External Prestressing: from Construction History to Modern Technique and Technology
by M.P. Virlogeux

Synopsis: The development of external prestressing has been one of the major trends in construction during the last ten years, along with the development of concrete cable-stayed bridges and with the increasing use of high strength concrete. But external prestressing has been used in the early ages of prestressing, in Germany with Franz Dischinger and Ulrich Finsterwalder, in Belgium with Magnel, and in France with Henri Lissier and Coignet. The idea came back in the United States with Jean Muller, and in France under the influence of SETRA. After a historical review, the main principles of the design of externally prestressed bridges are presented. The paper then details the influence of the construction method on the external tendon organization: bridges built span by span, bridges built by the cantilever method or by methods which are mechanically equivalent, and bridges built by the incremental launching method. Some practical problems are finally evoked, such as handling heavy jacks. A last chapter is devoted to composite structures, with concrete top and bottom slabs and with steel webs, prestressed by external tendons. The French experimental constructions of this type did not appear economically interesting, and prestressing classical composite structures is not yet considered as a good solution for the same reason. But external prestressing is now widely developed for concrete bridges in the United States, in France, and more recently in Belgium, in Switzerland, in Venezuela, in Germany and in Czechoslovakia.

Keywords: box beams; bridges (structures); cable-supported structures; cantilever bridges; composite construction (concrete and steel); concrete construction; history; prestressed concrete; prestressing; prestressing steels; reviews; segmental construction; structural design; unbonded prestressing
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INTRODUCTION

In the past ten years, numerous prestressed concrete bridges have been built in the United States of America and in France with external tendons, that is with tendons outside the concrete. More recently, this solution has been used in Belgium and in Venezuela, and will be used in Switzerland and in Germany.

The idea is not new. Several bridges were built in this way in the early days of pre-stressing, namely: 1) in Germany first, with the bridge at Aue in 1936 [5]. Designed by Franz Dischinger, the bridge, built before the wide development of prestressing steel, is prestressed by bars of high strength steel, the yield stress of which is about 500 MPa. 2) In Belgium then, under the influence of Magnel, with the Sclayn bridge in 1950 [8, 9]. And 3) in France between 1950 and 1952: the bridge at Villeneuve-Saint-Georges, designed by Lossier; the bridges at Vaux-sur-Seine and Port à Binson, built by Colignet; and the bridge at Can Bia. In these projects, external prestressing was developed to by-pass Freyssinet's patents on prestressing systems.

These first attempts did not produce excellent results: most of these externally prestressed structures suffered from corrosion.

Smoke from steam locomotives corroded the external prestressing bars of the bridge of Aue, which had to be cleaned and painted. It also required retensioning twice, in 1962 and 1980, to restore the losses caused by steel relaxation and concrete shrinkage and creep, too important for the low steel capacity [13, 14, 51].

For the construction of the Villeneuve-Saint-Georges bridge [12], Henri Lossier used large monostrands – similar to those used for suspension bridges – which were left bare in the box-girder, with a single coat of bitumen paint (figure 1). But Lossier had foreseen the need to make possible the external tendons replacement. This made it possible to replace one cable rusted by water running through an inspection chamber in the top slab. Due to an excellent maintenance, the bridge has not suffered other problems. But the deviations of the external tendons were enforced by very sophisticated and expensive struts in reinforced concrete or steel, which dissuaded engineers from further investigations in this field (figures 2, 3).

The arrangements of the other French structures were more economical, with simple concrete cross-beams for the external tendons deviations. The external tendons were made of small diameter parallel prestressing wires (5 to 7 mm).

In Port à Binson bridge, protection of the external tendons by a bitumen paint proved to be sufficient (figures 4, 5). It would have been in Vaux-sur-Seine bridge, too, if birds had not nested in the bridge, depositing acid droppings on the external tendons which thus suffered from corrosion (and from insufficient maintenance); the bridge had to be strengthened recently with additional external tendons [10, 11]. For the construction history, the tendons were placed and anchored at the abutments, and then only tensioned, by transverse deviations; at some selected points, the tendons were parted by jacks, and these deviations maintained by concreting deviation blocks; at
some other selected points, the tendons were drawn together by jacks, and these deviations maintained by steel braces (figure 6).

On the other hand, the poor design of the Can Bia bridge, with anchorages in the top slab, allowed water to flow along the external tendons which were never repainted after construction, causing major corrosion and rupture of wires making it necessary to close the bridge to traffic some years ago and consider its demolition (figures 7, 8).

This experience gave a poor image of external prestressing, and very few externally prestressed concrete bridges were built in the sixties and in the seventies except for a series of road bridges in Belgium, between 1960 and 1970; and in England the Bournemouth bridge [15], and the Exe and Exminster viaducts [17]. Few of these constructions were a success.

The Belgian road bridges were built with external tendons embedded in a cylinder of concrete; in some bridges, the concrete cracked and the tendons were corroded. These corroded tendons have been recently replaced.

In the Bournemouth bridge, the external tendons were large monostrands, as for Villeneuve-Saint-Georges bridge. But they corroded and had to be replaced some years after construction.

We must note that all these problems concerned prestressing steel corrosion and not poor mechanical behaviour of these bridges—due, for example, to the lack of bond between concrete and tendons. They always performed adequately from the structural point of view, even the Can Bia bridge despite the rupture of about 10 to 15% of prestressing wires.

The recent return to external prestressing, under the influence of French engineers—Jean Muller in the United States, and SETRA in France—, has been made possible by the development of the prestressing technology. The development of high-capacity tendons (12, then 19 strands of 15 millimeters and more) has resulted in a reduction in the number of external tendons which eases design and construction. And, above all, the experience of strengthening some classical prestressed concrete bridges, in which the initial prestressing forces were not great enough, has made it possible to put into use protective systems adapted to ensure resistance to corrosion of external tendons.

Furthermore, experience in strengthening these bridges, which perforce had to be by tendons outside the concrete, made designers aware of the advantages of external prestressing. This led them to consider its use in building new bridges. The principal advantages are the considerable simplification of the lay-out of the tendons and the large reduction in losses of prestress due to friction.

The need to allow for the replacement of the stays during the construction of the footbridges at Meylan and Illhof led to the idea that external tendons could be replaced, not knowing at the time that this had been foreseen by Henri Lossier, 30 years before.

Finally, some traditionally prestressed bridges were built by incremental launching with the help of some temporary external tendons—to increase central prestress during launching without increasing the cross-section weight (bridges at Aiguilly [69.2], Blière [45, 69.2], and more recently Kouilou [44]). And, in the Sathorn bridge, in Thailand, temporary tendons—partly inside the concrete, and partly outside the concrete above the top slab—were placed with a lay-out in opposition to the classical undulated tendons inside the concrete, so as to balance the moment created by internal tendons and produce a central prestress during launching [41].
All these ideas helped the development of external prestressing to the point it is today, in the United States and in France.

To understand the difference in design of externally prestressed bridges in these two countries, one must know the real objectives of this development in each of them.

In the United States, the primary objective is to reduce the construction cost, because contracts are almost always given to the lowest bids. External prestressing has then been a way to reducing the structures weight, mainly by reducing the webs thickness. And the prestressing technology is the simplest possible, the most economical, without considering external tendons replacement. In these conditions, Figg and Muller has been able to build a series of segmental bridges, competing with success against standard AASHTO beams.

The situation is completely different in France. External prestressing, developed under the influence of the Government, is first a way to improve construction quality. External prestressing leads to simple tendon lay-out, with very small construction deviations; reduces friction losses; improves the concreting conditions by eliminating ducts from the webs; improves the grouting conditions; and, above all for clients, external prestressing allows for external tendons replacement, and for an easy strengthening of structures. But the prestressing technology cannot be the same as in the United States and is more expensive, reducing largely the economical advantages of external prestressing.

External prestressing is now widely utilized in France, and practically all large bridges are now built with external tendons; even when the contract comes from an alternative design. This proves that there is still an economical advantage when tendons replacement is an obligation.

Recently, some bridges have been built with external tendons in Belgium, with designs from René Greisch and Bruno Cremer (Ben Ahin and Wandre bridges), and in Venezuela, with a Figg and Muller design [68]. The interest in external prestressing is developing in Switzerland, in Germany and in Czechoslovakia.

DESIGN OF EXTERNALLY PRESTRESSED CONCRETE BRIDGES

General Consideration of the Design of the Tendon Lay-out

Tendons external to the concrete pose problems of how to spread the forces to the anchorages and of local bending at the anchorages, which are much greater than with traditional tendons, since the forces no longer go towards the concrete mass of the structure. In particular, it is much more difficult to anchor external tendons to isolated bosses than to anchor internal tendons to them. The anchorage force creates major shear forces at the junction between the boss and the concrete panels, as well as introducing bending which cannot be ignored (figure 9). Logic thus led designers to anchor external tendons to massive elements and thus, from preference, to the existing members – bracings on piers.

The simplest cable system thus consists of the use of tendons in each bay, anchored to the cross-beams on the piers, and crossing within these cross-beams. Almost all structures have been built in accordance with this principle, adopted for the first time for the Long Key bridge (figures 10, 11) [19, 20, 21, 29].
Clearly, for external tendons to have the best lay-out so far as bending and shear are involved, they must be deviated in the span so as to be at the bottom fibres in the key zone. This deviation has been arranged, in the classic box sections, sometimes by small deviation blocks at the junction of the lower slab and the web, and sometimes by cross-beams.

The solution using deviation blocks has been used for numerous bridges: all the American bridges of Jean Muller, and the viaducts of Vallon des Fleurs and la Banquière, built in France just before we developed our action in this field [50, 69.1]. It enables the tendons to be deviated one by one, selecting the anchorage point for the different tendons in the cross-beams on the piers in such a way that the deviation force is introduced without bending – so far as possible – in the web and the lower slab. The lay-out of the tendons can then be in line with the force diagrams, leading to very light structures. But, as every tendon can only be thus deviated near to the web, it is necessary to get those tendons which have already been deviated away from the web, resulting in a non-linear lay-out in plan (figure 11).

This led us to a preference for deviating the tendons by means of special cross-beams – lighter than the cross-beams on the piers –, seeking simple lay-outs, each tendon being placed in a vertical plane (or slightly inclined) with occasional slight alterations at the anchorages. It is then preferable to give the same lay-out in elevation to all the tendons, to reduce the angular deviations. The choice of thickness of the web is linked to the number of cross-beams: if there are only two deviation cross-beams per bay, it is not possible to have a perfect lay-out for shear and it is necessary to have relatively thick webs; on the other hand, if one accepts four deviation cross-beams per bay, this results in extremely thin webs if the cross-beams are properly placed and the cables at the right levels. The extreme case is that of the bridge at La Flèche, where the extremely reduced height of the box called for the placing of numerous deviation cross-beams to limit the web thickness (figures 14 to 17) [27, 39, 69.2]. Almost all the bridges in France with external tendons have been built in accordance with these principles.

But the situation is rather different for box-girders of constant height, than for box-girders of variable height.

The first bridges which had been built were of constant height. With two deviation cross-beams in each bay, the cross-beams must be placed not far from the third points of the span to fit with bending conditions. The ideal design for shear forces is, of course, with tendons radiating from the cross-beams on the piers, which progressively reduce the shear forces produced by the structure weight and loads; as is done in the Figg and Muller designs, which are equivalent from a shear forces point of view to designs using a series of deviation cross-beams in the spans (figure 18).

The situation is very different with beams where the height is parabolically variable. Due to the height variation, the deviation cross-beams – if there are two in each bay – must be very close to the key zone. But the shear force reduction is then very small, unless the box-girder height is very important in the key section, as it is for the bridge near Chalon (figure 22) which had been designed for very heavy nuclear transports (650 metric tons) [45] or for the Ré Island bridge. If the key section height is small, it is necessary to improve the shear forces design by adding some deviation cross-beams. There is no need for an important shear force reduction near the piers, due to the rapid height variation, and it is favourable to increase the tendons inclination in the intermediate parts of the bay. But the best solution is a modification of the height variation, with the two ends of the bay having a parabolically varying height, each of them about 20 to 25% of the span length, and with a central part of the bay of constant height, evidently greater than at the key section with a classical height variation (figure 19). The bridge at La Flèche and the Pont à Mousson viaduct (figures 20, 21).
were designed like this for aesthetic purpose, before we understood the advantages of this solution.

Finally, with the construction of the viaduct at Pont à Mousson on the river Moselle, we extended external tendons over two bays, to reduce the number of anchorages in the cross-beams on the piers, to limit the forces to be spread in the cross-beams and from there into the cross-section; and also to limit the geometric congestion due to a great number of anchorages. This resulted in an economical advantage since the cost of the anchorages is important for replaceable external tendons.

**General Consideration of Construction Problems**

The problem of the construction method then arises.

If the bridge is built bay by bay, and if it is desired to place all the tendons outside the concrete with anchorages in the cross-beams on the piers, it is necessary to do without the prestressing forces during the construction and to provide something else to support the weight of concrete during construction. Numerous solutions have been employed:

- building the whole deck on centering, or bay by bay on centering;
- building on temporary supports with the help of temporary prestressing bars;
- using removable centering;
- using temporary stays...

Concerning bridges built by the cantilever method, we developed the idea of a mixed prestressing system, with some tendons within the concrete, their number being strictly limited to that required for construction, to balance the cantilevers self-weights. External tendons are only installed — from on-pier section to on-pier section — after completion of the final key section.

External prestressing has been applied only recently to incrementally launched bridges, and several solutions have been put forward.

**Bridges built Bay by Bay**

The simplest case is where a bridge is built bay by bay, one at a time or in groups of bays, or the whole deck in one single operation. In all cases the method of construction must make it possible to balance the forces due to self-weights of the span (or of the spans) without using its final prestress, which can only be placed and tensioned when the construction of the bay (or of the bays) is complete. At which time all tendons can be placed outside the concrete and can be anchored in the cross-beams on the piers.

Following is a list of the different methods developed for the construction of bridges bay by bay, or in groups of bays, according to this principle:

- bridges built in one single operation on general centering or scaffoldings: Martinville viaduct, also called Lyons viaduct [45, 69.2], and Serignan bridge over the river l'Aigue [45], in France; the scaffoldings support the concrete weight during construction;
- bridges built bay by bay on scaffoldings; the scaffoldings support the weight of the built bay; they are dismantled when the bay has been prestressed, and installed for
the construction of a new bay; for example, the small spans of the Pont à Mousson viaduct (figures 20, 21) [37, 69.2], and the viaducts of the second line of the Lille Mass Transit System (VAL), in France; note that a great part of the tendons are internal in the small spans of the Pont à Mousson viaduct, though they could have been all external;

- bridges built bay by bay with the help of a moveable erecting beam on which the precast segments of the span to build are installed; this is the method used for the greater part of the American bridges designed by Jean Muller: Long Key, Channel Five and Niles Channel bridges (figures 10, 11) [19, 20, 21, 29], Dauphin Island bridge access spans, Wiscasset bridge, and the two viaducts for the Mass Transit System in Atlanta [32], to mention the first ones;  

- in the same way, bridges built span by span on a moveable centering system, on which each bay could be concreted;  

- bridges built span by span with the help of a launching beam; this could be imagined for cast in situ bridges, each bay being concreted on the beam in the adapted position; but this solution has been used with precast segments: for the Seven Mile bridge, and for the access spans of the Sunshine Skyway bridge; and for the Bubiyan bridge in Kuwait designed and built by Bouygues [24, 25, 30]; in this last case, the launching beam was used like a gigantic crane suspending all the segments of the span to build (figures 12, 13); the latest application, with a very classical launching beam, is the construction of four viaducts for the Romulo Bettancourt highway, in Venezuela, designed by Jean Müller [68];  

- bridges built span by span with the help of a temporary cable-staying designed by Campenon Bernard: the Vallon des Fleurs and La Banquière [50, 69.1], viaducts near Nice, and, recently, the Frébuge viaduct and the Francin bridge, all of them in France;  

- and finally, bridges built span by span with the help of multiple temporary supports and of prestressing bars, with the example of the viaduct in Saint Agnant, between Rochefort and La Rochelle, in France [35, 69.2].

These methods are mainly adapted to very long bridges, what explains that they have been much more developed in the United States than in France.

Bridges built by the Cantilever Method

The principle of the mixed cabling relies on balancing, with the fewest tendons possible, forces due to the cantilever self-weights (and to the carriages weight) by tendons within the concrete, segment after segment, with a lay-out which is not far from the lay-out of classical cantilever tendons. But these cantilever tendons remain horizontal and are situated and anchored in the upper nodes, at the junctions between the webs and the top slab.

This makes it possible to build the successive cantilevers, and to place the external tendons — from on-pier section to on-pier section in one bay — just after closing the span, for the typical intermediate spans.

Many bridges have been built this way in France: the main spans of the Pont à Mousson viaduct (figures 20, 21) [37, 69.2]; the bridges near Toul, near Rumilly [45, 69.2] and near Chinon (figure 22) [45]; the main spans of the Roquebillière viaduct near Cahors (figures 30 to 34), and of the Poncin viaduct on the A 40 highway (figure 35 to 40); more recently the Arrêt Darré viaduct near Tarbes (figure 23, 24, 25), the Persan-Beaumont bridge over the river Oise, and the Ré Island bridge which is the largest construction of this type. This method is currently being used for the Champ du Comte viaduct near Chambéry and the Cheviré bridge near Nantes, which will be of the same importance as the Ré Island bridge; both are under construction.
Some of these bridges are wide, with box-girders about 20 meters wide, such as the Poncin and Arrêt Darré viaducts, and the Cheviré bridge. The webs are about 50 to 60 centimeter thick, which is not much as compared to the cross-section.

