line of the outermost closed transverse torsional reinforcement. Eq. (18) may under-estimate the torsional strength of lightly reinforced small members by up to 40 percent and over-estimate the torsional strength of heavily reinforced large members by up to 20 percent.

The transverse torsional bars required by Eqs. (14) and (17) or (18) should be in the form of hoops or closed stirrups. They should meet the maximum spacing requirement of $s \leq p_h/8$ or 12 in.

Longitudinal torsional steel—The torsional longitudinal steel can be designed according to Eq. (11) assuming the yielding of steel, $f_\ell = f_{y\ell}$, where $f_{y\ell}$ is yield strength of longitudinal torsional reinforcement. However, a more convenient equation relating the torsional longitudinal steel to the torsional transverse steel can be derived by equating Eq. (11) to Eq. (12). Noticing in Eq. (11) that $\overline{N}_\ell = A_\ell f_{y\ell}$, where A_ℓ is now defined as the total area of torsional longitudinal steel in the cross section, and assuming that $p_o = p_h$, where p_h is defined as the perimeter of the center-line of the outermost hoop bars, we derive the ACI equation:

$$A_l = \frac{A_t}{s} p_h \left(\frac{f_{yv}}{f_{yl}} \right) \cot^2\theta \tag{19}$$

The angle θ is the same one used for transverse torsional steel in Eq. (14).

Minimum longitudinal torsional steel—In order to avoid a brittle torsional failure, a minimum amount of torsional reinforcement (including both transverse and longitudinal steel) is required in a member subjected to torsion. The basic criterion for determining this minimum torsional reinforcement is to equate the post-cracking strength T_n to the cracking strength T_{cr}:

$$T_n = T_{cr} \tag{20}$$

Taking the angle $\theta = 45$ deg, the post-cracking torsional strength T_n of both solid and hollow sections can be predicted from Eq. (14):

$$T_n = \frac{2A_o A_t f_{yv}}{s} \tag{21}$$

The cracking torque of solid sections subjected to combined torsion, shear and bending, T_{cr}, can be predicted by (Hsu and Hwang, 1977):

$$T_{cr} = 4\sqrt{f'_c}\frac{A_{cp}^2}{p_{cp}} \tag{22}$$

where f'_c and $\sqrt{f'_c}$ are in pounds per square inch (psi). In the case of hollow sections, Mattock (1995) suggested a simple relationship between the cracking torque of a hollow section, $(T_{cr})_{hollow}$, and that of a solid section with the same outer dimensions, $(T_{cr})_{solid}$:

$$\frac{(T_{cr})_{hollow}}{(T_{cr})_{solid}} = \frac{A_g}{A_{cp}} \tag{23}$$

where A_g is the cross-sectional area of the concrete only and not including the hole(s), while A_{cp} is the area of the same hollow section including the hole(s). For solid sections, $A_g = A_{cp}$. The cracking torque of solid and hollow sections can then be expressed by one equation:

$$T_{cr} = 4\sqrt{f'_c}\, A_g \frac{A_{cp}}{p_{cp}} \tag{24}$$

Inserting Eqs. (24) and (21) into Eq. (20) gives

$$A_t f_{yv} = \frac{2\sqrt{f'_c}\, A_g A_{cp} s}{A_o p_{cp}} \tag{25}$$

Multiplying both sides of Eq. (25) by p_h/s gives

$$A_t f_{yv} \frac{p_h}{s} = 2\sqrt{f'_c}\, A_g \left(\frac{A_{cp}}{A_o}\right)\left(\frac{p_h}{p_{cp}}\right) \tag{26}$$

The ratios A_{cp}/A_o and p_h/p_{cp} on the right hand side of Eq. (26) vary somewhat with the size of cross sections, because the concrete cover is usually a constant specified by the code for fire and corrosion protections. For sizes of cross sections normally used in buildings, the ratios A_{cp}/A_o and p_h/p_{cp} can be taken as 1.5 and 0.83, respectively. Eq. (26) can then be simplified to become:

$$A_t f_{yv} \frac{p_h}{s} = 2.5\sqrt{f'_c}\, A_g \tag{27}$$

Minimum torsional reinforcement requires not only transverse steel, but also longitudinal steel. The total yield force of longitudinal torsional steel is obtained from Eq. (19) assuming $\theta = 45$ deg:

$$A_\ell f_{y\ell} = A_t f_{yv} \frac{p_h}{s} \tag{28}$$

Substituting $A_t f_{yv} p_h/s$ from Eq. (27) into Eq. (28) gives

$$A_\ell f_{y\ell} = 2.5\sqrt{f'_c}\, A_g \tag{29}$$

Adding Eqs. (27) and (29) results in

$$A_\ell f_{y\ell} + A_t f_{yv} \frac{p_h}{s} = 5\sqrt{f'_c}\, A_g \tag{30}$$

The symbol A_ℓ in Eq. (30) is the total area of minimum longitudinal steel, $A_{\ell min}$. Rearranging Eq. (30) gives the ACI equation:

$$A_{\ell,min} = \frac{5\sqrt{f'_c}\, A_g}{f_{y\ell}} - \left(\frac{A_t}{s}\right) p_h \left(\frac{f_{yv}}{f_{y\ell}}\right) \tag{31}$$

To limit the value of $A_{\ell min}$, the transverse steel area per unit length, A_t/s, in the second term on the right-hand side of Eq. (31) needs not be taken less than $25b_w/f_{yv}$.

