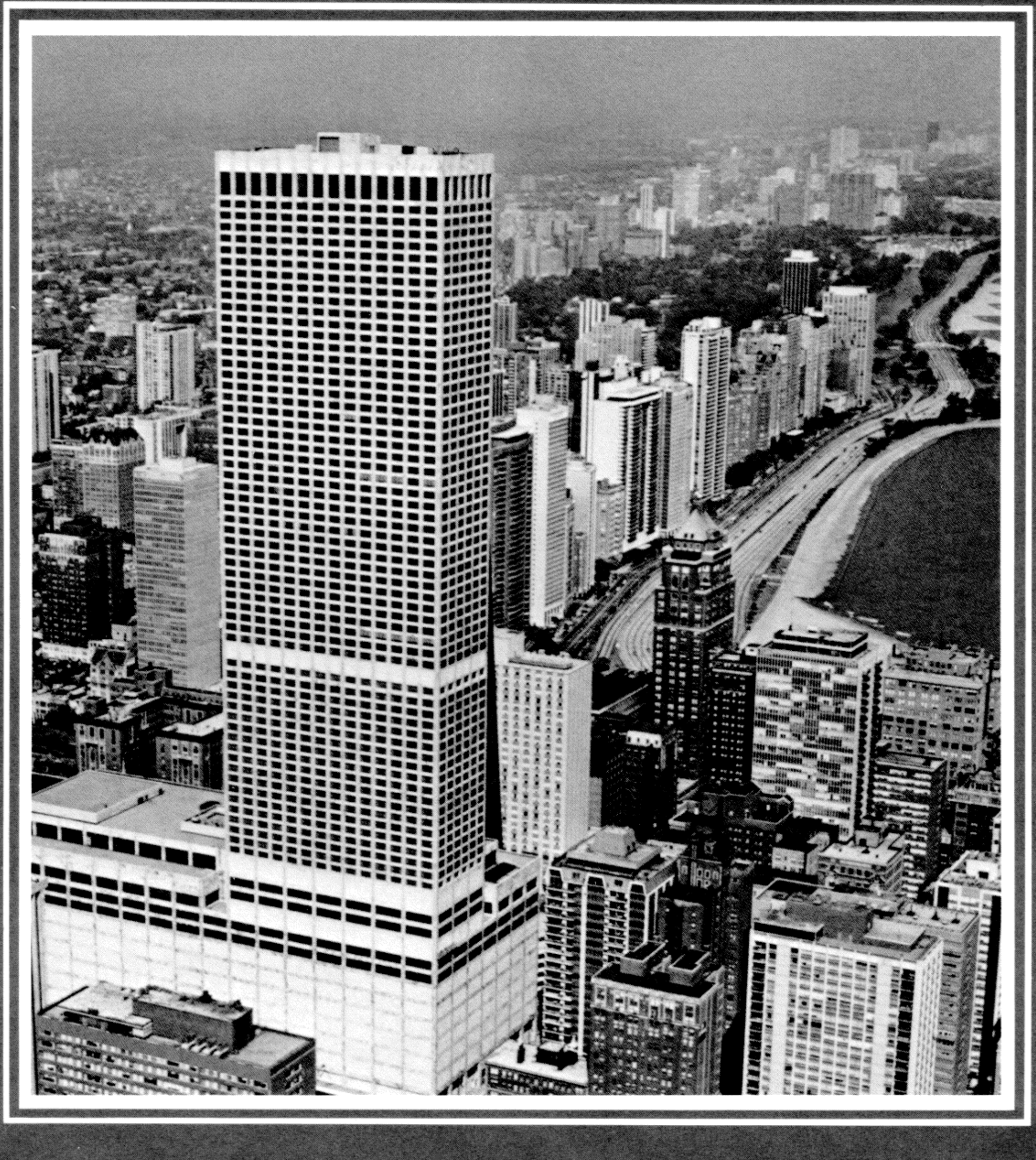
Column Shortening in Tall Structures—Prediction and Compensation

by Mark Fintel, S. K. Ghosh, and Hal Iyengar



Library of Congress catalog card number: 85-63541
ISBN 0-89312-083-9

This publication is based on the facts, tests, and authorities stated herein. It is intended for the use of professional personnel competent to evaluate the significance and limitations of the reported findings and who will accept responsibility for the application of the material it contains. The Portland Cement Association disclaims any and all responsibility for application of the stated principles or for the accuracy of any of the sources other than work performed or information developed by the Association.

Contents

About the authors:

Mark Fintel was formerly director, Advanced Engineering Services, Portland Cement Association, Skokie, Illinois.

S. K. Ghosh was formerly principal structural engineer, Advanced Engineering Services, Portland Cement Association, Skokie, Illinois. He is currently Associate Professor of Civil Engineering, University of Illinois at Chicago.

Hal Iyengar is a general partner, Skidmore, Owings and Merrill, Chicago, Illinois.

Column Shortening in Tall Structures—
Prediction and Compensation

INTRODUCTION

The effects of column shortening, both elastic and inelastic, take on added significance and need special consideration in design and construction with increased height of structures. Differential column shortenings are magnified by the quest for optimum economy through use of high-strength materials and, in some instances, the use of composite structural systems. These, in turn, change the initial positions of the slabs. As a consequence the partitions, mechanical equipment, cladding, architectural finishes, and built-in furnishings are also affected.

The strains in the columns of low as well as ultra-high-rise buildings are similar if the stress levels are similar; however, the overall column shortening is cumulative and depends upon the height of the structure. For example, in an 80-story steel structure, the total elastic shortening of the columns may be as high as 7 to 10 in. (180 to 255 mm) due to the high design stress levels of modern high-strength steels. By comparison, in an 80-story concrete building, the elastic shortening of columns would amount to only about 2.5 in. (65 mm); however, the total length change of the reinforced concrete columns may be 7 to 9 in. (180 to 230 mm) due to shrinkage and creep.

The potentially harmful effects of these large shortenings can be contained by providing details at each level that will allow the vertical structural members to deform without stressing the cladding, partitions, finishes, and so on. However, such details cannot eliminate the structural consequences of the relative shortening between adjacent vertical members; this shortening distorts the slab supported by the vertical members from its intended position.

Differential elastic shortenings of vertical members result from differing stress levels. Differential creep strains in concrete vertical members result from differing stress levels, loading histories, ratios of reinforcement, volume-to-surface ratios, and environmental conditions. Differential shrinkage strains are independent of stress levels; they depend only upon ratios of reinforcement, volume-to-surface ratios, and environmental conditions.

SCOPE

This report is concerned with the prediction of strains in columns from the moment the columns become part of a structure. It also attempts to predict differential column movements in reinforced concrete or composite structures of significant height.

In reinforced concrete structures (Fig. 1a), differential movements between shearwalls and columns and between neighboring columns due to elastic, shrinkage, and creep shortening are of concern. In composite structures (Fig. 1b, c) that combine either an interior core or peripheral beam-column frames made of concrete with the rest of the structure framed in structural steel, the shortening of the steel columns relative to the reinforced concrete core or the reinforced concrete peripheral system is of concern.

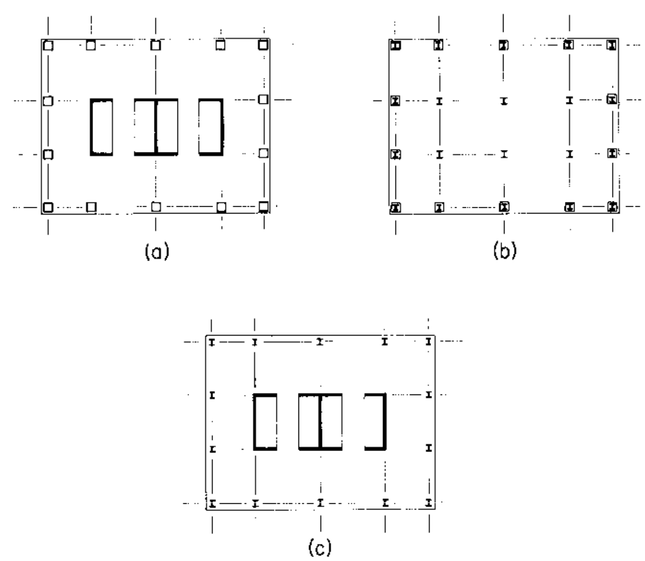


Fig. 1. Structural systems for tall buildings: (*a*) concrete shearwall-frame interactive system, (*b*) interior steel framing with peripheral concrete framing, (*c*) interior concrete shearwalls with exterior steel framing.

While the reinforced concrete vertical members are subject to elastic, creep, and shrinkage shortening, the steel columns are subject to elastic shortening only.

EFFECTS OF COLUMN SHORTENING

The shortening of columns within a single story affects the partitions, cladding, finishes, piping, and so on, since these nonstructural elements are not intended to carry vertical loads and are therefore not subject to shortening. On the contrary, partitions and cladding may elongate from moisture absorption, pipes from high temperature of liquid contents, cladding from solar radiation, and so on. Details for attaching these elements to the structure must be planned so that their movement relative to the structure will not cause distress.

The cumulative differential shortening of columns causes the slabs to tilt with resulting rotation of partitions, as shown in Fig. 2. Modern dry-wall partitions can be detailed with sufficient flexibility along their peripheries and at the vertical butt joints to permit their distortion without visible distress (Fig. 3). Plaster and masonry partitions, which were quite common in the past, are characteristically rigid and brittle and have limited ability to undergo distortions without cracking. When a slab carrying such partitions is subject to differential support displacements, the partitions must be detailed around their peripheries to allow movement relative to the frame.

MOVEMENTS RELATED TO CONSTRUCTION SEQUENCE

Considering a slab at level N of a multistory building (Fig. 4), each of its supports consists of N single-story segments.

During the construction process, each of the story-high support segments undergoes elastic shortening due to all the loads applied after placing or installation of the segment. In addition, concrete and composite columns begin to shrink from moisture loss and to creep as a result of the applied compressive forces.

The major objective of the procedure reported in this publication is to assure a proper final position for each slab within the building. The first step is to determine the elevations of the tops of supports at the time of slab installation to assure that proper compensation can be made for installation of the slab at a predetermined position. Next to be considered are the changes of support elevations subsequent to slab installation due to elastic stresses caused by additional loads, creep caused by additional and existing loads, and shrinkage.

The time of slab installation at its initial position becomes the dividing line between the following two types of support shortening:

1. Those taking place up to the time the slab is installed (preinstallation shortenings). Either analytical estimates or on-the-spot measurements of these shortenings are needed to adjust the support elevations so that the slab is installed at a predetermined position.
2. Those taking place after the slab is installed (postinstallation shortenings). Analytical estimates of these shortenings are needed for corrective measures to assure that the slab will be in the desired, predetermined position after all loads have been applied and after all shrinkage and creep have taken place.

In cast-in-place reinforced concrete structures (Fig. 1a), the amount of support shortening before slab installation is of no importance, since the forms are usually leveled at the time the concrete for each floor slab is placed. However, information is needed on how much the slab will change its position after placing from subsequent loads and subsequent volume changes. This information can then be used to tilt the formwork in the opposite direction so that in the future the slab will end up in the

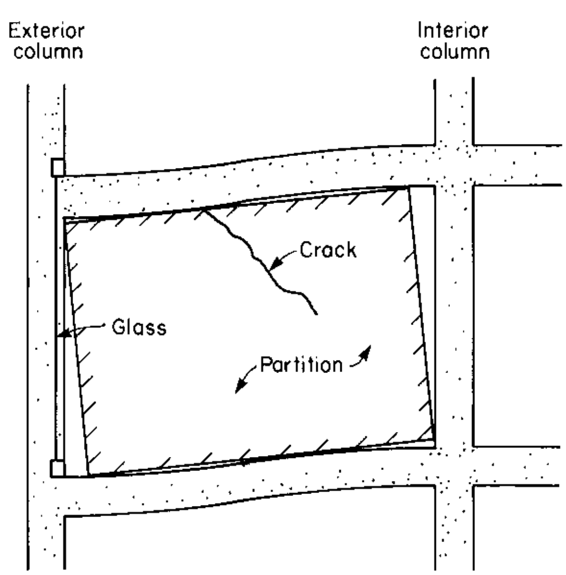


Fig. 2. Effect of tilted slabs.

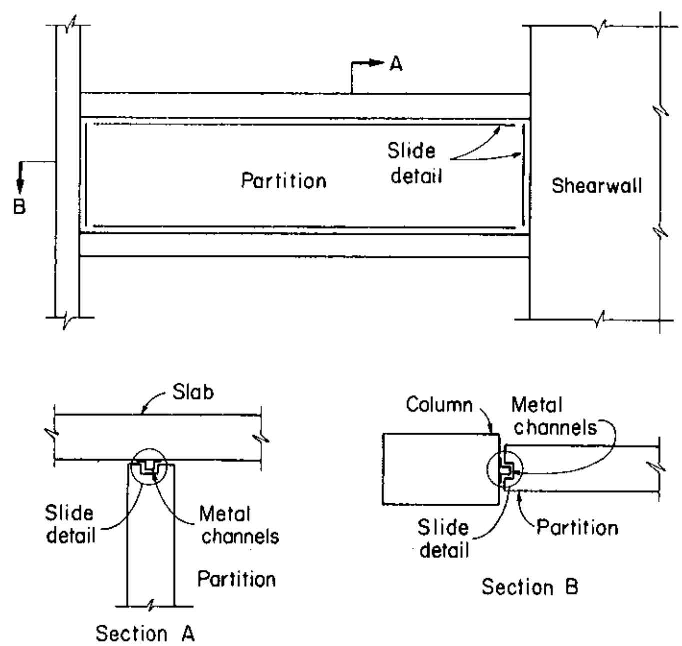


Fig. 3. Isolation of partitions from structural framing.

desired position. Depending upon circumstances, it may be decided that most of the time-dependent shortening (shrinkage and creep) be compensated for at the time of construction. In that case, at initial occupancy, the slab may have a reverse tilt that will gradually disappear. Or, it may be decided, for example, to compensate for only the shortening that is expected to take place within two years after construction. Thus, in two years the slab will be level and from that time on only the remaining shrinkage and creep will cause the slab to tilt.

In structures in which columns are fabricated to exact lengths (steel columns, Fig. 1c, or light steel erection columns that are later embedded in concrete, Fig. 1b), the preinstallation support shortening is of consequence since the attachments to receive the slabs are part of the shop-fabricated columns. To assure the predetermined initial slab elevation, the preinstallation length changes of these columns need to be known and compensated for. The postinstallation support shortenings (elastic and inelastic) also must be considered.

RELATIVE MOVEMENTS BETWEEN ADJACENT ELEMENTS IN VARIOUS STRUCTURAL SYSTEMS

Reinforced Concrete

As mentioned, only the differential movements that will occur between the supports of a slab after it has been placed are of concern, since they will change the slab's position.

The total shortening is rarely of practical interest. Even in cases of elevator rails attached to the shaft structure or vertical pipes, only the shortening that will occur after their attachment is of concern.

Differential movements in concrete buildings (Fig. 1a) are primarily those between isolated columns and shear-walls due to different ratios of vertical reinforcement, different stress levels, and different volume-to-surface ratios. Differential movements between neighboring columns may also occur. Differential shortenings are of particular concern where the distance between differentially shortening elements is small, causing significant tilt of the beam or the slab. Strain differentials of up to 200 μ in. per inch have been observed in cases where all the contributing factors increase the differential strains in the same direction. Such a strain differential amounts to a shortening of 0.0288 in. (0.73 mm) in a 12-ft story, and in a 50-story building would amount to 1.44 in. (36.5 mm) of total differential shortening. Obviously, such differential length changes need to be compensated for—at least the part that will occur within a certain initial period (say, two years, during which the majority of shrinkage and creep will have taken place), thus leaving only a minor portion of the total shrinkage and creep movement to cause slab or beam tilt.

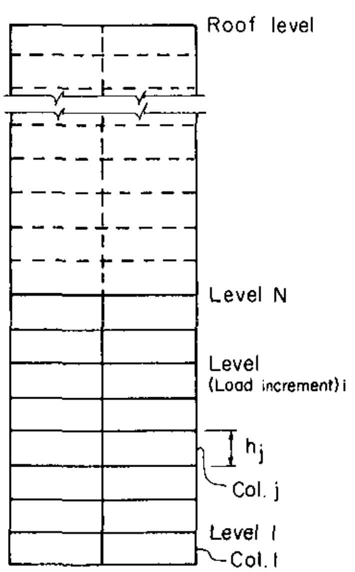


Fig. 4. Schematic section of a multistory building.

Composite Structures

Two types of composite (steel and concrete) structural systems are currently used; both utilize structural slabs consisting of steel beams supporting corrugated decks topped with concrete. One type has a reinforced concrete core with exterior structural steel columns (Fig. 1c); the other consists of a peripheral reinforced concrete column-beam system with structural steel interior columns (Fig. 1b). A variant of the latter system that facilitates a more efficient construction procedure is a basic steel structure with light peripheral columns (called erection columns) that are later encased in concrete. A third type of composite structural system consisting of a heavy structural steel core and exterior composite columns (steel erection columns encased in reinforced concrete) has also been used in recent times. For the purposes of this report, there is no essential difference between this system and the one shown in Fig. 1(b). Thus, the third system is not considered separately.

In composite structures with erection columns, the progress of peripheral concrete column casting follows behind the top lift of steel erection by a certain number of stories, say 9. This 9-story steel skeleton (running ahead of the peripheral concrete columns) consists of (a) 3 stories of steel columns and girders, (b) 3 stories of steel columns and girders plus steel deck, and (c) 3 stories of steel columns and girders plus steel deck plus concrete topping. As a new lift of steel columns and girders is added at the top, simultaneously 3 stories below, a steel deck is added. 3 stories farther down, concrete topping is added to a steel deck; and 3 stories still farther down, a lift of concrete peripheral columns is cast. Thus, there is always a 9-story steel structure above the story on which concrete columns have been cast (except at the very top where concrete lifts gradually approach the roof level after steel lifts have been erected for the entire building).

Composite Structures with Erection Columns

The erection columns are designed to carry only 9 to 12 stories of construction loads consisting of 3 to 4 stories each of the stages (a), (b), and (c) described above. These columns are then embedded into the peripheral concrete columns and become part of their vertical reinforcement.