Jacques Mathivat—working for the contractor on the SETRA design of the Cheviré bridge—decided to utilize some of the cantilever internal tendons in the webs to increase the shear force reduction. Due to the web thickness, this in no way worsened the concreting conditions. After finalization, this idea led to a very nice shear-forces distribution.

Of course, these ideas are also used for bridges built by methods which are producing the same force distribution as the cantilever method. At first for bridges built from the two sides by the incremental launching method, such as the Cergy Pontoise bridge, over the river Oise, near Paris [45, 69.2]; the Amouguez bridge in Morocco, built by Spie-Batignolles; and the bridges for the G12 highway interchange, near Paris. But also for bridges placed by rotation, such as for the bridge over the river Loir at La Fèche [27, 39, 69.2], which is the first bridge built with a mixed prestressing system, and which must be considered as the prototype of this solution, such as the Long Key bridge is for bridges built bay by bay (figures 14 to 17).

In all cases, analysis has shown the need for some internal continuity tendons (from one to three or four tendons per web), to avoid too high a requirement for the external prestress, excessive over the piers because of the presence of the internal tendons. These few internal tendons help for immediately closing the spans, providing the necessary strength capacity, because short internal tendons can be placed and tensioned much quicker than long external tendons.

Some other ideas have been developed for the external tendon lay-out in bridges built by the cantilever method.

At first, Bouygues and Philippe Lecroq decided to reproduce a traditional tendon lay-out for the Sermenaz viaduct, over the river Rhône near Lyon, with cantilever tendons and continuity tendons, all external, and with anchorages distributed in the spans, placed in vertical anchorage ribs (figure 26) [36]. But, in the end, the multiplicity and weight of these anchorage ribs outbalanced the webs lightening, and this solution has not been reproduced.

On the other hand, Jacques Fauchart considered the excess of prestressing forces in the pier zones of our designs, and managed to double the number of external tendons in the key zones by anchoring them in deviation cross-beams in the spans, with an overlap of tendons in the key zones. In such a situation, external tendons only pass through the cross-beams on the piers, without anchorage there (figure 27). The Darse du Gaz de France viaduct at Alfortville, on the A 86 highway near Paris, was built like this [43, 69.2]. With this design, however, the shear force reduction is only half as much, and the deviation cross-beams are more numerous and heavier; which results in somewhat heavier structures (figure 28).

This idea has been partly used by Jacques Combault for the Très Cassés bridge, over the river Garonne, near Montauban. Some external tendons are anchored in the cross-beams on the piers and some in the cross-beams in the spans. But this results in a very complicated geometry and in an increase of reinforcement. A somewhat similar situation can be found with the Venant viaduct, on the A 71 highway.

As a conclusion, despite the necessity of a greater quantity of prestressing steel, the simplest solution appears the best.
Finally, we cite the Sylans and Glacières viaducts on the A 40 highway, built by Bouygues according to Pierre Richard's ideas [59, 63]. As for the Bubiyan bridge, the structure is a prestressed concrete spatial truss, this time built by the cantilever method. The tendon arrangement is rather complex, with:

- temporary, straight cantilever tendons, external;
- few final straight cantilever tendons, internal;
- final, undulating cantilever tendons, external;
- some additional, undulating external tendons, placed after construction of the cantilevers;
- and some additional internal tendons, straight, placed in the nodes to the SETRA demand to improve the ultimate behaviour.

This looks a bit sophisticated, but an excellent distribution of prestressing forces is necessary in such bridges, due to the relatively reduced shear force capacity of triangulated structures.

**Incremental Launching**

The most recent developments in the field of external prestressing concern bridges built by the incremental launching method.

The first solution proposed was based on the idea of mixed cabling; launching tendons within the concrete, horizontal and placed in the upper and lower nodes – at the junctions between the webs and the slabs – and external, undulating tendons placed after launching and anchored in the cross-beams on the piers (figure 29). As compared to traditional internal tendon lay-out – where the launching tendons are placed in the upper and lower slabs, to free the webs for the internal, undulating tendons, placed after launching –, this solution allows for lightening both the slabs and the webs. What results is a fantastic economy in weight. This idea was put forward for several projects: the basic scheme for the Jules Verne viaduct at Amiens, and tenders from Spie-Batignolles for the lake Mosjø bridge in Norway and for the Biéroué bridge over the river Cher in France. But it has only been used, in this simplified form, for the access spans of the Roquebilière viaduct, near Cahors, which had been launched with a temporary support in each span (figures 30 to 34).

This solution has been improved by Claude Servant, who added some temporary launching tendons outside the concrete, these being straight, to reduce the number of the internal, final launching tendons, which are also straight. Of course, the final, undulating external tendons are placed only after launching. This solution has been used for the construction of the access spans of the Poncin viaduct, over the river Ain, also launched with a temporary support in each span (figure 35 to 40).

Of course, in both cases the internal horizontal launching tendons could be divided in segments according to the division of the beam in successive segments, cast on the prefabrication bed on one bank. This made it possible to detension and remove some of these tendons, in the Roquebilière viaduct, in the zones where they are unfavourable for flexural forces after launching. This was not necessary in the Poncin viaduct, due to the existence of temporary external tendons, but some straight tendons have been placed and tensioned after launching in the favourable zones: upper nodes in the pier zones, lower nodes in the spans.

Another family of solutions derives from an idea developed for the construction of the Sathorn bridge, in Bangkok [41]. The internal tendons were placed during
construction with a lay-out as for a bridge cast on centering, passing in the upper part of the webs on the piers, and in the lower part at mid-span. To result in a central prestress during launching, the moment which they created had to be balanced by placing temporary tendons in opposition. These temporary tendons passed also in the webs, but they were placed above the deck in the zones of positive moment in final position.

The basic idea consists of balancing the prestressing moment due to final tendons, having a classical lay-out and tensioned before launching, by temporary compensating tendons, so as to obtain a central prestress during launching (figure 41). This idea can be applied whether the final tendons are inside or outside the concrete, and, of course, the temporary compensating tendons must be external.

The idea was adopted by Michel Placidi for the construction of bridge numero 33 on the Autoroute du Littoral at Marseilles by placing during construction the final, undulating internal tendons in the webs, balanced by temporary external, undulating and compensating tendons. Some horizontal final tendons were also placed inside the concrete – tensioned segment by segment on the prefabrication bed –, and some additional external, horizontal temporary tendons were placed in the forward bay. Once the structure was in place and the compensating tendons had been stripped, external undulating tendons completed the prestress (figure 42, 43, 44).

The idea was simplified and improved by Jacques Combault for the construction of the viaduct over the river Durance, the Val de Durance viaduct [57, 61], with all tendons external. One part of the tendons, final and undulating, was placed during construction, balanced by temporary compensating tendons, also external; some temporary horizontal, external tendons completed the prestress during launching. After launching, the temporary tendons – horizontal and compensating – were removed and the final external prestress was completed. We must note that the bridge was launched with a temporary support in each span, only used for the passage of the front-bay (figure 45).

Jacques Combault finalized this solution with the construction of the Amiens viaduct, almost one kilometre long [57, 61]. Some of the final external tendons, tensioned at the construction on the prefabrication bed, are horizontal, not far from the top slab (figure 46).

This solution is extremely efficient, because of the large compression stresses during launching, due to the addition of final and compensating tendons. This gives a high shear capacity to the box-girder, allowing very thin webs: 30 centimetres for the Amiens viaduct, with spans more than 50 meter long; compared to the 50 to 60 centimetres that we had ten years ago in the same situation.

But, personally, we have a preference for keeping some horizontal tendons inside concrete, in the nodes, as done by Claude Servant for the Charix viaduct, launched with spans 64 meter long [66], because nodes constitute a very good place for some internal tendons, and because keeping some internal tendons limits the geometric congestion of external tendons in the box-girder and improves the bridge ultimate behaviour (figure 47).

To these new ideas we have added, with Michel Placidi, the re-use of temporary tendons as final prestressing tendons. For the bridge numero 33 in Marseilles, which has two parallel decks, the temporary tendons for the first deck were re-used as temporary tendons for the second. Even though they were not used in Marseilles, specifications were laid down for their re-use as final tendons, and some technological dispositions and operations were later invented to make this re-use possible and easy. But the increasing competitiveness of composite bridges has not yet provided the occasion to put them into practice.
In fact, the last bridge incrementally launched in France was the viaduct on the river Sioule, on the A71 highway. But we cannot consider its tendon arrangement as a progress as compared to those of the Val de Durance viaduct, the Amiens viaduct or the Charix viaduct.

We have to wait until next year, for the beginning of the construction of the access spans of the Normandy bridge, to see what we now consider as the finalized solution, with:

- internal horizontal tendons, in the upper and lower nodes, placed and tensioned during construction;
- a limited number of final undulated tendons, placed and tensioned during construction;
- and the adequate number of temporary antagonist undulated tendons, detensioned after launching and re-used one after another, in the same web, as final tendons.

**Handling and Placing Jacks**

The jacks used for tensioning external tendons of high capacity (19 strands of 15 millimetres and more) are extremely heavy, from 800 kilograms to more than one metric ton.

In the span-by-span construction, this led designers to place active anchorages on the front pier of the progressing construction, where the access is free to the on-pier cross-beam before placing (or concreting) the segments of the next span; and to place passive anchorages on the rear on-pier segment, on the last but one pier. This makes it possible to handle heavy jacks with small mobile cranes from outside, which is not possible in the box-girder on the rear on-pier segment. This solution has been systematically used by Jean Muller in all his bridges, and re-used as often as possible, for instance for the Saint-Agnant viaduct. It was slightly adapted for the Bubiyan bridge, in which the cables were coupled, on the rear on-pier segment, to those of the previous span, and tensioned from a small mobile carriage in front of the progressing construction on the front pier.

For small three-span bridges built in a single operation (such as the Martinville and Sérignan bridges), or so built that all external tendons could be tensioned in a single operation (such as the La Flèche and Cergy-Pontoise bridges), the normal situation consists in placing external tendons from one abutment to the other, anchored on the extreme cross-beams. Thus jacks can be handled from the outside, with small cranes. However, fewer tendons are usually necessary in the side spans than in the main one; for the Chinon bridge, Grands Travaux de Marseille imagined to place tendons on two spans only, symmetrically, in such a way that their number is double in the central span (with other longer external tendons, from one abutment to the other); but they placed passive anchorages on the intermediate supports, to keep all active anchorages at the abutments and prevent handling problems.

The construction of the Pont à Mousson viaduct evidenced that handling and placing heavy jacks inside a box-girder was not easy, due to their weight, due to the existence of the deviation and on-pier cross-beams, which makes the transport of jacks difficult, and due to the small distance between anchorages and top slab (figures 48, 49). For later applications, the designers had to organize the tendon layout in such a way that the distance between anchorages and top slab could be increased; and the prestressing system suppliers had to devise methods for handling and placing heavy jacks inside the box girder when necessary, for all long bridges built by the cantilever or the incremental launching methods.
Spie-Batignolles for LH system (figure 50), VSL and Freyssinet designed separately a mobile handling equipment, able to place the jack in an upper corner of the box girder; and SETRA designed, for the Ré Island bridge, a permanent rail, from which the tensioning jack could be suspended and moved along the bridge, to the on-pier anchorages (figures 51, 52).

But this problem also led to a purely technological solution: to eliminate the problem raised by handling and placing heavy jacks, Freyssinet imagined to tension the external tendons strand by strand, with a very light jack which can be moved by workers without any equipment (its weight is about 30 kilos). The idea has been to create bunches of coated-greased strands. But in the deviations the pressure produced by the external layers cut the coat of the internal strands. Freyssinet then imagined to place the coated strands in a continuous high density polyethylene duct (HOPE), injected with cement grout before tensioning the strands. There technology meets design.

**Some last Remarks on Dimensioning**

This presentation of a great number of examples evidences important differences in dimensioning. They mainly come from the Code-specifications: the live loads from the American Code are much lighter than those in the French one, what leads to much lighter structures. But the differences also come from the construction philosophy and organization.

The French Bridges Administration prefers designing rather comfortable structures, for easing construction and increasing durability. For this reason we do not take all possible advantages from external prestressing, and, for example, we do not design webs as thin as they could be. We can, in the French system, specify for each bridge a lower limit of the web thickness.

We also try to limit shear forces in the service limit state conditions, and to balance the greatest possible part of the bending moments due to permanent loads by antagonist prestressing moments, as recommended by Renaud Favre for example. Because deformations, cracks and durability are practically only conditioned by the situation in the structure produced by permanent loads, dead loads and prestressing forces.

This leads to a rather great number of undulating tendons, greater than it could be to only stick to Code-specifications. In the case of bridges built by the incremental launching method, it could be more economic to increase the number of final straight tendons - internal or external – than we do, because it would limit the number of temporary antagonist tendons and thus tendon handling. But the final distribution of stresses, under permanent loads, would not be so favourable.

We can evaluate the distribution of permanent forces in bridges of constant height with a classical organization of tendons:

- tendons all external and undulating, each of them flying on one span from pier to pier (or on two spans);
- or some external and undulating tendons, with in addition continuous straight tendons - internal or external -, flying on one complete span, or on several spans.

For long highly hyperstatic bridges with regular spans, the effect of straight tendons is a centered prestress, whatever their precise position is. And we can assimilate the bending effect of the undulating external tendons to a lifting uniform density of forces, given by:
\[ P_{\text{prec}} = \frac{8 F_{\text{und}} \Delta e}{L^2} \]

where \( F_{\text{und}} \) is the prestressing force produced by undulating tendons, \( \Delta e \) the undulation total amplitude, and where the span is noted \( L \). This estimate is the most precise as the external tendon layout is closer to a parabola (figure 23).

This lifting action of the undulating tendons balances a part of the dead loads, the uniform density of which is noted \( P_{\text{dead}} \). The ratio \( \frac{P_{\text{prec}}}{P_{\text{dead}}} \) is a good indicator of the stress distribution under permanent loads. We try to balance this way at least 60 to 70 per cent of the permanent loads, and we reach an extremely good situation with a ratio between 0.70 and 0.80. We hope to have from there the best conditions for a good durability of the externally prestressed structures.

This approach is not in disagreement - to our opinion - with the philosophy of partial prestressing for cast-in-situ bridges; because such a situation under permanent loads leads to some residual compression under frequent loads, and avoids any fatigue problem; and because there is no contradiction with limited tension under extreme live loads. Even if these tensions perhaps seem too limited to some colleagues.

**PRESTRESSING COMPOSITE BRIDGES**

In their search for lightening structures, French engineers have invented the substitution of much lighter steel webs for the concrete webs of classical box-girders. The pioneer in this field is Pierre Thivans, from Campenon Bernard, who developed the idea of a box-girder made of steel folded webs and concrete slabs.

Of course, these structures have to be prestressed so that the concrete slabs - top slab and bottom slab - are subjected to compressive stresses under the effect of permanent and frequent loads.

The prestressing tendons can be within the concrete of the slabs - in the bottom slab at least in the central part of the spans, and in the top slab at least near the piers. But the prestressing tendons can be also outside the concrete. Due to the development of external prestressing in France, this second solution has been immediately adopted for the first applications, with these advantages:

- a mechanical continuity of prestressing forces;
- an excellent distribution of forces, for bending forces as well as for shear forces;
- and a shear force reduction, leading to some possible economy in the web thickness and the connection.

The prestress design of these bridges follows the same principles as for prestressed concrete bridges, presented above.

But these ideas were coming from designers normally interested in prestressed concrete bridges, who simply introduced steel webs in their usual way of thinking. On the other side, some designers normally interested in steel construction - at least partially - had already thought of prestressing traditional composite bridges.

This results in the two directions that we shall cite.
New Composite Bridges: Steel-Prestressed Concrete

Four small bridges have been built these last years in France, following the ideas initiated by Pierre Thivans. We shall not present them in the chronological order, but in the logical order, from the simpler to the most sophisticated.

Fougerolle has built a small bridge, which is an isostatic span over the A 71 highway near La Ferté Saint Aubin, not far from Salbris [11.5]. The box-girder is made of two concrete slabs, connected by two steel plane webs, traditionally stiffened, vertically and horizontally. The bridge is prestressed by external tendons, normally undulating, anchored in concrete cross-beams on the abutments, and deviated by two cross-beams in the span, also in reinforced concrete. The bridge was launched after complete construction on a prefabrication bed, on one side. The final undulating tendons were straight during launching; and then detensioned and reinstalled in their final position, one by one, after launching (figures 53, 54).

Dragages et Travaux Publics and Société Générale d'Entreprise joined to build the Arbois bridge over the river Cuisance in Jura [11.6]. This is a small bridge with three spans, made of two concrete slabs connected by two steel trusses, in lieu of the classical webs. In addition, the central rib stiffening the concrete top slab is supported by steel struts, but these central steel elements do not constitute a flexural truss and do not take part in the longitudinal capacity of the bridge. Of course, this bridge is prestressed by longitudinal external tendons, with a now classical lay-out: on each side of the open box-girder, two external tendons are anchored in the extreme concrete cross-beams, on the abutments; and two are anchored in the cross-beam on the abutment on one side, and in the cross-beam on the pier of the other side; symmetrically for the two tendons, to overlap in the main span. These undulating tendons are deviated by the concrete cross-beams on the piers and in the spans. In addition, there is a straight tendon anchored from one abutment to the other (figures 55, 56, 57).