Although Eq. (31) is derived from non-prestressed solid and hollow sections, this equation is considered to be valid for prestressed sections, because the level of prestress is assumed to have a small effect on the minimum reinforcement.

The longitudinal torsional bars required by Eqs. (19) or (31) should be distributed uniformly along the perimeter of the cross section. They should meet the maximum spacing requirement of $s_\ell \leq 12$ in. and the minimum bar diameter of $d_b \geq s/16$ or No. 3 bar (Mitchell and Collins, 1976).

Shear steel

Transverse shear steel—The basic philosophy of shear design in the ACI Code remains unchanged in the 1995 ACI Code. The shear resistance V_n is assumed to be made up of two terms: V_s contributed by steel and V_c contributed by concrete. The simplified expressions of V_c are given as follows: For reinforced concrete

$$V_c = 2\sqrt{f'_c}\, b_w d \tag{32}$$

For prestressed concrete

$$V_c = \left(0.6\sqrt{f'_c} + 700\frac{V_u d}{M_u}\right) b_w d \tag{33}$$

where $2.0\sqrt{f'_c}\, b_w d \leq V_c \leq 5.0\sqrt{f'_c}\, b_w d$

and $V_u d/M_u \leq 1$.

The shear force V_s resisted by steel is

$$V_s = \frac{V_u}{\phi} - V_c \tag{34}$$

Assuming the yielding of steel, $f_v = f_{yv}$, and the angle $\theta = 45$ deg, the shear web steel can be designed according to Eq. (5):

$$\frac{A_v}{s} = \frac{V_s}{d f_{yv}} = \frac{V_u - \phi V_c}{\phi d f_{yv}} \tag{35}$$

The transverse shear steel calculated by Eq. (35) is required in the vertical legs of the cross section. The spacing s is limited to $d/2$ when $V_s \leq 4\sqrt{f'_c}\, b_w d$, and $d/4$ when $V_s > 4\sqrt{f'_c}\, b_w d$. The angle θ is taken as 45 deg in the ACI code. It would be physically more logical to use the same θ in Eq. (35) and Eq. (14) in the case of combined shear and torsion. This inconsistency between the new torsion provisions and the old shear provisions will have to be resolved in the future.

Longitudinal shear steel—According to the equilibrium truss model, shear stress also demands longitudinal shear steel according to Eq. (4). In the 1995 ACI Code, however, longitudinal shear steel continues to be designed indirectly by the so-called "shift rule" (Hsu, 1993). In this indirect method, the bending moment diagram is shifted toward the support by a distance of d, the effective depth. As a result, the longitudinal bars required by bending are each extended by a length d to take care of the longitudinal shear steel.

MAXIMUM SHEAR AND TORSIONAL STRENGTH

Maximum shear strength

The maximum shear strength of a cross section can be derived from Eq. (6) by taking $h = b_w$, $d_v = 0.9d$, and $\theta = 30$ deg. The compressive strength of concrete σ_d in Eq. (6) was found to be softened by the principal tensile strain in the perpendicular direction. Quantifying the softening effect by a softening coefficient ζ, then $\sigma_d = \zeta f'_c$ and the maximum shear strength becomes:

$$V_{n\ max} = 0.39\zeta f'_c b_w d \quad (36)$$

Recent tests at the University of Houston (Zhang and Hsu, 1996) indicate that the softening coefficient ζ is inversely proportional to $\sqrt{f'_c}$ for concrete up to f'_c = 15,000 psi (100 MPa). Assuming a conservative value of $\zeta = 26/\sqrt{f'_c}$ we arrive at the time-honored ACI provision for maximum shear stress of non-prestressed members:

$$\frac{V_{n\ max}}{b_w d} = 10\sqrt{f'_c} \quad (37)$$

Maximum torsional strength

The maximum torsional strength of a cross section can be derived from Eq. (13) by taking $h = 0.9A_{oh}/p_h$, $A_o = 0.85A_{oh}$, θ = 30 deg and $\sigma_d = \zeta f'_c$:

$$T_{n\ max} = 0.66\zeta f'_c \frac{A_{oh}^2}{p_h} \quad (38)$$

Again, assuming that $\zeta = 26/\sqrt{f'_c}$ gives the maximum torsional stress:

$$\frac{T_{n\ max}\, p_h}{A_{oh}^2} = 17\sqrt{f'_c} \quad (39)$$

Eq. (39) can be verified by the PCA torsion tests with concrete strengths from 2100 psi to 6500 psi (Hsu, 1968a). The shear panel tests at University of Houston seems to suggest that Eq. (39) is valid for concrete strength up to 15,000 psi (100 MPa).

Interaction of shear and torsion

Hollow sections—In a large hollow box structure normally used in guideways and bridges, Fig. 4(c), the shear stress due to shear and the shear stress due to torsion will be additive in one of the vertical walls. Consequently, a linear interaction relationship between the shear stress in Eq. (37) and the torsional stress in Eq. (39) is adopted by the ACI Code:

$$\left(\frac{V_u}{b_w d}\right) + \left(\frac{T_u p_h}{1.7A_{oh}^2}\right) \le \phi\left(\frac{V_c}{b_w d} + 8\sqrt{f'_c}\right) \quad (40)$$

In Eq. (40) $V_c/b_w d$ can be conservatively taken as $2\sqrt{f'_c}$ for non-prestressed members and the right hand side becomes $\phi(10\sqrt{f'_c})$.

