High-Strength Fiber Reinforced Concrete Utilizing Closely Spaced Reinforcing Bars

by B. Aarup

Synopsis:

CRC (Compact Reinforced Composite) is the designation for a special type of fiber reinforced concrete with high strength (150-400 MPa) and closely spaced reinforcing bars. The dense matrix - with water/powder ratios of typically 0.16 - provide a good bond to fibers and reinforcing bars, and the large content of steel fibers provide the ductility necessary for utilizing reinforcement effectively. The steel fiber content is typically 2-6% by volume and the content of reinforcing steel is 2-10% by volume. The improved durability of the matrix - due to a high micro silica content - makes it possible to use a concrete cover to the reinforcement of only 10 mm in aggressive environments, improving the effectiveness of the reinforcement.

The CRC concept was developed in 1986 and aimed specifically for use in structures such as beams, columns and joints, but most of the applications so far have been in the security industry, for corrosion protection and in heavily loaded floors. However, in the last few years CRC has also been applied in structures.

One of these applications, production of 40,000 drain covers for a tunnel as a replacement for cast-iron covers, is described as an example of a project where the properties of high performance fiber reinforced concrete were utilised.

Keywords: fiber reinforced concretes; high-strength; precast; structural applications
Bendt Aarup (M.Sc., Civil Engineer) has been involved in cement and concrete research since 1988 at the Cement & Concrete Laboratory of Aalborg Portland A/S, PO Box 165, DK-9100 Aalborg. He has mainly worked with High Performance Concrete and Fiber Reinforced Concrete and is now manager of the CRC Marketing Department.

**INTRODUCTION**

High Strength or High Performance Concretes (HSC or HPC) are used increasingly for a range of structural applications and standards in a number of countries are being revised to accommodate these improved materials. As HSC are often more brittle than conventional concretes this can lead to the use of larger factors of safety for these materials and in order to provide these materials with improved ductility it is in some cases advantageous to incorporate fibers in the materials.

Fiber Reinforced Concretes (FRC) have mostly been used in non-structural applications such as slabs, floors, roads and architectural concrete, and typically less than 1% by volume of fibers are used. It is often prohibitive - for reasons of cost as well as in order to ensure acceptable workability and homogeneity - to include larger contents of fibers, but it has been tried with success in a few cases, usually by combining the advantages of fiber reinforcement and HPC, and in these cases other properties besides ductility can be improved as well (1). The high fiber contents have even been used in connection with considerable main reinforcement - as for Slurry Infiltrated Fiber CONcrete (SIFCON)(2) - or with prestressing - as for Reactive Powder Concretes (RPC)(3).

The improved ductility which can be obtained by using fiber reinforcement is necessary to utilize the high compressive strength which can be achieved in HSC in structures where tensile or bending loads are applied. If large contents of main reinforcement are used in brittle concretes, structures will exhibit large cracks at relatively small loads, whereas they will be able to show highly improved behavior if the necessary ductility is achieved. This has been the design philosophy behind Compact Reinforced Composite (CRC), which was developed at the Cement & Concrete Laboratory of Aalborg Portland in 1986 based on experience with DSP-concretes, a type of concrete developed in 1978 (4).
COMPACT REINFORCED COMPOSITE

CRC is the designation for a special type of Fiber Reinforced High Performance Concrete (FRHPC) with high strength (150-400 MPa). The matrix has a very large content of micro silica and water/powder ratios of typically 0.16. A typical composition is shown in table 1.

The binder contains cement, micro silica and a dry superplasticizer. As the cement and micro silica content is quite large there is considerable strength development after 28 days. The maturity curve also differs from that observed for conventional concrete as heat curing is more effective with CRC. One day at 80 °C corresponds to one year of curing at 20 °C.

The type of fibers used have a length of 12.5 mm, a diameter of 0.4 mm and a tensile strength of 1600 MPa. Other types of fibers have been used in earlier research projects with good results - especially brass coated fibers produced by Bekaert with 0.15 mm diameter, 6 mm length and a tensile strength of 2950 MPa. However, while a compressive strength of 180 MPa can easily be achieved with these fibers the price is so high that it would only be realistic to use these fibers for very special applications.

The combination of micro silica, a very low water content and steel fiber contents of 2-6% by volume provides CRC with a good bond as well as with a large ductility and has the effect of ensuring crack-free behavior in e.g. beams at tensile strains of up to 3 mm/m, more than 10 times the strain capacity of conventional concrete.

As an example of the behavior in bending a load-deformation curve for a CRC beam with 10% by volume of main reinforcement is shown in fig. 1. In this case prestressing wires with a yield strength of 1800 MPa have been used as main reinforcement. The wires have not been prestressed, but have been used as conventional reinforcement in order to utilize the high quality of the steel. Even at the very large deformations - 60 mm center deflection on a span of less than 2 meters - only very small cracks were observed. At this stage of loading the bending stress was 320 MPa.

One key factor in utilizing the high quality matrix in FRHPC is to be able to use large contents of reinforcement - and also reinforcement of higher quality - without experiencing large cracks. Another important factor is to be able to place the reinforcement to full advantage. In CRC the size of the fibers and the largest grains of the CRC matrix dictate the distance between reinforcing bars and the cover layer to the reinforcement. This is the reason for typically using a mortar composition for CRC. Often a cover layer of
10-15 mm and a similar distance between individual bars are used. Especially the durability has been the subject of a number of research projects because of these small cover layers, but it has been demonstrated on a number of occasions that 10 mm of cover layer will provide sufficient protection of the reinforcement in an aggressive environment (5).

With the high fiber contents CRC is most suitable for pre-cast applications, but in-situ cast concrete with 6% by volume of fibers has also been produced - for joints between slabs in conventional concrete - using a poker vibrator.

As the properties of FRHPC are not taken into account in existing standards and recommendations, it has been necessary to provide extensive documentation on the properties of CRC, and the material has been the subject of a number of research projects. These have often been carried out in cooperation with universities in Denmark, but CRC has also been the subject of international research projects, such as COMPRESIT, a EUREKA project with partners from Denmark and England and sponsored by the two national Departments of Trade and Industry (6), and MINISTRUCT, a BRITE/EURAM project with partners from Denmark, France and Spain and sponsored by the European Commission (7).

APPLICATIONS

CRC was developed with applications in bridges and tall structures in mind, but some of the first applications of this combination of large fiber contents and closely spaced reinforcement were non-structural, as the concept was used in the security industry, for machine components and in heavily loaded floors where limited space was available.

It was anticipated that CRC could be used as a replacement for steel or cast-iron due to the favorable strength/weight ratio of the material, and this had also been the subject of the first international research project on CRC - COMPRESIT - in which design, production and testing was made of segmental tunnel-linings to be used as replacement for cast-iron linings. The project was concluded in 1993, at the same time as a major infrastructure project was carried out in Denmark - a joint bridge-tunnel project linking the two major islands of Denmark under the name of the Great Belt Link. As problems were anticipated with availability of the cast-iron linings, an inquiry was made to Aalborg Portland as to the availability of CRC-linings. This contact did not lead to the use of CRC-linings as the problems with availability of cast-iron were solved and the designers could maintain the original design, but the contact lead to the application of CRC in another
part of the two tunnels, a part which may seem of limited significance in the larger scale, but a part which was nevertheless well suited as a demonstration of the advantages of FRHPC, and an application which will be used as an example in the following in an attempt to describe the properties which can be of interest to the designers.

**DRAIN COVERS FOR THE GREAT BELT LINK**

The covers placed over the drain channels in the Great Belt tunnels were originally designed in cast-iron, but it was discovered that this could cause problems with stray currents for the electrical installations in the tunnel. The drain channels were an integrated part of the lining design, so the dimensions of the covers were fixed with a thickness of 40 mm and a span of 500 mm. As the covers were to be placed under the rails, they had to sustain an ultimate static load of 47 kN/m which would necessitate the use of reinforcement. The small thickness, however, was not compatible with the requirement for a cover of at least 45 mm to ensure a design life of 100 years in the aggressive environment.

A cross-section of the design made to suit these criteria is shown in fig. 2. The covers have the dimensions 590x412x40 mm. The reinforcement - 8 mm bars with a specified yield strength of 550 MPa - is closely spaced and the cover to the reinforcement is 10 mm. Compressive strength of the matrix was 150 MPa with 6% of fibers with dimensions 12.5x0.4 mm.

With regard to resistance to intrusion of chloride ions, measurements had been made on specimens under load exposed to chlorides, and these investigations show that the diffusion coefficient for chloride ions measured after two years of exposure under accelerated conditions was only $8 \times 10^{15}$ m$^2$/s, in which case corrosion should not be a problem. Tests performed later indicate that the chloride threshold value is also considerably higher for these very dense concretes.

Production of a total of 40,000 covers were carried out by a pre-cast manufacturer and the covers were installed in the tunnel in the spring of 1995. As the design was not exactly according to the current standards, it was decided to adhere to a rather strict program of quality control, which included testing of 1% of the covers produced in three-point bending after 7 days, after which a control was made of the cover to the reinforcement. Also compressive strength was measured on 100x200 cylinders cured at 80 $^\circ$C for one day. Average compressive strength of the reference specimens tested during the production was slightly above 150 MPa, while average strength of the more than 400 drain-covers tested was 216 kN/m. This
strength was considerably higher than necessary for the static case, but the high strength was used to ensure the capacity in fatigue as well, as $3.2 \times 10^8$ load cycles had to be considered in the design life of 100 years.

**DISCUSSION**

The application of CRC for the drain-covers of the Great Belt Link is an example of an application of FRHPC where properties of durability and fatigue as well as bending strength have been utilized.

Even though the properties of FRHPC are well documented for a number of compositions, the first applications are probably going to be similar to this case, where CRC was considered due to a problem encountered after the design was made - for trouble-shooting. Another solution for the owner could have been to electrically isolate the cast-iron covers. However, the risk of using a new material in this application was moderate, as the design allowed for full-scale testing of the individual covers, a strict quality control was maintained and failure of the covers would - while very costly - not lead to dangerous situations. Finally, the cost of the CRC covers was only about 60% of the cost for the cast-iron covers which could also have been of some significance in the considerations.

In the last few years CRC has been used for other structural applications, mostly as pre-cast components such as balcony slabs, manhole covers and facade elements, and as the components are increasing in size the results from these applications - combined with results from the joints which are cast on-site and results from applications of other types of FRHPC such as SIFCON, SIMCON and RPC - should provide designers with the references necessary for considering the use in columns and large beams where the properties of FRHPC can also be utilised.

**REFERENCES**


### TABLE 1—TYPICAL CRC COMPOSITION AND PROPERTIES MEASURED AT 28 DAYS.

<table>
<thead>
<tr>
<th></th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>binder</td>
<td>940 kg/m³</td>
</tr>
<tr>
<td>quartz sand 0-0.25 mm</td>
<td>170 kg/m³</td>
</tr>
<tr>
<td>0.25-1 mm</td>
<td>340 kg/m³</td>
</tr>
<tr>
<td>1-4 mm</td>
<td>680 kg/m³</td>
</tr>
<tr>
<td>water</td>
<td>150 kg/m³</td>
</tr>
<tr>
<td>steel fibers</td>
<td>475 kg/m³</td>
</tr>
<tr>
<td>compressive strength [MPa]</td>
<td>140</td>
</tr>
<tr>
<td>bending strength [MPa]</td>
<td>20</td>
</tr>
<tr>
<td>Young's modulus [GPa]</td>
<td>47</td>
</tr>
<tr>
<td>density [kg/m³]</td>
<td>2750</td>
</tr>
</tbody>
</table>
Fig. 1—Load-deformation behavior of CRC beam with prestressing wires as main reinforcement.

σ_b = 323 MPa

Fig. 2—Cross section of CRC drain covers used for the Great Belt Link.
Structural Behavior of Steel Fiber Reinforced Concrete Beams in Shear

by B. Oh, D. Lim, K. Hong, S. Yoo and S. Chae

Synopsis: The structural behavior of steel fiber reinforced concrete beams in shear is studied. A comprehensive experimental program has been set up and several series of reinforced concrete beams with steel fibers have been tested. The test variables include the volume contents of steel fibers and stirrups. The fiber contents varies from 0% to 2% by volume. It is seen from these tests that the cracking and ultimate shear strengths increase as fiber contents increase. The present study indicates that fiber reinforcement can reduce the amount of shear stirrups to obtain same strength. The combination of fibers and stirrups may accomplish strength requirements as well as ductility requirements. A theoretical approach is proposed to predict the shear strength of reinforced concrete beams containing steel fibers and good correlation is obtained with test data. The present study allows more efficient structural application of steel fibers for shear reinforcement in reinforced concrete structures.

Keywords: cracking shear strength; fiber reinforced concrete; shear analysis; shear reinforcements; steel fibers; ultimate shear strength
Byung I-Iwan, Oh, Ph. D, ACI member, is a professor of Civil Engineering at Seoul National University, Seoul, Korea. He received his Ph. D from Northwestern University in 1982. Dr. Oh has been the editor-in-chief of several journals in Korea. His research interests include analysis of reinforced and prestressed concrete structures, mechanical behavior of fiber-reinforced concretes and high performance concretes.

Dong Hwan, Lim, Ph. D, is a lecturer at the Dong Seo University, Pusan, Korea. He received his Ph. D from Seoul National University in 1994. His research interests include analysis of reinforced and prestressed concrete structures.

Kyung Ok, Hong is a Ph. D candidate in the Department of Civil Engineering at Seoul National University, Seoul, Korea. He is also working on the flexural and shear behavior of high strength concrete members.

Sung Won, Yoo is a Ph. D candidate in the Department of Civil Engineering at Seoul National University, Seoul, Korea. He is currently working on the analysis of prestressed structures as well as the fiber reinforced concrete.

Sung Tae, Chae is a graduate research assistant in the Department of Civil Engineering at Seoul National University, Seoul, Korea. He is currently working on the analysis of underground structures as well as the fiber reinforced concrete.

INTRODUCTION

The brittle nature of concrete causes collapse in unreinforced beams shortly after the formation of the first crack. The addition of steel fibers aids in converting the brittle characteristics to a ductile one. The principal role of fibers is to resist the formation and growth of cracks by providing pinching force at the crack tip. In addition, a marginal improvement in tensile strength also results and fiber reinforced concrete have higher ultimate strain than plain concrete (1-12).

Many studies have been conducted to determine the flexural behavior of steel fiber reinforced concrete beams. The authors recently published a paper concerning the flexural analysis of reinforced concrete beams containing steel fibers as Ref.1. However, only a few studies (5,10,11) are available on the shear behavior of reinforced concrete beams with steel fibers and the shear behavior of those beams is not well established yet. The purpose of the present study is therefore to explore the mechanical behavior of steel fiber reinforced concrete beams in shear for structural
The tests were carried out on concrete beams reinforced with stirrups and steel fibers. The purpose of the present study is (1) to examine the effect of fiber addition on the shear behavior of reinforced concrete beams, (2) to study the potential use of fibers to replace the stirrups, and (3) to investigate effective combinations of stirrups and steel fibers for improvement of ultimate and cracking shear strengths as well as ductility. Based on the present test results and earlier published studies, a method of predicting ultimate shear strength for beams reinforced with both stirrups and steel fibers is proposed.

RESEARCH SIGNIFICANCE

The addition of steel fibers in reinforced concrete beams results in a substantial increase in strength and ductility. These phenomena have been verified for fiber reinforced concrete beams in flexure [1]. The behavior of reinforced concrete beams containing steel fibers under shear loading is not relatively well established. Moreover, quantitative amounts of shear resistance due to steel fibers must be clarified in order to determine the shear capacity of fiber reinforced concrete beams.

The purpose of the present study is to investigate a possibility of optimum combination of steel fibers and shear stirrups to achieve the strength requirement as well as ductility requirement. To accomplish this goal, a comprehensive experimental study was planned and conducted in the present study.

DIMENSION OF TEST MEMBERS

The test program here consists of testing several series of reinforced concrete beams having identical rectangular cross sections of 100mm × 180mm. The span length of the members is 1300mm and the shear span length is 400mm. The effective depth of the beam is 150mm. A total of 9 beams were tested to investigate the influence of fiber reinforcement on the mechanical behavior of reinforced concrete beams in shear. The major test variables are the amount of steel fibers and the volume of shear stirrups. The fiber contents were varied from 0% to 2% of the concrete total volume and the ratios stirrups from 0% to 100% of the code-required values. Table 1 describes the details of the beams and the major test variables.

The present tests are to examine the influence of fiber addition on the shear behavior of reinforced concrete beams and to determine the effectiveness of steel fibers on the shear resistance.
MATERIALS AND FABRICATION OF TEST MEMBERS

The ordinary portland cement (TYPE I) was used and the pea gravel with maximum aggregate size of 10mm was used. The sand fineness modulus is about 2.21. The round straight steel fibers with 0.7mm diameter, 42mm length and ultimate strength of 1784MPa are used. The aspect ratio of steel fiber is 60. The material properties of steel fibers are summarized in Table 2. The longitudinal deformed steel, 16mm diameter (tensile steel), 10mm diameter (compressive steel) with yield strength of 420MPa and 6mm diameter steel for stirrups were used. The actual yield stress of stirrups was 359 MPa.

The mix proportion of concrete of test members is listed in Table 3. The mix was designed to obtain the compressive strength of 35MPa at 28days. The aggregates, cement, and water were batched by weight and mixed in a drum mixer. The fibers were introduced last and dispersed uniformly. Although the fibrous mixes were less workable than plain concrete, the mix procedures proved satisfactory in that the dispersion of fibers was found to be uniform and there was no significant fiber balling. The slump value was about 12cm. The 100 φ X 200mm concrete cylinder specimens for compression and split-tensile test and 100 X 100 X 400 beam specimens mold for flexural tests were also cast from the same mix as were the corresponding.

All test beams were vibrated by a 25 mm diameter internal poker vibrator until satisfactory compaction was achieved. Following casting, the concrete beams and control specimens were covered with wet burlap. All specimens were stripped from their molds after 24 hours and then stored in a curing water tub.

TEST PROCEDURES AND MEASUREMENTS

The four-point loading method has been applied to the test beam as two equal concentrated loads by means of a steel spread beam (see Fig.1). A calibrated load cell was placed between the jack and spread beam. Three dial gages with 0.01mm accuracy were used to measure the deflections under increasing loads at several locations. Steel strain gages were attached on longitudinal steel and stirrups, and concrete strain gages were attached for searching concrete strains and principal stress direction. Locations of measuring sensors are shown in Fig.1. The loads were applied step by step in small increments up to the ultimate load. The amount of incremental load at each step was 5 kN (500kg). All the measuring values were automatically scanned and stored in the computer at each loading step. These test results were analyzed later by the computer.
TEST RESULTS AND DISCUSSIONS

Strength Characteristics and Modes of Failure

Compressive strength—The compressive strengths of test cylinders were measured for various cases of fiber contents. Fig. 2 shows the relative strength of concrete in compression due to the addition of steel fibers. The strength increase was about 25% when fibers are introduced into concrete up to 2% by volume. The actual strength values are summarized in Table 4.

Flexural strength—The present test indicate that the flexural strength of fiber reinforced concrete is greatly enhanced due to the addition of steel fibers. The increase of flexural strength was about 55% when the fiber content was increased to two percent (see Fig. 3). One more important feature in flexural behavior is that the fiber concrete showed great ductility and energy absorption capacity.

Splitting tensile strength—Figure 4 shows the increase of tensile strength due to steel fibers. It can be seen that splitting tensile strength was increased more than twice at 2-percent fiber volume. Figure 5 shows a comparison of various relative strengths for various fiber contents. The relative strength indicates the normalized value to the strength of plain concrete without fibers. It can be seen from this figure that the rate of strength increase due to fiber addition is greatest in splitting tensile strength. This means that the steel fiber greatly enhances the tensile properties of concrete and improves the resistance to cracking.

Modes of failure—All the beams exhibited similar linear behavior from initial loading up to the occurrence of first hair-line crack. The beam S0V0, which is without shear reinforcement, failed soon after the formation of the diagonal crack. The cracks initiated and progressed along the compressive stress trajectories in the shear span and led very rapidly to a catastrophic type of failure. On the other hand, the beams with fiber reinforcement (beams S0V1 and S0V2) continued to resist higher shear stress and a considerable ductility was observed. As shown at the failure modes of the S0-beam series, the mode failure changed from shear to flexural types as fibers contents increase. The ductility is also enhanced greatly with the addition of fibers. It can be also seen that the beams with 50% of conventional code-required stirrups and 1% of steel fiber contents had a flexural type of failure. The modes of failure are shown at Fig. 6. It can be seen from this figure that fiber contents of 1% is critical point of transferring failure modes (from shear mode to flexural mode). In beams with stirrups(S0.5V0, S0.75V0, S1V0), some spalling occurred at ultimate stage. But the inclusion of steel fibers eliminated the spalling of concrete : steel fibers enhanced the capacity to hold parts of concrete together in post cracking stage, thus preventing the spalling even at ultimate failure.
Load-Deflection Characteristics

The load-deflection curves for the test beams are shown at Fig. 7-9. No displacements were observed at supports since the supports were fixed. From these figures, it can be seen that all the beams exhibit linear behavior from initial loading up to the occurrence of first diagonal crack. After formation of cracks, all the beams reveal nonlinear load-deflection characteristics.

Comparison of peak deflections at ultimate loads reveals that there was great improvement in the ductility due to use of fibers. Beams without stirrups exhibit much improvement in the ultimate strength (Fig. 7), but beams with stirrups (Fig. 8, 9) exhibit small improvement in ultimate strength due to fiber addition compared with the beam without stirrups. These phenomena can be clearly seen from Fig. 7 - Fig. 9.

Cracking Shear Strength

One of predominant effects due to steel fibers is the increase of cracking shear strength. Generally, cracking shear strength of fiber reinforced concrete is higher than that of conventional reinforced concrete.

Fig. 10 shows cracking shear strength as it varies with fiber contents. From these test results, it can be seen that fiber inclusion increases cracking shear strength greatly. This means that, for the increase of cracking shear strength and ductility, the addition of steel fibers is better than an increase of stirrups.

Ultimate Shear Strength

Fig. 11 shows ultimate shear strength according to fiber contents for each beam. From this figure, it can be seen that fiber reinforced concrete beams exhibit much improvements in ultimate shear strength compared with the companion beams without fibers. The increase of shear strength due to fibers becomes larger for the beams with lower shear stirrups. This means that the steel fibers play a much more role for lightly reinforced conventional beams.

It is found from Fig. 11 that the combination of 75% of full conventional stirrups with 1% steel fiber volume contents (S0.75V1) gives the same ultimate shear strength as the conventional beam (S1V0), and that the combination of 50% of full conventional stirrups with 2% fiber contents (S0.5V2) also gives almost similar shear resistance to the beam with full stirrups without fibers (S1V0).

From these test results, it can be concluded that fiber reinforcement can reduce the needed amount of shear stirrups and
that the combination of fibers and stirrups may accomplish the strength requirement as well as the ductility requirement.

THEORETICAL PREDICTION OF SHEAR STRENGTH FOR STEEL FIBER REINFORCED CONCRETE BEAMS

Fig. 12 shows the free body diagram of parts of the shear span of a simply supported fiber reinforced concrete beam. From this figure, the total shear forces, \( V_u \), can be written as follows,

\[
V_u = V_c + V_{ay} + V_d + V_{fy}
\]  

where, \( V_c \) : shearing force across the compression zone resisted by concrete,

\( V_{ay} \) : aggregate interlocking force (vertical component),

\( V_d \) : dowel action force,

\( V_{fy} \) : vertical component of fiber pull out force along the inclined crack.

Considering the shear resistance of concrete without web reinforcement, \( V_{uc} \), the equation of ultimate shear strength can be written as follows (13),

\[
V_{uc} = (10 \rho f'c \frac{d}{a_s}) \frac{1}{3} bd \quad \text{for} \quad \frac{a_s}{d} \geq 2.5
\]

\[
V_{uc} = (160 \rho f'c \frac{d}{a_s}) \frac{4}{3} bd \quad \text{for} \quad \frac{a_s}{d} \leq 2.5
\]

where, \( \rho \) = reinforcement ratio, \( f'c \) = compressive strength of concrete (MPa), \( d \) = effective depth (m), \( b \) = width of the beam (m), \( a_s \) = shear span (m).

However, as our tests conducted on reinforced concrete beams with web reinforcements, one must consider the contribution of the stirrups, \( V_{us} = A_v f_v d/s \). Thus the equation of ultimate shear strength must be written as \( V_u = V_{uc} + V_{us} \), i.e.,

\[
V_u = (10 \rho f'c \frac{d}{a_s}) \frac{1}{3} bd + \frac{A_v f_v}{s} d \quad \text{for} \quad \frac{a_s}{d} \geq 2.5
\]

\[
V_u = (160 \rho f'c \frac{d}{a_s}) \frac{4}{3} bd + \frac{A_v f_v}{s} d \quad \text{for} \quad \frac{a_s}{d} \leq 2.5
\]

where, \( A_v \) = area of stirrups (m²), \( f_v \) = yield strength of stirrups (MPa), \( s \) = spacing of stirrups (m).

To estimate the contribution of the steel fibers to the shear resistance, the neutral axis depth (c) has to be calculated first. The external moment (\( V_u \cdot a_s \)) must be equal to internal ultimate resisting moment (\( M_u \)).
\[ M_u = 0.85f'_c ab(d - a/2) + A_{s'}f'_s(d - d') \]  

Where, \( A_{s'} \) = area of compressive steel, \( f'_s \) = compressive steel stress, and \( d' \) = cover of compressive steel.

From Eq.(6), one can determine rectangular stress block depth "a", and the neutral axis depth is calculated as \( c = a/\beta \).

Referring to Fig. 12, the length of the inclined shear crack is equal to \((h-c)/\sin \alpha\), and the area through which the steel fibers contribute to shear resistance of the beam is \( b(h-c)/\sin \alpha\). The number of fibers, \( n \), crossing a unit area of the crack may be taken as \((5,10)\)

\[ n = 0.5 \frac{V_f}{\pi r_f^2} \]

Where, \( V_f \) = volume fraction of fiber, \( r_f \) = radius of fiber.

At failure, fiber pull-out invariably occurs, since the fiber length \( (l_r) \) is usually less than the critical length necessary to develop the ultimate tensile strength of fiber, and also due to the relative displacement of two faces of the crack. Since failure is by fiber pull-out, it has been shown \((4,5)\) that the mean fiber pull-out length is \( l_r/4 \). The average pull-out force \( F_0 \) for a fiber is then given by;

\[ F_0 = \tau \pi d_t \frac{l_r}{4} \]

Where, \( \tau \) = average bond strength which is defined as half the ultimate bond strength, i.e., \( \tau_u/2 \), where \( \tau_u = 4.15 \text{N/mm}^2 \) given by Swamy et al. \((3,4)\).

The ultimate force sustained by steel fibers per unit area of crack at failure is therefore given by;

\[ F_1 = nF_0 = n \tau \pi d_t \frac{l_r}{4} \]

Substituting Eq.(7) into Eq.(9), the following equation is obtained for \( F_1 \),

\[ F_1 = 0.5 \tau V_f \frac{l_r}{d_t} \]

The total force \( F_t \) perpendicular to the crack is therefore;

\[ F_t = F_1 \left( \frac{b(h-c)}{\sin \alpha} \right) \]

The vertical component of this force is equal to the increase in shear resistance of the beam due to the presence of steel fibers and
is equal to:
\[ V_{ty} = F_1 \sin \alpha = F_1 b (h-c) \]  (12)

Finally, the total shear strength of fiber reinforced concrete beam can be calculated as the sum of Eq.(4) and Eq.(12). [or Eq.(5) and Eq.(12)]
\[ V_u = V_{uc} + V_{us} + V_{ty} \]  (13)
or Eq.(13) can be written in stress form as
\[ v_u = \frac{V_u}{bd} \]  (14)

COMPARISONS OF THE THEORY WITH TEST RESULTS

Table 4 and Fig.13 represent the results of the comparison of the observed ultimate shear strength \( V_{uo} \) with the predicted ultimate shear strength \( V_{up} \) given by this analysis method. The predicted values using this method give acceptable results.

The mean value of the ratio of the observed ultimate shear strength to predicted ultimate shear strength for test beams was about 0.92 with a standard deviation of 0.08 and coefficient of variation of 8.7 percent.

CONCLUSIONS

The structural behavior of steel fiber reinforced concrete beams subjected to shear is studied. A series of beams have been tested to investigate the influence of fiber reinforcement on the mechanical behavior of reinforced concrete beams in shear. The major test variables are the amount of steel fibers and the volume of shear stirrups. The fiber contents were varied from 0% to 2% by volume and the amount of stirrups varied from nothing to full reinforcement of code-required values.

From the present experimental and analytical study on the shear behavior of steel fiber reinforced concrete beams, the following conclusions may be drawn.

1. The addition of steel fibers increases compression strength, flexural strength and splitting tensile strength, with the rate of strength increase greatest in splitting tensile strength. This means that the steel fibers greatly enhance the tensile properties of concrete and improve the resistance to cracking.

2. The cracking shear strength increased significantly due to the
addition of fibers, and the increase of the strength was about 100% when the fiber content was increased from 0% to 2%. Thus, the addition of fibers is effective in controlling shear cracks.

3. The mode of failure of conventional reinforced concrete beam underreinforced in shear changed from shear failure to flexural failure types when the volume fraction of fibers exceeds 1%. This means that fiber reinforcement increases shear capacity greatly.

4. The use of steel fibers can reduce the amount of shear stirrups needed, and a combination of fibers and stirrups may accomplish strength requirement as well as ductility requirement. It is found that either the combinations of 75% of full conventional stirrups and 1% fiber volume content or 50% of full conventional stirrups and 2% fiber content may be good optimized combinations for shear ductility and strength.

5. A theoretical method to predict the shear strength of reinforced concrete beams containing steel fibers is presented and good comparisons are made with test results. The method will allow more realistic shear analysis of reinforced concrete structural members containing steel fibers.

REFERENCES


TABLE 1—TEST BEAM DETAILS AND TEST VARIABLES.

<table>
<thead>
<tr>
<th>Identification of beams</th>
<th>Spacing (mm)</th>
<th>( \rho_v ) (%)</th>
<th>Aspect ratio ( (\ell/d) )</th>
<th>( V_t ) (%)</th>
<th>( f'_c ) (MPa)</th>
<th>( f_{sp} ) (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>S0V0</td>
<td>-</td>
<td>0.000 (0%)</td>
<td>60</td>
<td>0</td>
<td>34.0</td>
<td>2.5</td>
</tr>
<tr>
<td>S0.5V0</td>
<td>80</td>
<td>0.007 (50%)</td>
<td>60</td>
<td>0</td>
<td>34.0</td>
<td>2.5</td>
</tr>
<tr>
<td>S0.75V0</td>
<td>60</td>
<td>0.009 (75%)</td>
<td>60</td>
<td>0</td>
<td>34.0</td>
<td>2.5</td>
</tr>
<tr>
<td>S1V0</td>
<td>40</td>
<td>0.014 (100%)</td>
<td>60</td>
<td>0</td>
<td>34.0</td>
<td>2.5</td>
</tr>
<tr>
<td>S0V1</td>
<td>-</td>
<td>0.000 (0%)</td>
<td>60</td>
<td>1</td>
<td>38.7</td>
<td>4.0</td>
</tr>
<tr>
<td>S0.5V1</td>
<td>80</td>
<td>0.007 (50%)</td>
<td>60</td>
<td>1</td>
<td>38.7</td>
<td>4.0</td>
</tr>
<tr>
<td>S0.75V1</td>
<td>60</td>
<td>0.009 (75%)</td>
<td>60</td>
<td>1</td>
<td>38.7</td>
<td>4.0</td>
</tr>
<tr>
<td>S0V2</td>
<td>-</td>
<td>0.000 (0%)</td>
<td>60</td>
<td>2</td>
<td>42.4</td>
<td>5.1</td>
</tr>
<tr>
<td>S0.5V2</td>
<td>80</td>
<td>0.007 (50%)</td>
<td>60</td>
<td>2</td>
<td>42.4</td>
<td>5.1</td>
</tr>
</tbody>
</table>

NOTE: \( \rho_v = A_v/bs = \) shear reinforcement ratio. * The values in parenthesis represent the percent ratio of shear stirrups to the code-required full shear reinforcements.

TABLE 2—MATERIAL PROPERTIES OF STEEL BARS AND STEEL FIBERS.

<table>
<thead>
<tr>
<th></th>
<th>Yield strength (MPa)</th>
<th>Tensile strength (MPa)</th>
<th>Modulus of elasticity ( (\times 10^5 ) MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Longitudinal Steel</td>
<td>420</td>
<td>545</td>
<td>2.0</td>
</tr>
<tr>
<td>Stirrup</td>
<td>359</td>
<td>534</td>
<td>2.0</td>
</tr>
<tr>
<td>Steel fiber</td>
<td>1,303</td>
<td>1,784</td>
<td>2.0</td>
</tr>
</tbody>
</table>
### Structural Applications of Fiber Reinforced Concrete

21

#### TABLE 3—PROPORTIONS OF MIX DESIGN.

<table>
<thead>
<tr>
<th>Contents</th>
<th>Cement</th>
<th>Water</th>
<th>Sand</th>
<th>Aggregate</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>450</td>
<td>171</td>
<td>707</td>
<td>1,060</td>
</tr>
</tbody>
</table>

#### TABLE 4—COMPARISON OF THE OBSERVED ULTIMATE SHEAR STRENGTH $V_{uo}$ WITH THE PREDICTED ULTIMATE SHEAR STRENGTH $V_{up}$ FOR VARIOUS TEST SERIES.

<table>
<thead>
<tr>
<th>Identification of beams</th>
<th>Stirrup Spacing (mm)</th>
<th>$\rho_v$ (%)</th>
<th>$\ell/d$</th>
<th>$V_v$ (%)</th>
<th>$f'_c$</th>
<th>$f'_{sp}$</th>
<th>$V_c$</th>
<th>$V_uo$</th>
<th>$V_{up}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>S0V0</td>
<td>-</td>
<td>0.000( 0%)</td>
<td>60</td>
<td>0</td>
<td>34.0</td>
<td>2.5</td>
<td>0.78</td>
<td>3.56</td>
<td>3.38</td>
</tr>
<tr>
<td>S0.5V0</td>
<td>80</td>
<td>0.007( 50%)</td>
<td>60</td>
<td>0</td>
<td>34.0</td>
<td>2.5</td>
<td>1.09</td>
<td>5.35</td>
<td>5.34</td>
</tr>
<tr>
<td>S0.75V0</td>
<td>60</td>
<td>0.009( 75%)</td>
<td>60</td>
<td>0</td>
<td>34.0</td>
<td>2.5</td>
<td>1.18</td>
<td>6.36</td>
<td>6.30</td>
</tr>
<tr>
<td>S1V0</td>
<td>40</td>
<td>0.014(100%)</td>
<td>60</td>
<td>0</td>
<td>34.0</td>
<td>2.5</td>
<td>1.25</td>
<td>7.00</td>
<td>7.77</td>
</tr>
<tr>
<td>S0V1</td>
<td>-</td>
<td>0.000( 0%)</td>
<td>60</td>
<td>1</td>
<td>38.7</td>
<td>4.0</td>
<td>1.49</td>
<td>4.49</td>
<td>4.98</td>
</tr>
<tr>
<td>S0.5V1</td>
<td>80</td>
<td>0.007( 50%)</td>
<td>60</td>
<td>1</td>
<td>38.7</td>
<td>4.0</td>
<td>1.69</td>
<td>5.73</td>
<td>7.00</td>
</tr>
<tr>
<td>S0.75V1</td>
<td>60</td>
<td>0.009( 75%)</td>
<td>60</td>
<td>1</td>
<td>38.7</td>
<td>4.0</td>
<td>1.84</td>
<td>7.00</td>
<td>7.70</td>
</tr>
<tr>
<td>S0V2</td>
<td>-</td>
<td>0.000( 0%)</td>
<td>60</td>
<td>2</td>
<td>42.4</td>
<td>5.1</td>
<td>1.81</td>
<td>5.73</td>
<td>6.36</td>
</tr>
<tr>
<td>S0.5V2</td>
<td>80</td>
<td>0.007( 50%)</td>
<td>60</td>
<td>2</td>
<td>42.4</td>
<td>5.1</td>
<td>2.60</td>
<td>6.80</td>
<td>8.50</td>
</tr>
</tbody>
</table>

Note: $f'_c$=compressive strength, $f'_{sp}$=splitting tensile strength, $V_c=V_{cr}/b_d$=cracking shear strength, $V_uo=V_{uo}/b_d$=observed ultimate strength, $V_{up}=V_{up}/b_d$=predicted ultimate shear strength, $\rho_v=A_v/b_s$=shear reinforcement ratio. * The values in parenthesis represent the percent ratio of shear stirrups to the code-required full shear reinforcements.
Fig. 1—Test setup and the locations of strain gages installed.

Fig. 2—Relative compressive strength of concrete due to the addition of steel fibers.

Fig. 3—Relative flexural strength of concrete due to the addition of steel fibers.
Fig. 4—Relative splitting tensile strength of concrete due to the addition of steel fibers.

Fig. 5—Comparison of various relative strength of concrete due to the addition of steel fibers.
Fig. 6a—Failure modes (cracking pattern) of test beams.
Test beam: S0VO—Test beam: S0.5VO—Test beam: S0.75VO

Fig. 6b—Test beam: S1VO—Test beam: S0V1—Test beam: S0.5V1

Fig. 6c—Test beam: S0.75V1—Test beam: S0V2—Test beam: S0.5V2
Fig. 7—Load-deflection curves for the beams without stirrups.

Fig. 8—Load-deflection curves for the beams with 50 percent full conventional stirrups according to fiber contents.

Fig. 9—Load-deflection curves for the beams with 75 percent full conventional stirrups according to fiber contents.
Fig. 10—Cracking shear strength according to fiber contents.

Fig. 11—Ultimate shear strength according to the fiber contents.

Fig. 12—Free body diagram of a part of shear span of a simply supported fiber reinforced concrete beam.
Fig. 13—Comparison between the observed ultimate shear strength $V_{uo}$ with predicted ultimate shear strength $V_{up}$. 
Fiber Reinforced High-Performance Concrete Beams: Material and Structural Behavior

by M. Marazzini and G. Rosati

Synopsis: The mechanical behavior of a few plain and fiber-reinforced high-performance concretes ($f_c=80-130$ MPa) is studied here by means of direct tensile tests and three-point bending tests, and a special “identification” procedure is adopted in order to cleanse the stress-strain and stress-displacement curves of any undesired structural effect. Then the overall behavior of two P/C beams typifying the sub-elements of a hollow-core slab is examined, with and without fibers, to study crack formation and propagation (by optical interferometry) and structural ductility.

Keywords: bond; concretes; cracking; hollow-core slabs; steel fibers; structural ductility; tensile behavior; test methods (concrete)
Maddalena Marazzini obtained a MS degree in Architecture at the Politecnico di Milano, Italy, in 1996, with a thesis on how high- and ultra high-performance concretes can improve the mechanical behavior of the structural members.

Gianpaolo Rosati is an assistant professor at the Politecnico di Milano, Italy, where he obtained both his MS and Ph.D. degrees in Structural Engineering with a thesis on tension stiffening and crack cohesion; his main field of research includes bond, concrete fracture mechanics and very high-strength concretes. Dr. Rosati is a member of the CEB Task Group TG 2/5 (Bond Models).

INTRODUCTION

In many engineering fields, the study of composite materials is now an important topic [1,2]. Among these materials, high performance cementitious composites are specifically suited to Civil Engineering, owing to their durability, abrasion resistance, early strength and volume stability [3]. Compared to normal concretes, high-performance concretes are more expensive and may increase the initial cost of a structure, but the extra cost is generally more than offset by their better long-term behavior (i.e. better durability with lower maintenance costs and longer service life), more affordable environmental impact and lower structural weight (since the greater strength leads to a generalized size reduction) [4].

Unfortunately, high-performance concretes containing silica fume (high-strength and ultra high-strength concretes and microconcretes) are very brittle compared to normal concretes, but this drawback can be overcome by introducing medium-to-high fiber contents into the mix [6] ($v_f > 1\%$ by volume, and typically $v_f \geq 4\%$, steel fibers).

The added toughness of the material increases the structural ductility, which is instrumental in preserving the overall performances of a structure, especially where and when unrepairable and irreplaceable members are in issue (e.g. bridge pylons and underwater tunnel liners).

To obtain long-term durability, concretes having very low permeability are required, and since concrete permeability is related to pore size and distribution, diffusion and transport processes must be fully understood.

In this paper the mechanical behavior of a few plain and fiber-reinforced high-performance concretes ($f_c = 80-130$ MPa) is studied by means of direct tensile tests and three-point bending tests, and a special “identification” procedure is adopted in order to cleanse the stress-strain and stress-displacement curves of any undesired structural effect. Then the overall behavior of two P/C beams is examined, with and without fibers, to study crack formation and propagation (by optical interferometry) and structural ductility. These precast, prestressed beams typify the single sub-elements of the hollow-core P/C slabs that are often found in medium-span bridge decks, where the durability and fatigue resistance provided by HPCs and FRCs are of great importance.
MATERIALS, INSTRUMENTATION AND TEST SETUP

The scope of the tensile and bending tests carried out by the authors on ultra high-strength concretes \( (f_c = 130-150 \text{ MPa}) \) was the characterization of the material properties for different fiber contents. The mix-design is reported in Table 1. Carbon steel microfibers, diameter 0.15 mm, length 6 mm, were used.

The mechanical properties refer to a curing period in water of 28 days.

The mix-design of the two prestressed beams, built in a precast-concrete factory, are given in Table 2. The first beam was made of a typical normal concrete \( (f_c = 46.9 \text{ MPa}) \). The second beam was made of a fiber-reinforced concrete with a fiber content of 1% by volume \( (f_c = 81 \text{ MPa}, \text{ steel fibers}) \). The tendons were tensioned before casting at a stress level of 1250 MPa. The tests were carried out one week after casting.

To carry out the tensile and bending tests, a suitably-modified 100 kN Instron machine was used. The machine is provided with an electromechanical actuator, with a minimum speed of 2 \( \mu \text{m} \) per hour; among the three control channels, one gives the possibility of choosing the feedback signal, in order to allow a stable control of the test; the closed-loop control with integral and derivative gain, makes it possible to remove the effects of the finite axial stiffness of the machine, so as to avoid unstable failures.

In the tensile tests, the displacements were measured using two LVDT's, having a sensitivity of 0.2 \( \mu \text{m} \), set across the notch, with a 50 mm base and arranged at 180° around the specimen axis (Fig.1: the average value was used as feedback signal for machine control). Furthermore, the relative displacement of the machine heads was measured at four points, by means of four LVDT's. In the part outside the notched zone, two electrical strain gauges were set at two diametrically-opposite points.

The moiré fringe pattern was continuously recorded during the whole test, by means of a video-camera, which was connected to a video-recorder. All the recorded images were analyzed by means of a software package, in order to
valuate the displacement field and crack opening. The outputs of the transducers, as well as those of the load cell, were automatically stored in a data logger. Fig. 5 shows the test setup and the instrumentation.

TENSILE TESTS

The direct tensile tests were performed on cylindrical specimens, cast in special Plexiglas moulds. All of the specimens had an external circumferential notch 7 mm deep, obtained by inserting a stiff rubber ring into the mould (Fig.1). The fresh concrete was poured into the moulds in several layers in order to favor concrete compaction. The tests were controlled with two linear voltage differential transformers (LVDT) placed parallel to the longitudinal axis of the specimen and diametrically opposed.

The complete load-displacement curves for each specimen and for each material are shown in Fig.6. The specimens with no steel micro-fibers showed typical brittle behavior. The response was linear up to roughly 60% of the peak load, and became always very steep in the post-peak phase.

The fiber-reinforced specimens showed a much gentler softening response, owing to the cohesive behavior of multiple cracking, as well as to the bridging action of the fibers across the localized cracks, up to fiber pull-out.

From the mechanical point of view, it is interesting to observe that the fiber-reinforced specimens showed a critical crack opening (= crack width with no stress transfer at the interface), about forty times larger than in unreinforced specimens.

With reference to Young’s modulus (which was measured with strain-gauges having a 20 mm base length), the fiber content has no practical influence (Fig.7).

BENDING TESTS

Three-point bending tests were performed on 100x100x400 mm specimens, with a simply-supported span of 360 mm.

The response of the unreinforced specimens was very brittle and the post-peak behavior was characterized by a “snap-back”. This phenomenon was fully controlled since the feedback signal to the servo-controller was the crack mouth opening displacement (CMOD), which is a monotonically-increasing function in this type of tests (Figs 8,9,10). The fibers also increased the peak load, besides inducing a more ductile kind of behavior.

In Fig.11 a comparison between the nominal tensile and bending strengths is shown, for different fiber contents.

The fiber reinforcement contributes markedly to the nominal strength since the fibers bridge the cracks, allowing significant tensile stresses to be transmitted in the damaged regions. As a result, larger stresses can develop in the compressive region, with a better exploitation of the improved compressive strength of the fiber-reinforced concrete.
INTERPRETATION OF THE EXPERIMENTAL CURVES IN TENSION

To assess the material properties in tension from the experimental curves, the identification procedure presented in [8] is applied here.

The load-elongation curves (see Fig.12) exhibit an initial linear ascending branch up to more than 50% of the peak load, a second increasingly non-linear branch up to the peak load, a first steep descending branch down to about one third of the peak load and, finally, a very flat branch down to total unloading. A close interrelation of these four branches with the increase in the notched-section rotation allows us to attribute a clear physical meaning to each branch.

As far as the non-linear elastic stress - strain curve is concerned, a three-parameter (E_c, ε_max and σ*) relation is adopted:

\[ E_c = \sigma \left( 1 - \left( \frac{\sigma}{\sigma^*} \right)^2 \right)^{-1/2} \quad \text{where} \quad \sigma(\varepsilon_{\max}) = f_{\text{ct}} \]  

where \( f_{\text{ct}} \) is the effective material tensile strength, \( E_c \) is Young's modulus, \( \varepsilon_{\max} \) is the maximum tensile strain before the onset of cracking, and \( \sigma^* \) is the asymptotic value of the stress corresponding to an infinite strain.

Firstly, the Young's modulus \( E_c \) is evaluated referring to the initial ascending branch, where the material response is linear elastic. The load \( P_M \) at the limit of linearity (point A in Fig.12) corresponds to a sudden increase in the rotation increments in the notched section. This increase is due to the non-linear material response at the onset of microcracking. The Young's modulus can, therefore, be calculated:

\[ E_c = \frac{P_M L_b \delta_{\text{M}}^{-1} A_i^{-1}}{\varepsilon_{\text{M}}} \]  

where \( A_i \) is the notched-section area, \( \delta_{\text{M}} \) is the average elongation at the limit of linearity and \( L_b \) is the base length of the transducers.

Secondly, the maximum longitudinal strain \( \varepsilon_{\max} \) is evaluated at peak load \( P_{\text{max}} \) (point B in Fig.12). Being affected by notch effects, \( \varepsilon_{\text{M}} \) will be greater than the measured nominal strain \( \varepsilon_{\text{nM}} \) at peak load. If \( 2\rho_e \) is the longitudinal length, astride the notch, where strain concentration occurs, and \( \varepsilon_{\text{M}} \) is the corresponding strain in the region unaffected by the notch, the effective value of \( \varepsilon_{\max} \) can be formulated as follows (Fig.13):

\[ \varepsilon_{\max} = \varepsilon_{\max} \left( \varepsilon_{\text{nM}}, \varepsilon_{\text{M}}, \rho_e, L_b \right) \]  

Thirdly, the value of \( \sigma^* \) can be calculated with the following theoretical relationship:

\[ \sigma^* = \left( \frac{\alpha_k^2}{2\alpha_e^2} \right)^{1/2} \varepsilon_{\text{nM}} \]  

where \( \alpha_k^* \) is the stress concentration factor at the tip of the notch for an ideal linear-elastic material, and \( \alpha_e \) is the non-linear strain concentration factor:
\[ \alpha_e = \frac{E_c \varepsilon_{\text{max}}}{\sigma_{\text{NM}}} \]  

and \( \sigma_{\text{NM}} \) is the nominal tensile strength, corresponding to the limit of linearity (point A in Fig.12 and point M in Fig.14):

\[ \sigma_{\text{NM}} = \frac{P_M}{A_i} + \frac{M(\varphi_M)}{W_i} = 4\frac{P_M}{(\pi D_i^2)} + \frac{E_c D_i}{(2L)} \varphi_M \]  

where \( D_i \) is the diameter of the notched section, \( L \) is the specimen length, \( W_i \) is the resistant modulus, \( M \) is the secondary bending moment provoked by non-intentional eccentricities and \( \varphi_M \) is the rotation in the notched section (corresponding to point A in Fig.12 and point M in Fig.14).

