A Research Report Sponsored by the Reinforced Concrete Research Council

Design Indications from Tests of Unbraced Multipanel Concrete Frames

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This paper summarizes the design recommendations and suggested Building Code changes stemming from studies of reinforced concrete columns in multipanel frames and a complementary study of isolated columns under controlled lateral deformation loading. The study supports a liberalization of the limiting strain criterion for column design which would permit efficient utilization of higher strength reinforcement (f_y = 80 to 90 ksi) and may provide significant design and economic opportunities.

The studies also provide a basis for improved computer analysis of complex reinforced concrete frames. Improved procedures for computing moment-curvature relationships for concrete columns and for incorporating such techniques into a nonlinear matrix analysis are suggested. The curvature relationships reflect the considerable ductility noted in the tests.

Multipanel frame studies clarified the nature of column and frame stability failures and verified the concept of using story moment magnification ($\Sigma P/\Sigma P_c$) in unbraced frames. Desirable changes in the moment magnification procedures are suggested to clarify the application to unbraced frames. These changes will generally reduce the gross conservatism of many present applications, may considerably lower the column design moments in unbraced frames, and will have important implications on the design of unbraced structures subject to combined gravity and lateral loads.

Keywords: axial loads; bending moments; building codes; **columns (supports)**; diagrams; ductility; **frames; long columns**; reinforced concrete; slenderness ratio; **stress-strain relationships;** structural analysis; structural design; tied columns; ultimate strength.

Introduction

Extensive laboratory investigations from 1930 to 1950 documented the strength and response of concentrically and eccentrically loaded isolated columns. Since then, column interaction diagrams have become familiar tools for most structural engineers. The general assumptions and principles of ACI 318-77,[1] Section 10.2 and 10.3, on which column section capacity is based, stem from pioneering studies of the behavior of isolated columns such as that of Hognestad.[2]

Beginning in approximately 1960,[3-7] attention focused on supplementing studies of individual columns with investigations of the effect of restraints and of overall frame behavior. Under the auspices of the Reinforced Concrete Research Council, tests were carried out to better define the role of the column as an integral frame member.[8-11] These pioneering tests began to illustrate the complex interaction of the column with other frame members, but were usually restricted to simple single bay rectangular frames with a single beam and column at each joint. In 1964, Wood[12] stated:

> . . . the major problem centers around what is going to be the performance of complete multistory frames versus individual members. Concentrated studies are needed on the whole framework and it is here that there is an alarming absence of information. . . . Although the designer has been told so often what to do, he has never been shown what is likely to happen once he starts making the calculations.

Design recommendations[13-14] on which the slender column provisions of ACI 318-71 and ACI 318-77 were based relied heavily on the computer and physical test studies of the 1960's cited above. In development of some provisions, extensive extrapolation or application of engineering judgment was necessary because of limited documentation of the behavior of reinforced concrete columns in realistic multibay and multistory frames.

When this Reinforced Concrete Research Council project was conceived in the 1967-1969 period, designers and those formulating design regulations were interested in questions such as:

(1) What is the collapse load of laterally loaded frames which also have loads on the beams?

Up to that time lateral load studies had involved only single-bay rectangular frames with axial column loads and lateral loads. Moments at the column ends in such frames are equal and little redistribution was possible before collapse. For the more realistic case with gravity loads on the beams in addition to lateral loads on the frame, the real collapse load is more involved. The dead load moments on the beams help create the hinging moment at one end but at the same time reduce and delay the possibility of hinging at the other end. Frame strength then depends on the extent to which redistribution of column shears and moments is possible.

(2) Is the heavily loaded column ductile enough so that the frame can readjust its load-carrying pattern? Would additional horizontal load go to stiff elements after the most critically loaded column reached its nominal capacity as defined by the interaction diagrams?

The violent explosions and shattering often evident when compression failures occurred in isolated column tests made such ductility suspect.

(3) Is the lateral stability of lighter columns in a story adequately protected by the presence of heavier columns?

Elastic analysis indicated that many story heights can be adequately braced by a few oversize columns. This formed the basis for the ACI Building Code use of an averaged story magnifier ($\Sigma P/\Sigma P_c$) in unbraced frames. This implicitly recognized the possibility of shear or moment redistribution in columns but was not based on any experimental evidence.

(4) What is the correct application of the moment magnification procedure for unbraced frames which have both gravity (nonsway producing) loads and lateral (sway producing) loads?

Since previous tests and computer studies had not treated the combined loading case, the design rules formulated were vague and somewhat misleading. A clear interpretation based on experimental evidence might considerably lower column design moments with important economic benefits.

(5) What procedures could be used in computer simulation of such frames to ensure correct modeling of the complex material and geometric nonlinearity and failure conditions of actual multipanel frames?

Development of realistic computer simulation could reduce the cost of experimentation and allow complex design cases to be studied in a more rational manner.

(6) What is the proper compressive strain limit for eccentrically loaded columns?

Isolated eccentrically loaded column tests[2] had suggested a value of 0.0038 in./in., which seemed to be in reasonable agreement with the 0.003 in./in. assumed for design use by the ACI Building Code. The use of the 0.003 in./in. limit prevented the economic use of reinforcement with f_y = 80 to 90 ksi in columns.[15-16] However, if the limiting strain could be relaxed to 0.004 in./in., Grade 90 reinforcement could be fully developed in most columns. Experiments with high strength reinforcement[17] had shown it feasible for use in eccentrically loaded columns.

(7) Could limit design procedures with a greater reliance on equilibrium based moments be used for column design?

This might possibly simplify design and allow more freedom in placement of reinforcement to reduce congestion. The lack of understanding of column redistribution capacity greatly impeded such studies.