The maximum thickness of the shear flow zone (corresponding to the maximum torsional resistance of a cross section) was found to be $0.8A_{cp}/p_{cp}$ (Hsu, 1993). This required thickness is taken conservatively as A_{oh}/p_h in the ACI Code. Therefore, if the actual wall thickness t is less than A_{oh}/p_h, the torsional stress in Eq. (40) should be increased proportionally by substituting $A_{oh}/p_h = t$ in the second term that results in $T_u/1.7A_{oh}t$. The thickness t is taken at the location where the stresses are being checked.

Solid sections—In a member with solid cross section as shown in Fig. 4 (a), the core of the cross section could be used to resist shear stress due to shear, leaving the outer ring area to resist shear stress due to torsion. Therefore, the stresses due to shear and torsion need not be additive. This favorable condition is reflected in the ACI Code by using a circular interaction relationship between the shear stress in Eq. (37) and the torsional stress in Eq. (39):

$$\sqrt{\left(\frac{V_u}{b_w d}\right)^2 + \left(\frac{T_u p_h}{1.7A_{oh}^2}\right)^2} \le \phi\left(\frac{V_c}{b_w d} + 8\sqrt{f'_c}\right) \quad (41)$$

Eqs. (40) and (41) are used to check the cross section of a member. If these conditions are not satisfied, the cross section must be enlarged.

OTHER DESIGN CONSIDERATIONS

Compatibility torsion

In the case of compatibility torsion, where a torsional moment in a statically indeterminate structure can be redistributed to other adjoining members after the formation of a plastic hinge, ACI code allows the torsional moment to be reduced to the cracking torsional moment under combined loadings. For non-prestressed members, the cracking torsional moment under combined loadings is expressed by Eq. (24). For prestressed concrete, however, the expression on the right-hand side of Eq. (24) must be multiplied by a factor which reflects the increase of cracking strength by the longitudinal prestress. Using the well-known square-root factor derived from either the Mohr stress circle (Hsu, 1984) or from the skew bending theory (Hsu, 1968b), the cracking torsional moment of hollow prestressed members is expressed as follows:

$$T_u = \phi T_{cr} = \phi 4\sqrt{f'_c}\, A_g \frac{A_{cp}}{p_{cp}} \sqrt{1 + \frac{f_{ce}}{4\sqrt{f'_c}}} \quad (42)$$

where f_{ce} in the square-root factor is the average compressive stress at the centroidal axis due to effective prestress after allowing for all losses. In the case of non-prestressed solid members, f_{ce} = 0 and the square-root factor becomes unity.

Threshold torque

In order to simplify the design processes, ACI Code allows a small torsional moment in a structure to be neglected. In the 1989 ACI Code (ACI 318, 1989) this "threshold torque" was taken for solid sections as 25 percent of the cracking torque which results in a torque of $\phi\sqrt{f'_c}\,(A_{cp}^2/p_{cp})$.

In the case of hollow sections, it is proposed (Hsu, 1996) that the threshold torque $\phi\sqrt{f'_c}\,(A_{cp}^2/p_{cp})$ for solid sections be multiplied by a factor for hollow sections, $(A_g/A_{cp})^2$, resulting in:

$$T_u = \phi \sqrt{f'_c} \frac{A_g^2}{p_{cp}} \quad (43)$$

Eq. (43) is applicable to nonprestressed and prestressed girders, because prestress does not significantly increase the ultimate torques of girders. When the threshold torque for a solid beam is viewed as a certain percentage (< 25 percent and depending on reinforcement) of the ultimate torque, then the threshold torque expressed by Eq. (43) for a hollow section will be a certain percentage (depending on reinforcement and wall thickness) of the ultimate torque, regardless of prestress.

Location of torsional steel in shear flow zone

The internal torsional moment of a member is contributed by the circulatory shear stresses acting along the centerline of shear flow, Fig. 3. To be theoretically correct, the centroidal line of the steel cage should be designed to coincide with the centerline of shear flow. Because a steel cage is made up of hoop bars and longitudinal bars, the centroidal line of the steel cage is best represented by the inner edge of the hoop bars, Fig. 4 (a) and (b). Define $\bar{c}$ as the distance measured from the outer face of cross section to the inner edge of the hoop bars. When the center line of the steel cage defined by $\bar{c}$ lies in the middle of the shear flow zone with a thickness of t_d, then the theoretically correct case of design is:

$$\bar{c} = 0.5 t_d \quad (44)$$

In the case of a hollow beam as shown in Fig. 4 (b), the inner concrete cover measured from the inside face of wall to the inner edge of hoop bars should also be $0.5t_d$ in theory. This theoretical requirement of the inner concrete cover is replaced in the 1995 ACI Code by a provision (Section 11.6.4.4) specifying that the distance from the inside face of wall to the centerline of the hoop bars shall be not less than $0.5A_{oh}/p_h$. This requirement is conservative, because the thickness $t_d = A_{oh}/p_h$ represents the maximum thickness required to resist a maximum torque for the given outer cross section. It is obvious that this provision is not intended to apply to the inner cover of large hollow cross sections with two layers of hoop steel as shown in Fig. 4 (c).