Finally, the material tensile strength \( f_{\text{ct}} \) is obtained:

\[ f_{\text{ct}} = \frac{E_c \varepsilon_{\text{max}} \sigma^*}{\sqrt{\sigma^*^2 + (E_c \varepsilon_{\text{max}})^2}} \]  

The transformation of the nominal into the effective \( \sigma-\varepsilon \) relation (Eq.1) is shown in Fig.14.

As far as the cohesive stress - crack opening curve is concerned, a two-parameter \((w_c \text{ and } \kappa)\) hyperbolic relation \([9]\) is adopted.

\[ \sigma_w / f_{\text{ct}} = \left( \frac{d_{\text{max}} (w_c - w)}{w_c (d_{\text{max}} + \kappa w)} \right) \text{ for } 0 \leq w \leq w_c ; \]
\[ \sigma_w / f_{\text{ct}} = 0 \text{ for } w \geq w_c \]  

where \( \sigma_w \) are the stresses transferred along the crack surfaces, \( w_c \) is the critical crack width beyond which no cohesive stress \( \sigma_w \) is transferred, \( \kappa \) is a coefficient related to the slope of the hyperbolic curve and \( d_{\text{max}} \) is the maximum aggregate size. Note that -for a given fracture energy \( G_f \)- the value of \( \kappa \) can be calculated by matching the definite integral of Eq.8 (evaluated from zero to \( w_c \)) with \( G_f \), whose value represents the area underneath the \( \sigma_w - w \) curve.

Let us now make a few remarks: (a) across a fully propagated crack in the notched section no stress and strain concentration occurs; (b) assuming a linear distribution for the local crack width \( w(\rho, \theta) \) over the cracked section, the four measured elongations \( \varepsilon \) (see Fig.1) make it possible to assess the crack-width distribution at each loading step; (c) the value of both \( \kappa \) and \( w_c \) can be predicted by means of Eq.8, so that a possible \( \sigma_w - w \) relation can be identified at each loading step, where \( w = \max w(\rho, \theta) \); and (d) by minimizing the standard deviation between the theoretical and experimental \( \sigma_w - w \) curve, the most probable cohesive relation is identified.

**BENDING TESTS TO EVALUATE THE TENSILE PROPERTIES OF CONCRETE**

The most spontaneous way to determine the behavior of concrete in tension is to carry out direct tensile tests. Nevertheless, the specimen response is highly influenced by its geometry and size, material heterogeneity, boundary and loading
conditions. Furthermore, the collapse can develop in a sudden and unstable way, with a localization of the strain which may be incompatible with the base length of the usual instruments. As a consequence, the test results may not represent the actual material properties, unless the above-mentioned disturbances are minimized. Thus, indirect tensile tests (such as three-point bending tests and splitting tests) are often preferred since their results are more reliable and less dispersed. However, at least in principle indirect tests lead to less exact results, owing to their more complex stress and strain state, which makes these tests a sort of “structural” tests, with solid, microcracked, plasticized, fracturing and fractured material, and step-by-step variable loading and geometrical conditions. As a matter of fact, it is not possible to identify exactly each contribution to the global response of each specimen. To analyze the structural effects correctly, it is necessary to know: (a) the local constitutive laws, (b) their governing parameters; (c) the structural effects and the consequent stress and strain redistribution; and (d) the interaction between the fractured and solid regions.

According to the sketch in Fig. 15, the overall beam response is indicated by the experimental load-displacement curve; the local response of the mid-span notched-part is represented by the experimental bending moment-rotation curve; the notch effects are described by the experimental bending moment-CMOD curve; and the fracture process by the experimental CMOD-crack penetration curve.

By fitting the experimental load-displacement curves (obtained via three-point bending tests) by means of a specific theoretical approach based on the material properties measured in direct tension [10], it is possible to identify the “objective” or “true” values of the parameters which govern the stress-strain and stress-crack opening laws of the concrete in tension.

**ELIMINATION OF THE STRUCTURAL EFFECTS**

In the present analysis the following scheme was adopted (Fig. 16):

- all the experimental load-displacement curves (tensile and bending tests) were treated according to the above-mentioned procedure, for each mix design, curing and age;
- many structural effects were filtered out both by placing suitable instruments in the most significant zones of the specimen, and by adopting sophisticated devices to reduce any disturbing interaction between the testing equipment and the specimen;
- for both the tensile and bending behavior, the above procedure was adopted, making it possible to compensate for the lack of information about the actual stress and strain distribution, and fracture process occurring during the tests;
- by adopting the tensile-test response as input data and the bending-test response as a reference, it was possible to develop an algorithm for a more reliable and accurate formulation of the constitutive law in tension [11].

For $\nu_r = 0$ and 4% by volume, the “objective” concrete properties are given in Table 3.
STRUCTURAL RESULTS

Two tests were carried out on pretensioned beams subjected to three-point bending (see the test set-up, Fig.5). The first P/C beam was made of fiber-reinforced concrete (1% by volume of smooth microfibers $\phi\ 150\ \mu m/ L\ 13\ mm$, beam size $8\times20\times300\ cm$, $f_c = 81\ MPa$, Fig.18), while the second P/C beam had no fibers (beam size $16\times20\times300\ cm$, $f_c = 46.9\ MPa$, Fig.19).

Both beams had a simply-supported span of 290 cm and were designed for the same ultimate capacity in bending ($P_u = 18\ kN$, reinforcement: one single straight 9.5 mm tendon). Furthermore, the FRC beam (Fig.18) was devised to simulate in some way a portion of a typical hollow-core slab (Fig.17).

The response curves are presented in Fig. 20. The moiré fringe patterns (fiber-reinforced specimen) are shown in Figs.21 and 22. With reference to point A of the load-displacement curve, the first crack appeared close to the middle cross-section. The behavior of the beam changed from linear to non linear, as can be seen from the load-deflection curve. With reference to point B, a second crack appeared. The crack widths were 0.075 and 0.05 mm at point C, and 0.1 and 0.075 mm at point D, respectively. Finally, at point E, (deflection = 4 cm) the crack widths were 0.4 and 0.341 mm, respectively (load level = 75% of the ultimate load). The P/C no-fiber beam showed the first crack at point A (Fig.23). Afterwards, no more cracks appeared. Beyond the peak load, the beam showed a sudden drop in the load carrying capacity. At point B, the crack opening was 0.54 mm, (load level = 40%, falling branch). All cracks formed at right angles to the principal tensile stress; in the FRC beam, the cracks originating in the central part of the beam propagated toward one of the loading points.

DISCUSSION OF THE RESULTS

Figs. 21 and 22 show the moiré fringe patterns at five load levels (no load = initial mismatch and points A, B, D, E of Fig.20, left) for the FRC beam, and Fig. 23 shows the moiré fringe patterns at three load levels (no load = initial mismatch, point A and point B of Fig.20, right) for the beam with no fibers. The beams, at the load level corresponding to first cracking, showed a uniform deformation along the bottom face, with a deformation of 0.0005 m/m in the fiber-reinforced beam, 2.5 larger than the deformation observed in the plain concrete beam.

Fiber-reinforced concrete seems to ensure a tendon-to-concrete bond about twice as large as that of plain concrete, as can be observed in Fig. 24, where the applied load is plotted against the draw-in of the tendons. Thus the beams failed because of bond reaching its ultimate limit state.

Finally, it is worth noting that the two beams exhibit roughly the same load at the end of the mostly linear ascending branch (9.2 kN and 11.2 kN respectively), but the subsequent behavior is entirely different (Fig.20).
CONCLUDING REMARKS

The results can be summarized as follows:

- Ultra High-Strength microconcretes \( (f_c \geq 130 \text{ MPa}) \) with small-diameter quartzitic aggregates are very brittle, in terms of both tensile strength \( (f_{ct}f_c = 1/30) \) and toughness; however, by adding 2 to 4% of smooth metallic fibers (by volume) the material becomes stronger in tension \( (f_{ct}f_c = 1/20) \) and definitely tougher (the fracture energy increases by two orders of magnitude, from 130 to 8000 N/m).

- In the UHS microconcretes studied here, the Young's modulus is not significantly affected by fiber amount, since fiber contents as large as 2-4% by volume increase \( E_c \) by less than 10%.

- The difference between the values of the Young's modulus in tension and compression is negligible \( (\sigma_t \leq f_{ct} / 3, \sigma_c \leq f_c / 30) \).

- By controlling the Crack-Mouth Opening Displacement (CMOD), the notched specimens can be tested successfully in direct tension even in the case of ultra high-strength concretes (with or without fibers, \( v_f = 0-4\% \)).

- The nominal tensile strength in bending varies from less than 2 to more than 3 times the tensile strength in direct tension, by adding up to 4% of fibers.

- By adopting a special procedure the results in direct tension can be cleansed of any undesirable structural effect (lack of concrete uniformity in the notched section, rotation of machine platens), and the constitutive laws (stress-strain and stress-crack opening) can be identified by means of an algorithm capable of combining the curves in direct tension and those in bending.

- The post-elastic behavior of typical prestressed, pretensioned beams made of a high-performance Concrete (same nominal ultimate capacity in bending) becomes completely different by adding smooth steel fibers (up to 1% by volume): in the case of plain concrete (no fibers) there is a steep falling branch followed by an extended plateau (-30% in bending capacity); with 1% of steel fibers, there is a plastic plateau, followed by a further increase in bending capacity (+30%).

- The better performance of pretensioned FRC beams seems to be mostly due to the better concrete-to-tendon bond, which reduces definitely the tendon drawing and contributes markedly to structural ductility.

Finally, it may be observed that the same nominal material may behave differently in a test specimen and in a real structure, both cast and tested in lab conditions. As a matter of fact, the structural behavior of the specimen often overshadows the "true" mechanical properties of the material, making specimen response different from material response. Consequently, in order to achieve "objectivity", all structural effects in specimen behavior have to be identified and removed via a suitable procedure based on a thorough analysis of both stress and strain concentrations in the region where crack and damage localize.
ACKNOWLEDGEMENTS: The financial support of Precompressi Centro Nord (Cerano-Novara, Italy) and Italcementi (Bergamo, Italy) was instrumental in carrying out successfully the tests, and their contribution to this research project is gratefully acknowledged.

REFERENCES


TABLE 1—MIX DESIGN OF TEST SPECIMENS.

<table>
<thead>
<tr>
<th>MIX-DESIGN</th>
<th>NO FIBERS</th>
<th>FIBERS (2/4% by volume)</th>
</tr>
</thead>
<tbody>
<tr>
<td>cement 52.5</td>
<td>696 kg/m³</td>
<td>667 kg/m³</td>
</tr>
<tr>
<td>silica fume (SF/C)</td>
<td>174 kg/m³ (25%)</td>
<td>167 kg/m³ (25%)</td>
</tr>
<tr>
<td>quartz 0.06-3.2 mm</td>
<td>1300 kg/m³</td>
<td>1246 kg/m³</td>
</tr>
<tr>
<td>superplasticizer (SP/C)</td>
<td>14.4 kg/m³ (2.1%)</td>
<td>13.8 kg/m³ (2.1%)</td>
</tr>
<tr>
<td>water (W/C+SF)</td>
<td>196 kg/m³ (22.5%)</td>
<td>187 kg/m³ (22.4%)</td>
</tr>
<tr>
<td>steel fibers (vr %)</td>
<td>//</td>
<td>162/324 kg/m³ (2.1/4.2%)</td>
</tr>
</tbody>
</table>

TABLE 2—MIX DESIGN OF THE PRESTRESSED BEAMS.

<table>
<thead>
<tr>
<th>MIX-DESIGN</th>
<th>NO FIBERS</th>
<th>FIBERS (1% by volume)</th>
</tr>
</thead>
<tbody>
<tr>
<td>cement 52.5</td>
<td>352 kg/m³</td>
<td>695 kg/m³</td>
</tr>
<tr>
<td>silica fume (SF/C)</td>
<td>//</td>
<td>174 kg/m³ (25%)</td>
</tr>
<tr>
<td>sand</td>
<td>813 kg/m³</td>
<td>652 kg/m³</td>
</tr>
<tr>
<td>gravel 3-6 mm</td>
<td>527 kg/m³</td>
<td>652 kg/m³</td>
</tr>
<tr>
<td>gravel 5-10 mm</td>
<td>592 kg/m³</td>
<td>//</td>
</tr>
<tr>
<td>superplasticizer (SP/C)</td>
<td>11.7 kg/m³ (3.3%)</td>
<td>46.4 kg/m³ (6.6%)</td>
</tr>
<tr>
<td>water (W/C+SF)</td>
<td>97 kg/m³ (27.5%)</td>
<td>200 kg/m³ (23%)</td>
</tr>
<tr>
<td>steel fibers (vr %)</td>
<td>//</td>
<td>78 kg/m³ (1%)</td>
</tr>
</tbody>
</table>
TABLE 3—CONCRETE PROPERTIES IN TENSION FROM THE IDENTIFICATION PROCEDURE.

<table>
<thead>
<tr>
<th>FIBER CONTENT</th>
<th>E_c (MPa)</th>
<th>f_{et} (MPa)</th>
<th>G_F (N/m)</th>
<th>w_c (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0% by volume</td>
<td>50000</td>
<td>9.6</td>
<td>130</td>
<td>0.15</td>
</tr>
<tr>
<td>4% by volume</td>
<td>51000</td>
<td>14.7</td>
<td>8000</td>
<td>3</td>
</tr>
</tbody>
</table>

Fig. 1—Testing in tension: notched specimens and instrumentation (see also [7]).

Fig. 2—Flow chart of the tensile testing procedure, with the feedback control.
Structural Applications of Fiber Reinforced Concrete

Fig. 3—Prismatic specimens for three-point bending tests.

Fig. 4—Instrumentation of the three-point bending tests (dimensions in mm).
Fig. 5—Instrumentation and loading equipment for the bending tests on prestressed beams (dimensions in cm).

Fig. 6—Load displacement curves in direct tension (notched specimens).

Fig. 7—Young's modulus in tension and compression: effects of fiber content.
Fig. 8—Test specimens in bending, with no fibers (left) and with fibers (right, 4 percent by volume).

Fig. 9—Test specimens in bending without fibers: load CMOD and load deflection curves.

Fig. 10—Test specimens in bending with fibers (4 percent by volume): load CMOD and load deflection curves.
Fig. 11—Nominal tensile and bending strength: influence of fiber content.

Fig. 12—Load displacement and load rotation curves; stress and c.o.d. distributions.
\[ \varepsilon(x) = \alpha e^{-\beta x} \]

\[ \varepsilon_{\text{eff}} = \varepsilon_{\text{max}} \]

Fig. 13—Longitudinal strain distribution astride the notch in a tension test.

Fig. 14—Typical stress-strain curves cleansed of the bending effects (left) and notch effects (right).

Fig. 15—Physical phenomena (top) and response curves (bottom) in a bending test.
Fig. 16—Algorithm for the identification of a more accurate tensile law for concrete.
Fig. 17—P/C hollow slab often adopted in a bridge deck design.

Fig. 18—Section of the P/C fiber reinforced beam (dimensions in cm).

Fig. 19—Section of the P/C no-fiber beam. (dimensions in cm).

Fig. 20—P/C beams in bending: load deflection curve, fiber reinforced concrete (left) and plain concrete (right).
Fig. 21—P/C fiber reinforced beam in bending: moiré fringe patterns at increasing load levels 0 percent (PO), 44 percent (PA), and 50 percent (PB); the load levels are referred to the nominal carrying capacity (18 kN).
Fig. 22—P/C fiber reinforced beam in bending: moiré fringe patterns at increasing load levels 62 percent (PC), 62 percent (PD), and 75 percent (PE); PE represents the maximum load attained during the test.
Fig. 23—P/C no-fiber beam in bending: moiré fringe patterns at increasing load levels 0 percent (PO), 41 percent (PA, ascending branch) and 34 percent (PB, falling branch); the load levels are referred to the nominal carrying capacity (18 kN).
Fig. 24—Prestressed beams in bending: load versus tendon draw-in.
High-Strength Concrete Beams Submitted to Shear: Steel Fibers Versus Stirrups

by P. Casanova and P. Rossi

Synopsis: The use of steel fibers as shear reinforcement in reinforced concrete beams is very promising. In this paper, an optimized high-strength concrete with steel fibers (100 kg/m³) is used in rectangular beams (2.3 x 0.25 x 0.125 m), reinforced with longitudinal bars. This solution is compared with classical reinforced concrete. Five specimens are tested in four-point bending. The 28-day mean compressive strength of concrete is 90 MPa measured on cylinders. The global behaviour of the beams is the same for both solutions. The load at the onset of cracking is equal for all tested beams but the crack opening is smaller with steel fibers. No problems were encountered concerning ductility.

Keywords: fiber reinforced concrete; high-strength concrete; shear; stirrups
**Dr Pascal Casanova** is graduate from the Ecole Polytechnique of Paris and from Ecole National des Ponts et Chaussées (Master of Science). He was PhD student at the Laboratoire Central des Ponts et Chaussées, Paris, France. He now works as a civil engineer.

**Dr Pierre Rossi** is research director and head of the Concrete and Cement Division at the Laboratoire Central des Ponts et Chaussées (LCPC) in Paris. He is member of ACI committee 544, Fiber Reinforced Concrete, member of RILEM committee TC 162-TDF, president of the technical and scientific committee of the french National Project on Steel Fiber Concretes. His research interests include modelling of the cracking of concrete structures, fiber reinforced and high strength concretes and delayed concrete behavior.

**INTRODUCTION**

During the last fifteen years, two new materials have seen growing development in the industry of civil engineering: high-strength concrete (HSC) and fiber reinforced concrete (FRC). Owing to its high compressive strength, its rheological properties and the improvement of durability, the first one has been more and more used in large structures. Nevertheless, one of the main problems with HSC is its relative brittleness.

The main advantages of FRC are to offer a multidirectional reinforcement and a reduced construction time. This explains its strong development, mainly in industrial floors and tunnel reinforcement using shotcreting. Unfortunately, fibers are still rarely used in truly structural applications. This is partly due to a low post-cracking residual stress and concerns about corrosion in cracked sections.

It appears that combining the two materials could be beneficial (1). As a matter of fact, the local and multidirectional reinforcement of a HSC by fibers would limit its brittleness by seaming the cracks from their onset. Concurrently, the fiber-matrix interface is improved in a good matrix and leads to a higher post-cracking residual stress. Furthermore, a good bond limits crack opening and, so, risks of corrosion. Optimizing a high-strength fiber reinforced concrete (HSFRC) consists in getting the highest post-cracking residual stress, while keeping a good ductility. One of the best kinds of fiber to achieve this goal is the **drawn steel fiber**.
These established facts are the origin of an experimental study dealing with a promising industrial application: shear reinforcement. As a matter of fact, replacing all the stirrups and passive reinforcement in beams would allow the automation of the production. In this study, stirrups and steel fibers have been compared in beams submitted to shear. In order to get a quantitative comparison, a characterization of HSFRC in tension is also presented. Using a block mechanism analysis, an equivalence is proposed between the two reinforcements.

MATERIALS STUDIED

Two materials were used in this study: a plain HSC for the concreting of classical RC beams and a HSFRC. The aim was to get a characteristic compressive cylinder strength of 80 MPa. With this strength short fibers have to be used to avoid their rupture after cracking. But the fibers must be long enough to give ductility and a high post-cracking residual stress. Hooked-end drawn steel fibers with yield strength of 1200 MPa, 30-mm long and 0.5 mm in diameter were used to fulfil these requirements. These fibers are glued together in about 30-fibers plaquettes to facilitate concrete mixing, to avoid balling. This fiber geometry leads to a large number of fibers for a given volume. This reduces the material heterogeneity and improves the composite effect.

Mixing and concreting

The concrete fabrication sequence is as follows:

- mixing of cement, dry gravels and sand for 30 seconds,
- pouring of the water combined with one third of the superplasticizer and mixing for 1 minute,
- introduction of the fibers with the remaining 2/3 of the superplasticizer and mixing for 2 min.

This procedure allows good homogeneity of the FRC. In order to optimize the compactness and workability, a method currently used for normal concrete is employed: the Baron-Lesage method (2). This method is based on the postulate that a mix with optimal compactness has optimal workability. The workability is measured by the flowing time of 0.03 m$^3$ of concrete under vibration. Our approach consists in comparing optimized concretes meeting the same industrial requirements: constituents, workability, compressive strength. Tables 1 and 2 describe the mix obtained respectively for normal and fiber reinforced concretes.
FRC is known to be very sensitive to the way of concreting: it leads to anisotropy of the fiber distribution. In order to get an orthotropic distribution of the fibers - isotropic distribution in the plane perpendicular to the direction of concreting - which enables a good seam of the shear cracks, the HSFRC is poured perpendicularly to the plane of the beam - instead of vertically as is usually done.

**Mechanical characterization**

The compressive strength and the modulus of elasticity are measured on cylinders as usual.

The mean values obtained at 28 days are given in table 3.

It is well known that steel fibers, for the quantity used in this study, do not affect significantly the tensile strength of the material: the fibers act after the cracking of the matrix (3). Consequently, the best way to quantify the action of fibers is to perform a uniaxial tension test and to measure the load - crack opening relationship (4). A beam of the same geometry and cast in the same conditions as the tested beams is made. The tension samples are then cored from this specimen to ensure compliance with the distribution of the fibers and the quality of the matrix, both greatly dependent on casting conditions.

Figure 1 shows the mean diagrams corresponding to samples taken respectively horizontally, vertically and inclined at 45° in the plane of the beam. It shows that the fiber distribution is homogeneous in the plane. Figure 2 shows the minimum, mean and maximum post-cracking residual stress vs. crack opening diagrams corresponding to 45° inclined samples. It shows that the scatter is limited owing to the large number of fibers.

**EXPERIMENTAL PROGRAM**

**Experimental plan**

The objective of this study was to compare the contribution of steel fibers to the action of stirrups in RC beams. Five beams were tested:

- two beams made of plain reinforced HSC. The transverse reinforcement of beam HSRC1 (resp. HSRC2) is provided by stirrups ($8 \text{ mm}$)
spaced of 18 mm (resp. 14 mm) and made of high bond steel - 0.2% proof test of 490 MPa, and ultimate tensile strength of 600 MPa.  
- three beams made of HSFRC. Two of them - HSFRC1 and 2 - were tested to get an idea of the scatter in shear failure conditions. The third one - HSFRC3 - was designed to get a bending ductile failure.

**Structural design of the beams**

Previous work (5) has shown a scale effect due to the height of the section: the critical crack opening at failure is proportional to the height. So, because of the softening behaviour of FRC (fig. 2), it is better to use this material in structures of relatively small height. A beam 2.3 x 0.25 x 0.125 m (L x h x b), representative of building precast beams, is a good representation.

The aim of the design was to ensure shear failure to quantify the contribution of the transverse reinforcement. But the longitudinal reinforcement should not be excessive in relation to the beam geometry - Ø25 mm, 0.2% proof test of 565 MPa, and ultimate tensile strength of 670 MPa. This lead to the reinforcement presented on fig. 3.

Beams HSFRC1 and 2 had neither stirrups, nor compression reinforcement, but were longitudinally reinforced with 2 bars - dia. 25 mm. Beam HSFRC3 was longitudinally reinforced with 2 bars Ø20 mm - 0.2% proof test of 540 MPa, and ultimate tensile strength of 640 MPa - to ensure bending failure in tension and to check that the beam would behave accordingly to standard design procedures: a ductile bending failure with small inclined shear cracks.

**Testing device**

The beams were tested under four-point bending to allow for possible shear and bending failures. The load was applied using a hydraulic jack of 500 kN according to fig. 4.

The test was controlled by the rate of the central deflection. This was measured by a LVDT (linear variable displacement transducer) supported by a bar, parallel to the neutral axis, fixed at the level of the beam supports to avoid extraneous deformations of the whole frame. The load was applied at a rate of 150 μm/min.
Strain gages were bonded on the longitudinal reinforcement and on the top compression face to measure the bending strain state of some section of the beams. To evaluate the opening of the inclined shear cracks, 6 LVDTs were bonded inclined at 45° on the axis of the beam, in the area of varying bending moment. Some acoustic emission measurements were made to get the load at the onset of shear cracks and to study correlations with the LVDTs measurements.

EXPERIMENTAL RESULTS

Load capacity and failure modes

The load vs. central deflection diagrams are reported in fig. 5. Beam HSFRC3 stands apart because of a lower longitudinal reinforcement that lead to lower rigidity and tension bending failure.

Let us consider beams HSRC1,2 and HSFRC1,2. The similarity of the diagrams must be first noticed. Beam HSRC1 had a lower rigidity and load capacity because of a lower transverse reinforcement which lead to larger crack openings and shear failure by diagonal tension. Beam HSRC2 failed by bending in the compression zone but the stirrups yielded. Beam HSFRC1 failed by shear - diagonal tension -, without brittleness. It confirms the capacity of steel fibers to control shear cracking. Beam HSFRC2 failed by bending in the compression zone, with small shear cracks.

Information given by extensometry

Some information given by extensometry is reported in table 4 for two values of the applied load - 200 and 300 kN. For beams 1 and 2, 200 kN is a little higher than the serviceability limit state calculated using the French design rules. If normal exposure conditions are considered, a maximum crack width of 0.3 mm is required. This condition is fulfilled by HSFRC beams but not by HSRC beams. This is due to the fact that shear cracks which open between the stirrups are seamed only when they reach the reinforcement. On the contrary, fibers seam the crack as soon as it appears. It leads to greater rigidity and greater durability, particularly using HSC.
Beam HSFRC3 was designed to fail in bending under a load high enough to induce shear cracks. This beam shows a large ductility due to bending failure in tension. The maximum shear crack opening at ultimate limit state is 0.3 mm, which firstly, corresponds to quite acceptable structural cracking and, secondly, allows very good action of the fibers - see fig. 2. So, the beam geometry and design are appropriate for industrial use.

Acoustic emission measurements have shown, for both reinforcement types, an onset of shear cracks for a load of about 100 kN. It is well correlated with a change in the slope of the load vs. shear crack opening diagrams.

ANALYSIS OF AN SFRC BEAM SUBMITTED TO SHEAR

Many parameters are involved in the shear behaviour of a beam but it seems that two categories may be distinguished: material and structural parameters. The latter corresponds to experimental aspects - such as shear span-to-depth ratio (a/d), beam geometry and main reinforcement - the influence of which has been noticed to be the same as on reinforced concrete (6,7). At the material level, effects of fiber type, fiber content, compressive strength and so on, have been studied. The main conclusion drawn from the literature available is the importance of the post-cracking stress carried out by the fibers: it integrates the influence of all material parameters.

Block mechanism

We consider here the case of a SFRC beam, with classical longitudinal reinforcement, at the ultimate state. It means that a main inclined crack has propagated and leads to a block mechanism. Considering either a RC or a SFRC beam, the shear load capacity, $V_u$, of a beam may be divided into two terms:

$$V_u = V_c + V_l$$

where $V_c$ is the structural part (due to the compression zone, longitudinal reinforcement, aggregate interlocking ...) and $V_l$ is the part directly carried by the transverse reinforcement.

As mentioned above, comparison of experimental data (on RC and SFRC beams) shows that the structural part $V_c$ is the same for RC and
SFRC beams of given geometry, concrete strength and longitudinal reinforcement. It means that the new aspect in designing a SFRC beam submitted to shear is determining the its transverse part, $V_t = V_f$.

Fig. 6 illustrates the block mechanism in a rectangular SFRC beam with classical longitudinal reinforcement. The main crack is supposed to be inclined at 45° and its width varies linearly from a maximum $w_m$ at the bottom of the beam to zero at the bottom of the compression zone.

$V_f$ is then calculated by integrating the post-cracking residual stress of SFRC along the crack and projecting it vertically to equilibrate a part of the shear load $V$. This leads to eq. (2):

$$V_f = \int_0^{0.9d/\sqrt{2}} \sigma_f \left( \frac{s}{0.9d/\sqrt{2}} \cdot w_m \right) \cdot b \cdot \frac{\sqrt{2}}{2} \cdot ds$$

$$V_f = 0.9bd \cdot \int_0^{w_m} \sigma_f (w) \cdot dw$$

where $\sigma_f (w)$ is the post-cracking residual stress of SFRC for a crack width $w$ and $s$ is the curvilinear absissa. $\bar{\sigma}_p (w_m)$ is called the equivalent post-cracking residual stress at the crack width $w_m$. It is in fact the mean value of the post-cracking residual stress between zero and $w_m$.

Eq. (3) shows that a uniaxial tensile test is necessary to quantify the part due to the fibers and that a definition of $w_m$ is necessary to quantify the ultimate load capacity of the beam submitted to shear.

**Equivalence relationship**

This approach may be applied to classical vertical transverse reinforcement. In this case, the part due to vertical stirrups, $V_s$, is easily calculated by considering the equilibrium of the blocks as in fig. 6. This leads to eq. (4) and (5):
\[ V_u = V_c + V_s \]  
\[ V_s = 0.9bd \cdot \rho_t \cdot f_y \]

where \( \rho_t \) is the transverse reinforcement ratio (transverse reinforcement area / beam width \cdot stirrups spacing) and \( f_y \) the steel yielding stress. In this study, the part due to stirrups is:

\[ V_s = 56 \text{ kN for beam HSRC1}, \]
\[ V_s = 72 \text{ kN for beam HSRC2}. \]

Eq. (1), (2) and (5) lead to an equivalence relationship between stirrups and SFRC:

\[ \overline{\sigma}_p (w_m) = \rho_t \cdot f_y \]  

Assuming that \( \tau_t \) is high enough to ensure a ductile behaviour of the girder, an equivalent SFRC post-cracking tensile stress, \( \overline{\sigma}_p (w_m) = \rho_t \cdot f_y \), would provide the same loading capacity of the member.

**Calculation of the fiber part**

The experiments carried out with different geometries of steel fiber reinforced concrete beams, reinforced with classical longitudinal bars, have shown the onset of inclined macrocracks separating the concrete struts in compression. The spacing of these macrocracks is roughly equal to the inner lever arm of the beam (5).

The ultimate limit state in tension is defined by the value of \( w_m \). The ultimate crack opening is experimentally shown to be proportional to the height of the beam (5). As the average crack spacing is roughly equal to the inner lever arm, 0.9d, and the crack opening is controlled by the longitudinal reinforcement (fig. 6), we propose to define the maximum crack opening as follows:

\[ w_m = \varepsilon_s \cdot 0.9d \]  

where \( \varepsilon_s \) is the strain of the longitudinal reinforcement.
According to the RC French design rules, this strain is limited to 1%. Thus the ultimate crack opening allowable in shear, \( w_{mu} \), is:

\[
w_{mu} = 1\% \cdot 0.9d
\] (8)

For the beams considered in this study, \( w_{mu} = 1.9 \text{ mm} \). It corresponds to an equivalent post-cracking residual stress \( \overline{\sigma}_p(w_{mu}) \) equals to 2MPa.

**Calculation of the structural part**

The French RC design rules propose a structural part \( V_c = 0.3 \cdot f_t \), where \( f_t \) is the tensile strength of concrete. But this approach is conservative, so we prefer to refer to the work of Kordina (8) who performed a statistical analysis of about a thousand tests to propose a mean value of the structural part:

\[
V_c = 0.85 \cdot h \cdot b \cdot \Delta\tau \cdot k_c \cdot k_L
\] (9)

where \( h \) is the height of the beam, \( b \) the width of the beam, \( \Delta\tau \) a corrective factor of the structural part, \( k_c \) the concrete factor, \( k_L \) the longitudinal reinforcement factor:

\[
k_L = \rho_L^{1/3} (\rho_L, \text{longitudinal reinforcement})
\]

\[
\Delta\tau = 0.24 \text{ (mean value)}
\]

\[
k_c = (f'_c)^{1/2} (f'_c, \text{compressive strength})
\]

For the beams considered in this paper, it leads to the following value:

\[
V_c = 91 \text{ kN}
\]

**Calculations vs. experiments**

Table 5 compares the maximum loads carried by the beams \( (V_{exp}) \) to the calculations based on the presented analysis \( (V_u) \). It shows a very good prediction concerning RC, confirming the validity of Kordina equations for HSC.
Then, considering that the structural part is well calculated, the lower prediction for HSFRC may be explained by the value of \( \bar{\sigma}_p(w_{\text{mu}}) \). As a matter of fact, the maximum shear crack opening measured at the peak load was roughly 1 mm, which is lower than \( w_{\text{mu}} \) (1.9 mm). If we consider a crack opening of 1 mm, it leads to an equivalent post-cracking residual stress of 2.5 MPa. The calculated ultimate shear load is then equal to 150 kN, closer to the experimental values \( (V_u/V_{\text{exp}} = 0.96) \).

**CONCLUSION**

The aim of this study was to compare the behaviour of HSRC and HSFRC beams, with the same longitudinal reinforcement, submitted to shear. The main conclusions which may be drawn are:

1. Combining HSC and drawn steel fibers is a feasible technique. It provides ductility, a high post-cracking residual stress and small crack openings.
2. The use of HSFRC as shear reinforcement in beams is possible at an industrial level. Indeed, the same global behaviour was obtained using a volume of 1.25% of fibers or 1.1 % of conventional steel (stirrups plus compression bars).
3. HSFRC provides a better durability to the structure than HSRC because crack openings are smaller.
4. A method of analysis is proposed which leads to an equivalence relation between stirrups and steel fiber reinforced concrete.

**REFERENCES**


**TABLE 1—CONSTITUENTS OF CONCRETE.**

<table>
<thead>
<tr>
<th>Gravel</th>
<th>Sand 1</th>
<th>Sand 2</th>
<th>Cement</th>
<th>Silica fume</th>
<th>Superplasticizer</th>
</tr>
</thead>
<tbody>
<tr>
<td>4-10 mm crushed</td>
<td>0-5mm</td>
<td>0-4 mm</td>
<td>high-strength</td>
<td>(SF) densified silica fume</td>
<td>(SP) melamine</td>
</tr>
<tr>
<td>limestone from</td>
<td>crushed</td>
<td>river sand from</td>
<td>strength</td>
<td>from</td>
<td></td>
</tr>
<tr>
<td>Boulonnais</td>
<td>limestone from</td>
<td></td>
<td>Type II portland cement</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Boulonnais</td>
<td></td>
<td>from the Seine</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**TABLE 2—MIX PROPORTIONS (kg/m³).**

<table>
<thead>
<tr>
<th>Concrete</th>
<th>Gravel</th>
<th>Sand 1</th>
<th>Sand 2</th>
<th>Cement</th>
<th>SF</th>
<th>Water</th>
<th>SP</th>
<th>Fibers</th>
</tr>
</thead>
<tbody>
<tr>
<td>HSC</td>
<td>985</td>
<td>412</td>
<td>412</td>
<td>430</td>
<td>43</td>
<td>160</td>
<td>8</td>
<td>-</td>
</tr>
<tr>
<td>HSFRC</td>
<td>765</td>
<td>497</td>
<td>497</td>
<td>430</td>
<td>43</td>
<td>170</td>
<td>10.6</td>
<td>100</td>
</tr>
</tbody>
</table>

**TABLE 3—COMPRESSIVE STRENGTH (f'c) AND MODULUS OF ELASTICITY (E) (3 SPECIMENS).**

<table>
<thead>
<tr>
<th>Concrete</th>
<th>f'c (MPa)</th>
<th>E (GPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>HSC</td>
<td>91</td>
<td>47</td>
</tr>
<tr>
<td>HSFRC</td>
<td>90</td>
<td>48</td>
</tr>
</tbody>
</table>
TABLE 4—MAXIMUM MEASURES FOR F = 200 OR 300 kN.

<table>
<thead>
<tr>
<th>beam</th>
<th>deflection (mm)</th>
<th>shear crack opening (mm)</th>
<th>concrete compressive strain (10^{-3})</th>
<th>longitudinal bars tensile strain (10^{-3})</th>
</tr>
</thead>
<tbody>
<tr>
<td>HSRC1</td>
<td>8.7</td>
<td>0.55</td>
<td>1.5</td>
<td>2.2</td>
</tr>
<tr>
<td>HSRC2</td>
<td>8.2</td>
<td>0.32</td>
<td>0.67</td>
<td>4.1</td>
</tr>
<tr>
<td>HSFRC1</td>
<td>7.6</td>
<td>0.14</td>
<td>0.94</td>
<td>2</td>
</tr>
<tr>
<td>HSFRC2</td>
<td>8.1</td>
<td>0.13</td>
<td>0.59</td>
<td>2.2</td>
</tr>
<tr>
<td>HSFRC3</td>
<td>13.7</td>
<td>0.315</td>
<td>2.5</td>
<td>18.4</td>
</tr>
<tr>
<td>load (kN)</td>
<td>200</td>
<td>300</td>
<td>200</td>
<td>300</td>
</tr>
</tbody>
</table>

TABLE 5—COMPARISON OF EXPERIMENTAL AND CALCULATED ULTIMATE SHEAR LOAD

<table>
<thead>
<tr>
<th>Beam</th>
<th>V_u (kN)</th>
<th>V_exp (kN)</th>
<th>V_u/V_exp</th>
</tr>
</thead>
<tbody>
<tr>
<td>HSRC1</td>
<td>147</td>
<td>150</td>
<td>0.98</td>
</tr>
<tr>
<td>HSRC2</td>
<td>163</td>
<td>155</td>
<td>1.05</td>
</tr>
<tr>
<td>HSFRC1</td>
<td>138</td>
<td>157</td>
<td>0.88</td>
</tr>
<tr>
<td>HSFRC2</td>
<td>138</td>
<td>156</td>
<td>0.88</td>
</tr>
</tbody>
</table>

Fig. 1—Mean tensile diagrams for three coring directions (six samples each).
Fig. 2—Minimum, mean, and maximum tension tests from 12 diagonal samples.

Fig. 3—HSRC-beams reinforcement.

Fig. 4—Testing device.
Fig. 5—Load versus deflection diagrams.

Fig. 6—Block mechanism at failure.
Structural Applications Using Ultra High-Strength Fiber Reinforced Concrete

by G. Bernier, M. Behloul, and N. Roux

Synopsis:

Considerable progress has recently been achieved in strength and ductility of concretes. The use of superplasticizers and large amounts of silica fume led to densified cementitious matrices and improved adherence to the fiber reinforcement. These two properties are obtained with Compact Reinforced Composite (CRC) developed at Aalborg Portland and closely studied during a 3-year EC.

The investigations reported in this paper cover the application of ultra-high strength-fiber reinforced concrete to enhance performance of beams, columns and beam to column connections. Mechanical tests were performed on full scale structural elements. Beams of 13 m in length, columns of 2.9 m in compression with and without eccentricity of the load, and beam to column connections were tested. In all cases, concrete strengths of more than 150 MPa were achieved.

Due to CRC’s high compacity and its extreme resistance to the penetration of aggressive elements, the CRC cover to the reinforcement was typically reduced from 30 mm to at least 12 mm. It has been shown that a reduction in concrete cover to the reinforcement is compatible with the requirements of structural applications.

The tests carried out have shown the possibility of using ultra-high strength concrete for large-scale structural concrete elements and opens new fields of applications. This contributes to saving raw materials, weight and volume and to improving ductility and durability.

Keywords: beam; column; concrete cover; connection; fiber reinforcement; shear strength; ultra high-strength concrete
Gérard Bernier (Associate Professeur) is working at the Civil Engineering Department at the Ecole Normale Supérieure de Cachan, 61, avenue du Président Wilson, F-94235 Cachan Cedex. He is personally active within concrete work on structural design.

Mouloud Behloul (Ph.D., Civil Engineer) is working at the Laboratoire de Mécanique et Technologie at the Ecole Normale Supérieure de Cachan, 61, avenue du Président Wilson, F-94235 Cachan Cedex. He has worked on fiber reinforced concrete and studied their mechanical behavior.

Nicolas Roux (Material Engineer) has been involved in concrete research and development since 1991 at the R&D Department of Bouygues, 1, avenue Eugène Freyssinet, F-78061 St. Quentin en Yvelines Cedex. He is active in the development of ultra high-strength fibre reinforced concretes.

INTRODUCTION

High-strength concrete with compressive strengths of more than 60 Mpa came into use after 1980 (1, 2). Its contribution in conventional structural applications has helped to reduce the column dimensions, to increase the capacity of long span members and enhance the durability of the concrete. The improvement of the compressive strength has led to higher material brittleness which in some cases prohibits the use of sufficient amounts of reinforcement to fully utilize the strength of these concretes.

CRC (Compact Reinforced Composite) - developed in 1986 at Aalborg Portland - is a special concept for high performance reinforced concretes, whereby ductility is achieved through incorporation of a large content of metallic fibers (3). This ductility combined with high compressive strength and exceptional durability (4) makes it possible to utilize a large amount of reinforcement, thus giving new structural possibilities compared to conventional high strength concrete.

The possibility of having an alternative to steel and conventional concrete gives designers and builders more freedom in design, providing better and more cost effective solutions (5, 6).

Using earlier research (3) on the mechanical behavior of CRC, the development of appropriate structural calculation methods and design rules for civil engineering applications was achieved (5). A range of structural members was produced and tested in the course of this project in order to evaluate the structural analysis.

Three different structural members were loaded to failure: beams, columns, and beam to column connections. They are supposed to be representative of the most frequent structural elements encountered in civil
COMPACT REINFORCED COMPOSITE (CRC)

CRC is an ultra strong and extremely tough concrete covering a range from about twice the strength of conventional reinforced concrete up to that of structural steel. The most obvious difference between conventional reinforced concrete and CRC are the high bending stresses that can be achieved with virtually no visible cracks and the extreme ductility.

CRC is made of three components, the cementitious matrix, the metallic fibres and the steel reinforcement. The cementitious matrix is a silica fume ultra high strength concrete developed by Aalborg Portland. It is made of white rapid hardening cement mixed with silica fume added at equivalent to 25% of the total mass and quartz aggregates of up to 4 mm in size. The addition of water is kept to a minimum and thanks to the use of superplasticizers, water/cement ratios of 0.18 to 0.20 are employed.

The cementitious matrix is a rather brittle material, but adding steel fibers to the matrix transforms it into a ductile material. The use of steel fibers contributes to the anchorage of reinforcement bars and enables the bending capacity of structures to be increased. Plain steel fibers are added to the cementitious matrix in the concrete mixer. With an average strength of 900 MPa, the fiber diameter is 0.4 mm and the length 13 mm.

The mechanical properties of the matrix reinforced on include with 6% by volume are, of fibres encompass a compressive strength of 150 MPa, a Young’s modulus of 50 GPa and a modulus of rupture measured on normalized prisms of 40 x 40 x 160 mm of 35 MPa.

The extreme ductility obtained with fibre reinforcement is combined with a closely spaced main reinforcement, yielding exceptionally high strengths and crack-free behavior. Volumes of reinforcement of up to 20% are employed.

The low amount of water introduced in CRC together with the large quantities of ultrafine silica fume particles added to the cementitious matrix result in an excellent durability (4). Resistance of the material to the principal aggressive agents, typically CO₂ and chloride ions, was shown to be several times higher than in ordinary concretes.

To take advantage of it and to benefit from the high ductility of the fibre reinforced matrix, the possibility was considered of reducing the cover thickness.

The corrosion of the reinforcement was investigated on specimens with concrete covers with a depth of 10 mm loaded to a constant center point deflection and exposed to chlorides. No correlation was observed between the
load applied and the rate of chloride diffusion. For loaded and non-loaded CRC beams, the diffusion coefficients measured were several orders of magnitude lower than values measured for conventional concrete.

With emphasis on structural reliability and in order to maintain tolerances during the placing of the reinforcement, the design of structural members was performed with concrete covers of 15 mm, which is two times less than recommended in most standards. In spite of the important reduction of the concrete cover, reinforcement steel remains perfectly protected against external attack and corrosion.

TEST PROGRAM

The possibilities of using CRC for structural applications in civil engineering were demonstrated in the MINISTRUCT project. Full scale tests were carried out to verify the design rules and to investigate and optimize the production, transportation, placing and curing techniques. The full scale tests described herein, include the following structural members:

1) Six columns with 5 and 15 % by volume of passive reinforcement respectively
2) One beam of 13m in length tested in bending,
3) Two beams tested in shear, with and without shear reinforcement,
4) Two beams to column connections.

CRC used to produce the test specimens was obtained with a water/binder ratio of 0.18, except for the beam, which was produced with a water/binder ratio of 0.19. The fiber content was kept constant at 6% by volume. Fibers were introduced at the end of the mixing process after the preparation of the cement.

Cylindrical and prismatic control specimens were produced for each batch and stored together with the CRC members. They were tested two or five days before the full scale tests were carried out and at least 28 days after casting. Average compressive strengths obtained varied between 132 and 146 MPa, depending on the age and on the water/binder ratio of the concrete.

The formwork was vibrated continuously during filling. External vibration was used in order to densify the matrix and to eliminate entrained air. At the end of the filling process, wet sacking was placed on the CRC members to avoid drying and surface cracking. After demoulding, 24 hours after concreting, a curing compound was sprayed over the concrete to reduce drying.
COLUMNS

The properties of CRC, which are of particular interest in the design of columns are:

1. High compressive strength. The use of CRC instead of conventional concrete leads for equivalent column sections to important profits in column carrying capacity.

2. High ductility. Conventional concrete columns require lateral confining reinforcement to improve the structural ductility. The addition of steel fibres to the cementitious ultra high-strength matrix leads to improvement in the direct tensile strength and to considerable material ductility (5). Stirrups, which are used traditionally for the confinement of the concrete and necessary so as to avoid buckling of the longitudinal reinforcement, can thus be omitted. The thickness of the concrete cover can thus also be reduced without risk of spalling.

The column design without stirrups and with low reduced cover thickness was tested on rectangular members with a section of 120 x 130 mm and a height of 2900 mm (figure 1), with two different reinforcement ratios and layouts. The strength of the reinforcement bars was guaranteed at 500 Mpa.

In order to verify the theoretical behavior of the columns and their sensitivity to the application of eccentric loads, mechanical tests in pure compression and with an eccentricity of 20 mm were carried out. In total three columns were tested for each reinforcement ratio, one in pure compression, the other two with an eccentrically applied load.

Since columns are principally solicited in compression, the column design was carried out with a parabolic representation of CRC behavior in compression:

parabolic stress-strain relationship according to

$$\frac{\sigma}{\sigma_u} = \sin\left(\frac{\pi E}{2 E_c}\right)$$

The chosen column geometry leads to a ratio of theoretical buckling load over the compressive load, - also called the slenderness index $\alpha$ -, over equal to 199. The critical load expected to be sustained by the tested columns was calculated using Euler’s equation $N_{cr} = \left(\frac{\pi}{L}\right)^2EI$ and a finite element software package taking into account non linearities of the structural behavior (7).

The six columns were tested on a 3000 kN testing machine. The load was transferred from the bearings to the columns by two steel caps, one at the
bottom, the other at the top of the column. The lower cap was built on a one-directional swivel-joint, the upper one on a bi-directional ball-joint. The cap geometry was designed in order to distribute stresses homogeneously over the entire column cross-section. It also permitted application of the load with an eccentricity of 20 mm.

The column instrumentation consisted of a load cell placed between the upper cap and the bearing, two displacement sensors to determine the longitudinal and the transversal column displacement and four strain gages fixed at mid-height on the four columns’ surfaces.

For each reinforcement ratio one centrally and one eccentrically loaded column were tested in three stages at 50, 75 and 100% of the expected ultimate load. For each reinforcement ratio, two other columns with the same eccentricity, were sustained five consecutive loading cycles at 60, 70, 80, 90 and 100% of the ultimate calculated carrying capacity, called hereafter olygocyclic loading. Table I provides the load at failure of the columns.

The mechanical behavior of the tested columns depends basically on the reinforcement ratio. For each column, the linear elastic stage is observed at least up to 80% of the ultimate carrying capacity of the columns.