The series of frame tests and computer studies summarized in Parts 2 and 3[18-19] and the isolated column tests under controlled lateral deformation loading summarized in Part 1[20] were designed to answer these types of questions. In the early phases of this RCRC study, an attempt was made to answer these questions by tests of single panel rectangular frames. In almost all of the single panel cases, effective redistribution was severely limited by frame lateral instability failures upon development of hinging in one column. Maximum strains measured in columns tended to be low because of the instability mode of failure. When tests began of more realistic multipanel frames, it became immediately apparent that concrete columns could sustain very large deformations if lateral instability could be prevented. This led to the series of isolated column tests under controlled lateral deformation loading in which a much clearer understanding of column failure conditions was obtained.

Objectives

This multipart study investigated the behavior, strength, deformation capacity, and stability of heavily loaded reinforced concrete columns included in a complex structural frame system. The objectives of the phase of the study which involved testing nine cantilever columns under controlled lateral deformation loading conditions were to:

(1) Experimentally determine the complete axial load-moment-curvature (P-M-ϕ) relationship for columns with large axial loads and minimal ties.

(2) Develop an analytical technique for predicting P-M-ϕ relationships.

(3) Evaluate the ACI Building Code 0.003 in./in. ultimate strain criterion.

Results were reported in Part 1.[20]

The main phase of the overall study was an experimental and analytical investigation of nine single story, multipanel frames. These frames had four columns of unequal stiffnesses and were loaded with beam, column, and lateral loads. The objectives of the multipanel frame investigation were to:

(1) Study the behavior and moment redistribution capability of highly indeterminate frames.

(2) Evaluate the ACI Building Code moment magnification technique for multiple slender columns of varying stiffnesses in unbraced frames.

(3) Determine the accuracy and applicability of a selected nonlinear analysis computer program.

Methodology for these tests and analyses was reported in Part 2[18] and results were summarized in Part 3.[19]

The objectives of this paper are to summarize the design recommendations and suggested Building Code changes resulting from the project. A few example figures are provided to illustrate major findings but the bulk of the documentation is in Parts 1-3 of the report series (References 18, 19, and 20).

Maximum compression strain

In the 1977 ACI Building Code, Section 10.2.3 specifies: "Maximum usable strain at extreme compression fiber shall be assumed equal to 0.003."

This assumption is one of long standing[21] and has been recognized as a conservative approximation of the limiting strain noted in beam tests. Many authors, including ACI-ASCE Committee 428,[22] Corley,[23] and Kaar and Corley,[24] have proposed expressions recognizing the effect of variables such as member width, moment gradient, and lateral reinforcement confinement on maximum strain. Except for the virtual elimination of the use of high strength reinforcement ($f_y > 60$ ksi) in columns, there has been little practical restriction in design because of the use of the conservative 0.003 in./in. limit. The moderate provisions for redistribution of negative moments in flexural members contained in ACI 318-77, Section 8.4 are based on ensured ductility by reinforcement percentage limitations. They do not specify any strain limit, although rotations may imply a larger strain. The flexural capacity for such under-reinforced members is primarily governed by the tensile reinforcement strength and is not sensitive to ε_u.

In column tests the maximum values of measured strains have been greatly influenced by the type of specimen, the type of instrumentation, and especially by the stability of the specimen after hinging begins. Fig. 1 indicates that current impressions that lightly tied columns with large axial loads are extremely brittle are based on tests aimed at determination of strength rather than deformation capacity. Most test specimens were either statically determinate, or only 1 or 2 deg indeterminate, and were loaded such that they became unstable immediately or shortly after the maximum moment had been reached at a critical location. In contrast to the usual "controlled load" tests, the "controlled deformation" cantilever column tests of this study indicate that the values reported as "maximum strains" in other tests are typical of the strains at development of maximum moment. In the cantilever column tests of this study, maximum strains at material failure of the cross section were more like 0.010 to 0.015 in./in.[20] These observed strain values were generally confirmed in the multibay frame tests and particularly in the test of FC8, which suffered a material failure before post-yielding frame instability could govern.[19]

Rectangular frames with failures near midheight show maximum strains similar to the pinned-base isolated columns. This is due to two factors. Stability of a column in a braced frame under single curvature loading depends essentially on the flexural restraint of the beams and only limited redistribution between columns is possible. In addition, there is no appreciable moment gradient in such a column. The results of this investigation clearly confirm work of earlier investigators[22, 23] who showed that maximum strain increases for members with moment gradients.

The results of this study, combined with recent tests reported by Kaar and Corley,[24] indicate that a realistic and generally conservative expression for ultimate strain is:

$$\varepsilon_u = 0.003 + 0.02\frac{b}{z} + [\varrho_h'' f_y/14.5]^2 \qquad (1)$$

where

b = width of specimen

z = distance between points of maximum and zero moment

ϱ_h'' = volumetric ratio of hoop (ties) reinforcement to confined concrete

f_y = yield stress of hoops in ksi

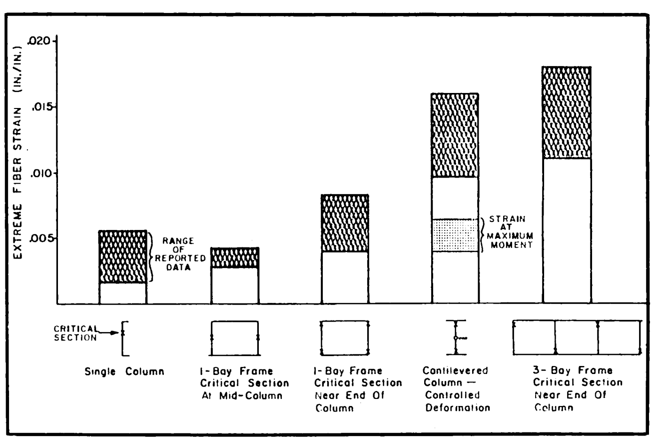
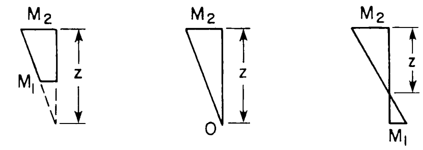


Fig. 1 — Maximum measured strains in specimens with different loading patterns, loading techniques, and degrees of indeterminancy.