Campenon Bernard also built a three span bridge at Cognac, over the river Charente [11.7]. This time, the concrete webs of the traditional box-girders have been changed to steel folded webs, 8 mm thick. This bridge, built on general scaffolding like the bridge at Arbois, is prestressed by external tendons, all anchored in the concrete cross-beams on the abutments, on each side. These undulating tendons are deviated by the concrete cross-beams on the piers and in the spans, as at Arbois. Due to the very great longitudinal flexibility of the steel folded webs, prestressing forces are concentrated in the concrete slabs, which is evidently efficient and economical. There is no need, in particular, to stiffen the steel webs which have not to support compression forces (figure 58).

These three bridges have been built in experimental conditions by direct agreement between the contractors and the owners - COFiROUTE, a private highway company, for the first of them, and the National Roads Authority (Direction des Routes) for the two others - under SETRA supervision. The fourth bridge - also built for the Direction des Routes - has been contracted after a call for bids that limited the type to this family of structures.

Campenon Bernard won the contract for the Vallon de Maupré bridge, near Charolles, with an original triangular box-girder, the bottom slab being changed to a steel tube, filled with concrete; and the concrete webs of traditional box-girders being changed to steel folded webs [9, 10, 11.8]. This bridge has been built by the incremental launching method. During construction - on a prefabrication bed on one bank - and launching, the bridge was prestressed by straight, final external tendons, placed just beneath the top slab, and anchored span by span in a transversal
rib stiffening the top slab on the piers. After completion, the prestress was completed by four undulating tendons, anchored in the extreme sections, in the concrete cross-beams on the abutments. These undulating tendons were deviated by steel saddles supported by the steel cross-beams on the piers, and by two steel saddles in each bay, connected to the bottom tube. Due to the great length of these tendons, their tension has been increased by jacking the steel saddles, a computed distance on the central pier (figures 59 to 62).

Some other ideas of the same kind are developpping in other countries, such as in Italy.

Prestressing Traditional Composite Bridges

And finally, why not prestress traditional composite bridges?

The French Code – and others also – don't consider the concrete top slab in the pier zones, where it is in tension. So, more than fifteen years ago, when the first codes had been written for composite bridges, some engineers invented to stress the top slab to avoid this limitation. It has been partly done, at times, by imposing variations in level on the supports to the just completed structure. And some engineers, for instance François Ciolina and Philippe Lecroq, invented to prestress the top slab, in the pier zones, by internal tendons.

But it is not so easy to place tendons and anchorages in thin slabs! External tendons are a much better solution.

We worked on the idea with Gilles Causse, and he designed a solution of this type for the Hospital sur Rhins bridge, over the small river Rhins, near Roanne. Unfortunately, it will not be built: a classical composite bridge was slightly cheaper, and the Owner did not accept the "risk" of an innovative design.

Jean Muller also proposed a solution of this type for the bridge on the river Nive, at Bayonne, but another contractor won the job.

From our experience we can give some preliminary conclusions:

- with a classical lay-out for the external tendons, we develop favourable bending moments and shear force reduction;
- this shear force reduction allows for reducing the web thickness and the connection;
- on the other hand, the prestressing forces, if they are too important, can create buckling problems in the webs; to avoid it, we must not produce compressive forces in the steel beams in the unloaded structure; we must simply limit tension forces;
- we cannot reduce the web thickness too much, due to patch-loading problems when launching the steel structure;
- we conclude that this solution can be applicable only for wide bridges, when the web thickness is relatively important;
- unfortunately, we cannot take all the advantage of the favourable action of prestressing in flexure, for two reasons: the first being that the concrete slab must be built before tensioning the tendons, so that the structure self-weight must be totally supported by the steel structure; and the second is that we cannot reduce too much the bottom flange, due to the increasing importance of fatigue specifications;
- finally, it is not easy to design anchorage elements for this longitudinal prestress, nor the deviation elements.
Contractors devoted to steel construction add that the economy on steel elements is at marginal cost – only the material cost –, and not the addition of tendons, anchorage and deviation elements...

But perhaps steel and concrete contractors could learn to work together. Prestressed composite bridges have already been built in the United States by T.Y. Lin (we don't know exactly how), perhaps in Austria, and surely in Belgium (viaduc du Chatelet) with a design which is not so far from what we imagined.

The coming years will give the final answer.

CONCLUSION

In a seven years time, we have built in France more than thirty bridges in prestressed concrete with external tendons; this number amounts to forty approximately if we add composite bridges - steel-prestressed concrete - built with external tendons at La Ferté Saint-Aubin, Arbois, Cognac, Charolles, and recently near Compiègne.

This gave us an opportunity to develop, step by step, convenient designs adapted to each of the main construction techniques:

- the span-by-span erection technique, segmental or not,
- the cantilever construction method,
- and the incremental launching method.

This has been also an occasion to develop adapted technologies for external prestressing, with many inadequate or uneconomical solutions which had to be abandoned after some experimental applications. This trial and error process has finally led to a limited number of good solutions, the description of which would be out of the theme of this paper.

These last years, practically all large prestressed concrete road bridges have been built in France with external tendons. This demonstrates that, besides and above the impulse of the French Administration in this direction, external prestressing offers great economical advantages for contractors. Now this technique is developed in Germany under the influence of Professor Eibl and in Switzerland; in Belgium, under the influence of René Greisch and Bruno Cremer; and in Czechoslovakia.

Even though we can still progress, in the field of technology mainly, we can state that we master the technique; it is no longer experimental for us, but the normal way of building large concrete bridges.
BIBLIOGRAPHY
ONEXTERNALLY PRESTRESSED CONCRETE BRIDGES


[61] J. COMBAULT, A. LEVEILLE, P. NERON and J.L. THIBONNET. Incrementally launched bridges with total external prestressing. IABSE Paris-Versailles


[69] Bulletins annuels de l'Association Française pour la Construction.


BIBLIOGRAPHY
ON PRESTRESSED COMPOSITE BRIDGES


  – A. ATTAL. Le point de vue de l’entreprise et du bureau d’étude, p. 31 à 38.
  – Ph. BERTHO. Le point de vue du laboratoire, p. 39 à 45.

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External Prestressing—a State of the Art

by A.S.G. Bruggeling

Synopsis: In the last decade the use of external tendons for prestressing of concrete structures has been developed considerably for a variety of applications. This paper deals with the technical problems related to the use of external prestressing in order to emphasize the development and the investigation of critical questions.

The following aspects are discussed:

1. The behavior of concrete structure at the Serviceability and the Ultimate Limit States.

2. The required minimum area of conventional (nonprestressed) reinforcement.

3. General protection of the tendons.

4. Corrosion protection of the prestressing steel.

5. Design of the saddles — deviation zones —:
   - Effect of tendon curvature on its strength.
   - Transfer of the prestress forces to the structure.

6. Anchorage of the tendons and the zone of transfer of the prestressing force into the structure.

Keywords: anchorage (structural); box beams; bridges (structures); cable-supported structures; history; prestressed concrete; prestressing; prestressing steels; reviews; segmental construction; serviceability; supports; unbonded prestressing
BRIEF HISTORICAL REVIEW

Already in the early fifties the prestressing tendons in some concrete structures were not positioned within but outside the concrete cross-section. The well-known Prof. G. Magnel from Belgium, one of the pioneers of prestressed concrete, designed several projects with such external tendons.

In these projects, after concreting of the structure, the prestressing wires were placed outside the webs of girders or box-girders [1]. They passed through holes in steel plates, which were connected with the concrete structure, for transfer to the girder of the forces resulting from the depression of the tendons.

After stressing, these wires were concreted in with mortar composed of sand, cement and water. This mortar was compacted in a mould and fixed around the prestressing steel. See Fig. 1.

The aim of this construction procedure was to substitute uncontrollable grouting of ducts for concreting of the stressed wires under full visual control. The role of the concrete was also to "bond" the prestressing steel to the concrete structure in order to ensure interaction of steel and concrete at overloading and, as a result, to ensure an acceptable factor of safety against failure of the structure.

Prof. F. Leonhardt introduced external cables in prestressed concrete box-girder bridges, constructed with the so-called launching method [2]. In France, external prestressing was also used in several structures [3].

In the Dutch recommendations for prestressed concrete [4], published in 1962, rules were given for the design of concrete structures with external prestressing. In these recommendations much attention was paid to the need of adequate "bond" between the external tendons and the concrete structure. In this way the prestressing steel contributed to the ultimate bending moment in the same extent as prestressing steel in grouted ducts within the concrete section.
In the 1960s and 70s, the use of external prestressing did not break through as a generally accepted construction method. The causes for it are not easy to ascertain; however, some draw-backs can be mentioned from practice.

Several structures with external prestressing showed, some time after completion, corrosion problems due to insufficient protection of the prestressing steel by the compacted mortar. In other cases the pulling of prestressing steel through holes in the steel deviators, during construction, and the stressing of the steel itself caused problems due to friction in the deviation zones.

Perhaps also the growing concern with structures to resist fire did not support the development of this prestressing technique.

Two developments, in the end of the sixties, can be mentioned in connection with the re-introduction of external tendons, namely the use of unbonded single strand tendons in the U.S.A. and the design of cable-stayed concrete bridges with stays of prestressing steel.

The introduction of unbonded single strand tendons made it necessary to develop a basic approach to the design of concrete structures without bond between the prestressing steel and the concrete, especially in the ultimate limit state.

In the development of cable-stayed concrete bridges real exterior tendons composed of prestressing steel were introduced. In these structures it was of paramount importance to guarantee the corrosion protection of the cable stays and to design carefully the connection in the anchorages between the cable stays and the concrete. Also, attention had to be paid to the behavior of the cable stays under dynamic loading [5].

**RE-INTRODUCTION OF THE USE OF EXTERNAL CABLES**

Several reasons can be given to re-introduce external prestressing. In the applications to be described here, "external tendons" are defined as tendons composed of prestressing steel with a tendon profile which, except at the deviation and the anchorages zones, has no relation to the shape of the concrete structures. The tendons can, for example, be located in the space of box-girders.

The difference in the tendon profile with cable stays of prestressing steel is that the defined exterior tendons have, in general, several deviation zones (saddles) and are mostly situated within the concrete structure, e.g. in the inner space of box-girder bridges.

Some reasons for the development of this type of external cables are:

1. The demand for methods to repair prestressed concrete bridges with corroded prestressing tendons in the concrete structures [6].
2. The development in practice of methods to strengthen concrete bridges or other structures already in use, due to the increase of traffic loads.

3. The need for less complex construction techniques on site by avoiding complicated profiles of prestressing tendons in the concrete structures. Simplifying tendon profiles has its effect on the concreting of the structure, the stressing of the tendons (friction problem) and the grouting of ducts (interaction between adjacent ducts).

4. The growing interest in methods in which the prestressing steel on the site is less exposed to an aggressive environment and the need to reduce the influence of workmanship on the overall quality of the realised concrete structures.

5. The need to maintain and repair concrete structures, especially bridges, without harming the use of the structure, e.g. by closing off the traffic on the bridge or viaduct. Bridges often play such an important role in the infrastructure of a region that it is hardly acceptable to limit their functioning during a period of time. The economic and social effects of reducing the capacity of bridges and viaducts are often of vital importance. In this respect it is of primary importance that the prestressing tendons can be inspected easily and, if necessary, replaced without harming the function of the bridge or viaduct in consideration. In such cases, the use of external cables offers a very good solution.

6. New developments in bridge design and bridge construction are resulting in the use of external cables.

This can be illustrated with new developments in France, where several bridges are prestressed with external cables [7]. In other applications the bridge-structure is composed of prefabricated segments, concreted on a casting yard, and put together on the launching bed (see Fig. 4).

The joints between the elements are filled with epoxy glue before connecting them to adjoining elements. In several American bridges, instead of glued joints, dry joints are used. These joints are not filled with epoxy glue. A direct contact exists between the concrete faces of the adjoining elements without any provision for the interface.

In using other construction methods, French engineers developed the use of external prestressing, especially in bridge design. In this respect the construction of a composite (concrete - steel) segmental construction can be mentioned, as well as the reduction of the thickness of the webs of box-girders, the use of concrete trusses or steel structures as webs in box-girder bridges and the use of diagonal trusses of prestressed concrete in the box-girders of stay bridges. The development of so-called "extrados external prestressing" (external tendons, outside the space of the concrete structures - short pylons) can also be mentioned as an interesting application.

It is not possible to describe fully these very interesting structures. Therefore, reference to literature [7] may be made. These applications show, however, that there is a tendency of a break-
through in bridge design due to the development of new con- cepts, related to external prestressing. From the incidental strengthening of weakened bridges new types of bridge structure are generated!!

It can be concluded that in the use of external prestressing - with tendons connected to the concrete structure in only a few zones - the following trends can be observed:

1. Simple longitudinal tendon profile.
   The conventional approach to the tendon profile is abandoned.
2. Use of tendons with special means of corrosion protection (ducts, "grout”).
3. Specially designed deviation and anchorage zone.
4. Reduction of dimensions of the concrete cross-section, especially reduction of thickness of webs, resulting in more freedom in detailing the vertical elements between compression and tension zone of the structure.
5. Possibility of replacing tendons without interrupting the use of the structure. This is only possible if the corrosion protection of the tendons allows release of the prestressing force in the tendon without inadmissible resistance by this protection to shortening of the tendon.

Of course these developments make it necessary to consider very carefully the consequences for the design of structures with external cables.

QUESTIONS RELATED TO THE USE OF EXTERNAL PRESTRESSING

The Behavior of the Concrete Structures

Serviceability limit state—Concrete structures with external prestressing are to be considered typical examples of "structural concrete" [8]. Structural concrete has been defined by the author as "Structures built of reinforced concrete which can - optionally in combination with artificial loading through prestressing tech- niques - resist, in a controlled way, all the actions exercised on these structures by loads, imposed deformations and other influences (earthquakes, explosions, etc.) and which must be constructed in a safe and economical way."

In this respect, through external prestressing, the concrete structure is loaded artificially in the vertical as well as in the horizontal direction (see Fig. 2). The shape of the tendon profile is not determined in the conventional way. This means that in certain zones concrete tensile stresses may occur. In the authors opinion, every concrete structure should contain a rational minimum rein- forcement.
In this respect reinforcement is defined as:

- conventionally used reinforcing bars;
- tendons which are bonded to the concrete section by fully grouting prestressed wires or strands in internal ducts;
- prestressing wires and strands directly bonded to the concrete section in the case of factory produced pretensioned prestressed concrete elements.

Of course the bond behavior of large grouted tendons is different from the bond of reinforcing bars or pretensioned single wires or strands. In the design of the concrete structure the real bond behavior should be taken into account [8].

Minimum reinforcement in concrete structures is necessary to control crack distribution and to limit crack width in the period of construction, in the serviceability limit state due to loads and/or imposed deformations and in the zones where, due to the simplified tendon profile, tensile stresses (and possible cracks) may occur.

It will be necessary to study and discuss the question under which conditions the bridge structures, mentioned before, where no reinforcement, as defined, is crossing the joints between prefabricated concrete segments, are structurally acceptable. In such a design, if no precautions are taken, wide cracks (in the joints) may occasionally appear due to unforeseen situations such as overloading, imposed deformations and so on.

In this respect the results of a German research project on prestressing without bond in segmental bridge structures are very important [13]. The following statements are made.

"This project has shown that in the case of (pure) prestressing without bond no control of crack width is possible. In such structures it should be tried to prevent cracking of the concrete by increasing the prestressing force. It is necessary to make provisions, in the construction phase, to allow simple repair of damage due to cracking in the bridge structures in use.

Only with conventional reinforcement (reinforcing bars), crossing the joints, crack width can be controlled in segmental structures. As a result the structure is able to deform in such a way that the effect of impedent imposed deformations can be reduced importantly. Several cracks will develop in the tensile zone. This crack pattern is preventing the creation of single hinges, in large cracks, in the bridge structure."

Ultimate limit state—Structures with external prestressing should primarily be calculated in the same way as concrete structures with unbonded tendons. This means that, at overloading, the forces in the external tendons are increasing only slightly because the longitudinal deformation of the structure at failure is relatively small and cannot result, by imposed elongation, in a large increase
of the tensile force in the tendons. However, due to the deflections of the structure in the ultimate limit state, the artificial loading by the external prestressing will increase because, as a result of these deflections, the angle of deviation of the tendons in the saddles may increase considerably.

The already mentioned German research project [13] has shown that there is no difference in behavior at overloading between segmental bridge structures with external tendons and monolithic concrete structures with internal unbonded tendons.

"The increase of the tensile force in the tendons from serviceability limit state to failure is the same in both cases.

This increase of the tensile force in the tendons can be reduced due to the behavior of the structure in shear. If, in the ultimate limit state, the crack width cannot be controlled - in the case of (pure) prestressing without bond - it is only possible to transfer the shear force by the compressive zone. In these structures it is therefore necessary to avoid cracking of the tensile zone - also in the ultimate limit state - of the section of the structure loaded in high shear. As a result the deformations of the bridge structure and the increase in the tensile force in the tendons will be reduced considerably.

With bonded reinforcement crossing the joints the interlock effect in cracked joints can, however, be activated, resulting in large deformations of the structure at failure."

This shows that the role of normal reinforcement in the concrete structure is, also in the ultimate limit state, very important because it may also contribute to an acceptable plastic behavior of the structure (rotational capacity) near failure. It is questionable if segmental bridge structures without any bonded reinforced crossing joints will have sufficient rotational capacity because no inclined cracks will appear at overloading.