At the time of embedment, an erection column is stressed to about $0.6 \times f_y$ (where f_y is the yield strength of the column steel) and has experienced a corresponding elastic shortening. Thus, the story-high lifts of the erection columns should be made longer to compensate for the preembedment differential shortening between them and the interior columns that are subjected to significantly lower stress levels. The loads carried by erection columns before their embedment are carried directly in steel-to-steel bearing, starting from the foundation-level base plate. At the time of casting, each lift of a composite column is subjected to the column dead load and one added full-floor load consisting of a concrete topping 3 stories above, a steel deck 3 stories farther up, and a new lift of steel columns and girders 3 stories still farther up. During construction of the top 9 stories of the building, the additional loads imposed on new lifts of composite columns are progressively smaller than full-floor loads, as the roof is approached. For example, when a column just below the roof level is cast, the only additional load on it is that of the column itself.

With the progress of time after construction, the elastic and inelastic strains of a composite column are added to the initial strains that the erection column had prior to embedment. At some point, the embedded erection column may begin to yield; further loads will then be carried by the concrete and the vertical reinforcement not yet yielded. It should be noted that such load shifting between reinforcing steel and concrete does not affect the overall load-carrying capacity of a column; only the proportion of load carried by the steel and the concrete changes.

Composite Structures with Concrete Cores

The composite-type structure consisting of a reinforced concrete central core and peripheral steel columns might have the core slipformed ahead of the steel structure; or the core might be built using jump forms proceeding simultaneously with the steel structure; or the core might follow behind steel erection by 9 to 12 stories, as described in the previous section. From the point of view of relative shortening between the core and the steel columns, all three cases are quite different. In the case of the slipformed core, at the time a steel story is erected some of the creep and shrinkage of the core will already have taken place. Thus, the post-slab-installation portion of shrinkage and creep is smaller. In cases where the core construction proceeds simultaneously with or lags behind the steel structure, all of the shrinkage and creep of the core contributes to the differential shortening relative to the steel columns.

ESTIMATES OF ELASTIC, SHRINKAGE, AND CREEP STRAINS IN COLUMNS

To carry out an analysis for elastic, shrinkage, and creep strains in reinforced concrete columns, information is required on the modulus of elasticity and the shrinkage and creep characteristics of the concrete mixes to be used in the structure being considered. Discussion of these properties follows.

Modulus of Elasticity

The effect on modulus of elasticity of the concrete's age at time of loading is taken into account implicitly when strength of the concrete at time of loading, rather than at 28 days, is inserted into the modulus of elasticity expression given in the ACI code[1]* (see Fig. 5):

$$E_{ct} = 33w^{1.5}\sqrt{f'_{ct}} \qquad (1)$$

where

E_{ct} = time-dependent modulus of elasticity of concrete in psi

w = unit weight of concrete in pcf

f'_{ct} = time-dependent concrete strength in psi

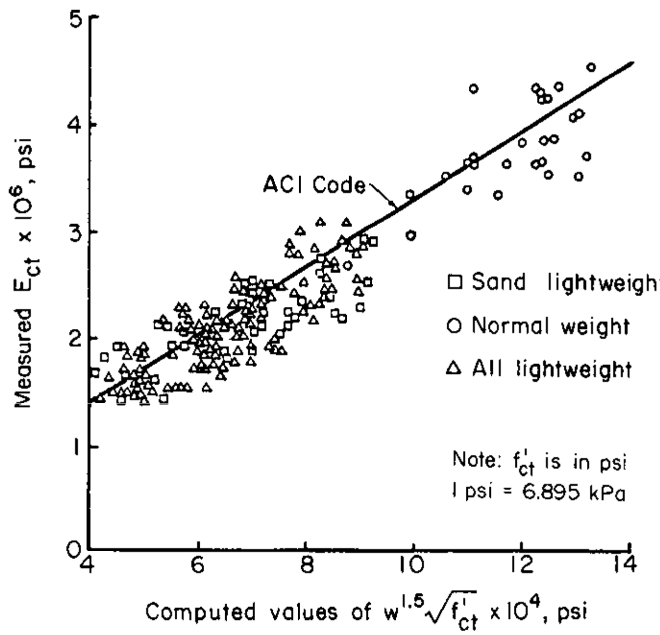


Fig. 5. Measured and computed moduli of elasticity of concrete.[2]

American Concrete Institute (ACI) Committee 209[2] has recommended the following expression for the time-dependent strength of moist-cured (as distinct from steam-cured) concrete using Type I cement:

$$f'_{ct} = \frac{t}{4.0 + 0.85t} f'_c \qquad (2)$$

*Superscript numbers in parentheses denote references at the end of this text.

where

t = time in days from placing to loading

f'_c = 28-day compressive strength of concrete

A more recent version of Fig. 5, applicable to concretes with compression strengths of up to 12 ksi (the ACI Code formula for modulus of elasticity was originally derived for concretes with compression strengths of 6 ksi or less), is now available in References 3 and 4.

The validity of equation (2) at very early concrete ages may be open to question. A fair amount of detailed information on the compressive strength of concrete at early ages is now available,[5, 6] although such information is probably still far from sufficient.

Equation (1) for normal-weight concrete when converted to International System of Units (SI) and rounded off becomes

$$E_{ct} = 5000\sqrt{f'_{ct}} \qquad (1a)$$

where E_{ct} and f'_{ct} are in newtons per square millimeter (the nearly exact SI equivalent of $E_{ct} = 57{,}000\sqrt{f'_{ct}}$ is $E_{ct} = 4730\sqrt{f'_{ct}}$). Fig. 6. shows a comparison of equation (1a) with the experimental information currently available on the modulus of elasticity of concrete at early ages.[6] Concrete strengths of 1 N/mm^2 (145 psi) and less are encountered only during the first few hours after placing concrete and are of little interest. In the range of practical application, the ACI equation relating modulus of elasticity and strength agrees quite favorably with the trend of experimental results.

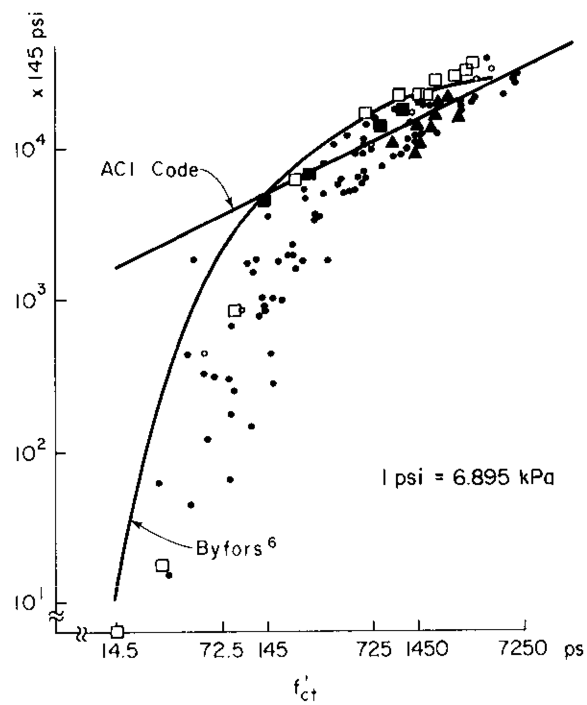


Fig. 6. Modulus of elasticity of concrete at early ages.[5]

Shrinkage of Unreinforced (Plain) Concrete

Shrinkage of concrete is caused by evaporation of moisture from the surface. The rate of shrinkage is high at early ages and decreases with an increase in age until the curve becomes asymptotic to the final value of shrinkage. The rate and amount of evaporation and consequently of shrinkage depend greatly upon relative humidity of the environment, size of the member, and mix proportions of the concrete. In a dry atmosphere, moderate-size members (24-in. or 600-mm diameter) may undergo up to half of their ultimate shrinkage within two to four months, while identical members kept in water may exhibit growth instead of shrinkage. In moderate-size members, the inside relative humidity has been measured at 80% after four years of storage in a laboratory at 50% relative humidity.

Basic Value of Shrinkage

Let $\epsilon_{s\infty}$ denote the ultimate shrinkage of 6-in.-diameter (150-mm) standard cylinders (volume-to-surface or $v{:}s$ ratio = 1.5 in. or 38 mm) moist-cured for 7 days and then exposed to 40% ambient relative humidity. If concrete has been cured for less than 7 days, multiply $\epsilon_{s\infty}$ by a factor linearly varying from 1.2 for 1 day of curing to 1.0 for 7 days of curing.[2]

Attempts have been made in the past to correlate $\epsilon_{s\infty}$ with parameters such as concrete strength. In view of experimental data now available,[7] it appears that no such correlation may in fact exist. The only possible correlation is probably that between $\epsilon_{s\infty}$ and the water content of a concrete mix (Fig. 7). In the absence of specific shrinkage data for concretes to be used in a particular structure, the value of $\epsilon_{s\infty}$ may be taken as between 500×10^{-6} in. per inch (low value) and 800×10^{-6} in. per inch (high value). The latter value has been recommended by ACI Committee 209.[2]

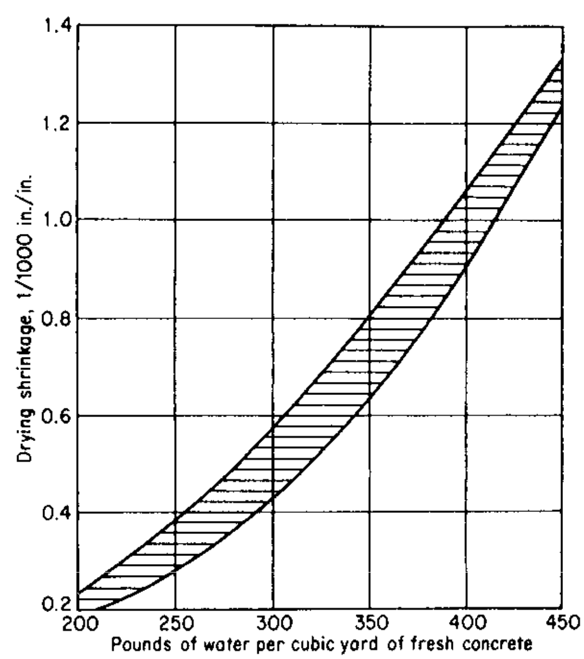


Fig. 7. Effect of water content on drying shrinkage.[8]

Effect of Member Size

Since evaporation occurs from the surface of members, the volume-to-surface ratio of a member has a pronounced effect on the amount of its shrinkage. The amount of shrinkage decreases as the size of specimen increases.

For shrinkage of members having volume-to-surface ratios different from 1.5, $\epsilon_{s\infty}$ must be multiplied by the following factor:

$$SH_{v:s} = \frac{0.037(v:s) + 0.944}{0.177(v:s) + 0.734} \qquad (3)$$

where $v:s$ is the volume-to-surface ratio in inches.

The relationship between the magnitude of shrinkage and the volume-to-surface ratio has been plotted in Fig. 8 based on laboratory data.[9] Also plotted in Fig. 8 are curves depicting European recommendations.[10, 11] It is interesting to note that the European curves and the one for Elgin gravel aggregate concrete[9] are not very far apart. The European recommendations may have been based on the Hansen-Mattock data, although this is not known for a certainty. Equation (3) is based on the curves presented in Fig. 8.

Much of the shrinkage data available in the literature was obtained from tests on prisms of a 3x3-in. (75x75-mm) section ($v:s$ = 0.75 in. or 19 mm). According to equation (3), the size coefficient for prisms of that size is 1.12. Thus, shrinkage measured on a prism of a 3x3-in. (75x75-mm) section must be divided by 1.12, before the size coefficient given by equation (3) is applied to it. It should be cautioned, however, that as the specimen size becomes smaller, the extrapolation to full-size members becomes less accurate.

Effect of Relative Humidity

The rate and amount of shrinkage greatly depend upon the relative humidity of the environment. If ambient relative humidity is substantially greater than 40%, $\epsilon_{s\infty}$ must be multiplied by

$$\begin{aligned} SH_H &= 1.40 - 0.010H \text{ for } 40 \leqslant H \leqslant 80 \\ &= 3.00 - 0.030H \text{ for } 80 \leqslant H \leqslant 100 \end{aligned} \qquad (4)$$

where H is the relative humidity in percent. Average annual values of H should probably be used. Maps giving average annual relative humidities for locations around the United States are available (Fig. 9). However, if locally measured humidity data are available, they are likely to be more accurate than the information in Fig. 9 and should be used in conjunction with equation (4).

Equation (4) is based on ACI Committee 209 recommendations.[2] A comparison with European recommendations[10] is shown in Fig. 10.

If shrinkage specimens are stored under jobsite conditions rather than under standard laboratory conditions, the correction for humidity, as given by equation (4), should be eliminated.

Progress of Shrinkage with Time

Hansen and Mattock[9] established that the size of a member influences not only the final value of shrinkage but also the rate of shrinkage, which appears to be only logical. Their expression giving the progress of shrinkage with time is

$$SH_t = \frac{\epsilon_{st}}{\epsilon_{s\infty}} = \frac{t}{26.0e^{0.36(v:s)} + t} \qquad (5)$$

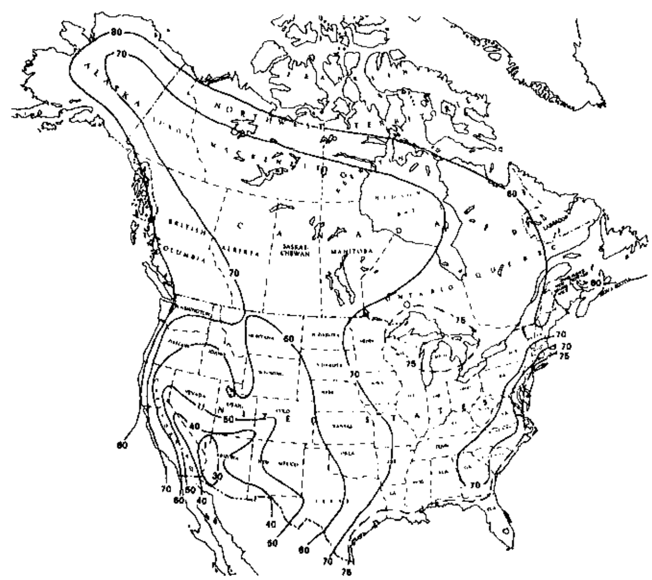


Fig. 9. Annual average ambient relative humidity in the United States.[12]

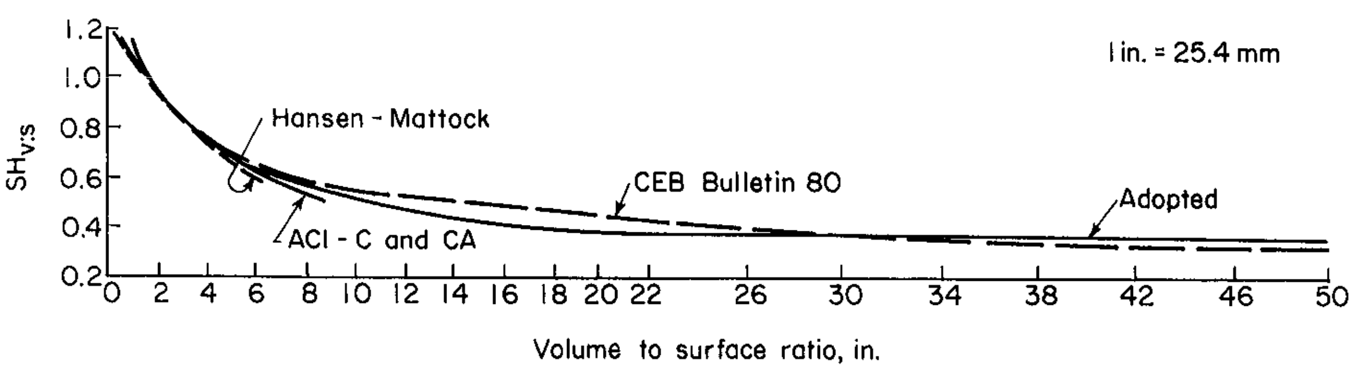


Fig. 8. Effect of member size on drying shrinkage.

where ϵ_{st} and $\epsilon_{s\infty}$ are shrinkage strains up to time t and time infinity, respectively; and t is measured from the end of moist curing.

Equation (5) is compared in Fig. 11 with the progress of shrinkage curve from Reference 10. Also shown in Fig. 11 is a comparison of equation (5) with the progress of shrinkage relationship recommended by ACI Committee 209.[2] It should be noted that both the ACI-Cement and Concrete Association (C&CA)[10] and the ACI Committee 209[2] relationships are independent of the volume-to-surface ratio.

Creep of Unreinforced (Plain) Concrete

Creep is a time-dependent increment of the strain of a stressed element that continues for many years. The basic phenomenon of creep is not yet *conclusively* explained. During the initial period following the loading of a structural member, the rate of creep is significant. The rate diminishes as time progresses until it eventually approaches zero.

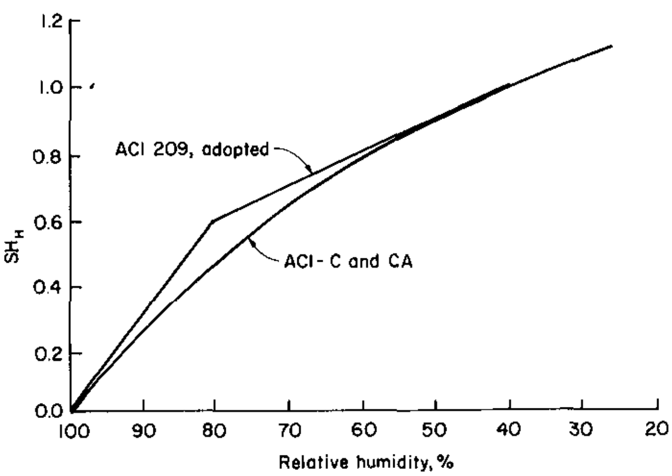


Fig. 10. Effect of relative humidity on drying shrinkage.

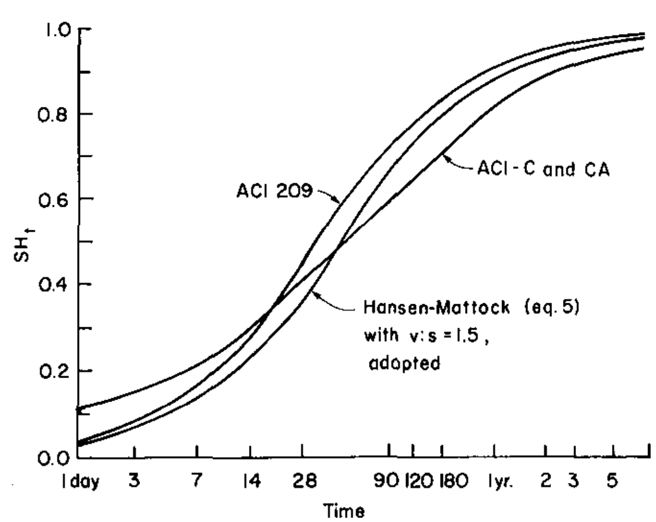


Fig. 11. Progress of shrinkage with time.