TABLE 1 — Effect of moment gradient term

$\varepsilon = \frac{0.02b}{z}$



Assume h = 10 in., b = 10 in. (Narrow column is most critical case)

	$\frac{M_1}{M_2} = 1$	$\frac{M_1}{M_2} = 0.5$	$\frac{M_1}{M_2} = 0$	$\frac{M_1}{M_2} = -0.5$	$\frac{M_1}{M_2} = -1.0$	l_u	$\frac{l_u}{r}$
z (in.)	∞	192	96	64	48	8′	32
$\frac{0.02b}{z}$ (in./in.)	0	0.0010	0.0021	0.0031	0.0042		
z (in.)	∞	288	144	96	72	12′	48
$\frac{0.02b}{z}$ (in./in.)	0	0.0007	0.0014	0.0021	0.0028		
z (in.)	∞	384	192	128	92	16′	64
$\frac{0.02b}{z}$ (in./in.)	0	0.0005	0.0010	0.0016	0.0021		
z (in.)	∞	480	240	160	120	20′	80
$\frac{0.2b}{z}$ (in./in.)	0	0.0004	0.0008	0.0012	0.0016		

l_u = unsupported column height
r = column radius of gyration
z = distance between points of maximum and zero moment

The practical implication of this equation is that the ACI Building Code limiting strain criterion can be relaxed to 0.004 in./in. for columns with moment gradients which produce reversal of moments in the column length. This is the common governing condition both for unbraced frames and for the case of a braced frame with maximum axial load from factored loads on all floors or roof and the maximum moment from factored loads on a single adjacent span. A change to 0.004 in./in. with no specific reliance on hoop confinement means that the second term of Eq. (1) must be equal or greater than 0.001 in./in. This is true for cases in which a moment reversal occurs ($M_1/M_2 \leqslant 0$) for typical column slenderness ratios as seen in Table 1. This table illustrates the variation of the term $0.02b/z$ for various moment gradients and weak axis slenderness ratios for a very narrow column. Values for which the approximation that $0.02b/z$ = 0.001 in./in. (or ε_u = 0.004 in./in.) are unconservative are underlined in the table. Any wider column would result in increased or more conservative values. It can be seen that the only cases of practical concern are the single curvature columns $M_1/M_2 > 0$. This case can and does occur in braced frames, but seldom governs design. However, even in cases where there is no appreciable moment gradient, the use of ε_u = 0.004 in./in. is still possible by making the third term of Eq. (1) equal 0.001 in./in. This can be done by requiring a minimum volumetric percentage of hoop (tie) reinforcement when $\varepsilon_u > 0.003$ in./in.

For

$$\left(\frac{\varrho_h'' f_y}{14.5}\right)^2 \geqslant 0.001 \qquad (2a)$$

$$\frac{\varrho_h'' f_y}{14.5} \geqslant 0.032 \qquad (2b)$$

$$\varrho_h'' \geqslant \frac{0.46}{f_y} \text{ where } f_y \text{ is in ksi} \qquad (3a)$$

or

$$\varrho_h'' \geqslant \frac{460}{f_y} \text{ where } f_y \text{ is in psi} \qquad (3b)$$

Thus, even for single curvature columns, ε_u = 0.004 in./in. is conservative if the volumetric percentage of ties is $460/f_y$. This is not an unrealistic requirement for use of extra high strength reinforcement in those columns where a single curvature loading case may govern or where designers do not want to check the single curvature loading case for a strength limit imposed by ε_u = 0.003 in./in.

For example, in a 18 in. by 18 in. column with approximately 4 percent longitudinal reinforcement consisting of eight #11 bars, present tie requirements would call for #4 ties at 18 in. maximum spacing. A requirement of $\varrho_h'' \geqslant 460/f_y$ for 2 in. clear cover would require that Grade 60 ties be spaced not more than 7½ in. for #4 ties or 11½ in. for #5 ties. The designer could then decide whether possible economies associated with use of Grade 80 or Grade 90 longitudinal reinforcement justified the increase in ties.

In order to provide the flexibility to the designer who wishes to use higher grade steel in columns while not unduly complicating procedures for the general case where the 0.003 in./in. limit is satisfactory, the following Building Code changes are suggested:

(1) Revise Section 10.2.3 as follows:

10.2.3 — Maximum usable strain at extreme concrete compression fiber shall be assumed equal to 0.003 except that for compression members with $M_1/M_2 \leqslant 0$ or $\varrho_h'' \geqslant 460/f_y$ the value may be assumed equal to 0.004.

(2) Revise Section 10.3.2 by deleting "of 0.003" and adding "as specified in Section 10.2.3."

Stress-strain curve and moment-curvature diagrams

The relatively modest increase in ε_u from 0.003 to 0.004 in./in. recommended in the previous section does not reflect the true nature of ductility available in highly indeterminate concrete frames. Continuing studies in concrete frame analysis, computer simulation of frame behavior, and investigations of actual structural behavior often require more refined and realistic mathematical models. Utilization of the concrete stress-strain curve shown in Fig. 2, which incorporates the maximum strain limit of Eq. (1), produces P-M-ϕ relationships which are in excellent agreement with those measured at the critical sections in the controlled deformation cantilever column tests. Typical agreement is shown in Fig. 3. When compared with the Hognestad stress-strain curve with ε_u = 0.0038 in./in., the suggested block is seen to much more accurately represent the deformation capacity of the column. The maximum moment is virtually the same for the proposed and the Hognestad block theoretical results using $f_c'' = 0.95f_c'$, as is appropriate for these horizontally cast columns. The maximum strains measured are in very good agreement with the 0.0095 in./in. predicted by Eq. (1) for this column. The strain at the critical station at development of maximum moment was 0.0059 in./in., which further confirms the reasonableness and conservatism of the suggested 0.004 in./in. design value. The moment-curvature diagrams obtained differ appreciably in deformation capacity from those previously used in studies of columns. Historically, these have been similar to the curve using the Hognestad block. While the marked difference is due in part to the moment gradient in these columns, it