**Protection of the Tendon**

The ducts of external tendons are very important elements in the concrete structure.

These ducts should fulfil the following requirements [9]:

1. Resistance against environmental attack - corrosion protection of the prestressing steel - over the whole length (connections between sections and at the deviation zones).
2. Waterproof - see 1.
3. Compatibility with prestressing steel and its corrosion protection.
4. Resistance against damage during construction and installation.
5. Resistance against damage in service conditions (sabotage).
6. Fireproof in cases where fires cannot be excluded.
7. Controlled creep behavior of the materials of the duct.
8. Resistance against transverse forces (deviations).
9. Replaceability.

Ducts can be composed of several materials including rigid steel pipes, flexible plastic or steel reinforced plastic pipes, and corrugated metal ducts.

The effectiveness of all these materials in respect to their requirements should be investigated carefully.

**Corrosion Protection of the Prestressing Steel**

The prestressing steel of the external tendons should be protected to avoid corrosion under service conditions.

The sheathing (ducts) around the prestressing steel can act as a primary barrier against environmental attacks.

A second barrier should be provided by a protective "grout" in the ducts around the prestressing steel.

These protection materials should fulfill the following requirements:

1. Completely fill the ducts (no air pockets or as few and small as possible).
2. Bond to the prestressing steel over the entire tendon length.
3. Act as a continuous ductile layer on the prestressing steel during the lifetime of the tendon. No cracks caused by longitudinal deformation of the tendons are allowed.
4. No time-dependent or temperature-dependent deformations or settlement should occur.
5. Not contain impurities which are harmful for the prestressing steel.
6. Allow replacement of the tendons.

The protection material can be cement grout, grease, wax, bitumen or plastics.

Cement grout has been used since the development of the conventional prestressing technique in concrete structures. Therefore sufficient know-how is available based on many years of experience and research.

Nevertheless, many drawbacks have been reported on behalf of bad or incorrect filling of ducts. In practice the efficiency of grouting has shown to be difficult to control afterwards (incomplete filling of ducts). Also bleeding of grout, especially important in the
top of curved ducts, has resulted in poor protection of parts of the prestressing steel. New grouting techniques has been developed since e.g. pressure grouting for strands, and using the space between the seven wires of a strand as a drain. These developments are rather new and at the moment experience is available of limited number of jobs. The results looks, however, very promising.

In external cables the result of grouting is much easier to control because the ducts (tubes) can be simply inspected because they are accessible. If some defects are determined they can be repaired because small holes can be drilled in the ducts to add new grout.

If external cables are grouted it will be very difficult to replace them because at release the tensile force in the prestressing steel will be transmitted to the grout which will - of course - fail. It is questionable if the wedges in the anchorages can be loosened because the cement grout should also fill the space between the wedges and the anchorage elements. Therefore the FIP-commission considers cement grouted tendons as not replaceable.

It depends on the quality of the hardened grout if cracks may appear in the grout due to shrinkage, but also due to vibrations. It shows that cement grout is not the ideal material to fulfil all the requirements given above.

Bituminious products, tar-epoxies and greases have shown in some practical applications not to be stable. They show settlement in the ducts after some time. This settlement is resulting in partly incomplete filling of the ducts.

In this respect the experience available from cable stays consisting of prestressing steel, can be mentioned. This experience can also be used in the case of external tendons. From some information one may conclude that wax behaves very well as a "grout" for external tendons. Experience with ducts filled with wax is, however, limited.

It will be clear that in this respect more consistent information is necessary to avoid draw-backs in the future due to malfunction of these materials.

Prestressing Steel and New Materials for Prestressing

The discussion of the problems related with corrosion protection of prestressing steel shows that it is very important to improve the grouting technique. On the other hand one may ask if the problems mentioned above do not show that one should investigate the possibilities of new materials which can be used in external tendons instead of prestressing steel.
In this respect the use of glass fibers and aramide fibers can be mentioned. At least one case is known in which glass fibers are used as external cables.

The possibilities of some new materials are promising. However, one should be aware that at the moment no experience over long periods of use is available and that it is questionable if these materials are economic now. As an indication the costs of such materials, from some information collected, will be at least, per unit of prestressing force, 10 times the cost of prestressing steel.

Research and carefully testing of these materials and these anchorages is necessary prior to use of them on a large scale in external tendons.

The Anchorage of the Prestressing Steel

One can distinguish at least two different practical applications of prestressing systems for external cables and their anchorages.

System 1--Cables with strands into ducts which are preferably of high density polyethylene (HDPE) but occasionally of metal. The ducts are passing the anchorage block by means of metal tubes cast into the concrete structure (see Fig. 5).

These metal tubes or sleeves are also used to pass cross-beams or diaphragms with HDPE-ducts.

After stressing of the tendons the ducts and the anchorages are cement grouted.

This application is rather conventional.

System 2--Cables with greased monostrands, each strand in a polyethylene sheathing. These monostrands are widely used as, so-called, unbonded tendons, e.g. in flat slabs. These monostrands are orderly grouped, and fixed in the right place, in thick HDPE-ducts (see Fig. 6).

These ducts are directly concreted in the anchorage blocks and cross-beams or diaphragms.

Prior to one by one stressing of the monostrands, the duct is grouted and each monostrand encased in cement grout. The monostrands are stressed one by one.

To avoid any cracking of the grout in the duct, due to the effect of stretching, the monostrands should be straight and show no wobbles.
In this case the greased strands, each in their own sheathing, can be replaced as has been proved with unbonded tendons. If the ducts are correctly filled with cement grout and each monostand is embedded correctly, the strands will pass one by one the deviation zone without being pressed to another one.

The ducts have a larger diameter than in case 1 due the larger diameter of the monostands with sheathing and the necessity of fully embedment of each monostand in the grout.

Ulterior adjustment of the prestress or full release of prestress is only possible if the length of the strands behind the anchorage allow gripping by jacks and is left uncut after the stressing operation. This means that special provisions are necessary in this zone.

The Design of the Saddle-Deviation Zones

In most structures, external cables will be deviated in certain zones (see Fig. 3 and Fig. 4). In this type of structure, several types of tendon profile and overlapping are possible.

The deviation points or saddles consist of metal tubes or sleeves concreted in cross-beams or diaphragms. The HDPE-ducts of the external cables of system 1 are passing the structural elements. The radius of the cable depends on the shape of the metal tubes or sleeves (see Fig. 5).

In the case of system 2 the HDPE-ducts of the external cables can be concreted in directly (see Fig. 5). Of course they can also pass the cross-beams or diaphragms by means of metal tubes or sleeves as shown in Fig. 6.

In other cases the deviation points consist of a set of curved steel shells (parts of a steel pipe) or precast concrete elements with a curved gully. The sheathing of polyethylene fits in these curved parts. Of course this solution is only possible if the tendon is pressed, in the deviation point, into the curved gully.

In the deviation zones transverse forces are acting on the tendons. These forces are combined with relative movements of the prestressing steel in longitudinal direction during the stressing operation.

The wall thickness of the polyethylene duct shall therefore in system 1 be strong enough to resist the high transverse pressure of the prestressing steel in the deviation point.

The saddles should not harm the strength of the prestressing steel during the lifetime of the structure.
**Stressing operation**—During the stressing operation the saddles should not damage the prestressing steel or develop too high friction in the contact zone between saddle and prestressing steel. Special precautions should be taken to avoid every possible damage (friction or hardening effects) of the prestressing steel.

To reduce the friction in the deviation points, and as a result also to avoid damage of the prestressing steel, for some applications neoprene cushions with teflon are developed to admit sliding of the prestressing steel along the saddle with only small friction between steel and saddle.

**Experimental investigations**—Experimental investigations have shown that, due to curvature effects, the strength of a strand reduces the function of its curvature.

If prestressing strands are used with a so-called D-value of 28 [10], the reduction of the actual tensile strength of a strand can be limited to 5% if the radius of curvature \( R_{\text{end}} \) is larger than \( a \cdot \varphi_n \cdot N/n \) [12].

The D-value is established in a test — the deflected tensile test — standardized by F.I.P. to check the behavior of a seven wire prestressing strand under multiaxial stresses. In places where the tendon is deflected or deviated the prestressing steel is subjected to these multiaxial stresses.

The D-value is the reduction (in %) of the strength of the strand if this strand in a deflected shape is subjected to a tensile test. For a quality controlled produced prestressing steel the maximum value of the coefficient D can be limited to 28% \( (D = 28) \), as is resulting from the available test data.

In this formula:  
\( \varphi_n \) = the nominal diameter of the strand;  
\( N \) = total number of strands in the tendon;  
\( n \) = number of strands transferring the radial component of the tendon force to the supporting deviator in a cable like system 1 of Fig. 5.  
If a tendon consist of 20 strands, only, for example, 3 strands will be pressed to the internal sheathing of the deviation point. The other strands are transferring their radial component to the deviator by the 3 strands in the contact zone. In this case \( n = 3 \) and \( N = 20 \);  
\( a \) = factor with the order of magnitude of:  
20 for smooth ducts  
40 for ribbed (corrugated) ducts.
Remark: The formula for $R_{\text{tend}}$ is given a magnification factor $(a \cdot N/n)$ of $\phi_n$. Therefore every unit system can be used (mm, inch, etc.).

Attention should be drawn to the fact that in the deviation zone the actual strength of critical strands is assumed to be reduced by 5%. If only 1% reduction is acceptable the value $a$ should be multiplied by 5.

Example: $\phi_n = 12.5$ mm, $N = 20$, $n = 3$ - Smooth ducts

$R_{\text{tend}} \geq 1700$ mm (5% reduction of strength of critical strands)

With 1% reduction of strength:

$R_{\text{tend}} \geq 8500$ mm.

To avoid the problems of reduction of strength in the deviation zone, system 2, as shown in Fig. 6, is adequate.

The transfer of deviation force to the concrete structure

The transverse force – and a horizontal force – should be transferred, with a sufficient margin of safety, from the deviation zone to the concrete structure. Because the angle between the axis of the tendon on both sides of one deviation point is relatively large and the tensile force in the tendons is high, the magnitude of the resulting transverse force is considerable. The external tendons are preferably situated inside the concrete structure (box-girder). Therefore, the connection between deviation zone and concrete structure should transfer large forces (see Fig. 8). If diaphragms are used to carry these forces, considerable shear forces may develop between these diaphragms and the main structure (see Fig. 9).

These parts of the concrete structure should be detailed very carefully. One has to take into account the fact that locally concentrated loads are acting in the original concrete structure. Also, these loads should be distributed safely over the whole structure. These zones of transfer of the prestressing force of the structure should be reinforced adequately.

Anchorage of the Tendons

With the exception of so-called unbonded tendons, the ducts of conventional internal prestressing tendons are grouted after stressing of the prestressing steel. If this grouting has been carried out carefully, the transfer of the prestressing force in the tendon may, in case of damage of the original anchorage, be partly taken over by the bond of this tendon – the grouted duct – to the structure. The strength of the anchorage of a tendon is therefore of less importance for most of the lifetime of the structure than in the period before grouting of the ducts.
In the case of external tendons, however, the anchorage of the tendon is of paramount importance during the whole lifetime. This is also the case if the ducts of these tendons are grouted after stressing because this grout only protects the prestressing steel against corrosion and does not assure bond of the entire tendon length to the concrete structure.

As a result, the strength of the structure depends mainly on the strength of the anchorages.

Because tendons should be replaceable, the zone behind the anchorages is not concreted in after stressing the tendons. They should be situated in the structure in such a way that:

1. the place where the tendon leaves the anchorage should be easily accessible for inspection during the lifetime of the structure.
2. the tendons must be released in order to be replaced. Therefore, it is necessary to provide permanent space for the jacks and the uncut prestressing steel behind the anchorages. As already mentioned cement grouted tendons are not considered to be replaceable. These tendons can only be dismantled. Removal of the steel still under high tensile stress will be dangerous.
3. the anchorages and the prestressing steel should be protected sufficiently against corrosion, fretting fatigue, fire and sabotage.

The requirements for these anchorages are more or less the same as for the anchorages of stay cables. F.I.P. has published a very interesting paper on stay cables and their anchorages by its commission on "Prestressing Steel and Systems" [11].

Because the tensile forces of the external tendons are transferred by the anchorages to the concrete, attention must be paid to the detailing of the reinforcement (and prestressing) of these zones of transfer. The ducts, penetrating these zones of transfer, are from steel with an internal cable sheathing made of polyethylene (see Fig. 5).

In Fig. 7 an intermediate anchorage of an external prestressing cable is shown in an anchorage block.

In this case the connection between this block and the concrete structure should transfer the total prestressing force. In this considerable shear forces may develop between this anchorage block and the main structure. This connection should be detailed very carefully.

One has to consider the situation of the structure in case some tendons are destressed to replace them. In these temporary situations it might be possible that in certain zones larger tensile stresses are allowed.
One remark regarding anchorages and cables may be repeated here. Contrary to standard post-tensioning systems with fully bonded tendons, in external prestressing the anchored tendon is supposed to withstand extreme conditions such as vibrations. The free length of the external tendons should therefore be limited in order to avoid unacceptable vibrations. In literature, a maximum distance of 7-8 m between two deviation points of a tendon is mentioned. The own frequency of the tendon is then at least 7-10 hZ.

CLOSING REMARKS

External prestressing was already a mode of construction in the fifties. After lying dormant for some time, it has been rediscovered as an attractive application of prestressing, adopting recently developed possibilities of prestressing technology.

Several applications of this new technology can be mentioned [9]:

- Reconstruction, strengthening and repair of prestressed concrete structures.
- Prestressed trusses.
- "Understressed" beams with external tendons (stays) below the bottom of the concrete structure. In these structures, several solutions for the deviation zone of the tendons are developed (nodes with crossing of two straight tendons overlapping at columns and special deviation devices).
- Cable-stayed bridges, also with short pylons (extrados prestressing).
- Provisional prestressed concrete structures.
- Free supported span by span temporally prestressed bridge structures, composed of prefabricated segments, which are, after placing on their piers, made continuously by external prestressing.
- Incremental launching procedures with the aim of centric internal prestressing and continuity tendons as external prestressing (without bond).
- Span by span in situ construction.
- Continuity tendons of bridges executed according to the cantilevering construction method.
- Prestressing of steel or timber structures.

A variety of applications of this new technique has already been tried-out in practice. This new approach is, however, related to the introduction of several new technologies. Therefore, it is necessary to study and to investigate these new technologies very carefully.
The following advantages of external prestressing can be mentioned:

- Simple construction methods.
- Simple tendon profile, resulting in simple construction on the site.
- Reduction of the dimensions of the concrete structure (width of web).
- Few or no problems with grouting of tendons.
- Possibility of inspection of the tendons during the lifetime of the structure with x-rays or other detection techniques.
- Replaceability of tendons.

Disadvantages of this method are:

- Vulnerability of the structure to fire, damage, sabotage.
- Reduced contribution of the prestressing steel to the failure load and the stability of the structure.

Points of special attention are:

- Deviation zones.
- Anchorages.
- Ducts.
- Corrosion protection of the prestressing steel.
- Minimum reinforcement needed in the concrete structure.

But nevertheless...........

External prestressing offers a challenge for concrete designers to develop new structural concepts and construction methods.

REFERENCES


Fig. 1 External tendons - Magnel

Fig. 2 Artificial loading

Fig. 3 Shape of an external tendon in a box-girder bridge
Fig. 4  External tendons in a box-girder bridge in France

Fig. 5  System 1 - Multistrand cable
Fig. 6  System 2 - Use of monostrands in PE-sheathing

Fig. 7  Intermediate anchor [3]

Fig. 8  Deviation block [3]
Fig. 9  Cross-section bridge for pedestrians, span 40 m, designed by the author. Use of concrete diaphragms

Fig. 10  Transfer of prestressing forces into a concrete box-girder
Development of External Prestressing in Bridges—Evolution of the Technique

by P. Jartoux and R. Lacroix

Synopsis: After ten or so years of research and practical application, the external prestressing of concrete is now becoming a normal procedure, soon to be codified in some countries.

Among the technical solutions applied to a large number of works, in the USA and France particularly, some emerge as being the best examples both from the point of view of performance and of economy. This is the case where normal strands in HDPE ducts are used with an injected cement grout. This technique can be used equally well with removable external prestressing and with external prestressing which is partially bonded.

In difficult cases, such as very long structures (bridges with lengths exceeding 200m or 600 ft) or structures with high curvature (tanks and various vessels) or where it is difficult to bring large jacks up to the anchorages, an external cable, formed of protected strands, gathered together in the same HDPE duct and isolated from one another, is a very effective and elegant solution with an unequalled degree of protection and with the opportunity to check the prestressing force throughout the entire life of the structure.

Keywords: bridges (structures); ducts; friction; galvanized materials; grout; long span; prestressed concrete; prestressing; prestressing steels; reviews; unbonded prestressing
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External prestressing is defined as prestress produced by cables placed outside the load-bearing structure over the greater part of their length; only the anchorage heads and those parts of the cables close to the anchorages are built into the structure (Figure 1). Deviators may be within the structure or may be external. This type of prestress applies both to new structures and to those being strengthened, whether in reinforced or prestressed concrete, in masonry, steel or composite.