Creep consists of two components:

1. Basic (or true) creep occurring under conditions of hygral equilibrium, which means that no moisture movement occurs to or from the ambient medium. In the laboratory, basic creep can be reproduced by sealing a specimen in copper foil or by keeping it in a fog room.
2. Drying creep resulting from an exchange of moisture between the stressed member and its environment. Drying creep has its effect only during the initial period under load.

Creep of concrete is very nearly a linear function of stress up to stresses that are about 40% of the ultimate strength. This includes all practical ranges of stresses in columns and walls. Beyond this level, creep becomes a nonlinear function of stress.

For structural engineering practice it is convenient to consider specific creep, which is defined as the ultimate creep strain per unit of sustained stress.

Value of Specific Creep

Specific creep values can be obtained by extrapolation of results from a number of laboratory tests performed on samples prepared in advance from the actual mix to be used in a structure. It is obvious that sufficient time for such tests must be allowed prior to the start of construction, since reliability of the prediction improves with the length of time over which creep is actually measured.

A way of predicting basic specific creep (excluding drying creep), without testing, from the modulus of elasticity of concrete at the time of loading was proposed by Hickey[13] on the basis of long-term creep studies at the Bureau of Reclamation in Denver. Hickey's proposal was adopted by Fintel and Khan.[14, 15] A simpler suggestion is made in this study.

Let $\epsilon_{c\infty}$ denote the specific creep (basic plus drying) of 6-in.-diameter (150-mm) standard cylinders ($v:s = 1.5$ in. or 38 mm) exposed to 40% relative humidity following about 7 days of moist-curing and loaded at the age of 28 days. In the absence of specific creep data for concretes to be used in a particular structure, the following likely values of $\epsilon_{c\infty}$ may be used:

$$\epsilon_{c\infty} = 3/f'_c \text{ (low value) to } 5/f'_c \text{ (high value)} \qquad (6)$$

where $\epsilon_{c\infty}$ is in inch per inch per kip per square inch if f'_c is in ksi; or in inch per inch per pound per square inch if f'_c is in psi. The lower end of the proposed range is in accord with specific creep values suggested by Neville.[16] The upper end agrees with laboratory data obtained by testing concretes used in Water Tower Place[7] in Chicago, Illinois.

Effect of Age of Concrete at Loading

For a given mix of concrete the amount of creep depends not only on the stress level, but also to a great extent on the age of the concrete at the time of loading. Fig. 12 shows the relationship between creep and age at loading, as developed by Comité Européen du Béton (CEB), using available information from many tests.[11] The coefficient

CR_{LA} relates the creep for any age at loading to the creep of a specimen loaded at the age of 28 days. The 28-day creep is used as a basis of comparison, the corresponding CR_{LA} being equal to 1.0.

Fig. 12 also depicts the following suggested relationship between creep and age at loading:

$$CR_{LA} = 2.3 t_{LA}^{-0.25} \tag{7}$$

where t_{LA} is the age of concrete at the time of loading, in days. The form of equation (7) is as suggested by ACI Committee 209.[2] Equation (7) gives better correlation with the CEB mean curve than the corresponding equation suggested by Committee 209. Fig. 12 also shows comparison with a few experimental results.[7,17] According to equation (7), the creep of concrete loaded at 7 days of age is 41% higher than that of concrete loaded at 28 days.

Effect of Member Size

Creep is less sensitive to member size than shrinkage, since only the drying-creep component of the total creep is affected by the size and shape of members, whereas basic creep is independent of size and shape.

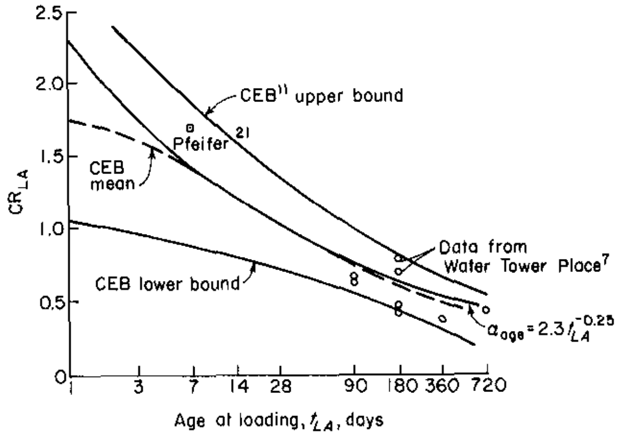


Fig. 12. Creep versus age of concrete at time of loading.

For members with volume-to-surface ratios different from 1.5 in. or 38 mm, $\epsilon_{c\infty}$ should be multiplied by

$$CR_{v:s} = \frac{0.044(v:s) + 0.934}{0.1(v:s) + 0.85} \tag{8}$$

where $v:s$ is the volume-to-surface ratio in inches.

The relationship between the magnitude of creep and the volume-to-surface ratio has been plotted in Fig. 13 based on laboratory data obtained by Hansen and Mattock.[9] Also plotted in Fig. 13 are curves based on European investigations.[10, 11] The European curves and the one for Elgin gravel aggregate concrete (the Hansen-Mattock curve) are not very far apart. Equation (8) is based on the curves presented in Fig. 13.

Much of the creep data available in the literature was obtained by testing 6-in.-diameter (150-mm) standard cylinders wrapped in foil. The wrapped specimens simulate very large columns. Equation (8) yields a value of $CR_{v:s}$ equal to 0.49 for $v:s = 100$. Thus, it is suggested that creep data obtained from sealed specimen tests should be multiplied by $2(1/0.49 \simeq 2)$ before the modification factor given by equation (8) is applied to such data.

Effect of Relative Humidity

For an ambient relative humidity greater than 40%, $\epsilon_{c\infty}$ should be multiplied by the following factor, as suggested by ACI Committee 209:[9]

$$CR_H = 1.40 - 0.01H \tag{9}$$

where H is the relative humidity in percent. Again, it is suggested that the average annual value of H should be used.

Progress of Creep with Time

The progress of creep relationship recommended by ACI Committee 209[2] is given by the following expression:

$$CR_t = \frac{\epsilon_{ct}}{\epsilon_{c\infty}} = \frac{t^{0.6}}{10 + t^{0.6}} \tag{10}$$

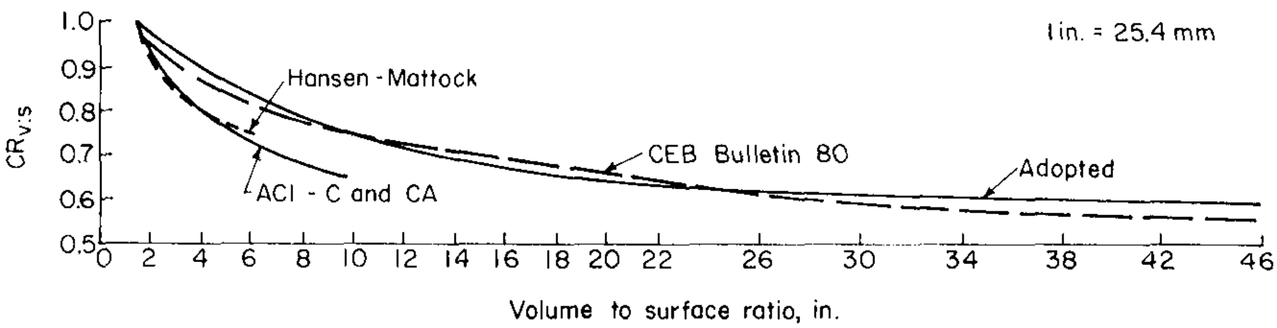


Fig. 13. Effect of member size on creep.

where ϵ_{ct} is creep strain per unit stress up to time t, and t is measured from the time of loading.

The above relationship is plotted in Fig. 14 where it is seen to compare well with the creep versus time curve suggested in European recommendations.[10] Equation (10) has been adopted in the present investigation.

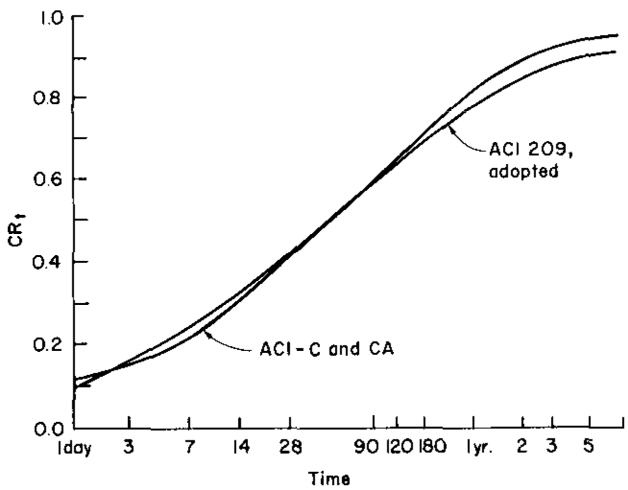


Fig. 14. Progress of creep with time.

Residual Shrinkage and Creep of Reinforced Concrete

In a reinforced concrete column, creep and shrinkage of the concrete are restrained by the reinforcement. Tests have shown that when reinforced concrete columns are subjected to sustained loads, there is a tendency for stress to be gradually transferred to the steel, with a simultaneous decrease in the concrete stress. Long-term tests by Troxell and others[18] showed that in columns with low percentages of reinforcement, the stress in the steel increased until yielding, while in highly reinforced columns, after the entire load had been transferred to the steel, further shrinkage actually caused some tensile stresses in the concrete. It should, however, be noted that despite the redistribution of load between concrete and steel, the ultimate load capacity of the column remains unchanged.

The residual creep strain of a reinforced concrete column segment can be calculated by the following formula first proposed by Dischinger in 1937[19] and used earlier by Fintel and Khan:[14, 15]

$$\epsilon^R_{c\infty} = \epsilon_{c\infty} \cdot CR_R = \frac{\epsilon_{c\infty}}{p\epsilon^*_{c\infty}E_s}\left(1 - e^{\frac{-pm}{1+pm}\epsilon^*_{c\infty}E_{ct}}\right) \quad (11)$$

where

$\epsilon^R_{c\infty}$ = total residual (ultimate) creep strain per unit stress in reinforced concrete

CR_R = residual creep factor

p = *reinforcement ratio of the cross* section of the column segment

$\epsilon^*_{c\infty}$ = specific creep of plain concrete, adjusted for age at loading, volume-to-surface ratio, and humidity

m = time-dependent modular ratio, E_s/E_{ct}

E_s = modulus of elasticity of steel

Since $\epsilon^*_{c\infty}$ (which includes adjustment for age at loading), E_{ct}, and m are all time-dependent, the factor CR_R, as calculated by equation (11), will be different for each load increment applied to a column segment. As to shrinkage, it is suggested that for a column segment subjected to k load increments, residual shrinkage strains be calculated as follows:

$$\epsilon^R_{s\infty} = \epsilon_{s\infty} \cdot SH_R = \epsilon_{s\infty}\frac{1}{k}\sum_{i=1}^{k} CR_{R,i} \quad (12)$$

where

$\epsilon^R_{s\infty}$ = total residual (ultimate) shrinkage strain of reinforced concrete

$CR_{R,i}$ = residual creep factor corresponding to the ith load increment

and

SH_R = average residual factor for shrinkage, which is a load-independent phenomenon

DETERMINING ELASTIC, SHRINKAGE, AND CREEP SHORTENING OF COLUMNS

Fig. 4 shows a schematic section of a multistory building with reinforced concrete or composite columns, with the slabs up to level N already cast. The slabs above level N will be cast as construction proceeds. This section presents mathematical expressions for the cumulative elastic, shrinkage, and creep shortening of all segments of a column up to level N (called solution-floor level in the remainder of this paper). In other words, expressions are given for the vertical displacement of one support of the slab at level N.

Several possible load stages are considered. The initial loads are those that start acting immediately on construction. These come on the structure in as many increments as there are floors. One set of subsequent loads may be those due to the installation of cladding and partitions. Installation of such items would normally proceed story by story, so that the loads would come on in as many increments as there are stories. The final set of loads may be live loads that start acting as the building is occupied. Occupation may proceed story by story or in some other sequence. The following expressions allow specification of the time of application of each stage of each floor load separately. Each type of shortening caused by the initial loads, as discussed previously, is computed separately *up to and subsequent to* the casting of the slab at level N.

The postulated and confirmed principle of superposition of creep states:

> Strains produced in concrete at any time by a stress increment are independent of the effects of any stress applied either earlier or later. The stress increment may be either positive or negative, but stresses that approach the ultimate strength are excluded.

Thus, each load increment causes a creep strain corresponding to the strength-to-stress ratio at the time of its application, as if it were the only loading to which the column were subjected. This principle of superposition is applied to determine the total creep strains in a column subjected to a number of load increments by totaling the creep strains caused by each of the incremental loadings.

Elastic shortening of steel columns can be computed in exactly the same way as that of reinforced concrete or composite columns, except that the computation is somewhat simpler due to the absence of any effect of age on strength and due to the absence of shrinkage and creep.

Elastic Shortening (denoted by superscript *e*)

Due to Initial Loads (denoted by subscript 1)

Up to Casting of Solution-Floor Level (denoted by subscript *p*)

$$\Delta^e_{1,p} = \sum_{j=1}^{N}\sum_{i=j}^{N} \frac{P_i h_j}{A_{t,ij} E_{ct,ij}} \tag{13}$$

with

$$E_{ct,ij} = 33w^{1.5}\sqrt{f'_{ct,ij}} \quad \text{—from equation (1)} \tag{13a}$$

$$f'_{ct,ij} = \frac{f'_{c,j}(t_i - t_j)}{4.0 + 0.85(t_i - t_j)} \quad \text{—from equation (2)} \tag{13a'}$$

and

$$A_{t,ij} = A_{g,j} + A_{s,j}(m_{ij} - 1) \tag{13b}$$

$$m_{ij} = E_s / E_{ct,ij} \tag{13b'}$$

where

i = a particular floor level or load increment

j = a particular column

P = applied load

h = floor height

A_t = time-dependent transformed area of column cross section

E_{ct} = time-dependent modulus of elasticity of concrete

w = unit weight of concrete

f'_{ct} = time-dependent cylinder strength of concrete

t = time of casting or load application (starting from casting of foundation)

A_g = gross area of column cross section

A_s = total area of reinforcing steel in column cross section

m = time-dependent modular ratio

E_s = modulus of elasticity of steel

Subsequent to Casting of Solution-Floor Level (denoted by subscript *s*)

$$\Delta^e_{1,s} = \sum_{j=1}^{N}\sum_{i=N+1}^{n} \frac{P_i h_j}{A_{t,ij} E_{ct,ij}} \tag{14}$$

where n = total number of floors

Due to Subsequent Load Application(s)(denoted by subscripts 2, 3, and so on)

$$\Delta^e_2 = \sum_{j=1}^{N}\sum_{k=j}^{n} \frac{P_k h_j}{A_{t,kj} E_{ct,kj}} \tag{15}$$

with

$$E_{ct,kj} = 33w^{1.5}\sqrt{f'_{ct,kj}} \quad \text{—from equation (1)} \tag{15a}$$

$$f'_{ct,kj} = \frac{f'_{c,j}(t_k - t_j)}{4.0 + 0.85\,(t_k - t_j)} \quad \text{—from equation (2)} \tag{15a'}$$

where k = a particular floor level or load increment

$$\Delta^e_3 = \sum_{j=1}^{N}\sum_{\ell=j}^{n} \frac{P_\ell h_j}{A_{t,\ell j} E_{ct,\ell j}}, \quad \text{and so on} \tag{15'}$$

where ℓ = a particular floor level or load increment.

Shrinkage Shortening (denoted by superscript s)

Up to Casting of Solution-Floor Level (denoted by subscript p)

$$\Delta^s_p = \sum_{j=1}^{N} h_j \cdot \epsilon_{s\infty,j} \cdot SH_{v:s,j} \cdot SH_H \cdot SH_{t,j} \cdot SH_{R,j} \tag{16}$$

with

$$SH_{v:s,j} = \frac{0.037(v{:}s)_j + 0.944}{0.177(v{:}s)_j + 0.734} \quad \text{—from equation (3)} \tag{16a}$$

and

$$SH_{t,j} = \frac{t_N - t_j - t'_j}{26.0e^{0.36(v:s)_j} + (t_N - t_j - t'_j)} \quad \text{—from equation (5)} \tag{16b}$$

where t'_j is the period of moist-curing of column j, SH_H is from equation (4), and $SH_{R,j}$ (see equation 12) is defined as follows:

$$SH_{R,j} = \frac{\sum_{i=j}^{n} CR_{R,ij}}{n-j+1} \tag{16c}$$

$CR_{R,ij}$ is given by equation (18d).