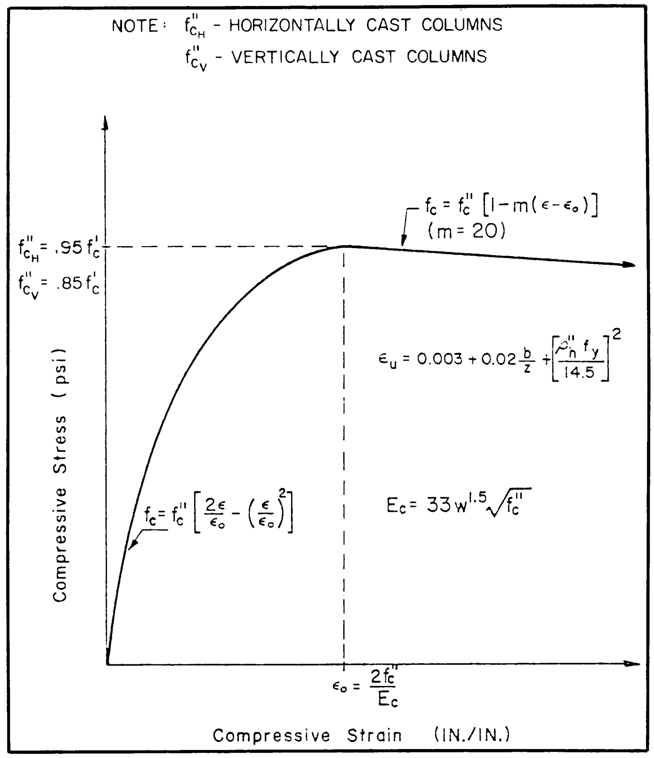


Fig. 2 — Proposed concrete compressive stress-strain curve.

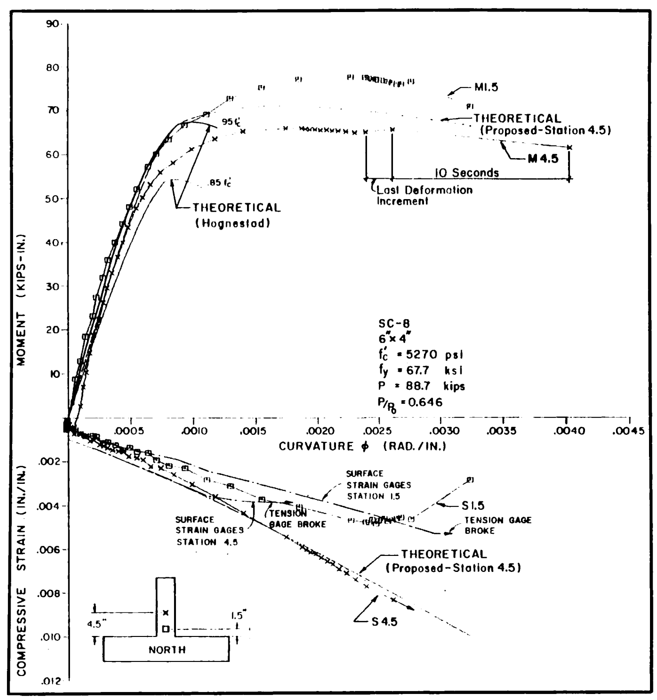


Fig. 3 — Load-moment-curvature (P-M-ϕ) relationship for Specimen SC-8.