HISTORY

External prestressing was virtually created in France with the first applications of embedded prestress carried out by Eugène Freyssinet. In the years immediately after the Second World War; perhaps as a way of dealing with patents. We will not dwell on earlier applications previously quoted by M. VIRLOGUEUX [1a], but in an article on the development of this technique, it is important to place the origin. We will simply recall that these applications were not very successful. The CAN BIA Bridge at ARLES SUR TECH was destroyed recently, and the CHILTEPEC Bridge in Mexico, built in 1965 with bare wires protected merely by painting, was saved recently by the complete replacement of the prestressing. This last example is fully representative of modern solutions provided by external prestressing for the repair of structures. It is, moreover, this field of structural activity which is the origin of new developments in external prestressing.

Starting in the 1960s, in Belgium, under the impulse of two design offices aiming to simplify cable lay-outs - done by using small units with large deviations on plan - about thirty bridges were built with a combination of internal and external cables. These structures are still in service and, in general, have performed very well.

During the 1970s, in France, a large number of structures have had to be strengthened by additional prestress, in order to meet new design requirements. These structures have made it possible to improve the technology as applied either to new structures or to strengthening operations (Figure 2).

But it was Jean MULLER who, in 1979 with the bridges in the Florida Keys (LONG KEY Bridge) [23], proved all the advantages of fully external prestressing in lightening structures formed of prefabricated segments. In France, from the 1980s and with the support of SETRA (Service Technique des Routes et Autoroutes -Official Technical Design Service of Roads and Highways) and M. VIRLOGUEUX in particular, numerous structures have been desi-
gned and built with external prestressing, with the additional feature that external tendons are removable.

DIFFERENT WAYS OF IMPLEMENTING EXTERNAL PRESTRESSING

Numerous systems for implementing external prestressing have been developed within the last ten years. They can be divided into two main classes:

- external prestressing bonded to the structure,
- unbonded external prestressing

External prestressing bonded to the structure

When external prestressing bonding is used on civil engineering structures it is provided by cables external to the concrete over their entire length, except in the neighborhood of the anchorages (in the diaphragms, pier segments or abutments) and in the span’s deviators. There are two basic types of lay-out:

External cables with all-steel ducts—(Figures 3 and 4). A cold-bent deviator tube set in the concrete during pouring, is connected to the external duct of the cable by socketing, and by welding with or without crimping and resin waterproofing. The cable is then placed in the traditional manner with a cement grout injection; the grouting bonds the cable when passing through the concrete. Several strengthening jobs have been carried out with this type of cable lay-out.

External cables with steel and HOPE ducts—(Figures 3 and 4). A deviator tube, cold-bent or a corrugated sheath in strip metal, is embedded in the concrete, and connected to the duct tubes in HDPE (high density polyethylene* or polypropylene). The polyolefine tubes are preferably used in straight lengths, connected by thermo welding ("mirror" or sleeves) rather than in coils, because of residual deformations.

The steel-to-HDPE connection is a delicate operation, requiring tested procedure or adapted connectors (Figure 5). The cable is then placed in the traditional manner as in the preceding method. A large number of applications of this technique have been carried out either in repair work or in new works.

In these types of application, very special attention has to be paid to the frictional properties of the steel deviator tube, since the radii of curvature are generally small (3 to 5 m).

* HDPE in the continuation of the text.
or 10 to 16 ft) and the contact pressures are very high, thus introducing the risk of seizing-up.

**Unbonded external prestressing**

On civil engineering structures, unbonded external prestressing has taken multiple forms, all of which permit more or less simple replacement of the prestressing without any demolition or drilling parts of the structure.

These forms can be classified in accordance with the type of protective injection - no injection, unrigid injection, rigid injection.

**Unbonded cables, not protected by injection**—These are cables formed from self-protected strand, generally met in two types: galvanised strand and strand protected by HDPE sheath and grease or wax.

**Galvanised strand**—(Figure 6). These strands are obtained by bringing together wires galvanised by immersion in a bath of molten zinc. This provides an effective protection, but this effectiveness depends on the aggressiveness of the surrounding atmosphere; the application calls for meticulous surveillance of cables used in the structure.

Several structures in France have used it either as additional prestress or for the original prestress.

When this type of self-protected strand is used, it is reasonable to envision protection of the same type for the anchorage equipment, to ensure homogeneity. As a safety measure, where the distances between deviators are large, it is usual to place a free sheath on these cables, preventing the cables from rubbing and still permitting detailed inspection. Anti-vibration collars are used to fix the cables to the concrete structure.

**Strand protected with coating and HDPE sheath**—This type of strand, as yet little used in civil engineering works, is developed for buildings and is frequently used in the USA and Northern Europe (in this case the strand is called unbonded single-strand tendon). It is manufactured from standard strand, coated with grease or paraffin wax, and then the entire strand is covered by a layer of extruded HDPE. This is excellent protection against corrosion although a little brittle in handling. Use of such strands in bundles calls for special arrangements at the deviators, preventing interactions between the strands which can damage the sheath.

When used, this system isolates completely one strand from another, and thus it is possible to stress large cables with single-strand jacks. This is especially advantageous in strengthening structures, as bringing up multi-strand jacks and carrying out stressing operations always presents major pro-
blems. Furthermore, the very low coefficient of friction of these strands (0.05) makes it possible to introduce prestress with excellent efficiency, even when using very long and/or very curved cables. [6].

The FREYSSINET system, for example, (Figure 7) uses a cable formed from a bundle of protected strands without classification; mechanical insulation is provided by a cement grout placed by injection, before stressing, in a thin pipe of HDPE.

It should be emphasized that the cement grout injection and the duct only act here as means of ensuring strict maintenance of spacing, in a manner exactly suited to the nature of the bundle of strands after threading. The conditions for grouting are much easier and have nothing in common with the necessary requirements to guarantee the protection of the bare strands. The system offers a guarantee of good resistance to the different aggressive elements surrounding the cables.

This system has been subjected to research and tests prior to being put into operation on site. Two problems had to be solved:

- interaction between strands protected by sheaths in a cable where they are in bundles, while the device was intended to be used alone, embedded in concrete;
- threading of sheath-protected strands into a duct, the equipment used for a bare strand damaging the polyethylene envelope.

* Interaction between strands - A full-scale test was carried out on a 19T15 tendon on the FREYSSINET curved beam. This test is described in detail [6] (Figure 8).

It will simply be recalled that the aim was to verify the behaviour of strands and their protection when they are formed into a cable within the same duct, the regularity of classification not being guaranteed - mechanical insulation of the strands was provided by a cement grout injection prior to stressing.

Stressing of the cable was done strand by strand up to 85% of ultimate tensile strength and all of the strands were displaced several times in their individual sheath by stressing; after these operations, the cable was removed for examination and the following observations were noted:

- at no point had the sheath with a thickness of 1.5 to 1.75 mm (about 1/16") been perforated, since the electrical insulation measurements between the strands showed no contact.

- after being dismantled the sheath was seen to be in good condition - only the strands inside the curve (radius 2.5m, about 8 ft) had left imprints of the order of seve-
ral tenths of a mm on the polyethylene.
- the controlled-setting grout had provided excellent filling.
- a duration of hardening of the length of 1 to 2 days is sufficient to ensure the effectiveness of the grout as a spacer. Although the individual strand sheaths are in contact, the grout cover is sufficient to block the system and ensure the independence of all the strands.

* Threading of the sheath protected strands-Apart from the absence of standard equipment for threading, the high coefficient of friction of the polyethylene duct on the individual sheath of the strand raised a problem.

FREYSSINET carried out a systematic research programme on lubrication to reduce this friction; the final choice fell on an emulsion of wax in water which reduced the frictional forces at threading by about 90%. A full-scale test on a 19T15 cable 100m (about 300 ft) in length proved the effectiveness of this lubricant (threading force for one strand below 300 kg - 600 lbf). It also made it possible to perfect the technique of threading using a winch alternately, first at one end and then at the other with the same individual cable (Figures 9 & 10).

Several applications have already been made either in strengthening structures or in new structures. The strands used in all cases were coated with grease (16).

Unbonded cables protected by unrigid injection-The technological arrangements made depend on the method of injection of the flexible product, whether cold or hot, and not on the material - grease or wax. The expression "cold or hot" is not sufficiently precise; it is necessary to state the physical nature of the supple product at the time of injection. A cold or tepid product in the viscous state or a hot product in the fluid state?

Unbonded cables protected by a soft protection product injected in the viscous state-(Figure 11). This application calls for relatively high pressures (more than 1.5 N/sq mm - 210 PSI) and numerous injection points along the tube; the duct must be in steel with a placing technique known as "central heating". In addition, the ducting must be provided with chambers allowing the flexible product to expand during very hot weather.

Unbonded cables protected by a soft protection product injected in the liquid state-(Figure 12). This is a technique more widely used than the previous one, and is a natural result of applications which were made on the containments vessels of nuclear power stations (BECHTEL schemes). The sheathing is in thin steel tubing (1 to 2 mm - .04 to .08", depending on the diameter of the duct) or in a HOPE tube. The difficulties associated with this type of application relate to the watertightness of the piping during injection and the softening of the
polyethylene piping during injection which call for special precautions with the injection ports. Supple products injected may be either grease or paraffin wax; wax has the advantage of a melting point slightly below that of the grease and there is no occluded oil which, when using grease, always tends to sweat and seep through the pipe joints. This technique has been applied in a number of French bridges [15]. All prestressing cables protected by soft protection products are demountable before cutting of stressing overlengths, cables can be destressed and afterwards dismantled; in other cases, de-tensioning is a little more harsh. Arrangements must then be made to protect the cable surroundings from grease or wax extracted with the strands. Cleaning of the piping should be done before the cable is rethreaded. It does not appear that there has been any replacement of cables to date although the precautions to be taken have been exhaustively examined.

Unbonded cables protected by rigid injection—This is the system of demountable prestressing which has been used a great number of times in France. Cement grout is always the injection product used and the tubing is either in a thin steel tube or HDPE tube (nominal pressure of .6 N/sq mm or 85 psi). In both cases, the absence of bond (or the ability to be demounted) is obtained by the use of double tubing in the concrete diaphragms. The first tube is a sleeve—generally in thin material—embedded in the concrete of the cross-braces or the deviators or introduced after concreting [4]. The second tube is the duct tubing which surrounds the cable in a continuous and watertight manner, from one anchorage to the other.

Cables with a thin steel duct—(Figure 13). This is a solution highly appreciated, surely more because of the assumed defects in HDPE (ageing) than because of the qualities of steel. In opposition, it is possible to indicate difficulties inherent in the use of steel tubes:

- joining separate tubes close to concrete surfaces is not always easy;
- steel rusts when it is not painted and if this happens, the end result is not always very good.

Some structures have been built with this technique [10] [1a].

Cables with a continuous HDPE duct—(Figure 10). The basics of the present technique were established during the construction of the PONT-A-MOUSSON Bridge. Fears which existed were that the polyethylene of the duct would flow in the deviators, under the action of the forces due to curvature of the cables, leading to contact pressures much higher than those normally accepted for this type of material. FREYSSINET took part in the perfecting of this technique by carrying out numerous tests. Three main problems had to be solved:
- effectiveness of the arrangements made for removal in the anchorage zones,
- strength of the HDPE duct in the deviators,
- effectiveness of an anchorage against failure when it is injected with cement grout.

* During the construction of the bridges at PONT-A-MOUSSON and LE VENANT, tests [7] were carried out to verify the extent to which the duct could be demounted and its strength (Figures 15 & 16). Two models were made and showed that:

- demounting is very easy provided the watertightness of the grouting is well guaranteed in the anchorage zone, so as to avoid seepage in the double sheath.
- the cable certainly marks the polyethylene, but given a reasonable delay prior to injection no damage needs be feared. It has since been established that this delay could be much longer - there is undoubtedly a binding effect from the HDPE pinched between the sheath tube and the stressed strands. On the other hand, it has been shown that the positioning of the sheaths in the concrete of the deviators is not easy if it is required to guarantee a good lay-out of the cable, with no sharp point. Several solutions have since been found to overcome this difficulty (toric shape, for example (Figure 17)).

* Demountable prestressing is, by definition, unbonded. The entire efficiency and longevity of the prestressing thus depends on the performance of the anchorage.

This is not a new problem; USA has seen the development of unbonded cables both for containment vessels (BECHTEL schemes) and for the floors in buildings (unbonded single strand tendon). A slightly new idea in France consisted of the cradle of prestressing and of the idea of cement grout bonding. Cement grout bonding, when properly done, guarantees preservation of the cables and their efficiency up to ultimate load, due to the mobilisation of the bond forces.

Many examples can be listed and it is worthwhile to mention that this type of prestressing has been associated with all types of construction [4, 5, 10, 12, 14, 15].

ADVANTAGES OF EXTERNAL PRESTRESSING

External prestressing advantages have been the object of many examinations and publications. We will mention briefly only particular aspects of setting and technology: the reduction in thickness of the webs, ease of concreting, ease of checking during and after installing, the quality of execution of the
prestressing and simplification of the cable lay-out. This particular point calls for additional comment, since external prestressing makes it possible to simplify the cable lay-out provided that a few simple requirements are met:

- do not attempt too rigorously to match the cable lay-out with the bending moment envelope by increasing the number of deviators and small cables;
- do not terminate cables a few metres from the end of a structure in order to reduce the ratio steel-concrete, since the handling of the jacks becomes considerably more difficult.
- select a unit size adapted to the size of the structure; all jacks are heavy to manipulate beyond a mass of 30 kg (66 lbs) but it is easier to place a jack weighing 1.5t with properly planned equipment than to place a jack of 100kg (220 lbs) with no equipment. Similarly, it is difficult to manhandle jacks even for small units, across a multitude of external cables.

To summarise to some extent the advantages of external prestressing, it should be said that its great merit is to separate two activities which go uneasily together - that of the civil engineering contractor who wishes to place his concrete under the simplest possible conditions and that of the prestressing works which are becoming more and more the mechanical activities of Public Works. The quality of the final work depends on this state of affairs.

**DISADVANTAGES OF EXTERNAL PRESTRESSING**

Like for the advantages, we will touch only on operating systems and technology.

The only disadvantages of external prestressing are those which arise from the fact that it is outside the concrete and sometimes even outside the whole structure. Two disadvantages in particular may be mentioned:

- vulnerability to acts of vandalism when the cables are accessible; in each particular case consideration must be given to the prohibitions to be enforced (effective closing of access to box sections, barrier grids on openings where the cables could be accessible etc.);
- vulnerability to fire; wherever there is an urban structure (intersection, overhead motorway) this problem must be envisaged. This is an important problem which must be examined, but it should not be a brake on the development of these techniques, since the problem of fire is not new and exists on suspension and cable-stayed bridges and has caused many less accidents than corrosion.
DEMOUNTING OF UNBONDED EXTERNAL PRESTRESSING INJECTED WITH CEMENT GROUT

Unbonded external prestressing is often referred to as "demountable". The system of double sheaths, if tight against the injection of the cement grout and associated with deviators in a single arc of a circle, makes it possible to remove the stressed cable section by section; although the stress must be released. For the moment there is little experience, but there are two cases worthy to be mentioned.

The first case is that of FREYSSINET with the former external tie-rods for the arch of Studios d'EUROPE N°1 in FELSBERG [8], and the second is that of CITRA on the Viaduct of CHAFFIX (rendered necessary as a result of an error in placing two cables). The principle consists in cutting the cable progressively, making sure that each section cut can be de-tensioned; this means that the binding of the grout by the duct must be freed so that it is destroyed by compression as each strand is cut. This operation is not dangerous but calls for a labour force fully trained in the task and aware of the desired end.

REGULATIONS

At present, there is no special agreement or set of regulations defining systems of unbonded external prestress.

Approval for its use is given step by step for minor modifications with accepted standard prestressing anchorages. The French Interministerial Commission on Prestressing has set up a working group which has to suggest specific regulations. This should come to light in the coming months.

CONCLUSIONS

This rapid survey of the development of external prestressing systems has made it possible to examine the majority of possible solutions; thus, the tendencies can be forecasted. It is clear that the HDPE duct grouted cement is by far the most economical and the most useful at the present time, whether the prestressing is bonded or unbonded. Other solutions, strands for unbonded tendons for example, introduce elegant solutions in special cases such as very long cables, or difficult accesses for stressing.

It should be emphasised that external prestressing, particularly the unbonded type with a HDPE duct, is of such a quality
that:

- one is well on the way to guarantee the design forces in every section of the structure, as the friction between the cable and polyethylene is low, precisely known, and unaffected by placing;

- the quality of the protection and the independence of the structure guarantee the durability of the prestressing for a very long time.

Paradoxically, to finish with the past and attempt to look into the future, let us say that external prestressing, born in the immediate post-War period, has experienced in the 1980s a rejuvenation which has made it possible to increase performance and quality of prestressed structures.