In practice, it is sometimes difficult to satisfy Eq. (44), $t_d = 2\bar{c}$, particularly in small cross sections where the relatively large outer cover is dictated by fire resistance and corrosion requirements. If $t_d < 2\bar{c}$, the lever arm area A_o calculated by Eq. (17) may overestimate the true A_o, because the true A_o depends not only on the location of the concrete struts but also on the location of the steel cage. Tests of pure torsional members have shown (Hsu and Mo, 1985b) that the calculated torsional strengths will be safe if

$$t_d \geq 1.33\bar{c} \quad (45)$$

If this condition is considered, the thickness t_d calculated by Eq. (15) may be limited to a minimum of $t_d = 1.33\bar{c}$. This condition, however, is not serious because a small t_d means a small torque T_n. A small error of a small torque is not a concern in a member subjected to shear and torsion. For this reason, the requirement of Eq. (45) is neglected.

Box sections with outstanding flanges

When the outstanding flanges are very thin as compared to the height of the box section, the parameter A_{cp}^2/p_{cp} for the flanged section may be less than the same parameter for the box section without flanges. This is conceptually wrong because the addition of flanges is supposed to increase the torsional resistance which is proportional to this parameter. Physically, this inconsistency means that the cross section is not "bulky" enough and that St. Venant torsional stresses can not flow into the flanges. When this happens, the outstanding flanges should be neglected in the calculation of the cross-sectional properties of A_{cp}, A_g and p_{cp} (see section "Check Outstanding Flanges" in the example problem).

DESIGN PROCEDURES

The design procedures for shear and torsion are given in a flow chart in Fig. 5. The design steps are as follows:

(1) Calculate the shear force diagram and the torsional moment diagram. Check the factored shear force V_u and the factored torsional moment T_u at the critical sections. For a beam subjected to uniformly distributed load on the upper surface, the critical sections are taken at a distance d from the support for reinforced concrete, and at a distance $h/2$ from the support for prestressed concrete. In the case of compatibility torsion, T_u is calculated by Eq. (42).

(2) Check the factored torsional moment T_u by Eq. (43). Torsion can be neglected in design if T_u is less than that calculated by this equation.

(3) Check the size of cross section by the interaction Eqs. (40) or (41), as appropriate. If the appropriate combination of shear stress and torsional stress is greater than the specified maximum stress, enlarge the cross section.

(4) Calculate the transverse torsional steel, A_t/s, by Eqs. (14) and (17) or (18). This transverse torsional steel must be in the form of hoops or closed stirrups and must satisfy the maximum spacing requirement.

(5) Determine the longitudinal torsional steel, A_ℓ, by Eq. (19) based on strength, or by Eq. (31) based on ductility. This longitudinal torsional steel must be uniformly distributed along the perimeter of the cross section and must meet the requirements of maximum spacing and minimum bar diameter.

(6) Calculate the transverse shear steel, A_v/s, by Eqs. (32) to (35). This transverse shear steel is required in the vertical legs of cross section and must satisfy the maximum spacing requirement.

(7) Arrange the transverse steel caused by shear and torsion in two ways: (i.) In the case of small cross sections normally used in buildings, Fig. 4 (a) and (b), both the torsional steel and shear steel are designed in the form of closed stirrups. As a result, the combined area for two legs is $2A_t + A_v$. This combined area must satisfy the minimum requirement of $2A_t + A_v \geq 50b_w s/f_{yv}$. (ii.) In the case of large hollow sections normally used in guideways and bridges, Fig. 4 (c), shear steel and torsional steel are designed separately for each wall. The combined area (for one wall) of $A_t + A_v/2$ is used for the vertical wall where shear and torsion are addi-