The columns with 5% reinforcement show significant linear elastic stage followed by a structural ductility. The columns with 15% of reinforcement show, after the linear elastic stage, non-linear elastic behavior with more important ductility of the structure. The transversal displacement of the centrally and eccentrically loaded columns with 5 and 15% of fibres by volume is illustrated in figure 2 and compared to the calculated values.

The ultimate carrying capacity of the columns was in good agreement with the expected theoretical values, except for the centrally loaded column with 5% of reinforcement. The load at failure exceeded the calculated ultimate load by 20%. Unlike all other columns, which failed in bending (compression failure of the concrete, see figure 3), this specimen failed due to buckling of the reinforcement in the upper section. Strain measurements at mid-height of the column revealed equal deformations on all four column faces, which indicates rupture in mode 2.

These tests demonstrated that buckling of the principal reinforcement can be prevented by using a fibre reinforced highly ductile cementitious matrix with a concrete cover layer of 15 mm.

BEAMS / SHEAR TESTS

The design of conventional concrete beams requires on the one hand longitudinal reinforcement and on the other hand shear reinforcement. For
this material, the use of fibres contributes efficiently to the improvement of beam shear behavior (8, 9). The possibility of designing a CRC beam without shear reinforcement in the web was investigated. Shear tests were carried out in order to compare results when using fibers only with results when adding a specific shear reinforced with CRC in the web.

Preliminary tests were carried out to study the shear behavior of beam segments. The beams measured 2 600 mm in length and 560 mm in height, with a web of 80 mm in width, as illustrated in figure 4. The beams were tested in three point bending. The span between the bearings, 3.5 times the height of the tested beam, was 1 980 mm in order to guarantee the shear failure of the segment. The segment was supported on two steel cylinders in order to allow for the horizontal displacements of the bearings during the loading. The test was load-controlled and the vertical displacement at mid length of the segment was measured. The beams were loaded to failure.

As expected both beam segments failed in shear. The beam without shear reinforcement in the web reached an ultimate carrying capacity of 875 kN. With 1.5 % by volume specific shear reinforcement in the web, the beam failed at 1 456 kN. This corresponds to an improvement of 66% in the carrying capacity. With shear reinforcement, two cracks propagated symmetrically from the bearings to the point where the segment was loaded. In the case of the beam without shear reinforcement, only the propagation of one major crack was observed. The major crack was accompanied by secondary cracks which showed up in the mid-section of the beam segment (figures 5 and 6).

The test carried out on a beam segment without reinforcement in the web, enabled the shear strength of the fibre reinforced CRC matrix to be calculated. The equation used for the calculation is:

\[ \tau_w = \frac{3 F_u}{4 h b} \]

with:
- \( F_u \) = ultimate load
- \( h \) = beam height
- \( b \) = web width
- \( \tau_w \) = shear strength

Here the calculated shear strength was 14.7 MPa, which is slightly higher than the tensile strength of 13.8 MPa measured by Nielsen (10).

**BEAM / BENDING TEST**

A bending test was carried out on a 13 000 mm long beam similar in geometry to the one tested in shear. The beam was expected to carry a moment at mid-span of approximately 600 MNm. The beam was reinforced with 8 longitudinal steel reinforcement bars of 20 mm in diameter. The steel
reinforcement was anchored at both beam ends using transverse reinforcements.

The longitudinal reinforcement steel was kept in position by using concrete spacers of 15 mm thickness placed at intervals of 1 000 mm each. The formwork was filled with three successive concrete layers which were each compacted by external vibrator. The formwork was stripped 24 hours after casting. In order to avoid concrete drying, a curing compound was sprayed on the concrete surface.

The beam was loaded at 65 days after casting on a test rig equipped with 8 jacks positioned symmetrically along the beam at a distance of 1 230 to 1 380 mm, as illustrated on figure 7. The beam was supported by two steel cylinders placed at each beam end to compensate longitudinal displacements of the bearings during the loading process. To avoid the risk of lateral buckling at the failure stage, lateral guides were fixed to the test rig at a distance of 5 to 10 mm from the beam.

To improve the visual detection of the crack pattern, the beam was painted in white with a brittle product. The test equipment included load cells and displacement transducers at mid-length of the beam, as well as five strain sensors distributed at different heights of the beam section.

The beam (figure 8) was loaded to four different load levels, 15, 30, 50, 60% of the ultimate load in successive cycles, before running the beam to failure (figure 9). The test was load controlled, the load being increased gradually. Each cycle lasted approximately 30 minutes.

The beam failed in bending at mid-length. The measured bending moment of 635 MNm was slightly higher than the expected beam carrying capacity which was estimated at 600 MNm. Linear elastic behavior was observed up to 85% of the ultimate load. After that stage, crack formation was observed. Failure of the tested CRC beam is characterized by an significant structural ductility reflected in the load-deflection curve of figure 9.

**BEAM TO COLUMN CONNECTIONS**

The excellent anchorage properties of CRC (11) due to a high bond strength between the cementitious matrix and the steel reinforcement allows the reduction of the transfer length. Moreover, the high ductility of CRC and the possibility of using high reinforcement bar ratios, owing to small aggregate size, are favorable factors.

Two T-shaped beam to column connections (figure 10) were produced by assembling prefabricated beam and column segments produced from a fibre
reinforced CRC matrix. The arrangement of the longitudinal and lateral reinforcement was optimized, to reach a maximum bending capacity of the joint at a given geometry and to simplify the casting procedure. The geometrical arrangement of the used steel reinforcement is illustrated in figure 11.

To increase the bond capacity between the hardened and fresh CRC, the roughness of the hardened concrete surface was increased by using a surface desactivator when casting the beam and column assemblies. After hardening of the concrete, the surface of the concrete was washed out with at high pressure.

The T-shaped specimens with a width of 140 mm (figure 11, 12) were tested in bending at a span of 2 000 mm between the bearings. To avoid transversal forces, the test specimens were supported by two bi-directional swivels at both ends. Load, displacement and crack opening at the joints were recorded during loading. A first loading cycle to 50% of the calculated failure load was carried out before testing the specimen to failure.

Failure loads of 320 and 321 kN were measured respectively. Failure of the connection was caused by pull-out of the longitudinal steel reinforcement bars located at the center of the connection. The failure loads lead to a flexural capacity of 144 MNm. This value corresponds to 64% of the calculated ultimate bending moment of the beam sections used for the connection.

The crack pattern was very similar for both test specimens, which shows a satisfactory test reproducibility. This is illustrated in figure 12.

The deflection curves of the beam-to-column connections presented in figure 12 show a ductile and progressive failure process of the structure. The larger deflection of the second beam to column connection resulted from a non-symmetric opening rate of one of the connection joints.

CONCLUSIONS

Full-scale structural elements made of ultra high-strength fibre reinforced concrete, called CRC, were produced and tested. Columns, beams and joints were designed to take advantage of the mechanical properties of this material and to verify the general structural behavior.

The experimental work on the different structural members led to a number of results, which are:

Columns: Despite the absence of stirrups and a concrete cover of
only 15 mm, no buckling of the longitudinal reinforcement and no spalling of the concrete was observed.

Beam: Shear strength of the fibre reinforced CRC matrix was estimated at 15 MPa. Although shear reinforcement increases significantly the shear capacity of beams, a ductile failure was observed on a 13 m long beam tested in bending without any shear reinforcement.

Connections: The anchorage properties of CRC are well adapted for beam to column connections. High structural ductility is obtained using significant re-bar ratios (about 4% by volume, 2% from each beam) and an optimized arrangement of the re-bars.

The use of CRC in structure with the same characteristics as traditional structures allows savings in the material used while loading to high-quality value products such as beams, columns, connections and other structures. For example, for the beam, its cross-section of 50 000 mm² can be compared to a cross-section of 210 000 mm² for a comparable conventional reinforced concrete beam. This represents savings of up to 76% in materials.

ACKNOWLEDGMENTS

The tests presented herein were carried out within a project sponsored by EC, European Commission Project MINISTRUCT “MINimal STRUCTures using ultra high-strength concrete”. Funding of the project by the European Commission with the Britc-EuRAm Program is gratefully acknowledged.

The project was conducted by Aalborg Portland, Denmark, and partners involved in this project were Bouygues (France), Carl Bro (Denmark) and CSIC. The authors would like to thank Denis Parmentier and Joël Cuny for their active participation in this experimental work.

REFERENCES


**TABLE 1—ULTIMATE LOADS OF CRC COLUMNS COMPARED TO THE EXPECTED FAILURE LOAD.**

<table>
<thead>
<tr>
<th>Reinforcement ratio and load distribution</th>
<th>Critical load</th>
<th>Average theoretical load</th>
</tr>
</thead>
<tbody>
<tr>
<td>5 vol-%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Central load</td>
<td>1.53 MN</td>
<td>1.28 MN</td>
</tr>
<tr>
<td>Excentricity 20mm</td>
<td>0.56 MN</td>
<td>0.57 MN</td>
</tr>
<tr>
<td>Oligocycl. excentr. 20mm</td>
<td>0.59 MN</td>
<td>0.57 MN</td>
</tr>
<tr>
<td>15 vol-%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Central load</td>
<td>1.43 MN</td>
<td>1.48 MN</td>
</tr>
<tr>
<td>Excentricity 20mm</td>
<td>0.75 MN</td>
<td>0.79 MN</td>
</tr>
<tr>
<td>Oligocycl. excentr. 20mm</td>
<td>0.79 MN</td>
<td>0.79 MN</td>
</tr>
</tbody>
</table>
Fig. 1—Reinforcement layouts for column test.

Fig. 2—Transversal displacement of CRC reinforced with 5 and 15 percent, respectively, of longitudinal reinforcement and loaded centrally (Centr.) and eccentrically (Exc.).
Fig. 3—Ductile failure of reinforced CRC column.

Fig. 4—Section of the beams tested in shear and position of the reinforcement.

Fig. 5—Shear cracks in beam with shear reinforcement.
Fig. 6—Shear cracks in beam without shear reinforcement.

Fig. 7—Test rig geometry for beam test.

Fig. 8—CRC beam of 13 000 mm length.
Bending moment [kNm]

Displacement [mm]

Fig. 9—Vertical displacement of the beam during the loading process.

Fig. 10—Test of beam to column assembly.
Fig. 11—Reinforcement layout used for the beam to column assembly.
Fig. 12—Crack pattern observed on the beam column assembly.

Fig. 13—Deflection curves of beam to column assembly.
Steel Fiber Reinforcement for Extruded Prestressed Hollow Core Slabs

by C. Peaston, K. Elliott, and K. Paine

Synopsis: An experimental investigation was conducted to assess the potential of steel fibres as secondary reinforcement in prestressed hollow core slabs. Following a brief laboratory study and a feasibility trial, a series of fibre reinforced extruded slabs were made at the premises of a local manufacturer and subsequently tested in shear: one of a number of potential modes of failure which cause concern in this type of slab because of the lack of shear or secondary reinforcement. The addition of the fibres increased both the ultimate strength and toughness of the slabs leading to safer and more controlled failures. The predictive equations of other researchers were shown to accurately estimate the shear strength in the case of plain hollow core slabs, but to overestimate the shear enhancement due to adding steel fibres.

Additionally, the effect of the manufacturing process, in which the concrete is compacted by rotating augers, on the fibre distribution and orientation was investigated. Whilst fibres were found to be randomly distributed within the cross-section, a tendency to align vertically within the webs was observed. This has particular relevance to the vertical shear performance.

Keywords: extrusion; fiber reinforced concrete; hollow core slabs; precast concrete; prestressed concrete; reinforcing materials; shear strength; shear testing; steel fibers
Chris H. Peaston is a Lecturer in the Department of Civil Engineering, University of Nottingham. He has research interests in fibre reinforced and sprayed concrete and supplementary cementitious materials, and has authored a number of papers on these subjects.

Kim S. Elliott is a Senior Lecturer in the Department of Civil Engineering, University of Nottingham. He is a member of the British Precast Concrete Federation’s Technical Committee on frames and flooring, of the FIP Commission on Prefabrication and has published several papers on precast concrete frame behaviour.

Kevin A. Paine is a Research Student in the Department of Civil Engineering, University of Nottingham, where he is currently studying for a PhD. His research interest is in prestressed fibre reinforced concrete.

INTRODUCTION

Hollow core floor slabs are amongst the most advanced products in the precast and prestressed concrete industry. They are used in commercial, industrial and residential buildings as well as for car parks and sports stadiums. The slabs are between 110 mm and 400 mm in depth and usually have a nominal width of 1200 mm. The combination of low self-weight and longitudinal prestressing allows spans of up to 20m.

The slabs are manufactured by either long-line extrusion, or by slip-forming: both methods impose restrictions on the placing of secondary reinforcement. Hollow core slabs are therefore exempt from the minimum shear reinforcement requirements found in design codes (1,2,3). The lack of secondary and shear reinforcement may lead to: problems in resisting shear and torsion; an inability to support axial column loads (punching shear); restrictions on the use of shotfired ceiling fixings; cracking around large voids; and a high incidence of accidental damage during handling, transporting and building-in.

Additionally some slabs, particularly those extruded with large circular cores, are manufactured with webs that are less than 40 mm in thickness. The principal tensile stress occurs in this region, and reductions in the shear capacity, as a result of the additional shear stresses $\tau_y$ and $\tau_x$ (Figure 1) which occur when slabs are placed on flexible supports, are of particular concern (4). This paper presents the results of an investigation conducted to assess the potential of
randomly distributed short steel fibres for improving the shear performance of hollow core slabs.

The performance of a variety of fibre reinforced concretes (FRCs) under shear have been previously reported (5). Generally the shear capacity increases with increased fibre content, fibre aspect ratio and with improved fibre-matrix interfacial bond. Fibres have a greater effect on the ultimate shear strength than the cracking shear strength. Increases in cracking shear strength are attributable to increases in the tensile strength of the composite which are likely to be small. The enhanced post-cracking performance with possible higher ultimate strength, is associated with the energy that is required to pull the fibres out of the cracked matrix.

The behaviour of prestressed hollow core floor slabs subjected to shear forces is a complex three dimensional problem, one which is best studied by full scale testing of slabs manufactured according to normal industrial practice. Although steel fibres are commonly added to concrete, the unique geometry and material properties of these slabs meant that both the manufacturing technique and structural performance had to be investigated. This paper describes how a series of trials was conducted at the premises of a local manufacturer using the 'Spiroll' extrusion technique, and presents the results of the subsequent shear tests.

BACKGROUND

Two modes of shear failure can occur in hollow core slabs: flexural shear and web shear tension. Flexural shear occurs when a flexural crack develops into a shear crack and leads to a relatively ductile failure. Web shear tension is of greater concern because the failure is brittle and may not be preceded by any warning of impending failure. The shear tests reported in this paper were conducted to investigate the effects of fibre reinforcement on web shear tension failures in hollow core slabs.

Web shear tension failure occurs when the principal tensile stress in the web, due to a vertical shear load, reaches the tensile strength of the concrete. The resulting diagonal crack usually occurs in the middle of the web, where it is thinnest, and at a distance of h/2 from the inner face of the bearing (6); h being the depth of the slab. The crack propagates quickly at an angle of approximately 35° to the direction of the slab causing almost immediate failure.

Traditionally the shear strength \( V_{co} \) of a prestressed concrete beam failing by web shear tension is calculated using the tensile strength of the concrete \( f_{ct} \) and
the effective prestress at the centroid of the section due to prestressing ($\sigma_{cp}$) by an equation of the type

$$V_{co} = \frac{I_b}{Ay} \sqrt{f_{ct}^2 + \sigma_{cp}f_{ct}}$$

(1)

where $Ay$ and $I_e$ are respectively the first and second moments of area about the centroidal axis, and b is the effective width. In pretensioned members $\sigma_{cp}$ is reduced because the section critical for cracking falls within the prestress transmission length.

Equation 1 is simplified in BS 8110 (2) to

$$V_{co} = 0.67bh \sqrt{f_{ct}^2 + 0.8\sigma_{cp}f_{ct}}$$

(2)

The value of 0.8 is included as a safety factor and, for hollow core slabs, b is taken as the sum of the width of all the webs in the element. This simplification leads to conservative designs in hollow core slabs because 0.67bh is less than $I_b/Ay$. Despite this Pisanty (7) has shown that it is still necessary to reduce $V_{co}$ by a further 10% to bring the calculations into line with experimental results. In BS 8110 the tensile strength of concrete is calculated empirically from the compressive cube strength ($f_{cu}$) by

$$f_{ct} = 0.3\sqrt{f_{cu}}$$

(3)

where a safety factor of 1.5 present in BS 8110 has been omitted.

A large number of tests on pretensioned (8) and post-tensioned (9) FRC beams have shown that the ultimate shear strength of these beams is greater than in equivalent plain prestressed beams and, that the ultimate shear capacity of prestressed beams under web shear tension can be predicted by the following equation

$$V_{co} = 0.67bh \sqrt{f_{sp}^2 + \sigma_{cp}f_{sp}}$$

(4)

Here the tensile strength of concrete is taken as the ultimate split cylinder strength, $f_{sp}$. Additionally it was suggested that $f_{sp}$ can be found empirically from the compressive strength of 100mm cubes using the expression
where \( A \) and \( B \) are dimensional constants equal to 0.7 N/mm\(^2\) and 1 N/mm\(^2\) respectively. The fibre factor \( F \) differentiates between the reinforcing abilities of various fibre types and is given by

\[
F = \eta_b, V_f, \lambda_f
\]

based on the fibre volume fraction (\( V_f \)), fibre aspect ratio (\( \lambda_f \)), and a bond factor (\( \eta_b \)) which was used to account for the various bond and anchorage capabilities of different fibres.

The use of the split cylinder strength has been regarded with some scepticism in FRC design, however, because the stresses after cracking are complicated and not accurately understood (10). Additionally the use of the split cylinder strength in plain hollow core slab design has been shown to produce exaggeratedly high results (7). Equations for shear strength of prestressed FRC, with the tensile strength derived indirectly from flexural strength or toughness may be more acceptable.

Another possible approach is to add a supplement to Equation 1 based on a fibre reinforcement mechanism, which accounts for the additional strength provided by fibres. Such supplements have been suggested for use with reinforced concrete beams (11,12), but these need to be adaptable to all fibre types and show the gradual reduction in improved performance when adding further fibres. At present the approach based on ultimate split cylinder strength appears to produce the best correlation with test results for fibre reinforced prestressed beams with traditional approaches still used for plain beams.

MANUFACTURE OF EXTRUDED SLABS

Production

The slabs were produced using the ‘Spiroll’ extrusion technique (Figure 2) in which a zero slump, dry mix concrete, which is self-supporting over the voids, is compacted by rotating augers to produce the required cross-section. The most important feature is the action of the concrete in the webs, which is dominant in organising the orientation of the fibres. The concrete in four of the seven webs is compacted against augers rotating in opposite directions (Figure 3), thereby inducing a shearing action on the concrete. Helical seven-wire strand, or plain
or indented wire is drawn along the length of the prestressing bed and pretensioned. A localised interruption to the distribution of fibres is possible because the reinforcement is located directly beneath the centre line of the web.

Handling of the slabs usually takes place within 12 hours of casting when the compressive cube strength is around 35 N/mm². 28 day strengths are about 90 N/mm², although a value of 60 N/mm² is used in design. Typical mixes are given in Table 1. The free water/cementitious material ratio is 0.27 for the plain mix. Apart from a slight increase in the water content to allow for the steel fibres, as explained later, the mix proportions were nominally identical in the plain and fibre reinforced slabs. However, some adjustments in the quantities do occur on a day-to-day basis at the factory to account for small variations in aggregate quality, aggregate moisture content, ambient weather conditions and individual machines.

The fibre reinforced hollow core slabs were made in a total of four trials at the premises of a local manufacturer. A preliminary trial was conducted to assess whether the inclusion of steel fibres in hollow core slabs was feasible, and to assess the benefits. Problems were found to exist with adding the fibres, compacting the concrete, contamination of other slabs being produced at the same time and additionally safety implications were raised. Having considered and reduced or eliminated these problems, three further trial production runs were carried out from which six fibre reinforced and two plain 2m long slabs were sawn to assess the shear performance of steel fibre reinforced hollow core slabs.

1200 mm wide by 200 mm deep units were produced in all the trials, since these are the most common size of slab produced in the UK. The slabs were reinforced with seven prestressed helical strands with nominal diameters of 11 mm and 12.5 mm, for the preliminary and main trials, respectively. Strands were of relaxation class 2 (13) and initially prestressed to 70% of their nominal tensile strength (1770 N/mm²). The fibres used were 30 mm long by 0.5 mm diameter hooked end drawn steel wire fibres collated into bundles with a water soluble adhesive. The first three columns of Table 2 show the slab designation and fibre reinforcement at each trial.

Mixing Procedure

Different mixing procedures for adding steel fibres were investigated in the laboratory at the University of Nottingham (14). To adequately separate the fibres and prevent balling in dry, zero slump mixes it was found necessary to add all the dry materials including the steel fibres before the addition of any free water. This method was adopted in the subsequent trials.
At the factory the fibres were added by hand through an observation port in the mixer which took between 3 and 4 minutes to complete. Dry mixing was therefore longer than normal, but this did reduce the chances of fibre balling which can occur when fibres are added too quickly to a harsh mix. The mixer was of the pan type and had a capacity of 0.65 m$^3$. After the addition of water the concrete was discharged into a skip for delivery to the extruder. Visual inspection of the material in the skip showed the fibres to be well separated and distributed.

Compaction

For the preliminary trial three batches of material were made, each producing a six metre length of slab. For the first batch the fibres were added to a traditional plain mix. Although the resulting fibre reinforced material passed through the rotating augers with no apparent difficulties, it was clear that problems existed with the compaction as the top surface had a very obvious honeycombed texture. For the second and third batches additional water of 5 kg/m$^3$ and 8 kg/m$^3$, respectively was added to improve compaction to the same quality as the plain mix slabs produced at the same time.

The higher water content was used in the three main trials and lead to a slight reduction in the measured compressive strengths (Table 2). It is possible that a superplasticizer may improve the ease of compaction of concrete and maintain the compressive strength. However, at present it is unclear which superplasticizers are suitable for use in zero slump extruded concrete (15). It should be noted that the operation of the machine was not adjusted to allow for the presence of fibres, and nor was the portal above the augers opened wider to allow an easier passage of material. After detensioning, the slabs were cut to the lengths required for testing and stored in the usual way prior to transporting to the University of Nottingham for testing.

During slab production 100 mm cubes were made for testing the compressive strength. Since the mix is too dry for normal gravity vibration compaction methods, a mechanical hammer with a footprint of 100 mm square was used. The harsh nature of the mix prevented the making of larger specimens for tensile or flexural tests. The 28 day cube strengths were used to derive the splitting strength indirectly from Equations 5 and 6 taking the bond factor, $\eta_b$ as 0.75 (8) and aspect ratio, $\lambda_t$ as 60 (16). The effective prestress was calculated semi-empirically from assumed initial prestressing values and measured geometrical values. These values are all presented in Table 2.
DISTRIBUTION OF FIBRES

Fibre distribution was investigated by wash-out tests on samples taken immediately after casting, and by taking cores from the hardened concrete. The wash-out tests consisted of taking samples of the fibre reinforced concrete from the end unit on the prestressing bed from each of the locations shown in Figure 4. Collection of a representative sample is difficult because the concrete is dry and well compacted, with samples around the strand being especially difficult to collect. The samples were weighed and then washed through a 300μm sieve until the retained material was free of cement. A magnet was used to extract the fibres. The weights of the sample and the extracted fibres were then converted into their respective volumes, and the fibre volume fraction calculated.

The results (Figure 4) show that the fibres were evenly distributed throughout the slab in the preliminary trial and trial 1. However, the fibre contents found in trial 1 were greater than expected. The four results for trial 2 are widely spread although their average equates to the 1% fibre volume added. It appears that wash-out tests give a qualitative indication of the fibre distribution, but are rather variable due to the nature of the sample.

Additionally, a 70 mm diameter core bit was used to cut specimens, at three separate locations, vertically through the webs of each slab from trials 1 to 3, to a point just above the level of the strands. The saturated density of the resulting, non-cylindrical specimen, was determined prior to crushing which enabled both the density of the plain concrete, in the critical region of the slab, and the volume of fibres to be accurately determined. These results (Table 2) showed that the intended fibre volume was present in the slabs throughout the entire length and width, and that fibres were not lost in production. The density of the plain concrete allows a comparison of the compaction achieved in plain slabs and fibre reinforced slabs. Although rather low for the slabs in Trial 1 this value is otherwise consistent at around 2440 kg/m³ showing that the same levels of compaction can be obtained in fibre reinforced slabs as in plain slabs.

ORIENTATION OF FIBRES

A random three dimensional fibre orientation was considered unlikely due to the narrow thickness of the flanges and webs, and to the polarisation of the compaction of concrete as the extrusion machine travels along the bed. The orientation of fibres is critical in the web zone, where a predominant inclination between 60° and 90° to the horizontal might help resist diagonal tension more effectively.
To evaluate the changes in fibre orientation throughout the depth of the slab, two sections were cut vertically from the web of a preliminary trial slab, as shown shaded in Figure 4. These sections were cut into prisms of about 40 mm in height such that the fibres in each of the orthogonal directions were exposed. The number of fibres intercepting each of the six faces were visually counted and an orientation factor, $\eta_0$, the average ratio of the projected fibre length in each direction to the actual fibre length, calculated from the following relationship (17)

$$\eta_0 = \frac{N \cdot V}{\sum L_f}$$  \hspace{1cm} (7)

where $N$ is the number of fibres per unit area; $V$ is the sample volume found by hydrostatic weighing of the specimen; and the total length ($\Sigma L_f$) of all the fibres in the specimen is obtained by crushing the specimen and collecting the fibres. Where a 3-dimensional random distribution occurs the fibre has an equal probability of orienting in any direction; this gives a theoretical orientation factor of 0.405 (18).

The average results from the two sections (i.e. four faces) are shown in Figure 5. Fibre orientation in the top flange is more pronounced in the longitudinal x direction (the direction that the slab is produced). This is probably due to the drag of the machine moving over the top surface reorienting many vertically aligned fibres; such fibres may provide resistance to cracks caused by tensile stresses at the release of prestress. The situation is significantly different in the web where the rotating augers cause fibres to orient vertically, at the expense of the longitudinally and transversely aligned fibres. This might be expected to improve the ultimate shear capacity of the slabs because web shear tension, which starts in the middle of the webs and propagates quickly in the vertical plane, leads to a brittle failure in plain mix slabs.

There appears to be no favourable orientation at the bottom of the sections with the values close to that expected for a random distribution. However, because of the location of the prestressing strand the prism cut from the bottom of the slab was about 60mm high and the orientation effects that might have been expected in the vicinity of the boundary have been masked. An alignment in the horizontal plane might have been anticipated and this would increase ductility in transverse flexural type failures which occur when slabs are placed on flexible supports.
SHEAR TEST DETAILS

Shear tests were carried out on each of the 2m long slabs loaded over their full width. The tests were performed on both ends of the slab, referred to as the A Test and B Test. The slabs were simply supported over effective spans of 1900 mm and 1470 mm for the A and B tests, respectively. A bearing of 100 mm was used on both ends of the slab to imitate typical hollow core slab applications. The general arrangement for the tests is shown in Figure 6 and details of the full test programme are given in Table 3. The test variables were fibre volume fraction and shear span to effective depth ratio (a/d). Two nominally identical tests were conducted for each combination of variables.

Linear Potentiometers (LP) were placed under the beam to measure deflections at the load-point and at positions close to the supports. Two LPs were used, one on either side of the slab, at each location to provide an average reading and to compensate for any twisting. Two LPs were also placed close to the support to record support settlements. In later tests, a deflection bar was used to suspend the LPs from the slab so that net deflections free of extraneous deformations could be obtained.

The load was applied with a hydraulic jack attached to a manual pump and a 500 kN load cell, and transmitted to the slab through a spreader beam. The load was applied in increments of up to 10 kN until first crack, after which increments of deflection were used. Tests were stopped when the average crack width equalled 15 mm (equal to half the length of a fibre).

SHEAR TEST RESULTS

General Test Behaviour

In all of the tests the ultimate failure was by web shear tension accompanied by strand slip. For the plain slabs the load-deflection curve was linear up to the first crack at which point there was an instantaneous failure, with cracking in all the webs, resulting in a large loss of strength. In the fibre reinforced slabs, cracking always occurred in the central webs before spreading to the edges where it could be more clearly monitored. During this initial cracking phase, where some webs were less stiff than others, the slab tended to behave in a three dimensional manner and transverse cracks, as well as torsion cracks, appeared across the sawn end of the slab. Although this problem was present in the plain slabs it was less severe, because once a web failed its load carrying capacity was
substantially reduced, thus overloading adjacent webs and causing instantaneous failure. When crack widths exceeded about 5 mm the slab tended to hinge about the load point causing the cracks to open widest at the soffit.

Crack patterns varied for the two a/d ratios. For a/d = 2.0 the location of the critical crack - taken as the position of the crack at the thinnest part of the web - occurred at a distance roughly equal to h/2 from the edge of the bearing and ran at approximately 45° to the direction of the slab (Figure 7). At a/d = 2.8, however, the critical crack occurs further into the slab at about 0.8h - 1.0h from the edge of the bearing and subtends an angle of approximately 35° to the horizontal (Figure 8).

Shear strengths

The results of the shear tests are given in Table 3. For each test two shear strength values have been defined: the first crack strength ($V_{cr}$) indicates the load at which the slab started to fail in shear and the ultimate shear strength ($V_{ult}$) refers to the highest value of shear the slab withstood. Table 3 also gives the semi-empirical theoretical shear strength $V_{th}$ which was calculated from Equation 4 for fibre reinforced slabs, and from Equation 2, with the safety factor omitted, for plain slabs. The values of $\sigma_{cr}$ given in Table 2 were used in these calculations along with the empirically derived values for $f_{sp}$ in the case of the fibre slabs. Individual results for nominally identical tests showed some variation which might have been anticipated due to the complex nature of the slabs.

The average shear strength for the plain slabs corresponded well with the predicted values at both a/d ratios; the slab tested on the shorter a/d was expected to have a lower shear capacity because of its geometry. However, the predicted shear strength takes no account, either of the fact that the slabs at a/d = 2.8 failed in a region where the prestress has been able to develop more than the slabs at a/d = 2.0 or, more significantly, of the enhanced shear strength close to the support. Since Equation 2 accurately estimates the shear strength of the plain hollow core slabs tested here, the 10% reduction in strength advocated by Pisanty (7) is inappropriate. This is probably because Pisanty tested over an a/d = 3.5 where the enhanced shear strength close to supports was not as significant.

A small increase in the cracking strength, $V_{cr}$, of the fibre reinforced slabs over that of the plain slabs was observed at the larger a/d ratio. The mean values for $V_{cr}$ for the plain and 0.5% fibre reinforced slabs at a/d = 2.8 were 152 kN and 157.8 kN respectively, an increase of only 4%. The corresponding values at the shorter shear span were 141 kN in the plain and 184.6 kN in the fibre slabs, an
increase of 31%. This improvement was unexpectedly high and was attributed in part to the observation that the cube strengths of slabs F1 and F2, which where used for three of the five tests at a/d = 2.0, were higher than that of the plain slabs.

The most striking feature in the fibre reinforced slabs was an average increase of 9% in $V_{ult}$ (average 172.4 kN) over $V_{ult}$ (average 157.8 kN) for a/d = 2.8, which is not replicated in the case of the 0.5% slabs at a/d = 2.0. Indeed, for the shorter shear span, first crack and ultimate load occurred simultaneously in all but one of the 0.5% slabs, and for the two tests conducted on the 1.0% slab the average increase was only 5%.

These observations are attributed to the different types of shear behaviour that occur with changing shear span/depth ratios. At the longer a/d ratio the dominant failure mechanism is one of shear - tension and cracking occurs as a result of the complementary shear stresses. However, at shorter shear span to depth ratios a shear - compression type failure becomes more dominant with the diagonal cracking forming as a result of the bursting stresses which occur due to the compressive strut action near the support.

There is no apparent reason why fibres should be any more effective at increasing the first cracking strength in shear - compression as opposed to shear - tension and the experimentally observed difference is attributed to the limited number of tests conducted on plain slabs and the experimental variables discussed above. However the significant difference observed in the post cracking behaviour may be explained by considering the two failure modes. In shear - compression failure tends to be more brittle and occurs as a result of the concrete crushing at the load point, a mechanism over which the fibres will have little influence. In the case of shear - tension the failure is less brittle and is accompanied by a widening of the diagonal tension crack, a process which is more significantly affected by the fibres bridging the crack.

The work that lead to the development of Equation 4 (8) was conducted on similar shear span to depth ratios as the test programme described here, although it is clear that the equation overestimates the shear capacity of fibre reinforced hollow core slabs. This may in part be attributable to the unique behaviour of the slabs observed by Pisanty(7). However, his 10% factor was shown to be inappropriate for the plain hollow core slabs and can not explain the observation that the experimental ultimate shear strengths are on average only 73% and 76% of the predicted values for a/d = 2.8 and a/d = 2.0 respectively.

The most likely reason for this observation lies with the method used to predict the tensile splitting strength of the concrete. Equation 5 was developed for FRC with a maximum compressive strength of 50 N/mm$^2$ (8) and is clearly inappropriate for the high strength, brittle concrete used in hollow core slabs.
By comparing Equation 3 and Equation 5 for plain concrete \((F = 0)\) it can be clearly seen that Equation 5 gives higher values for the tensile splitting strength and that the difference between the two values of tensile splitting strength increases with increasing compressive strength. An alternative method for deriving the splitting tensile strength of high strength FRCs is therefore necessary and is currently being studied.

Post-Peak Behaviour

Typical shear load - deflection curves for the two a/d regimes are shown in Figure 9. The post-peak behaviour of the plain slabs is characterised by a large reduction in load carrying capacity occurring simultaneously with a large deflection and wide diagonal shear cracks. In the majority of the fibre reinforced slabs there was only a small fall in load after the peak load as the fibres crossing the shear crack were able to carry most of the released stresses. Some slabs such as in Test F6A did show a fairly brittle behaviour but the initial losses in load were not as great as in the plain slabs.

An interesting comparison is that of the percentage of the ultimate load the slabs can carry as the crack width increases (Figure 10). Figure 10b shows the relationship for the a/d = 2.0 slabs, showing that the plain slabs are very brittle with only about 15% of the ultimate load remaining when the average crack width has increased to 6 mm. This compares with a value of about 70% for the fibre reinforced slabs. At a/d = 2.8 the plain slabs are not so brittle with around 50% of the ultimate load still being retained at a crack width of over 12 mm; this still compares unfavourably with the fibre slabs (Test F4A) which carry as much as 85% of the ultimate load.

In general these observations are consistent with the arguments propounded in the previous section. Some individual test results, such as Test F6A which does not show much improvement over the plain slabs in terms of post-peak behaviour, were inconsistent with the general trend, but this was attributed to the more variable nature of fibre reinforced concrete which occurs as a result of the variable distribution of fibres throughout the sample.

CONCLUSIONS

1. The manufacture of extruded hollow core slabs reinforced with steel fibres has been shown to be practicable. However, to ensure adequate compaction of fibre reinforced concrete using the ‘Spiroll’ extrusion technique it was necessary to make the mix slightly wetter than usual.
2. The fibres distributed themselves consistently within the cross-section of the slabs providing reasonably homogeneous reinforcement. However, the orientation of the fibres was not random and was strongly influenced by the manufacturing process. Of particular interest was the tendency for the fibres to align vertically within the web.

3. The addition of steel fibres to hollow core slabs increased the first crack and ultimate shear capacity. The exact nature of the improvement, however, was dependent on the a/d ratio with different behaviour being observed for the two a/d ratio’s investigated. Post-cracking ductility of fibre reinforced slabs was also substantially improved in comparison with plain slabs with safer, controlled failures being observed.

4. At relatively low a/d ratios the code equation for predicting the shear strength of plain hollow core slabs corresponds with the test results. However, the changes to this equation, proposed by other researchers, which allow the prediction of the shear strength of prestressed FRC beams, overestimates the strength of fibre reinforced hollow core slabs. This is because the equation used to derive the splitting tensile strength of FRC appears not to be applicable to high strength concrete.

NOTATION

\[ a = \text{shear span} \]
\[ A_y = \text{first moment of area} \]
\[ b = \text{effective width} \]
\[ d = \text{effective depth} \]
\[ h = \text{depth of member} \]
\[ I_c = \text{second moment of area} \]
\[ F = \text{fibre factor} \]
\[ f_{cu} = \text{compressive cube strength} \]
\[ f_{ct} = \text{tensile strength} \]
\[ f_{sp} = \text{splitting tensile strength} \]
\[ L_f = \text{fibre length} \]
\[ N = \text{number of fibres per unit area} \]
\[ V = \text{total composite volume (orientation tests)} \]
\[ V_{co} = \text{theoretical shear strength (of a section uncracked in flexure)} \]
\[ V_{cf} = \text{experimental first crack shear strength} \]
\[ V_f = \text{fibre volume fraction} \]
\[ V_{ult} = \text{experimental ultimate shear strength} \]
\[ \eta_b = \text{fibre bonding factor} \]
Structural Applications of Fiber Reinforced Concrete

\[ \eta_0 = \text{fibre orientation factor} \]
\[ \lambda_f = \text{fibre aspect ratio} \]
\[ \sigma_{cp} = \text{compressive stress at the centroidal axis due to prestressing} \]

ACKNOWLEDGEMENTS

The authors wish to thank N.V. Bekaert for the fibres used in this study, and are especially grateful to the hollow core manufacturer Richard Lees Ltd., Derby, England.

REFERENCES

1. ACI Committee 318, “Building Requirements for Reinforced Concrete (ACI 318-89),” American Concrete Institute. 1989


## Structural Applications of Fiber Reinforced Concrete

### TABLE 1—TYPICAL MIXES USED FOR HOLLOW CORE SLABS IN MAIN TRIALS.

<table>
<thead>
<tr>
<th></th>
<th>Plain Mix (kg/m³)</th>
<th>Fibre Mix (Vᵣ = 0.5%) (kg/m³)</th>
<th>Fibre Mix (Vᵣ = 1%) (kg/m³)</th>
</tr>
</thead>
<tbody>
<tr>
<td>14 mm crushed limestone</td>
<td>523</td>
<td>519</td>
<td>516</td>
</tr>
<tr>
<td>10 mm crushed limestone</td>
<td>677</td>
<td>674</td>
<td>688</td>
</tr>
<tr>
<td>Sand</td>
<td>749</td>
<td>735</td>
<td>735</td>
</tr>
<tr>
<td>Rapid Hardening Portland Cement</td>
<td>308</td>
<td>306</td>
<td>309</td>
</tr>
<tr>
<td>Pulverised-Fuel Ash</td>
<td>92</td>
<td>89</td>
<td>92</td>
</tr>
<tr>
<td>Water</td>
<td>108</td>
<td>112</td>
<td>112</td>
</tr>
<tr>
<td>Steel Fibres</td>
<td>-</td>
<td>39</td>
<td>78</td>
</tr>
</tbody>
</table>

### TABLE 2—DETAILS OF TEST SLABS.

<table>
<thead>
<tr>
<th>Trial</th>
<th>Slab No.</th>
<th>Intended Vᵣ (%)</th>
<th>Measured Vᵣ (%)</th>
<th>Density of Plain Concrete (kg/m³)</th>
<th>fₑᵤ (N/mm²) (eqn. 5)</th>
<th>fₛₑ (N/mm²) (eqn. 5)</th>
<th>σₛₑ (N/mm²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Prelim. Plain</td>
<td>0</td>
<td>-</td>
<td>2451</td>
<td>94.0</td>
<td>-</td>
<td>6.1</td>
<td></td>
</tr>
<tr>
<td>Batch 2</td>
<td>0.6</td>
<td>-</td>
<td>2350</td>
<td>91.0</td>
<td>5.9</td>
<td>6.1</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>F1</td>
<td>0.5</td>
<td>0.46</td>
<td>2408</td>
<td>86.5</td>
<td>5.7</td>
<td>6.0</td>
</tr>
<tr>
<td></td>
<td>F2</td>
<td>0.5</td>
<td>0.54</td>
<td>2418</td>
<td>86.5</td>
<td>5.7</td>
<td>6.0</td>
</tr>
<tr>
<td>2</td>
<td>F3</td>
<td>0.5</td>
<td>0.54</td>
<td>2440</td>
<td>79.5</td>
<td>5.3</td>
<td>6.0</td>
</tr>
<tr>
<td></td>
<td>F4</td>
<td>0.5</td>
<td>0.51</td>
<td>2444</td>
<td>79.5</td>
<td>5.3</td>
<td>6.0</td>
</tr>
<tr>
<td></td>
<td>F5</td>
<td>1</td>
<td>0.91</td>
<td>2435</td>
<td>91.2</td>
<td>6.2</td>
<td>6.0</td>
</tr>
<tr>
<td></td>
<td>F6</td>
<td>0.5</td>
<td>0.53</td>
<td>2430</td>
<td>79.5</td>
<td>5.3</td>
<td>6.0</td>
</tr>
<tr>
<td>3</td>
<td>P1</td>
<td>0</td>
<td>-</td>
<td>2440</td>
<td>78.0</td>
<td>-</td>
<td>6.0</td>
</tr>
<tr>
<td></td>
<td>P2</td>
<td>0</td>
<td>-</td>
<td>2446</td>
<td>78.0</td>
<td>-</td>
<td>6.0</td>
</tr>
</tbody>
</table>

*a* mean of 3 results  
*b* mean of 2 results
### TABLE 3—RESULTS OF SHEAR TESTS ON PLAIN AND FIBER REINFORCED SLABS.

<table>
<thead>
<tr>
<th>Test No.</th>
<th>a/d</th>
<th>$V_r$</th>
<th>Span (mm)</th>
<th>$V_{cr}$ (kN)</th>
<th>$V_{lt}$ (kN)</th>
<th>$V_{co}$ (kN)</th>
<th>$V_{ult} / V_{co}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>P2A</td>
<td>2.8</td>
<td>0</td>
<td>1900</td>
<td>147</td>
<td>147</td>
<td>156</td>
<td>0.94</td>
</tr>
<tr>
<td>P2B</td>
<td>2.8</td>
<td>0</td>
<td>1470</td>
<td>157</td>
<td>157</td>
<td>156</td>
<td>1.01</td>
</tr>
<tr>
<td>F1A</td>
<td>2.8</td>
<td>0.5</td>
<td>1900</td>
<td>165</td>
<td>180</td>
<td>260</td>
<td>0.69</td>
</tr>
<tr>
<td>F4A</td>
<td>2.8</td>
<td>0.5</td>
<td>1900</td>
<td>164</td>
<td>164</td>
<td>231</td>
<td>0.71</td>
</tr>
<tr>
<td>F4B</td>
<td>2.8</td>
<td>0.5</td>
<td>1470</td>
<td>160</td>
<td>161</td>
<td>231</td>
<td>0.70</td>
</tr>
<tr>
<td>F6A</td>
<td>2.8</td>
<td>0.5</td>
<td>1900</td>
<td>159</td>
<td>183</td>
<td>231</td>
<td>0.79</td>
</tr>
<tr>
<td>F6B</td>
<td>2.8</td>
<td>0.5</td>
<td>1470</td>
<td>141</td>
<td>174</td>
<td>231</td>
<td>0.75</td>
</tr>
<tr>
<td>P1A</td>
<td>2.0</td>
<td>0</td>
<td>1900</td>
<td>113</td>
<td>113</td>
<td>142</td>
<td>0.80</td>
</tr>
<tr>
<td>P1B</td>
<td>2.0</td>
<td>0</td>
<td>1470</td>
<td>169</td>
<td>169</td>
<td>142</td>
<td>1.19</td>
</tr>
<tr>
<td>F1B</td>
<td>2.0</td>
<td>0.5</td>
<td>1470</td>
<td>200</td>
<td>200</td>
<td>261</td>
<td>0.77</td>
</tr>
<tr>
<td>F2A</td>
<td>2.0</td>
<td>0.5</td>
<td>1900</td>
<td>219</td>
<td>219</td>
<td>259</td>
<td>0.85</td>
</tr>
<tr>
<td>F2B</td>
<td>2.0</td>
<td>0.5</td>
<td>1470</td>
<td>194</td>
<td>194</td>
<td>259</td>
<td>0.75</td>
</tr>
<tr>
<td>F3A</td>
<td>2.0</td>
<td>0.5</td>
<td>1900</td>
<td>159</td>
<td>159</td>
<td>230</td>
<td>0.69</td>
</tr>
<tr>
<td>F3B</td>
<td>2.0</td>
<td>0.5</td>
<td>1470</td>
<td>151</td>
<td>170</td>
<td>230</td>
<td>0.74</td>
</tr>
<tr>
<td>F5A</td>
<td>2.0</td>
<td>1</td>
<td>1900</td>
<td>220</td>
<td>232</td>
<td>277</td>
<td>0.84</td>
</tr>
<tr>
<td>F5B</td>
<td>2.0</td>
<td>1</td>
<td>1470</td>
<td>195</td>
<td>202</td>
<td>277</td>
<td>0.73</td>
</tr>
</tbody>
</table>

**Fig. 1**—Stress component in the web of a hollow core slab.
Fig. 2—"Spiroll" extrusion machine.

Fig. 3—Typical section showing direction of rotation of augers.

<table>
<thead>
<tr>
<th>Location</th>
<th>Prelim.</th>
<th>Trial 1</th>
<th>Trial 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0.63</td>
<td>0.66</td>
<td>0.67</td>
</tr>
<tr>
<td>B</td>
<td>0.59</td>
<td></td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>0.54</td>
<td></td>
<td></td>
</tr>
<tr>
<td>D</td>
<td></td>
<td>1.15</td>
<td></td>
</tr>
<tr>
<td>E</td>
<td>0.70</td>
<td>0.56</td>
<td>1.31</td>
</tr>
<tr>
<td>F</td>
<td>0.58</td>
<td></td>
<td></td>
</tr>
<tr>
<td>G</td>
<td>0.66</td>
<td></td>
<td></td>
</tr>
<tr>
<td>H</td>
<td>0.55</td>
<td></td>
<td></td>
</tr>
<tr>
<td>I</td>
<td>0.64</td>
<td></td>
<td>1.05</td>
</tr>
<tr>
<td>J</td>
<td></td>
<td>0.71</td>
<td></td>
</tr>
<tr>
<td>average</td>
<td>0.60</td>
<td>0.65</td>
<td>1.05</td>
</tr>
</tbody>
</table>

Fig. 4—Location and results of wash-out tests.
average standard deviation of each data point = 0.05

**Fig. 5**—Orientation factor in each orthogonal direction through the depth of slab.

![Fig. 5](image)

**Fig. 6**—Shear test set-up.

![Fig. 6](image)

**Fig. 7**—Typical crack in slab at \(a/d = 2.0\).
Structural Applications of Fiber Reinforced Concrete 107

Fig. 8—Typical crack in slab at a/d = 2.8.

Fig. 9—Shear load versus deflection curves for (a) a/d = 2.8; (b) a/d = 2.0.

Fig. 10—Relationship between percentage of ultimate strength remaining as crack width increases for (a) a/d = 2.8; (b) a/d = 2.0.
Effect of Various Types of Fibers on Bond Capacity—Experimental, Analytical, and Numerical Investigations

by K. Noghabai

Synopsis: This paper is based on two different experiments:
1) Splitting tests on thick-walled concrete rings subjected to an inner pressure
2) Pull-tension tests on deformed rebars embedded in concrete prisms.

The experiments consider different aspects of bond between deformed rebars and fiber reinforced high-strength concrete (FRHSC). Tests according to 1) study mechanisms for spalling of the concrete cover, due to the radial pressure exerted to the concrete by the ribs of a rebar in tension. There are analytical models (based on nonlinear fracture mechanics) for capacity assessment of the rings. In the experiments according to 2), the bond forces decide the development of transverse cracks in the concrete. By these tests, the so-called tension-stiffening effect on rebars may be studied. In composing the FRHSC, five different types of fibers were used: 2 steel fibers (crimped and straight), 2 polyolefin fibers and one carbon fiber. The fibers were added to a HSC with an expected 28-days compressive strength of 120 MPa. Volume fraction of fibers was 1% in all cases.