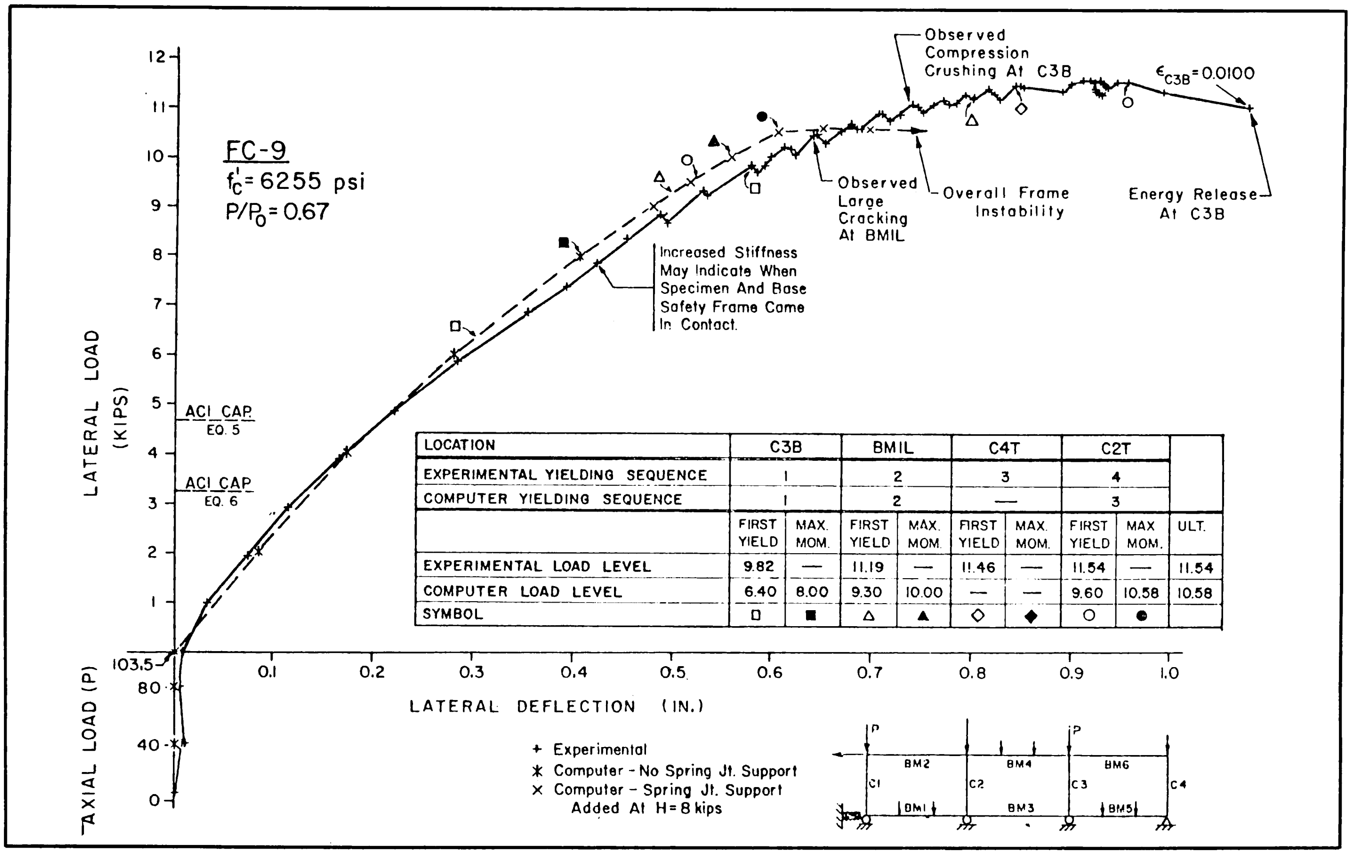


Fig. 4 — FC9 load versus lateral deflection.

also reflects the prevention of lateral instability upon formation of the first critical region. Utilization of these type moment-curvature diagrams will greatly improve studies of frame behavior. A relatively simple and fairly accurate approximation is to assume the P-M-ϕ curves as flat-topped beyond maximum moment. This was done indirectly in the analytical solution used to model the frame tests by inserting plastic hinges in a member whenever a negative member stiffness occurred after a maximum moment capacity was reached. The mathematical model developed for the frames[18] used a tangent stiffness matrix solution based on these P-M-ϕ curves, a joint stiffening to reflect actual column failures occurring about a distance h from the face of the beam, and an assumption of elastic joints. Results agreed very well with frame test results, as shown in Fig. 4. Not only was the overall response in close agreement but the number and sequence of hinge formations agreed favorably. This type analytical tool is not suited for actual design application but could be used to advantage for further parameter studies to assist in development of design aids and procedures.

In its present form, using the hinge insertion technique, a limiting strain or curvature criterion is not automatically checked. Consequently, the only failure mode that can be predicted by the program is instability. Sufficient data can be output so that a deformation criterion can be checked manually. The program is an excellent research tool which deserves further modification to eliminate the negative stiffness limitation.

Column post-yielding redistribution capacity

A central focus of the study was the question of whether the heavily loaded concrete column could evidence the ductility needed for post-yielding column shear and moment redistribution. Most prior experimental evidence indicated that the lightly tied column with low eccentricities typical of many actual structures failed dramatically, suddenly and catastrophically. In general, the test results of the isolated controlled deformation loading cantilever columns and the multipanel frames showed surprising ductility and energy absorption. All specimens were purposely designed with ACI Code minimum column ties and were loaded to very high ratios of axial load to ensure that compression-type failures would occur and that secondary moment ($P\Delta$) effects would be significant. All loading was monotonic. While results should be applicable to most gravity and wind loading conditions, they are not applicable for earthquake loading. For recommendations for unbraced frames in high seismic areas, tests are required on multipanel specimens with column hoop details typical of seismic regions and under reversed cyclic loading with higher lateral forces.

All nine of the frames tested demonstrated the ability to redistribute moments. Eight of the nine

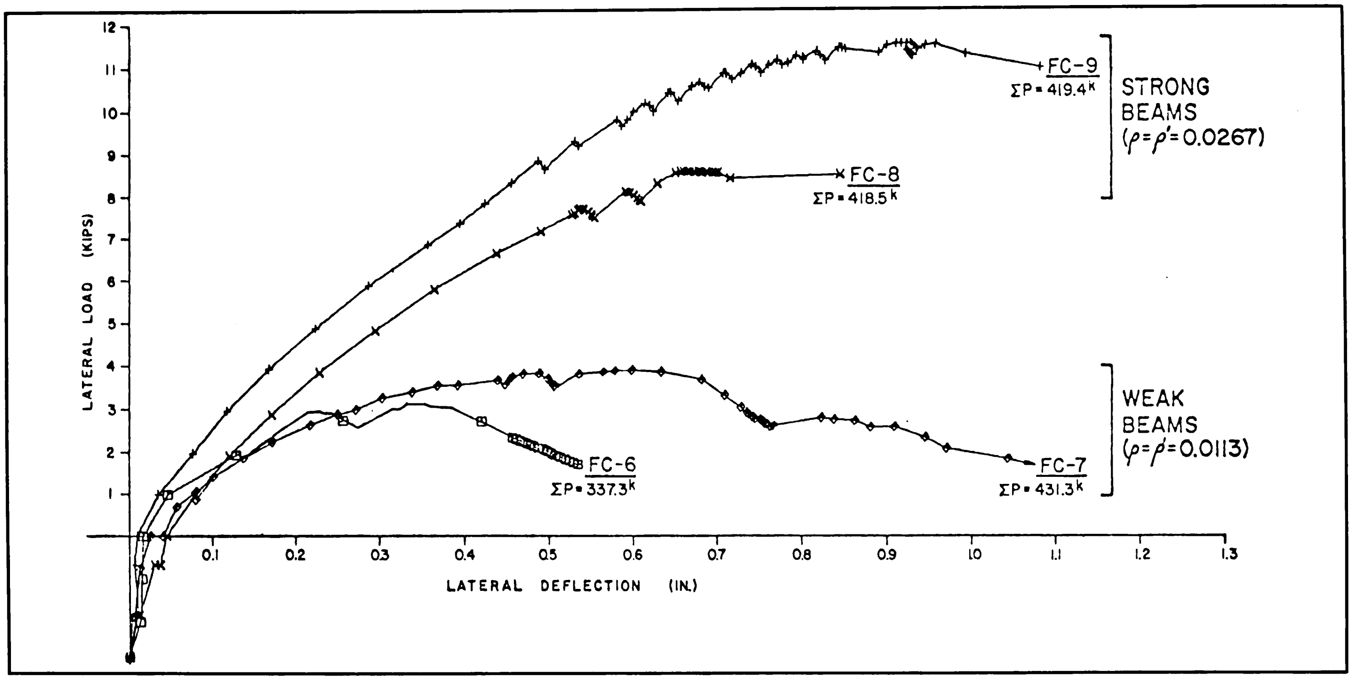


Fig. 5 — Load versus lateral deflection curves for FC6 through FC9.