Subsequent to Casting of Solution-Floor Level (denoted by subscript s)

$$\Delta_s^s = \sum_{j=1}^{N} h_j \cdot \epsilon_{s\infty,j} \cdot SH_{v:s,j} \cdot SH_H \cdot (1 - SH_{t,j}) \cdot SH_{R,j} \tag{17}$$

Creep Shortening (denoted by superscript *c*)

Due to Initial Loads (denoted by subscript 1)

Up to Casting of Solution-Floor Level (denoted by subscript *p*)

$$\Delta_{1,p}^c = \sum_{j=1}^{N} \sum_{i=j}^{n} \frac{P_i CR_{LA,ij}}{A_{t,ij}} \cdot \epsilon_{c\infty,j} \cdot h_j \cdot CR_{v:s,j} \cdot CR_H \cdot CR_{t,j} \cdot CR_{R,ij} \tag{18}$$

where

$$CR_{LA,ij} = 2.3(t_i - t_j)^{-0.25} \quad \text{—from equation (7)} \tag{18a}$$

$A_{t,ij}$ has been defined by equations (13a), (13a′), (13b), and (13b′)

$$CR_{v:s,j} = \frac{0.044(v:s)_j + 0.934}{0.1(v:s)_j + 0.85} \quad \text{—from equation (8)} \tag{18b}$$

CR_H is given by equation (9)

$$CR_{t,j} = \frac{(t_N - t_j)^{0.6}}{10.0 + (t_N - t_j)^{0.6}} \quad \text{if } t_N \geqslant t_j$$

$$= 0 \quad \text{if } t_N \leqslant t_j \quad \text{—from equation (10)} \tag{18c}$$

and

$$CR_{R,ij} = \frac{1 - e^{\frac{-p_j m_{ij}}{1 + p_j m_{ij}} \cdot \epsilon^*_{c\infty,ij} \cdot E_{ct,ij}}}{p_j \cdot \epsilon^*_{c\infty,ij} \cdot E_s} \quad \text{—from equation (11)} \tag{18d}$$

$$\epsilon^*_{c\infty,ij} = \epsilon_{c\infty,ij} \cdot CR_{LA,ij} \cdot CR_{v:s,j} \cdot CR_H \tag{18d′}$$

$$p_j = A_{s,j}/A_{g,j} \tag{18d″}$$

Subsequent to Casting of Solution-Floor Level (denoted by subscript *s*)

$$\Delta_{1,s}^c = \sum_{j=1}^{N} \sum_{i=j}^{n} \frac{P_i \cdot CR_{LA,ij}}{A_{t,ij}} \epsilon_{c\infty,j} \cdot h_j \cdot CR_{v:s,j} \cdot CR_H \cdot (1 - CR_{t,j}) \cdot CR_{R,ij} \tag{19}$$

Due to Subsequent Load Application(s)(denoted by subscripts 2, 3, and so on)

$$\Delta_2^c = \sum_{j=1}^{N} \sum_{k=j}^{n} \frac{P_k CR_{LA,kj}}{A_{t,kj}} \cdot \epsilon_{c\infty,j} \cdot h_j \cdot CR_{v:s,j} \cdot CR_H \cdot CR_{R,kj} \tag{20}$$

$$\Delta_3^c = \sum_{j=1}^{N} \sum_{\ell=j}^{n} \frac{P_\ell CR_{LA,\ell j}}{A_{t,\ell j}} \cdot \epsilon_{c\infty,j} \cdot h_j \cdot CR_{v:s,j} \cdot CR_H \cdot CR_{R,\ell j} \tag{20′}$$

EXAMPLES OF COLUMN SHORTENING ANALYSIS

Using the methodology described in the previous sections, examples of analyses for differential column length changes in tall buildings featuring two different structural systems are presented below.

80-Story Composite Structure

Fig. 15 shows a quarter of the plan of an 80-story composite building with a peripheral beam-column system of reinforced concrete to provide lateral rigidity and interior structural steel columns supporting a slab system of structural steel beams, a corrugated deck, and a concrete topping. The large peripheral columns have structural steel erection columns embedded in them. Table 1 gives the loads and the section properties for a typical exterior and a typical interior column throughout the height of the building. The computed components of shortening due to elastic stresses, shrinkage, and creep for the exterior composite column and the interior steel column are shown in Fig. 16.

Curve *a* in Fig. 16a (showing the elastic shortening of the interior steel column) indicates that the vertical column displacements at the various floor levels up to the time of slab installation at those levels increase up the height of the building, because the loads from each added floor shorten all the column segments below that level.

Subsequent to slab installation at the various levels, the vertical column displacements at those levels initially increase, but then decrease with increasing height (curve *b*, Fig. 16a). This is because loads from fewer and fewer stories contribute to post-slab-installation shortenings as construction progresses toward the top of the building. At the roof level, only the mechanical bulkheads above contribute to column shortening subsequent to the installation of the roof slab.

Curves 1 and 5 in Fig. 16b (showing the elastic shortening of the exterior composite column) exhibit the same trends as curves *a* and *b*, respectively, of Fig. 16a. The shortenings of exterior columns are based on assumed moderate values of shrinkage and creep coefficients (Table 1). As mentioned, light erection columns are embedded in the peripheral concrete columns. Their elastic shortenings prior to embedment become part of the total differential shortenings between exterior and interior columns (Fig. 16b).

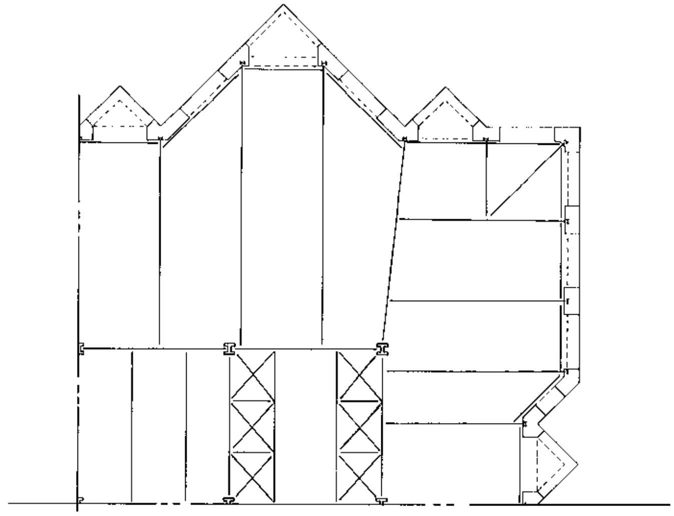


Fig. 15. Typical floor plan of 80-story steel-concrete-composite building.

Table 1. Properties of Exterior Composite Column and Interior Steel Column in 80-Story Composite Building

Speed of Construction: 7 days per floor
Story Height: 18-ft bottom story; 13-ft, all other stories

Exterior Composite Column

Floor levels	Concrete strength, ksi	Gross col. area, in.^{2}	Steel area,* in.^{2}	Floor load, kips	Subsequent floor load,** kips	Volume-to-surface ratio, in.	Ultimate shrinkage, x10^{-6} in./in.	Specific creep, x10^{-6} in./in./psi
1-9	7	2822.4	75.5	63.8	15.9	12.7		0.200†
10-19	7	2419.2	69.4	54.9	13.7	10.6		0.400††
20-29	6	2419.2	44.4	54.9	13.7	10.6		0.250†
30-39	6	1987.2	40.0	49.2	12.3	9.0	600†	0.450††
40-49	6	1987.2	40.0	44.7	11.2	9.0	800††	
50-59	5	1987.2	40.0	44.7	11.2	9.0		0.300†
60-69	5	1987.2	30.1	45.5	11.4	9.0		0.500††
70-76	5	1987.2	30.1	47.8	11.9	9.0		
77	5	1987.2	30.1	161.7	40.4	9.0		

*Includes erection column area
**Assumed to start acting 300 days after casting of column
†Low value ††High value

Interior Steel Column

Floor levels	Gross area, in.^{2}	Floor load, kips	Floor levels	Gross area, in.^{2}	Floor load, kips
1	431.5	81.3	56-57	134.0	75.1
2-7	410.5	81.3	58-59	125.0	75.1
8-9	391.5	81.3	60-61	117.0	80.0
10-13	391.5	78.8	62-63	109.0	80.0
14-19	356.0	78.8	64-65	101.0	80.0
20-25	356.0	77.9	66-67	83.3	80.0
26-29	234.0	77.9	68-69	75.6	80.0
30-37	234.0	82.3	70-71	68.5	73.9
38-39	215.0	82.3	72-73	62.0	73.9
40-45	196.0	76.1	74-75	51.8	73.9
46-47	178.0	76.1	76-77	42.7	73.9
48-49	162.0	76.1	78	35.3	73.9
50-51	162.0	75.1	79	35.3	651.0
52-55	147.0	75.1			

1 in. = 25.4 mm
1 in.^{2} = 645 mm^2
1 in./in. = 1 mm/mm
1 in./in./psi = 0.145 mm/mm/kPa
1 kip = 4.448 kN
1 ksi = 6.895 MPa

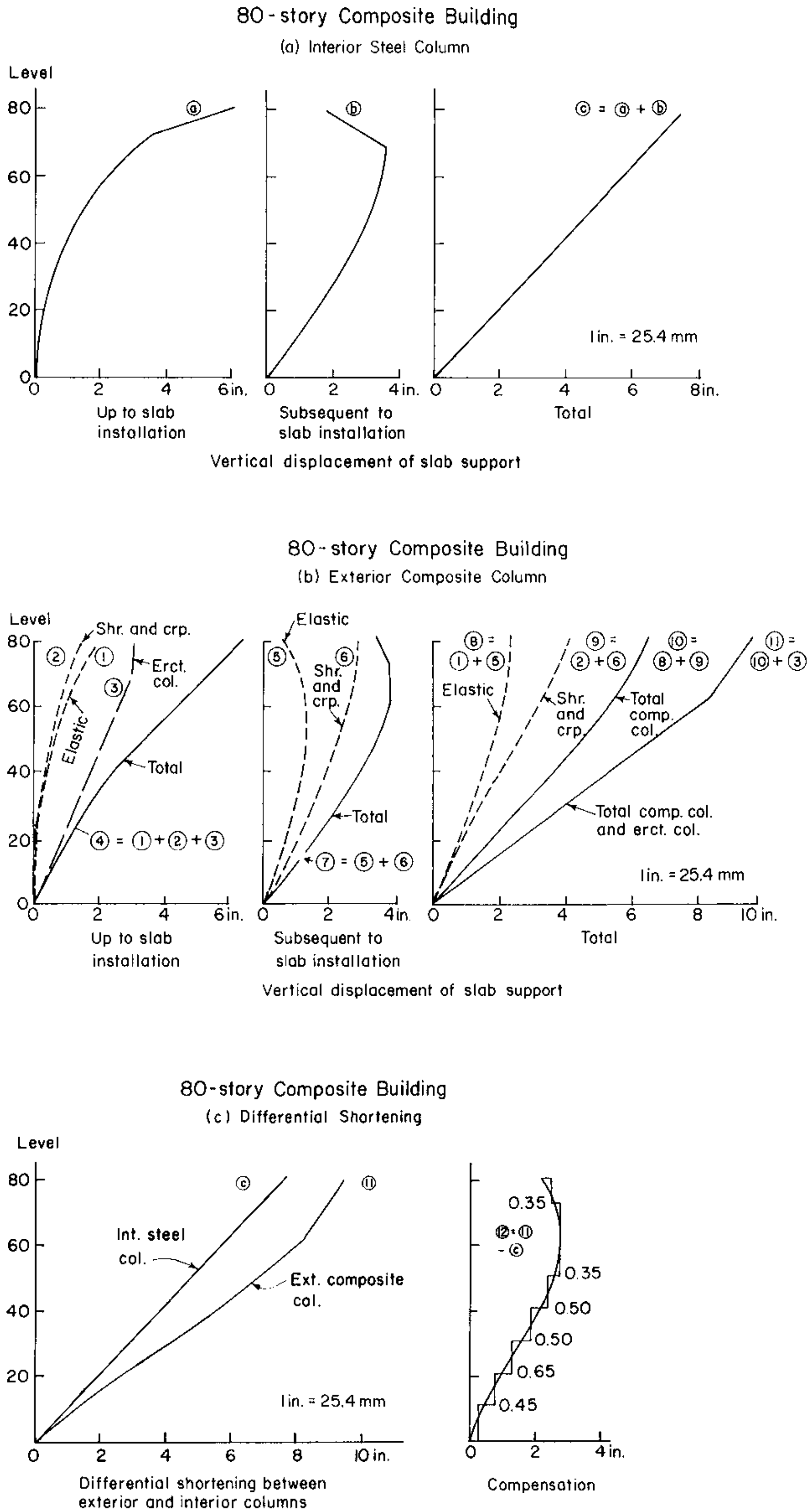


Fig. 16. Column length changes in 80-story building with composite steel-concrete structural system, assuming low shrinkage and creep coefficients.

Fig. 16c shows the differential shortenings at the various floor levels between the interior steel and the exterior composite columns considered in Fig. 16a and b. The differential shortenings up to and subsequent to the installation of slabs at the various floor levels have been added for compensation purposes. The values shown on the right side of Fig. 16c are needed to detail the columns for fabrication, so that after all loads have been applied and shrinkage and creep have taken place, the slabs will be horizontal. The largest differential shortening over the height of the structure is 2.65 in. (67.3 mm); this maximum occurs at the 60th floor level.

Fig. 17a and b are similar to Fig. 16b and c, respectively, and show shortenings of the exterior composite column for assumed high values of shrinkage and creep (Table 1). The maximum differential shortening over the height of the structure is now 4.3 in. (109.2 mm) occurring at the 70th floor level; obviously corrective measures are required during construction to avoid tilted slabs. For this particular building, it is suggested that at every 10th story the interior column lift be shortened as shown on the compensation curve.

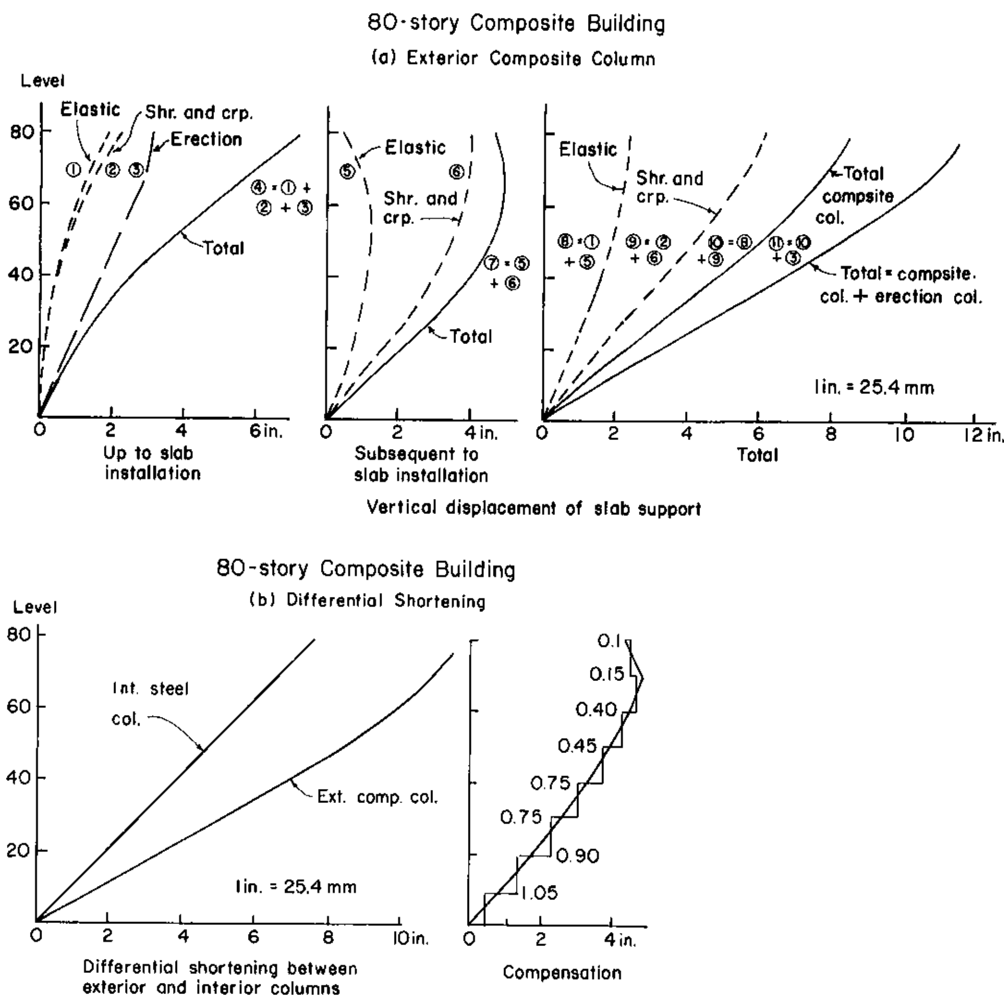


Fig. 17. Column length changes in 80-story building with composite steel-concrete structural system, assuming high shrinkage and creep coefficients.

70-Story Reinforced Concrete Frame-Shearwall Building

Fig. 18 shows the plan of a 70-story reinforced concrete frame-shearwall building. In Table 2, concrete strengths, loads, amounts of reinforcement, and other properties for a typical peripheral column and a typical interior shearwall segment are given. As can be judged from the plan, the potential differential shortening between the peripheral columns and the central shearwall core may cause tilting of the slabs and should be investigated.

The components of elastic, shrinkage, and creep shortenings of the peripheral column and the interior shearwall segment are shown in Fig. 19 for relatively high assumed values of shrinkage and creep (Table 2). The deformations that occur before the casting of a slab are of no consequence, since the formwork for each slab is usually installed horizontally; thus, the pre-slab-installation differentials are automatically compensated for. Only the post-slab-installation deformations (right side of Fig. 19b) may need compensation, if the predicted amount is more than can be tolerated.

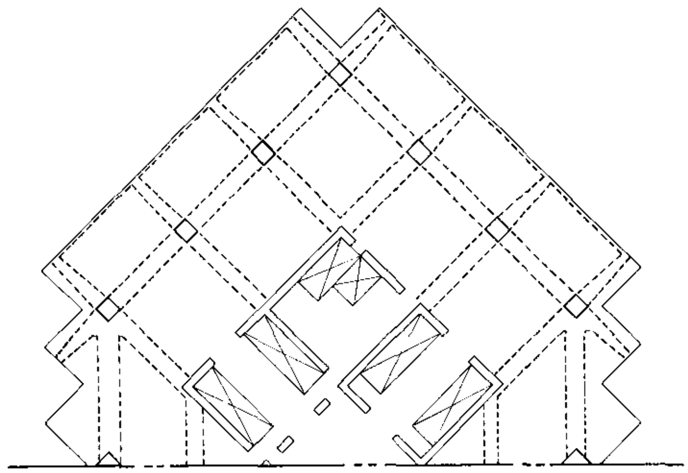


Fig. 18. Typical floor plan of 70-story concrete frame-shearwall building.