FRHSC is here characterized by its response under uniaxial tension. By using the tensile load-deformation curve in available analytical and/or numerical tools (such as FEM), the effect of a fiber inclusion on the structural response may be explained. Both the analytical and FE-models used herein, seem capable to describe the physical events that take place in the experiments. Thus, it is promising that in order to achieve a desired performance in the test pieces (and consequently with respect to a desired bond capacity), a fibrous concrete may be tailor-made for the purpose.

Keywords: bond; carbon fibers; high-strength concrete; fiber; non-linear fracture mechanics; polyolefin fibers; splitting; tension stiffening
Keivan Noghabai is a Ph.D. student in the Division of Structural Engineering at the Luleå University of Technology. He received his MS degree at the Luleå University of Technology and returned back to obtain a Tekn. Lic. degree after working as a building designer for three years. His research interests involve fracture mechanics, bond of rebars, environmental effects, fibrous and non-fibrous HSC.

INTRODUCTION

Bond between concrete and reinforcement in general and adequate concrete cover in particular, is paramount for design and application of reinforced concrete structures (RCS). This is with respect to the load bearing capacity as well as from a durability point of view, since the concrete-rebar interaction is decisive for the distribution and widths of cracks in RCS. In use of high-strength concrete (HSC), the design of concrete cover and bond issues are given a renewed concern as they also are essential to the competitiveness of the building material. Two major beneficial effects on the functions of the concrete cover are anticipated, when fibers are added to the RCS. Firstly, a premature splitting of the cover is prevented and the bond capacity between concrete and the tension rebar is generally enhanced. Secondly, due to their crack bridging effect, the fibers provide crack control and consequently the rebar is protected against environmental exposure.

In a sense, fibers are added to concrete in order to improve the very functions a deformed rebar is designed to fulfill. Both types of reinforcement also share the mechanisms by which they interact with the surrounding material. Hence, the fiber-matrix interaction involve the local bonding capacity between a single fiber and the matrix, as fibers are meant to function after rupture of the concrete matrix. As the general bond between the constituents of a HSC (including fibers) is enhanced, the strength and/or deformability of the fibers bridging over the cracks are decisive for the crack widths and the overall ductility of RCS. Since good models for bond between deformed bars and concrete is lacking, certainly the local bond behavior of each individual fiber and concrete matrix is still more unknown. The problem is magnified due to the diversity in shapes, materials, orientations, amounts and interaction of fibers in combination with the composition of the concrete.

Due to this addition to the complexity of the problem, it is hard to exactly foresee how a certain amount of a certain type of fiber will perform in a certain HSC mixture. In view of the fact that the structural dimensions are substantially larger than the constituents of the material, fiber reinforced HSC may be regarded as the "homogenous" material FRHSC. Thus, the mechanisms at a micro-level
are incorporated in continuum models. In this context, a load-deformation curve for the specific FRHSC (or any fiber reinforced cementitious material for that matter) under the relevant loading condition, may suffice as a constitutive or “material” model. This supposition must be verified. The verification procedure should preferably involve experiments, numerical methods (by e.g. the finite element method, FEM) and analytical models. It is believed that a phenomenon may be understood, only by a sound interaction between all three methods.

As a means for verification, numerical modeling offers a versatile tool for investigating the effect of the fiber-concrete interaction on the performance of structures of various shapes and sizes. From an analytical point of view, the computational modeling may also offer insight to the mechanisms by which the stresses and strains distribute in the structure. Consequently, sound analytical or empirical models may be derived, which also would be amenable to the effect of different structural sizes. The discussion in this paragraph, tacitly requires that the material description employed in the model is conceptually correct and physically sound.

To be able to verify models and in order to acquire knowledge on the behavior of FRHSC various types of fibers of different material, strength, length and configuration are added to HSC mixtures of approximately the same composition. From both an economical and a feasibility aspect the volume fraction of fiber is set to 1% for all FRHSC. It must be emphasized that the choice of fibers at this stage does not aim at optimizing the FRHSC, but is done primarily to examine the generality of existing analytical and/or numerical tools for a variation in material behavior. Two types of experiments are treated herein, both intending to describe the fracture behavior of concrete surrounding a single deformed bar in tension. Thus they indirectly study the effects of bond between deformed rebars and concrete. The experimental procedure and analytical as well as computational models below are elaborated in the authors Tekn. lic. and Ph.D. thesis (1, 2).

CONCRETING FIBER REINFORCED HSC

Types of Fibers

The types of fibers used in this study were steel fibers, polyolefin fibers and carbon fibers. Two types of steel fibers were used: crimped fibers of length $l=30$ mm and diameter $\varnothing=0.6$ mm and straight fibers with $l=6$ mm and $\varnothing=0.15$ mm (henceforth denoted $30/0.6$ and $6/0.15$). Polyolefin fibers were of dimensions $50/0.63$ and $25/0.38$. It is admitted that carbon fibers may not be suitable in use with aggregate fractions exceeding several millimeters. The configuration and other data on the chosen fibers are summarized in Table 1.
Apart from a reference batch of plain concrete, 7 batches of FRHSC were cast. The concrete had a cement content of 490 kg/m$^3$ and a maximum aggregate size of 16 mm. The water-to-binder ratio was 0.31 and the silica-fume-to-binder ratio was 0.09. Superplasticizer was added to an amount of 0.9-1.0% of the cement content. In all cases the fibers were dry mixed with the aggregates. There were 3 batches with steel fibers, 2 batches with polyolefin fibers and 2 batches with carbon fibers. In two of the steel fiber reinforced HSC batches, the two types of fibers were mixed to an amount of 80 kg/m$^3$ concrete. In a third batch, both types of steel fibers were added to the concrete in an equal amount (i.e. 40 kg/m$^3$ of each type of fiber). In both batches of carbon fiber reinforced HSC, the same type of carbon fiber was used. For the sake of convenience the different batches will be denoted HSC indexed by the denominations given in Table 1 (S = steel and P = polyolefin). For the non-fibrous case the index REF is used and the two batches of carbon fibers are distinguished by roman numbers I and II following the index. The batch where both the steel fibers are used is denoted HSC$^\text{smix}$.

Apart from specimens for determination of material properties (see below), one cylindrical specimen for use in the splitting test and one prism embedding a deformed bar were produced from each batch.

**MATERIAL PROPERTIES OF FRHSC**

The experiments in this study, as bond problems, are typical for a cross-section of a RCS subjected to tension. The tensile characteristics of concrete is usually determined by indirect methods. The by far most common method in classifying a fibrous concrete, is by means of the toughness indices obtained from four-point bending test on small beams according to ASTM C1018. It offers a good opportunity for comparing different types of fibrous concrete, however the true tensile characteristics of the material are reflected only indirectly in the flexural structural response. As was mentioned in the introduction, the strategy is to characterize a FRHSC by its response under a typical loading condition (pure tension) and fracture mode (Mode I). The elastic range of deformation is described by the modulus of elasticity, $E_c$, up to the onset of failure at the tensile strength, $f_{ct}$. At this stage a fracture plane is localized while the bulk outside this plane is unloaded. The deformations in post-failure regime are now concentrated to the crack opening (COD) and the descending, softening branch reflects the course of rupture of the internal bond between the constituents of the material. Hence, all the relevant material properties may be determined by a uniaxial tensile test on notched concrete prisms conducted in displacement control. Due to the inherent
instability of quasi-brittle materials, such experiments require highly sophisticated testing apparatus and control arrangement in order to cover the entire softening curve. The boundary conditions are also important, as the desired material characteristics in tension should not be influenced by geometrical instability caused by eccentricities and factors related to the shape of the specimen. Test methods and their pros and cons are further discussed in (1, 2, 3, 4).

The test pieces of the uniaxial tensile test were prepared from cylindrical 74 mm cores that were drilled out of two small beams. At least two notched specimens of length 85 mm were produced from each core (i.e. at least 4 specimens per batch). Due to presence of fibers a saddle notch was preferred rather than a sharp notch (3), see Fig 1. All test pieces were stored in water for as long as possible. The ends of the specimens were bonded by a two-component epoxy adhesive to platens that were attached to the testing machine. The tests were conducted in displacement control, using a closed-loop servo-hydraulic test machine. The rate of deformation applied to the specimen was decided by three different monitoring procedures of the displacement. In the most critical range of deformation (the descending branch after peak-load up to 1 mm deformation) the control was based on the mean value of four extensometers placed symmetrically over the notch (denoted COD in Fig 1). The measuring length was approximately 30 mm. After the deformations had exceeded 1 mm, the displacement of the active cross-head, monitored by a LVDT, was used for the control (up to 10 mm). At this point the concrete matrix is separated and the remaining resistance of the fibers is overcome by elongating the specimen at a cross-head travel rate. The displacement rates were varied during the whole test: more cautious steps were taken at peak load and a more rapid pace was allowed on the tail of the descending branch.

Although this control procedure proceeded smoothly in general, the moderate values of the tensile strength cause some confusion. This was based on the observation that for some cases, where fibers should not have influenced the tensile strength (such as for steel and polyolefin fibers), markedly higher strengths were obtained, compared with specimen of HSC_{REF} and HSC_{Carbon}-batches. In the latter cases, a typical distinct peak was absent. These cases also displayed weaker elastic moduli (reflecting extensive micro-cracking in the pre-peak stage) and more prolonged softening curves. This behavior is believed to be primarily caused by keeping the specimens in water. To what extent this has affected the results, is not certain and is currently being investigated. It must be mentioned that uniaxial tensile tests such as described here have been successfully performed earlier in (3), on FRHSC with the same mixing proportion as used herein, but never on specimens kept under these conditions. A preliminary test was performed on dry specimens of HSC_{REF}. The obtained load-deformation curves were in much better agreement with the experience of (3). It is possible that the stress concentration caused by the saddle notches becomes more effective, when the specimens are cured in water. However, for dry specimen the concrete is still
believed to be fairly notch insensitive. The influence of fibers on the behavior of the HSC is quite clear and typical curves are given in Fig 2. All the experiments were performed at concrete ages beyond 70 days. The compressive strength were determined at an age of 28 days, from compression tests on four 100 mm cubes. The strengths are given (in MPa) in connection to Fig 2.

There is a decrease in the compressive strength of carbon fiber reinforced HSC compared with HSC\textsubscript{REF}. This is due to the markedly higher porosity of the concrete both concerning larger, visible voids as well as diffused, small pores. According to Fig 2, the HSC\textsubscript{S6/0.15} shows increased strength, which is believed to be caused by this particular fiber. Due to its small dimensions, the S6/0.15 fibers have changed the elastic behavior of the HSC, both regarding the \( E_c \) and \( f_{ct} \). This explains its rather brittle post-peak behavior, compared with other cases where steel fibers are used, see Fig 2. The contribution of the longer steel fibers to the ductility of HSC (for both HSC\textsubscript{S30/0.6} and HSC\textsubscript{Smix}) is more immediate, which is manifested in a more shallow descent of the softening curves. The polyolefin fibers act differently mainly due to the low modulus of elasticity. The specimens respond like a plain concrete, at the beginning of their descending branches. However, at a crack width \( w = 0.2 \) mm the fibers become active, resulting in a bump on the \( \sigma - w \) curves (e) and (f) in Fig 2. It is believed that the tensile strength varies between 5-5.5 MPa and the modulus of elasticity ranges from 35 to 45 GPa for all cases except for the carbon reinforced HSC and HSC\textsubscript{S6/0.15} where lower and higher values are expected for the respective cases, see Fig 2.

**EXPERIMENTS**

**Splitting test on thick-walled concrete rings**

This set of experiments aims at analogizing the in-plane splitting behavior of concrete covers. As a deformed bar embedded in concrete is tensioned, the ribs of the bar will exert a hydrostatic pressure to the surrounding concrete. An analytical model has been developed for assessment of the capacity of a ring against splitting, which is based on the concept of nonlinear fracture mechanics (NLFM). The model and the experimental procedure are elaborated in (2).

**Casting and curing of specimens** -- The specimens were produced from cylinders of diameter 313 mm with a height of 175 mm. They were submerged in water for at least 28 days. After the concrete had cured, concentric circular holes were drilled out of each cylinder shaping the rings as shown in Fig 3. The specimens were kept in water until up to a week before testing.
Testing procedure -- The pressure is applied to the ring by a sleeve. The sleeve consists of a steel mandrel coated with a 2-3 mm thick membrane of polyester based polyurethane elastomer. Oil is pumped into the cavity between the mandrel and the inner membrane wall using a pressure controlled hydraulic pump. The radial displacements were measured in two orthogonal directions, symmetrically over the hole.

Observations -- Two radial longitudinal cracks occurred on all rings at the maximum pressure $p_{i,\text{max}}$, according to Fig 4, except for the HSC$_{\text{Smix}}$ ring. Very thin cracks were produced in the rings of HSC$_{\text{Smix}}$ and HSC$_{60/0.15}$ whereas larger cracks were observed on the other specimens. Earlier experiments (1, 3) have indicated that in rings of softening materials two cracks localize at $p_{i,\text{max}}$ and additional cracks are produced as the material approaches a plastic behavior at the range of crack openings pertinent to this problem. Referring to Fig 2, this may explain the third, very thin crack observed on the HSC$_{\text{Smix}}$ specimen.

Tension Test on Tie Elements

The aim of this second type of experiment is to map the crack distribution along a concrete prism, as a bar embedded in its centre is tensioned. The setup of the experiment is in accordance with the RILEM Round-Robin Test and Analysis on Bond recommended by the committee FMB-147 (5), although it does not prescribe fibrous concrete. Recently, this type of experiment has been performed on plain HSC by (6, 7, 8).

Casting and curing of specimens -- A deformed $\varnothing$16 mm bar of the Swedish grade Ks500 (nominal yield strength $f_y = 500$ MPa) is cast in a concrete prism of dimension $80 \times 80 \times 960$ mm$^3$ (i.e. the concrete cover is $2 \times \varnothing$, the prism length is $60 \times \varnothing$ and the reinforcement ratio is 0.031). A schematic view of the specimen is given in Fig 5. The specimens were cast horizontally and cured in water for seven days. Then they were stored in approximately 60% relative humidity at about 20°C and kept in this condition up to one week before testing.

Testing procedure -- The tie elements were tested in a servohydraulic testing machine with a capacity of 60 tons. The experiments were performed in displacement control. The displacement of the specimen was monitored by 4 LVDTs. The relative movement of the concrete prism was recorded on two opposite sides, on the top and the bottom of the specimen, see Fig 5. During testing, the crack widths were measured along the specimen.

Observations -- In Fig 6 the crack patterns on the tie elements are given for specimens of (from top downwards): REF, S30/0.6, S6/0.15, Smix, P50/0.63, P25/0.38, Carbon I and Carbon II. It must be noted that the state shown in Fig 6, corresponds to a loading level at or in the vicinity of strain hardening of the steel. Thus the crack widths in the picture are of no relevance, however the number of
the major cracks and their location on the different specimens should be noticed. Most of the primary cracks occurred at loading levels between 15-30 kN. The lowest load was obtained for the HSC\text{Carbon II} specimen. For the specimens of HSC\text{REF}, HSC\text{P50/0.63}, HSC\text{P25/0.38} and HSC\text{Carbon II}, the crack load (at which the first crack appears) varied between 19-22 kN, while for the specimens with steel fibers the crack occurred at load levels between 25-30 kN (HSC\text{S6/0.15} showed the highest load). The crack loads seem to correspond well with the tensile strength of the FRHSC used. Additional cracks appeared along the specimen, at very short intervals. At the creation of each crack, a sudden drop in load (roughly a couple of kilonewtons) could be observed. In Fig 9, the load-elongation (F-I) responses are given for HSC\text{REF}, HSC\text{P25/0.38} and HSC\text{S30/0.6}. The tie elements of carbon fiber, behave almost identical to HSC\text{REF}. On the tie element of HSC\text{P25/0.38}, the first crack widths measuring 0.2 mm was detected at a load level around 40 kN. This crack width roughly corresponds to the onset of the bump on \( \sigma-w \) curve (f) in Fig 2, and would explain the tendency towards a stiffer response for the HSC\text{P25/0.38} specimen, according to Fig 9. However, this behavior was not present for the HSC\text{P50/0.68} case, which responded like HSC\text{REF}. This may be due to a poor fabrication of this specimen. The P50/0.63 fibers generally showed a tendency to float to the surface when casting the concrete. This is explained by the low density in combination with the higher aspect ratio of this particular fiber, see Table 1.

Just prior to steel yielding the main cracks have localized to 8-10 crack planes. This agrees with the experiments of (7), where the setup and specimen dimensions were similar to the tests here, but the prism had a circular cross-section. At deformations near the yielding of the steel (at approximately 110 kN) a splitting could be detected at the ends of some specimens. This occurred at load levels exceeding 90 kN, which is explained by the Poisson's effect as the bonding is radically diminished at substantial elongation of the steel at the ends of the specimen (9). Depending on the bonding capacity and the quality of the surrounding concrete the localized cracks will grow in turn, as the steel yielding progresses along the tensioned rebar from one end to the other. The manner the yielding develops along the rebar could be retraced by monitoring the crack widths. This is reflected in a serrated curve where usually each drop represents the stage at which the resistance of a single crack is overcome. With increasing deformations, the tie element is severely damaged and longitudinal crack branches connect the previously created crack planes. As bond stresses start to develop along the rebar in the separated blocks, the splitting phenomenon reappears due to the mentioned Poisson’s effect on the rebar.
ANALYTICAL AND NUMERICAL MODELS

Crack localizes in softening materials (as was the case in the uniaxial tension tests). If a specimen of softening material is restrained to deform freely, e.g. in presence of rebars, multiple cracks will occur. The crack localization makes the formulation of constitutive laws more complicated. This puts special requirements on the FEM that is chosen as a tool of analysis. The FEM model used herein is based on the concept of the Inner Softening Band (ISB), which is an element-embedded crack model. Some basic traits of the method will be mentioned here, in order to substantiate why the approach is preferred. The concept of ISB is elaborated in (10, 11, 12). The method neither requires a predetermined crack patterns (contrary to the discrete crack models), nor a re-meshing procedure. This is a great advantage, since prediction of the crack distribution and widths, is a main concern of this study. In the ISB model, after initiation of a crack, the crack evolution process is described in the traction space along the surface of the created “crack” and not in terms of volumetric strains (as is commonly used in the smeared crack models). Crack mentioned within quotation marks refers to the concept of fictitious crack (13), and only implies that a strength criterion is met. As a consequence of the chosen material and element descriptions, the ISB model only needs a minimum amount of input parameters, which all have a physical relevance and are experimentally attainable. Also, there is no need for a calibration of the model or additional non-physical parameters (e.g. an internal length). The ISB approach has proved to give very accurate results especially for Mode I fracture. The ultimate capacity is mostly predicted with good accuracy and the crack patterns at a state of collapse is strikingly congruent with the experiments, see further (1, 10, 11, 14). The references also indicated that the method is quite mesh insensitive. The input parameters needed for description of the concrete in tension are: the modulus of elasticity \( E_c \), the Poisson’s ratio \( v \) (which for concrete is set to 0.15 when needed), the tensile strength \( f_{ct} \) and the shape of the softening curve. The importance of an accurate description of the softening curve becomes evident in the light of the two objects of analysis treated herein. In the thick-walled ring (and with respect to design of concrete cover) the cracks propagate with virtually no visible widths, hence the incipient curve is of more important. On the other hand, in the tie element, cracks gradually develop to considerable widths. Further examples on the influence of the shape of softening curve are given in (1, 2).

Thick-Walled Concrete Rings

The ring problem has previously been examined by FEM, using the discrete crack model, the smeared crack model and the ISB approach (1, 14, 15). This section focuses on an analytical model for assessment of the splitting capacity of a ring. The model is elaborated in (1). The model gave accurate predictions of the capacity of the rings that were tested in (1). The model considers the equilibrium of a crack front, \( r_c \), propagating into an elastic surrounding due to increasing inner pressure, \( p_i \), in the circular hole (with radius \( r_i \)), see Fig 7. Thus, the ring comprises one cracked, inner ring, in which the tension softening takes place, and
an elastic outer ring. The tangential stresses equal the tensile strength, $f_{ct}$, at the boundary of the two rings. Assuming a linear softening curve in the inner ring, the fracture mechanics properties are accounted for by the so-called characteristic length, $l_{ch} = G_fE_c/f_{ct}^2$ according to (13). $G_f$ is the total amount of fracture energy, roughly equal to the area under the $\sigma$-$w$ curves. Based on the assumptions made above, the equilibrium of the two rings may be expressed as Eq (1), see (2). The first term in Eq (1) is the contribution of the elastic ring while the second term accounts for the softening of the cracked inner ring. The ultimate capacity $p_i, max$ occurs at a critical crack depth, $r_c = r_{c, crit}$ when $\partial p_i/\partial r_c = 0$.

$$\frac{p_i}{f_{ct}} = \frac{r_c}{r_i} \cdot \frac{r_0^2 - r_c^2}{r_o^2 + r_c^2} + \frac{n \cdot l_{ch} - \pi \cdot r_c}{\pi \cdot r_i} \cdot \ln \left( \frac{n \cdot l_{ch} - \pi \cdot r_i}{n \cdot l_{ch} - \pi \cdot r_c} \right)$$

The different radii in Eq (1) is according to Fig 7 and the factor $n$ indicates the number of cracks. Keeping the dimensions of the ring constant, a concrete composition with greater $l_{ch}$ turns the response of Eq (1) into a plastic mode. With decreasing $l_{ch}$ the contribution of the cracked ring is diminished and the behavior becomes brittle. This effect can be expressed in terms of a brittleness number, $B$, defined as the ratio between a structural size, e.g. the outer radius of the ring $r_0$, and the $l_{ch}$. Some experimental results presented herein as well as previous results (1, 3) are presented in Fig 8. The model should also be applicable to a varying wall thickness (i.e. concrete covers). Fig 8 also shows the results of ring models based on other theories, according to elastic and perfectly-plastic models and linear elastic fracture mechanics (LEFM), see further (1).

Evidently, a linear softening relation is a crude assumption for fibrous concrete, due to the substantial increase in $G_f$. Thus, the shape of the $\sigma$-$w$ curve is more important than the total amount of fracture energy, $G_f$. However, assuming that a linear $\sigma$-$w$ relationship still suffices for rings of plain concrete (or concrete behaving in a similar manner) based on (1), the poor results of HSC$_{REF}$ give rise to doubts about the applicability of the result of the uniaxial tensile test on wet cured specimens. This doubt seems justified in view of the fact, that new parameters obtained from dry specimens of HSC$_{REF}$ give a much better agreement, see Fig 8. From the wet cured specimens of HSC$_{REF}$ the following data could be obtained based on mean values of three specimens: $E_c=35.3$ GPa, $f_{ct}=4.08$ MPa and $G_f=173$ N/m giving a $l_{ch} = 0.368$ m. For tensile test on two dry specimens (with the same shape as before) a $l_{ch}$ of 0.357 m is calculated from the material properties $E_c=41.5$ GPa, $f_{ct}=5.01$ MPa and $G_f=216$ N/m. Although the $l_{ch}$ is not altered much, the low $f_{ct}$ obtained from the soaked specimens do make a difference, also due to a change in the slope of the softening $\sigma$-$w$ curve, cf. Fig 2.
Tie Elements

FE-analysis on the pull-tension tests on rebars embedded in concrete, have earlier been treated in e.g. (16). The tie element here is modeled by a randomly generated mesh comprising 308 CST plane-stress elements and 44 bar elements that connect the existing nodes along a plane of symmetry (i.e. perfect bond is assumed). The assumption of symmetry was necessary, in order to avoid spurious bending actions to occur in the model. The load is applied to the model by a prescribed displacement of the node at the end of the bar. The FE-analysis here aim at investigating the development of cracks as a phenomenon, and is compared with the experimental observations only in this sense.

The input to the model is the $E_c$, $f_{ct}$, $v$ and $\sigma$-$\omega$ curve for the FRHSC, and a stress-strain curve for the steel rebar. Two different $\sigma$-$\omega$ curves of FRHSC have been used in the ISB model: one resembling that of a HSC $P_{50/0.63}$ and another for a HSC with 2% volume fraction of straight fibers with surface indentation (denoted HSC $2\%$) according to data from (3). The material behavior is more drastically changed by this choice. HSC $P_{50/0.63}$ is seen to fit, since it at first behaves almost as a plain concrete and then shifts its behavior and becomes more ductile, cf. Fig 2. The influence of 2 volume percent of fibers on the softening of HSC is almost immediate, which is reflected in a shallow $\sigma$-$\omega$ curve.

In Fig 9, the force-elongation ($F$-$\delta$) responses obtained by the ISB-model are given. For the case with $P_{50/0.63}$ fibers, also the active ISBs (i.e. cracks) along the concrete prism are shown (the at the time passive cracks are not shown), at the load levels indicated in the $F$-$\delta$ diagram. For each given state, the numerically obtained steel strains and their distribution along the rebar are presented. Each peak on the steel strain curves, corresponds to a localized crack and a higher strain level indicates a larger crack width. Surpassing the elastic stage, inclined conical cracks are induced at each end and additional cracks develop inwards up to point a) in Fig 9. At this point the crack propagation becomes less influenced by the boundaries at the free ends of the specimen and the first crack will localize anywhere along the midspan of the prism. In creation of the first and subsequent cracks severe difficulties are encountered in obtaining numerical convergence. Each crack corresponds to an abrupt drop (i.e. instability) in load, manifested by serrations in the $F$-$\delta$ curve at points (a) to (e), in Fig 9. This behavior occurred in the case with $P_{50/0.63}$, but did not appear for HSC $2\%$. This phenomenon is related to the stability of the structure and material itself, i.e. on the steepness of the softening branch, see also (16). All major cracks are localized to up to 10 distinct planes, whereafter the stable crack growth can proceed, cf. point (f) in Fig 9. At the onset of yielding of the steel rebar, yield strains will localize at the location of a crack. This phenomenon was reproduced in the analysis on HSC $2\%$, manifested once again by a serrated curve. All this are in agreement with and explains the experimental observations (cf. the inlaid photograph in Fig 9 illustrating the crack patterns on the authentic case of HSC $P_{50/0.63}$).
CONCLUSIONS

The following conclusions can be drawn from this study:

1. The two types of experiment treated here, consider different aspects of bond between concrete and deformed rebars. Splitting tests on thick-walled rings of FRHSC, reflect the case of spalling of concrete covers due to the radial pressure exerted to the concrete by the ribs of a rebar in tension. In the pull-tension tests on rebars embedded in concrete prisms, the bond forces decide the development of transverse cracks in the concrete. The pull-tension tests provided new insight in the tension-stiffening effect. The tension-stiffening and crack control is markedly improved by use of 1 percent steel fibres per volume concrete. With respect to spalling of concrete covers, the increase in splitting capacity is moderate for ring specimens of the actual size, since inclusion of 1% fibers have not changed the tensile strength much.

2. The experiments above were studied by means of analytical and numerical models. The models require the tensile characteristics of the FRHSC, in terms of a modulus of elasticity ($E_t$), a tensile strength ($f_{ct}$) and a post-peak softening ($\sigma$-$w$) curve. These material properties were determined by uniaxial tension tests, and the results were implemented in the analytical and numerical models. The FE-modeling of the tie elements by the Inner Softening Band (ISB) approach, convincingly reproduced evident phenomena. Credible variation in the tension-stiffening effect was obtained, only by introducing new $\sigma$-$w$ curves. Together with the previous results in (1, 14), both the analytical and FE-models used herein, seem capable to describe the physical events that take place in the experiments. Thus, it is promising that in order to achieve a desired performance in the test pieces (and consequently with respect to a desired bond capacity), a fibrous concrete may be tailor-made for the purpose.

3. When a specific material quality is prescribed for attainment of optimal structural performance, the level of analysis may shift into the microscopic scale, in order to account for the mechanical aspects of the fiber-matrix interaction.

4. The conclusions above underline the increased demands on the testing arrangement and the procedure for obtaining the relevant material properties. Analysis of structures with well known behavior may reveal flaws in the testing procedures and suggest improvements. For instance, the analysis of thick-walled concrete ring, indicated that material properties obtained from uniaxial tension tests on specimens cured in air, gave better agreement between the model predictions (by both analytical and FEM) and the experimental results (compared with results based on properties of specimens stored in water).
REFERENCES


### TABLE 1—FIBERS USED IN THIS INVESTIGATION.

<table>
<thead>
<tr>
<th>Indices</th>
<th>Material Type</th>
<th>Configuration</th>
<th>Aspect ratio l/Ø</th>
<th>Density ρ kg/m³</th>
<th>Strength f_{fy} MPa</th>
<th>Elastic modulus E_f GPa</th>
</tr>
</thead>
<tbody>
<tr>
<td>S30/0.6</td>
<td>Steel</td>
<td>Ø=0.6</td>
<td>50</td>
<td>7800</td>
<td>1100</td>
<td>200</td>
</tr>
<tr>
<td>S6/0.15</td>
<td>Steel</td>
<td>-</td>
<td>40</td>
<td>7800</td>
<td>2600</td>
<td>200</td>
</tr>
<tr>
<td>P50/0.63</td>
<td>Polyolefin</td>
<td>79.4</td>
<td>910</td>
<td>275</td>
<td>2.6</td>
<td></td>
</tr>
<tr>
<td>P25/0.38</td>
<td>Polyolefin</td>
<td>65.8</td>
<td>910</td>
<td>275</td>
<td>2.6</td>
<td></td>
</tr>
<tr>
<td>Carbon</td>
<td>Carbon fiber</td>
<td>(=10⁶)</td>
<td>1600</td>
<td>4350</td>
<td>230</td>
<td></td>
</tr>
</tbody>
</table>
Fig. 1—Specimen and experimental setup of uniaxial tensile test.
Fig. 2—Responses of various FRHSC to uniaxial tension.

Fig. 3—Experimental setup of splitting tests on thick-walled concrete rings.
REF S6/0.15 P50/0.63

<table>
<thead>
<tr>
<th>Specimen</th>
<th>REF</th>
<th>S30/0.6</th>
<th>S6/0.15</th>
<th>Smix</th>
<th>P50/0.63</th>
<th>P25/0.38</th>
<th>Carbon I</th>
<th>Carbon II</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_{u,\text{max}}$ MPa</td>
<td>29.5</td>
<td>28.7</td>
<td>26.4</td>
<td>32.8</td>
<td>32.9</td>
<td>30.0</td>
<td>29.3</td>
<td>28.1</td>
</tr>
</tbody>
</table>

Fig. 4—Crack patterns at given ultimate splitting pressures.

Fig. 5—Experimental setup of pull-tension tests on reinforcing bars embedded in concrete prisms (tie elements).
Fig. 6—Crack patterns on tie elements.

Fig. 7—Principles of the thick-walled ring model.

Fig. 8—Experimental results on ring versus different analytical models.
Fig. 9—Results of experiments and FE-analysis for tie elements.
Bond of Deformed Bars to Concrete: Effects of Specialty Cellulose Fibers

by P. Soroushian and S. Ravanbakhsh

Processed cellulose fibers provide high levels of elastic modulus, tensile strength, bond strength to concrete, and durability. Their fine diameter also yields a close fiber spacing at relatively low fiber volume fractions, and allows them establish a strong presence in the interface zones between reinforcing bars and concrete. Specialty cellulose fibers have been recently developed for convenient dispersion into normal concrete mixtures using conventional mixing procedures. This research project investigated the effect of specialty cellulose fibers at volume fractions of about 0.1% on the strength and toughness of bond between deformed bars and concrete. The experimental results were indicative of the effectiveness of specialty cellulose fibers in enhancing bond strength and toughness. The positive impact of specialty cellulose fibers on bond strength was more pronounced as fiber volume fractions increased to the upper limit of 0.18% considered in this investigation.

Keywords: bond; cellulose fibers; concrete; deformed bars; pull-out; strength; toughness
INTRODUCTION

The structural performance of reinforced concrete partly depends on the bond strength and bond stress-slip behavior at the interfaces between reinforcing bars and concrete. Deformed surfaces of reinforcing bars cause various local and global damage mechanisms when they are pulled against the surrounding concrete. Fine discrete fibers which are closely spaced in the close vicinity of reinforcing bars in concrete would be expected to enhance the performance of concrete under these local and global damaging effects and thus benefit the bond strength and toughness of concrete. The work reported herein determines the effects of specialty cellulose fibers on the bond behavior of deformed bars in concrete.

RESEARCH SIGNIFICANCE

Bond of deformed bars to concrete is critical to various aspects of reinforced concrete behavior, including flexural strength, toughness, and seismic resistance. Improvement of the bond behavior with low volume fractions of specialty cellulose fibers would thus have important implications for structural applications of fiber reinforced concrete.

NATURE OF BOND BEHAVIOR

The bond resistance between deformed bars and concrete is provided by a
combination of chemical adhesion, friction, and interlocking (Figure 1a). Once the adhesion of steel surfaces to concrete is lost, friction and most importantly interlocking govern the bond behavior. The bar deformations interlocked within concrete apply bearing pressures to concrete at an inclined direction (Figure 1b). This generates local damage in the form of cracks initiating from the bar deformations. The inclined nature of these bearing pressures also produces an internal radial pressure against concrete (Figure 1c), which leads to the generation of splitting tensile cracks (Figure 1d). Heavy confinement of concrete around bars as well as large concrete cover thicknesses tend to promote failure by increasing local damage (Figure 1c).

Low confinement levels and small cover thicknesses, on the other hand, promote more brittle failure modes associated with the formation of splitting cracks (Figure 1d).

**SPECIALTY CELLULOSE FIBERS**

Processed cellulose fibers have found growing applications in thin reinforced cement products. These applications have confirmed the high reinforcement efficiency and durability of processed cellulose fibers in cement-based materials. Conventional cellulose fiber reinforced cement composites are produced using special prefabrication techniques where excess moisture and mechanical efforts are used to ensure uniform dispersion of fibers in the matrix. Recently, specialty cellulose (fabroset™) fibers have been developed which are readily dispersible in conventional concrete and mortar mixtures using normal mixing procedures. These fibers can be added to the mix at any stage, including after the addition of other mix ingredients, and the simple mixing action used with concrete would be sufficient to uniformly disperse the fibers in concrete.

Table 1 compares some key attributes of the specialty cellulose fibers with those of commercially available polypropylene fibers. Specialty cellulose fibers are distinguished by their high elastic modulus and bond strength, and small diameter (i.e. high surface area). The hydrophilic surface of specialty cellulose fibers facilitates their dispersion in concrete and enhances the interface characteristics. It should be noted that the effective diameter of specialty cellulose fibers is not much greater than the particle size of cement; this minimizes any disturbance of the packing of cement particles and the distribution of cement hydration products resulting from the presence of fibers, and thus enhances the bulk properties of the matrix. Fiber densities less than that of water (1 g/cm³ or 62 lb/ft³) encourage
segregation due to floating, the density of specialty cellulose fibers (1.5 g/cm³ or 93 lb/ft³) thus favors their case in concrete. The specialty cellulose fibers used in this project are highly durable in the alkaline environment of concrete; their effect on fresh mix slump is comparable to that of polypropylene fibers.

In fiber reinforced concrete it is essential to achieve close fiber spacings, high fiber count, and large fiber surface area. Polypropylene fibers at about 1.2 kg/m³ (2 lb/yd³) are commonly used as secondary reinforcement in concrete. This dosage is equivalent to 0.133% fiber volume fraction. An equal weight dosage of cellulose fibers, given the higher specific gravity of these fibers, accounts for 0.08% fiber volume fraction. Table 2 compares the fiber spacings, fiber counts and fiber surface areas achieved through the addition of 1.2 kg/m³ (2 lb/yd³) of cellulose and polypropylene fibers to concrete. The smaller diameter of cellulose fibers clearly benefits these key measures of reinforcement efficiency in concrete.

REINFORCING ACTION OF SPECIALTY CELLULOSE FIBERS

Specialty cellulose fibers enhance the properties of concrete through interfering with the processes of microcrack propagation and cracking in concrete. With a modulus of elasticity that is higher than that of concrete, the closely spaced specialty cellulose fibers with strong bonding to concrete resist the concentration of strains near crack tips (Figure 2a) and also at the edge of bar deformations (Figure 2b). This delay in crack formation and propagation is expected to increase the bond strength of deformed bars in concrete.

Specialty cellulose fibers with their close spacing, high surface area, and high tensile and bond strengths also force a tortuous path of crack propagation (Figure 2c) and resist widening of cracks by bridging across them (Figure 2d). Finally, the fineness of cellulose fibers allows them reinforce the mortar fraction of concrete which occupies the critical thin zone between bar deformations. These effects of specialty cellulose fibers are expected to provide for a ductile failure of the bond between deformed bars and concrete.

EXPERIMENTAL RESULTS WITH POLYPROPYLENE AND NYLON FIBERS

Bond properties of reinforcing bars embedded in polypropylene and nylon fiber reinforced concrete have been reported in References 3 and 4, respectively. In
both cases, bars were pulled out from concrete cubes, and fiber volume fractions were below 0.2% (following the commercial practice). The results indicated that the addition of polypropylene and nylon fibers does not influence the bond strength or the bond stress-slip behavior of deformed bars in concrete.

EXPERIMENTAL PROGRAM

We subjected deformed reinforcing bars of Grade 60 with 414 MPa (60 ksi) yield strength and 19 mm (0.75 in) diameter to the pull-out test of ASTM C 234 (Figure 3). The concrete used in this project had a cement content of 332 kg/m$^3$ (560 lb/yd$^3$) and a water-to-cement ratio of 0.56. The coarse aggregate was crushed limestone with 25 mm (1 in) maximum particle size, and the fine aggregate was river sand. The mix was air-entrained with about 6% air content. This concrete had a slump of 150 mm (6 in), and its compressive and flexural strengths were 31 and 4 MPa (4.5 and 0.6 ksi), respectively. The pull-out tests were repeated with specialty cellulose fibers added to concrete at different dosages; the test program is presented in Table 3.

TEST RESULTS AND DISCUSSION

The deformed bars pulled out of plain and fiber reinforced concrete specimens without causing splitting cracks. Failure seemed to have initiated by the formation of inclined cracks as shown in Figure 1 c, and then continued by shearing off the concrete between bar deformations. Typical experimental bond stress-slip relationships resulting from pull-out tests are presented in Figure 4. Bond stress was obtained by dividing the pull-out load by the normal interface area between the bar and concrete. The average values of ultimate bond strength and toughness (defined as the total area under the bond stress-slip curve) are presented in Figure 5. These results indicate that low dosages of specialty cellulose fibers are effective in enhancing the strength and specially toughness of the bond between deformed bars and concrete.

The test results presented above support the hypothesis that the effectiveness of specialty cellulose fibers in the control of crack formation and propagation, resulting from their close spacing and high elastic modulus, tensile strength and bond strength to concrete, benefits the bond strength and toughness between deformed bars and concrete. This is true even at fiber volume fractions below 0.1%.
CONCLUSIONS

Specialty cellulose fibers, developed recently for convenient dispersion in normal concrete mixtures using conventional mixing procedures, are effective in enhancing the bond strength and toughness between deformed bars and concrete, even when added to concrete at volume fractions below 0.1%. At such low fiber volume fractions, fiber reinforced concrete offers fresh mix workability characteristics which are comparable to those of plain concrete; the cost implications of using such low fiber contents in concrete are also limited. The effects of specialty cellulose fibers on bond strength and toughness were more pronounced as the fiber volume fraction was increased to the upper value of 0.18% considered in this investigation. At such low fiber volume fractions, polypropylene and nylon fibers have not caused any improvements in bond behavior in past investigations. Specialty cellulose fibers, offering relatively high elastic modulus, tensile strength and bond strength to concrete, with fine diameters and thus close fiber spacings in the vicinity of reinforcing bars, provide practical and economical means of enhancing the performance characteristics of reinforced as well as plain concrete.

ACKNOWLEDGMENT

The authors are thankful to DPD, Inc. of Lansing, Michigan for providing the specialty cellulose (fabroset™) fibers used in this investigation.

REFERENCES


### TABLE 1—FIBER CHARACTERISTICS.

<table>
<thead>
<tr>
<th>PROPERTY</th>
<th>PREFERENCE</th>
<th>FIBER TYPE</th>
<th>SPECIALTY CELLULOSE</th>
<th>FIBRILATED POLYPROPYLENE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Elastic Modulus GPa (ksi)</td>
<td>High</td>
<td>60 (8,700)</td>
<td>4 (850)</td>
<td></td>
</tr>
<tr>
<td>Bond Strength MPa (ksi)</td>
<td>High</td>
<td>1.5 (0.20)</td>
<td>0.4 (0.06)</td>
<td></td>
</tr>
<tr>
<td>Tensile Strength MPa (ksi)</td>
<td>High</td>
<td>500 (72)</td>
<td>600 (87)</td>
<td></td>
</tr>
<tr>
<td>Effective Diameter µm (in)</td>
<td>Low</td>
<td>15 (0.0006)</td>
<td>60 (0.0024)</td>
<td></td>
</tr>
<tr>
<td>Aspect (length-to-diameter) Ratio</td>
<td>High</td>
<td>200</td>
<td>200</td>
<td></td>
</tr>
<tr>
<td>Hydrophilic/Hydrophobic Surface</td>
<td>Hydrophilic</td>
<td>Hydrophilic</td>
<td>Hydrophilic</td>
<td></td>
</tr>
<tr>
<td>Density g/cm³ (lb/ft³)</td>
<td>Slightly greater than 1.0</td>
<td>1.5</td>
<td>0.9</td>
<td></td>
</tr>
<tr>
<td>Alkali Resistance</td>
<td>High</td>
<td>High</td>
<td>High</td>
<td></td>
</tr>
</tbody>
</table>

### TABLE 2—VALUES OF FIBER SPACING, COUNT AND SURFACE AREA (ACI COMMITTEE 544, 1995).

<table>
<thead>
<tr>
<th>PROPERTY</th>
<th>PREFERENCE</th>
<th>FIBER TYPE</th>
<th>SPECIALTY CELLULOSE</th>
<th>FIBRILATED POLYPROPYLENE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fiber Spacing mm (in)</td>
<td>Low</td>
<td>0.53 (0.021)</td>
<td>2.8 (0.11)</td>
<td></td>
</tr>
<tr>
<td>Fiber Count I/cm³ (1/in³)</td>
<td>High</td>
<td>90 (1,475)</td>
<td>0.5 (8)</td>
<td></td>
</tr>
<tr>
<td>Specific Surface Area I/cm² (1/in²)</td>
<td>High</td>
<td>0.13 (0.33)</td>
<td>0.033 (0.083)</td>
<td></td>
</tr>
</tbody>
</table>

1 Spacing = d/√V,F, where d = fiber diameter, and V,F = fiber volume fraction
2 Fiber Count = 0.077 V,F / (l,t, d²), where l,t = fiber length
3 Specific Surface Area = 0.244 V,F / d,t

### TABLE 3—THE EXPERIMENTAL PROGRAM.

<table>
<thead>
<tr>
<th>Fiber Dosage kg/m³ (lb/yard³)</th>
<th>Equivalent Fiber Volume Fraction (%)</th>
<th>Number of Pull-Out Test Specimens</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>0.9 (1.5)</td>
<td>0.06</td>
<td>3</td>
</tr>
<tr>
<td>1.35 (2.25)</td>
<td>0.09</td>
<td>2</td>
</tr>
<tr>
<td>1.8 (3)</td>
<td>0.12</td>
<td>2</td>
</tr>
<tr>
<td>2.7 (4.5)</td>
<td>0.18</td>
<td>2</td>
</tr>
</tbody>
</table>
Fig. 1—The bond behavior of deformed bars in concrete.
Fig. 2—Mechanisms of action of specialty cellulose fibers in concrete.
Fig. 3—The deformed bar pull-out test setup.

Fig. 4—Typical bond stress-slip relationships.
Structural Applications of Fiber Reinforced Concrete 139

Fig. 5—Bond strength and toughness values.
Compressive Toughness Characterization of Normal and High-Strength Fiber Concrete Reinforced with Steel Spirals

by G. Campione and S. Mindess

Synopsis: Steel, polyolefin and carbon fiber reinforced concretes were combined with traditional transverse steel reinforcement in the form of steel spirals. The complete stress-strain relationship and the ductility of the concrete in compression in both the unconfined and confined states was evaluated. The compressive toughness was evaluated both according to the Japanese Standard JSCE - SF5 and according to a new method proposed in the present work. The experimental program consisted of testing concrete cylinders under compression at two different strength levels: normal (48 MPa) and high strength (70 MPa), reinforced with different volume percentages $V_f$ (1.5, 2.0 and 3.0) of steel, polyolefin and carbon fibers. These tests were then repeated with the addition of spiral reinforcement, $\phi_s = 5$ mm diameter, with different pitches $s$ (25 and 50 mm).

It was found that by combining fibers and steel spirals it is possible: (1) to obtain a high level of fracture energy dissipation, which could previously be obtained only by using a high volume percentage of spiral steel; and (2) to improve the maximum strain of the concrete, corresponding to the first failure of the spiral steel.

Keywords: compressive toughness; high-strength fiber concrete; steel spirals
Giuseppe Campione is a Ph.D. student in the Department of Structural and Geotechnical Engineering at the University of Palermo, Sicily, Italy. This work was carried out while he was a visiting research engineer in the Department of Civil Engineering at the University of British Columbia, Vancouver, B.C., Canada.

Sidney Mindess, F ACI, is a professor in the Department of Civil Engineering at the University of British Columbia, Vancouver, B.C., Canada. He is member of ACI Committees 370, Short-Duration Dynamic and Vibratory Load, and 446, Fracture Mechanics, and is also a member of the Co-ordinating Committee of RILEM.

INTRODUCTION

Reinforced concrete structures should possess both adequate strength and adequate ductility. If the compressive zone of a member (beam, column etc) is confined by closely spaced transverse reinforcement in the form of closed stirrups, ties, hoops or spirals, the apparent ductility of the concrete may be greatly improved, and a more ductile performance of the member at ultimate load will result (1). At low levels of compressive stress, the transverse reinforcement is little stressed and the behaviour of the concrete is unaffected by the lateral reinforcement. At stresses approaching the uniaxial concrete strength, the transverse strains in the concrete increase rapidly because of the progressive internal cracking, and the concrete expands against the transverse reinforcement. The restraining pressure applied by the reinforcement to the concrete considerably improves the stress-strain characteristics of the concrete at higher strains. The ductility of reinforced beams and columns can thus be substantially increased by lateral confinement.

In recent years, engineers have become very interested in using high strength concrete (HSC), where the increase in strength is accompanied by an increase in brittleness compared to that of normal strength concrete (NSC). If HSC is to be used for seismic-resistant concrete structures, large amounts of longitudinal and transverse reinforcing steel are required in specific regions according to ACI Committee 318 (2), for instance beam-column joints, or the fixed bases of shear walls or columns, to withstand the shears, the axial forces and the bending moments induced by an earthquake. The placing of such longitudinal and transverse steel entails much effort and cost.

Recent studies (3-6) have shown that it is possible to combine conventional reinforcement with short fibers to obtain the same level of fracture energy dissipation as that obtained using only a high volume percentage of spiral steel, which is generally used in real structures. In this way it is possible to achieve an increase in the apparent compressive strength of the concrete due to
confinement, that should offset the strength loss due to cover spalling, and the members in compression should then be able to sustain large deformations without a dramatic loss in strength.

In the case of fiber reinforced concrete (FRC) an improvement in the tensile strength, and consequently an increase in the shear capacity, was observed. Also, with FRC the dissipative capacity in bending increases (7). This was related to the high deformation capacity in compression of FRC, with only a small reduction in load-bearing capacity, that was found when using a high volume percentage of fibres (3-6). However, for the rational design of concrete structures, the complete stress-strain curves including both the loading and unloading branches should be determined for all of the constituent materials, including FRC in combination with conventional reinforcement (8-13). In this case the stress-strain relationship for confined concrete may be assumed to indicate the distribution of compressive stress in the compression zone of a member with confined concrete. It is also important to determine the ultimate strain in compression and the toughness capacity of the members in compression.