One can state without doubt that external prestressing will become a technological must in the 1990s, with the development of high-strength concrete. Furthermore, the beginning of the next century will be the start of a combination of composite materials and external prestressing, which will free the civil engineers from the problems associated with the protection of steel.
BIBLIOGRAPHY

(1) Annales ITBTP du 25.11.82.
   Annales n° 420 Décembre 83.
   a) La Précontrainte extérieure par M. VIRLOGEUX
   b) La construction à l'avancement par J. COMBAULT

(2) Revue Travaux - Janvier 85
   Le Viaduc de SERMENAZ
   par MM. PHAM-TAO - J. PIRON - Ph. LECROCQ

(3) Revue Travaux - Janvier 85
   Le Viaduc de ST AGNAN par MM. J. COMBAULT - D. POINEAU -
   D. GOBINET - M. DUVIARD - F. EDON - M. VIRLOGEUX

(4) Revue Travaux - Janvier 85
   La construction du 2ème Pont sur la Moselle à PONT-A-MOUSSON
   par MM. G. CAUSSE - T. DUCLOS - M. VIRLOGEUX - J.M. BONNET -
   J.C. HUMBERT - C. LECLERC - J.P. AUBRY - J.P. CAZENAVE -
   A. GHENASSIA

(5) Revue Travaux - Supplément de Janvier 1986 - 10e CONGRES FIP
   à NEW DELHI
   Améliorations des connaissances sur la mise en oeuvre de la
   précontrainte
   par MM. A. CHABERT - R. AMBROSINO - J. LAVIGNE - A. REMY -
   B. FARGEOT - P. JARTOUX

(6) Revue Travaux - Supplément de Janvier 1986 - 10e CONGRES FIP
   à NEW DELHI
   La précontrainte extérieure FREYSSINET INTERNATIONAL -
   Rétrospective d'une évolution par P. JARTOUX

(7) Revue Travaux - Janvier 1983
   Réparation du bâtiment émetteur d'Europe n°1 au FELSBERG
   (Sarre) par P. XERCAVINS

(8) Revue Travaux - Janvier 1986
   Viaduc du franchissement de la DARSE D'ALFORTVILLE par l.au-
   toroute A86 par MM. A. HEUSSE et BONNEAU

(9) Revue Travaux - Janvier 1986
   Caractéristiques principales des ouvrages suivants :
   Pont de CHINON sur la VIENNE
   Pont de RUMILLY sur le CHERAN
   Autoroute A 51 - OA 82 franchissement de la DURANCE
   Viaduc OA 33 sur l'Autoroute du Littoral de MARSEILLE
   Pont de CERGY-PONTOISE
   Pont sur l'AIGUES à SERIGNAN
   Passage supérieur n° 8 sur A 71
   Viaduc de ST AGNAN

(10) Revue Travaux - Octobre 86
    Les ouvrages n° 14 et 15 de la Voie Express G12
    par Ph. MEYRAND
(11) Chantiers de France - Septembre 86
Autoroute A 71 - Viaduc sur le VENANT

(12) Chantiers de France - Octobre 1987
Pont de l'ILE DE RE

(13) Chantiers de France - Novembre 1987
Pont sur la Charente à COGNAC
Viaduc de MAUPRE à CHAROLLES

(14) IABSE SYMPOSIUM - PARIS-VERSAILLES 1987
Ponts poussés à précontrainte totalement extérieure
par MM. J. COMBAULT - P. NERON - A. LEVEILLE -
J.L. THIBONNET
Les viaducs de SYLANS et GLACIERES
par MM. J. BOUDOT - PX. THAO - B. RADIGUET
Viaduc de PONCIN par M.C. SERVANT
Viaduc de l'ARRET DARRE par M.C. SERVANT

(15) IABSE SYMPOSIUM - HELSINKI 1988
The introduction of unbonded greased or waxed tendons in
external prestressing
by A. CHABERT - P. JARTOUX - R. VILLETTE

(16) Revue Travaux - Supplément de Janvier 1986 - 10e CONGRES
FIP à NEW DELHI
Précontrainte extérieure en béton - Comportement jusqu'à
rupture de poutres à voussoirs préfabriqués.
par B. FOURRE et M. VIRLOGEUX

(17) Revue Travaux - Supplément Janvier 1986 - 10e CONGRES FIP
à NEW DELHI
Utilisation de la cire pétrolière dans le domaine du Génie
Civil par A. CHABERT

(18) FIP NOTES 1987/2
External prestressing of concrete by M. VIRLOGEUX

(19) FIP RECOMMENDATIONS 1986
Corrosion protection of unbonded tendons

(20) C.E.B. - G.T.G.17 "State of the art report on internal and
external unbonded tendons" - Draft report to be published

(21) FIP COMMISSION "Prestressing Steel and Systems"
External prestressing - Draft report

(22) FIP SYMPOSIUM JERUSALEM (1988)
External Prestressing by A.CHABERT - B.CRETON - M.VIRLOGEUX

(23) PCI JOURNAL - November-December 1980
Construction of LONG KEY Bridge by J. MULLER
Design features and prestressing aspects of LONG KEY Bridge
by T.M. GALLAWAY
Anchorage

Parts of Tendon outside of the Concrete

Deviations in Concrete Diaphragm (Bonded or Unbonded)

Fig. 1

EXTERNAL PRESTRESSING FOR REPAIRS
Typical Arrangement With Concrete Freyssinet Anchorage

Concrete Diaphragm

Cement Grout

Steel or HDPE Pipe

Drilled Hole

External Anchorage

12 & 7 Wires Cable

Fig. 2
Partially Bonded External Prestressing

Anchor Deviator’s Zone

Standard Guide

Embedded Galvanised (Steel Pipe or Corrugated Sheath)

Concrete Diaphragm

Steel Pipe (Painted or Galvanised)

or HDPE Pipe

Tight Sleeve

60 psi

Fig. 3

Partially Bonded External Prestressing

Deviator’s Zone

Concrete Diaphragm

Embedded Galvanised Steel Pipe

Steel Pipe (Painted or Galvanised)

or HDPE Pipe

Tight Sleeves

(Permanent or Reusable)

.4 N/Sq mm - 60 psi

Fig. 4
External Prestressing Connections of Sheath Steel Pipes

- Welding

- Socketing with Resin

- Socketing with Thermo-retractable Sleeve

- 2 Half Shells Sleeve (Reusable)

Fig. 5

UNBONDED EXTERNAL PRESTRESSING
Galvanised Tendons

See Detail 1

Deviator (Galvanised) Steel Pipe

Galvanised Strands

Anti-vibrating Device

Overlength Detension Retension

Protection Sleeves Against of Strands Breaking

Fig. 6
UNBONDED EXTERNAL PRESTRESSING
HDPE Coated Greased Strand

Stuffing Box Device

Pressing Bolts  Seals  Bearing Plate

HDPE Coated Greased Strands

Grouting Tube

Fig. 7

FRICTION TEST BENCH

1 x 1  Sqm

Tested Tendon

Tendon Locally Embedded in Curves

R = 4 m
R = 2.5 m
R = 12 m
Length = 35 m

Fig. 8
UNBONDED EXTERNAL PRESTRESSING
HDPE Coated Greased Strand
Test for Threading Through HDPE Sheath

R = 4 m
12'

TESTING BENCH
R = 2.5 m
8'

45m~140'
45m~140'

3m
10'

Backwards and Forwards
Threading Cable

THREADING EQUIPMENT
HDPE Sheath

Winch
Lubricant Box
Reverse Coupler
Lubricant Box

Fig_9

UNBONDED EXTERNAL PRESTRESSING
HDPE Coated Greased Strand

BASIC FRICTION TESTS

Adjustable loading device

Fn

HDPE Runway

Ft

Strand sheath sample

Time (min)

Without Lubricant

With Lubricant

Fn = cte

Fig_10
UNBONDED EXTERNAL PRESTRESSING

Cold Grease Injection

- Overlength
- Detension Retension
- Steel Pipe
- Galvanised or Painted
- Expansion Chamber
- Standard Strands
- Grease
- Injection Pressure > 15 N/sq mm
- 210 psi
- Threaded Sleeve (Central Heating or Welding Type)
- Ports For Injection (All 30 feet - 10 m)

Fig. 11

UNBONDED EXTERNAL PRESTRESSING

Hot Wax Injection

- Overlength
- Detension Retension
- Standard Strands
- Galvanised Steel Pipe
- Grease or Wax
- HDPE Pipe
- Special Sleeves (High Tightness)
- Painted or Galvanised
- 6 N/sq mm
- 85 PSI

Fig. 12
UNBONDED EXTERNAL PRESTRESSING
Steel Sheathing
1/12"

See Detail 3 Steel Pipe ~ 2mm Thick Standard Strands

Cement Grout Steel Deviator (Galvanised)

Fig. 13

UNBONDED EXTERNAL PRESTRESSING
HDPE Sheathing

See Detail 4 HDPE Pipe 1.6 N/sq mm 85 PSI Standard Strands

Cement Grout Steel Deviator (Galvanised)

Fig. 14
UNBONDED EXTERNAL PRESTRESSING
Pont-a-Mousson Bridge. 19 x 6" tendon
FEASIBILITY TEST

External Grouted Tendon
HDPE Pipe
Translation RAM (Simulation of Long Tendon)
Tensioning RAM
Stuffing Box
R=4m=12'

Fig. 15

UNBONDED EXTERNAL PRESTRESSING
Venant Bridge. 12 x 6"
Pulling out of Tendon

TEST BENCH

Fig. 16
Unbonded External Prestressing Deviators

- Embedded Types -

Over Bended Pipe

\[ D > d \quad r < R \]

Standard Type

Sawn Cut

Type for Precast Segments

Foamsleeve

Fig_17

PULLING TEST BENCH

Testing of External Prestressing Cable

Fixed Bloc

Pulling Jack

Movable Bloc

Grouting Cap

Multi-use Anchor

Tightness Seal

Grouted External Prestressing Cable

(3m~10')

Fig_18
UNBONDED EXTERNAL PRESTRESSING
Cement Grouted Cable

Jaws Free to Move

Hand Pump

Before Grouting

Soft Protection Product
(Wax, Grease, ...)

Cement Grout

Fig_19

UNBONDED EXTERNAL PRESTRESSING
Setting of Jaws

1: Standard Test
2: Ultimate Tensile Test with Blocked Jaws
3: Ultimate Tensile Test Jaws Free to Move

Fig_20
Properties of Polyaramid Ropes and Implications for Their Use as External Prestressing Tendons

by C.J. Burgoyne

Synopsis: The paper describes the properties of parallel-lay ropes with a polyaramid (Kevlar 49) core, with particular reference to the long term properties which are of importance to the designers of prestressing systems. The anchorage and prestressing systems are described, and results are given for stress-strain, relaxation, creep, stress-rupture and fatigue behaviour.

Durability and thermal response are also considered, and it is inferred that the lack of corrosion, in addition to the high strength and high stiffness, makes these materials ideal for use as prestressing tendons where the concrete cannot be used to provide corrosion protection to steel.

Descriptions are given of tests on beams prestressed with external tendons, which show that a ductile response can be achieved in a beam made from two brittle materials.

It is concluded that these materials will extend the range of structures that can be built with prestressed concrete, and will at last allow the realisation of the full potential of externally prestressed concrete.

Keywords: beams (supports); corrosion resistance; creep properties; durability; fatigue (materials); modulus of elasticity; plastics, polymers, and resins; prestressed concrete; prestressing; strength; stress relaxation; thermal properties
Dr Chris Burgoyne is a lecturer at Cambridge University, and has been carrying out research into the properties of aramid ropes for a number of years. His other research interests include the application of expert systems to the design of prestressed concrete, and the general philosophy underlying structural design. He was formerly employed at Imperial College, London, where much of the work described here was carried out.

INTRODUCTION

The use of external tendons in prestressed concrete is an idea that has been tempting designers almost since the first use of prestressing. The biggest benefit is the saving of weight in the webs. They can be reduced to the thickness needed to carry the shear forces, without the necessity of providing cover for the tendons. In addition, the cables are accessible for inspection and, potentially, replacement.

The drawbacks lie in the absence of the corrosion protection that is normally provided by the concrete. Without the passivating environment provided by the highly alkaline cement matrix, the steel will corrode very rapidly unless extensive measures are taken to prevent corrosion from occurring.

In the United Kingdom, as elsewhere, there have been problems with corrosion in external tendons. At Braidley Road viaduct, the external tendons had to be replaced and provided with additional corrosion protection after some of the tendons failed after only 12 months (1).

Even internal tendons can corrode. A recent report on Ynys-y-Gwas bridge in Wales (2), which collapsed in 1985 under the action of dead load only, attributes failure to corrosion of the prestressing tendons. This is despite the fact that the tendons are internal, that most ducts were properly grouted and that the concrete was of adequate quality. Furthermore, the bridge was regularly inspected and there were no indications of anything awry before failure. The bridge was of segmental construction, with mortar joints between the segments and no in-situ topping. Water, almost certainly containing de-icing salts, penetrated these joints and caused severe localised corrosion, which eventually led to failure.

A recent report on a condition survey of the stays in cable supported bridges (3,4) has shown that many of these are visibly in a bad state, and other work has shown that many failures in multi-strand cables begin on the inner wires, rather than the outer ones (5). It must be presumed that many prestressing cables are deteriorating in a similar way.

The conclusion that can be drawn is that new and existing structures prestressed with external steel tendons are very susceptible to corrosion. Many existing structures prestressed with internal steel tendons must also be very suspect, but how their condition can determined, without causing the corrosion that we wish to prevent, is a separate problem that is being investigated.
Attempts have been made to realise some of the advantages of thinner webs, without resorting to external tendons, by keeping all the tendons in the flanges, but at the expense of more awkward tendon arrangements. Redheugh Bridge over the River Tyne at Newcastle uses this system (6), and the nearby bridge over the River Coquet extended the idea by using precast panel elements for the webs, with in-situ concrete flanges (7). These are only intermediate stages however.

There is thus a proven need for a non-corroding prestressing tendon, which can be used externally, either in new construction, or as additional prestressing in structures in need of repair. The remainder of this paper describes the properties of such a material.

**PARALLEL LAY ARAMID ROPE.**

Aramid fibres, with an elastic modulus (approx 124 kN/mm$^2$; 18,000,000 psi) approaching that of steel and a strength (2760 N/mm$^2$; 400,000 psi) exceeding that of prestressing steel, offer a combination of properties that are suitable for prestressing tendons. These fibres were developed under the name Kevlar (by Du Pont) in 1973, and similar, though not identical, materials are now produced by Akzo (under the name Twaron) and by Teijin (under the name Technora).

The fibres themselves are very fine, being supplied as a yarn consisting of 1000 individual filaments, each filament being of 2.13 denier (equivalent to a cross sectional area of 0.000163 mm$^2$; $2.53 \times 10^{-7}$ sq.ins). The individual fibres must be aggregated to form tendons; conventional laid ropes, which maintain their integrity by twisting together many yarns, are not suitable, since the individual fibres follow helical paths along the rope. These would act like springs when stretched, and the axial stiffness of the rope would be very low by comparison with that of the constituent fibres (8). On the other hand, parallel-lay ropes allow virtually the full stiffness of the fibres to be realised without the need to introduce resin, but require an external sheath to maintain integrity.

This paper is concerned with one such parallel-lay rope, manufactured by ICI Linear Composites Ltd, under the name Parafil. Various fibres can be incorporated within the core, but all the results quoted here were obtained from Type G Parafil ropes, which have a core of Kevlar 49 fibres.

The ropes were originally developed for other purposes, most notably mooring buoys and offshore platforms in very deep water. The combination of properties, especially when using stiffer fibres like Kevlar, makes the ropes suitable for structural applications.

**Anchorage**

The ability of an element to carry significant tensile forces is only as good as the mechanism for getting the force into the member; a number of methods are possible, including external wedges, bond or cast resin cones. For a variety of reasons, however, including the desire to avoid resins because of creep and the poor response to high temperatures, one method is clearly preferable (9). This is the internal wedge (or spike) which provides a radial
gripping force between the spike and the external body, so that all the fibres are anchored. The length of the spike can be chosen to ensure that the transverse stresses are within the capacity of the fibres. As with the rope itself, there is no need to introduce resin anywhere in the termination.

Figure 1 shows a typical termination for parallel lay aramid ropes. Once the load has been transmitted from the fibres to the terminal body, further connection to the structure can be made by fitting a variety of devices, such as clevis pins, anchorage plates, or whatever is required. The figure shows such a terminal modified for use as a prestressing tendon. The terminal body has two threaded regions; the inner thread is used for connection to a pull rod which is attached to the jack during stressing, while the outer thread is used to provide a connection for a permanent back nut, which also allows some adjustment to take account of slack. The anchorage is capable of achieving the full strength of the rope. Reasonable care must be taken to ensure that the spike is fitted centrally within the rope (to ensure even load-sharing between fibres), but otherwise no special skills are needed. Anchorages for parallel-lay aramid ropes with capacities between 1 and 1500 tonnes (2.2 to 3300 kips) have been provided using this system. All the results quoted here have been obtained on ropes fitted with anchorages in this way.

It is normal practice, where practicable, to preload the rope to ensure that the central spike is fully drawn into the termination. This makes sure that no subsequent movement occurs in the termination, either on loading or unloading.

**Stressing procedure**

The stressing procedure for tendons is straightforward (Figure 2). A pull-rod is connected to the internal thread on the terminal body, and passed through a centre-hole jack. This is loaded against the concrete by means of a trestle, and when the correct jack force is achieved, a back-nut is placed on the external thread to provide the permanent anchor.

**ROPE PROPERTIES**

The rope properties of most interest to prestressing engineers are the strength, elastic modulus, relaxation, creep & stress rupture, and durability. The response to temperature changes and the fatigue behaviour are also relevant, as is any bond between the ropes and concrete. There is no space here to go into full details of these properties, but a broad outline of the results, and the way they were obtained, will be given.