Table 2. Properties of Exterior Column and Interior Shearwall in 70-Story Concrete Building

Speed of Construction: 8 days per floor
Story Height: 18-ft bottom story; 13-ft, all other stories

Exterior Column

Floor levels	Concrete *strength,* ksi	*Col. size,* in.xin.	Gross *area,* in.^{2}	Steel area, *% of* gross area	Floor *load,** kips	Volume-to- *surface* ratio, in.	Ultimate shrinkage, 10^{-6} in./in.	Specific creep, 10^{-6} in./in./psi
1-10	8	72x72	5184	2.47	254.7	18.0		
11-20	8	72x72	5184	1.23	254.7	18.0		0.175**
21-30	8	60x60	3600	2.22	254.7	15.0		0.275†
31-40	7	60x60	3600	1.78	254.7	15.0	500** 800†	0.200** 0.300†
41-50	6	60x60	3600	1.33	254.7	15.0		
51-62	6	44x44	1936	1.65	254.7	11.0		
63-67	6	32x32	1024	1.22	254.7	8.0		0.250**
68-70	6	32x32	1024	1.22	196.8	8.0		0.350†

Interior Shearwall

Floor levels	Concrete strength, ksi	Wall thickness, in.	Gross area, in.^{2}	Steel area, in.^{2}	Floor load,* kips	Volume-to-surface ratio, in.	Ultimate shrinkage x10^{-6} in./in.	Specific creep x10^{-6} in./in./psi
1-14	6	24	9936	109.2	245.6	11.34		
15-18	6	20	8280	88.9	231.6	9.54		0.250**
19-30	6	20	8280	88.9	237.0	9.54	500	0.350†
31-40	6	16	6624	70.0	222.6	7.70	800	
41-46	4.5	12	6624	70.0	222.6	7.70		0.350**
47-72	4.5	12	4968	42.0	208.6	5.83		0.450†

*Dead load plus 10 psf live load
**Low value †High value

1 in. = 25.4 mm
1 in.^{2} = 645 mm^2
1 in./in. = 1 mm/mm

1 in./in./psi = 0.145 mm/mm/kPa
1 kip = 4.448kN
1 ksi = 6.895 MPa

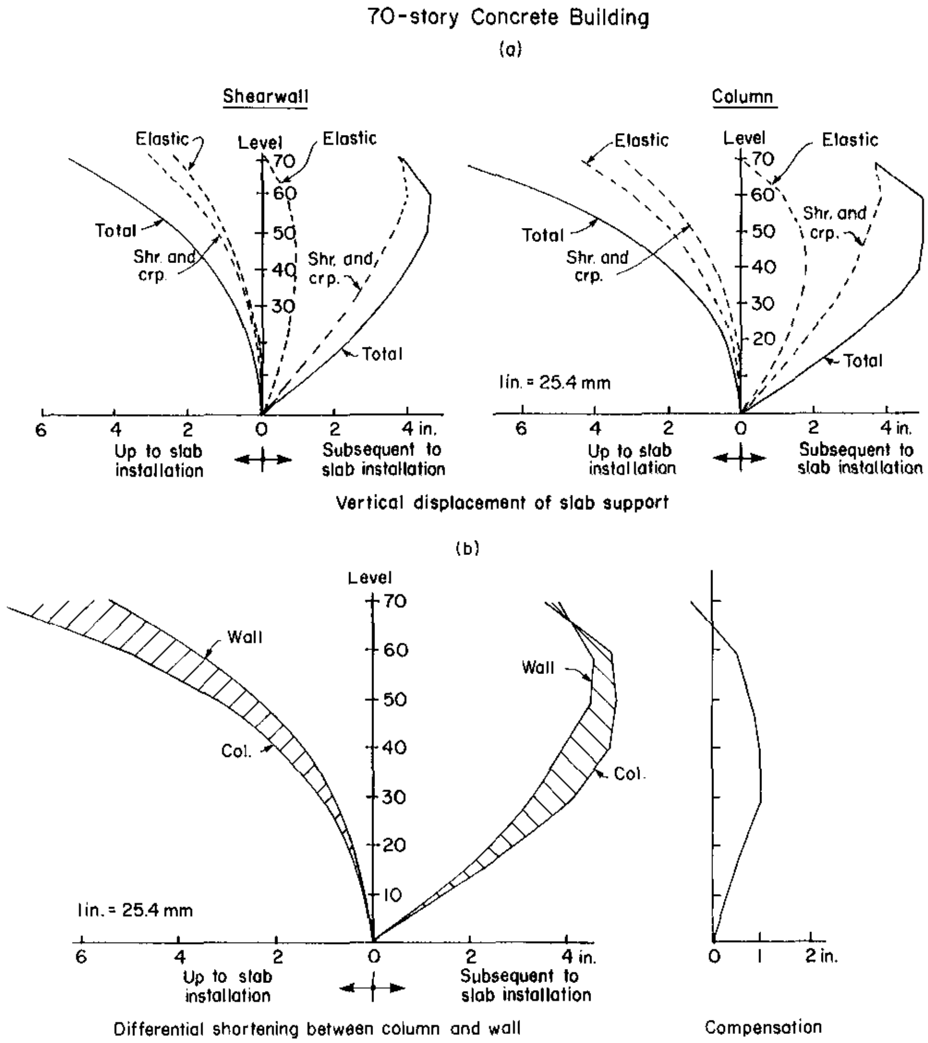


Fig. 19. Changes in the lengths of columns and shearwalls in 70-story concrete building, assuming high shrinkage and creep coefficients.

To determine the effect of a range of material properties, the shortenings of the exterior column and the interior shearwall segment also were computed for relatively low assumed values of shrinkage and creep (Table 2); these are presented in Fig. 20. The right side of Fig. 20b shows the differential shortenings (after slab casting) for low values of shrinkage and creep.

As can be seen, the maximum differential shortenings for the 70-story building are at about the 40th floor level, ranging between 0.85 in. (21.6 mm) and 1.05 in. (26.7 mm) for low and high values of shrinkage and creep, respectively.

It is quite simple to compensate for differential shortening in concrete structures by raising the forms along the peripheral columns.

Whether to compensate for a maximum differential shortening of only 1.0 in. (25.4 mm) is a question that should be decided by comparing the disadvantages of the tilted slab against the cost and inconvenience of cambering the slabs during construction.

SENSITIVITY OF MOVEMENTS RELATIVE TO MATERIAL CHARACTERISTICS AND OTHER FACTORS

As is apparent from the preceding sections, the following variables have an effect on the total shortening of a column:

A. Material characteristics (initial and ultimate values and their time-evolution curves)
   1. Modulus of elasticity
   2. Shrinkage
   3. Specific creep
B. Design assumptions
   1. Cross-sectional area
   2. Volume-to-surface ratio
   3. Percentage of reinforcement
C. Loading assumptions
   1. Progress of construction
   2. Progress of occupancy
   3. Environmental conditions (temperature and humidity)

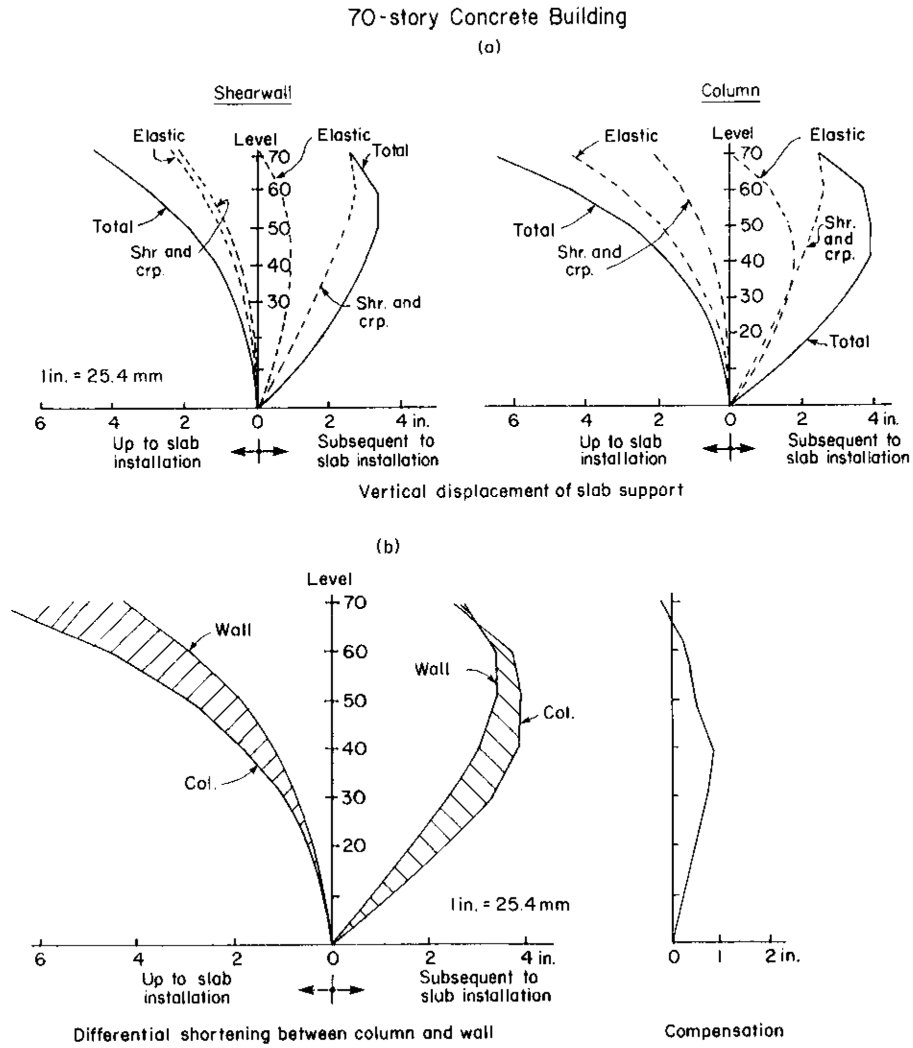


Fig. 20. Changes in the lengths of columns and shearwalls in 70-story concrete building, assuming low shrinkage and creep coefficients.

Only rarely are the values of ultimate shrinkage and specific creep known at the time of preliminary design when the structural system for a building is determined and when preliminary information on differential shortening is needed to decide the feasibility of a particular structural configuration.

While the designer has a degree of control over his design assumptions and modulus of elasticity seems reasonably well defined as a function of the specified strength of the concrete, it is necessary to find out during the preliminary stage how sensitive the potential column length changes may be to variations in the values of ultimate shrinkage and specific creep. Variations in the speed of construction may also have a pronounced effect on the amounts of creep and shrinkage that will occur after a slab is in place.

Sensitivity studies were carried out for the previously described 80-story composite structure and 70-story shearwall-frame interactive structure to determine the effects of low and high values of ultimate shrinkage and specific creep. The results for the 80-story composite structure are shown in Figs. 16 and 17 in the form of differential length changes between a composite and a steel column. As is seen, the computed tilt of the slabs from their original position in the upper stories of the composite building ranges from 2.6 to 4.3 in. (66 to 109 mm) for the low and the high shrinkage and creep, respectively, if no precautions are taken in the detailing and erection of the steel columns.

The results for the 70-story concrete building in the form of differential length changes between a column and a shearwall are presented in Figs. 19 and 20. As is seen, the maximum differential shortenings (at about the 40th floor level) are 0.85 in. (21.6 mm) and 1.05 in. (26.7 mm) for the low and the high values of shrinkage and creep, respectively.

The results for the two structural systems indicate a high sensitivity of the differential column shortening to values of shrinkage and creep in composite construction and a less than significant sensitivity in all-concrete structures. The reason is that while in the composite building only one of the two differentially shortening elements is subject to shrinkage and creep (the steel column shortens only elastically), both differentially shortening elements in the concrete building shrink and creep.

RESTRAINING EFFECT OF THE SLAB SYSTEM

The differentially shortening supports that have been discussed cause deflections of the supported slab system. In reinforced concrete structures, the deflecting slabs respond to settling supports with resistant shears acting back on the supports, thus decreasing the unrestrained differential shortening. This decrease in shortening is the result of the resistance of the frame and depends on the out-of-plane stiffness of the slab system and the axial stiffness of the columns.

The moments in the slabs due to the differential settlement of supports cause a redistribution of loads between supports, which in turn creates new modified stress levels for creep. A refined consideration of the load redistribution at each stage of construction (after each slab has created, in effect, a new frame) is rarely warranted even in structures with rigid slabs. A relatively simple analysis to determine moments in the slabs due to differential settlement of supports is considered sufficient in view of the existing uncertainties in material properties and stiffness assumptions.

The analysis as given in Reference 15 starts with the known differential settlements of supports (unrestrained by frame action) at each level. A simplification is introduced by substituting a ten-story one-column reference frame for the real structure. Reference 15 gives graphs to determine the equivalent stiffnesses for the one-bay frame and the resulting residual shortening from which the moments in the slabs and the columns can be determined. As can be seen in the graphs, the residual shortening (as a percentage of unrestrained shortening) depends upon the relative slab-to-column stiffness ratio and upon the number of stories. For flat-plate-type slabs there is an insignificant restraint resulting in a high residual shortening. With increasing slab stiffness and number of floors, there is less residual shortening.

Effect of Creep on Slab Moments Caused by Differential Settlement of Supports

A restrained member, as shown in Fig. 21, subjected to an instantaneous differential settlement of supports, Δ, will respond with restraint moments, $\pm M$. Creep of the concrete will cause relaxation of the moments with time as shown qualitatively by curve A. The rate of creeping out of the moment depends upon the creep properties of the member, the change in the effective stiffness of the member caused by progressive cracking, if any, and the increase of the modulus of elasticity with time.

If the same settlement, Δ, is applied over a period, T, the induced moments will change with time as shown by curve B. They will start from zero and reach a maximum at time T and then continue to creep out after the peak.

The elastic and inelastic differential shortening of supports of a slab in a multistory building does not occur instantaneously. The elastic shortening takes place over the period it takes to construct the structure above the slab under consideration. The creep and shrinkage shortening continues for years at a progressively decreasing rate.

The relationship between the magnitude of internal forces caused by settlements and the length of time it takes to apply the settlement was studied experimentally and analytically by Ghali and others.[20] The time during which the settlement, Δ, was applied was varied up to five years. The studies show that for settlements applied during a period of more than 30 days, little change occurs in the maximum value of the parasitic forces, as shown in Fig. 22 taken from Reference 20. Based on this study, it seems reasonable to combine the effects on the frame of the elastic and inelastic shortening of supports.

For practical design of buildings, the authors suggest that the maximum value of the differential settlement moments be assumed at 50% of the moments that would occur without relaxation due to creep. These moments should then be used with appropriate load factors in combination with the effects of other loads. The 50% reduction accounts only for creep relaxation during the period of settlement. Beyond this time a further creeping out of settlement moments takes place.

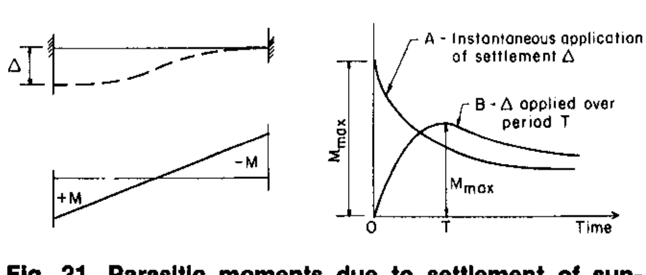


Fig. 21. Parasitic moments due to settlement of supports.[15]

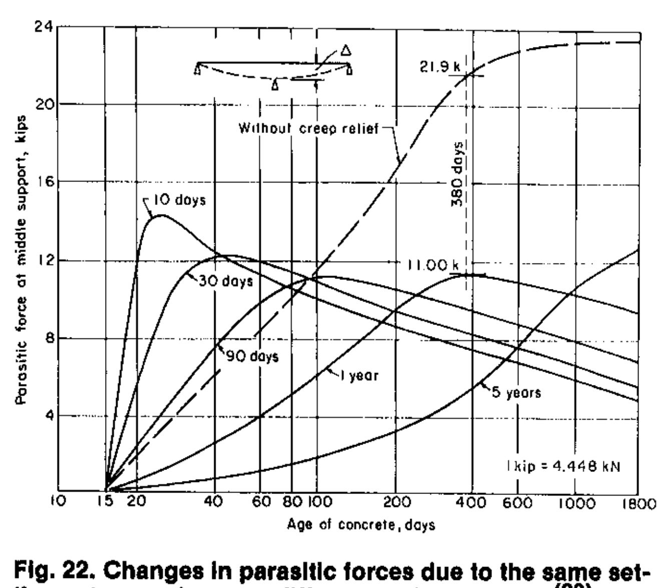


Fig. 22. Changes in parasitic forces due to the same settlement occurring over different periods of time.[20]

Load Transfer Between Adjacent Differentially Shortening Elements

In a frame with differentially settling supports, the support that settles less will receive additional load from the support that settles more. The transferred load is

$$V_i = \frac{M_{i1} - M_{i2}}{L_i} \tag{21}$$

where M_{i1} and M_{i2} are the settlement moments (reduced due to creep relaxation) at the two ends of the horizontal element and L_i is the span.

The load transferred over the entire height of the structure is a summation of loads transferred on all floors. The load transfer is cumulative starting from the top of the structure and progressing down to its base.

Stress

The stresses due to differential shortening should be treated as equivalent dead-load stresses with appropriate load factors before combining them with other loading conditions. When choosing a load factor, it should be kept in mind that the design shortening moments occur only for a short time during the life of the structure and they continue to creep out after that.

RELATION TO OTHER PERMANENT AND TRANSIENT MOVEMENTS

In addition to the elastic and inelastic length changes due to gravity loads and shrinkage as discussed in this publication, columns and walls in tall structures are subject to length changes due to wind, daily and seasonal temperature variations, and foundation settlements.

To establish design criteria specifying limits on distortions that can be accepted at various locations without impairing the strength and the function (serviceability) of the structure, these effects should be combined based on the probabilities of their simultaneous occurrence.

Length changes of columns due to the environmental effects of wind and temperature are transient. Foundation settlements and slab deflections due to gravity loads are permanent and have immediate and long-term components; the latter stabilize with progress of time.

While some of the movements add to the column length changes from elastic stress, creep, and shrinkage, other effects (wind) may cause transient elongations that temporarily mitigate the effects of column shortening.

The tilting of slabs due to column length changes caused by shrinkage and gravity loads is permanent; these length changes can and should be compensated for during construction. The permissible tilt of slabs can then be set aside for unavoidable and uncorrectable transient effects of wind and temperature variations.

Wind Movements

Transient wind distortions of a structure basically have no long-term effects. Under wind forces, the columns of a structure are subject to length changes (compression and tension) and to horizontal translation. Only the shortening of the leeward columns is of consequence, since it is in addition to the column shortening caused by gravity loads. In tall and slender buildings this shortening should be considered.

Temperature Movements

Both seasonal and daily temperature fluctuations cause length changes in exterior columns if they are unprotected from the ambient temperatures. The exterior columns elongate when their temperature is higher than the temperature of the interior, protected columns. When these peripheral exposed columns are colder than the interior columns, their thermal shortening is additive to the gravity shortening.

Thermal length changes of columns, like shortening caused by gravity loads, are cumulative. Therefore, cooling of the exposed columns in tall structures (during the winter) combined with column shortenings due to gravity may become a critical design consideration. The daily temperature fluctuations have a less aggravating effect, because the rapid temperature fluctuations do not penetrate sufficiently into the depth of the concrete to cause length changes. Only average temperatures over a number of days (depending on the column size) can have an effect on column-length changes.