had redistribution after a column critical yield region (hinge) formed. The other frame failed after multiple beam hinges had formed. Clearly, there was proven column redistribution ability. The amount of redistribution which took place was generally limited by post-yielding lateral frame instability. In only one frame was lack of deformation capacity the primary cause of failure. However, many factors can affect the utilization of the column redistribution capacity.

Effect of combining gravity beam loads and lateral loads

A fundamental question of interest is whether the frame with beam loads as well as lateral loads has more potential for redistribution when compared to a frame with only lateral loads. This question was studied directly in Frames FC1, FC2, and FC3. Frames FC1 and FC2 were virtually identical, except that FC1 had no beam loads while FC2 had beam loads. FC3 was a retest of FC1 with improved joint details, which confirmed that possible joint weakness had not affected the results. The ratios of ultimate load to load at first yielding were 1.14 and 1.13 for the frames with only lateral loads and 1.60 for the frame with beam loads and lateral loads. This was an apparent 45 percent increase in redistribution potential. This clearly demonstrated the improved redistribution possible when the beam moments can combine to reduce the sway moments at one end of the column. The apparent improvement in redistribution is somewhat misleading, since the ultimate lateral load of the frame with beam and lateral loads was actually less than those of the frames with lateral load only. The beam loading had caused the end of the column where beam moments and lateral load moments were additive in sign to yield at approximately 60 percent of the load at which the frame yielded under lateral load alone.

The practical application in design of the increased capacity for redistribution due to combined beam and lateral loads is further reduced when one considers a basic load combination that often governs for lateral loading such as wind. ACI 318-77, Eq. (9-3) specifies $U = 0.9D + 1.3W$. Since dead load would not be patterned, there would be no gravity load moments in interior columns of symmetrical regular structures. Consequently, interior column moments would be due to lateral loads only and both ends of the columns would become critical simultaneously. Thus, the reserve redistribution would not be mobilized. In view of this important case, there does not seem to be a general way to utilize the apparent redistribution potential from the combination of these moment diagrams. Only the exterior columns which would be in reverse curvature due to dead load would have such redistribution potential. This might be important in structures with only a few bays.

Effect of beam strength

Comparison of the lateral load-lateral deflection curves for FC6 through FC9 shown in Fig. 5 indicates the dramatic effect of beam strength on frame stiffness, toughness, and strength. While more redistribution was possible with the weak beams which yielded earlier, the strength and stiffness of the weak beam frames was greatly reduced. It is interesting to note that for virtually identical columns and loading conditions, increasing the beam reinforcement from $\rho = 0.0113$ to $\rho = 0.0267$ (an increase of 136 percent) resulted in increases in the average lateral load at first yield of 254 percent and

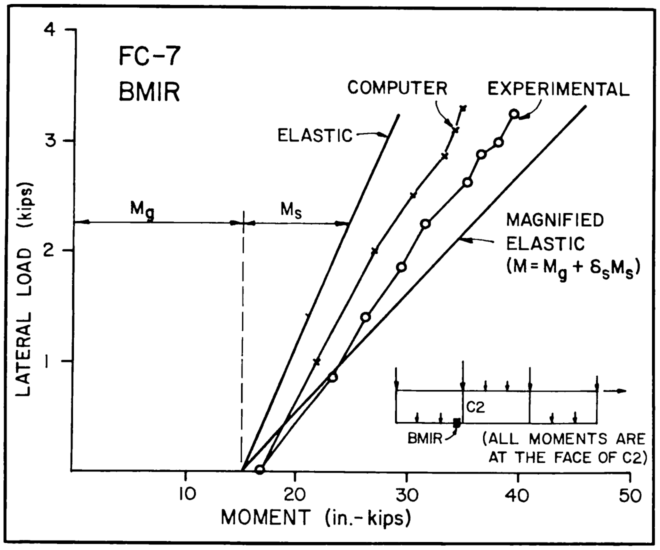


Fig. 6 — FC7 lateral load versus moment at BM1R.

ultimate load of 188 percent. Beam reinforcement thus has a very significant role in frame behavior.

ACI 318-77, Section 10.11.6.4 requires that:

"In frames not braced against sidesway, flexural members shall be designed for the total magnified end moments of the compression members at the joint."

The importance of this requirement is clearly visible in Fig. 6 which compares experimentally measured beam moments to those predicted by a first order elastic analysis, the computer model, and the magnified elastic moments. Clearly, the use of unmagnified beam moments would result in very unconservative design values while the moment magnification procedure based on an average story magnification factor gives very realistic predictions of the actual moments. This figure also confirms the suggested procedure of distributing the magnified column moments into the beams in proportion to their stiffness and the application of moment magnification in unbraced frames which will be discussed in the next section.

Application of the moment magnifier to unbraced frames

Section 10.11 of ACI 318-77 contains provisions for an approximate computation of the additional column secondary or $P\Delta$ moments due to slenderness effects. A distinction is made between the magnifier (δ) computations for braced and unbraced frames but no distinction is made regarding the moment to be magnified. The present Code format strongly implies an extremely conservative procedure for calculating the moments for proportioning the column.