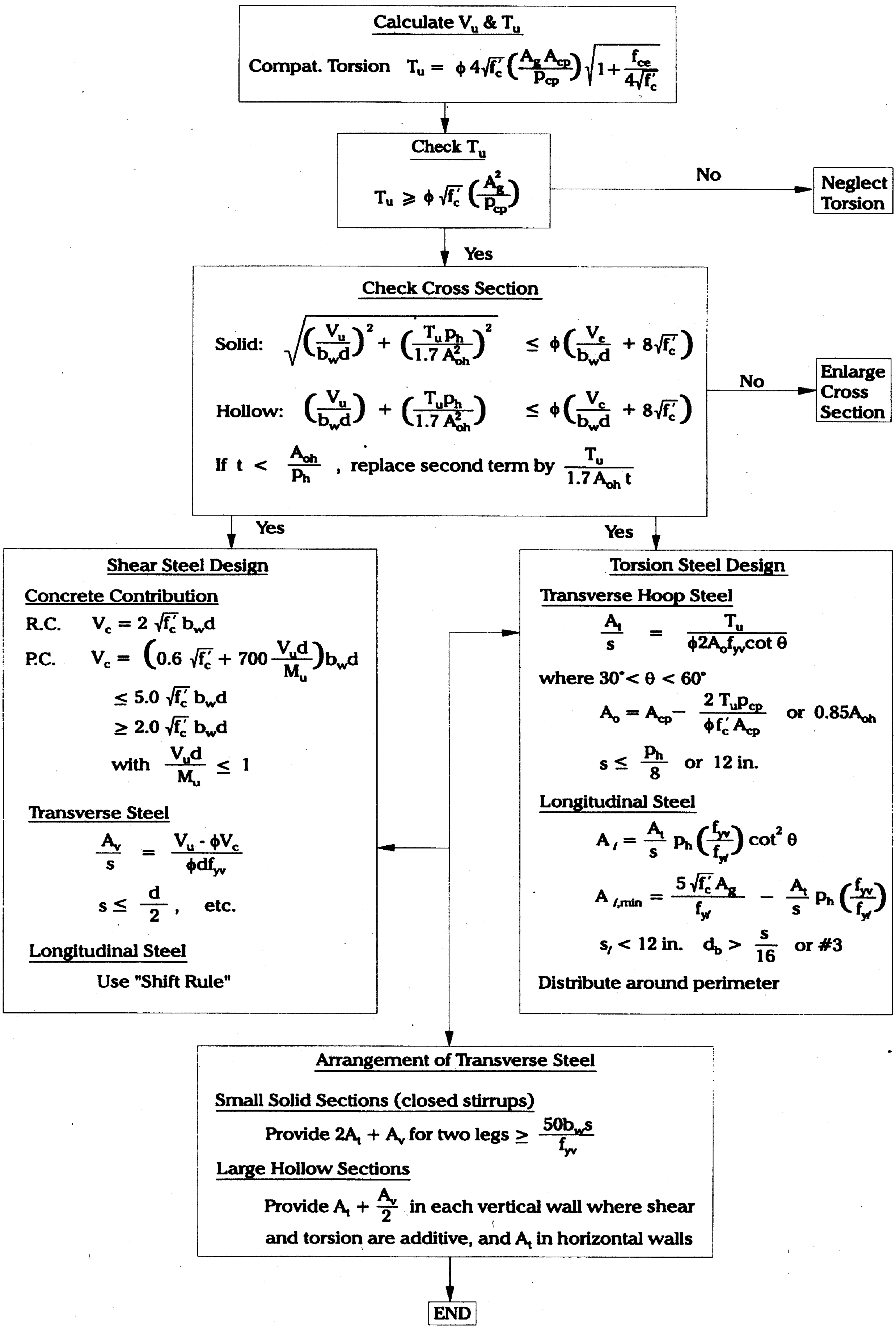


Fig. 5—ACI shear & torsion design procedures

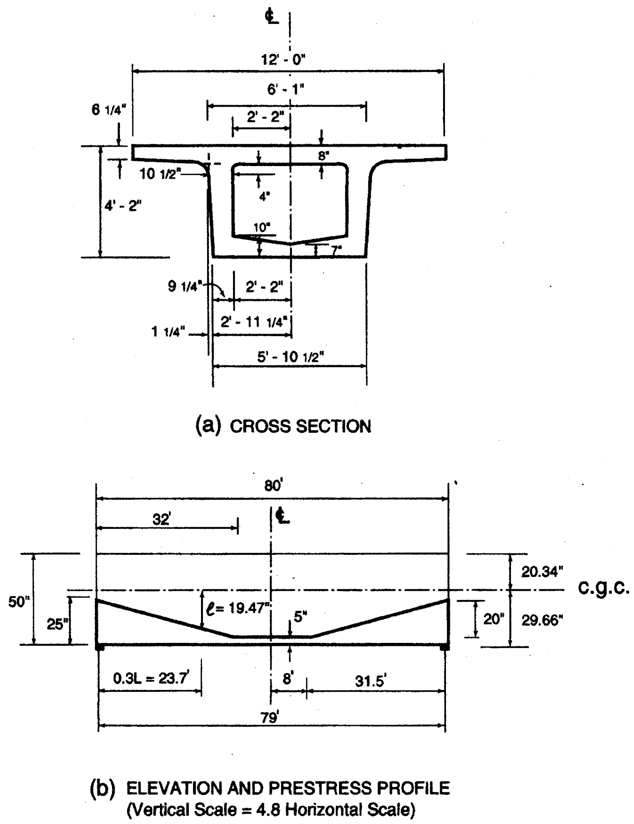


Fig. 6—Cross section and elevation of box girder

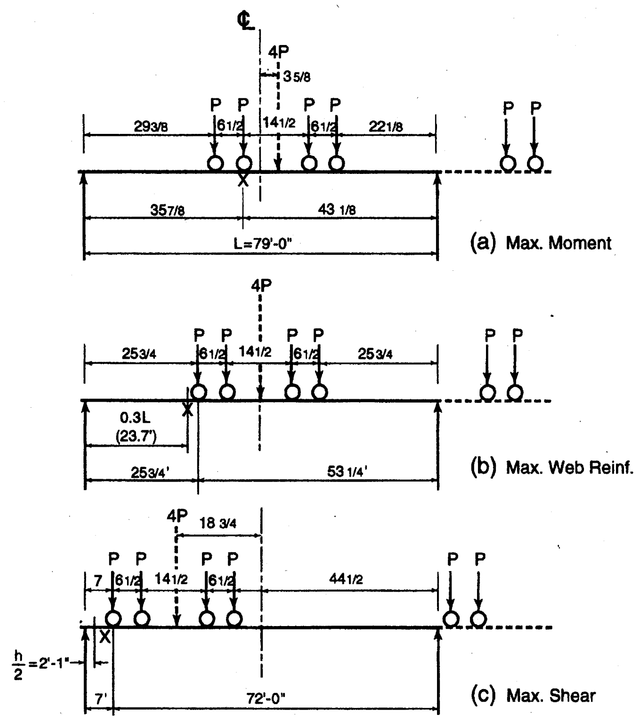


Fig. 7—Loading conditions and sections for design

tive. This combined area must satisfy the minimum requirement of $A_t + A_v/2 \geq 25 b_w s/f_{yv}$. The horizontal walls, which are subjected only to torsional shear stress, can be designed for A_t only.