EXPERIMENTAL PROCEDURES

Material Properties

Two different mixes were used to investigate the influence of fibers, at different percentages, both in the presence and in the absence of traditional spiral steel reinforcement. The normal strength concrete (NSC) and the high strength concrete (HSC) mix designs are given in Table 1. The materials consisted of Type I Portland cement, sand from a local source, crushed stone with maximum size of 10 mm, silica fume (SF), and a superplasticizer at a dosage of 1.5 % by weight of cement to maintain good workability. No attempt was made to optimize the workability, since the same sand content was used for the different fiber types and fiber volumes. The fibers were added to the mix after all of the other ingredients had been batched and mixed for several minutes. The fibers used were straight polyolefin and carbon fibers and hooked steel fibers; the properties of the fibers are given in Table 2.

Test specimens

Steel, polyolefin and carbon fibers at volume fractions $V_f = 1.5, 2.0$ and $3.0 \%$ in HSC and in NSC were used. For each test series three $100 \text{ mm} \times 200 \text{ mm}$ cylindrical specimens were cast. The spiral confinement consisted of closed circular spirals of $\phi_s = 5 \text{ mm}$ diameter; it was constructed of steel wire having
The stress-strain curves of the various concrete specimens were measured in uniaxial compression. Specimens were moist cured at room temperature for at least 28 days before testing. The 100x200 mm cylindrical specimens were tested using an open loop machine with a maximum load capacity of 160,000 kg. To obtain the complete curve, a slow rate of loading of 0.3 MPa/s was employed. The axial deformations in the ascending branch of the curves (elastic range) were measured using two LVDT's which were mounted on opposite sides of the specimen with a gauge length of 100 mm (Fig. 1) according to JSCE-SF5 (14). The same procedures were used to evaluate the modulus of elasticity. Three LVDT's measuring the axial deformation over the entire length of the specimen were placed on the top of the platen of the compressive testing machine as shown in Fig. 2.

EXPERIMENTAL RESULTS AND DISCUSSION: EFFECTS OF DIFFERENT VARIABLES ON CONFINED CONCRETE BEHAVIOUR

The results presented below are focused on the softening branches of the \( \sigma-\varepsilon \) curves in compression of the FRC, comparing the effects of the different kinds of fibers used, their volume percentages, and the volumetric ratio of spiral reinforcement.

Monotonic loading of normal strength FRC

For monotonic loading of normal strength concrete, the effects of steel, polyolefin and carbon fibers are shown in Figs. 3 and 4 and in Table 5. Using high percentages of fibers, the maximum strengths \( f'c \) and the corresponding deformations \( \varepsilon_0 \) changed with respect to those of the plain concrete. The initial modulus of elasticity of FRC with steel fibers increased compared to the plain concrete, but with polyolefin and carbon fibers it decreased. The maximum strains corresponding to the peak stresses increased compared to the plain concrete by factors of 1.26, 2.60 and by 5.42 for polyolefin, steel and carbon fibers, respectively, at an addition rate of 3.0 \% by volume.
The decreases in maximum strength compared to the plain concrete were more marked for carbon and polyolefin fibers than for steel fibers. In all cases, the decrease in strength was due to the fact that, with high fiber contents, it was not possible to compact the mix fully, leading to higher porosities. This was particularly the case for the 3.0% fiber mixes. The decrease in strength, however, was balanced by an increase in the energy absorbed by the specimens during the compressive test. This energy, represented by the total area under the stress-strain curve, increased with increasing fiber volume. For 1.5% fibers, the energy absorbed increased by factors of 3.47, 7.60 and 3.89 for polyolefin, steel and carbon fibers, respectively. For 3.0% fibers, the increases in energy absorbed were even more marked.

FRC with steel fibers dissipated more energy than FRC with carbon or polyolefin fibers. When high percentages of fibers were used, non-linear behaviour and strain hardening in the ascending branches of the σ-ε curves were more pronounced. The softening branches showed an increase in residual strength; in the case of 3.0% fibers it was possible to observe pseudo-ductile behaviour up to a value of strain of 0.035, corresponding to the maximum value of strain that it is generally possible to achieve with high volumes of transverse reinforcement.

Monotonic Loading of High Strength FRC

In the case of monotonic loading of high strength FRC, the effects of steel, polyolefin and carbon fibers are shown in Figs. 5 and 6 and in Table 6. The initial modulus of elasticity for FRC with steel fibers increased with respect to the plain concrete, for both 1.5% and 3.0% fibers. In the case of polyolefin and carbon fibers the modulus of elasticity did not change significantly with respect to the plain concrete. The maximum strain corresponding to the peak stress increased with respect to the plain concrete by factor of 1.37, 1.72 and by 2.06 for 3.0% polyolefin, steel and carbon fibers, respectively.

In all cases an increase in the energy absorbed by the specimens was observed with respect to the plain concrete. For 1.5% polyolefin, steel and carbon fibers, the energy absorbed increased by factors of 3.08, 6.51, 2.86 respectively. For 3.0% fibers, the corresponding increase in energy absorbed was by factors of 4.92, 8.27 and 9.41. Thus, for carbon fibers, more energy was dissipated than with the steel and the polyolefin fibers.

It is interesting to note that the same volume percentages of fibers had different effects on the behaviour of normal and high strength fiber reinforced concretes. The small reduction in strength combined with the large increase in energy absorbed and in ductility for HSC, suggests a more effective use of fibers in high strength concrete than in normal strength concrete for structural
applications. In particular, the carbon fibers were better than the steel and polyolefin fibers in HSC. This is probably due to the better bond developed between the matrix and carbon fibers when silica fume is added to the matrix (7).

Monotonic Loading of Normal and High Strength FRC Containing Steel Spirals

Figures 7 and 8 and Tables 5 and 6 show the results of tests on normal and high strength concrete cylinders containing carbon and steel fibers at volume percentages of 2.0%, in conjunction with transverse reinforcement provided by 5 mm diameter steel spirals with volumetric ratios $\rho = 0.02592$ and $\rho = 0.01296$, corresponding to pitches of 25 and 50 mm, respectively.

For NSC the effects of 2.0 % carbon fibers with different percentages of steel spirals are shown in Fig. 7. The maximum compressive strength, the strain corresponding to the peak load, and the strain corresponding to the first yielding of the spirals all increased. The energy absorbed also increased, compared to the case of plain concrete with lateral reinforcement. For 2.0 % carbon fibers, the energy increased by factors of 1.58 and 1.36 for $\rho = 0.02592$ and $\rho = 0.01296$, respectively. When fibers and steel spirals were both used, the strain corresponding to the peak strength increased. The softening branch indicates that fibres tended to improve the ductility of the concrete.

It is interesting to observe that:

i) the energy absorbed by the matrix with $\rho = 0.01296$ spiral steel and 2.0 % carbon fibers was comparable to the energy absorbed by the plain matrix with $\rho = 0.02592$ spiral steel, but a higher strain corresponding to the peak stress was observed for the combination of fibers and spiral steel.

ii) the load corresponding to the first yielding of the spirals increased when fibers and spirals were both used at high volume percentages.

For HSC, Fig. 8 shows the effects of steel and carbon fibers with a constant percentage $\rho = 0.02592$ of spiral steel. The maximum compressive strength, the strain corresponding to the peak load and the strain corresponding to the first rupture of the spirals increased with respect to those of the plain concrete with spirals. For 2.0 % carbon fibers an high increase of energy absorbed (evaluated up to the first rupturc of the spirals) was observed compared to the plain concrete with spirals. The strain corresponding to the peak strength and the strain corresponding to the first failure of the steel spirals increased as well.
According to JSCE Standard SF-5 (14), the energy absorbed in compression by fiber reinforced concrete is evaluated by the area under the load-deformation curve out to the point at which the deformation $\delta_{0.75}$ (measured on a 100 mm gauge length) reaches 0.75 mm for cylindrical specimens of dimensions 100 mm x 200 mm. As shown in Fig. 9, from this measure of compressive toughness ($T_c$), a compressive toughness factor ($\bar{\sigma}_c$) is evaluated, defined as:

$$\bar{\sigma}_c = \frac{4 \cdot T_c}{\pi \cdot d^2 \cdot \delta_{0.75}}$$  \hspace{1cm} (1)

where $\bar{\sigma}_c$ is the compressive toughness factor (N/mm²), $T_c$ is the compressive toughness (Nm), $\delta_{0.75}$ is the deformation corresponding to 0.75% converted to strain (mm) and $d$ is the diameter of the specimen.

Note that the compressive toughness $T_c$ has the units of work, so that its value indicates the energy expended by the external load in opening the cracks and bringing about failure of the specimen. In the case of plain concrete this work is expended primarily to overcome the closing pressures in the fracture process zone. If fibers or steel spirals are used, additional work is expended for the pull-out of the fibers from the matrix, or to overcome the confining pressure of the spirals. In these cases the maximum value of deformation suggested in JSCE SF-5 (14) for the calculation of $T_c$ is purely arbitrary, and might overestimate or underestimate the effective dissipative capacities of the concrete if steel confinement and fibers are both used. Also, it might be more appropriate to distinguish between the energies absorbed by the specimen before and after the peak load, and to evaluate the post-peak energy by using a deformation corresponding to a load which is some appropriate fraction of the maximum load capacity of the specimens. The compressive toughness factor ($\bar{\sigma}_c$) has units of stress, so that its value indicates the post-matrix cracking residual strength in compression of the material when loaded to an arbitrary deformation corresponding to 0.75% converted to strain.

The JSCE technique to evaluate the compressive toughness factor is specimen geometry dependent, and also depends on the gauge length used, which makes an exact correlation with the field performance of FRC rather difficult. Figs. 10 and 11 show the values of the compressive toughness factor evaluated using (1), but with different values of $\delta_{0.75}$. It may be seen that the maximum value of strain (0.75%) assumed by JSCE-SF5 for normal strength steel fiber reinforced concrete to evaluate the toughness in compression sometimes overestimates the
real toughness capacity of the composite when low percentages of fibers are used.

Here, a new method is proposed to evaluate the compressive toughness factor. This computation considers only the post-peak area under the load versus deflection curve, and defines a post-crack strength $PCS_m$. To analyse a given load-deformation curve using this method one should:

i) Locate the peak load and divide the curve into two regions: the pre-peak region and the post-peak region. Determine the peak load and measure the area under the curve up to the peak load. This area represents the pre-peak energy, $T_{pre}$. If high fiber volume fractions are used, two load peaks may occur: one at the end of the matrix contribution and the other when the fibers reach their ultimate capacity. In this case, the first load peak, corresponding to the end of the matrix contribution, should be considered.

ii) Locate points on the curve in the post-peak region at specimen deformations $\delta_i$ corresponding to various fractions of the maximum load ($P_{peak/m}$), considering the ultimate deformation. This value was assumed as 3.5 mm, when only fibers are used in the matrix, or as the deformation corresponding to the first rupture in yielding of the spirals, if fibers and spirals are both used. The suggested displacements $\delta_i$ lie between the displacement corresponding to $0.85 \sigma_c$ and the ultimate deformation. The areas under the curve up to these deformations are denoted as $T_{post,m}$.

iii) Calculate the Post Crack Strength ($PCS_m$) in the post-peak region at the various deformations. The Post Crack Strength at a deformation $\delta_i$ is defined as:

$$PCS_m = \frac{(T_{post,m}) \cdot L}{(L/m - \delta_{peak}) \cdot b \cdot h^2} \quad (2)$$

where $PCS_m$ is the compressive toughness factor (N/mm²) at a deformation of $\delta_i$ assumed serviceability limit deformation, $T_{post,m}$ is the compressive post peak toughness (Nm), that is the area under the load deflection curve in the post-peak region up to the serviceability limit, $\delta_{peak}$ is the deformation at the peak load, $\delta_i$ is the deformation corresponding to a load that is some percentage of the maximum load or a deformation corresponding to failure of the first hoop, and $d$ is the diameter of the specimen.

Tables 7 and 8 show the values of compressive toughness according to JSCE-Standard SF5 while Tables 9 and 10 show the Post Cracking Strength values according to the proposed new method. It may be seen that the $PCS_m$ values better describe the behaviour of the composite than do the $\sigma_c$ values calculated from JSCE SF-5. That is, they permit comparisons amongst different fiber types.
and volumes to be made at different values of post-peak deformations such as may be chosen for any particular application, rather than at a fixed strain of 0.75%.

**CONCLUSIONS**

Compressive tests on cylinders of normal and high strength concrete with fibers and steel spirals showed:

1. An increase in the energy absorbed by the FRC with respect to the plain concrete.

2. With fibers, concrete reinforced with steel spirals showed an increase in the strength and in the energy absorbed compared to plain concrete confined by steel spirals.

3. The maximum strain corresponding to the failure of the steel spirals increased with increasing fiber content.

4. In the case of high percentages of fibers, multiple failures of the steel spirals occurred at different transverse sections, indicating the capacity of FRC to absorb energy and to achieve high values of ductility.

5. When 2.0% fibers were used in conjunction with spirals $\rho = 0.01296$ ($\rho_s = 5/50$ mm), it was possible to obtain similar values of absorbed energy as with spirals $\rho = 0.02592$ ($\rho = 5/25$ mm) and no fibers, but with more strain capacity.

6. The evaluation of the toughness index and the residual strength according to JSCE SF-5 for the fibers reinforced concrete may overestimate or underestimate the effective dissipative capacities of FRC or FRC with transverse steel when high percentages of fibers are used.

7. The new method proposed to evaluate compressive toughness gives more useful values.

8. Finally, the use of fibers combined with steel spirals permits the achievement of high ductility, and high maximum strain corresponding to the failure of the first spirals.
ACKNOWLEDGEMENTS

The research reported here was supported by the Natural Sciences and Engineering Research Council of Canada. It represents a co-operative program between the University of Palermo and the University of British Columbia.

REFERENCES


2. ACI Committee 318 (1984), Building Code Requirement for Reinforced Concrete and Commentary (ACI 318-89/ACI 318RM-89), American Concrete Institute, Detroit, pp. 1079-1102


NOTATION

\begin{itemize}
  \item $V_f$: volume fraction of fibers (%)
  \item $L_f$: length of fiber (mm)
  \item $\phi$: diameter of fiber (mm)
  \item $d$: diameter of the specimen tested
  \item $L_f/\phi$: aspect ratio
  \item $\rho_s$: volumetric ratio of spiral steel measured from out-to-out of lateral steel
  \item $\phi_s$: diameter of steel spiral
  \item $f'\text{c}$: compressive strength of concrete at 28 days
  \item $E$: elastic modulus in compression of concrete at 28 days
  \item $f_y$: specific yield strength of spiral steel
  \item $s$: spiral spacing
  \item $\varepsilon_0$: concrete strain corresponding to compressive strength
  \item $\varepsilon_u$: concrete strain corresponding to the first rupture of the spirals (or assumed 0.035 for FRC).
  \item $\delta_{0.75\%}$: deformation corresponding to 0.75 % converted to strain (mm)
  \item $\delta_i$: deformation corresponding to fraction of the maximum load converted to strain (mm).
  \item $T_c$: compressive toughness (Nm) evaluated from the area under the load-deformation plot up to a load point maximum deformation $\delta_{0.75\%}$
  \item $T_u$: full energy absorbed (Nm) evaluated from the area under the load-deformation plot up to a maximum deformation $\delta_u$.
  \item $\sigma_c$: compressive toughness factor (N/mm$^2$), related to $T_c$
  \item $PCS_m$: Post Crack Strength (MPa) in the post-peak region at the various deformations
\end{itemize}
### TABLE 1—MIX PROPORTIONS FOR NORMAL AND HIGH-STRENGTH CONCRETES.

<table>
<thead>
<tr>
<th>Mix</th>
<th>Cement kg/m</th>
<th>Water kg/m</th>
<th>Coarse aggregate kg/m</th>
<th>Sand kg/m³</th>
<th>Silica fume kg/m</th>
<th>Superplasticizer kg/m</th>
</tr>
</thead>
<tbody>
<tr>
<td>Normal strength concrete</td>
<td>300</td>
<td>165</td>
<td>1050</td>
<td>850</td>
<td>/</td>
<td>/</td>
</tr>
<tr>
<td>High strength concrete</td>
<td>400</td>
<td>150</td>
<td>1050</td>
<td>720</td>
<td>55</td>
<td>8</td>
</tr>
</tbody>
</table>

### TABLE 2—CHARACTERISTICS OF THE FIBERS.

<table>
<thead>
<tr>
<th>Type of fibre</th>
<th>Longitudinal profile</th>
<th>Cross-section shape</th>
<th>Effective diameter (mm)</th>
<th>Length Lf (mm)</th>
<th>Tensile Strength (N/mm²)</th>
<th>Weight density (kg/m³)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Polyolefin</td>
<td>straight</td>
<td>Circular</td>
<td>0.80</td>
<td>25</td>
<td>375</td>
<td>900</td>
</tr>
<tr>
<td>Carbon</td>
<td>straight</td>
<td>Rectangular</td>
<td>0.78</td>
<td>20</td>
<td>800</td>
<td>1340</td>
</tr>
<tr>
<td>Steel</td>
<td>hooked-end</td>
<td>Circular</td>
<td>0.80</td>
<td>30</td>
<td>1115</td>
<td>7860</td>
</tr>
</tbody>
</table>

### TABLE 3—CHARACTERISTICS OF THE SPECIMENS AND PROPERTIES OF FRESH NSC.

<table>
<thead>
<tr>
<th>Specimen Designation</th>
<th>Type of concrete</th>
<th>Type of fiber</th>
<th>Vf (%)</th>
<th>ρs</th>
<th>Air content (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>N1</td>
<td>NSC</td>
<td>/</td>
<td>/</td>
<td>/</td>
<td>1.5</td>
</tr>
<tr>
<td>N2</td>
<td>&quot;</td>
<td>/</td>
<td>/</td>
<td>0.01296</td>
<td>1.5</td>
</tr>
<tr>
<td>N3</td>
<td>&quot;</td>
<td>/</td>
<td>/</td>
<td>0.02592</td>
<td>1.5</td>
</tr>
<tr>
<td>NC1</td>
<td>&quot;</td>
<td>carbon</td>
<td>1.5</td>
<td>/</td>
<td>2.6</td>
</tr>
<tr>
<td>NC2</td>
<td>&quot;</td>
<td>&quot;</td>
<td>2.0</td>
<td>/</td>
<td>2.8</td>
</tr>
<tr>
<td>NC3</td>
<td>&quot;</td>
<td>&quot;</td>
<td>3.0</td>
<td>/</td>
<td>3.1</td>
</tr>
<tr>
<td>NC4</td>
<td>&quot;</td>
<td>&quot;</td>
<td>2.0</td>
<td>0.01296</td>
<td>2.8</td>
</tr>
<tr>
<td>NC5</td>
<td>&quot;</td>
<td>&quot;</td>
<td>2.0</td>
<td>0.02592</td>
<td>2.8</td>
</tr>
<tr>
<td>NS1</td>
<td>&quot;</td>
<td>steel</td>
<td>1.5</td>
<td>/</td>
<td>3.8</td>
</tr>
<tr>
<td>NS2</td>
<td>&quot;</td>
<td>&quot;</td>
<td>3.0</td>
<td>/</td>
<td>4.6</td>
</tr>
<tr>
<td>NP1</td>
<td>&quot;</td>
<td>Polyolefin</td>
<td>1.5</td>
<td>/</td>
<td>2.4</td>
</tr>
<tr>
<td>NP2</td>
<td>&quot;</td>
<td>&quot;</td>
<td>3.0</td>
<td>/</td>
<td>7.4</td>
</tr>
</tbody>
</table>
TABLE 4—CHARACTERISTICS OF THE SPECIMENS AND PROPERTIES OF FRESH HSC.

<table>
<thead>
<tr>
<th>Specimen Designation</th>
<th>Type of concrete</th>
<th>Type of fiber</th>
<th>$V_f$ (%)</th>
<th>$\rho_s$ (%)</th>
<th>Air content (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>H1</td>
<td>HSC</td>
<td>/</td>
<td>/</td>
<td>/</td>
<td>1.0</td>
</tr>
<tr>
<td>H2</td>
<td>&quot;</td>
<td>/</td>
<td>/</td>
<td>0.01296</td>
<td>1.0</td>
</tr>
<tr>
<td>H3</td>
<td>&quot;</td>
<td>/</td>
<td>/</td>
<td>0.02592</td>
<td>1.0</td>
</tr>
<tr>
<td>HC1</td>
<td>&quot;</td>
<td>carbon</td>
<td>1.5</td>
<td>/</td>
<td>1.7</td>
</tr>
<tr>
<td>HC2</td>
<td>&quot;</td>
<td>&quot;</td>
<td>2.0</td>
<td>/</td>
<td>1.9</td>
</tr>
<tr>
<td>HC3</td>
<td>&quot;</td>
<td>&quot;</td>
<td>3.0</td>
<td>/</td>
<td>3.0</td>
</tr>
<tr>
<td>HC4</td>
<td>&quot;</td>
<td>&quot;</td>
<td>2.0</td>
<td>0.01296</td>
<td>1.9</td>
</tr>
<tr>
<td>HC5</td>
<td>&quot;</td>
<td>&quot;</td>
<td>2.0</td>
<td>0.02592</td>
<td>1.9</td>
</tr>
<tr>
<td>HS1</td>
<td>&quot;</td>
<td>steel</td>
<td>1.5</td>
<td>/</td>
<td>1.4</td>
</tr>
<tr>
<td>HS2</td>
<td>&quot;</td>
<td>&quot;</td>
<td>2.0</td>
<td>/</td>
<td>1.6</td>
</tr>
<tr>
<td>HS3</td>
<td>&quot;</td>
<td>&quot;</td>
<td>3.0</td>
<td>/</td>
<td>1.9</td>
</tr>
<tr>
<td>HS4</td>
<td>&quot;</td>
<td>&quot;</td>
<td>2.0</td>
<td>0.01296</td>
<td>1.6</td>
</tr>
<tr>
<td>HS5</td>
<td>&quot;</td>
<td>&quot;</td>
<td>2.0</td>
<td>0.02592</td>
<td>1.6</td>
</tr>
<tr>
<td>HP1</td>
<td>&quot;</td>
<td>polyolefin</td>
<td>1.5</td>
<td>/</td>
<td>2.3</td>
</tr>
<tr>
<td>HP2</td>
<td>&quot;</td>
<td>&quot;</td>
<td>3.0</td>
<td>/</td>
<td>4.4</td>
</tr>
</tbody>
</table>

TABLE 5—PROPERTIES OF HARDENED NSC.

<table>
<thead>
<tr>
<th>Specimen</th>
<th>$V_f$ (%)</th>
<th>$\rho_s$ (%)</th>
<th>$f_c$ (MPa)</th>
<th>$\varepsilon_0$</th>
<th>$\varepsilon_{cu}$</th>
<th>E (GPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>N1</td>
<td>0</td>
<td>0</td>
<td>48.88</td>
<td>0.0035</td>
<td>/</td>
<td>31</td>
</tr>
<tr>
<td>N2</td>
<td>/</td>
<td>0.01296</td>
<td>41.13</td>
<td>0.0051</td>
<td>0.026</td>
<td>32</td>
</tr>
<tr>
<td>N3</td>
<td>/</td>
<td>0.02596</td>
<td>55.37</td>
<td>0.0120</td>
<td>0.039</td>
<td>34</td>
</tr>
<tr>
<td>NC1</td>
<td>1.5</td>
<td>/</td>
<td>37.15</td>
<td>0.0069</td>
<td>/</td>
<td>28</td>
</tr>
<tr>
<td>NC2</td>
<td>2.0</td>
<td>/</td>
<td>38.93</td>
<td>0.0068</td>
<td>/</td>
<td>29</td>
</tr>
<tr>
<td>NC3</td>
<td>3.0</td>
<td>/</td>
<td>30.01</td>
<td>0.0160</td>
<td>/</td>
<td>25</td>
</tr>
<tr>
<td>NC4</td>
<td>2.0</td>
<td>0.01296</td>
<td>46.19</td>
<td>0.0093</td>
<td>0.033</td>
<td>35</td>
</tr>
<tr>
<td>NC5</td>
<td>2.0</td>
<td>0.02596</td>
<td>58.59</td>
<td>0.0190</td>
<td>0.047</td>
<td>32</td>
</tr>
<tr>
<td>NS1</td>
<td>1.5</td>
<td>/</td>
<td>42.43</td>
<td>0.0078</td>
<td>/</td>
<td>36</td>
</tr>
<tr>
<td>NS2</td>
<td>3.0</td>
<td>/</td>
<td>39.00</td>
<td>0.0091</td>
<td>/</td>
<td>29</td>
</tr>
<tr>
<td>NP1</td>
<td>1.5</td>
<td>/</td>
<td>38.49</td>
<td>0.0041</td>
<td>/</td>
<td>26</td>
</tr>
<tr>
<td>NP2</td>
<td>3.0</td>
<td>/</td>
<td>33.41</td>
<td>0.0044</td>
<td>/</td>
<td>24</td>
</tr>
</tbody>
</table>
### TABLE 6—PROPERTIES OF HARDENED HSC.

<table>
<thead>
<tr>
<th>Specimen</th>
<th>Vf (%)</th>
<th>ρs</th>
<th>tc (MPa)</th>
<th>ε₀</th>
<th>εcu</th>
<th>E (GPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>H1</td>
<td>0</td>
<td>0</td>
<td>70.23</td>
<td>0.0029</td>
<td>/</td>
<td>33</td>
</tr>
<tr>
<td>H2</td>
<td>/</td>
<td>0.01296</td>
<td>54.57</td>
<td>0.0066</td>
<td>0.020</td>
<td>32</td>
</tr>
<tr>
<td>H3</td>
<td>/</td>
<td>0.02596</td>
<td>75.60</td>
<td>0.0110</td>
<td>0.039</td>
<td>37</td>
</tr>
<tr>
<td>HC1</td>
<td>1.5</td>
<td>/</td>
<td>74.55</td>
<td>0.0037</td>
<td>/</td>
<td>35</td>
</tr>
<tr>
<td>HC2</td>
<td>2.0</td>
<td>/</td>
<td>73.66</td>
<td>0.0047</td>
<td>/</td>
<td>34</td>
</tr>
<tr>
<td>HC3</td>
<td>3.0</td>
<td>/</td>
<td>61.57</td>
<td>0.0060</td>
<td>/</td>
<td>36</td>
</tr>
<tr>
<td>HC4</td>
<td>2.0</td>
<td>0.01296</td>
<td>61.60</td>
<td>0.0041</td>
<td>0.038</td>
<td>33</td>
</tr>
<tr>
<td>HC5</td>
<td>2.0</td>
<td>0.02596</td>
<td>78.15</td>
<td>0.0140</td>
<td>0.064</td>
<td>30</td>
</tr>
<tr>
<td>HS1</td>
<td>1.5</td>
<td>/</td>
<td>75.57</td>
<td>0.0040</td>
<td>/</td>
<td>41</td>
</tr>
<tr>
<td>HS2</td>
<td>2.0</td>
<td>/</td>
<td>72.41</td>
<td>0.0050</td>
<td>/</td>
<td>42</td>
</tr>
<tr>
<td>HS3</td>
<td>3.0</td>
<td>/</td>
<td>68.80</td>
<td>0.0110</td>
<td>/</td>
<td>36</td>
</tr>
<tr>
<td>HS4</td>
<td>2.0</td>
<td>0.01296</td>
<td>80.12</td>
<td>0.0088</td>
<td>0.024</td>
<td>39</td>
</tr>
<tr>
<td>HS5</td>
<td>2.0</td>
<td>0.02596</td>
<td>88.97</td>
<td>0.0100</td>
<td>0.035</td>
<td>43</td>
</tr>
<tr>
<td>HP1</td>
<td>1.5</td>
<td>/</td>
<td>60.00</td>
<td>0.0032</td>
<td>/</td>
<td>34</td>
</tr>
<tr>
<td>HP2</td>
<td>3.0</td>
<td>/</td>
<td>51.00</td>
<td>0.0040</td>
<td>/</td>
<td>32</td>
</tr>
</tbody>
</table>

### TABLE 7—JSCE-SF5 TOUGHNESS INDEX AND RESIDUAL STRENGTH IN COMPRESSION FOR NSC.

<table>
<thead>
<tr>
<th>Specimen</th>
<th>Vf (%)</th>
<th>ρs</th>
<th>Tc (Nm)</th>
<th>$\overline{\sigma}_c$ (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>N1</td>
<td>0</td>
<td>/</td>
<td>/</td>
<td>/</td>
</tr>
<tr>
<td>N2</td>
<td>/</td>
<td>0.01296</td>
<td>173</td>
<td>29.23</td>
</tr>
<tr>
<td>N3</td>
<td>/</td>
<td>0.02596</td>
<td>190</td>
<td>33.53</td>
</tr>
<tr>
<td>NC1</td>
<td>1.5</td>
<td>/</td>
<td>163</td>
<td>28.47</td>
</tr>
<tr>
<td>NC2</td>
<td>2.0</td>
<td>/</td>
<td>172</td>
<td>28.73</td>
</tr>
<tr>
<td>NC3</td>
<td>3.0</td>
<td>/</td>
<td>155</td>
<td>17.16</td>
</tr>
<tr>
<td>NC4</td>
<td>2.0</td>
<td>0.01296</td>
<td>147</td>
<td>26.72</td>
</tr>
<tr>
<td>NC5</td>
<td>2.0</td>
<td>0.02596</td>
<td>142</td>
<td>25.02</td>
</tr>
<tr>
<td>NS1</td>
<td>1.5</td>
<td>/</td>
<td>190</td>
<td>32.31</td>
</tr>
<tr>
<td>NS2</td>
<td>3.0</td>
<td>/</td>
<td>171</td>
<td>29.36</td>
</tr>
<tr>
<td>NP1</td>
<td>1.5</td>
<td>/</td>
<td>159</td>
<td>27.05</td>
</tr>
<tr>
<td>NP2</td>
<td>3.0</td>
<td>/</td>
<td>139</td>
<td>25.67</td>
</tr>
</tbody>
</table>
### TABLE 8—JSCE SF-5 TOUGHNESS INDEX AND RESIDUAL STRENGTH IN COMPRESSION FOR HSC.

<table>
<thead>
<tr>
<th>Specimen</th>
<th>( V_r (%) )</th>
<th>( \rho_s )</th>
<th>( T_c (Nm) )</th>
<th>( \sigma_c ) (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>H1</td>
<td>0</td>
<td>/</td>
<td>/</td>
<td>/</td>
</tr>
<tr>
<td>H2</td>
<td>/</td>
<td>0.01296</td>
<td>268</td>
<td>45.74</td>
</tr>
<tr>
<td>H3</td>
<td>/</td>
<td>0.02596</td>
<td>295</td>
<td>50.66</td>
</tr>
<tr>
<td>HC1</td>
<td>1.5</td>
<td>/</td>
<td>197</td>
<td>33.61</td>
</tr>
<tr>
<td>HC2</td>
<td>2.0</td>
<td>/</td>
<td>239</td>
<td>48.86</td>
</tr>
<tr>
<td>HC3</td>
<td>3.0</td>
<td>/</td>
<td>258</td>
<td>46.19</td>
</tr>
<tr>
<td>HC4</td>
<td>2.0</td>
<td>0.01296</td>
<td>297</td>
<td>50.57</td>
</tr>
<tr>
<td>HC5</td>
<td>2.0</td>
<td>0.02596</td>
<td>295</td>
<td>49.05</td>
</tr>
<tr>
<td>HS1</td>
<td>1.5</td>
<td>/</td>
<td>310</td>
<td>51.00</td>
</tr>
<tr>
<td>HS2</td>
<td>2.0</td>
<td>/</td>
<td>305</td>
<td>51.00</td>
</tr>
<tr>
<td>HS3</td>
<td>3.0</td>
<td>/</td>
<td>275</td>
<td>47.80</td>
</tr>
<tr>
<td>HS4</td>
<td>2.0</td>
<td>0.01296</td>
<td>283</td>
<td>51.43</td>
</tr>
<tr>
<td>HS5</td>
<td>2.0</td>
<td>0.02596</td>
<td>262</td>
<td>43.50</td>
</tr>
<tr>
<td>HP1</td>
<td>1.5</td>
<td>/</td>
<td>206</td>
<td>42.50</td>
</tr>
<tr>
<td>HP2</td>
<td>3.0</td>
<td>/</td>
<td>207</td>
<td>35.64</td>
</tr>
</tbody>
</table>

### TABLE 9—POST-CRACK STRENGTH VALUES (PCS) FOR NSC.

<table>
<thead>
<tr>
<th>Specimen</th>
<th>Deformation at Peak (mm)</th>
<th>Peak Load (KN)</th>
<th>Post Crack Strength (MPa)</th>
<th>( T_{absorbed} ) (Nm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>N1</td>
<td>0.35</td>
<td>381</td>
<td>/</td>
<td>/</td>
</tr>
<tr>
<td>N2</td>
<td>0.54</td>
<td>320</td>
<td>39.75</td>
<td>/</td>
</tr>
<tr>
<td>N3</td>
<td>1.19</td>
<td>431</td>
<td>53.64</td>
<td>51.33</td>
</tr>
<tr>
<td>NC1</td>
<td>0.60</td>
<td>300</td>
<td>37.91</td>
<td>36.96</td>
</tr>
<tr>
<td>NC2</td>
<td>0.69</td>
<td>290</td>
<td>36.15</td>
<td>34.55</td>
</tr>
<tr>
<td>NC3</td>
<td>1.61</td>
<td>234</td>
<td>29.77</td>
<td>27.57</td>
</tr>
<tr>
<td>NC4</td>
<td>2.15</td>
<td>349</td>
<td>42.82</td>
<td>41.55</td>
</tr>
<tr>
<td>NC5</td>
<td>2.15</td>
<td>449</td>
<td>55.97</td>
<td>52.73</td>
</tr>
<tr>
<td>NS1</td>
<td>0.77</td>
<td>332</td>
<td>41.61</td>
<td>35.36</td>
</tr>
<tr>
<td>NS2</td>
<td>1.01</td>
<td>303</td>
<td>38.02</td>
<td>37.13</td>
</tr>
<tr>
<td>NP1</td>
<td>0.41</td>
<td>300</td>
<td>37.59</td>
<td>36.12</td>
</tr>
<tr>
<td>NP2</td>
<td>0.42</td>
<td>260</td>
<td>32.64</td>
<td>31.55</td>
</tr>
</tbody>
</table>
### TABLE 10—POST-CRACK STRENGTH VALUES (PCS) FOR HSC.

<table>
<thead>
<tr>
<th>Specimen</th>
<th>Deformation at Peak (mm)</th>
<th>Peak Load (KN)</th>
<th>Post Crack Strength (MPa)</th>
<th>$T_{absorbed}$ (Nm)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>PCS95</td>
<td>PCS85</td>
</tr>
<tr>
<td>H1</td>
<td>0.28</td>
<td>549</td>
<td>/</td>
<td>/</td>
</tr>
<tr>
<td>H2</td>
<td>0.88</td>
<td>425</td>
<td>53.85</td>
<td>52.45</td>
</tr>
<tr>
<td>H3</td>
<td>0.91</td>
<td>590</td>
<td>73.28</td>
<td>70.89</td>
</tr>
<tr>
<td>HC1</td>
<td>0.34</td>
<td>582</td>
<td>72.74</td>
<td>71.14</td>
</tr>
<tr>
<td>HC2</td>
<td>0.48</td>
<td>555</td>
<td>68.47</td>
<td>65.80</td>
</tr>
<tr>
<td>HC3</td>
<td>0.71</td>
<td>472</td>
<td>58.94</td>
<td>53.72</td>
</tr>
<tr>
<td>HC4</td>
<td>0.49</td>
<td>481</td>
<td>59.80</td>
<td>56.69</td>
</tr>
<tr>
<td>HC5</td>
<td>1.41</td>
<td>611</td>
<td>76.21</td>
<td>73.59</td>
</tr>
<tr>
<td>HS1</td>
<td>0.37</td>
<td>593</td>
<td>73.10</td>
<td>72.95</td>
</tr>
<tr>
<td>HS2</td>
<td>0.48</td>
<td>556</td>
<td>68.04</td>
<td>65.82</td>
</tr>
<tr>
<td>HS3</td>
<td>1.16</td>
<td>539</td>
<td>67.82</td>
<td>63.82</td>
</tr>
<tr>
<td>HS4</td>
<td>0.75</td>
<td>615</td>
<td>78.10</td>
<td>76.61</td>
</tr>
<tr>
<td>HS5</td>
<td>1.00</td>
<td>681</td>
<td>86.00</td>
<td>84.08</td>
</tr>
<tr>
<td>HP1</td>
<td>0.29</td>
<td>476</td>
<td>60.62</td>
<td>60.04</td>
</tr>
<tr>
<td>HP2</td>
<td>0.36</td>
<td>398</td>
<td>48.46</td>
<td>46.45</td>
</tr>
</tbody>
</table>

Fig. 1—Test set-up for measuring the initial modulus of elasticity.
Fig. 2—Test set-up for measuring the full stress-strain curve.

Fig. 3—Stress-strain curves for NSC at 1.5 percent by volume percentage of fibers.
Fig. 4—Stress-strain curves for NSC at 3.0 percent by volume of fibers.

Fig. 5—Stress-strain curves for HSC at 1.5 percent by volume percentage of fibers.
Fig. 6—Stress-strain curves for HSC at 3.0 percent by volume of fibers.

Fig. 7—Stress-strain curves for confined NSC at different fiber volumes and steel spiral volumes.
Fig. 8—Stress-strain curves for confined HSC at different fiber volumes spiral reinforcement $p=0.02592$ — failure of steel spirals.

Fig. 9—Load deformation curve in compression, from JSCE-SF5 (14).
Fig. 10—Compressive toughness factor for HSC for 1.5 percent fibers according to JSCE-SF5, for different values of $\delta_{tc}$.

Fig. 11—Compressive toughness factor for HSC for 3.0 percent fibers according to JSCE-SF5, for different values of $\delta_{tc}$. 
Compressive Strength and Ductility of Steel Fiber Reinforced Concrete

by B. Massicotte, B. Mossor, A. Filiatrault, and S. Tremblay

Synopsis: It is known that Steel Fiber Reinforced Concrete (SFRC) has advantages over plain concrete. In particular, fiber reinforcement makes concrete tougher and more ductile. Although these attributes are appealing for earthquake resisting structures, design codes do not yet incorporate specifications relative to the use of SFRC for structural applications. Recent developments have indicated a good potential for SFRC in structural and seismic applications. In the first part of this paper, the beneficial effects of SFRC in the seismic design of columns are briefly reviewed. The paper then presents an overview of an ongoing research project on column strength and ductility in which 18 columns were tested in uniaxial compression. The variables considered were the fiber content of 0%, 0.5% and 1.0% per volume, the amount of transverse reinforcement for confining the column core, and the confinement provided by the fibers in the cover. It is shown that SFRC improves significantly the post-peak behavior of columns for all hoop spacings and that SFRC can be juxtaposed with reduced traditional confining reinforcement under the same seismic design philosophy. Although SFRC in the cover delay its spalling, no noticeable confining contribution of the cover was observed. Fibers do not really confine concrete, but rather change the failure mode by limiting the progression of cracks and enhancing the aggregate interlock along failure planes.

Keywords: columns; confinement; ductility; experimental testing; reinforced concrete (SFRC); seismic resistance; steel fibers
ACI member Bruno Massicotte is associate professor of civil engineering at École Polytechnique in Montreal. He is member of ACI Committee 514, Fiber Reinforced Concrete, and ACI Committee 342, Bridge Evaluation. His research interest are bridges, composite structures, structural applications of fiber concrete, and nonlinear finite element modeling of concrete structures.

Bart Mossor, a bridge engineer for Inter-Rives Inc. in Quebec, was formerly a graduate research assistant in Civil Engineering at École Polytechnique in Montreal. His interests are bridge design and construction.

ACI member André Filiatrault is professor of civil engineering at École Polytechnique in Montreal. He is chairman of Epicentre, École Polytechnique earthquake engineering research group. His research interests include fiber reinforced concrete, earthquake engineering, and structural dynamics.

Stéphane Tremblay is a senior researcher at IREQ, Hydro-Quebec Research Institute. His research interests are the rehabilitation, instrumentation and injection of concrete dams and related structures.

INTRODUCTION

In seismic design, the need for confining the concrete is an essential requirement. This is achieved by providing passive confinement through additional transverse hoops or ties at critical locations in the structure. This transverse reinforcement becomes active when the concrete expands laterally in compression. Through years of development and considerable research effort, the structural engineering community around the world now benefits from design rules applicable to a wide variety of concrete elements and structures built in active seismic zones.

Although it is recognized that steel fibers improve the mechanical characteristics of concrete, their beneficial contribution in structural applications has not yet been incorporated in design codes. Many practicing engineers still perceive fibrous concrete as a good material for non-structural applications where additional durability is needed, such as in concrete pavements, slab on grade, bridge deck overlay, shotcrete, etc. This perception exists mainly because the tensile contribution of concrete, which has been traditionally neglected in structural applications, is used for crack control in non-structural applications of SFRC. Therefore, it can be difficult for practicing engineers to realize that fiber concrete can be used economically to partially replace conventional reinforcement without jeopardizing structural safety or be added to conventionally reinforced concrete members to enhance significantly their ductility.
Nevertheless, fibrous concrete should enjoy a wider field of applications than commonly assumed by the engineering community. Many seismic code requirements, regarding the confinement and aiming to improve ductility and energy dissipation, can be relaxed by the use of SFRC. The cornerstone of the capacity seismic design philosophy of reinforced concrete structures is the choice of critical elements which exhibit a ductile mode of failure. To maintain this approach with SFRC structures, the effect of fibers must be incorporated not only on the resistance side, but also on the loading side if fibers are used in critical elements. The addition of fibers in critical elements, to increase their ductility, may be also detrimental to the structural behavior if they increase the strength of these elements and, thereby, the demand on the other, non-critical, elements. If a ductile failure mode can be accomplished in critical elements without fibers, and is viable economically, then SFRC should be concentrated in non-critical elements to increase their strength and reduce their detailing requirements without compromising their last resort ductility. For this reason, the use SFRC in columns is considered to be more efficient than in beams of earthquake resistant structures designed according to the capacity design philosophy.

CODE REQUIREMENTS AFFECTED BY THE USE OF SFRC IN COLUMNS

Transverse steel in reinforced concrete columns serves three purposes (Park et al. 1982): confining the compressed concrete, preventing lateral buckling of the longitudinal reinforcing steel, and acting as shear reinforcement. In the last two decades, the confinement of concrete columns by rectangular ties has been extensively studied by numerous research groups (Sheickh and Uzumeri 1980, Ozcebe and Saatcioglu 1987, Mander et al. 1988). These studies led to the existing code requirements for earthquake resistant columns. Among the numerous requirements for ductile and non-ductile members, those applying to columns in ductile frame systems can be potentially relaxed by the use of SFRC combined with reduced amount of conventional reinforcement. Two sets of requirements could particularly benefit from the addition of fibers to concrete: those related to the area and spacing of hoops, and those related to the splices of longitudinal reinforcement. Generally codes require for ductile frames that the maximum hoop spacing in a compression member cannot exceed: i) one quarter of the minimum member’s dimension (d/4), ii) six times the diameter of the smallest longitudinal bars, iii) 100 mm, or iv) 48 tie diameters. These requirements are more severe near beam-column joints or in the first floor where plastic hinges may form. The maximum spacing of hoops in columns can be increased due to the beneficial effect of SFRC. From the experimental results of Geiken and Ramsay (1989), a conservative maximum spacing for hoops in SFRC columns could be set at 1.5 to 1.6 times the current maximum spacing specified by the code. They suggest that a minimum fiber content, such as 1% per volume, be specified to take advantage of this increased spacing.
The minimum hoop spacing requirement directly related to confinement (d/4 and 100 mm) can be relaxed. Requirements associated with reinforcing bar buckling must be treated separately.

REVIEW OF PAST STUDIES ON SFRC IN COMPRESSION MEMBERS

Although the literature on SFRC is abundant, the amount of information published on the compression behavior of the material or structural elements is limited. Fanella and Naaman (1985) tested cylinders in compression to determine the stress-strain characteristics of fiber reinforced mortar. They concluded that adding fibers produces a noticeable increase of the peak stress and strain, and a significant increase in the ductility in the post-peak portion of the stress-strain curve. More recently Traina and Mansour (1991) studied the effect of SFRC on the peak stress and strain under uniaxial and biaxial stress conditions. They concluded, in the case of uniaxial compression, that SFRC exhibits an increase in the ultimate strength compared to plain concrete for fibers with a large aspect ratio. The maximum strength increase observed was 22% at fiber content of 1.5% per volume. However, in biaxial conditions, the increase in strength was up to 85% for the same volume of fibers. They concluded that SFRC is efficient to provide confinement to the concrete. They did not study the effect of SFRC on the post-peak stress-strain curve. Chern et al (1992) studied the behavior of SFRC under triaxial loading. They concluded that fibers change the failure mode at low confinement (under 10 MPa), which is typical of confinement level in columns.

Columns incorporating up to 2 percent fiber per volume were tested by Craig et al (1984). They concluded that the strength only is slightly increased by adding fibers. However their results show a significant increase of ductility, particularly in the presence of high shear forces combined with axial loads. Ganesan et al (1990) and Alsayed (1992) carried out a series of tests on axially loaded columns containing rectangular hoops and SFRC. Their results indicate an increase of the stress and strain at peak load and a better ductility when the columns incorporated fibers compared with columns which had hoops only.

SCOPE OF THE RESEARCH PROJECT

Hoop confinement in concrete columns under axial compression increases the peak load and corresponding strain. However, the main role of confinement is to improve the post-peak strength and ductility of columns. Therefore the scope of this research project is to study the beneficial contribution of fiber concrete to enhance the ductility of columns in a ductile frame system or in a nominally
ductile frame system. All requirements related to column design will be considered: minimum confinement for axial strength and ductility, shear and moment resistance under axial load, splice requirements. This paper deals only with the lateral confinement requirement under axial compression. The final objective of this ongoing research project is to determine the role played by the fibers in columns, expressed in code specification format and as a function of material properties of SFRC.

EXPERIMENTAL PROGRAM

Geometry

The experimental program consisted of fabricating and testing eighteen large scale columns with and without steel fibers. All columns had nominal dimensions of 250x250x1000 mm. However, in order to study separately the confinement provided by SFRC in the core and in the cover, nine specimens were fabricated without cover with a core dimension of 170x170 mm. The main reinforcement consisted of 4-15M reinforcing bars (200 mm² each). The transverse hoops were made of 10M reinforcing bars (100 mm²) spaced at d, d/2 and d/4, where d is the nominal column size, which corresponds to spacings of 250 mm, 125 mm and 62.5 mm respectively. The longitudinal reinforcement was welded to 12 mm thick steel plates at the two ends of the specimens. The specimen geometry is shown in Fig. 1, while Table 1 gives the details of each specimen.

Material properties

The nominal concrete strength was 35 MPa at 28 days. The longitudinal reinforcement came from two groups of bars having different yield stress and stress-strain curve, as indicated in Table 1: one with a well defined yield plateau and the other without plateau. Transverse reinforcement had a yield stress of 489 MPa. Hook-end steel fibers, 30 mm long and 0.5 mm in diameter (Dramix ZC-30/0.5), were used in fourteen columns, at volume content of 0.5 or 1.0 percent. Concrete was mixed in a 0.1 m³ mixer and fibers were added slowly into the concrete to avoid balling. Cylinders with and without fibers, and prisms were fabricated for each mix. The specimens were cast horizontally and concrete was vibrated with a needle. The specimens were tested between two and four weeks after they were fabricated. Material properties were measured just before testing. Measured concrete properties are given in Table 1 while the concrete mix design is indicated in Table 2.
Specimen axial deformations were measured on a 775 mm gauge length using four linear variable differential transformers (LVDTs) located on each face of the specimen. Aluminum collars at top and bottom were attached to the specimen by means of four screws to support the LVDTs. The columns were loaded on a rigid MTS displacement controlled testing machine having a capacity of 4500 kN in the Hydro-Quebec Concrete Testing Laboratory. The end conditions are considered rotationally fixed. The displacement of the machine head, controlled at a rate of 4000 Hz, was set at 0.05 mm/sec, which corresponds to a strain rate of 50 microstrain per second for a 1000 mm long specimen.