Design of compression members uses the factored axial load P_u from a conventional frame analysis and a magnified factored moment M_c defined by

$$M_c = \delta M_2 \quad (4)$$

M_2 is the larger factored end moment on the compression member as calculated by a conventional elastic frame analysis. This definition strongly implies that in unbraced frames M_c is the sum of the moments due to gravity loading (which generally would not produce major sidesway) and the moments due to lateral loading (which would produce sidesway).

Code provisions do provide for different methods of calculating δ for the braced and unbraced cases. For the braced frame the magnifier δ uses effective length factors of 1.0 or less, while for the unbraced case they are greater than 1.0. C_m factors for braced frames can be from 0.4 to 1.0, while for unbraced frames 1.0 is used. Thus, critical loads P_c are higher and magnification factors are smaller for the braced case. For unbraced frames, Code Section 10.11.6 requires that two values of δ be computed and the larger value used. To check the effects of story stability, the unbraced value of δ is computed as an averaged value for the entire story based on use of $\Sigma P / \Sigma P_c$. This reflects the interaction of all columns in the story on the $P\Delta$ effects, since the lateral deflection of all columns in the story must be equal in the absence of twist. This assumption is quite valid, as will be discussed in a later section. In addition, since it is possible that a particularly slender individual column in an unbraced frame could have substantial midheight deflections even if adequately braced against lateral end deflections by the other columns in the story, the Code requires that each individual column be also checked using the braced frame magnifier value. The specific Code wording implies that one calculates the two δ values, selects the larger, and applies it to the value of M_2.

A fundamental problem is that the Code uses a single symbol δ, without subscript, for two very different cases, the braced or nonsway magnifier (δ_b) and the unbraced or sway magnifier (δ_s). An additional problem is that the Code uses a single symbol M_2 for the moment to be magnified and defines it in such a way that it strongly implies M_2 is the sum of factored nonsway moments and factored sway moments.

Basic stability theory indicates that moments which produce sway should be magnified by a sway magnifier (δ_s) and moments which do not produce sway should be magnified by a braced magnifier (δ_b). In addition, it must be realized that the maximum magnified moments produced by δ_s occur at column ends and the maximum magnified moments produced by δ_b occur at different locations, i.e., somewhere along the column length but not necessarily at the column end.

Measurements made in the frame tests (see Fig. 7) clearly support the differentiation between sway or nonsway moments in computation of maximum design moment for the gravity plus lateral load case. Results indicate that

$$M_c = M_g + \delta_s M_s \quad (5)$$

should be used rather than the usual interpretation

$$M_c = \delta M_2 = \delta_s (M_g + M_s) \quad (6)$$

Note that using Eq. (6) would imply that the gravity moments should be magnified even if no lateral load is applied. In Fig. 7, the zero lateral load values show this to be clearly unnecessary. Results show Eq. (6) to be extremely conservative when compared to Eq. (5). All supportive evidence in the frame tests and the computer analyses indicated that sway magnification of the elastic gravity movement by the sway magnifier is unwarranted. A number of frames could theoretically have carried no lateral load if such a procedure was used.

If the gravity moments are significantly larger than the lateral load moments and δ_b is large, the maximum moment for the gravity plus lateral load case can theoretically occur at some midheight region of the column. The specific location is not known and a conservative approximation can be made by recognizing that, while $\delta_b M_g$ and $\delta_s M_s$ do not occur at the same location, the actual moment cannot exceed their sum. Hence, a conservative approximation of the design moment would be

$$M_c = \delta_b M_g + \delta_s M_s \quad (7)$$

Note from Fig. 7 that Eq. (7) gives slightly improved agreement when $H = 0$ and is slightly more conservative when higher levels of H were imposed. It has two distinct advantages over the present procedure. It will almost always reduce the design moments from the extremely conservative levels of current interpretations and it more clearly indicates the application of the two magnification factors now required to be computed for unbraced frames. It does not introduce any additional computation above that now required.

This change in procedure can be effected by the following suggested Building Code changes:

(1) Revise Section 10.11.5.1 to read as follows:

10.11.5.1 — Compression members shall be designed using the factored axial load P_u from a conventional frame analysis and a magnified moment M_c defined by

$$M_c = \delta_b M_{2_{ns}} + \delta_s M_{2_s} \quad \text{(new 10-6)}$$

where

$$\delta_b = \frac{C_m}{1 - \dfrac{P_u}{\phi P_c}} \geqslant 1.0 \quad \text{(new 10-7a)}$$

$$\delta_s = \frac{1}{1 - \dfrac{\Sigma P_u}{\phi \Sigma P_c}} \geqslant 1.0 \quad \text{(new 10-7b)}$$

and

$$P_c = \frac{\pi^2 EI}{(kl_u)^2} \quad (10\text{-}8)$$

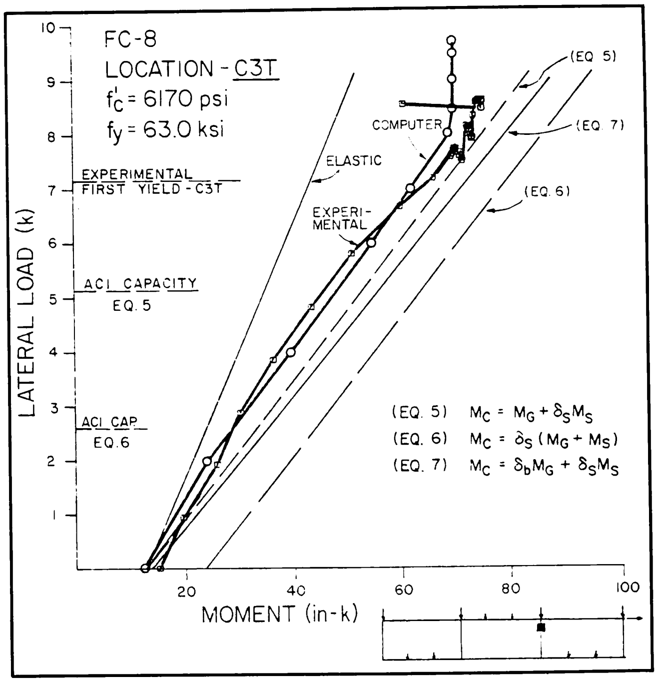


Fig. 7 — FC8 lateral load versus moment at C3T.