EXAMPLE PROBLEM

Design the shear and torsional reinforcement of a guideway girder. A 12 ft-wide and 4 ft 2 in.-deep box girder with overhanging flanges, Fig. 6 (a), was designed as an alternative to the double-tee girder used in the Dade County Rapid Transit System, Florida. The standard prestressed girder in this 22-mile guideway is simply supported and 80 ft long. It is prestressed with sixty two 270K, 1/2 in., seven-wire strands as shown in Fig. 6 (b). The total prestress force is 1366 kips after prestress loss. The design of flexural steel is omitted for simplicity. The net concrete cover is 1.5 in. (3.81 cm) and the material strengths are f'_c = 7000 psi (48.2 MPa) and f_y = 60,000psi (413MPa).

Sectional properties

L = 79ft
h = 50 in.
d = 40 in. at $0.3L$ from support
t = 9.875 in. (average of stem width)
b_w = 19.75 in.
A = 2361.4 in.^{2}
I = 768,336 in.^{4}
y_t = 20.34 in.
y_b = 29.66 in.

Loading criteria

The standard girders are designed to carry a train of cars, each 75', 0" long. Each car has two trucks with a center-to-center distance of 54', 0". Each truck consists of two axles 6', 6" apart. The crush live load of each car is 115.5 kips. The maximum amount of web reinforcement was obtained at the section $0.3L$ from the support under a derailment load, which consists of two truck loads located symmetrically at a distance 10', 6" from the midspan, Fig. 7 (b). Each axle load is taken as one-fourth of the crush live load (115.5 kips/4) with 100 percent impact and a maximum sideshift of 3 ft. The load factor is taken as 1.4.

The girder is also subjected to a superimposed dead load caused by the weight of the track rails, rail plinth pads, power rail, guard rail, cableway, acoustic barrier, etc. At derailment, this superimposed dead load is assumed to produce a uniform vertical load of 0.88 kip/ft and a uniformly distributed torque of 0.71 kip-ft/ft. This torque is neglected in the calculation because the magnitude of the distributed torque is small and because the torque is acting in a direction opposite to the derailment torque.

Derailment load per axle

$$P_{u,L} = 1.4\left(\frac{115.5}{4}\right)(2) = 80.85 \text{ kip/axle}$$

Derailment torque per axle

$$T_{u,L} = 1.4\left(\frac{115.5}{4}\right)(2)(3) = 242.5 \text{ k-ft/axle}$$

Girder weight (assumed concrete density of 144 lb/ft^3 and load factor of 1.4)

$$w_{u,g} = 1.4\frac{2361.4(144)}{144(100)} = 3.31 \text{ k/ft}$$

Superimposed dead weight (load factor of 1.4)

$$w_{u,s} = 1.4(0.88) = 1.23 \text{ k/ft}$$

Factored shear, torque, and bending moment

Refer to Fig. 7(b), V_u, T_u and M_u at $0.3L$ from support are:

$$V_u = (w_{u,g} + w_{u,s})(0.2L) + 2P_{u,L}$$

$$= (3.31 + 1.23)(0.2)(79) + 2(80.85) = \underline{233.4 \text{ k}}$$

$$T_u = 2T_{u,L} = 2(242.5) = \underline{485 \text{ k-ft}}$$

$$M_u = \frac{1}{2}(w_{u,g} + w_{u,s})L(0.3L) - \frac{1}{2}(w_{u,g} + w_{u,s})(0.3L)^2 +$$

$$2P_{u,L}(0.3L)$$

$$= \frac{1}{2}(3.31 + 1.23)(79)(23.7)$$

$$- \frac{1}{2}(3.31 + 1.23)(23.7)^2 + 2(80.85)(23.7)$$

$$= \underline{6807 \text{ k-ft}}$$

Check outstanding flanges

Refer to Fig. 6 (a), the parameter A_{cp}^2/p_{cp} is determined as follows:

Neglect overhanging flanges

$$A_{cp} = 73(8) + \frac{1}{2}(73 + 70.5)(42) = 3597 \text{ in.}^2$$

$$p_{cp} = 73 + 70.5 + 2(50) = 243.5 \text{ in.}$$

$$\frac{A_{cp}^2}{p_{cp}} = \frac{3597^2}{243.5} = \underline{53{,}135 \text{ in.}^3}$$

Include overhanging flanges

$$A_{cp} = 73(8) + \frac{1}{2}(73 + 70.5)(42) + 2(35.5)(7.125)$$
$$= 4103 \text{ in.}^2$$

$$p_{cp} = 144 + 70.5 + 2(6.25) + 2(78.75) = 384.5 \text{ in}$$

$$\frac{A_{cp}^2}{p_{cp}} = \frac{4103^2}{384.5} = 43{,}783 \text{ in.}^3 < 53{,}135 \text{ in.}^3 \text{ N.G.}$$