The applied load, the testing machine head displacement and the LVDTs readings were recorded at a rate of 10 Hz. Prior to carrying out the test, the specimen alignment in the testing machine was achieved by trials and errors. A load corresponding to 25% of the uniaxial strength of concrete was applied and removed until all four LVDTs measured axial displacements within 10 percent of their average value. During the test, the LVDTs were kept on the specimens during the post-peak unloading response until the load had dropped at least 50 percent below the maximum load sustained by the specimen. At that point the test was interrupted and the LVDTs removed while maintaining the axial displacement constant. This operation was usually accompanied by a reduction of the applied axial load. In resuming the test, the load carried out by the column before the interruption could be reached again. From this point, only the testing machine head displacement were recorded and used for computing the axial displacement.

**TEST RESULTS**

Fig. 2 shows the schematic appearance of the eighteen specimens after testing. Fig. 3 presents the total applied axial load versus displacement of all eighteen specimens. The displacement readings after removing the LVDTs are those of the testing machine modified according to a calibration on the post-peak measurements of the LVDTs while still in place.

**Global response**

As shown in Fig. 3, the specimens exhibited a more ductile post-peak response for smaller hoop spacing and higher fiber content. Some specimens (C6 and C9) experienced a slight strain hardening. Only one specimen without fibers (C3)
had a ductile post-peak behavior as expected since the hoop spacing (62.5 mm) meets the requirements for a fully ductile member. On the other hand, fibers added to the ductility of all specimens. Even at a fiber content of 0.5 percent per volume, the specimens exhibited a good ductility. This ductility improved even more at a fiber content of 1.0 percent per volume. Peak load and deformation were increased by the presence of fibers. However, problems encountered in mixing the 0.5 percent mix led to almost no strength increase and a large scatter in the cylinder strength. Specimens at 1.0 percent fiber content behave more as expected and the cylinder strengths were much more consistent.

The specimens with cover (C10 to C15) all contained fibers and showed a good ductility. Due to the premature failure of the cover, the member strength decreased more rapidly. Although no tests were carried out with a cover and no fibers, it is expected that the cover would have spalled rapidly and that only the core would have remained effective. In that perspective, fibers contributed in keeping the cover efficient at larger deformations.

The three unreinforced specimens (C16 to C18) had 0, 0.5 and 1.0 percent fiber volume respectively. They show the effect of the fibers in modifying the failure mode. Specimen C16 failed in a very fragile manner with the formation of a longitudinal crack. The failure of specimens C17 and C18 was due to the formation of a diagonal crack which was more horizontal at the higher fiber content. The fibers added considerable amount of ductility in these cases.

Effect of fibers on ductility

In Fig. 4a to 4c, the effect of fibers for the specimens without cover is presented as a function of the average stress sustained by the core. The axial load carried by concrete was first obtained by removing from the total axial load the portion taken by the longitudinal reinforcement. Bar buckling was considered by a mechanical model accounting for the actual shape of the stress-strain curve and the formation of a mechanism with plastic hinges between two adjacent hoops. The average stress on the column was computed assuming a core dimension equal to the outside perimeter of the hoops whereas the full cross section was used in the presence of cover in Fig. 4d to 4f. The concrete strength without fiber, $f_c'$, was used as reference value to compare the columns.

Except for column C5 which did not behave as expected, one can see that adding fibers contributes significantly to the ductility. Fibers improved considerably the ductility, even for the column designed according to the full seismic code requirements. However, at a hoop spacing of 250 mm, the effect, although important, is less pronounced. Results for the columns with cover are similar to those without cover. The cover contribution was computed by subtracting the core load-displacement response obtained for the corresponding
cases without cover to the values including the cover. It can be seen that the cover carries significant load. However, it was clear during the tests that the cover did not participate very much in the confinement of the core since the reinforcing bars buckled, and thus pushed against large pieces of SFRC that were still carrying some load without contributing to the confinement of the columns.

In order to compare the columns, the area under the stress-displacement curve of each column, which is the energy per unit of surface, has been computed up to a displacement corresponding to a core stress of $0.5f_c$ in the descending branch, and compared to the elastic energy per unit area in the ascending branch, $0.5f_c^2L_0/E_0$, where $L_0$ is the gauge length of the axial displacement measuring device and $E_0$ is the initial Young's modulus. The ratios of the total energy to the elastic energy are given in Table 3. Although these values cannot be related directly to the column performance during an earthquake, they provide a good indication of the relative efficiency of each column configuration. One can see, based on these measures, that 1.0% fiber volume with hoop spacing of 125 mm ($d/2$) dissipated an amount of energy comparable with a column without fibers with hoops spaced at 62.5 mm. An intermediate case with hoop spacing of 96 mm (6 longitudinal bar diameter) would have probably given the same amount of energy. Another interesting observation is that adding fibers to columns with closely spaced hoops improves considerably their energy dissipation. This suggests that if additional ductility is required above code requirements, it could be better to use SFRC instead of adding hoops. This may also indicate that spiral spacing requirement in circular columns could be relaxed and still get comparable performance if SFRC is used. However this issue must be addressed separately.

However, one should not forget, for earthquake resisting structures, that columns are subjected to cyclic loading with load reversal. Past experimental investigations at Ecole Polytechnique (Filiatrault et al 1994 and 1995) indicated that, as cracks open, fibers pull out and then buckle as cracks close, which deteriorates rapidly their efficiency. Therefore the maximum crack opening and the minimum amount of conventional reinforcement still remain to be determined in order to obtained the expected behavior.

**STRESS-STRAIN CURVE**

**Axial strain computation**

Computing the axial compressive strain in the post-peak softening branch requires to determine the length of the zone that undergoes strain softening. Figure 5 illustrates the observed column behavior and the associated response.
According to several authors (Bazant 1989; Markeset and Hillerborg 1995) it is clear that not all the concrete softens: some undamaged regions are unloading, whereas the remaining part of the column softens. Also, compression failure are often accompanied with a diagonal failure plane that takes place during the softening of the columns. It was observed that the higher the confinement the higher was the load associated with the onset of the shear failure (Cusson et al 1996). However the main assumption that need to be made is the length of the damaged region, \( L_d \). Selecting a value too short leads to a post-peak curve too soft, whereas a value too large underestimates the softening characteristics of the material. For pure axial loading, it has been observed in our experiments that the better the confinement, the longer the damaged zone. However, in the case of cyclic loading involving bending moments in double curvature, the damaged region would be restricted to smaller area of better defined dimensions. In order to define a moment curvature relationships to analyze actual columns bent in double curvature, the softening response is needed. Based on our observations, we made the following assumptions for the specimens without cover for which the failure mechanism was more easily observable: 1) the length of the damaged zone is determined by visual observation and is equal to a multiple of the hoop spacing; 2) longitudinal reinforcements were consider to buckle only in the damaged zone, at a stress determined according to the hoop spacing and the actual bar stress-strain curve.

For the first nine specimens without cover, the length of the damaged zone was estimated from visual observation (Fig. 2) and used to compute the average damage strain (\( \varepsilon_d \)) as indicated in Fig. 5. It was also possible to determine the strain (\( \varepsilon_{d_{\text{max}}} \)) and stress (\( \sigma_d \)) at the onset of shear failure mechanism since their was always a point in the \( \sigma-\varepsilon_d \) curve at which the load started to drop as shown in the third component illustrated in Fig. 5. The quantities should not be perceived as definite values since their determination is slightly subjective. However they can be considered as good indication of the actual behavior. In Table 4, values of the softening branch modulus (\( E_{d_{\text{max}}} \)), the maximum damage strain (\( \varepsilon_{d_{\text{max}}} \)) and the corresponding stress (\( \sigma_{d_{\text{max}}} \)) are indicated. As mentioned before, column C5, with 0.5 percent volume of fibers, did not behave as expected, probably due to human error in mixing the concrete.

It can be seen from these results that the slope of the softening branch is slightly affected by the addition of fibers whereas stirrup spacing is the main governing parameter. Similarly, the strain at which the shear failure occurs (\( \varepsilon_{d_{\text{max}}} \)) is not affected consistently by the presence of fibers. However this value was determined by visual observations and more accurate measurements on the specimens should be used to determine the actual value. The maximum strain in an axially loaded column may differ from the corresponding value observed if bending moments are applied simultaneously. A larger value could be expected in this case since the onset of the shear failure plane would be delayed by the strain gradient. As an indication, Park et al (1982) do not put any limitation on the maximum strain. Finally, the onset of the shear failure mechanism show the
same trend as for the cumulative energy: the closer the hoop spacing and the higher the fiber content, the higher the maximum stress at which shear failure begins.

**Measured stress-strain curve**

In combining the first two stress-strain curves of Fig. 5, one obtains a general stress-strain curve illustrated in Fig. 6 that could be used for analysis. The parameters defining the stress-strain curve based on the tests of this study are given in Table 5. These values could be used to define a parametric expression for the stress-strain curve. However, to be a general expression, these relationships must be linked to actual material properties. In this respect, it is difficult to make a correlation between toughness indices measured in four-points bending tests and the behavior of columns in compression. For this reason, it was judged more appropriate at this moment not to try to derive an equation without going into the details of the failure mechanism of SFRC in compression.

**CONCLUSION AND DISCUSSION**

The first part of this ongoing experimental program allowed to show the beneficial effects of steel fibers for earthquake resistant structures. The main conclusions are:

1) fibers increase significantly the ductility of columns in compression, particularly when combined with a reasonable amount of conventional reinforcement;
2) adding fibers could allow to increase the maximum hoop spacing, especially in moderate seismic zones;
3) fibers in the cover do not provide additional confinement but contribute to postpone the cover failure;
4) although no spiral columns were tested, it is believed that fibers could possibly be added to spiral columns to reduce the spiral spacing for axially loaded columns and still keep the same ductility; this would be particularly beneficial for large columns;
5) hoop spacing could have been increased to probably 96 mm (6 15M bar diameter) with the presence of fibers of at least 0.5 percent volume. However more tests are needed to propose a definite value.

This first test series showed the potential of using SFRC in columns in order to improve their ductility. More research is still needed particularly to determine if the slope of the softening branch is independent of the specimen length (or the length of the damaged region). Also, tests under cyclic loading must be carried
out. The two percentage of fibers used are practical. Below 0.5 percent of fiber volume the beneficial effects are not significant. Placing SFRC at volume content beyond 1.0 percent is hardly feasible in field conditions but higher percentages would be possible in precast situations. Therefore, tests could also be done at higher percentages to determine the optimal fiber content in term of ductility and workability.

Finally, through the observation of failure modes in this test series, one cannot affirm that fibers confine the concrete although it seems to do so. Fibers rather limit the progression of cracks in the paste between aggregates and thereby, maintain an integrity of the material at larger strain. In that respect, relating the fracture energy in tension of SFRC to the compressive strength of concrete, as do Markeset and Hillerborg (1995) for unreinforced concrete, would be the ideal avenue. Such a fundamental approach could bring more light into the failure mechanism of concrete in compression.

REFERENCES


ACKNOWLEDGMENTS

The authors gratefully acknowledge the financial support of NSERC. The authors also acknowledge the support of Hydro-Quebec in providing the accessibility to their Concrete Laboratory. The steel fibers were provided by Bekaert Steel Wire Corp.

The authors would like to thank the team of technicians and undergraduates students of the Structures Laboratory at Ecole Polytechnique for their valuable assistance in carrying out the experimental part of this research program.
### TABLE 1—DETAILS OF COLUMNS REINFORCEMENT.

<table>
<thead>
<tr>
<th>Specimen</th>
<th>Hoop spacing (mm)</th>
<th>Cover (mm)</th>
<th>Volume of fibers (%)</th>
<th>( f'_c ) (MPa)</th>
<th>( f'_f ) (MPa)</th>
<th>Long. reinf.</th>
</tr>
</thead>
<tbody>
<tr>
<td>C1</td>
<td>250</td>
<td>0</td>
<td>0</td>
<td>40.5</td>
<td>NA</td>
<td>4-M15&lt;sup&gt;1&lt;/sup&gt;</td>
</tr>
<tr>
<td>C2</td>
<td>125</td>
<td>0</td>
<td>0.5</td>
<td>43.3</td>
<td>45.3</td>
<td>4-M15&lt;sup&gt;1&lt;/sup&gt;</td>
</tr>
<tr>
<td>C3</td>
<td>62.5</td>
<td>0</td>
<td>1.0</td>
<td>35.5</td>
<td>47.4</td>
<td>4-M15&lt;sup&gt;1&lt;/sup&gt;</td>
</tr>
<tr>
<td>C4</td>
<td>250</td>
<td>0</td>
<td>0.5</td>
<td>43.3</td>
<td>45.3</td>
<td>4-M15&lt;sup&gt;1&lt;/sup&gt;</td>
</tr>
<tr>
<td>C5</td>
<td>125</td>
<td>40</td>
<td>0.5</td>
<td>43.3</td>
<td>45.3</td>
<td>4-M15&lt;sup&gt;1&lt;/sup&gt;</td>
</tr>
<tr>
<td>C6</td>
<td>62.5</td>
<td>40</td>
<td>1.0</td>
<td>35.5</td>
<td>47.4</td>
<td>4-M15&lt;sup&gt;1&lt;/sup&gt;</td>
</tr>
<tr>
<td>C7</td>
<td>None</td>
<td>Unreinf.</td>
<td>0</td>
<td>41.2</td>
<td>NA</td>
<td>None</td>
</tr>
<tr>
<td>C8</td>
<td>None</td>
<td>250x250</td>
<td>0.5</td>
<td>41.2</td>
<td>45.6</td>
<td>None</td>
</tr>
<tr>
<td>C9</td>
<td>None</td>
<td>250x250</td>
<td>1.0</td>
<td>41.2</td>
<td>45.6</td>
<td>None</td>
</tr>
<tr>
<td>C10</td>
<td>None</td>
<td>Unreinf.</td>
<td>0</td>
<td>41.2</td>
<td>NA</td>
<td>None</td>
</tr>
<tr>
<td>C11</td>
<td>None</td>
<td>250x250</td>
<td>0.5</td>
<td>41.2</td>
<td>45.6</td>
<td>None</td>
</tr>
<tr>
<td>C12</td>
<td>None</td>
<td>250x250</td>
<td>1.0</td>
<td>41.2</td>
<td>45.6</td>
<td>None</td>
</tr>
</tbody>
</table>

1: \( f_y = 525 \) MPa  
2: \( f_y = 475 \) MPa

### TABLE 2—CONCRETE MIX DESIGN.

<table>
<thead>
<tr>
<th>10 mm aggregate (kg/m&lt;sup&gt;3&lt;/sup&gt;)</th>
<th>Sand (kg/m&lt;sup&gt;3&lt;/sup&gt;)</th>
<th>Cement (kg/m&lt;sup&gt;3&lt;/sup&gt;)</th>
<th>Water (kg/m&lt;sup&gt;3&lt;/sup&gt;)</th>
<th>W/C</th>
<th>Superplasticizer (ml/m&lt;sup&gt;3&lt;/sup&gt;)</th>
</tr>
</thead>
<tbody>
<tr>
<td>903</td>
<td>886</td>
<td>365</td>
<td>167</td>
<td>0.46</td>
<td>4200</td>
</tr>
</tbody>
</table>
TABLE 3—RATIO OF THE TOTAL ENERGY TO THE ELASTIC ENERGY FOR THE COLUMNS WITHOUT COVER.

<table>
<thead>
<tr>
<th>Hoop spacing (mm)</th>
<th>Volume of fibers (percent)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0</td>
</tr>
<tr>
<td>750</td>
<td>1.00</td>
</tr>
<tr>
<td>250</td>
<td>6.08</td>
</tr>
<tr>
<td>125</td>
<td>12.04</td>
</tr>
<tr>
<td>62.5</td>
<td>32.93</td>
</tr>
</tbody>
</table>

1: Ratio for an axial displacement of 35 mm at which test was stopped.

TABLE 4—CHARACTERISTICS OF THE SOFTENING RESPONSE.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Hoop spacing (mm)</th>
<th>Volume of fibers (percent)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0</td>
<td>0.5</td>
</tr>
<tr>
<td>$E_a$ (MPa)</td>
<td>250</td>
<td>-2860</td>
</tr>
<tr>
<td></td>
<td>125</td>
<td>-1710</td>
</tr>
<tr>
<td></td>
<td>62.5</td>
<td>-128</td>
</tr>
<tr>
<td>$\varepsilon_{\text{max}} \times 10^6$</td>
<td>250</td>
<td>13 100</td>
</tr>
<tr>
<td></td>
<td>125</td>
<td>5 600</td>
</tr>
<tr>
<td></td>
<td>62.5</td>
<td>11 400</td>
</tr>
<tr>
<td>$\frac{\sigma_{\text{cf}}}{f_c}$</td>
<td>250</td>
<td>0.17</td>
</tr>
<tr>
<td></td>
<td>125</td>
<td>0.95</td>
</tr>
<tr>
<td></td>
<td>62.5</td>
<td>1.20</td>
</tr>
</tbody>
</table>

TABLE 5—OBSERVED STRESS-STRAIN CURVES FOR CONFINED SFRC.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Hoop spacing (mm)</th>
<th>Volume of fibers (percent)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0</td>
<td>0.5</td>
</tr>
<tr>
<td>$\varepsilon_{\text{cf}}$ (MPa)</td>
<td>250</td>
<td>33 820</td>
</tr>
<tr>
<td></td>
<td>125</td>
<td>34 020</td>
</tr>
<tr>
<td></td>
<td>62.5</td>
<td>34 180</td>
</tr>
<tr>
<td>$f'_{\text{cf}}$ (MPa)</td>
<td>250</td>
<td>44.4</td>
</tr>
<tr>
<td></td>
<td>125</td>
<td>47.8</td>
</tr>
<tr>
<td></td>
<td>62.5</td>
<td>50.1</td>
</tr>
<tr>
<td>$\varepsilon_{\text{cf}} \times 10^6$</td>
<td>250</td>
<td>2 000</td>
</tr>
<tr>
<td></td>
<td>125</td>
<td>2 700</td>
</tr>
<tr>
<td></td>
<td>62.5</td>
<td>3 600</td>
</tr>
<tr>
<td>$\varepsilon_{\text{max}} \times 10^6$</td>
<td>250</td>
<td>14 000</td>
</tr>
<tr>
<td></td>
<td>125</td>
<td>8 000</td>
</tr>
<tr>
<td></td>
<td>62.5</td>
<td>15 000</td>
</tr>
<tr>
<td>$\sigma_{\text{max}}$ (MPa)</td>
<td>250</td>
<td>44.2</td>
</tr>
<tr>
<td></td>
<td>125</td>
<td>47.8</td>
</tr>
<tr>
<td></td>
<td>62.5</td>
<td>50.0</td>
</tr>
</tbody>
</table>

1: This specimen was probably not fabricated or loaded correctly.
Fig. 1—Specimen geometry.

Fig. 2—Failure mechanisms.
Fig. 3—Columns load-displacement response.
Fig. 4—Concrete stresses in columns without and with cover.
Fig. 5—Three components for the axial deformations.

Fig. 6—Parametric description of the stress-strain curve.
Ultimate Punching Shear Strength Analysis of Slab-Column Connections With Steel Fibers

by D. Theodorakopoulos and R. Swamy

Synopsis: A simple analytical model is presented to predict the ultimate punching shear strength of slab-column connections made with steel fiber concrete. The model is based on the physical behavior of the connection under load, and is therefore applicable to both lightweight and normal weight concrete as well as to concrete without fibers. The model assumes that punching is a form of shearing without concrete crushing, and occurs when the tensile splitting strength of the concrete is exceeded. The theory is applied to predict the ultimate punching shear strength of forty seven slab-column connections tested by the authors and other researchers over a period of several years and designed to fail in shear, involving a wide range of fiber variables, concrete strength, tension steel ratio and loaded area. The results show very good agreement between the predicted and experimental values. The uniqueness of the model is that it incorporates many physical characteristics of the slabs and their failure behavior.

Keywords: analytical model; fiber concrete; lightweight concrete; plain concrete; punching shear strength; slab-column connections; structural design; tests
Dr Theodorakopoulos is an Associate Professor in the Department of Civil Engineering at the University of Patras, Greece. He took his PhD from the University of Sheffield, and his specialist research interests are in structural analysis, modelling and design. He is actively involved in both research and design, as a consultant.

ACI Fellow Professor R. Narayan Swamy is at the Structural Integrity Research Institute and the Centre for Cement and Concrete, University of Sheffield, England. His research interests are in construction materials and concrete structures. He is the Founder/Editor of the Journal Cement and Concrete Composites. He has received many national and international awards for his research including the George Stephenson Gold Medal of the Institution of Civil Engineers, UK, and the Robert E. Philleo Award of the Concrete Research Council, USA.

INTRODUCTION

The ultimate strength of a reinforced concrete slab under concentrated load is often determined by the shear failure load, rather than the flexural load calculated by theories such as the yield line theory. Many variables have a marked effect on the punching shear strength of slabs. These include the concrete strength, the ratio of the column size to slab effective depth, the ratio of shear strength to flexural strength, the column shape and the lateral constraints. At present, the mechanism of shear failure of a reinforced concrete slab still remains unsolved. Although there have been extensive experimental data, and many empirical equations have been developed, there is still considerable argument on several aspects of punching shear failure such as the stress distribution around the column and the role of the dowel action effect.

When steel fibers are incorporated in the concrete, there are significant changes to the behavior of the slabs. Extensive tests on one-to-one scale models (1800 x 1800 x 125 mm) of slab-column connections show that fibers reduce all deformations at all stages of loading, increase ultimate punching shear loads, and produce ductile shear failures [1-3]. The presence of fibers also enhances substantially the ductility and energy absorption capabilities of the structural elements. These characteristics have also been supported by tests of smaller scale models [4-5]. The role of fibers in structural members carrying very high shear stresses which critically influence the resulting failure modes is now well understood. The fibers bridge the cracks, prevent their excessive opening and significantly raise the threshold of ultimate shear strength [6-8].
There are, however, no simple theoretical models which can satisfactorily predict the ultimate punching shear strength of reinforced concrete slab-column connections, and which can also be used by designers. The purpose of this paper is to present a simple analytical model for the punching shear strength of slab-column connections. The concepts are developed in such a way that the model can be applied to both lightweight and normal weight concrete connections, and then be extended to slabs made of concrete incorporating steel fibers. In this way, a single approach for conventional slab-column connections is modified to include connections made of fiber concrete. It is shown that the analysis presented here advances a good theoretical model to predict the punching shear strength of reinforced concrete slab-column connections made with lightweight and normal weight concrete whether or not they incorporate steel fibers.

EXPERIMENTAL BACKGROUND TO THEORY

The analytical model is applied to both lightweight and normal weight concrete slab-column connections tested in the laboratories of the second author [1,3] as well as to those reported in the literature [5,9,10]. The slab-column connections were all designed to fail in punching shear. Altogether, the punching shear strength of forty seven slabs is predicted and compared to experimental data - 21 of the slabs were made of lightweight concrete with sixteen incorporating steel fibers, seventeen normal weight concrete connections, fourteen of which incorporated steel fibers and nine micro-concrete with eight incorporating steel fibers.

The twenty six slab-column connections tested by the authors were all of the same size, and represented a one-to-one scale model of a prototype flat-plate structure with a column spacing of 4.0m center to center in both directions. The overall slab depth was 125mm, and the test specimens were 1800mm x 1800mm in size, simulating the prototype boundary conditions, namely, the area located within the negative bending moment region around an interior column and inside the line of contra-flexure. The average effective depth of the slabs was 100mm, and they were tested simply supported along all four edges with the corners free to rise. The slabs were cast monolithically with a square column stub 250mm high through which they were loaded. Fig. 1 shows the typical slab geometry and test arrangement, and details of reinforcement of a typical slab connection.

The main variables investigated in the tests included fiber volume, reduction in both tension and compression steel, type of steel fiber at constant volume, location of the fiber concrete, variation in loaded area and concrete cube strength. Table 1 and 2 give full details of the slabs including the full (i.e. without reduction) amount of tension and compression steel, the fiber volume
and type of fiber. Table 1 refers to slab connections made of lightweight concrete, and Table 2, to slabs made with normal weight concrete.

One of the major factors varied in these tests was the location of the fiber concrete, since regions of high critical shear stress occur only around the column head. This is highlighted in Table 1 and 2. In particular, in the lightweight concrete slabs, the steel fibers were distributed over the whole depth, but only within a square of 1100 x 1100 mm in the central area of the slab, except for the slab FS-20 where they were distributed throughout the entire slab (Table 1). In the normal weight concrete slabs, the location of the steel fiber concrete was varied as shown in Table 2.

The structural lightweight aggregate concrete mix used in the connections of Table 1 consisted of sintered fly ash aggregate and sand, with 30% of the cement replaced, mass for mass, by a quality controlled low calcium Type F fly ash. The concrete in the slabs of Series 1 to 4 of Table 1 was designed to give 28 day cube strength of about 45 MPa; the concrete in the slabs of Series 5 was designed to give a 28 day cube strengths of about 20 MPa, 35 MPa and 60 MPa. The normal weight concrete mix used in the connections of Table 2 was also designed to give 28 day cube strength of about 45 MPa; these mixed also contained 30% replacement of cement with the same Type F fly ash. All the concrete mixes were proportioned to give very workable, cohesive mixes without fiber balling, based on previous studies on lightweight and normal weight concretes containing fly ash and steel fibers [11-15]. The cube compressive strength of the concrete in the various slabs is shown in Table 1 and 2. The steel reinforcement used in the slabs was cold-worked high tensile deformed bars with a minimum specified characteristic yield strength of 460 MPa. The amount of compression and tension steel used in each slab is also shown in Tables 1 and 2.

**FAILURE CHARACTERISTICS OF SLAB CONNECTIONS**

All the slabs were tested to failure, beyond their maximum loads, so that the failure behavior in the strain softening region could be established. All the slabs were also extensively instrumented to provide quantitative data on their failure behavior such as concrete and steel strains, deflections, rotations and the mode and nature of fracture. The maximum loads sustained by the slabs are shown in column 7 of Table 1 for the lightweight concrete slabs and Table 2 for the normal weight concrete slabs. All punching failure occurred along a surface formed by inclined cracks in the immediate vicinity of the column resulting in truncated cone-shaped surfaces, starting from the column faces at the compression surface of the slab and extending outwards as shown in Fig. 2a. In many cases, the failure surfaces at the tension face of the slabs were irregular and incomplete with fibers bridging the cracks, and the failure perimeter not following any consistent shape or pattern as shown in Fig. 2b.
In general, in all the lightweight and normal weight concrete slabs incorporating fibers, the punching perimeter was larger, compared to those made with plain concrete, resulting in a decrease of the angle of the failure surface to the horizontal. In the lightweight concrete slabs, the failure surface perimeters at the tension face level in the plain concrete (i.e. without fibers) connections were located at distances of 1.90h to 2.48h (h = total depth of slab) from the column face, implying angles of failure surfaces from $22^\circ$ to $28^\circ$. In the fiber concrete slab, the angle of the failure surface was decreased by a maximum of $3^\circ$. In the normal weight concrete slabs, the angle of the failure surface was $24^\circ$ for the plain concrete slabs, and $19^\circ$ to $23^\circ$ for the fiber concrete slabs. All the observed data concerning the location of the punching shear perimeter and the computed angle of the failure surface are shown in Tables 1 and 2 for the lightweight and normal weight concrete connections respectively.

PROPOSED ULTIMATE STRENGTH ANALYSIS

Since the emphasis of this paper is on ultimate strength analysis, the only experimental data reported here are the failure loads of the slabs and the nature of their failure modes and failure surfaces. These data are provided in Tables 1 and 2. The tests showed visible evidence of the steel fibers bridging and controlling cracking, and thereby enhance serviceability behavior and ultimate loads. As shown in Tables 1 and 2, the fiber concrete slabs carried 30% to 40% more loads at failure, but above all the fibers transformed brittle and sudden shear failures into ductile punching shear failures. This is a very unusual and unknown aspect of shear failure, and emphasizes the role of steel fibers when they are incorporated in concrete. These and other aspects of the behavior of fiber concrete slabs are discussed elsewhere [1,3].

SLAB-COLUMN CONNECTION FAILURE MECHANISM

In the slabs tested in this study, whether the concrete incorporated fibers or not, failure occurred in the compression zone by splitting along the line AB shown in Fig. 3a, and there was no sign of concrete crushing. Other investigators have reported similar observations for both plain and fiber reinforced concretes [1, 16]. Punching is thus a form of shearing without shear-compression failure occurring.

The main flexural cracking in a slab-column connection is seen along radial lines at the tension face. Inclined cracking develops in the immediate vicinity of the loaded area, since a possible area resisting shear corresponding to a typical perimeter (area = length of perimeter x depth of the slab) increases with the distance from the column. It is likely that inclined cracking develops
first at the corners of the column where high stress concentration occurs, and it then propagates laterally in the plane of the slab with increasing load. After the opening of inclined cracking in the slab, its propagation is prevented by the compression zone above the top of the crack, and by the dowel action of the tension reinforcement acting in a perimeter where its length is greater than that at which the crack is initiated. Since the inclined cracking in a slab always forms close to the loaded area, as mentioned previously, the compression zone above the inclined crack is effectively strengthened by the triaxial state of stress at that section, which can lead to higher shear stresses in slabs than those obtainable in beams. Thus in slabs the ultimate strength can be considered as the usable strength compared to the inclined cracking strength in beams, especially with long shear spans, where the diagonal tension cracks form at some distance away from the position of the applied load.

Once inclined shear cracking has developed, the load is resisted by the vertical components of the force carried in the concrete compression zone above the crack $V_C$, the aggregate interlock force $V_a$, and the dowel action $V_d$ of the flexural reinforcement, Fig. 3b. Thus the total shear resistance of a slab connection without shear reinforcement is given by

$$V_u = V_C + V_a + V_d$$

In real slab-column connections these components do not remain isolated quantities, but co-exist together and, therefore, their contributions do not reach their maximum values at the same stage of loading [17].

The portion of the load resisted by the compression zone is dependent upon the area of slab at a perimeter away from the column, and the shear resistance of concrete. The aggregate interlock effect, which activates only after the appearance of inclined cracking, depends on concrete properties, crack width and the relative displacement between the two faces of the crack due to rotation about the head of crack [17,18]. In slab connections, the movement across the crack is largely vertical, and the residual interlock forces can therefore be considered to be negligible. The dowel action effect depends mainly on the tensile resistance of the concrete along the splitting plane and the bending resistance of the steel. Since $V_C$ is the critical component in equation (1), the problem of calculating the ultimate punching shear strength of a slab connection becomes mainly a problem of calculating the ultimate contribution of the compression zone, $V_C$ above the inclined cracking.

The effect of the inclusion of fibers in concrete is to check the upward movement of the neutral axis. Fibers also increase the strength of concrete, such that the net effect is an increase in the value of the component $V_C$. The net influence of fiber reinforcement on aggregate interlock force is rather limited or insignificant [6]. Fibers in reinforced concrete increase the tensile strength of the composite, and improve the stiffness and deformation behavior.
of the member including the concrete cover, which assists the tension steel in resisting the bending due to dowel action [1,6].

**Slab-Column Connections without Fiber Concrete**

Neglecting aggregate interlock contribution, equation (1) can be written as:

\[ V_u = V_c + V_d \]  

The resistance offered by the concrete compression zone, \( V_c \) (vertical component) is equal to the area of concrete confined between the plane of the slab-column junction and the neutral axis plane, the depth of which depends on the strength properties and amount of the reinforcement, multiplied by a critical shear stress, \( v_c \). Referring to Fig. 4a, this area of concrete, \( ARC \), is given by

\[ ARC = (A_1B_1C_1D_1) \cdot (AE') \]  

or,

\[ ARC = 4(0.5 \times \cot \theta + r + 0.5 \times \cot \theta) \cdot \frac{X}{\sin \theta} \]

or, in the case of a circular column,

\[ ARC = \pi(0.5 \times \cot \theta + D + 0.5 \times \cot \theta) \cdot \frac{X}{\sin \theta} \]

where \( A_1B_1C_1D_1 \) represents the average perimeter, at mid-section of the N.A. depth, i.e. midway between the two perimeters \( ABCD = 4r \) at the column junction and \( A'B'C'D' = 4(\cot \theta + r + \cot \theta) \) at the neutral axis plane, and, where

- \( AB = r \)
- \( \theta = \) inclination of failure surface
- \( X = \) neutral axis depth

Thus, the vertical component of the contribution to shear resistance from the compression zone, \( V_c \), (Fig. 4b) is given by:

\[ V_c = V_{cc} \cdot \cos \theta \]

i.e.,

\[ V_c = (v_c \cdot ARC) \cdot \cos \theta \]

and from equations (3.1) and (3.2), the \( V_c \) contribution becomes

\[ V_c = v_c \cdot (A_1B_1C_1D_1) \cdot (X \cdot \cot \theta) \]

The dowel action effect depends upon a number of parameters [19], but basically it can be said that it is a combination of two effects:
1. The tensile resistance of the concrete along the splitting plane, and
2. the bending resistance of the bars, that is, the tensile strength and amount of the reinforcement.

The resistance offered by dowel action in two way slabs is reported to be about 25-30% of the ultimate resistance, based primarily on experimental evidence \[1, 20, 21\] since a purely theoretical evaluation of the magnitude of the dowel effect is difficult with any degree of accuracy.

Only one effort has been made for an equation to be derived \[22\], where the dowel action effect for the case of a circular column is given by:

\[
V_d = \sigma_t \cdot \pi \left( d \cot \theta + D + d \cot \theta + L_0 \right) L_0
\]

where
- \(\sigma_t\) = concrete splitting tensile strength
- \(D\) = diameter of circular column
- \(d\) = effective depth of slab
- \(\theta = 30^\circ\)
- \(L_0\) = range of dowel action

and \(\pi(d \cot \theta + D + d \cot \theta)\) represents the perimeter at the reinforcement level.

Based on equation (3.4), equation (2) becomes

\[
V_u = v_c \cdot \left( A_1 B_1 C_1 D_1 \right) \cdot X \cdot \cot \theta + V_d \tag{4.1}
\]

Equations (3.4) and (3.5) can be seen to be similar in their structure and nature, and imply that both the concrete compression zone and dowel action contributions depend on the same or similar parameters. It is therefore rational, for the sake of simplicity and practicability, to consider these two contributions as a whole and to combine the two actions together, where the resulting equation should be of the same nature as equations (3.4) and (3.5), i.e.

\[
V_u = \text{stress} \times \text{perimeter} \times \text{depth}
\]

This combined action can thus be represented by considering a larger "fictitious" perimeter for use in equation (3.4). It is logical to consider this large perimeter at the level of the reinforcement (i.e. at the slab effective depth) where the dowel action is initiated. This concept is supported by the perimeters at distances 1.5h and 1.5d from the column face incorporated in BS CP110 and BS 8110 respectively which take into account the dowel action contribution of the slab's flexural reinforcement (23).
In the light of these considerations and using the perimeter at 1.5d from the column face, equation (3.4) becomes:

\[ V_u = v_e \cdot 4(1.5d + r + 1.5d) \cdot X \cot \theta \]

or

\[ V_u = v_e \cdot (4r + 12d) \cdot X \cot \theta \] (4.2)

To apply equation (4.2) to determine the ultimate punching shear strength of a slab-column connection, it is necessary to know the values of \( v_e \), \( \theta \) and \( X \).

**Evaluation of \( v_e \) and \( \theta \)**

In slab-column connections reference is often made to the strength of concrete in shear. However, concrete being a granular material, the unqualified term "concrete shear strength" is meaningless. It can be anything between the tensile strength (unconstrained concrete subjected to pure shear loading) and several times the compressive strength (concrete subjected to pure shearing deformation under plane strain conditions). Since punching is a form of splitting or shearing of the compression zone, and further, since in simply supported slabs there is no membrane action present prior to formation and completion of the full yield line mechanism [24-27], it is the unconstrained shear loading which is critical to punching failure. Thus the so-called shear strength is related to the tensile strength \( \sigma_t \) of concrete, but like the latter it is usually expressed as a function of the compressive strength [21]. From a wide range of test results for lightweight and normal weight concretes obtained at Sheffield, and by the other investigators [16,28-31], the following relationship between tensile splitting and compressive strength \( f_{cu} \) is assumed.

\[ \text{sand-lightweight concrete: } v_e = \sigma_t = 0.44 \sqrt{f_{cu}} \]  
\[ \text{normal weight concrete: } v_e = \sigma_t = 0.50 \sqrt{f_{cu}} \] (5)

where \( f_{cu} \) is the compressive strength. Fig. 5 shows the validity of this relationship for normal weight concrete, compared with the values of limiting shear stress of the compression zone proposed by Regan [32] and Nielson [33].

The angle of the failure surface \( \theta \) in the plain slab-column connections of both lightweight and normal weight concrete [1-3] varied from 22° to 28°, these values being the average ones along the inclined surface. In practice, the inclined failure surface occurs with a steeper slope near the column-slab junction as shown in Fig. 2a, which is the critical region in question, with a tendency for flattening near the tension face of the slab. Values ranging from 24° to 30° have also been reported by various investigators [16,20,22,34], although failures at steeper angles can also occur [21]. In this analysis a value of \( \theta \) equal to 30° is assumed.
Evaluation of neutral axis depth, $X$

The depth $X$ of the compression zone at failure has been studied for beams failing in shear \([32,35]\) and it has been shown that this can be related to the depth of the compression zone at flexural failure.

In slab connections, at the final stages of the loading history, the inclined crack as well as the flexural cracking are prevented from further propagation by the compression zone above the top of the cracks. The two types of cracks are at the critical sections of the slab, for shear and moment respectively. These critical sections are both at or very close to the perimeter of the loaded area, as shown schematically in Fig. 6 and hence it would be expected that moment-shear interaction would occur, as described elsewhere in detail \([19]\). Test results show that the shearing stress for failure decreases as the amount of reinforcement, and hence the flexural resistance of the slab increases \([19]\). However, it does not follow that the shear failure mechanism is related physically to the flexural failure mechanism. A question can, therefore, arise as to which depth should be used in equation (4.2) - the compression zone depth corresponding to the shear critical section, $X_s$, or, the one corresponding to flexural critical section, $X_f$, or better, an "average" value, $X$, of these two values? An answer would be possible only if the mechanism of the moment-shear interaction is known. Since this behavior is difficult to understand, what is needed is a prescription for evaluating the "average", or otherwise called the "fictitious value" of the N.A. depth in terms of $X_s$ and $X_f$, which is given by:

$$\frac{1}{X} = \frac{1}{2X_s} + \frac{1}{2X_f}$$

(5.1)

The background to this equation is discussed in the Appendix.

Slab-Column Connections with Fiber Concrete

Equation (4.2) above can also be used to estimate the ultimate punching strength of fiber concrete slabs provided values of $v_c$, $X$ and $\theta$ appropriate to these slabs are known and used. However, the tensile splitting strength of fiber reinforced concrete cannot be expressed in the form shown in equation (5). In the analysis presented here, it is therefore assumed that the ultimate punching shear strength of a fiber concrete slab-column connection is given by:

$$V_{up} = V_u^p + V_F$$

(6)

where $V_u^p$ is the contribution to shear of the "plain" concrete slab given by equation (4.2) and $V_F$ is the shear resistance offered by fibers when the compression zone fails by shearing along the line B'B shown in Fig. 4b. This
resistance $V_F$ is taken to be equal to the vertical component of unit shear (tensile) resistance of fiber concrete $\tau_{cu}$ acting in a direction parallel to inclined cracking multiplied by the area of the compression zone given in Fig. 4a. Referring to Figs 4a and 4b

$$V_F = V_F \cdot \sin \theta$$
or,

$$V_F = \sigma_{cu} \cdot (4 \cdot 0.5 \cdot X \cdot \cot \theta + r + 0.5 \cdot X \cdot \cot \theta) \cdot \frac{X}{\sin \theta} \cdot \sin \theta$$
or,

$$V_F = \sigma_{cu} \cdot (4r + 4X \cdot \cot \theta) \cdot X$$

(7)

The shear strength of fiber concrete is nearly the same as the ultimate tensile strength of that concrete (i.e. $\tau_{cu} = \sigma_{cu}$). The composite tensile strength is given by [36,37].

$$\sigma_{cu} = \eta_0 \eta_L \sigma_{fu} V_f$$

where

$\eta_0$ = fiber orientation factor,
$\eta_L$ = length efficiency factor,
$\sigma_{fu}$ = fiber fracture stress,
$V_f$ = fiber volume %

The computation of $\sigma_{cu}$ is shown in the Appendix.

Thus, for fiber concrete slab connection, the ultimate punching shear strength is given by:

$$V_{uf} = V_c \cdot (4r + 12d) \cdot X \cdot \cot \theta + \sigma_{cu} (4r + 4X \cdot \cot \theta) \cdot X$$

(9)

It should be noted that the term "plain" concrete slab is used to indicate that the computation of its ultimate strength $V_{up}$ is based on equation (4.2) with $X$ given by equation (5.1), but the computation of $X_f$ is that of the fiber concrete section. In other words, the presence of fibers in the slab, as far as its ultimate strength is concerned, has two implications. The first is the increase in the flexural neutral axis depth, $X_f$, and consequently of the value of $X$, and the second one through the term $V_f$.

**TEST RESULTS AND DISCUSSION**

The ultimate strength of all slabs without and with fibers tested by the authors and failing in punching shear as well as those with fibers reported in literature, has been analysed by equations (2) and (9) presented above. The steps to be followed to evaluate the ultimate punching shear strength of a given slab are shown in the Appendix.
The results for lightweight concrete slabs without fibers are presented in Table 3. These results show good agreement between experimental and theoretically predicted values with the average calculated/test ratio and standard deviation being 1.042 and 0.034 respectively.

The results for fiber reinforced lightweight concrete slabs are presented in Table 4. These also show good agreement between experimental and theoretical values with the mean calculated/test average ratio of 0.912 and a standard deviation of 0.045.

**Application of Theory to Normal Weight Concrete**

Tables 3 and 4 show the validity of the theoretical approach presented for slab-column connections made with lightweight concrete and tested by the authors. The theory can, of course, be extended to normal weight concrete. Table 5 shows the application of the theory to normal weight concrete slabs failing in punching, reported by Swamy and Ali (1). In Table 5, slabs S-2 to S-4 had steel fibers incorporated in the whole slab; slabs S-5, S-8 and S-11 to S-13 had steel fibers distributed only for a distance of 3.0 to 3.5h from the column face. Slab S-6 had fibers distributed through the whole specimen for a depth of 60mm at the bottom of the tension face. The results in Table 5 give full support to the theory presented here with an average theory/test ratio of 0.955 and standard deviation of 0.029 respectively.

**Application of Theory to Other Test Data**

The data presented in Tables 3 to 5 relate to tests carried out in the laboratories of the second author. These tests were performed over a period of several years, quite independently of the model presented here. The real proof of the validity of a model lies, however, when it can predict equally satisfactorily the data reported by other investigators. This is done in Table 6 and 7 - Table 6 presents test results of normal and lightweight concrete slabs reported from the University of Delft (5), whilst Table 7 gives data on micro concrete, lightweight and normal weight concrete slabs tested in the UK and the United States (9,10).

The good correlation of the theoretical and experimental results shown in Tables 6 and 7 gives further strong support to the ability of the theory used to explain the test results. The average predicted/test ratios for these slabs vary from 0.844 to 0.954 with standard deviation ranging from 0.054 to 0.070. The application of the theory to the slabs tested by Narayan and Darwish (9) is seen to give a low average value of the theory/test ratio of 0.844. This is due to the fact the test slabs were one-third scale models without coarse aggregate and containing 10mm diameter steel bars as reinforcement. Scale-effects are particularly important in punching shear tests (3) but no attempts have been made to quantify the scale-effects of the test results shown in Table 7. It is
worth noting that the average theory/test ratio for fiber concrete slabs, for every investigation reported in these Tables, is of the same order of magnitude as the ratio for the corresponding plain concrete slab, thus indicating the reliability of the proposed model for the fiber concrete slabs.

It is also worth emphasizing that the slabs analysed and presented in Tables 3 to 7 cover all major variables that influence shear behavior such as the type of concrete, fiber type and percentage, concrete compressive strength, steel reinforcement ratio and size of loaded area. Bearing this in mind as well as the fact that the tests themselves are, in most cases, one-to-one scale models of the prototype, and the inevitable scatter of tests results in concrete behavior, the theoretical model developed here is an excellent representation of the physical behavior of slab connections. It is also to be noted that all the tests used to validate the theory were carried out quite independently over several years prior to the development of the theoretical concepts.

CONCLUSIONS

This paper presents a simple analytical model to predict the ultimate punching shear strength of reinforced concrete slab-column connections. The model incorporates a good representation of the physical behavior, under load, of the connections made from lightweight concrete, normal weight concrete and concrete containing steel fibers. Punching is considered as a form of shearing without concrete crushing; failure is thus assumed to occur in the compression zone above the inclined cracking when the shear stress equals the tensile splitting strength of concrete. The method thus involved the calculation of the depth of the compression zone, and incorporates the inclination of the fracture surface, which reflects the influence of many of the parameters which affect the punching strength. Dowel action is taken into account by using a critical perimeter larger than the column perimeter.

The predicted punching shear strengths are compared with a large number of test results of the authors and other investigators, and these show very good agreement with experimental results. The slabs analysed cover all the major variables that influence shear behavior such as type of concrete, concrete compressive strength, steel reinforcement ratio, size of loaded area and fiber type and volume. The predicted theory/test result average ratio for the fiber concrete slab is of the same order of magnitude as the ratio for the corresponding plain concrete slab indicating the reliability and the appropriateness of the model to predict punching shear strength.
REFERENCES


**NOTATION**

- **ARC** - Area of compression zone above inclined cracking.
- **D** - Diameter of circular column.
- **d** - Effective depth of slab.
- **d’** - Depth of compression reinforcement.
- **d_f** - Fiber diameter.
- **f_{cu}** - Concrete cube compressive strength.
- **F_c** - Compressive force on cross-section.
- **F_f** - Force due to fibers.
- **F_s** - Force due to tension steel.
- **F_{ls}** - Force due to compression steel.
- **h** - Slab overall depth.
- **k_1, k_2** - Stress block parameters, normal weight concrete.
- **k_1′, k_2′** - Stress block parameters, lightweight concrete.
- **L_c** - Fiber critical length.
L_f - Fiber length.  
r - Side dimension of column loading area  
v_c - Concrete shear stress.  
V_a - Contribution to shear resistance by aggregate interlock.  
V_cc - Contribution to shear resistance from compression zone, normal to inclined cracking.  
V_c - Contribution to shear resistance from compression zone, vertical component.  
V_d - Contribution to shear resistance by dowel action.  
V_FF - Contribution to shear resistance by fibers, parallel to inclined cracking.  
V_F - Contribution to shear resistance by fibers, vertical component.  
V_f - Fiber percentage by volume.  
V_u - Ultimate shear capacity in a plain concrete section.  
V_p - Ultimate shear capacity of a plain concrete section.  
U_uF - Ultimate shear capacity of a fiber concrete slab.  
X_f - Neutral axis depth of flexural critical section.  
X_s - Neutral axis depth of shear critical section.  
X - Fictitious value of the slab neutral axis depth.  
\varepsilon_{cu} - Maximum concrete compressive strain.  
\varepsilon_s - Tensile steel strain.  
\eta_b - Bond efficiency factor.  
\eta_c - Concrete type factor for bond strength.  
\eta_o - Fiber orientation factor.  
\eta_L - Fiber length efficiency factor.  
\theta - Inclination of failure surface.  
\sigma_{cu} - Ultimate tensile strength of fiber concrete.  
\sigma_{fu} - Fiber fracture stress.  
\sigma_1 - Concrete splitting tensile strength.  
\tau - Average fiber-matrix bond strength.

APPENDIX

Evaluation of the neutral axis depth, X

Arguments have been put forward in the main part of the paper that it is logical to use a "mean" or "fictitious" value of the neutral axis depth in term of X_s and X_f.

Considering the two critical sections shown in Fig. 6 at shear and flexural failure, it is reasonable to assume that only the value of the depth X_f is affected by the strength properties of the steel, to which X_f is directly proportional, while the value of X_s remains constant. Thus a proper formulation for the fictitious value X is highly desirable.
The most straightforward procedure to obtain the fictitious N.A. depth, \( X \), would be to assume that \( X \) is the arithmetic mean of \( X_s \) and \( X_f \), that is

\[
X = \frac{X_s + X_f}{2} \quad (A.1)
\]

However, this simple-minded approach leads to rather incorrect implications when variations to the amount of reinforcement are considered, and more specifically, in limiting cases, for example, of zero or very high amount of tension steel. In addition, this approach cannot explain how one can accurately represent the abrupt changes of the critical section depths that occur very close to or at the perimeter of the loaded area.