**Strength and elastic modulus**

Figure 3 shows a typical stress strain curve for a Type G Parafil rope with a nominal breaking load of 60 Tonnes (132 kips). The elastic modulus is about 118 kN/mm² (17,100,100 psi), while the strength is about 1950 N/mm² (283,000 psi). These values are lower than those observed in tests on individual fibres, but bundle theory, which relates the properties of an agglomeration of components to the properties and variability of the components themselves, adequately accounts for the majority of the difference (10,11). Figure 4 shows the effect of rope size on strength; as predicted by
bundle theory, small ropes (below about 6 Tonnes capacity (13.2 kips)) are stronger than larger ropes, but there is very little decrease in strength above 6 Tonnes capacity. The largest rope tested to date (1500 Tonnes; 3300 kips) failed at virtually the same stress as the 60 Tonne ropes described here. Since all ropes used for prestressing tendons would be of at least 60 Tonnes capacity, this indicates that size effects can be ignored in practice.

The manufacturer's quoted nominal breaking loads (NBL) for all sizes of rope are based on an assumed stress at failure of 1926 N/mm$^2$ (279,000 psi), and this principle has been followed in this paper. As can be seen from Figure 4, this is conservative for all rope sizes.

**Relaxation**

The relaxation of tendons is clearly crucial to the behaviour of prestressed concrete. Aramid fibres are better in this respect than most organic materials. Figure 5 shows the relaxation of 60 Tonne ropes, loaded to various proportions of their breaking load. After about 100 hours, the stress reduction response becomes approximately linear with the logarithm of time, and the following formula for the reduction in stress in a tendon has been proposed (10).

\[ r = 1.82 + 0.0403f + 0.67 \log_{10}(t-100) \quad \text{(for } t > 100), \]

where  
- $r$ is the stress-relaxation expressed as % NBL  
- $f$ is the initial stress expressed as % NBL  
- $t$ is the time in hours

This formula relates to the properties of the rope itself. Movement within the termination has been allowed for in deriving these figures, and in practice is eliminated entirely by the normal preloading of the rope/terminal connection.

At working load stresses of about one third of the breaking load, this formula predicts relaxation losses of about 7% NBL, but most of this occurs within the first few days, and restressing would virtually eliminate relaxation losses.

An extensive test programme is underway to provide more data on the stress relaxation behaviour under sustained extension over long periods of time, and with various amounts of restressing after a few days.

**Creep and stress-rupture**

The creep response of a material is clearly related to its relaxation behaviour, although the two are often treated independently since they are usually important in different circumstances and are measured in different ways. A typical creep curve for a rope loaded to a high proportion of its breaking load is shown in Figure 6. Primary creep, immediately after loading, settles down to creep at virtually constant rate (on a logarithmic time basis). For loads which are a significant proportion of the breaking strength of the rope, there follows a tertiary creep phase which leads to failure.

An approximate correlation between stress relaxation and (secondary) creep has been published for pultrusions of the aramid fibre Twaron (12), and some
consideration has been given to the subject for Parafil (13). A more detailed study is now underway at Cambridge University.

Creep to rupture, or stress-rupture as it is more commonly known, is undesirable and it is important to be able to predict the lifetime of ropes at different load levels. Tests have been carried out on 60 Tonne (132 kip) NBL ropes at high stress levels, under loading applied by hydraulic jacks, with 'times to break' of up to 5 months. This method of loading is unreliable and inconvenient for long term tests, so dead weight loads are used for lower stress levels. Such tests have been underway now for some time on 1.5 Tonne (3.3 kip) and 3 Tonne (6.6 kip) ropes, with the object of providing sufficient data for engineers to have confidence in the long-term properties of the material. Figure 7 shows the results obtained to date. Some ropes have been under load for nearly two years.

The most realistic theoretical model for the failure of such materials is one based on a reaction rate approach. This predicts a linear relationship between applied load and the logarithm of the 'time to break'. A large number of tests were performed at the Lawrence Livermore National Laboratory (LLNL) on Kevlar 49/epoxy composites (14), which confirmed this model, and also gave some indication of the scatter of the results. Certain empirical factors must still be determined; these have been found from the tests on 60 Tonne (132 kip) ropes, and the predictions of this work for the lifetime of the ropes are also shown Figure 7, represented as 5% and 95% confidence limits. The results obtained so far from the smaller rope tests indicate that these predictions could, if anything, be pessimistic, since many of these results lie above the 95% probability of failure line. It seems reasonable to predict that a Parafil rope would sustain a load equivalent to 50% of its short-term breaking strength for a period of 100 years.

Estimates, based on the scatter of the LLNL data, have been produced elsewhere (15) for the probability of failure at different load levels. These have been converted into load levels to give a $10^{-6}$ probability of failure at different lifetimes. A more recent estimate, based solely on tests on Parafil ropes (13), gives almost identical predictions for the lifetimes at typical working loads. A cumulative damage technique has been applied to estimate the effect of relaxation of the tendon and creep of the concrete in a prestressing application (10). The combined effect is that a prestressing cable, stressed initially to 49% of its breaking load, would have a $10^{-6}$ probability of failure after 100 years, whereas a rope loaded by a constant force of about 38% of its breaking load would have a similar probability of failure. This is because the high initial stress of 940 N/mm$^2$ (136,000 psi) reduces rapidly with creep of the concrete and relaxation of the Parafil to about 730 N/mm$^2$ (106,000 psi) in service. Analogous results can be produced for other loading regimes or desired lifetimes.

**Durability**

Kevlar is durable under most circumstances. It is affected by ultra-violet light, by a mechanism which involves breaking links within the polymer chains, thus reducing strength. In a Parafil rope, this will not cause problems, since UV light is excluded by the thick black sheath.
There is some data available on the hydrolysis of Kevlar in steam at elevated temperatures (>140°C), and also evidence of reduction in strength at ambient temperature in strong acids and alkalis. However, there is no evidence to indicate that there is any reduction in strength in fresh water, sea water or mildly acidic or alkali solutions at normal operating temperatures.

Nevertheless, a test programme has been initiated at Imperial College, which aims to determine what would be the first signs of deterioration. If these are absent in a normal environment, then there will be positive evidence for the durability of aramid fibres.

Fatigue

Work on aramid filaments themselves (16) has shown that fatigue failures due to direct tension-tension loading only occur at a large number of cycles, and then only at stress ranges well above those normally used in real applications. There is also work (17), mainly on other filaments such as Nylon, but also on Kevlar, which indicates that 'fatigue' failures are related to duration of loading, rather than the number of cycles. This would indicate that rope lifetimes are better estimated on a stress-rupture basis, rather than on a fatigue basis. For structures such as bridges, where there are many load cycles, but where each is of short duration, it is probable that failure of the ropes due to cyclic loading would be unlikely.

Tension-tension fatigue tests (18) and tension-bending fatigue tests (19) have been carried out on Parafil ropes. These indicate that failure is caused by inter-fibre fretting, either in the termination, or at the lateral loading point, and that this behaviour is far more significant than true fatigue of the fibre. Figure 8, taken from (18), shows that the tension-tension fatigue behaviour is better than that of conventional steel ropes.

Thermal response

The thermal properties of a material are important in two ways; the response to fire, and the response to normal temperature fluctuations.

Aramid fibres do not burn, but decompose at about 450°C. They retain about one half of their short term strength when heated to about 250°C (20). In the form of a Parafil rope, the sheath, which is thermo-plastic, will melt during a fire, but fire retardants can be incorporated in the formulation. The thermal conductivity of the fibres is extremely low, and this will enhance the material's ability to withstand a fire.

The coefficient of thermal expansion is negative, as determined by tests on bare yarns immersed in distilled water, and is a function of stress (13). At operating stresses of prestressing tendons (about 700 N/mm²; 100,000 psi), the coefficient of thermal expansion is about −6×10⁻⁶. This will have some effect on the design of structures prestressed with Parafil, since the Parafil and concrete will have different coefficients of expansion. However, because of the low conductivity, it is unlikely that daily temperature cycles will have a significant effect on the ropes. Only very slow temperature changes will need to be taken into account. Even when conducting the tests on Kevlar 49 yarns completely immersed in water, it was found that there was a very significant lag in the response of the yarn to temperature changes.
There is some evidence of absorption of water by aramids, which does not affect the strength, but which could have affected the values quoted here. These tests are to be repeated soon with the yarns dry, to separate the effects of temperature from those due to the water.

Bond

Various tests have been carried out to measure the bond between Parafil ropes and concrete (13). However, the bond strengths achieved were very low, (of the order of 0.15 N/mm²; 22 psi), and for all practical purposes should be ignored. This is not particularly surprising, since the individual yarns are not linked in any way, either to themselves or to the sheath. The sheath itself is smooth, and so cannot be expected to bond significantly to the concrete. Furthermore, since the sheath is made from a thermoplastic, (usually polyethylene), any sustained shear load passing through the sheath would cause large creep strains over a period of time, and any bond that existed would effectively be lost. There is thus no real possibility of using the ropes for pretensioning cables, or as reinforcing bars, where load transfer must be by bond.

IMPLICATIONS FOR PRESTRESSED CONCRETE BEAM DESIGN

Now that the properties of the material are available, the implications for the design of prestressed concrete beams can be considered.

1. The materials are durable, so there is no need to embed them in concrete to provide corrosion protection.

2. Aramid fibres are brittle, so it is not desirable for the ropes to pick up additional strains due to live loads. It would thus be a disadvantage if the ropes bonded to the concrete, so the difficulty of achieving such bond is not a problem. This result may at first sight be surprising, and needs a little explanation.

   In the vicinity of a flexural failure in a beam, cracks will open in the concrete. If the tendon is bonded to the concrete, it is forced, locally, to have very high strains. If the material is ductile, these do not cause a problem; the tendon yields but does not fail. For a brittle material, however, the tendon would snap, thus losing all its tensile force.

3. The working stresses are likely to be determined by stress-rupture criteria, rather than short-term strengths. Initial prestress loads of about 49% of the nominal breaking load would give a probability of failure of about $10^{-6}$ in 100 years.

The conclusion is that the material is suitable for use as external prestressing tendons for concrete. Indeed, unlike steel, there are no benefits to be gained by embedding the tendons in concrete, either for pre-tensioning or grouted post-tensioning. The ropes are equally useful for the repair of existing structures, as well as for the building of new ones.
BEAM TESTS

As part of the development process for the materials and to check the systems for applying loads to the tendons in practical situations, two beams have been built at Imperial College using Parafil ropes as prestressing tendons.

The first beam, with a length of 5m (16ft 5ins), was prestressed with a single straight 60 Tonne (132 kip) tendon which passed through a plastic duct on the centreline of the beam as shown in Figure 9 (10). There was, however, no attempt made to bond the tendon to the duct, and indeed the rope was wrapped in PTFE tape to reduce further the friction between duct and sheath. The terminals are obviously larger than the rope, and it is difficult to terminate the ropes in-situ, so the tendon was cut to length, fitted with terminals and installed in the duct, before the concrete was cast.

The second beam, on the other hand, was more representative of a practical application, with two 60 Tonne (132 kip) ropes mounted externally to the concrete, and deflected at saddles close to the loading points. This beam was 8m (26ft 3ins) long overall (13). In this case, larger holes were left for the anchors in the end block, so the tendons could be fitted after the beam had been cast. Figure 10 shows the overall beam layout. Full details of the beams and their geometry will be given elsewhere (21), but a summary of the procedures and results is given here.

The beams were simply supported close to their ends, and loaded by two point loads applied through a spreader beam. In both cases, a number of load cycles were applied, each at successively higher loads, until the beam failed. The maximum load on the first cycle was fixed when the first visible cracks were observed.

**Beam test results**

Figure 11 shows the load deflection curve from the first beam (10); the results from the second beam are similar. After the initial loading, the beam returned to its initial shape, as would be expected, under the action of the restoring force provided by the prestressing cable. Even after unloading from higher loads, when severe cracks were appearing in the beam, most of the deflection was recovered.

The final failure in both beams was characterised by large cracks opening in the tension face, with considerable deflection at virtually constant load. In both cases, final failure occurred by crushing of the top flange, followed by compressive failure in the concrete down the web as the beam tried to carry its own dead weight on a steadily reducing section. In the case of the first beam, the failure stopped when the beam came into contact with support trestles under the beam, with the bottom flange intact. For the second beam, failure continued right through the beam.

In both cases, the Parafil tendons did not appear to be affected by the failure of the concrete. In the first beam, the single tendon was found to be carrying a load of 33 Tonnes (72.6 kips) after failure (cf initial prestress of 42 Tonnes; 92.4 kips); this had to be destressed before the beam was dismantled. Unfortunately, the tendon had to be cut to remove it from the beam so it was not possible to test the tendon itself subsequently. In the
second beam, however, the tendons could be removed and were subsequently tested to failure in tension. They both failed at loads (69.9 and 68.1 Tonnes; 154 and 150 kips) in excess of the short term breaking load of the tendons (60 Tonnes nominal, normal mean figure about 61.5 Tonnes (135 kip)). Similar increases in strength have been observed after relaxation tests on Parafil (10) and other polymeric ropes. The reason for this increase in strength is not immediately clear, but it is possible that a sustained load (approx 50% of the nominal strength) applied to a rope for a significant period (in this case, about 2 months) will tend to even out the load sharing between filaments (due to differential creep), and thus lead to the bundle having a higher overall efficiency. The possible interaction of bundle theory and visco-elastic behaviour is currently under investigation.

CONCLUSIONS

Parallel-lay aramid ropes, such as Parafil, offer a new material for the designer of prestressed concrete. For the first time, they allow designers to make use of the inherent advantages of externally prestressed concrete without the need to worry about corrosion. Structures will become feasible that, hitherto, could not be built because of the dangers associated with corrosion of steel prestressing tendons.

References


1) Tendon installation
2) Jack attachment
3) Prestressing force application
4) Final arrangement

Figure 2. Stressing sequence for Parafil prestressing tendons.

Figure 1. Terminal for a 60 Tonne Parafil rope, modified for use as a prestressing anchorage.
Figure 3. Stress-strain curves for Type G Parafil and steel.

Figure 4. Effect of rope size on strength for Type G Parafil.
Figure 5. Relaxation behaviour of Type G Parafil ropes.

Figure 6. Typical creep behaviour at high load levels.
Figure 7. Stress rupture behaviour of Type G Parafil ropes.
Figure 8. Fatigue response of Parafil and wire ropes.
(from reference 18.)
Figure 9. Details of 5m Parafil prestressed beam.
Figure 10. 8m beam prestressed with two 60 Tonne Parafil tendons.

Figure 11. Load deflection curve for 5m Parafil prestressed beam.
The Application of External Prestressing of Bridges in Germany

by H.S. Svensson

Synopsis: The main advantage of external prestressing is that it facilitates the placement and vibration of the web concrete and thus permits thinner webs. The main disadvantages of unbonded external prestressing are its reduced ultimate capacity and its lack of contribution to crack control. The cost of bridges built with unbonded external tendons in Germany is currently higher than for those with bonded internal tendons, mainly because of code requirements for minimum web thickness and crack control, and due to higher costs for the external unbonded tendons.

External prestressing has only limited application in Germany. For new concrete bridges it has been used a few times since the 1950's for tendons bonded to the webs. The first experimental new bridge with unbonded external tendons in Germany is currently under construction.

Special applications of external tendons are for incrementally launched bridges, for the rehabilitation of bridges and as longitudinal force couplers.

In the future, unbonded external tendons will probably be used mainly for the rehabilitation of existing bridges in Germany.

Keywords: bridges (structures); concrete construction; corrosion; economics; grouting; history; limit design method; post-tensioning; prestressed concrete; prestressing; strains; structural design; unbonded prestressing
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INTRODUCTION

Virtually all major concrete bridges in Germany are built by segmental cast-in-place construction methods. An important quality requirement is that all construction joints are crossed by substantial rebar for crack control. Internal bonded prestressing tendons further improve the quality of construction joints. Partial prestress is generally applied.

External unbonded prestressing is currently under investigation in Germany to improve the quality of concrete bridges. Some advantages include:

- The placement and vibration of web concrete is facilitated by external tendons, resulting in improved concrete quality and permitting thinner webs.
- The corrosion risk of not completely grouted tendons can be reduced because the grouting of external tendons can better be checked.
- The tendons are not exposed to cracks in the concrete.
- The integrity of external tendons can be better controlled.
- External tendons can be exchanged.
- The friction losses of internal tendons are difficult to estimate.
- External tendons may be re-stressed at a later stage.
- External tendons facilitate the removal of obsolete bridges.

Some of the problems associated with internal tendons can be overcome by other means. Mistakes during grouting can be reduced by improved methods, workmanship and quality control. Replacement of internal tendons and increase of the prestressing force can be achieved by providing spare ducts and/or anchorage points for external tendons to be added later.

On the other hand, certain disadvantages are also connected with external unbonded tendons:

- They do not participate in local crack control.
- The eccentricity of external draped tendons is generally smaller than for internal tendons.
- Their ultimate capacity is smaller because the local strain of joint rotations is distributed over the full free length.
- The strain differential between concrete and steel may lead to movements of the tendons over the deviation saddles and thus to friction corrosion.
- External unbonded tendons might be more susceptible to fire damage.

These disadvantages can be improved by bonding external tendons to the webs as shown in Fig. 1, thereby retaining advantages of external tendons like ease of web concrete placement and control of friction.

Cost--The cost for bridges with external tendons in Germany is currently higher than for bridges with standard internal tendons, mainly for the following reasons:
- Current German codes specify minimum web thicknesses, see Fig. 2.
- The cost for the corrosion protection of unbonded external tendons is higher.
- More prestressing steel for the ultimate strength and more rebar for crack control during service is normally required.