Differential Foundation Movements

Relative foundation settlements that occur between the core and the exterior peripheral columns need to be considered, along with the differential column shortenings due to shrinkage and gravity loads, to determine the combined effects on the slab systems. In most cases a larger settlement occurs at the center of a building, regardless of whether individual footings or mat foundations are used. On the other hand, the gravity effects in reinforced concrete buildings usually cause larger shortenings of the peripheral columns relative to the core walls. Only a precise estimate of the two types of distortion can determine if they cancel each other, and whether their superposition can be omitted. It seems practical to consider each of the two effects separately, as if the other did not exist. This is advisable in view of the uncertainty in realistic prediction of differential foundation settlement. Rarely in the past could the predicted amount of differential foundation settlement be verified by actual field measurements. Further field observations of foundation settlement are needed before they can be considered in combination with other effects.

Vertical Deflections of Slabs Due to Gravity

Fig. 23 shows the superposition of slab deflections due to gravity loads on slabs and those due to differential shortening of the supporting columns. As is seen, the column shortening increases the tilt of the slab and therefore has an effect on partition distortions.

PERFORMANCE CRITERIA—LIMITATION ON DISTORTIONS

Effects of the tilt of slabs on partitions and finishes require limitations on column movements, although it is possible to structurally accommodate a considerable amount of column movement either by reinforcing for the effects of such movement or by having the slabs hinged. Serviceability criteria for slabs are expressed in terms of the ratio of the support settlement to the slab span length (angular distortion), as well as the absolute amount of movement.

From the previous discussion, it is evident that in considering limitations, the column shortenings due to gravity loads and shrinkage should be superimposed with other effects and limitations established on the combined movement. As is seen in Fig. 23, the column shortening due to gravity and that due to thermal effects and wind can be additions to vertical load deflections of the slab in their effects on partition distortions and on slab tilt.

Traditional British practice limits angular distortions caused by differential settlements to $L/360$ (L = span length of member) to protect brickwork and plaster from cracking; $L/180$ is considered tolerable in warehouses and industrial buildings without masonry. Although in our modern high-rise buildings the considerations of performance are totally different from that of cracking of masonry and plaster (these materials having almost disappeared from our modern buildings), limitations such as those in British practice may still be suitable, since they result in distortion limits usually customary in the construction industry. Thus, a suggested limitation of $L/240$ to be applied to the combined differential column shortening due to elastic stresses, creep, shrinkage, temperature variations, and wind would limit the slab distortion at midspan to $1/2 \times L/240$ plus the slab deflection due to gravity (see Fig. 23).

In keeping with standards used in some leading engineering offices, additional limits on the maximum differential settlement of supports of a slab, applicable regardless of the span length, can be suggested. The most commonly used limit appears to be 0.75 in. (19.1 mm) on the differential settlement caused by either temperature or wind or elastic stresses plus creep plus shrinkage, and 1 in. (25.4 mm) on the differential settlement caused by a combination of all the above factors.

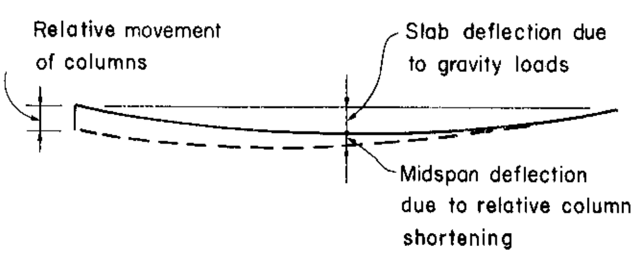


Fig. 23. Combined slab deflections due to gravity loads and column shortening.

COMPENSATION FOR DIFFERENTIAL SUPPORT SHORTENING

The main purpose in computing anticipated column shortenings is to compensate for the differential length changes during construction so as to ensure that the slabs will be horizontal in their final position. Such compensation for differential length changes is similar in concept to the cambering of slabs, since both processes are intended to offset deformations anticipated in the future.

At the time a slab is installed, each of the two slab supports has already undergone a certain amount of shortening. After slab installation, the supports will settle further, due to subsequent loads as construction progresses, and due to shrinkage and creep. In composite and concrete structures where the anticipated shortenings may take place over many years, full compensation can be made during construction provided the amount is not excessive. Where large differential length changes need to be compensated for, compensation can be made during construction to offset movements expected to take place over several years; subsequent shrinkage and creep will then cause tilt of the slab. Note that during the initial two years about 85% to 90% of all shrinkage and creep distortions will have taken place.

The methods of compensation are different in concrete structures and in each of the two types of composite structures.

Concrete Structures

In concrete structures, compensation is required only for the differential settlements of supports that are expected to occur after slab installation. Each time formwork for a new slab is erected, its position is adjusted to the desired level; the preinstallation settlements of the supports are thus eliminated.

The anticipated differential shortening of the slab supports to be compensated for consists of the following components:

Elastic—due to loads above (progress of construction)
due to loads of finishes below and above
due to live loads below and above
Inelastic—due to shrinkage and creep from the time of slab installation

Either complete or partial compensation of the differential length changes can be specified for the formwork, depending upon the magnitude of anticipated support settlements.

Composite Structures

Composite structures in which some of the columns are steel and others are concrete or composite have a considerably high potential for differential shortening, as seen previously. Therefore, neglecting to consider vertical distortions in very tall composite structures may cause behavioral problems, either immediately or sometimes years after the building is in satisfactory operation.

In composite structures consisting of interior steel columns and an exterior reinforced concrete beam-column system with embedded erection columns, the steel columns are usually fabricated to specified story-high lengths. Traditionally, after a number of floors (say, 6 to 10) have been erected, the elevations of the column tops are measured and shim plates to compensate for the differential length changes are added, to start all columns of the new story from the same elevation. Thus, compensation is made for pre-slab-installation shortenings; but post-slab-installation shortenings will cause tilting of the slabs. If an analysis for column shortening is available, it may be possible to upgrade this traditional procedure by modifying the thickness of the shim plates to include the differential shortenings anticipated after slab installation.

To avoid the costly shim-plate procedure and to ensure horizontal slabs in the finished building, column lengths can be detailed to compensate for the anticipated shortening due to all loads. The compensation can be detailed for every column lift; or if the computed column-lift differential is smaller than the fabrication tolerances, then after a number of floors (say, 6 to 10), a lift with corrected column lengths can be detailed.

As seen in Figs. 16 and 17, the differential shortening of columns in the 80-story composite structure includes the shortening of the erection columns prior to their embedment into the concrete in addition to the elastic and inelastic shortenings of the concrete in the composite columns. On the right side of Figs. 16c and 17b, the suggested modification of the lengths of the steel erection columns is shown. While the erection column at the 10th story needs to be longer by 1.05 in. (26.7 mm) according to Fig. 17b, this differential becomes gradually smaller and is 0.4 in. (10.2 mm) at the 60th story, and 0.15 in. (3.8 mm) at the 70th story. The erection column becomes shorter than the interior steel column by 0.1 in. (2.54 mm) at the 80th story.

In composite structures consisting of an interior concrete core and steel peripheral columns and steel slab beams, compensation for column and wall length changes can be made by attaching the support plates for the steel slab beams to the core (into pockets left in the wall), regardless of whether the core is constructed with jump forms proceeding simultaneously with steel erection or is slipformed. In both cases, the before-slab-installation shortening is eliminated if the steel beams are installed horizontally.

In jump-formed-core construction, the beam support plates can be embedded in the pockets at the desired location to accommodate anticipated support shortening after the slab has been installed, either by attaching them to the forms or by placing them on mortar beds after the wall is cast.

In slipformed cores, pockets are left to receive the slab beams. Sufficient tolerance needs to be detailed in the size and location of the pockets so that the beams can be placed to accommodate the anticipated shortening of supports.

With the above arrangements, the steel columns do not need compensation and can be manufactured to the specified story-high length; only the elevation of the support plates for the beam ends resting on the concrete walls needs adjustment to provide for the differential shortening that will occur after the slab is in place.

TESTING OF MATERIALS TO ACQUIRE DATA

Information on shrinkage, creep, and modulus of elasticity for various strengths of concrete for use in preliminary analyses of column shortening has been discussed. The information is based on American and European literature.

Should preliminary analyses indicate substantial differential length changes, it may be desirable for a more refined prediction of such length changes to determine the material characteristics of the actual concretes to be used in a laboratory testing program prior to the start of construction.

Standard 6-in. (150-mm) cylinders, 12 in. (300 mm) long and made of the actual design mixes, should be used to determine the strength, modulus of elasticity, shrinkage and creep coefficients, and the coefficient of thermal expansion. All specimens should be moist-cured for 7 days and then stored in an atmosphere of 73° F (23° C) and 50% relative humidity. A modest testing program would entail a total of 12 test cylinders for each of the concrete strengths to be used in the structure: six cylinders for strength (two each for testing at 28, 90, and 180 days); two cylinders for the shrinkage coefficient; two cylinders for the creep coefficient; one sealed cylinder (in copper foil) for thermal expansion; and one spare cylinder.

The creep specimens should be loaded at 28 days of age to a stress level expected in the structure, but not to exceed 40% of the cylinder strength. Measurement of shrinkage on the shrinkage specimen should start after 7 days of moist-curing. While the measurements can go on for several years, the 90-day interim readings can be used to extrapolate the ultimate values of shrinkage and creep, although the extrapolation will obviously be more accurate for data obtained over longer periods of time. The thermal coefficient should be measured on the sealed specimen at 28 days between temperatures of 40° F (4° C) and 100° F (38° C).

VERIFICATION

Two 6x6x36-in. (152x152x914-mm) lightweight concrete columns reinforced with four #5 (16-mm)) deformed 75-ksi (517-MPa) bars were tested in the laboratories of the Portland Cement Association under 50 equal weekly increments of load, each equaling 2% of the full load of 70 kips (311 kN), with the first increment applied at one week of age.[17] The loading simulated conditions encountered in a 50-story concrete building. The columns were moist-cured for 3 days and then sealed in thick copper foil to simulate idealized moisture conditions in large prototype columns. Such sealing also virtually eliminated shrinkage. The measured data from the incrementally loaded columns are shown in Fig. 24. Column strains were measured on three faces of each column, over 10-in. (254-mm) gage lengths at midheight.

Comparison cylinders, 6x12 in. (152x305 mm), were cast and continuously moist-cured until time of testing for compressive strength, modulus of elasticity, creep, and drying shrinkage characteristics as functions of time. Creep tests were conducted on sealed as well as unsealed specimens. The measured 28-day compressive strength of concrete was 6360 psi (44 MPa), and the measured modulus of elasticity was 3.34×10^6 psi (23,029 MPa). Both values represent average results from four cylinder tests. After 146 weeks of loading, the measured creep of sealed and unsealed cylinders loaded at 28 days of age was 250 and 725 millionths, respectively, under 1500 psi (10.34 MPa) of stress. The corresponding projected ultimate values (using equation 10) of sealed and unsealed creep are 289 and 838 millionths, respectively, for 1500 psi (10.34 MPa) of stress. These translate into sealed and unsealed specific creep values of 0.193×10^{-6} in. per inch per psi (0.280×10^{-7} mm per mm per kPa) and 0.559×10^{-6} in. per inch per psi (0.811×10^{-7} mm per mm per kPa), respectively.

Elastic plus creep strains for the tested columns were predicted using the procedures outlined in this report. Measured values of compressive strength and modulus of elasticity were used in the computations. Two sets of computations were made—one using the unsealed specific creep value of 0.559×10^{-6} in. per inch per psi (0.811×10^{-7} mm per mm per kPa) with a corresponding volume-to-surface ratio of 1.5 in. (38 mm) and the other using the sealed specific creep value of 0.193×10^{-6} in. per inch per psi (0.280×10^{-7} mm per mm per kPa) with a correspondingly large volume-to-surface ratio of 100 in. (254 mm). Since the tested columns were sealed, shrinkage was not considered in the computations. The analytical results are compared with the test data in Fig. 24. Excellent agreement is observed, irrespective of whether unsealed or sealed creep values are used in computations. It is obviously much simpler to test unsealed, rather than sealed, specimens. Testing of sealed specimens appears to be unnecessary under most circumstances. Realistic prediction of creep deformations appears to be possible on the basis of creep measured on unsealed specimens.

Fig. 24 provides verification of the proposed column strain prediction procedure including various components of the creep model. A direct verification of the shrinkage model, however, was not obtained because the tested columns were sealed.

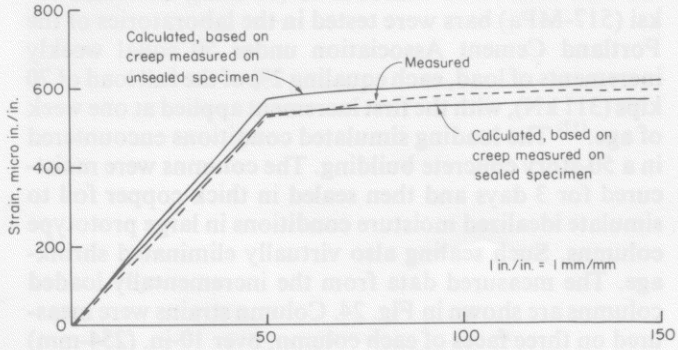


Fig. 24. Verification of proposed analytical procedure.

FIELD OBSERVATIONS

Water Tower Place

Water Tower Place (Fig. 25), a 76-story reinforced concrete building located in Chicago, Illinois, is 859 ft (262 m) tall. It exceeds in height any reinforced concrete building constructed up to this writing. The building was instrumented during construction for measurement of time-dependent deformations.[7] Vertical strains of columns and structural walls were measured at selected floor levels throughout the height of the building.

Paralleling the field study, laboratory tests were performed on concrete cylinders made in the field. The variations with time of compressive strength, elastic modulus, coefficient of expansion, creep, and shrinkage were measured from samples of concrete of each different strength used in the building.

To facilitate proper interpretation of the data, detailed construction records and loading histories of the instrumented columns were maintained.

The 13-story lower part of the structure contains commercial and office space and occupies a full city block, 214x531 ft (65.2x161.8 m). Four basements are below ground level. The 63-story tower, located at one corner of the base structure, measures 94x221 ft (28.7x67.4 m) in

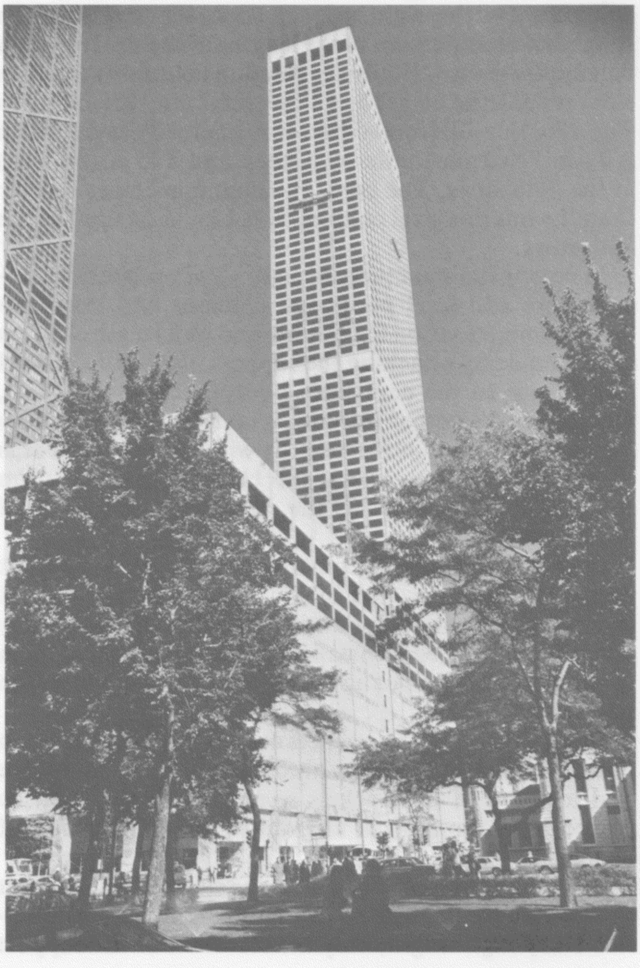


Fig. 25. Water Tower Place, Chicago.

plan. Details of the framing plan are given in Fig. 26. Transfer between the two framing systems occurs at the 14th floor level where 4-ft-wide (1219-mm-wide) by 15-ft-deep (4572-mm-deep) reinforced concrete transfer girders of 9-ksi (62 MPa) concrete spanning about 30 ft (9.14 m) are located.

Field measurements were made at six levels in the building. Instrumented locations were selected to provide measurements for each of the different concrete strengths. Five instrumented levels were located in the tower and one in the base structure below the tower. Within each concrete strength, the individual floors were selected so that access to the instrumentation would be possible for the longest period of time. Details of instrumented columns are given in Table 3 and their locations shown in Fig. 26.

Vertical shortening of individual columns and walls was measured with a 20-in. (508-mm) Whittemore mechanical strain gage. A minimum of three sets of strains were measured on circular columns and four on square columns. For every strain reading, a measurement of air temperature and concrete surface temperature was recorded. All Whittemore gage readings were corrected for changes in temperature of the concrete and a reference standard bar. Measured strains for all instrumented columns were adjusted to a column temperature of 73° F (23° C).

The discussion here on the comparison between measured and predicted column strains is made for all six instrumented levels (74-73, 58-57, 45-44, 32M-32, 16-15, and B4-B3) of Column D2 and for all instrumented columns (D2, C'2, C2-1/2, C'3, B'C3-3/4) on level 32M-32. The recorded loads on these columns are shown in Tables 4 and 5. Listed in Tables 4 and 5 are the increments of loads recorded at various column ages, rather than the total column loads recorded at these ages. The properties of concretes of various strengths used in the columns are listed in Table 6. The moduli of elasticity computed by equation (1) on the basis of the concrete design strengths were used in the computations. The ultimate specific creep values based on unsealed specimen tests, also listed in Table 6, were used. Comparisons between measured and predicted column strains are shown in Fig. 27 for the various levels of column D2 and in Fig. 28 for the various columns on level 32M-32.