Neglect outstanding flanges and use $A_{cp}^2/p_{cp} = \underline{53{,}135 \text{ in.}^3}$

Check threshold torque

$$A_g = 3597 - (52 \times 33.5) = 1855 \text{ in.}^2$$

$$T_u = \phi\sqrt{f'_c}\frac{A_g^2}{p_{cp}} = 0.85\sqrt{7000}\frac{1855^2}{243.5} = 1{,}005{,}000 \text{ in.-lb}$$
$$= \underline{83.7 \text{ k-ft}} < 485 \text{ k-ft}$$

Factored torsional moment needs to be considered in design.

Check cross section

Assume a clear concrete cover of 1.5 in. and No. 4 bars for web reinforcement:

$$A_{oh} = \frac{1}{2}[(73 - 3.5) + (70.5 - 3.5)](50 - 3.5) = 3174 \text{ in.}^2$$

$$p_h = (73 - 3.5) + (70.5 - 3.5) + 2(50 - 3.5) = 229.5 \text{ in.}$$

$$e = (29.66 - 5) - \frac{32 - 23.7}{32}20 = 19.47 \text{ in. at } 0.3L \text{ from support}$$

$$d = y_t + e = 20.34 + 19.47 = 39.81 \text{ in. at } 0.3L \text{ from support}$$

$$d = 0.8h = 0.8(50) = 40 \text{ in. governs}$$

$$b_w = 2t = 2(9.875) = 19.75 \text{ in.}$$

$$b_w d = 19.75(40) = 790 \text{ in.}^2$$

The interaction equation for hollow box sections is

$$\left(\frac{V_u}{b_w d}\right) + \left(\frac{T_u}{1.7A_{oh}t}\right) \le \phi\left(\frac{V_c}{b_w d} + 8\sqrt{f'_c}\right) \text{ when } t < \frac{A_{oh}}{p_h}$$

$$\frac{A_{oh}}{p_h} = \frac{3174}{229.5} = 13.8 \text{ in.} > t = 9.875 \text{ in.}$$

$$\left(\frac{V_u}{b_w d}\right) + \left(\frac{T_u}{1.7A_{oh}t}\right) = \frac{233{,}400}{790} + \frac{485(12{,}000)}{1.7(3174)(9.875)}$$

$$= 295 + 109 = \underline{404 \text{ psi}}$$

$$V_c = \left[0.6\sqrt{f'_c} + 700\frac{V_u d}{M_u}\right]b_w d \text{ where } \frac{V_u d}{M_u} \le 1$$

$$\frac{V_u d}{M_u} = \frac{233.4(40)}{6807(12)} = 0.1143 < 1 \text{ O.K.}$$

$$V_c = [0.6\sqrt{7000} + 700(0.1143)](790) = 102{,}900 \text{ lbs}$$
$$V_{c,min} = 2\sqrt{f'_c}\,b_w d = 2\sqrt{7000}(790) = 132{,}200 \text{ lbs}$$
governs.

$$\phi\left(\frac{V_c}{b_w d} + 8\sqrt{f'_c}\right) = 0.85\left(\frac{132{,}000}{790} + 8\sqrt{7000}\right)$$

$$= 0.85(167 + 669) = \underline{711 \text{ psi}} > 404 \text{ psi O.K.}$$

Design of torsional hoop steel

$$A_o = A_{cp} - \frac{2T_u p_{cp}}{\phi f'_c A_{cp}} = 3597 - \frac{2(485)(12)(243.5)}{0.85(7)(3597)}$$

$$= 3464 \text{ in.}^2$$

Assume $\theta = 37.5$ deg as recommended by the Code provision for prestressed members:

$$\frac{A_t}{s} = \frac{T_u}{\phi 2 A_o f_{yv} \cot\theta} = \frac{485(12)}{(0.85)(2)(3464)(60)(1.303)}$$

$$= \underline{0.0127 \text{ in.}^2/\text{in.}}$$

$$s_{max} = \frac{P_h}{8} = \frac{229.5}{8} = 28.7 \text{ in.} > 12 \text{ in. } 12 \text{ in. spacing}$$

12 in. spacing governs.

Design of torsional longitudinal steel

$$A_\ell = \frac{A_t}{s} p_h \left(\frac{f_{yv}}{f_{y\ell}}\right) \cot^2\theta = (0.0127)(229.5)(1)(1.303^2)$$

$$= 5.0 \text{ in.}^2$$

Check minimum limitation for A_t/s:

$$\frac{A_t}{s} = \frac{25 b_w}{f_{yv}} = \frac{25(19.75)}{60,000}$$

$$= 0.0082 \text{ in.}^2/\text{in.} < 0.0127 \text{ in.}^2/\text{in.}$$

$$A_{\ell,min} = \frac{5\sqrt{f'_c}\, A_g}{f_{y\ell}} - \left(\frac{A_t}{s}\right) p_h \left(\frac{f_{yv}}{f_{y\ell}}\right)$$

$$= \frac{5\sqrt{7000}(1855)}{60,000} - (0.0127)(229.5)(1)$$

$$= 12.9 - 2.9 = \underline{10.0 \text{ in.}^2} \text{ governs.}$$

Select 36 No. 5 longitudinal bars: $A_\ell = 36(0.31) = 11.2$ in.^{2} > 10.0 in.^{2}