Recognizing the physical similarities between the problem of evaluating a uniform depth of the compression zone and the problem of evaluating a uniform coefficient of thermal conductivity in a composite material consisting of two parts with different thermal properties, and bearing in mind that the shear and moment critical sections are at or very close to the column perimeter, the use of the harmonic mean of \( X_s \) and \( X_f \), rather than the arithmetic mean, is suggested for the calculation of \( X \), i.e.

\[
X = \frac{2}{\frac{1}{X_s} + \frac{1}{X_f}} \quad (A.2)
\]

or,

\[
\frac{1}{X} = \frac{1}{2X_s} + \frac{1}{2X_f} \quad (A.3)
\]

The effectiveness of the harmonic mean formulation can be quickly seen in the following two limiting cases

(A) Let \( X_f \rightarrow 0 \)

that is, the amount of tension steel tends to zero.

Then, equation (A.3) yields

\[
X \rightarrow 0
\]

This implies that the punching strength of a concrete slab, equation (4.2), without tension reinforcement becomes zero, as it should be expected. In reality the ultimate load of the slab, in such a case, would only be that contributed by the concrete through its tensile strength (in the tension zone before any crack occurs). On the other hand, the arithmetic-mean formation would have given a non-zero strength in this situation.

(B) Let \( X_f \gg X_s \)

Then

\[
X \rightarrow \frac{X_f}{1/2} = 2X_s \quad (A.4)
\]
This relationship indicates that, at limit, the value of $X$ is not at all dependent on $X_f$. This is reasonable to expect, because as the experimental evidence on flat slabs indicates, the punching strength of a connection increases with the amount of tension steel reinforcement but not proportionally, and therefore, the increase in the amount of steel above a limiting value offers negligible resistance to punching strength [19,21]. Once again, it is easily seen that the arithmetic-mean formula of equation (A.1) would have retained the effect of $X_f$ on $X$ (for very high values of $X_f$).

Another implication of equation (A.4), when $X_f \gg X_s$ is that $X$ is not equal to $X_s$ but rather twice its value. In other words, in the case of the presence of a very high amount of tension steel, the combined value $X$ of the two sections is equal, or better, tends to $2X_s$. But, at limit, it is reasonable to assume that $X$ is equal to 0.50d (that is, the neutral axis is midway the effective depth of the slab) in the case under consideration, i.e. in a slab where hardly any flexural cracks are created due to a large amount of steel.

Based on the above consideration, equation (A.4) gives
\[
X = 2X_s = 0.50d
\]
and hence
\[
X_s = 0.25d \quad (A.5)
\]
This value of $X_s$ given in equation (A.5) is considered to be valid for any slab provided because, as mentioned previously, the depth of shear critical section is clearly independent on the amount of tension steel.

It should be of great interest to consider the case of a slab for which the two depths of the critical sections are equal, i.e.
\[
X_s = X_f = 0.25d
\]
Then
1. From equation (A.3)
\[
X = X_s = X_f
\]
2. From equilibrium of forces in the cross-section considering the well-known Whitney's stress block or that in the British Code, we have:
\[
0.25 = \frac{X_f}{d} = \frac{\rho f_y}{0.85 \cdot (0.85 \cdot 0.79 \cdot f_{cu})}
\]
or
\[
\frac{f_{cu}}{\rho f_y} = 7.0
\]
where $\rho = $ tension steel ratio
fy = yield stress

Most of the test slabs in the literature which showed yielding of reinforcement before punching had $f_{cu}/\rho$ fy values varying from 5.0 to 9.0 with the majority from 6.0 to 8.0 with an average value equal to 7.0. It therefore can be said that for slabs with $f_{cu}/\rho$ fy values around 7.0 the depths of the two critical sections coincide, that is, the depth to be used in equation (4.2) is that of the flexural section.

The computation of the neutral axis depth of the flexural critical section $X_f$, is based on the conventional compatibility and equilibrium conditions used for plain concrete, except that the effect of steel strain hardening is recognized. The calculation is based on the compression stress block, either for normal or lightweight concrete, incorporated in the British Code as shown in Fig. 7. For fiber reinforced concrete sections the contribution of the steel fibers both in compression and tension zones are recognized, which are shown in Figs. 8 and 9. Referring to Fig. 9 the equilibrium conditions require:

$$F_s + F_f = F_c + F'_s$$

where

- $F_s$ = Force due to tension steel
- $F_f$ = Force due to fibers
- $F_c$ = Compressive Force on cross-section
- $F'_s$ = Force due to compression steel (if any).

The stress block parameters are as follows:

For Normal weight concrete

$$K_1 = \frac{\varepsilon_{cu} - \varepsilon_o / 3}{\varepsilon_{cu}}$$

$$K_2 = \frac{(2 - \varepsilon_o / \varepsilon_{cu})^2 + 2}{4(3 - \varepsilon_o / \varepsilon_{cu})}$$

For Lightweight concrete

$$K'_1 = \frac{\varepsilon_{cu} - 1.32\varepsilon_o / 3}{\varepsilon_{cu}}$$

$$K'_2 = \frac{(2 - 1.32\varepsilon_o / \varepsilon_{cu})^2 + 2}{4(3 - 1.32\varepsilon_o / \varepsilon_{cu})}$$

It is noted that $\varepsilon_{cu} = 0.0035$ and $K = 0.67$ for plain concrete

$\varepsilon_{cu} = 0.0045$ and $K = 0.72$ for fiber concrete

The neutral axis depth is obtained by an iterative process until equation (A.6) is satisfied considering the known stress-strain diagram of the reinforcing steel, and equilibrium and compatibility conditions at failure of a reinforced concrete section.

**Evaluation of $\sigma_{cu}$** [36,37]

The composite tensile strength is given by equation (8)

$$\sigma_{cu} = \eta_o \eta_L \sigma_{fu} V_f$$
Structural Applications of Fiber Reinforced Concrete

For uniformly distributed and randomly oriented fibers, the fiber orientation factor is 0.41 [36, 37]. The fiber length efficiency factor is given by [39].

\[ \eta_L = \frac{L_F}{2L_c} \text{ for } L_F < L_c \]  
\[ \eta_L = 1 - \frac{L_c}{2L_F} \text{ for } L_F > L_c \]

and

where \( L_F \) is the fiber length and \( L_c \) is the fiber critical length.

\[ L_c = \frac{d_f}{\sigma_{fu}} \frac{1}{2(\eta_b \eta_c \tau)} \]

where

\( d_f \) = fiber diameter
\( \tau \) = fiber-matrix interfacial bond stress
\( \eta_b \) = bond efficiency factor
\( \eta_c \) = concrete type factor for bond strength.

For slabs, it is more appropriate to use interfacial bond strength values based on flexural tests. A value for \( \tau \) of 4.15 MPa with \( \eta_c \) equal to 1.00 used for normal weight concrete and a value of 4.15 with \( \eta_c \) equal to 0.85 is applied for lightweight concrete and for concretes with \( f_{cu} \geq 40 \) MPa [36, 37]. For concretes with compressive strengths less than 20 MPa, a basic value for \( \eta_b \) of 4.15 MPa is suggested to account for the decreasing bond strength as compressive strength decreases. For values of \( f_{cu} \) between 20 and 40 MPa, a linear interpolation factor is applied. A bond efficiency factor is also assigned to take care of the geometry of the fibers. For the range of fibers used in the tests of this paper and those by Swamy and Ali [1], Table 8 gives the geometry of the fibers, their bond efficiency factor and the bond strength for normal and lightweight concretes. The \( \sigma_{cu} \) values evaluated by equation (8) are also presented in Table 8.

**Evaluation of Ultimate Punching Shear Strength**

The steps to follow to evaluate punching shear strength of a given slab without or with fibers are as follows:

1. For plain concrete slabs calculate \( v_e \) from equation (5).
2. For fiber concrete slabs, calculate \( \sigma_{cu} \) from equation (8) as shown.
3. Evaluate the neutral axis depth \( X_f \) by an iterative process as shown.
4. Compute the neutral axis depth, \( X_p \), from equation (A.5).
5. Evaluate the fictitious value of neutral axis depth, \( X \), from equation (5.1).
6. Calculate the ultimate punching shear strength from equation (4.2) or equation (9) as appropriate.
### TABLE 1—REINFORCEMENT DETAILS AND FAILURE CHARACTERISTICS OF LIGHTWEIGHT CONCRETE SLAB-COLUMN CONNECTIONS.

<table>
<thead>
<tr>
<th>Series No.</th>
<th>Slab No.</th>
<th>Column Size, r, in mm</th>
<th>Compression rein, and Tension rein.</th>
<th>Steel Fibre Type and Percentage by Volume</th>
<th>Cube Compress Strength MPa</th>
<th>Maximum punching failure load, kN</th>
<th>Distance of punching failure surface from column face (at the tension face level)</th>
<th>Punching failure angle</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>FS-1</td>
<td>150</td>
<td>7-8 mm 12-10mm</td>
<td></td>
<td>4.20</td>
<td>173.5</td>
<td>2.38h</td>
<td>23°</td>
</tr>
<tr>
<td></td>
<td>FS-2</td>
<td>150</td>
<td>7-8 mm 12-10mm</td>
<td>Crimped 0.5</td>
<td>42.50</td>
<td>225.0</td>
<td>2.47h</td>
<td>22°</td>
</tr>
<tr>
<td></td>
<td>FS-3</td>
<td>150</td>
<td>7-8 mm 12-10mm</td>
<td>Crimped 1.0</td>
<td>44.60</td>
<td>247.4</td>
<td>2.58h</td>
<td>21°</td>
</tr>
<tr>
<td>2</td>
<td>FS-4</td>
<td>150</td>
<td>-</td>
<td>Crimped 1.0</td>
<td>46.70</td>
<td>224.4</td>
<td>2.32h</td>
<td>23°</td>
</tr>
<tr>
<td></td>
<td>FS-5</td>
<td>150</td>
<td>7-8 mm 8-10mm</td>
<td>Crimped 1.0</td>
<td>47.50</td>
<td>198.1</td>
<td>2.20h</td>
<td>24°</td>
</tr>
<tr>
<td></td>
<td>FS-6</td>
<td>150</td>
<td>8-10mm</td>
<td>Crimped 1.0</td>
<td>44.60</td>
<td>174.5</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>FS-7</td>
<td>150</td>
<td>3-8 mm 8-10mm</td>
<td>Crimped 1.0</td>
<td>45.80</td>
<td>192.4</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>FS-19</td>
<td>150</td>
<td>7-8 mm 8-10mm</td>
<td></td>
<td>43.10</td>
<td>136.5</td>
<td>1.94h</td>
<td>27°</td>
</tr>
<tr>
<td></td>
<td>FS-20</td>
<td>150</td>
<td>8-10mm</td>
<td>Crimped 1.0</td>
<td>46.30</td>
<td>211.0</td>
<td>1.97h</td>
<td>27°</td>
</tr>
<tr>
<td>3</td>
<td>FS-8</td>
<td>100</td>
<td>7-8 mm 12-10mm</td>
<td></td>
<td>45.80</td>
<td>150.3</td>
<td>1.96h</td>
<td>28°</td>
</tr>
<tr>
<td></td>
<td>FS-9</td>
<td>100</td>
<td>7-8 mm 12-10mm</td>
<td>Crimped 1.0</td>
<td>44.50</td>
<td>216.6</td>
<td>2.03h</td>
<td>26°</td>
</tr>
<tr>
<td></td>
<td>FS-10</td>
<td>200</td>
<td>7-8 mm 12-10mm</td>
<td></td>
<td>45.50</td>
<td>191.4</td>
<td>2.48h</td>
<td>22°</td>
</tr>
<tr>
<td></td>
<td>FS-11</td>
<td>200</td>
<td>7-8 mm 12-10mm</td>
<td>Crimped 1.0</td>
<td>42.80</td>
<td>259.8</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>4</td>
<td>FS-12</td>
<td>150</td>
<td>7-8 mm 12-10mm</td>
<td>Japanese 1.0</td>
<td>45.10</td>
<td>217.5</td>
<td>2.70h</td>
<td>20°</td>
</tr>
<tr>
<td></td>
<td>FS-13</td>
<td>150</td>
<td>7-8 mm 12-10mm</td>
<td>Hooked 1.0</td>
<td>41.90</td>
<td>235.5</td>
<td>2.40h</td>
<td>23°</td>
</tr>
<tr>
<td></td>
<td>FS-14</td>
<td>150</td>
<td>7-8 mm 12-10mm</td>
<td>Paddle 1.0</td>
<td>43.70</td>
<td>239.5</td>
<td>2.60h</td>
<td>21°</td>
</tr>
<tr>
<td></td>
<td>FS-15</td>
<td>150</td>
<td>7-8 mm 12-10mm</td>
<td>Crimped 1.0</td>
<td>39.10</td>
<td>238.0</td>
<td>2.11h</td>
<td>25°</td>
</tr>
<tr>
<td>5</td>
<td>FS-16</td>
<td>150</td>
<td>7-8 mm 12-10mm</td>
<td>Paddle 1.0</td>
<td>34.90</td>
<td>227.8</td>
<td>1.86h</td>
<td>28°</td>
</tr>
<tr>
<td></td>
<td>FS-17</td>
<td>150</td>
<td>7-8 mm 12-10mm</td>
<td>Paddle 1.0</td>
<td>58.60</td>
<td>268.4</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>FS-18</td>
<td>150</td>
<td>7-8 mm 12-10mm</td>
<td>Paddle 1.0</td>
<td>17.80</td>
<td>166.0</td>
<td>1.16h</td>
<td>41°</td>
</tr>
</tbody>
</table>

NOTE: Slabs FS-6, FS-7, FS-11 and FS-17 failed in flexure.
<table>
<thead>
<tr>
<th>Series No.</th>
<th>Slab No.</th>
<th>Column Size mm</th>
<th>Comp. Tension reinf</th>
<th>Steel fibre Type and % vol.</th>
<th>Concrete Cube Strength MPa</th>
<th>Maximum failure load kN</th>
<th>Distance of punching failure surf from col. face</th>
<th>Punching failure angle</th>
<th>Location of steel fibre concrete</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>S-1</td>
<td>150</td>
<td>7-8mm, 12-10mm</td>
<td>-</td>
<td>50.7</td>
<td>197.7</td>
<td>2.32h</td>
<td>24°</td>
<td>Plain concrete control</td>
</tr>
<tr>
<td></td>
<td>S-2</td>
<td>150</td>
<td>7-8mm, 12-10mm</td>
<td>Crimped 0.6</td>
<td>48.7</td>
<td>243.6</td>
<td>2.56h</td>
<td>22°</td>
<td>Fibre concrete over whole section</td>
</tr>
<tr>
<td></td>
<td>S-3</td>
<td>150</td>
<td>7-8mm, 12-10mm</td>
<td>Crimped 0.9</td>
<td>47.2</td>
<td>262.9</td>
<td>2.88h</td>
<td>19°</td>
<td>&quot;</td>
</tr>
<tr>
<td></td>
<td>S-4</td>
<td>150</td>
<td>7-8mm, 12-10mm</td>
<td>Crimped 1.2</td>
<td>46.1</td>
<td>281.0</td>
<td>3.16h</td>
<td>18°</td>
<td>&quot;</td>
</tr>
<tr>
<td>2</td>
<td>S-5</td>
<td>150</td>
<td>7-8mm, 12-10mm</td>
<td>Crimped 0.9</td>
<td>47.3</td>
<td>267.2</td>
<td>2.52h</td>
<td>22°</td>
<td>Fibre concrete for 3h from column face</td>
</tr>
<tr>
<td></td>
<td>S-6</td>
<td>150</td>
<td>7-8mm, 12-10mm</td>
<td>Crimped 0.9</td>
<td>47.5</td>
<td>239.0</td>
<td>2.88h</td>
<td>19°</td>
<td>Fibre concrete 60mm depth tension face</td>
</tr>
<tr>
<td>3</td>
<td>S-8</td>
<td>150</td>
<td>7-8mm, 12-10mm</td>
<td>Crimped 0.9</td>
<td>51.4</td>
<td>255.7</td>
<td>2.52h</td>
<td>22°</td>
<td>Fibre concrete for 3.5h from column face</td>
</tr>
<tr>
<td></td>
<td>S-11</td>
<td>150</td>
<td>7-8mm, 12-10mm</td>
<td>Crimped 0.9</td>
<td>46.4</td>
<td>262.0</td>
<td>2.40h</td>
<td>23°</td>
<td>&quot;</td>
</tr>
<tr>
<td></td>
<td>S-12</td>
<td>150</td>
<td>7-8mm, 12-10mm</td>
<td>Hooked 0.9</td>
<td>46.0</td>
<td>249.0</td>
<td>2.40h</td>
<td>23°</td>
<td>&quot;</td>
</tr>
<tr>
<td></td>
<td>S-13</td>
<td>150</td>
<td>7-8mm, 12-10mm</td>
<td>Plain 0.9</td>
<td>49.1</td>
<td>236.7</td>
<td>2.12h</td>
<td>25°</td>
<td>&quot;</td>
</tr>
</tbody>
</table>
TABLE 3—COMPARISON OF EXPERIMENTAL AND CALCULATED ULTIMATE PUNCHING SHEAR STRENGTHS OF PLAIN LIGHTWEIGHT CONCRETE SLAB CONNECTIONS.

<table>
<thead>
<tr>
<th>Slab Number</th>
<th>Column Size, r, mm</th>
<th>% Tension reinforcement ratio</th>
<th>Cube compressive strength MPa</th>
<th>Ultimate punching shear strength Test, kN</th>
<th>Neutral axis depth, X, mm Eqn.(5.1)</th>
<th>Cotθ</th>
<th>Ultimate punching shear strength Theory, kN</th>
<th>Theory Test</th>
</tr>
</thead>
<tbody>
<tr>
<td>FS-1</td>
<td>150</td>
<td>0.5574</td>
<td>44.20</td>
<td>173.5</td>
<td>19.2</td>
<td>1.732</td>
<td>174.9</td>
<td>1.008</td>
</tr>
<tr>
<td>FS-8</td>
<td>100</td>
<td>0.5574</td>
<td>45.80</td>
<td>150.3</td>
<td>19.0</td>
<td>1.732</td>
<td>156.6</td>
<td>1.042</td>
</tr>
<tr>
<td>FS-10</td>
<td>200</td>
<td>0.5574</td>
<td>45.50</td>
<td>191.4</td>
<td>19.0</td>
<td>1.732</td>
<td>195.5</td>
<td>1.021</td>
</tr>
<tr>
<td>FS-19</td>
<td>150</td>
<td>0.3716</td>
<td>43.10</td>
<td>156.5</td>
<td>16.6</td>
<td>1.732</td>
<td>149.8</td>
<td>1.097</td>
</tr>
</tbody>
</table>

Mean Value 1.042
Standard Deviation 0.034
### Table 4—Comparison of Experimental and Calculated Ultimate Punching Shear Strengths of Lightweight Fiber Concrete Slab Connections.

<table>
<thead>
<tr>
<th>Slab No</th>
<th>Column size, r, mm</th>
<th>% Tension reinforcement ratio</th>
<th>Fibre Type</th>
<th>Fibre percentage</th>
<th>Cube comp strength, MPa</th>
<th>Ultimate punching shear strength Test, kN</th>
<th>$\sigma_{cu}$ MPa</th>
<th>Neutral axis depth X, mm Eqn. (5.1)</th>
<th>Ultimate punching shear strength Theory, kN</th>
<th>Theory Test</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>FS-2</td>
<td>150</td>
<td>0.5574</td>
<td>Crimped</td>
<td>0.5</td>
<td>42.5</td>
<td>225.0</td>
<td>0.868</td>
<td>20.3</td>
<td>181.5, 13.1, 194.6</td>
<td>0.865</td>
</tr>
<tr>
<td>FS-3</td>
<td>150</td>
<td>0.5574</td>
<td>Crimped</td>
<td>1.0</td>
<td>44.6</td>
<td>247.4</td>
<td>1.736</td>
<td>21.1</td>
<td>193.1, 27.3, 220.4</td>
<td>0.890</td>
</tr>
<tr>
<td>FS-4</td>
<td>150</td>
<td>0.5574</td>
<td>Crimped</td>
<td>1.0</td>
<td>46.7</td>
<td>224.4</td>
<td>1.736</td>
<td>19.5</td>
<td>182.9, 24.9, 207.8</td>
<td>0.926</td>
</tr>
<tr>
<td>FS-5</td>
<td>150</td>
<td>0.3716</td>
<td>Crimped</td>
<td>1.0</td>
<td>47.5</td>
<td>198.1</td>
<td>1.736</td>
<td>19.1</td>
<td>180.9, 24.3, 205.2</td>
<td>1.036</td>
</tr>
<tr>
<td>FS-9</td>
<td>100</td>
<td>0.5574</td>
<td>Crimped</td>
<td>1.0</td>
<td>44.5</td>
<td>216.6</td>
<td>1.736</td>
<td>21.1</td>
<td>171.6, 20.0, 191.6</td>
<td>0.885</td>
</tr>
<tr>
<td>FS-12</td>
<td>150</td>
<td>0.5574</td>
<td>Japanese</td>
<td>1.0</td>
<td>45.1</td>
<td>217.5</td>
<td>0.801</td>
<td>19.9</td>
<td>183.0, 11.8, 194.8</td>
<td>0.896</td>
</tr>
<tr>
<td>FS-13</td>
<td>150</td>
<td>0.5574</td>
<td>Hooked</td>
<td>1.0</td>
<td>41.9</td>
<td>235.7</td>
<td>1.665</td>
<td>21.4</td>
<td>189.5, 26.7, 216.2</td>
<td>0.917</td>
</tr>
<tr>
<td>FS-14</td>
<td>150</td>
<td>0.5574</td>
<td>Paddle</td>
<td>1.0</td>
<td>43.7</td>
<td>239.5</td>
<td>1.670</td>
<td>21.1</td>
<td>191.7, 26.3, 218.0</td>
<td>0.910</td>
</tr>
<tr>
<td>FS-15</td>
<td>150</td>
<td>0.5574</td>
<td>Crimped</td>
<td>1.0</td>
<td>39.1</td>
<td>238.0</td>
<td>1.562</td>
<td>21.7</td>
<td>186.3, 25.4, 221.7</td>
<td>0.932</td>
</tr>
<tr>
<td>FS-16</td>
<td>150</td>
<td>0.5574</td>
<td>Paddle</td>
<td>1.0</td>
<td>34.9</td>
<td>227.8</td>
<td>1.670</td>
<td>22.6</td>
<td>182.9, 28.6, 211.5</td>
<td>0.928</td>
</tr>
<tr>
<td>FS-18</td>
<td>150</td>
<td>0.5574</td>
<td>Paddle</td>
<td>1.0</td>
<td>17.8</td>
<td>166.0</td>
<td>1.006</td>
<td>23.4</td>
<td>134.8, 17.9, 152.7</td>
<td>0.920</td>
</tr>
<tr>
<td>FS-20</td>
<td>150</td>
<td>0.3716</td>
<td>Crimped</td>
<td>1.0</td>
<td>46.3</td>
<td>211.0</td>
<td>1.736</td>
<td>16.8</td>
<td>156.5, 20.9, 177.4</td>
<td>0.841</td>
</tr>
</tbody>
</table>

Mean: 0.912
Standard Deviation: 0.045
TABLE 5—COMPARISON OF EXPERIMENTAL AND CALCULATED ULTIMATE PUNCHING SHEAR STRENGTHS OF NORMAL WEIGHT SLAB CONNECTIONS MADE WITHOUT AND WITH FIBER CONCRETE.

<table>
<thead>
<tr>
<th>Investigator</th>
<th>Slab No</th>
<th>Column size, r, mm</th>
<th>%Tension reinforcement ratio</th>
<th>Fibre Type</th>
<th>Fibre percentage</th>
<th>Cube comp strength MPa</th>
<th>Ultimate punching shear strength Test, kN</th>
<th>$\sigma_{cu}$ MPa</th>
<th>Neutral axis depth X, mm</th>
<th>Ultimate punching shear strength Theory, kN</th>
<th>Theory Test</th>
<th>$V_{th}$</th>
<th>$V_F$</th>
<th>$V_{th} - V_F$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Swamy and Ali S. (1)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>S-1</td>
<td>150</td>
<td>0.5574</td>
<td>-</td>
<td>0.0</td>
<td>47.3</td>
<td>197.7</td>
<td>-</td>
<td>18.4</td>
<td>197.3</td>
<td>-</td>
<td>197.3</td>
<td>0.998</td>
<td></td>
<td></td>
</tr>
<tr>
<td>S-2</td>
<td>150</td>
<td>0.5574 Crimped</td>
<td>0.6</td>
<td>48.7</td>
<td>243.6</td>
<td>1.225</td>
<td>19.6</td>
<td>213.2</td>
<td>17.7</td>
<td>230.9</td>
<td>0.948</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>S-3</td>
<td>150</td>
<td>0.5574 Crimped</td>
<td>0.9</td>
<td>47.2</td>
<td>262.9</td>
<td>1.837</td>
<td>20.5</td>
<td>219.5</td>
<td>27.9</td>
<td>247.4</td>
<td>0.941</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>S-4</td>
<td>150</td>
<td>0.5574 Crimped</td>
<td>1.2</td>
<td>46.1</td>
<td>281.0</td>
<td>2.45</td>
<td>21.3</td>
<td>225.4</td>
<td>32.0</td>
<td>287.4</td>
<td>0.941</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>S-5</td>
<td>150</td>
<td>0.5574 Crimped</td>
<td>0.9</td>
<td>47.3</td>
<td>267.2</td>
<td>1.837</td>
<td>20.5</td>
<td>219.8</td>
<td>27.9</td>
<td>247.7</td>
<td>0.927</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>S-6</td>
<td>150</td>
<td>0.5574 Crimped</td>
<td>0.9</td>
<td>47.5</td>
<td>239.0</td>
<td>1.837</td>
<td>20.5</td>
<td>220.2</td>
<td>-</td>
<td>220.2</td>
<td>0.921</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>S-8</td>
<td>150</td>
<td>0.5574 Crimped</td>
<td>0.9</td>
<td>51.4</td>
<td>255.7</td>
<td>1.837</td>
<td>20.0</td>
<td>223.5</td>
<td>27.1</td>
<td>250.6</td>
<td>0.980</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>S-11</td>
<td>150</td>
<td>0.5574 Crimped</td>
<td>0.9</td>
<td>46.4</td>
<td>262.0</td>
<td>1.837</td>
<td>20.6</td>
<td>218.7</td>
<td>28.1</td>
<td>246.8</td>
<td>0.942</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>S-12</td>
<td>150</td>
<td>0.5574 Hooked</td>
<td>0.9</td>
<td>46.0</td>
<td>249.0</td>
<td>1.761</td>
<td>20.6</td>
<td>217.8</td>
<td>26.9</td>
<td>244.7</td>
<td>0.982</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>S-13</td>
<td>140</td>
<td>0.5574 Plain</td>
<td>0.9</td>
<td>49.1</td>
<td>236.7</td>
<td>1.531</td>
<td>19.9</td>
<td>217.4</td>
<td>22.5</td>
<td>239.9</td>
<td>1.014</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Average of ratios excluding S-1 slab = 0.955
Standard Deviation = 0.029
<table>
<thead>
<tr>
<th>Investigator</th>
<th>Slab No</th>
<th>Column size, D, mm</th>
<th>% Tension reinforcement ratio</th>
<th>Concrete Type</th>
<th>Fibre percentage</th>
<th>Concrete comp strength, MPa</th>
<th>Ultimate punching shear strength, kN</th>
<th>σcu, MPa</th>
<th>Neutral axis depth, X, mm (Eqn. (5.2))</th>
<th>Ultimate punching shear strength, Theory, kN</th>
<th>Theory Test</th>
</tr>
</thead>
<tbody>
<tr>
<td>Walraven Pat and Markov (5)</td>
<td>4</td>
<td>250</td>
<td>1.00</td>
<td>N.W</td>
<td>-</td>
<td>41.4</td>
<td>406</td>
<td>-</td>
<td>25.3</td>
<td>327.1</td>
<td>0.806</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>250</td>
<td>1.00</td>
<td>N.W</td>
<td>0.50</td>
<td>46.3</td>
<td>462</td>
<td>0.638</td>
<td>26.3</td>
<td>359.5</td>
<td>1.63</td>
</tr>
<tr>
<td></td>
<td>6</td>
<td>250</td>
<td>1.00</td>
<td>N.W</td>
<td>1.25</td>
<td>48.5</td>
<td>454</td>
<td>1.595</td>
<td>27.3</td>
<td>382.4</td>
<td>42.4</td>
</tr>
<tr>
<td></td>
<td>7</td>
<td>250</td>
<td>1.83</td>
<td>N.W</td>
<td>-</td>
<td>47.5</td>
<td>467</td>
<td>-</td>
<td>31.1</td>
<td>430.6</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>8</td>
<td>250</td>
<td>1.83</td>
<td>N.W</td>
<td>0.50</td>
<td>45.1</td>
<td>502</td>
<td>0.638</td>
<td>34.4</td>
<td>464.1</td>
<td>22.5</td>
</tr>
<tr>
<td></td>
<td>9</td>
<td>250</td>
<td>1.83</td>
<td>N.W</td>
<td>1.25</td>
<td>45.1</td>
<td>540</td>
<td>1.595</td>
<td>35.3</td>
<td>476.3</td>
<td>58.0</td>
</tr>
<tr>
<td></td>
<td>24</td>
<td>250</td>
<td>1.00</td>
<td>L.W</td>
<td>-</td>
<td>26.2</td>
<td>252</td>
<td>-</td>
<td>31.2</td>
<td>282.4</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>25</td>
<td>250</td>
<td>1.00</td>
<td>L.W</td>
<td>0.50</td>
<td>29.2</td>
<td>326</td>
<td>0.542</td>
<td>32.0</td>
<td>305.7</td>
<td>17.5</td>
</tr>
<tr>
<td></td>
<td>26</td>
<td>250</td>
<td>1.00</td>
<td>L.W</td>
<td>1.25</td>
<td>29.2</td>
<td>355</td>
<td>1.356</td>
<td>33.4</td>
<td>319.1</td>
<td>46.0</td>
</tr>
</tbody>
</table>

Average of ratios excluding 4, 7, 24 slabs 0.954
Standard Deviation 0.070

Average ratio of slabs 4, 7, 24 0.949
### TABLE 7—COMPARISON OF EXPERIMENTAL AND CALCULATED ULTIMATE PUNCHING SHEAR STRENGTHS OF SLAB CONNECTIONS TESTED BY NARAYANAN, DARWISH, AND CRISWELL.

<table>
<thead>
<tr>
<th>Investigator</th>
<th>Slab No</th>
<th>Column size, ( r ), mm</th>
<th>%Tension reinforcement ratio</th>
<th>Concrete Type</th>
<th>Fibre percentage</th>
<th>Cube comp strength, MPa</th>
<th>Ultimate punching shear strength, ( V_n ), kN</th>
<th>( \sigma_{cu} ), MPa</th>
<th>Neutral axis depth, ( X ), mm</th>
<th>Ultimate punching shear strength, Theory, kN</th>
<th>Test, Eqn.(5.2)</th>
<th>Test,kN</th>
<th>Theory,kN</th>
<th>Average of ratios excluding S1 slab</th>
<th>Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Narayanan and Darwish (9)</td>
<td>S1</td>
<td>100</td>
<td>2.01</td>
<td>Micro Concrete</td>
<td>-</td>
<td>54.1</td>
<td>86.5</td>
<td>-</td>
<td>12.6</td>
<td>75.5</td>
<td>-</td>
<td>75.5</td>
<td>0.873</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>S2</td>
<td>100</td>
<td>2.01</td>
<td>&quot;</td>
<td>0.25</td>
<td>65.1</td>
<td>93.4</td>
<td>0.51</td>
<td>11.5</td>
<td>75.5</td>
<td>2.8</td>
<td>78.3</td>
<td>0.838</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>S3</td>
<td>100</td>
<td>2.01</td>
<td>&quot;</td>
<td>0.50</td>
<td>55.9</td>
<td>102.0</td>
<td>1.02</td>
<td>12.3</td>
<td>74.9</td>
<td>6.1</td>
<td>81.0</td>
<td>0.794</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>S4</td>
<td>100</td>
<td>2.01</td>
<td>&quot;</td>
<td>0.75</td>
<td>57.5</td>
<td>107.5</td>
<td>1.53</td>
<td>12.3</td>
<td>75.9</td>
<td>9.1</td>
<td>85.0</td>
<td>0.790</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>S5</td>
<td>100</td>
<td>2.01</td>
<td>&quot;</td>
<td>1.00</td>
<td>66.2</td>
<td>113.6</td>
<td>2.04</td>
<td>11.9</td>
<td>78.8</td>
<td>11.7</td>
<td>90.5</td>
<td>0.797</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>S6</td>
<td>100</td>
<td>2.01</td>
<td>&quot;</td>
<td>1.25</td>
<td>66.3</td>
<td>122.2</td>
<td>2.55</td>
<td>12.1</td>
<td>80.20</td>
<td>14.9</td>
<td>95.1</td>
<td>0.778</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>S8</td>
<td>100</td>
<td>2.24</td>
<td>&quot;</td>
<td>1.00</td>
<td>58.6</td>
<td>111.1</td>
<td>2.04</td>
<td>13.0</td>
<td>81.0</td>
<td>13.0</td>
<td>94.0</td>
<td>0.846</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>S9</td>
<td>100</td>
<td>2.46</td>
<td>&quot;</td>
<td>1.00</td>
<td>54.4</td>
<td>111.3</td>
<td>2.04</td>
<td>13.6</td>
<td>81.6</td>
<td>13.7</td>
<td>95.3</td>
<td>0.856</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>S10</td>
<td>100</td>
<td>2.69</td>
<td>&quot;</td>
<td>1.00</td>
<td>59.5</td>
<td>113.3</td>
<td>2.04</td>
<td>13.6</td>
<td>85.4</td>
<td>13.7</td>
<td>99.1</td>
<td>0.875</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>S11</td>
<td>100</td>
<td>2.01</td>
<td>L.W</td>
<td>1.00</td>
<td>37.2</td>
<td>82.1</td>
<td>2.04</td>
<td>14.4</td>
<td>62.9</td>
<td>14.7</td>
<td>77.6</td>
<td>0.945</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>S12</td>
<td>100</td>
<td>2.01</td>
<td>L.W</td>
<td>1.00</td>
<td>40.5</td>
<td>84.9</td>
<td>2.04</td>
<td>14.0</td>
<td>63.8</td>
<td>14.3</td>
<td>78.1</td>
<td>0.920</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Criswell</td>
<td>No.4</td>
<td>114.3</td>
<td>1.88</td>
<td>Plain</td>
<td>1.0</td>
<td>52.3</td>
<td>97.86</td>
<td>1.021</td>
<td>13.3</td>
<td>88.9</td>
<td>7.5</td>
<td>96.4</td>
<td>0.985</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Average of ratios excluding S1 slab: 0.844
Standard Deviation: 0.054
TABLE 8—BOND STRENGTH AND ULTIMATE TENSILE STRENGTH OF FIBER CONCRETE.

<table>
<thead>
<tr>
<th>Concrete</th>
<th>Fibre Type</th>
<th>Fibre geometry mm</th>
<th>Bond eff. factor, $\eta_b$</th>
<th>Bond Strength MPa $\eta_b \eta_c \tau$</th>
<th>Fibre Volume %</th>
<th>Ultimate Tensile strength MPa, $\sigma_{cu}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lightweight</td>
<td>Crimped</td>
<td>0.50 x 50</td>
<td>1.20</td>
<td>4.23</td>
<td>1.0</td>
<td>1.736</td>
</tr>
<tr>
<td>Lightweight</td>
<td>Japanese</td>
<td>0.42 x 25</td>
<td>1.00</td>
<td>3.53</td>
<td>1.0</td>
<td>0.801</td>
</tr>
<tr>
<td>Lightweight</td>
<td>Hooked</td>
<td>0.50 x 50</td>
<td>1.15</td>
<td>4.06</td>
<td>1.0</td>
<td>1.670</td>
</tr>
<tr>
<td>Lightweight</td>
<td>Paddle</td>
<td>0.76 x 53</td>
<td>1.65</td>
<td>5.82</td>
<td>1.0</td>
<td>1.670</td>
</tr>
<tr>
<td>Normal weight</td>
<td>Crimped</td>
<td>0.5 x 50</td>
<td>1.20</td>
<td>4.98</td>
<td>0.9</td>
<td>1.837</td>
</tr>
<tr>
<td>Normal weight</td>
<td>Hooked</td>
<td>0.5 x 50</td>
<td>1.15</td>
<td>4.77</td>
<td>0.9</td>
<td>1.761</td>
</tr>
<tr>
<td>Normal weight</td>
<td>Plain</td>
<td>0.6 x 50</td>
<td>1.00</td>
<td>4.15</td>
<td>0.9</td>
<td>1.531</td>
</tr>
</tbody>
</table>
Fig. 1—(a) Test slab geometry and test arrangement; (b) Typical reinforcement details.
Fig. 2—(a) Typical punching shear failure truncated cone; (b) Typical punching shear failure surfaces on the tension face.

Fig. 3—Conditions after inclined cracking in a plain concrete slab.
$A B = r$
$A' B' = X \cot \theta + r + X \cot \theta$
$A_1 B_1 = \frac{1}{2} X \cot \theta + r + \frac{1}{2} X \cot \theta$
$ARC = (A_1 B_1 C_1 D_1) \cdot A' E'$

$A_1 B_1 C_1 D_1$ Plane at mid-depth of N.A. depth

---

Fig. 4—(a) Failure surface above neutral axis; (b) Compression zone and fiber shear resistance contributions along failure surface.
Fig. 5—Limiting shear stress of the compression zone in normal weight concrete.

Fig. 6—Schematic presentation of flexural and shear critical sections.
Fig. 7—Compressive stress block and strain distribution for: (a) plain normal weight concrete; (b) plain lightweight concrete.

Fig. 8—Compressive stress block and strain distribution for: (a) fiber normal weight concrete; (b) fiber lightweight concrete.

Fig. 9—Strain and stress distribution in fiber concrete.
Fiber Reinforced Concrete for Enhancing Structural Fire Resistance of Columns

by V. Kodur

Synopsis: This paper deals with the application of fibre-reinforced concrete to enhance structural fire resistance. Materials, such as fibre-reinforced concrete, have good fire resistance properties and, by properly designing the building elements, fire resistance, in the practical range, can be obtained.

The properties of steel fibre-reinforced concrete are discussed. Examples of some fire resistance applications of steel fibre-reinforced concrete in buildings are illustrated. Results from studies on the fire resistance of concrete-filled steel columns show that the addition of steel fibres in concrete filling improves the fire resistance of steel columns and eliminates the need for external fire protection. The application of fibre-reinforced concrete in enhancing the fire resistance of high strength concrete columns is also discussed.

Keywords: elevated temperatures; fiber reinforced concrete; fire resistance; high-strength concrete; material properties; structural applications
V. K. R. Kodur is a Research Associate at the National Fire Laboratory, Institute for Research in Construction of the National Research Council of Canada, Ottawa. He received his M.Sc. and Ph.D. degrees from Queen's University, Kingston, Ontario. His research interests include nonlinear analysis of concrete structures and evaluation of fire resistance of concrete and steel structures.

INTRODUCTION

In recent years, the construction industry has shown significant interest in the use of fibre-reinforced concrete, due to the improvements in structural performance it can provide compared to traditional plain concrete. These improvements, combined with the recent development of rational design approaches for the use of fibre-reinforced concrete, will result in a wider use of this material in building construction.

When used in buildings, structural members must be designed to satisfy appropriate fire resistance requirements, in addition to other structural requirements specified in building codes. These fire resistance requirements are included in codes on the premise that, when other measures of controlling the fire fail, structural integrity is the last line of defence.

Zollo (1) has indicated that, for sustained high temperature applications (in the range of 200-800°C), steel fibre-reinforced concrete is usually most effective. The properties of steel fibre-reinforced concrete, at elevated temperatures, are superior to those of plain concrete and improve structural performance under fire conditions (2, 3). This superior performance has led to the development of fire resistance applications (4, 5) for steel fibre-reinforced concrete in building elements which must have good structural performance when exposed to temperatures commonly encountered in fire. Preliminary studies (6, 7) have also indicated that the use of polypropylene fibres in high strength concrete prevents spalling of concrete at elevated temperatures, thus improving its fire performance.

In this paper, it will be shown that the use of steel and polypropylene fibre-reinforced concrete offers potential applications in enhancing the fire resistance of structural members. The performance of fibre-reinforced concrete at elevated temperatures is presented and compared to that of plain concrete. The properties of steel fibre-reinforced concrete that enhance the structural performance, when exposed to fire, are discussed. Examples of some fire resistance applications of steel fibre-reinforced concrete in buildings are illustrated. An overview of a current research program, at the National Research Council of Canada (NRC), to overcome the problem of spalling in high
performance concrete through the use of steel and polypropylene fibres is discussed.

**FIRE RESISTANCE**

Fire resistance is defined as the ability of a structural member to carry its applied load at the elevated temperatures that could be encountered in a fire. The time during which a structural member exhibits fire resistance is termed its fire resistance rating. Typical fire resistance rating requirements for different building elements are specified in building codes, such as the National Building Code of Canada (NBCC) (8).

The fire resistance of a structural member depends, in part, on the materials used in its construction. For some constructions, such as steel structural members, surface fire protection is applied to the member to obtain the required fire resistance ratings. This adds to the cost of construction. In the case of high strength concrete structural members, spalling of concrete, under fire conditions, is one of the major concerns and this reduces the fire resistance of the structural member. Materials, such as fibre-reinforced concrete, have good fire resistance properties and, by properly designing the building elements, fire resistance, in the practical range, can be obtained.

**PROPERTIES ENHANCING FIRE RESISTANCE**

The behaviour of a structural member when exposed to fire, is dependent, in part, on the thermal, mechanical, and deformation properties of the materials of which the member is composed. In order to understand and eventually predict the performance of structural members employing fibre-reinforced concrete, the material properties that determine the behaviour of the member at elevated temperatures must be known.

The thermal properties that influence the temperature rise and distribution in a concrete structural member are thermal conductivity and specific heat. The mechanical properties that determine the fire resistance of structural members are the strength and modulus of elasticity of the component materials. The deformation properties that govern the performance of a structural member are thermal expansion and creep. All these properties vary as a function of temperature. In addition, the spalling of concrete is often the determining factor in establishing the performance of a concrete structural member exposed to fire.
Material Properties

A study was undertaken at NRC to establish the properties of steel fibre-reinforced concrete at elevated temperatures. In this study, concrete strengths up to 45 MPa were considered. Detailed results from the studies are described in References (9, 10). Results from the study showed that, at elevated temperatures, steel fibre-reinforced concrete exhibits thermal properties that are similar to those of plain concrete. However, the mechanical and deformation properties of steel fibre-reinforced concrete, at elevated temperatures, produce superior fire resistance to that of plain concrete. The properties that primarily enhanced the fire resistance of steel fibre-reinforced concrete were:

Compressive strength: The compressive strength of steel fibre-reinforced concrete is higher than that of plain concrete at elevated temperatures (10). This can be seen in Figure 1 where the effect of temperature on the compressive strength of steel fibre-reinforced concretes is compared to that of plain concrete. The strength is expressed as a percentage of the compressive strength at room temperature. The strength of fibre-reinforced concrete exceeds the room-temperature strength for temperatures up to approximately 400°C. The strength of plain concrete decreases slightly with increasing temperature up to 400°C.

Above 400°C, the strength of both concretes decreases rapidly with increasing temperature. This loss of strength is due to enhanced crack formation in the concrete, which is initiated as a result of the expansion difference between the cement paste and the aggregate (11). For steel fibre-reinforced concrete, a contributing factor is the decrease of strength of the steel, which, at approximately 400°C, reduces at an accelerated rate (12). This higher compressive strength of steel fibre-reinforced concrete at elevated temperatures enhances the fire resistance of a structural member.

Tensile strength: The effect of temperature on the tensile strength of steel fibre-reinforced concretes is compared to that of plain concrete in Figure 2 (13). The tensile strength of both concretes decreases with increased temperature. However, the strength of steel fibre-reinforced concrete decreases at a lower rate than that of plain concrete throughout the temperature range. For temperatures up to about 350°C, the strength of steel fibre-reinforced concrete is significantly higher than that of plain concrete. The increased tensile strength delays the propagation of cracks in steel fibre-reinforced concrete structural members. This is beneficial when a structural member is subjected to bending stresses.

Ultimate strain: The ultimate strain for plain and steel fibre-reinforced concrete is shown as a function of temperature in Figure 3. At elevated temperatures, steel fibre-reinforced concrete attains higher ultimate strains than plain concrete. The increase in strain can be attributed to the increase in crack volume, caused by the steel fibres, which have a greater thermal expansion than
the concrete (10). This increased strain in steel fibre-reinforced concrete produces a higher ductility in a structural member.

Modulus of elasticity: The effect of temperature on the modulus of elasticity of plain and steel fibre-reinforced concrete is shown in Figure 4, where the ratio of the modulus of elasticity, expressed as a percentage of the modulus of elasticity at room temperature, is plotted. Similar to that of plain concrete, the modulus of elasticity of steel fibre-reinforced concrete decreases with increasing temperature.

The modulus of elasticity is affected more by the formation of cracks than the compressive strength. The formation of micro-cracks due to shrinkage of the cement paste, leads to a decrease in the modulus of elasticity (10, 11). Although the fibre-reinforcement increases the compressive strength of the concrete, the increase in strains, due to the steel fibres, results in a decrease in the modulus of elasticity. The rate of decrease in the modulus of elasticity is highest in the temperature range 0-300°C for both types of concretes.

Creep: At normal stresses and ambient temperatures, the deformation due to creep is not significant. At higher stress levels and at elevated temperatures, however, the rate of deformation caused by creep can be substantial. Hence, the main factors that influence creep are the temperatures of the concrete and the stress level.

The axial deformations at elevated temperatures of plain and steel fibre-reinforced concrete specimens, subjected to stresses of 0, 30, 45 and 60% of the initial compressive strength of these specimens, are shown in Figure 5 (10). The deformations, corresponding to a stress level of zero, represent the conventional expansion of the concrete. At 30% stress level, plain concrete specimens had premature failure.

The deformations of the plain and steel fibre-reinforced concrete specimens are similar in the temperature range under consideration. The effect of the stress on deformations is large, particularly at temperatures above approximately 550°C. This can be attributed to dehydration and shrinkage of the cement paste in the concrete (14). The steel fibre-reinforced concrete specimens failed at higher temperatures, however, due to the enhanced ductility of the concrete provided by the reinforcement.

Spalling: The spalling of concrete, under fire conditions, is one of the major concerns in the use of high strength concrete due to its lower water-cement ratio. The spalling of concrete exposed to fire has been observed in concrete structural members under laboratory and real fire conditions (7). Spalling, which results in loss of concrete during a fire, has the effect of exposing deeper layers of concrete to the maximum fire temperature, thereby increasing the rate of
transmission of heat to the inner layers of the structure, including to the reinforcement.

Spalling is theorized to be caused by the build up of pore pressure during heating. High strength concrete is believed to be more susceptible to this pressure build up because of its low permeability compared to normal strength concrete. The extremely high water vapour pressure, generated during exposure to fire, cannot escape due to the high density of high strength concrete and this pressure often reaches the saturation vapour pressure. At 300°C, the pressure reaches about 8 MPa. Such internal pressures are too high to be resisted by the high strength concrete mix having a tensile strength of about 5 MPa (15).

Preliminary studies indicate that the addition of fibre reinforcement to concrete reduces spalling (6, 7). Experimental studies on high strength reinforced concrete columns showed deep spalling and rupture after fire tests. However, only slight or no spalling was observed in fire tests on the same high strength concrete columns reinforced with polypropylene fibres. Figure 6 shows the condition of reinforced concrete columns, with and without fibre reinforcement, after a test (6). It can be seen that the presence of polypropylene fibres reduced spalling in the high strength reinforced concrete columns.