ΣP_u and ΣP_c are the summations for all columns in a story. For frames braced against sidesway, δ_s shall be taken as zero. In calculation of δ_b, k shall be computed according to Section 10.11.2.1, and in calculation of δ_s, k shall be computed according to Section 10.11.2.2.

(2) Eliminate present Sections 10.11.6.1 through 10.11.6.3. Renumber Section 10.11.6.4 as follows:

Section 10.11.6 — Moment magnification for flexural members

In frames not braced against sidesway, flexural members shall be designed for the total magnified end moments of the compression members at the joint.

(3) Add to Section 10.0 the following definitions

δ_b = magnification factor for braced frames and for magnification of moments which do not produce sway in unbraced frames.

δ_s = magnification factor for sway producing moments in unbraced frames.

$M_{2_{ns}}$ = value of larger factored end moment on compression member due to loads which result in no sidesway, calculated by conventional elastic frame analysis.

M_{2_s} = value of larger factored end moment on unbraced compression member due to loads which result in sidesway, calculated by conventional elastic frame analysis.

The present definition of δ in Section 10.0 would be eliminated.

Post-yielding lateral instability

The development of each yielded location affects the restraint provided to, and, consequently, the critical load of two or three columns in an unbraced story. The number of yielded regions sufficient to cause unbraced frame instability can be identified by determining those necessary to reduce the sum of the critical column loads (ΣP_c) below the total load applied to a story (ΣP). This technique is referred to as the incremental stability analysis using the $\Sigma P / \Sigma P_c$ ratio. In determining the critical load of each column, the effective length, k, can be determined from the unbraced frame alignment chart in the ACI Building Code Commentary if, in determining relative stiffnesses, ψ, the effect of reinforcement yielding is taken into account, as illustrated in Part 3.[19] The assumption of no restraint after yielding (perfectly plastic) provided accurate results for the frame specimens. In all specimens which became unstable, the $\Sigma P / \Sigma P_c$ technique for predicting the number of yielded locations required to produce instability agreed very closely with the experimental and computer predicted results. The relatively small number of computations required for this procedure are simple and do not require the aid of a computer to be feasible. However, to apply the procedure the yielded locations in the frame must be known; hence, some kind of second order analysis must be performed to locate the yielded locations. Although the procedure requires information on sequence of hinge development which is difficult to obtain, because of its computational simplicity it should be further investigated to determine if it can be beneficially used in design office practice for cases where an accurate estimate of frame capacity is required.

Utilization of redistribution potential in multipanel frames

The results of this investigation clearly indicate that the heavily loaded lightly tied column in an unbraced multipanel frame has considerable ductility and does not completely fail when the first column reaches its nominal axial load-moment capacity as indicated by the interaction diagram. However, the study also indicated that post-yield lateral instability and increased but still limited column deformation capacity prevent attainment of the full panel collapse mechanism indicated by simple plastic theory.

As discussed for the effect of combining gravity beam loads and lateral loads, the Dead plus Wind load combination most likely to govern in unbraced frames would produce column moment diagrams which often correspond more closely to test frames without beam loads. These frames had low ratios of ultimate load to first hinging.[1,14] In addition, frames with high strength concrete and strong beams had similar low ratios even with beam loads. Hence, reliance on post-yielding redistribution does not seem to have a substantial benefit and it is recommended that column design procedures remain based on considering the capacity of the frame as that load which produces axial load and moment combinations equal to the nominal capacity of the most highly stressed column.

In making this recommendation the authors are not dismissing redistribution and bracing of weak columns by stronger columns. Rather, it is apparent from the results of the frame tests that the use of the story moment magnifier ($\Sigma P / \Sigma P_c$) adequately reflects this bracing effect and provides a conservative but surprisingly accurate measure of the column interdependence. Should the magnifier for unbraced frames have been based on calculated individual column ratios of P/P_c, then substantial redistribution would have to be assumed to predict ultimate loads. The present $\Sigma P / \Sigma P_c$ moment magnification story criterion, when used with techniques which distinguish sway moments from nonsway moments as suggested, seems like a satisfactory tool for providing reasonable redistribution effects in multipanel frames.

Conclusions

This paper draws on the results of physical tests and analytical modeling of heavily loaded, lightly tied reinforced concrete columns to suggest improvements in design techniques. Specific changes to current Building Code provisions are presented which would allow an increase in the assumed ultimate compressive strain for columns when beneficial for use of higher strength steels. A major redefinition of the application of the moment magnification procedure for approximating column slenderness effects is presented which can substantially reduce design moments in unbraced frames. An improved model for analyzing complex frames is suggested along with a procedure for making incremental post-yield stability analyses of concrete frames. The effect of the suggestions would be to provide designers the opportunity to utilize proven reserve strength with modest additional computational effort and to clarify the proper procedures for design of unbraced frames under combined vertical and lateral loads.

Acknowledgments

The authors would like to acknowledge the important contributions to the study made by several groups and individuals. Financial support was provided by the Reinforced Concrete Research Council, the Army Office of Chief of Engineers, and the National Science Foundation under Grant GK 31069. Throughout the study, valuable suggestions and guidance were provided by the RCRC Special Task Committee 31 chaired by E. O. Pfrang. Valuable ideas and assistance were given by many faculty, graduate students, and technicians at The University of Texas Phil M. Ferguson Structural Engineering Laboratory, but the authors would especially like to mention the assistance of P. M. Ferguson, R. W. Furlong, and J. A. Yura in the study.

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Conversion factors — inch-pound to SI

inch = 25.4 mm
foot = 0.3048 m
pound per square inch = 6.895 kPa
kip = 444.8 N
kip per square inch = 6.895 MPa
inch kip = 0.1130 N·m

Received Sept. 19, 1980 and reviewed under Institute publication policies.

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