Fig. 27 shows that the agreement between the observed and the predicted column strains is most encouraging at all levels of Column D2, except at the very top level. The measured strains on the lightly loaded column at the very top are so erratic that they are probably unpredictable by any analytical model or technique. It should be noted that

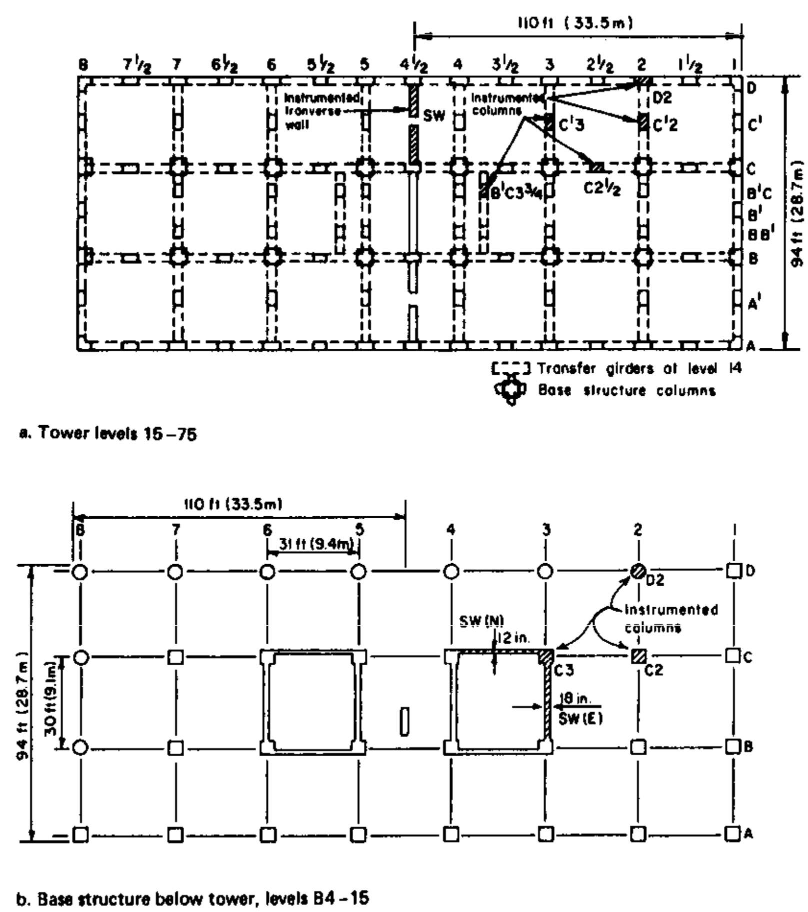


Fig. 26. Location of instrumented columns and walls in Water Tower Place.(7)

Table 3. Properties of Instrumented Columns and Wall of Water Tower Place

Level	Age at initial reading, days	Concrete design strength, psi	Instrumented column size, in., and reinforcement, %				
			Column mark				
			D2	C'2	C2½	C'3	B'C3¾
74-73	34	4000	25x44 0.28	18x24 0.41	18x24 0.41	18x24 0.41	18x18 0.54
58-57	21	5000	10x48 0.78	18x30 0.98	18x40 1.00	18x30 0.98	18x24 0.83
45-44	11	6000	14x48 0.79	18x40 1.11	18x40 2.22	18x40 1.11	18x30 0.89
32M-32	43	7500	16x44 0.88	18x54 0.82	18x54 1.03	18x54 0.82	18x36 2.16
16-15	13	9000	16x48 2.08	18x54 1.44	18x54 1.03	18x54 1.44	18x36 2.47
			D2	C2	C3		
B4-B3	23	9000	48 dia. 7.96	48x48 8.33	48x48 3.70	—	—

1 in. = 25.4 mm 1 psi = 6.895 kPa

Table 4. Incremental Loads on Various Levels of Column D2 of Water Tower Place

Level	74-73	Level	58-57	Level	45-44	Level	32M-32	Level	16-15	Level	B4-B3
Age, days	Incr. load, kips	Age, days	Incr. load, kips	Age, days	Incr. load, kips	Age, days	Incr. load, kips	Age, days	Incr. load, kips	Age, days	Incr. load, kips
15	11	29	182	15	108	15	32	24	171	247	1663
45	15	59	236	44	251	46	236	54	231	278	118
75	3	89	48	74	214	77	246	85	134	306	18
105	3	119	19	104	205	105	193	116	187	369	540
136	2	149	32	134	73	134	230	147	245	400	163
389	1	179	51	164	21	164	253	175	234	430	406
		210	20	194	32	195	49	204	217	461	557
		238	7	224	90	225	21	234	253	530	340
		350	5	255	36	256	32	265	73	561	375
		463	4	283	12	287	129	295	21	618	665
		576	5	395	8	318	51	326	32	619	298
				508	8	346	17	357	180	650	435
				621	8	458	11	388	72	680	505
						571	12	416	24	711	40
						684	12	641	32	741	149
								754	16	772	64
										803	1707
										834	358
										862	227
										974	335
										1087	335
										1200	335

1 kip = 4.448 kN

Table 5. Incremental Loads on Various Columns on Level 32M-32 of Water Tower Place

Age, days	Incremental Load, kips				
	Col. D2	Col. C'2	Col. C2½	Col. C'3	Col. B'C3¾
15	32	20	20	20	20
46	236	168	208	168	188
77	246	172	212	164	194
105	193	551	519	559	145
134	230	266	287	251	167
164	253	288	343	287	171
195	49	49	57	65	19
225	21	—	—	—	—
256	32	77	52	77	33
287	129	95	137	95	44
318	51	86	107	86	38
346	17	86	99	86	39
458	11	60	61	60	27
571	12	60	61	60	27
584	12	61	63	61	28

1 kip = 4.448 kN

Table 6. Properties of Concretes Used in the Columns and Walls of Water Tower Place

Concrete Design Strength, psi				
9000	7500	6000	5000	4000
Compressive Strength at 28 Days, psi*				
9150	7600	7510	7270	4710
Modulus of Elasticity at 28 Days, ksi*				
5260	4580	4270	4090	3920
Computed** Modulus of Elasticity Based on Design Strength, ksi				
5751	5250	4696	4287	3834
Ultimate Shrinkage, 10^{-6} in./in.†				
805	695	630	807	855
Ultimate Specific Creep, 10^{-6} in./in./psi Sealed Specimen Loaded at 28 Days†				
0.480	0.320	0.315	0.420	0.640
Ultimate Specific Creep, 10^{-6} in./in./psi Unsealed Specimen Loaded at 28 Days†				
0.540	0.602	0.700	0.925	1.275

*Average values based on results of three cylinder tests.
**Using equation (1)
†Average values based on results of two cylinder tests. Measurements taken over periods ranging from 3.6 to 5.3 years were projected to time infinity, using equations (5) or (10), as appropriate.

1 in./in. = 1 mm/mm
1 in./in./psi = 0.145 mm/mm/kPa
1 ksi = 6.895 MPa
1 psi = 6.895 kPa

in Reference 7 an attempt was made to analytically predict the field-measured strains. The attempt was far from successful in the case of Level 74-73 of column D2, as is seen in Fig. 29. As far as the other levels of column D2 are concerned, the predictions of this study, the measured column strains, and the analytical computations of Reference 7 can be compared from Figs. 27 and 29. It should be noted that the column strain measurements considered in this study in many cases extend over a longer period of time than the measured strains reported in Reference 7.

Fig 28 shows that the agreement between analytically predicted column strains and the column strains measured on the various instrumented columns on Level 32M-32 is also most encouraging. It should be mentioned, however, that equally good correlation between prediction and observation was not obtained in the case of all other instrumented columns not included in Figs. 27 and 28. It should also be noted that structural restraint to column shortening is not considered in the predictions of this study.

3150 Lake Shore Drive

Two isolated columns and two shearwalls on the second floor of 3150 Lake Shore Drive, Chicago, a 34-story reinforced concrete building, were instrumented to obtain actual creep and shrinkage strains of columns and walls and to compare them with the analytically predicted values according to the procedures of Reference 14.

Fig. 30 shows part of the second-floor plan of the building. Nine measuring points were installed on this level, which serves as a parking garage for the building and was chosen for future accessibility for observations without disturbing tenants. After the building was completed, the garage was enclosed and heated in the winter. Only points 4 and 9 are on isolated columns, all other points are on shearwalls. Shearwall A contains points 1, 2, and 3, and the T-shaped shearwall B contains points 5, 6, 7, and 8. The part of shearwall A containing point 2 terminates below the garage level, and only the two columns containing points 1 and 3 continue to the foundation. These shearwalls were chosen for instrumentation because it was expected that they might produce different creep characteristics due to their support conditions. The normal-weight concrete used for the columns and walls at the second-floor level was specified to have a strength of 5000 psi (34.5 MPa) at 28 days.

Columns and walls of the second floor were cast on November 5, 1962. The measuring points were installed several days later when formwork was removed, and the first readings were taken on November 27, 1962.

The roof of the penthouse was cast on August 25, 1963—9 months and 3 weeks after casting of the second-floor columns. Thus, the construction proceeded at an average rate of 8 calendar days per floor.

Readings were taken at a decreasing frequency until July 11, 1969. One last reading was taken on October 18, 1981. Thus, the total period of observation extended over 19 years.

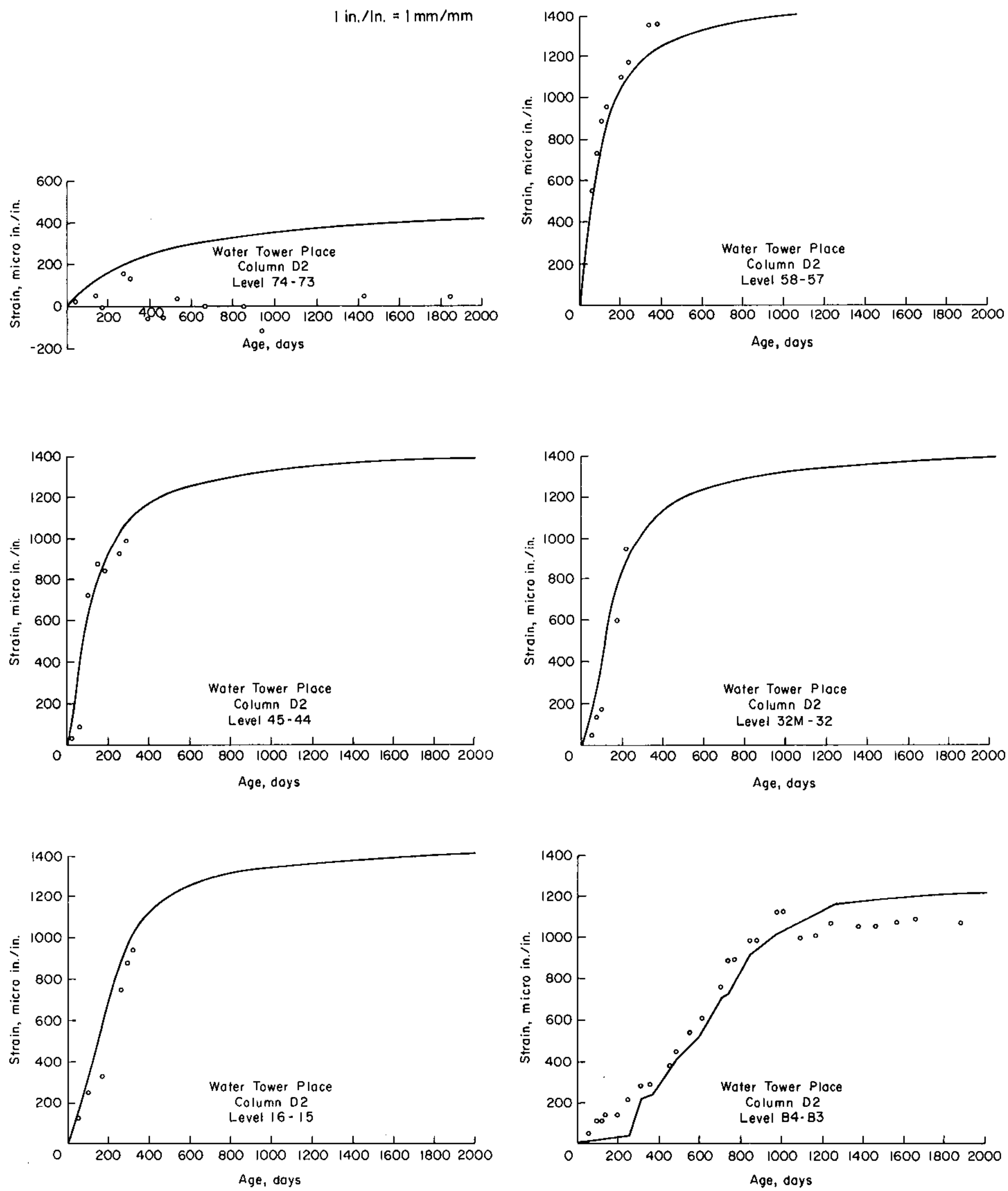


Fig. 27. Comparison of calculated and measured vertical strains for Column D2.

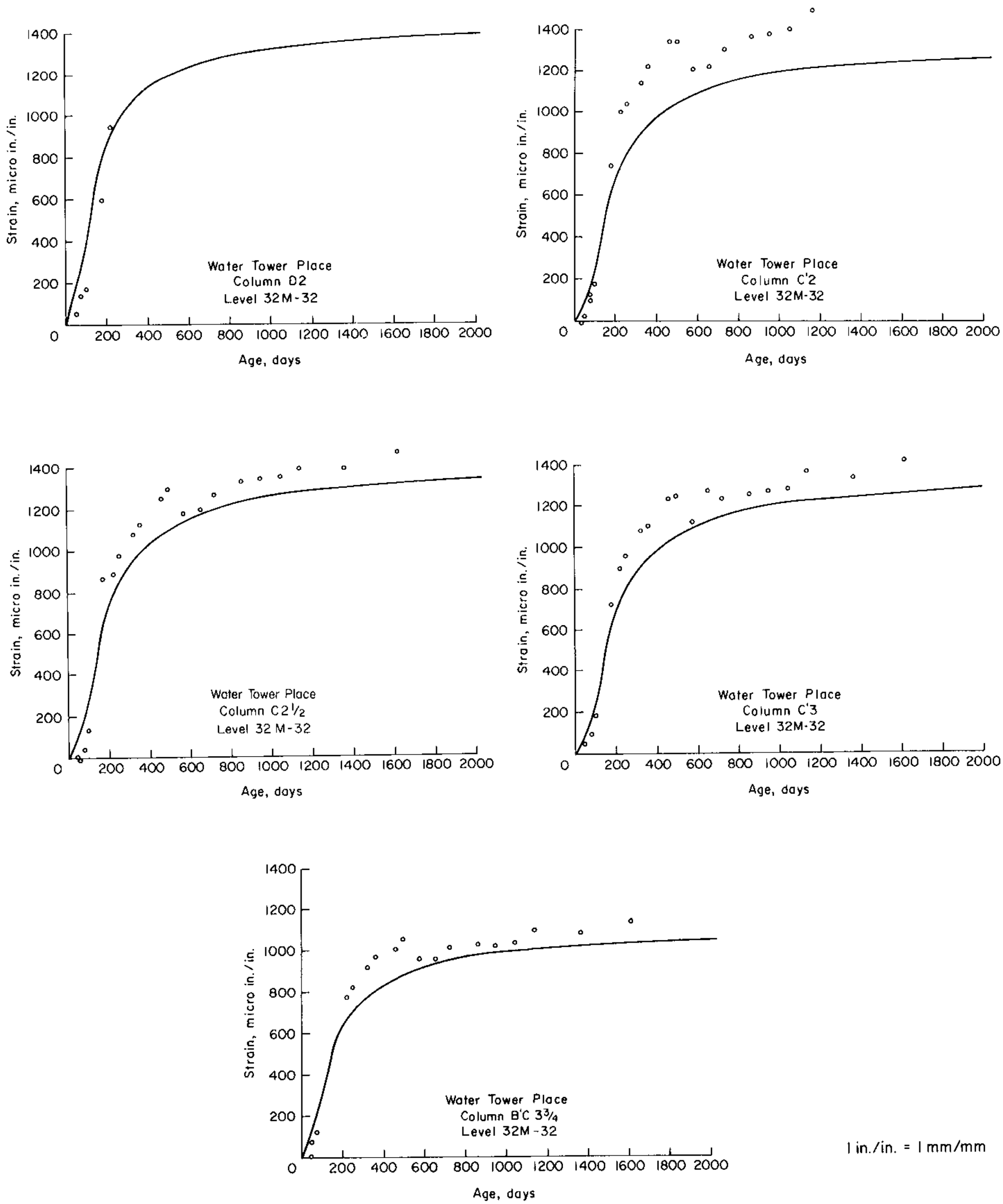


Fig. 28. Comparison of calculated and measured vertical strains for columns on Level 32M-32.

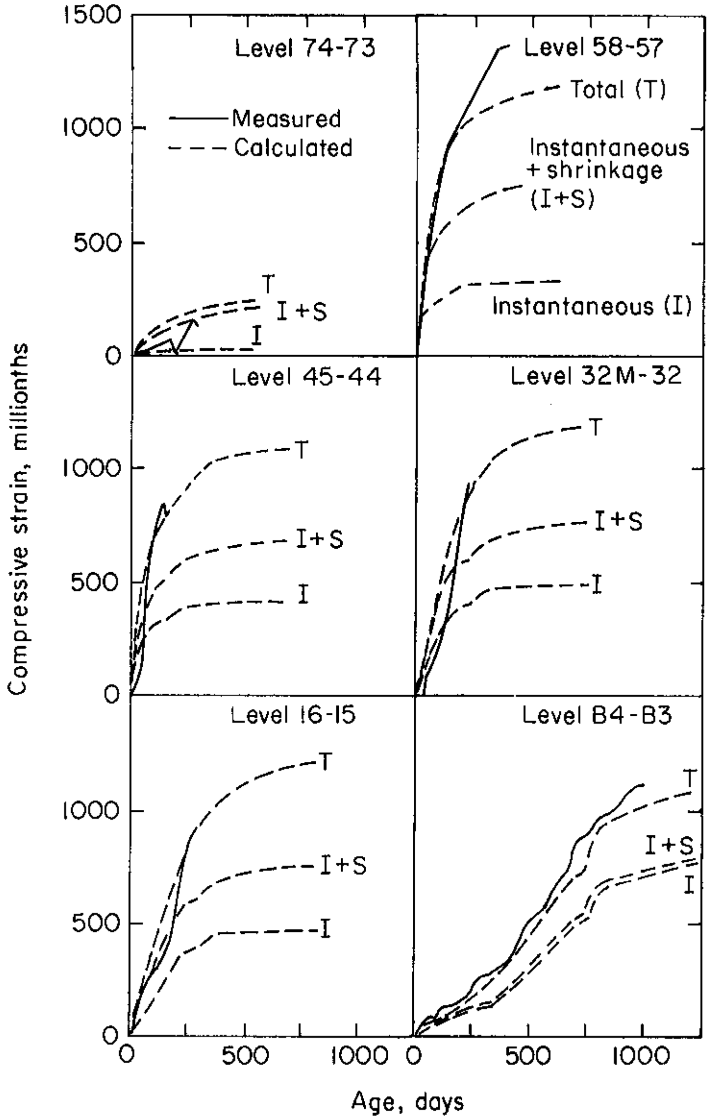


Fig. 29. Comparison of calculated and measured vertical strains for Column D2, as reported in Reference 7.