FIRE RESISTANCE APPLICATIONS

The use of fibre-reinforced concrete is gaining increasing popularity in a number of structural applications in buildings (16, 17). The superior performance of steel fibre-reinforced concrete, under fire conditions, compliments its other properties and makes it an attractive alternative for structural elements in buildings, such as columns, deep beams, and slabs, which have to satisfy fire resistance requirements. Polypropylene or steel fibres can also find wide application in high strength concrete construction to overcome the problem of fire-induced spalling.

Recent studies have extended the application of fibre-reinforced concrete to building elements which also have to perform satisfactorily when exposed to fire. Examples of some practical building applications are presented in this section.

HSS Columns

Steel hollow structural section (HSS) columns are structurally very efficient in resisting compression loads and are widely used in the construction of framed structures and in industrial buildings. However, these columns need to be
provided with additional fire protection to achieve the code-required fire resistance ratings, involving additional cost and construction time. By filling these columns with concrete, the load-bearing capacity can be increased substantially, while at the same time, creating a higher fire resistance without using external fire protection for the steel. Recent studies have resulted in practical solutions for obtaining the required fire resistance for steel columns through steel fibre-reinforced concrete-filling (4, 18). Concrete-filled HSS columns offer a number of advantages which often result in economical construction.

Experimental and numerical studies were carried out on concrete-filled HSS columns to investigate the influence of three types of concrete-filling: plain concrete (PC); bar-reinforced concrete (RC); and steel fibre-reinforced concrete (FC) (18). The concrete used in the study was of normal strength (30-50 MPa). The studies showed that HSS columns filled with steel fibre-reinforced concrete, had greater fire resistance than those filled with plain concrete and that their use permits more cost-effective construction.

Figure 7 shows the variation of axial deformation with time for a typical concrete-filled HSS column subjected to fire. The test results indicate that the fire resistance of HSS columns filled with plain concrete is limited to about two hours. Failure is due to rapid crack propagation in the concrete, resulting in premature failure of the concrete core. Test data also indicate that the fire resistance of these plain concrete-filled columns is sensitive to eccentric loads.

The limitations of plain concrete-filled HSS columns can be overcome by using either bar-reinforced or steel fibre-reinforced concrete-filling (4). As can be seen in Figure 7, fire resistance performance was significantly improved for the HSS columns filled with steel fibre-reinforced and bar-reinforced concrete.

Fire resistance ratings of up to three hours were obtained for steel fibre-reinforced concrete-filled columns without any reduction in the load. The presence of steel fibres, about 1.75% by mass of concrete, reduced cracking in the concrete, thereby preventing premature failure. Furthermore, test data indicate that the steel fibre-reinforced-concrete filling has the following benefits as compared to plain concrete filling (4).

- better deformation behaviour, resulting in gradual (ductile) rather than sudden (brittle) failure;
- increased (by about 10-20 percent) load carrying capacity;
- increased fire resistance, to about 3 hours, even under eccentric loads;
- decreased buckling;
- suitability for a wide range of column dimensions.
These beneficial effects can be attributed to the superior mechanical and thermal properties of steel fibre-reinforced concrete at elevated temperatures, and to the containment effect provided by the steel fibres in the concrete core. The increased cost of using steel fibres, rather than plain concrete filling, can often be justified by the numerous advantages of this composite material.

Bar-reinforced concrete-filled columns offer many of the advantages of steel fibre-reinforced concrete-filled columns (see Figure 7) and can carry higher loads. They are, however, more expensive to construct because of the labour involved in placing the reinforcing bars. They are also more difficult to work with in small dimension HSS columns (achieving sufficient concrete coverage of the reinforcing bars in such a restricted space is problematic).

High Strength Concrete

In recent years, high strength concrete has become an attractive alternative to traditional plain concrete. High strength concrete (HSC) has now been utilized in several high-rise buildings and its major application is in columns. With the increased use of high strength concrete in this application, concern has developed regarding its behaviour in fire, in particular, the occurrence of spalling at elevated temperatures. Further, the occurrence of explosive spalling when HSC is subjected to rapid heating, as in the case of a fire, was observed in some tests (7).

Recent results of fire tests, in a number of laboratories (7), have shown that there are well-defined differences between the properties of HSC and normal strength concrete at high temperatures. Data from the tests show that spalling of HSC is affected by the following factors:

- Original compressive strength
- Moisture content of concrete
- Concrete density
- Heating rate
- Specimen dimensions and shapes
- Loading conditions

The spalling can be minimized by creating pores through which water vapour can be relieved before vapour pressure reaches critical values. This is usually done by adding polypropylene fibres to the HSC. Also, by increasing the tensile strength, by adding steel fibres to HSC, spalling can be minimized and the fire resistance can be enhanced.

Preliminary tests were carried out at NRC to examine the spalling effect in high strength concrete walls exposed to fires. Both plain and polypropylene fibre-reinforced concrete specimens were exposed for two hours to a simulated high
intensity hydrocarbon fire. The spalling was significantly less in the case of polypropylene fibre-reinforced concrete specimens.

Studies are also in progress at NRC to develop the necessary fire resistance guidelines for hollow steel columns with HSC filling. Data from the fire tests indicate that the structural performance of the concrete filled HSS columns was significantly improved when the steel fibres were added to HSS. Further, the use of steel fibre-reinforcement in high strength concrete as a filling improves the fire resistance of HSS columns and eliminates the need for external fire protection (19). Data from these studies, as well as from other laboratories, are being used to develop guidelines for the application of fibre-reinforced concrete to control spalling in HSC structural members exposed to fire.

Test program: Studies are in progress at NRC to develop fire resistance design guidelines for high strength concrete for possible incorporation in codes and standards. The main objective of this research, being undertaken together with the Portland Cement Association (PCA), Canadian Portland Cement Association (CPCA), Concrete Canada, and the National Chiao Tung University in Taiwan (NCTU), is to study the behaviour of HSC at elevated temperatures and to develop solutions to minimize spalling. Both experimental and theoretical studies are being carried out.

As part of the experimental study, forty-eight full-scale reinforced concrete columns, are being tested by exposing the columns to fire under structural loads. These columns are made with two types of HSC, namely plain-HSC and fibre-reinforced-HSC. Both polypropylene and steel fibres are being considered in the study. Twenty columns with plain-HSC, eight columns with steel fibre-reinforced-HSC and twenty columns with polypropylene fibre-reinforced-HSC are being fabricated.

All columns are 3810 mm long and are of either square or rectangular cross section. Two sizes of square columns, 306 mm and 406 mm, are being considered in the study. The rectangular columns are 306 x 456 mm and 203 x 916 mm. The test variables are column section dimensions, tie spacing, load intensity, end conditions, concrete strength, aggregate type and reinforcement. Figure 8 shows elevation and cross-sectional details for a typical column considered in the study.

The construction of the columns with plain-HSC and with steel fibre-reinforced-HSC has been completed. The polypropylene fibre-reinforced concrete columns are expected to be constructed shortly. Two types of coarse aggregate, namely, siliceous aggregate and carbonate aggregate, are being used in the concrete mix to also study the influence of aggregate in fire performance of HSC. For steel fibre-reinforced concrete, steel fibres, 1.77 percent by mass, were mixed with the concrete. Table 1 gives the mix details for plain-HSC and steel fibre-
reinforced-HSC. Mix 1 and Mix 2 are plain-HSC while Mix 3 and Mix 4 use steel fibre-reinforcement. Mix 1 and Mix 3 are made with siliceous aggregate while Mix 2 and Mix 4 are made with carbonate aggregate. The compressive strengths of the concretes are in the range of 80-90 MPa.

The concrete was poured with the formwork horizontal and vibrators were used to consolidate the concrete. Thermocouples, with a thickness of 0.91 mm, were installed at the mid-height of the column to measure the temperature at different locations in the cross section.

The tests will be carried out by exposing the HSC columns to heat in a furnace specially built for testing loaded columns. The test furnace is designed to produce conditions such as temperature, structural loads and heat transfer, to which a member might be exposed during a fire. It consists of a steel framework with the furnace chamber inside it. The furnace facility includes a hydraulic loading system with a capacity of 1,000 t.

The columns will be tested under the maximum allowable load according to North American Building Codes for concrete structures. Most of the HSC columns will be subjected to constant concentric loads during testing. However, some columns will be tested under eccentric loads to study the influence of eccentricity on the performance of columns at high temperatures. The applied load on the columns will be calculated according to the specifications of Canadian Standard CSA-A23.3-M94 (20) and ACI-95 (21).

During the test, the column will be exposed, under a load, to heating controlled in such a way that the average temperature in the furnace follows, as closely as possible, the ASTM E119-88 (22) standard temperature-time curve. The furnace, concrete and steel temperatures as well as the axial deformations and rotations will be recorded until failure of the column.

**HSC Material properties:** Results from the fire tests will be used to develop computer models for predicting the behaviour of HSC columns exposed to fire. These computer programs require the thermal, mechanical and deformation properties of HSC at elevated temperatures. To establish these properties, a study is being carried out as part of a joint research project involving NRC and NCTU. HSC, with and without fibres, is being considered in this study. Experimental studies on thermal and deformation properties are being carried out at NRC while tests on mechanical properties are being carried out at NCTU. The data obtained from the studies will be used to develop thermal and mechanical relationships, as a function of temperature, for HSC. These relationships can be used as input in computer programs to determine the behaviour of HSC structural members at high temperatures.
Numerical studies: The development of computer programs for predicting the fire behaviour of high strength concrete columns is currently in progress at NRC (23). The steps, associated with the development of the models, involves the calculation of the fire temperatures and the cross-sectional temperatures, deformations and strength of an HSC column. The effect of spalling will be accounted for through pore pressure computations. The validity of the computer programs will be established by comparing the predictions from the computer programs with test data.

The computer programs will be used to carry out detailed parametric studies to study the influence of the various parameters such as cross-sectional shape, slenderness ratio, the concrete strength, the load intensity, type of aggregate, type of concrete and the spalling of concrete. Data from the parametric studies will be used to develop design guidelines to overcome the problem of spalling in HSC columns and for predicting the fire resistance of HSC columns.

The above studies, currently in progress at NRC, will facilitate the application of fibre reinforcement to HSC to enhance its structural performance at elevated temperatures.

SUMMARY

Steel fibre-reinforced concrete exhibits, at elevated temperatures, material properties that are more beneficial to fire resistance than those of plain concrete. The superior performance of steel fibre-reinforced concrete, under fire conditions, compliments the other beneficial properties such as resistance to crack growth, resistance to thermal shock and energy absorption. This makes steel fibre-reinforced concrete an attractive material in fire resistance applications.

In recent years, a number of building applications have been developed for fibre-reinforced concrete. The high fire resistance of steel fibre-reinforced concrete is advantageous in such applications, since most building components must satisfy fire resistance requirements. Fibre-reinforced concrete (both steel and polypropylene) has good potential for overcoming the problem of spalling in high strength concrete structural members.

ACKNOWLEDGEMENTS

The research described in this paper is the result of partnerships between the National Research Council of Canada and the Canadian Steel Construction Council, Portland Cement Association (PCA), Canadian Portland Cement
Association, Concrete Canada and the National Chiao Tung University in Taiwan. PCA has contributed both financially and technically to conduct experimental studies while the other partners have been providing technical expertise for the ongoing studies described in this paper.
REFERENCES


228 Kodur


21. ACI Committee 318. Building code requirements for reinforced concrete. ACI 318-95 and Commentary-ACI 318R-95. 1995. American Concrete Institute, Detroit, MI.

TABLE 1—BATCH QUANTITIES AND PROPERTIES OF HIGH-STRENGTH CONCRETE MIX.

<table>
<thead>
<tr>
<th>Property</th>
<th>Mix 1</th>
<th>Mix 2</th>
<th>Mix 3</th>
<th>Mix 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cement content (kg/m³)</td>
<td>500</td>
<td>500</td>
<td>500</td>
<td>500</td>
</tr>
<tr>
<td>Fine aggregate (kg/m³)</td>
<td>700</td>
<td>700</td>
<td>700</td>
<td>700</td>
</tr>
<tr>
<td>Coarse aggregate (kg/m³) (10 mm)</td>
<td>1100</td>
<td>1100</td>
<td>1100</td>
<td>1100</td>
</tr>
<tr>
<td>Aggregate type</td>
<td>Siliceous</td>
<td>Carbonate</td>
<td>Siliceous</td>
<td>Carbonate</td>
</tr>
<tr>
<td>Water (kg/m³)</td>
<td>140</td>
<td>140</td>
<td>140</td>
<td>140</td>
</tr>
<tr>
<td>Water - cement ratio</td>
<td>0.28</td>
<td>0.28</td>
<td>0.28</td>
<td>0.28</td>
</tr>
<tr>
<td>Retarding admixture (mL/m³)</td>
<td>966.7</td>
<td>1450</td>
<td>1450</td>
<td>1450</td>
</tr>
<tr>
<td>Silica fume (kg/m³)</td>
<td>45.7</td>
<td>50</td>
<td>42</td>
<td>42</td>
</tr>
<tr>
<td>Superplasticizer (mL/m³)</td>
<td>9000</td>
<td>13500</td>
<td>6750</td>
<td>11000</td>
</tr>
<tr>
<td>Steel fibre (kg/m³)</td>
<td>–</td>
<td>–</td>
<td>42</td>
<td>42</td>
</tr>
<tr>
<td>28 day compressive strength (MPa)</td>
<td>80.6</td>
<td>83</td>
<td>87.2</td>
<td>82.9</td>
</tr>
<tr>
<td>90 day compressive strength (MPa)</td>
<td>87.1</td>
<td>85.8</td>
<td>91.7</td>
<td>87.3</td>
</tr>
</tbody>
</table>

Fig. 1—Compressive strength of plain and steel fiber-reinforced concrete as a function of temperature.
Fig. 2—Tensile strength of plain and steel fiber-reinforced concrete as a function of temperature.

Fig. 3—Ultimate strain of plain and steel fiber-reinforced concrete as a function of temperature.
Fig. 4—Modulus of elasticity of plain and steel fiber-reinforced concrete as a function of temperature.

Fig. 5—Axial deformations of plain and steel fiber-reinforced concrete at elevated temperatures for various stress levels.
Fig. 6—View of high-strength reinforced concrete columns exposed to fire.

Fig. 7—Axial deflection as a function of time for three types of concrete-filled HSS columns.
Fig. 8—Elevation and cross-section of HSC columns.
Structural Application of Polyolefin Fiber Reinforced Concrete

by V. Ramakrishnan

Synopsis: This paper presents the construction of a bridge deck and jersey barriers with the newly developed polyolefin fiber reinforced concrete. This is the first time this synthetic fiber-reinforced concrete was used in the construction of reinforced concrete structural elements such as bridge deck and jersey barriers.

The mixture proportions used, the procedure used for mixing, transporting, placing, consolidating, finishing and curing are described. This new polyolefin fiber-reinforced concrete with enhanced fatigue, impact resistance, modulus of rupture, ductility, and toughness properties is particularly suitable for the construction of durable highway structures.

Keywords: bridge-deck; construction applications; fiber reinforced concrete; flexural toughness; impact strength; nonmetallic fibers; overlay; performance characteristics; polyolefin fiber; toughness indices
ACI Fellow, V. Ramakrishnan is Distinguished professor of Civil Engineering at the South Dakota School of Mines and Technology. He received his Ph.D degree in 1960 from the University of London, UK. He is currently Chairman of the TRB Committee A2E05 Admixtures and Cementitious Materials for Concrete and formerly Chairman of the ACI Committee 214 and the TRB Committee on Mechanical properties of concrete, and a member of several Committees in ACI, TRB and ASCE. He is author of two books and over 200 papers on concrete structures and concrete technology and is recipient of numerous awards.

INTRODUCTION

Due to a decaying infrastructure and tightening budget constraints, transportation engineers are challenged to rehabilitate existing facilities economically with an increase in performance. However, simultaneous improvements in cost and performance are unlikely unless new material technology can be exploited. The recently developed polyolefin fiber-reinforced concrete is one material that promises to provide many advantages, providing a practical approach to enhanced durability and cost-effectiveness in concrete compositions. It eliminates problems such as staining, inherent corrosion and potentially harmful protrusions. (1, 2) It has been shown in earlier research and publications by the author (6 to 9) that fiber reinforced concrete (FRC) with its enhanced properties beneficial in structural applications is a highly suitable material for the construction and/or rebuilding bridges and other transportation structures.

Polyolefin fiber-reinforced concrete incorporates 50 mm by 0.64 mm (2" by 0.025") fibers into the concrete mix. These fibers are longer and stronger than plastic fibers previously used to reinforce concrete, and a proprietary packaging technology enables rapid and uniform mixing into the concrete matrix at quantities up to 2% by volume. These volumes of fiber significantly alter the concrete's physical properties, especially toughness, impact resistance, fatigue strength, shear strength, ductility, and resistance to shrinkage cracking. The improved properties make polyolefin fiber reinforced concrete an attractive material for bridge deck construction and/or replacements (1, 2).

To be successful and long-lived, a new bridge deck or a replacement deck must be durable and resistant to fatigue, and must remain uncracked to resist intrusion of chlorides. In the past, transportation agencies throughout the nation have found these requirements (especially low cracking) difficult to
achieve. Several research projects have been undertaken to solve these problems, but with limited success. However, these challenges perfectly match polyolefin fiber-reinforced concrete's characteristics.

The South Dakota Department of Transportation has sponsored research to investigate the properties and practicality of polyolefin fiber-reinforced concrete. Through laboratory tests at the South Dakota School of Mines and Technology and construction of a segment of pavement, a bridge deck overlay, concrete barrier replacement, and a thin unbounded overlay of asphalt bridge approaches, the material proved to be workable and significantly more resistant to early cracking than ordinary concrete. The research results demonstrated increased fatigue capacity of 150%, crack width reductions below American Concrete Institute (ACI Committee 224 Report on Cracking) recommendations (10) for chloride intrusion, and skid resistant surface texture. The favorable research results warrant more widespread use of polyolefin fiber-reinforced concrete in other applications, including the construction of new bridge decks and barriers and the rehabilitation of deteriorating bridge structures. Both prestressed and reinforced concrete structural elements built with FRC, would have significantly increased toughness and ductility and would better resist earthquake forces and suddenly applied loads (3 to 5).

APPLICATIONS

The construction projects undertaken to evaluate the non-metallic polyolefin fiber-reinforced concrete were a part of repair, rehabilitation and construction of the following structures:

1. Thin bridge-deck bonded overlay on the bridge at Vivian (the bridge on the U.S. 83 number 43-026-195, over I-90 south of Pierre, SD).
2. Reinforced concrete Jersey barrier on the above referred bridge.
3. A total replacement of bridge-deck slab on the bridge at Spearfish, SD (bridge over I-90, exit 10).

The research activities involved were the development of mixture proportions, quality control testing, and advice on the construction, monitoring and evaluation of the above structures. The research activities also involved periodic condition surveys to evaluate the performance of the constructed bridge-deck overlays, barriers, and the bridge-deck slab.
RESULTS OF CONTROL TESTS AND DISCUSSION

Bridge-Deck Overlay Concrete Properties

The same mixture proportions were used for the plain and fiber reinforced concrete as shown in Table B1. Fresh concretes were tested for slump (ASTM C143), air content (ASTM C231), fresh concrete unit weight and yield (ASTM C138). The fresh properties and the concrete temperature are given Table B2. The table also includes the results of tests conducted by the DOT personnel. The slumps and air contents measured were satisfactory and they were within the range specified by DOT. The concrete temperatures varied between 25.6°C to 28.9°C (68°F to 84°F). The actual measured fiber content in the samples taken from the field concrete were close to the specified amounts.

The hardened concrete properties such as the compression strength, modulus of elasticity, and impact strengths are given in Table B3. The compressive strengths varied from 37.6 Mpa to 44.3 Mpa (5445 psi to 6420 psi) which is a tolerable variation in the field concrete. The variation of the elastic modulus was consistent with that of the compressive strength variation. The concrete was tested for impact strength by the drop weight test method (ACI Committee 544) (11). There was a high impact strength due to the addition of polyolefin fibers in concrete. The number of blows for ultimate failure in fiber concrete was above 200 whereas it was between 20 to 25 for plain concrete.

The first crack strength and modulus of rupture values are compared in Fig. B1. There was not a significant variation in the modulus of rupture (flexural strength) for different batches and it was about 5.5 MPa (800 psi). The flexural strength of fiber concrete at 28 days was about 4% higher than that of plain concrete.

The toughness indices calculated according to the ASTM C1018 standard procedures are shown in Fig. B2 and the residual strengths factor (R-values) are compared in Fig. B3. The first crack toughness is compared in Fig. B4. These comparisons showed positively that the addition of polyolefin fibers had increased the toughness of the concrete. The ASTM toughness values were approximately the same at 7 and 28 days which was normally expected. The R-values indicated a ductile behavior.

The Japanese Standard (JCI) flexural toughness factors are compared in Fig. B5. This comparison also confirms the increase in toughness and ductility of the concrete due to the addition of polyolefin fibers.
**Jersey Barrier Concrete Properties**

The standard class A45 concrete, as specified in the SD DOT Standard Specifications for Roads and Bridges Section 460 was used for the construction of the barriers. Polyolefin fibers were added to the basic mixture proportions as given in Table J1.

The same quality control tests for fresh and hardened concrete were done as described above. The fresh concrete properties are given in Table J2. As anticipated, the addition of fibers reduced the slump of the concrete. There was not a significant difference in the air contents of plain and fiber concretes and the unit weights were nearly the same. The actual fiber content in the field concrete was slightly less than the specified quantity in Mix J2 and slightly higher than the specified quantity in Mix J3.

The hardened concrete properties such as compressive strength, modulus of elasticity and impact strength are given in Table J3. There was not much difference in these values for the three mixes except the impact strength which was significantly higher for the fiber concretes. The average number of blows for ultimate failure were 128 and 232 respectively for polyolefin fiber concretes with 11.9 Kg/m$^3$ (20 lbs./cu.yd.) and 14.8 Kg/m$^3$ (25 lbs./cu.yd.).

The first crack strength and modulus of rupture values are plotted in Fig. J1. There was not a significant difference between the cracking strength and the ultimate strength due to the addition of fibers. The modulus of rupture was about 4.5 MPa (650 psi). The first crack toughness and the toughness indices calculated from the load-deflection curves according to ASTM C1018 Procedures are given respectively in Fig. J1 and J2. The residual strength factors (R-values) are plotted in Fig. J3 and the flexural toughness factors calculated according to Japanese Standard (JCI) are plotted in Fig. J4. As anticipated, the addition of polyolefin fibers greatly increased the toughness of the concrete. The ASTM toughness indices I5, I10, and I20 are approximately 4, 8, and 15 times higher than that of plain concrete.

**CONSTRUCTION DETAILS AND PERFORMANCE**

**Construction of Bridge-Deck Overlay**

A thin bridge-deck bonded overlay was constructed on the bridge at Vivian (on the U.S. 83 number 43-026-195, over I-90 South of Pierre, SD). One side of the bridge deck overlay was constructed with the polyolefin FRC and the other side with normal portland cement concrete (PCC). Different quantities of
fibers 11.9 and 14.8 kg/m³ (20 lbs./cu.yd. and 25 lbs./cu.yd.) were used. The concrete was batched at a central mixing plant and transported to the site in a truck mixer. The fibers were then added at site and mixed for additional time. The mixture proportions selected for the plain concrete mix and fiber quantities were worked out in the laboratory. The mixture proportions were such that the fiber concrete had the same workability and compatibility as ordinary low slump concrete. The slump and density were as per the SD DOT specifications. The same mixture proportion used for the plain, low slump dense concrete, as specified in the SDDOT Standard Specifications for Roads and Bridges, was used for the polyolefin FRC concrete. The slump was specified as 25 mm (1.0 inch) maximum.

Traces of corrosion on the entire deck except in spots were there was significant corrosion of the reinforcement, was removed by sand blasting. In some parts of the deck the concrete had severely deteriorated. At these locations, all the loose concrete was removed. The deck overlay on the east side was constructed with a low slump concrete. The concrete was mixed at the site using a mobile mixer. The bridge deck overlay on the west side was constructed with polyolefin FRC. Approximately one-half of the length of the deck on the north side was overlaid with polyolefin FRC containing 11.9 kg/m³ (20 lbs./cu.yd.) of fibers and the remaining length was overlaid with polyolefin FRC containing 14.8 kg/m³ (25 lbs./cu.yd.) of fibers. After testing for slump and air content, fibers were added and mixed for 4 to 5 minutes. The slump and air content were determined again. As the slump was almost zero, the concrete was retempered with water and the slump and air content were determined again and found to be satisfactory. The concrete was then discharged into a power buggie and transported to the site. For the subsequent trucks, fibers were added first and mixed, and the slump and air content were determined.

The same paving machine and the same procedure for placing, consolidating and finishing that were used for the east side (plain mobile mixer concrete) were used for the polyolefin FRC concrete also. There were no difficulties encountered in the placing, consolidating and finishing operations. The tining was done by turning the tining rake over so that the tines were not vertical to avoid the possible pulling out of the fibers. In addition, a little increased pressure was applied to obtain the proper tining depth and all tining was done in the same direction for the deck overlay. It is suggested that the plain concrete should also be tined in this way so that the tining operations will consolidate the concrete by pushing the aggregate pieces down instead of pulling the aggregate (both fine and coarse) pieces out (dislodging them) during the tining operation. The ready mixed polyolefin FRC concrete was as workable, placeable, and finishable as the low slump concrete mixed with the mobile mixer. The curing procedure used was as per the SD DOT specifications. The same procedures were used for both mobile mixed low slump concrete and ready-mixed polyolefin FRC concrete.
Inspection of the bridge-deck overlay showed no shrinkage cracks. The bonding appeared to be good. The appearance of the east side and west side of the bridge deck was almost the same, except for a few fibers visible on the west side. The tinning appeared to be the same. There was a good bond between the plain low slump and the polyolefin FRC concrete overlays along the centerline. The part of the deck slab that was chipped off showed a uniform distribution of the fibers over the entire depth of the overlay. The fibers were well bonded to the concrete matrix. There was no distress, popouts, scaling, sign of debonding or delamination on the deck slab.

Construction of Jersey Barrier

The standard class A45 concrete, as specified in the SD DOT Standard Specifications for Roads and Bridges Section 460 was used for the construction of the jersey barrier. The fine to coarse aggregate ratio was 40.8/59.2 and the cement content was 397 kg/m³ (670 lbs./cu.yd.) and the water to cement ratio was 0.4. The polyolefin FRC barrier on the west side was constructed in three days. The north half of the barrier on the west side of the bridge was constructed using polyolefin FRC with fiber equal to 11.9 kg/m³ (20 lbs./cu.yd.). It was decided to use the same mixture proportions for the construction of the Jersey Barrier on both sides of the bridge using plain concrete and polyolefin FRC concrete.

The ready mixed concrete was supplied by the Presho plant. After the trucks arrived at the site, slump and air content of the concrete were determined and the fibers were added and mixed for approximately 4 to 5 minutes. The concrete was slightly stiff and there was some difficulty in discharging the concrete from the truck. Since the Jersey barrier was heavily reinforced, the concrete did not flow easily; it had to be pushed down. Also, the shallow angle of the shoot did not provide the concrete to flow down the shoot. This was a result of the barrier height. The placement was slow since there was only one vibrator available. Since there was delay in placement, the concrete had to be retempered with superplasticizer. When the side form work was removed, it was noticed that the concrete had been well consolidated and there was no honeycombing. There were some fibers exposed and sticking out. There was no difficulty in finishing the surface. The exposed fibers could easily be burned off when necessary.

The barriers were carefully inspected as soon as the form work was removed and also after a week. During the inspection, it was observed that in both barriers with and without fiber-reinforced concrete, there were numerous shrinkage cracks. These were drying shrinkage cracks caused by the reinforcement restraint and no expansion, contraction, or construction joints over
a long length. Initially when the side forms were removed, there were no cracks. Hence these cracks are not considered as plastic shrinkage cracks. The length and width of these cracks were accurately measured using a crack comparator which can measure the width accurate to 0.05 mm (0.002 in.). These widths were also verified by a crack measuring, hand held, microscope with graduated cross wires. In longer cracks, the widths were measured at 3 to 5 locations and the average was taken.

The ages of the concretes were the same for both east and west barriers when the crack measurements were made. In the barrier with the fiber reinforced concrete, 88 cracks were observed and these were uniformly distributed. The widths of these cracks were very small, less than 0.18 mm (0.007 in.), except a few. In the barrier with the plain concrete, cracks could not be measured for a small part (about 10 ft.) due to an obstruction. In the remaining section of the barrier, 50 cracks were observed. Most of these cracks were wider than 0.18 mm (0.007 in.). The American Concrete Institute Committee 224 on cracking has recommended the maximum permissible crack widths for different conditions of exposure (10). If the cracks are narrower than recommended widths, then it can be assumed that the possibility of corrosion of the reinforcement due to moisture penetration is negligible and the concrete is durable. The maximum crack width that is permissible under the environmental conditions at the bridge (exposed surface subjected to deicing chemicals) is 0.18 mm (0.007 in.). Calculations had shown that only 7 percent of the cracks were wider than 0.18 mm (0.007 in.) in the barrier with the fiber-reinforced concrete. Whereas in the case of the plain concrete barrier, 85 percent of the cracks were wider than 0.18 mm (0.007 in.). It is known that any reinforcement, including randomly oriented fiber reinforcement, can not prevent cracking of concrete. The addition of fibers would prevent widening of the cracks once they were formed and the cracking would be more uniform. Therefore the observations made here were anticipated. The crack size distribution is shown in Fig. 16, and the average crack widths for barriers with and without fibers are given below.

<table>
<thead>
<tr>
<th>Number of Cracks</th>
<th>Crack Width in mm (inch)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Total</td>
</tr>
<tr>
<td>NMFRC</td>
<td>88</td>
</tr>
<tr>
<td>No Fiber (NF)</td>
<td>50</td>
</tr>
</tbody>
</table>
New Deck Slab Construction

The concreting for the bridge deck was done in two placements. The weather conditions were satisfactory during the construction of the deck slab. Concrete was mixed at Birdsall Sand and Gravel Plant in Spearfish, SD, which is close to the bridge site. Concrete was mixed and transported in trucks and delivered to the hopper of the pump. The air content and slump were checked. In the beginning, the contractor had difficulty in estimating the correct amount of air entraining agent to be added because of the pumping of the concrete involved. For the first few trucks, the air content was adjusted at site, for specified air content and slump. Later, the contractor was able to deliver the concrete for the specified air content and slump.

For the first placement of the bridge deck, the placing, consolidating, finishing and tining operations were carried out without any problems. For the second placement, the same mixture proportions and the same procedures as per the first placement were used for mixing, transporting, pumping, placing, consolidating, finishing, and tining operations. No problems were encountered in the beginning. During the middle of the placement, two trucks delivered concrete in which a few fiber bundles were not opened. Two minutes of additional mixing corrected this problem. The rest of the concreting was done without any problems. In general, the fibers were well mixed, and they were also uniformly distributed. In our opinion, no additional effort was needed on the part of the contractor, for transporting, placing and finishing operations. However, some additional expense was needed to add the fibers and mix in the concrete, when compared to plain concrete without fibers.

Barrier Construction

The barrier construction was not done in one operation because the contractor did not have adequate forms. For the first and second time, the concrete was pumped. The trucks could not go on the bridge deck, as the approach slab had not attained the required strength. Additional placements, were done using concrete from the truck. The contractor preferred to use a low slump concrete, about 50 to 64 mm (2 to 2-1/2 inches). Hence there was difficulty in placing and vibration. It had to be vibrated more and this slowed the construction of the barrier. In our opinion, for a thin, heavily reinforced section, such as the barrier, the concrete should have a slump of 4-1/2 to 5 inches. Then minimum amount of vibration will be needed and placement will be easier. This higher slump can be achieved without a reduction in compressive strength, with the addition of an appropriate amount of superplasticizer.
The total concrete used for construction of bridge deck, barrier and haunches over girders was 325 m$^3$ (425 cu.yd.). Total quantity of fibers required was 4812 kg (10,623 lbs.). Inspection of the bridge deck top surface indicated no cracks. There was no spalling, no scaling or any other distress. Fibers were visible at the surface; however they were well bonded to the concrete. It is expected that these exposed fibers would wear out in the course of time due to the traffic.

Inspection of the barrier indicated two hair line cracks less than 0.1 mm (0.0039 inch) wide in the already constructed portion of the barrier. Subsequent inspections revealed a few additional hairline cracks. They were not easily visible. After about 10 weeks, a detailed inspection was made using magnifying lenses. The crack widths were also measured. There were three hair line cracks on the east side barrier. One crack was 0.1 mm (0.0039 inch) wide and the other two were 0.08 mm (0.003 inch) wide. All these cracks were shrinkage cracks. The width of these cracks were less than 0.18 mm (0.007 inch), as per the ACI Committee 224 recommended maximum tolerable crack width under the environmental conditions at the bridge.

The deck slab bottom side was inspected using “snoopers” to look closely at the bottom side of the deck slab. A detailed and careful inspection of the entire deck slab indicated that there were totally 8 cracks. The length and width of all these cracks were measured.

There were a total of six cracks on the cantilevered part of the slab on the east side of the deck. Three of the cracks were wider than 0.18 mm (0.007 inch) and the other three were less than 0.18 mm (0.007 inch) in width. There were only three cracks in the main slab, two of them just above the two end pylon supports. The third one was a small crack about the middle of the bridge 38.1 mm (15 inch) long and 0.2 mm (0.0079 inch) wide. No crack extended over the entire width of the slab. The south end crack was 2 meters (6 ft. 8 in.) long and 0.47 mm (0.0183) wide and the north end crack was 2.56 m (8 ft. 5 in.) long and only 0.13 mm (0.005 inch) wide.

There was white efflorescence in all these cracks which enhanced the visibility of these cracks and made these cracks look wider and longer. All the cracks were perpendicular to the traffic direction and parallel to the main reinforcement. Therefore the cracks were not induced due to any bending action. These cracks were due to restrained shrinkage.

The purpose of adding fibers was not to increase the flexural strength (the plain concrete had adequate flexural strength), but primarily to reduce the initial plastic shrinkage cracking and to reduce the overall cracking in the structures due to drying shrinkage and thermal changes during the lifetime. The minimum and maximum temperatures may vary approximately from -35°C (-
20°F) to 43° C (110°F). The fibers were also added to enhance the fatigue strength, the impact strength, toughness and ductility of the concrete. The fiber dosage was selected to achieve the above stated objectives. The properties of the field concretes determined from control tests and the observed performance of the structures observed for 3 years had shown that the objectives were achieved. The newly constructed bridge deck with FRC had shown considerable less longitudinal cracking that normally occurs at the bottom side of the slab in such newly constructed bridges. A comparison of the visually observed cracks in the identical bridge built without the addition of fibers in the concrete at the same I-90 exit 10 over the east bound lanes has confirmed the above observation. There had been practically no shrinkage cracking on the top surface and the barriers.

CONCLUSIONS

It was possible to incorporate the newly developed high-tech, non-metallic, synthetic (polyolefin) fibers in concrete at 14.8 kg/m³ (25 lbs./cu.yd.) without causing any balling, clogging and segregation. The advantages of adding polyolefin fibers are:

1. More number of fibers are added in concrete ensuring more uniform distribution and consistent results.
2. Less chance for balling, segregation, bleeding or causing any other construction problems during mixing, placing, consolidating, finishing and tining operations.
3. Fibers are non-corrosive, non-hazardous, non-metallic, and non-magnetic. They do not protrude from the surface; if they do, they could easily be burned off.

Addition of polyolefin fibers at 11.9 kg/m³ or 14.8 kg/m³ (20 lbs./cu.yd. or 25 lbs./cu.yd.) enhanced the structural properties of concrete. There was a considerable increase in toughness, impact, fatigue, endurance limit, and post-crack load-carrying capacity. There was no difficulty or problem encountered during the mixing, transporting, placing, consolidating and tining of the average 75 mm (3 inch) thick overlay with this mix specially proportioned for the addition of polyolefin fibers. The required or specified workability and finishability could be achieved in polyolefin FRC.

The same construction techniques and construction equipment without any modification could be used in the construction of pavements, bridge deck overlays, barriers and white-topping using polyolefin FRC.
In the jersey barrier, the addition of fibers reduced the shrinkage crack width to a level permissible in the exposed surfaces (less than 0.18mm (0.007 inch) and a more desirable crack distribution (more number of uniformly distributed thinner cracks than a fewer number of wide cracks) was obtained. In the plain concrete jersey barrier, almost 85 percent of the cracks were wider than 0.18 mm (0.007 inch) whereas in polyolefin FRC barrier, there were only a few cracks (7 percent) that were wider than 0.18 mm (0.007 inch). In addition to reduction of crack width, the addition of fibers had increased the impact strength and toughness of the concrete.

ACKNOWLEDGMENT

The author gratefully acknowledges funding received from SD DOT and expresses gratitude to David Huft and the Technical Panel of the SD DOT for supporting and encouraging this research. The support and encouragement received for this research from the 3M Company is also gratefully acknowledged.

REFERENCES


4. Ramakrishnan, V., “Recent Advancements in Concrete Fibre Composites”, Concrete Lecture - 1993, Published by American Concrete Institute, Singapore Chapter, Singapore, 1993, 28 pages.


10. ACI Committee 224, "Control of Cracking in Concrete Structures, 224-84" American Concrete Institute, Detroit, 1989.

TABLE B1—MIXTURE PROPORTIONS USED FOR BRIDGE DECK OVERLAY CONCRETE.

<table>
<thead>
<tr>
<th>Mix Type</th>
<th>Mixture Proportions Kg/m³ (lbs/cu.yd)</th>
<th>AEA mL/m³ (ounce/cu.yd)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Cement Coarse Aggregate Fine Aggregate</td>
<td>Water</td>
</tr>
<tr>
<td>PLAIN B1</td>
<td>488 (823) 827 (1394) 827 (1394) 0</td>
<td>160 (270) 657.6 (17)</td>
</tr>
<tr>
<td>FRC B2</td>
<td>488 (823) 827 (1394) 827 (1394) 11.86 (20*)</td>
<td>160 (270) 657.6 (17)</td>
</tr>
<tr>
<td>PLAIN B3</td>
<td>488 (823) 827 (1394) 827 (1394) 0</td>
<td>160 (270) 657.6 (17)</td>
</tr>
<tr>
<td>FRC B4</td>
<td>488 (823) 827 (1394) 827 (1394) 14.83 (25*)</td>
<td>160 (270) 657.6 (17)</td>
</tr>
</tbody>
</table>

AEA - Air Entraining Agent mL/m³ (ounces/cu.yd).
* Polyolefin fibers 50.8 mm (2 in) long and 0.635 mm (0.025 in) diameter.

TABLE B2—PROPERTIES OF FRESH CONCRETE.

<table>
<thead>
<tr>
<th>Mix Type</th>
<th>Test #</th>
<th>Time a.m.</th>
<th>Slump (mm)</th>
<th>Air Content (%)</th>
<th>Concrete Temp (°C)</th>
<th>Unit Weight (Kg/m³)</th>
<th>Yield (m³)</th>
<th>Fiber Content (Kg/m³)</th>
</tr>
</thead>
<tbody>
<tr>
<td>PLAIN B1</td>
<td>1*</td>
<td>6.50</td>
<td>12.70</td>
<td>6.2</td>
<td>20.0</td>
<td>2303.7</td>
<td>---</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>2*</td>
<td>8.50</td>
<td>25.40</td>
<td>7.0</td>
<td>24.4</td>
<td>2232.9</td>
<td>---</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>3*</td>
<td>10.40</td>
<td>19.05</td>
<td>6.3</td>
<td>27.7</td>
<td>2297.3</td>
<td>---</td>
<td>0</td>
</tr>
<tr>
<td>FRC B2</td>
<td>1*</td>
<td>6.45</td>
<td>6.35</td>
<td>4.1</td>
<td>18.9</td>
<td>---</td>
<td>---</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>2*</td>
<td>6.50</td>
<td>12.70</td>
<td>5.4</td>
<td>18.9</td>
<td>---</td>
<td>---</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>7.00</td>
<td>3.17</td>
<td>5.4</td>
<td>20.0</td>
<td>2356.2</td>
<td>3.00</td>
<td>11.68</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>7.45</td>
<td>6.35</td>
<td>6.0</td>
<td>21.1</td>
<td>---</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>8.30</td>
<td>38.10</td>
<td>9.0</td>
<td>21.1</td>
<td>2310.1</td>
<td>3.06</td>
<td>11.28</td>
</tr>
<tr>
<td></td>
<td>6*</td>
<td>8.45</td>
<td>19.05</td>
<td>6.0</td>
<td>23.3</td>
<td>---</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>PLAIN B3</td>
<td>1</td>
<td>6.45</td>
<td>57.15</td>
<td>5.6</td>
<td>23.3</td>
<td>2376.1</td>
<td>---</td>
<td>0</td>
</tr>
<tr>
<td>FRC B4</td>
<td>1*</td>
<td>10.45</td>
<td>6.35</td>
<td>6.2</td>
<td>26.6</td>
<td>2376.1</td>
<td>3.73</td>
<td>15.37</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>11.30</td>
<td>12.70</td>
<td>6.2</td>
<td>26.6</td>
<td>2376.1</td>
<td>3.73</td>
<td>16.04</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>12.15</td>
<td>19.05</td>
<td>5.6</td>
<td>28.9</td>
<td>---</td>
<td>---</td>
<td>---</td>
</tr>
</tbody>
</table>

Notes * Tests conducted by SDDOT personnel.
--- No tests were done.

1 lb/cu.yd. = 0.593 Kg/m³
1 lb/ft³ = 16.02 Kg/m³
° C = 0.56 (° F - 32)

TABLE B3—HARDENED CONCRETE PROPERTIES.

<table>
<thead>
<tr>
<th>MIX TYPE</th>
<th>E (10⁶ psi)</th>
<th>f'c' (psi)</th>
<th>IMPACT STRENGTH** Number of blows First Crack Failure</th>
<th>JCI Equivalent Flexural Strength Mpa (psi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>PLAIN B1</td>
<td>36846</td>
<td>44,298</td>
<td>12</td>
<td>15</td>
</tr>
<tr>
<td>FRC B2</td>
<td>36777</td>
<td>42,504</td>
<td>37</td>
<td>200</td>
</tr>
<tr>
<td>PLAIN B3</td>
<td>36639</td>
<td>42,263</td>
<td>21</td>
<td>24</td>
</tr>
<tr>
<td>FRC B4</td>
<td>29394</td>
<td>37,571</td>
<td>31</td>
<td>231</td>
</tr>
</tbody>
</table>

NOTE: E = Static Modulus
f'c' = Compressive Strength** Average of three specimens
Average of five specimens
TABLE J1—MIXTURE PROPORTIONS USED FOR JERSEY BARRIER CONCRETE.

<table>
<thead>
<tr>
<th>Mix Type</th>
<th>Mixture Proportions Kg/m³ (lbs/cu.yd)</th>
<th>AEA ml/m³ (ounce/cu.yd)</th>
</tr>
</thead>
<tbody>
<tr>
<td>PLAIN J1</td>
<td>397 (670) 1025 (1728) 705 (1189) 0 161.3 (272)</td>
<td>464.2 (12)</td>
</tr>
<tr>
<td>FRC J2</td>
<td>397 (670) 1025 (1728) 705 (1189) 11.86 (20*) 161.3 (272)</td>
<td>464.2 (12)</td>
</tr>
<tr>
<td>FRC J3</td>
<td>397 (670) 1025 (1728) 705 (1189) 14.83 (25*) 161.3 (272)</td>
<td>386.8 (10)</td>
</tr>
</tbody>
</table>

AEA - Air Entraining Agent ml/m³ (ounces/cu.yd).

* - Polyolefin fibers 50.8 mm (2 in) long and 0.635 mm (0.025 in) diameter.

TABLE J2—PROPERTIES OF FRESH CONCRETE (JERSEY BARRIER).

<table>
<thead>
<tr>
<th>Mix Type</th>
<th>Test #</th>
<th>Time</th>
<th>Slump (mm)</th>
<th>Air Content (%)</th>
<th>Concrete Temp (°C)</th>
<th>Unit Weight (Kg/m³)</th>
<th>Yield (m³)</th>
<th>Fiber Content (Kg/m³)</th>
</tr>
</thead>
<tbody>
<tr>
<td>PLAIN J1</td>
<td>1*</td>
<td>3.15p.m</td>
<td>120.6</td>
<td>8.0</td>
<td>29.4</td>
<td>---</td>
<td>---</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>2*</td>
<td>3.15p.m</td>
<td>127.0</td>
<td>8.5</td>
<td>28.9</td>
<td>---</td>
<td>---</td>
<td>0</td>
</tr>
<tr>
<td>FRC J2</td>
<td>1*</td>
<td>3.25p.m</td>
<td>44.45</td>
<td>7.6</td>
<td>31.5</td>
<td>2296.8</td>
<td>4.59</td>
<td>10.887</td>
</tr>
<tr>
<td></td>
<td>2*</td>
<td>3.30p.m</td>
<td>60.32</td>
<td>8.0</td>
<td>29.4</td>
<td>---</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td></td>
<td>3*</td>
<td>3.50p.m</td>
<td>44.25</td>
<td>6.0</td>
<td>30.0</td>
<td>---</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>9.00a.m</td>
<td>31.75</td>
<td>7.0</td>
<td>26.1</td>
<td>2310.1</td>
<td>3.87</td>
<td>---</td>
</tr>
<tr>
<td></td>
<td>5*</td>
<td>9.10a.m</td>
<td>44.25</td>
<td>6.2</td>
<td>26.1</td>
<td>---</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>FRC J3</td>
<td>1*</td>
<td>8.25a.m</td>
<td>63.50</td>
<td>9.2</td>
<td>24.4</td>
<td>2237.5</td>
<td>4.74</td>
<td>17.339</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>8.25a.m</td>
<td>73.02</td>
<td>10.0</td>
<td>24.4</td>
<td>---</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td></td>
<td>3*</td>
<td>8.50a.m</td>
<td>57.15</td>
<td>7.8</td>
<td>24.4</td>
<td>---</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>9.00a.m</td>
<td>57.15</td>
<td>8.0</td>
<td>25.5</td>
<td>2257.2</td>
<td>4.00</td>
<td>---</td>
</tr>
<tr>
<td></td>
<td>5*</td>
<td>9.00a.m</td>
<td>38.10</td>
<td>6.6</td>
<td>25.5</td>
<td>---</td>
<td>---</td>
<td>---</td>
</tr>
</tbody>
</table>

Notes:
* Tests conducted by SDDOT personnel.
--- No tests were done.

Conversions:
1 (yd)³ = 0.765 m³
1 lb/cu.yd = 0.593 Kg/m³
1 lb/(ft)³ = 16.02 Kg/m³
0°C = 32 °F

TABLE J3—HARDENED CONCRETE PROPERTIES (JERSEY BARRIER).

<table>
<thead>
<tr>
<th>MIX TYPE</th>
<th>E Mpa (10⁶ psi)</th>
<th>fc' Mpa (psi)</th>
<th>IMPACT STRENGTH&quot;</th>
<th>JCI Equivalent Flexural Strength MPa (psi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>PLAIN J1</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>FRC J2</td>
<td>31050 (4.50)</td>
<td>30.670 (4445)</td>
<td>31 128</td>
<td>2.74 (397)</td>
</tr>
<tr>
<td>FRC J3</td>
<td>32085 (4.65)</td>
<td>32.906 (4769)</td>
<td>38 232</td>
<td>2.73 (395)</td>
</tr>
</tbody>
</table>

NOTE:
E = Static Modulus
fc' = Compressive Strength

* Average of three specimens
" Average of five specimens
Fig. B1—Comparison of first crack and flexural strength for different mixtures.

Fig. B2—Comparison of toughness indices 15, 110, 120 for different mixtures.

Fig. B3—Comparison of residual strength factors.
Fig. B4—Comparison of first crack toughness for different mixtures.

Fig. B5—Comparison of flexural toughness factor (JCI) for different mixtures.

Fig. J1—Comparison of first crack and flexural strength for different mixtures.
Fig. J2—Comparison of toughness indices, J5, J10, J20 for different mixtures.

Fig. J3—Comparison of residual strength factors.

Fig. J4—Comparison of first crack toughness for different mixtures.
Fig. J5—Comparison of flexural toughness factor (JCI) for different mixtures.

Fig. J6—Crack size distribution.