Design of shear steel

$$V_c = V_{c,min} = 132.2 \text{ kips}$$

$$\frac{A_v}{s} = \frac{V_u - \phi V_c}{\phi d f_{yv}} = \frac{233.4 - 0.85(132.2)}{0.85(40)(60)}$$

$$= \underline{0.0593 \text{ in.}^2}$$

$$4\sqrt{f'_c}\, b_w d = 4\sqrt{7000}(790) = 264{,}400 \text{ lbs} = 264.4 \text{ kips}$$

$$V_s = \frac{V_u}{\phi} - V_c = \frac{233.4}{0.85} - 132.2$$

$$= 142.4 \text{ kips} < 264.4 \text{ kips}$$

$$s_{max} = \frac{d}{2} = \frac{40}{2} = 20 \text{ in.} < 12 \text{ in.}$$

Spacing of 12 in. governs.

Transverse steel for vertical walls

Transverse steel in the vertical walls is contributed by both shear and torsion:

$$\frac{A_t}{s} + \frac{1}{2}\frac{A_v}{s} = 0.0127 + \frac{1}{2}(0.0593) = 0.0423 \text{ in.}^2/\text{in.}$$

$$\left(\frac{A_t}{s} + \frac{1}{2}\frac{A_v}{s}\right)_{min} = \frac{25(19.75)}{60,000}$$

$$= 0.00823 \text{ in.}^2/\text{in.} < 0.0423 \text{ in.}^2/\text{in. O.K.}$$

Select 2 layers of No. 5 bars in each vertical wall at 12 in. spacing:

$$\frac{2(0.31)}{12} = 0.0517 \text{ in.}^2/\text{in.} > 0.0423 \text{ in.}^2/\text{in. O.K.}$$

Transverse steel for horizontal walls

Transverse steel in the horizontal walls is contributed by torsion only:

$$\frac{A_t}{s} = 0.0127 \text{ in.}^2/\text{in.}$$

Select 2 layers of No. 3 bars in each horizontal wall at 12 in. spacing:

$$\frac{2(0.11)}{12} = 0.0183 \text{ in.}^2/\text{in.} > 0.0127 \text{ in.}^2/\text{in. O.K.}$$

The transverse steel in the top wall should be added to the flexural steel required in the top flange acting as a transverse continuous slab.

Arrangement of reinforcing bars

The arrangement of the reinforcing bars for shear and torsion is shown in Fig. 8. This steel arrangement could be conservatively used throughout the length of the girder.

CONCLUSIONS

This paper provides the background information for the new ACI shear and torsion provisions. This background information is presented in a systematic manner. First, from the equilibrium (plasticity) truss model we derive a set of three equations for shear and another set of three equations for torsion. These two sets of equilibrium equations are shown to be similar. In each set of three equations, the first equation relates the external shear (or torque) to transverse steel, the second to longitudinal steel and the third to concrete struts.

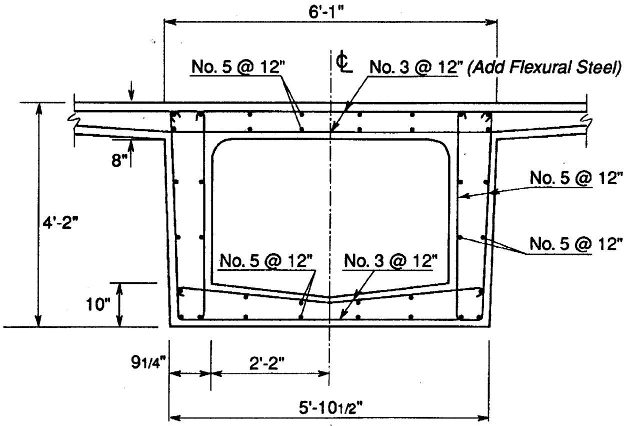


Fig. 8—Steel arrangement for shear and torsion

Second, torsion design equations for transverse and longitudinal steel are logically derived from the first two of the set of three equilibrium equations. In this derivation, the three physical concepts are emphasized, namely, the thickness of shear flow zone, the lever arm area, and the torsional steel required to prevent brittle failure. In contrast to the rationality of torsion design, the shear design method remains empirical. "Concrete contribution" is subtracted from the shear stress before the transverse steel is designed. The longitudinal steel is provided indirectly by the "shift rule," rather than directly from the equilibrium condition.

Third, the maximum shear and torsional strengths of a cross section are derived from the third of the sets of three equilibrium equations relating the external shear (or torque) to the stress in the concrete struts. Interaction relationships of shear and torsional strengths are given for hollow and solid sections. These interaction equations are used to check the size of cross section.

Fourth, various design considerations are presented, including compatibility torsion, threshold torque, the location of torsional steel in shear flow zone, and the outstanding flanges. The whole design procedure is summarized in a flow chart and illustrated by an example problem.

At present, three formulas in the 1995 ACI Code are applicable only to solid sections. These formulas are generalized to include hollow sections as shown in Eqs. (31), (42) and (43).

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