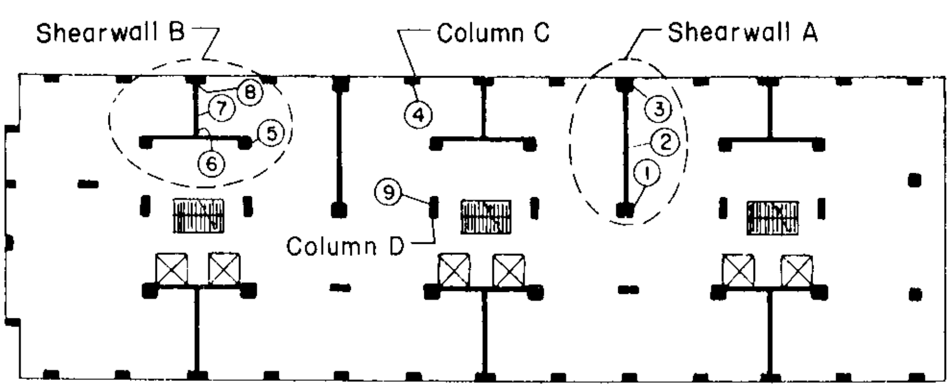


Fig. 30. Location of instrumented columns and walls in 3150 Lake Shore Drive building, Chicago.[13]

Each gage installation consisted of 3 brass knobs, forming two 10-in. (254-mm) gage lengths. The 1/2-in.-diameter (13-mm) brass knobs with holes drilled in their centers were set in grout flush with the surface to protect them from accidental removal. A Whittemore mechanical strain gage, 10 in. (254 mm) in length, was used to measure the distance between the holes in the knobs.

Only during the initial 14 months were the columns and walls subject to ambient temperature changes. After that the building was enclosed and heated in winter; thus, the subsequent temperature fluctuations were insignificant. No temperature corrections were made in the readings.

The results of the field investigation are presented in Table 7, reproduced in abbreviated form from Reference 15. The table lists the physical properties of the columns and walls, such as cross-sectional areas and ratios of reinforcement. The volume-to-surface ratios and the loads are also listed.

Table 8, also reproduced in abbreviated form from Reference 15, shows the analytically predicted inelastic strains, unrestrained by frame action, computed by the procedure in Reference 14. The ratios of measured inelastic strains (measured total strains less computed elastic strains) to the corresponding analytically predicted values are listed in Table 8. The ratios vary between 0.74 and 1.30 with an average of 1.04.

Table 9 shows the analytically predicted column strains, unrestrained by frame action, computed by the procedure in this study, which is basically a computerized and updated version of the earlier Fintel-Khan procedure.[14] The ratios of measured inelastic strains (ultimate values) to the corresponding analytically predicted values range between 0.74 and 1.21, with an average of 1.01, indicating that in the process of computerizing and updating the Fintel-Khan procedure for the present study, accuracy has not been lost—indeed, it may have improved a little.

Also shown in Table 9 are the average (total elastic plus inelastic) column strains based on the last set of measurements taken on October 18, 1981. The gage points on column *D* were missing on that date, so that no readings were obtainable. The corresponding predicted 19-year total strains are listed in Table 9. The ratio of measured to predicted values range between 0.70 and 0.84, indicating that the projected ultimate values of column strains were not closely approached even in 19 years. The predicted strain values in Table 9 do inspire a degree of confidence in the analytical procedure advanced in this study.

FURTHER VERIFICATION

The procedure needs to be further verified against additional field measurements. Some long-term strain measurements are available on the columns and walls of the 70-story Lake Point Tower in Chicago[21] (over a period of 12 years) and on the columns of the 60-story Marina City Towers in Chicago (over a period of 22 years). Some overseas column strain and deformation measurements have also come to the attention of the authors.[22] Instrumentation has been installed on a number of newer build-

Table 7. Column and Shearwall Sizes, Reinforcement, Loads, and Measured Strains from Reference 15

No. of point	Wall or column	Size, in. x in.	Gross area, in.^{2}	Reinforcement: Bars	Reinforcement: Percent, Individual	Reinforcement: Percent, Combined	Volume-to-surface ratio, in.	Dead load, kips	Measured total strain, elastic and inelastic: Indiv.	Measured total strain, elastic and inelastic: Avg.	Computed elastic strain	Measured inelastic strain = measured total strain less elastic strain	Extrapolated inelastic strain to $t = \infty$
1	Wall A	40x40	1600	28#18	7.00	2.60	8.05	3245	375	432	87	345	575
2		12x273	3280	31#4	0.19				430				
3		46x46	2120	16#18	3.0				490				
4	Col. C	16x26	416	24#11*	4.50*		4.95	732	638		303	335	559
5	Wall B	30x29	870	18#11	3.22	1.71	5.80	3654	604	565	123	442	737
6		8x210	1680	14#4	0.17				540				
7		10x126	1260	8#5	0.20				547				
8		24x46	1104	24#11	3.40				570				
9	Col. D	49x20	980	26#11	4.15		7.10	1335	703		239	464	773

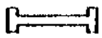
Notes: All strains are given in millionth in. per inch (mm/mm). Columns and shearwalls were cast on November 5-19, 1962. First reading taken on November 27, 1962. Last reading taken June 11, 1969. Total time under observation 6 years 7 months = 79 months. The final inelastic strain for $t = \infty$ is based on the assumption that measurements started at 22 days of age. The observed strain from 22 days until 79 months of age corresponds to 60% of the total inelastic strain.

*This is how the reinforcement in column C was indicated in Reference 13. However, 24#11 bars in a 16x26-in. section translate into 9.0% reinforcement. The computations of this study were based on 4.5% reinforcement in column C.

1 in. = 25.4 mm
1 in.^{2} = 645 mm^2
1 kip = 4.448 kN

Bar Sizes
#4 ≃ 13-mm dia. #11 ≃ 35-mm dia.
#5 ≃ 16-mm dia. #18 ≃ 57-mm dia.

Table 8. Predicted Shrinkage and Creep Strains (Unrestrained by Frame Action) from Reference 15

Column	Area, in.²	v:s ratio, in.	Percent reinf.	D. L., kips	Specific creep, ϵ_{C_∞}	Adjustment to specific creep for v:s ratio	Shrinkage, ϵ_{S_∞}	Predicted total shrinkage and creep, nonreinforced	Residual inelastic strain, reinforced	Measured and extrapolated inelastic strain from Table 7	Measured / Predicted
Wall A Pts. 1, 2, 3	 7000	8.05	2.60	3245	0.330	1.04	1035	98+550 = 648	542	575	1.06
Col. C Pt. 4	16x26 = 416	4.95	4.50	732	0.330	1.17	1035	386+694 = 1080	753	559	0.74
Wall B Pts. 5, 6, 7, 8	 5784	5.80	1.71	3654	0.330	1.10	1035	147+652 = 799	697	737	1.06
Col. D Pt. 9	49x20 = 980	7.1	4.15	1335	0.330	1.06	1035	265+590 = 855	596	773	1.30

Note: All strains are given in millionth in. per inch (mm/mm).

1 in. = 25.4 mm 1 in.² = 645 mm² 1 kip = 4.448 kN

Table 9. Prediction of Column Strains in 3150 Lake Shore Drive Building

Column	Gross area, in.²	v:s ratio in.	Reinf. area, in.²	Strength of normal wt. conc., ksi	Total load, kips	Load application	Specific creep,* ϵ_{C_∞}	Ult. shrinkage, ϵ_{S_∞}	Predicted ult. inelastic strain	Measured and extrapolated inelastic strain from Table 7	Measured / Predicted	Measured total strain @ 19 yrs.	Predicted total strain @ 19 yrs.	Measured / Predicted
Wall A Pts. 1, 2, 3	7000	8.05	182.20	5.0	3245	34 equal increments at 8-day intervals	0.441	1035	551	575	1.04	487	603	0.81
Col.C Pt. 4	416	4.95	18.72	5.0	732	"	0.451	1035	755	559	0.74	729	1043	0.70
Wall B Pts. 5, 6, 7, 8	5784	5.80	98.88	5.0	3654	"	0.437	1035	708	737	1.04	678	811	0.84
Col. D Pt. 9	980	7.10	40.56	5.0	1335	"	0.438	1035	638	773	1.21	—	—	—

*After adjustment for volume-to-surface ratios, these specific creep values are the same as those used (after adjustment for v:s ratio) in Reference 15.

Note: All strains are given in millionth in. per inch (mm/mm)

1 in. = 25.4mm 1 in.² = 145 mm² 1 kip = 4.448 kN 1 ksi = 6.895 MPa

ings like One Magnificent Mile in Chicago and Southeast Financial Center in Miami. Systematizing the field measurements and presenting them in an organized manner would be of immense benefit to the design profession.

SUMMARY AND CONCLUSIONS

In high-rise buildings, the total elastic and inelastic shortening of columns and walls due to gravity loads and shrinkage may be as high as 1 in. for every 80 ft of height. The possibly large absolute amount of cumulative column shortening over the height of the structure in ultra-high-rise buildings is of consequence in its effects on the cladding, finishes, partitions, and so on. These effects can be contained by providing details at every floor level that would allow the vertical structural members to deform without stressing the nonstructural elements. The differential shortening between adjacent columns may cause distortion of slabs leading to impaired serviceability. The tilting of slabs due to length changes of columns caused by gravity loads and shrinkage are permanent; they can and should be compensated for during construction. The permissible tilt of slabs can then be set aside for the unavoidable and uncorrectable transient effects of wind and temperature.

An analytical procedure has been developed to predict the anticipated elastic and inelastic shortenings that will occur in the columns and the walls both before a slab is installed and after its installation. This procedure has been verified and found to be in reasonable agreement with a number of field measurements of column shortening in tall structures that extended over periods of up to 19 years.

It should be noted that a differential between two summations, each of which consists of many components, is very sensitive to the accuracy of the individual components. A small change in an individual component may substantially alter the differential between the two summations.

Design examples of column shortening have been presented for the following structural systems used for ultra-high-rise buildings:

- Reinforced concrete, braced by a frame-shearwall system
- Composite, braced by a reinforced concrete beam-column periphery

The differential shortening between the columns and walls of the 70-story reinforced concrete structure studied was 0.85 and 1.05 in. (21.6 and 26.7 mm), for assumed low and high values of shrinkage and creep, respectively. For the 80-story composite structure investigated, the predicted differential shortening between the reinforced concrete and the steel columns was 2.6 and 4.3 in. (66.0 and 109.2 mm), for assumed low and high values of shrinkage and creep, respectively. Obviously, such differential length changes should be compensated for during construction and the fabricated column detailed accordingly.

The column shortenings predicted by the analytical procedure proposed here are computed column shortenings in the absence of any restraint caused by frame action. In a general case, these computed unrestrained column shortenings are to be input into a general structural analysis program such as ETABS,[23] which in turn will compute the actual restrained (residual) column shortenings.

Limitations on slab distortions caused by column shortening due to gravity loads and shrinkage effects are recommended. For several different structural systems, different construction methods to compensate for differential shortenings are suggested.

Compensation for differential column shortening in reinforced concrete structures is similar to cambering and is relatively simple: the formwork along one edge is raised by the specified amount.

In composite structures with a reinforced concrete peripheral beam-column bracing system, the steel columns are detailed to compensate for all length changes.

In composite structures braced with a reinforced concrete core, differential length changes are compensated for by adjusting the embedment levels of the support plates of the steel beams in the core; the steel columns can be fabricated to the specified story heights.

The apparent greater sensitivity of composite structures to differential shortening makes it desirable to consider such movements and their effects on the slab during design and to plan steps to be taken during construction to limit slab tilt to 3/4 in. (19 mm) or less.

REFERENCES

1. *Building Code Requirements for Reinforced Concrete,* ACI 318-83, American Concrete Institute, Detroit, Mich., 1983.
2. ACI Committee 209, "Prediction of Creep, Shrinkage, and Temperature Effects in Concrete Structures," *Designing for Effects of Creep, Shrinkage, and Temperature in Concrete Structures,* Publication SP-27, American Concrete Institute, Detroit, Mich., 1971, pages 51-93.
3. Martinez, S.; Nilson, A. H.; and Slate, F. O., *Spirally Reinforced High-Strength Concrete Columns,* Research Report No. 82-10, Department of Structural Engineering, Cornell University, Ithaca, N.Y., August 1982.
4. ACI Committee 363, "State-of-the-Art Report on High-Strength Concrete," *ACI Journal, Proceedings,* Vol. 81, No. 4, American Concrete Institute, Detroit, Mich., July-August 1984, pages 364-411.
5. RILEM Commission 42-CEA, "Properties of Set Concrete at Early Ages: State-of-the-Art Report," *Materials and Structures,* Vol. 14, No. 84, Paris, France, November-December 1981, pages 399-450.
6. Byfors, J., *Plain Concrete at Early Ages,* Swedish Cement and Concrete Research Institute, Fo. 3, No. 80, Stockholm, Sweden, 1980, 566 pages.

7. Russell, H. G., and Corley, W. G., *Time-Dependent Behavior of Columns in Water Tower Place,* publication RD052B, Portland Cement Association, Skokie, Ill., 1977, 10 pages.

8. Troxell, G. E.; Davis, H. E.; and Kelly, J. W., *Composition and Properties of Concrete,* 2nd ed., McGraw-Hill Book Co., New York, 1968.

9. Hansen, T. C., and Mattock, A. H., "Influence of Size and Shape of Member on the Shrinkage and Creep of Concrete," *ACI Journal, Proceedings,* Vol. 63, No. 2, American Concrete Institute Detroit, Mich., February 1966, pages 267-289.

10. *Recommendations for an International Code of Practice for Reinforced Concrete,* American Concrete Institute, Detroit, Mich., and Cement and Concrete Association, London, undated.

11. *Manual: Structural Effects of Time-Dependent Behavior of Concrete,* Bulletin D'Information, No. 80, Comité Européen du Béton (CEB), Paris, February 1972.

12. Zia, P.; Preston, H. K.; Scott, N. L.; and Workman, E. B., "Estimating Prestress Losses," *Concrete International: Design and Construction,* Vol. 1, No. 6, American Concrete Institute, Detroit, Mich., June 1979, pages 32-38.

13. Hickey, K. B., *Creep of Concrete Predicted from Elastic Modulus Tests,* Report No. C-1242, United States Department of the Interior, Bureau of Reclamation, Denver, Colo., January 1968.

14. Fintel, M., and Khan, F. R., "Effects of Column Creep and Shrinkage in Tall Structures—Prediction of Inelastic Column Shortening," *ACI Journal, Proceedings,* Vol. 66, No. 12, American Concrete Institute, Detroit, Mich., December 1969, pages 957-967.

15. Fintel, M., and Khan, F. R., "Effects of Column Creep and Shrinkage in Tall Structures—Analysis for Differential Shortening of Columns and Field Observation of Structures," *Symposium on Designing for Effects of Creep, Shrinkage, and Temperature in Concrete Structures,* Publication SP-27, American Concrete Institute, Detroit, Mich., 1971, pages 159-185.

16. Neville, A. M., *Properties of Concrete,* 3rd ed., Pitman, London, 1981, 779 pages.

17. Pfeifer, D. W., and Hognestad, E., "Incremental Loading of Reinforced Lightweight Concrete Columns," *Lightweight Concrete,* Special Publication SP29, American Concrete Institute, Detroit, Mich., 1971, pages 35-45.

18. Troxell, G. E.; Raphael, J. M.; and Davis, R. E., "Long-Time Creep and Shrinkage Tests of Plain and Reinforced Concrete," *Proceedings ASTM,* Vol. 58, 1958, pages 1101-1120.

19. Dischinger, F., *Der Bauingenieur,* Berlin, October 1937 and December 1939.

20. Ghali, A.; Dilger, W.; and Neville, A. M., "Time-Dependent Forces Induced by Settlement of Supports in Continuous Reinforced Concrete Beams," *ACI Journal, Proceedings,* Vol. 66, No. 11, American Concrete Institute, Detroit, Mich., November 1969.

21. Pfeifer, D. W.; Magura, D. D.; Russell, H. G.; and Corley, W. G., "Time-Dependent Deformations in a 70-Story Structure," *Symposium on Designing for Effects of Creep, Shrinkage, and Temperature in Concrete Structures,* Publication SP-27, American Concrete Institute, Detroit, Mich., 1971, pages 159-185.

22. Cridland, L.; Heiman, J. L.; Bakoss, S. L.; and Burfitt, A. J., *Measured and Predicted Strains and Deformations in a Column of a Tall Reinforced Concrete Building,* Civil Engineering Monograph No. C. E. 79/1 ST. E., The New South Wales Institute of Technology, School of Civil Engineering, Sydney, Australia, 1979, 68 pages plus figures.

23. ETABS 84—*Three-Dimensional Static and Dynamic Analysis of Building Systems,* Computers and Structures, Inc., Berkeley, Calif., 1984.

KEYWORDS: multistory concrete buildings, composite buildings, creep, shrinkage, elastic shortening, differential shortening, slab tilt, compensation.

ABSTRACT: A computerized procedure for prediction of elastic and inelastic column length changes in tall buildings has been developed. The procedure, applicable to concrete and composite structures, is presented and illustrated through practical examples. Idealizations of the elastic behavior, shrinkage, and creep of concrete that were used in the computerized procedure are discussed. The proposed procedure is verified against laboratory test results as well as against field observations.

Differential column length changes computed through the proposed analytical procedure can and should be compensated for during construction. Compensation techniques, which must vary with the type of structural system used, are suggested.

REFERENCE: Fintel, Mark; Ghosh, S. K.; and Iyengar, Hal, *Column Shortening in Tall Structures— Prediction and Compensation* (EB108.01D), Portland Cement Association, 1986.

PORTLAND CEMENT ASSOCIATION

An organization of cement manufacturers to improve and extend the uses of portland cement and concrete through scientific research, engineering field work, and market development.

5420 Old Orchard Road, Skokie, Illinois 60077-4321

Printed in U.S.A.

EB108.01D