HISTORICAL DEVELOPMENT

External unbonded tendons have been utilized in Germany for a few bridges at the beginning of prestressed concrete during the 1930's.

In 1934 Dischinger applied for a patent in which he envisaged tendons installed outside of the concrete cross-section of a bridge, Fig. 3. The tendons were arranged like the tension ties in a truss, (1). He designed the Aue Bridge in Saxony, Germany, Fig. 4, by using Grade 50 tie roads with 2 3/4 in. diameter. After completion in 1936, the tie-rods were re-stressed several times to overcome losses due to shrinkage and creep. (In 1962 the tie forces were found to be reduced to 20% of their original value, (2)).

Another early application of external tendons was patented by Finsterwalder in 1937, see Fig. 5. A beam with an articulated joint at midspan is post-tensioned by lowering and thereby elongating the bottom tie. He used this principle for a freeway crossing at Wiedenbrück, Germany, in 1938, see Fig. 6.

In the same year, however, the Oelde Bridge was completed, using girders pre-tensioned with internal tendons bonded to the concrete, Fig. 7. This method eliminated the need for long-term monitoring of the bridge and any corrective re-stressing, (2). The long-term corrosion protection of the tendons was also more easily achieved. Thus the principle of external unbonded prestressing was left at an early stage in Germany.

Virtually all prestressed concrete bridges built in Germany since 1950 have utilized tendons inside the concrete cross-section and bonded to it. A few major concrete bridges completed during the 1950's and 1960's used a post-tensioning system for concentrated tendons with forces up to 10,000 Kip. The concentrated tendons are located outside the webs inside the box. They are bonded to
the webs. Unbonded tendons are also used to couple railway bridge spans which might later have to be exchanged.

INCREMENTALLY LAUNCHED BRIDGES

The incremental launching method for concrete structures was developed by Leonhardt and Andrä in the early 1960's, (3). It started with a bridge across the Caroni River in 1961, Fig. 8. The entire 1,575 ft. long superstructure was cast as a unit on shore. It was post-tensioned with continuous straight external tendons during launching across the river. After the beam had reached its final position the external tendons were pushed upwards over the piers and downwards in the span, Fig. 9. The now draped tendons were then bonded to the webs as shown in Fig. 1.

All subsequent approximately 200 incrementally launched concrete bridges were cast in segments in a stationary form behind an abutment, see Fig. 10. During launching, straight internal tendons in the top and bottom slab are used to counteract the varying tensile stresses in the beam. After launching, additional draped tendons are required for live load, see Fig. 11. The amount of draped tendons depends on whether auxiliary piers are used during launching, Fig. 12. Without auxiliary piers, only a few draped tendons are required, which can easily be placed inside the webs, Fig. 13a. For the higher amount of prestress required with auxiliary piers, external tendons have been used, Fig. 13b. They were always bonded to the webs. Examples are the Kufstein Bridge, Fig. 14, and the Taubertal Bridge, Fig. 15, completed in 1968.

For the Taubertal Bridge, standard tendons requiring standard stressing equipment were used instead of concentrated tendons. During construction of the Mainflingen Bridge in Germany, the central span of 436 ft. had to be kept completely free of any obstructions to the traffic on the Main River, Fig. 16. Despite this restriction, the incremental launching method was applied advantageously by launching the bridge from both sides and by using auxiliary stays on top of the superstructure. The relatively high amount of draped tendons led to the use of external bonded tendons. Since then the minimum web thickness required in Germany, Fig. 2, and the rising labor costs have made external bonded tendons uneconomical.

Currently, the first experimental bridge in Germany with external unbonded tendons is under construction in Germany. The 7-span bridge with two superstructures is about 1000 ft long and is built by incremental launching. Only straight tendons are used in order to avoid potential problems with friction corrosion over deviation saddles. Prof. Eibl of the Karlsruhe University presents this bridge in detail in his contribution for this Symposium.
REHABILITATION

The main reasons for the deterioration of concrete bridges due to rebar and stress steel corrosion in Germany are:
- Deicing salt penetrates into the concrete.
- Lack of concrete cover together with carbonization of the concrete;
- Insufficiently grouted tendons in connection with intruding water;
- Cracking of concrete due to unaccounted temperature strains.

Of the about 30,000 concrete bridges built in Germany since 1950, about 5 post-tensioned bridges had to be replaced. About 25 bridges were rehabilitated by strengthening them with external tendons.

Schussenbridge

The Schussenbridge has a single span with 4 main girders over a length of 102 ft., see Fig. 17. Due to a malfunctioning bridge drainage system, deicing salt penetrated into the concrete, eventually reaching the stress steel and causing corrosion damage, Fig. 18.

Each web was strengthened with one tendon comprising 25 strands with 1/2 in. diameter. Inclined holes were drilled through the end-cross girders at the center of gravity through which the additional tendons run. At the underside of the cross-girders bent steel pipes were installed as deviation saddles, Fig. 19. PE-pipes are connected to these pipes and cover the distance between the saddles. The strands were threaded-in, stressed and cement grouted.

Danube Viaduct Untermarchtal

The viaduct was built in 1953 as a continuous 5-span girder with inner spans of 230 ft. and a total length of 1555 ft. Its beam has a T-cross-section with 2 webs, see Fig. 20. The bridge was post-tensioned with one concentrated internal tendon per web, consisting of 380 strands with 3/8 in. diameter and a breaking load of nearly 3000 Kip.

In 1983 it was detected that both tendons were partly corroded at locations where the grouting was incomplete. As a first step the concentrated tendons were injected with a corrosion inhibitor. In order to regain the original strength of the bridge, each web was strengthened by 3 external tendons with a breaking strength of nearly 1,500 K to replace the estimated 50% loss of the original stress steel.

The tendons are bonded to the webs in a 24 in. x 8 in. concrete slab connected to the web by dowels, Fig. 21. The external tendons terminate in corbels at the bridge ends which are
post-tensioned transversely against the webs with bars of
1 1/4 in. diameter, see Fig. 22.

LONGITUDINAL COUPLERS

Temporary coupling of bridge beams during construction with
external unbonded tendons has frequently been used. An example is
the bridge across the Shatt-Al-Arab River in Iraq, Fig. 23. The
swing span was connected to the remainder of the beam with tempo­
rary tendons during incremental launching.

Permanently installed external unbonded tendons have been
developed for railway bridges, Fig. 24. These railway bridges are
often more than 3,000 ft. long. It shall be possible, however, to
exchange individual bridge sections not exceeding 1,150 ft. in
length if that should ever be required during the life of the
bridge. The large longitudinal forces due to braking and friction
have to be transmitted by the superstructure across the joints to
the fixed point of the bridge. The coupler tendons are designed
for tensile forces up to 4400 Kip.

The tendons are 20 ft to 30 ft long and consist of parallel
wires inside a PE-pipe. The pipe is injected with a corrosion in­
hibitor similar to permanent rock anchors, Fig. 25. Control, force
adjustment and exchange is possible at any time. These unbonded
tendons have been specially developed. They were tested at the
Otto-Graf-Institute (OGI) at Stuttgart University for fatigue due
to rotation of the two adjacent bridge beams with very satisfac­
tory results, Fig. 26.

Such coupler tendons have meanwhile been installed in seve­
ral large railway bridges in Germany. One example is the Main
River railway bridge shown on Fig. 27 and 28. The incrementally
launched superstructure with a total length of 4200 ft. is divided
into 4 subsections by three coupler joints.

SUMMARY AND CONCLUSION

The main advantage of external prestressing is that it faci­
litates the placement and vibration of the web concrete and thus
permits thinner webs. The main disadvantages of unbonded external
prestressing are its reduced ultimate capacity and its lack of
contribution to crack control. The cost of bridges built with un­
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Germany. For new concrete bridges it has been used a few times since the 1950's for tendons bonded to the webs. The first experimental new bridge with unbonded external tendons in Germany is currently under construction.

Special applications of external tendons are for incrementally launched bridges, for the rehabilitation of bridges and as longitudinal force couplers.

In the future unbonded external tendons will probably mainly be used for the rehabilitation of existing bridges in Germany.

REFERENCES:


Fig. 1: External concentrated tendon bonded to the web.

Fig. 2: Minimum web thickness currently required in Germany.
Fig. 3: Dischinger's patent application from 1934

Fig. 4: Dischinger's Aue Bridge, Germany, 1936
Fig. 5: Finsterwalder's method of post-tensioning, 1937

Fig. 6: Finsterwalder's Wiedenbrück Bridge, Germany, 1938

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ELEVATION

PLAN

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External Prestressing in Bridges—
the Example of the
Sylans and Glacieres Viaducts (France)

by P.X. Thao

Synopsis

The use of tendons placed outside the concrete for the longitudinal prestressing of bridge decks is particularly well-suited to the triangulated trusses which have now proven effective on more than one project by the Bouygues Group.

Experience acquired first on the Tehran Stadium roofing (internal tendons) and then on the externally-prestressed Bubiyan Bridge in Kuwait have highlighted the numerous advantages to be derived from the latter system.

We have therefore re-used the technology, with external tendons, in the construction of the Sylans and Glacières viaducts located along the Mâcon - Geneva - Mont Blanc highway in France, on behalf of the Société des Autoroutes Paris Rhin-Rhône.

The two viaducts have a total length of 1500 m. Each consists of two parallel decks, 10.75 m wide. Typical spans are 60 m long.

The deck is precast in forms as a series of 4.66-m long elements, and erected by sequential cantilevering using a launching girder.

Keywords: bridges (structures); cantilever bridges; concrete construction; high-strength concretes; post-tensioning; precast concrete; prestressed concrete; prestressing; precast prestressing steels; segmental construction; stadiums; structural design; unbonded prestressing; viaducts
Pham Xuan Thao is an engineer, graduate of the Ecole Catholique des Arts et Métiers in Lyon (France). He joined the Bouygues Group in 1969 as a structural engineer, and is now a Department Head within the Public Works Design Office.

1.0 GENERAL

The Sylans and Glacières viaducts mark the third application of the post-tensioned concrete triangulated space frame developed by the Bouygues group.

The first in the series was the roofing of the Tehran stadium (Iran), built in 1974. The main structure consists of three-dimensional truss girders spanning 82.30 m, with a depth of 8 m. These 500-t girders are precast on the ground, then hoisted to a height of 30 m and shifted laterally into position. Post-tensioning is provided by tendons located inside the concrete.

ARYAMEHR STADIUM

The second structure built using this technique was the Bubiyan Bridge in Kuwait (1983). The structure is about 2400 m long, with 59 typical spans of 40.16 m, and a main span of 53.84 m.

The 18.20-m wide deck is composed of two slabs connected by eight trussed webs, triangulated in the cross section. All longitudinal post tensioning is external and ensures virtually uniform distribution of compression under permanent loads.

BUBIYAN BRIDGE
Finally, the Sylans and Glacières viaducts constitute the third project using a triangulated concrete truss, which differs slightly from that of the Bubiyan Bridge in terms of the structural design itself as well as the construction methodology.

In order to respect deadlines and quality imperatives, this latest project includes new features such as the internal post-tensioning of truss bars combined with external longitudinal post-tensioning, cantilever erection without closure pours and use of high-strength concrete.

2.0 GEOMETRY OF THE SYLANS AND GLACIERES VIADUCTS

In 1985, the Société des Autoroutes Paris Rhin-Rhône entrusted the Bouygues company with the construction of the Sylans-Glacières viaducts, located along the A 10 highway (Mâcon - Geneva - Mont Blanc), on the north rim of the Nantua-Bellegarde gorge.

The Sylans viaduct is 1,268 m long and borders the lake of the same name, while the Glacières viaduct, 215.20 m long, skirts around the lake on the west side.

Given the sheer walls of the gorge, each viaduct has two parallel decks, 10.75 m wide, and a twisted alignment to better follow the natural contours of the site.

The minimum curve radius is 425 m and cross fall ranges from -2.5 % to 6.5 %.

The low slenderness ratio designed for the structure was made possible by the lack of vertical clearance requirement and by the fact that web weight represents only a small proportion of total structural weight.

Indeed, making the deck deeper results in only a modest increase in deck weight, leading to savings on longitudinal post-tensioning.
3.0 Piers and Foundations

Piers are built on 4-m-diameter pile foundations anchored at depths of 6 to 35 m in the limestone bedrock, often at the edge of the cliff.

Piles are bored through a layer of loose rock, with support by concrete rings cast immediately following the boring operation.

At the top of the piles, semi-circular concrete shells, tied to the base of the piles, protect the piers from the loose rock.

The high cost of these foundations largely reflects the technical difficulty in building them. The reduction in deck weight achieved through triangulation of the truss allows longer spans than would a conventional design for a given pier load capacity. With fewer piers, savings on foundations are significant.

4.0 Deck

The deck is about 10.75 m wide, composed of two slabs connected by four slanted webs. The webs constitute a triangulation in the cross section of the bridge. Moreover, each web consists of a triangulated panel in the shape of an X, the members of which are post-tensioned using anchored tendons injected with cement mortar.
5.0 LONGITUDINAL PRESTRESSING

Longitudinal post-tensioning is mainly provided by external tendons, completed by some internal tendons in mid-span. They form 3 groups:

- **external tendons of the cantilever (12T15)**, slanted and anchored at the base of the webs in mid-span

- **internal tendons for structural continuity (12T15) in mid-span**

- **two types of additional tendons installed at the request of the Consultant**:
  - longitudinal internal tendons at each node called "assembly prestressing"
  - Additional external tendons for safety placed in mid-span, to ensure that the joints do not open under ultimate limit state loadings.
6.0 **SEGMENT PRECASTING**

For precasting purposes, the deck is composed of segments measuring 4.66 m in length, weighing 60 T each.

Precasting involves two phases, beginning with casting of the X-shaped truss elements.

Each arm of the X is 3.50 m long, with a cross section of 0.20 m x 0.20 m. A 40-mm-diameter tube is embedded in each arm, to house a 5 or 10 PHI 7 wire, tensioned after the slab is cast.

X diagonals are installed in the segment precasting form, and only the slabs remain to be cast.
All of the segments for a 423-m-long viaduct are match-cast without closure pours. This requires extremely precise geometrical control, measurements, on-site analysis of measurements and constant follow-up by the design department. In elevation, geometry deviation is on the order of a few millimeters; in plan it does not exceed 2 cm.

Precasting entails the use of two forms, each designed for continuous casting of one basic viaduct. Precasting proceeds at a rate of one segment per day.

**PRINCIPLE OF BALANCED CANTILEVER ERECTION**

Segments are erected by the cantilever method using the launching girder that had already served for construction of the St. Maurice interchange viaducts. During erection, the launching girder is supported on the pier segment as pairs of segments are erected symmetrically about the pier. This arrangement halves the overturning moments transferred to the piers.
The cantilever is erected 20 cm off-center with respect to its final position and placed on four temporary shims. At closure, sliding bearings are inserted and closure is achieved by lateral displacement of the cantilever section.

In order to avoid concentrating excessive forces along the segment end fibers as the cantilever is moved into position, the joint surfaces must be made parallel prior to closure by adjusting the angle and level of the cantilever and the previously erected span.

By eliminating closure pours, the related interruptions to the erection process (installing formwork, waiting for concrete to reach the required strength) can be dispensed with.

The method also eliminates virtually all force redistribution due to creep, thereby allowing savings on post-tensioning. On the other hand, it requires complete mastery of geometry, to avoid geometrical defects due to precasting.

The erection cycle allows construction of one span every three days, in two eight-hour shifts.
8.0 USE OF HIGH STRENGTH CONCRETE

The Contractor deemed it necessary to increase the characteristic concrete strength to 65 MPa, while designing the sections on the basis of 37 MPa. This change permits a reduction in storage and handling time. Moreover, the risks of cracking during hoisting and of deformation during tensioning are diminished. Finally, the use of high-strength concrete is a factor contributing to the longer service life of the structure.
9.0 DESIGN CRITERIA APPLICABLE TO SECTIONS

Detailed criteria were established during the preliminary design phase, then used as a basis for the construction design. Design criteria may be summarized as follows:

- The end fibers of the deck remain completely compressed at ultimate limit states.

- Post-tensioning of truss elements is such that under service load combinations, the normal force is always compression. Bending moments are taken by reinforcement; tensile stress of reinforcing steel is limited. At the ultimate limit state, the reinforcing steel and over-tensioning of cables offset tensile forces in the diagonals.

10.0 ADVANTAGES OF EXTERNAL TENDONS

As the arrangement of the diagonals doesn't permit an easy location of inclined internal tendons, the use of external prestressing is necessary in truss decks. However it isn't only a necessity, the use of external tendons presents many significant advantages:

- Reduction of time to put in place conducts and tendon supports. The time to place tendons is slightly longer but the works contingencies are reduced.

- Less problems of grouting: no risk of duct obstruction and grout slipping and the reinforcement is less disturbed, more repetitive for the prefabrication.

- Quantities savings: the quantity of prestressing is reduced due to the diminution of the friction losses in using external tendons.

- Regarding the maintenance problem, the inspection of tendons is easier and the possibility of replacing tendons is interesting.

- The quasi absence of steel overstressing in external tendons under live loads increases their durability, and their bearing capacity regarding fatigue phenomena.

These features were used advantageously in the construction of the Sylans and Glacières Viaducts. So, for Bridge construction, the external prestressing system can be considered as a technical progress, an experience to extend.
11.0 PARTICIPANTS

The main bodies and firms that participated in the design and construction of the Bridge are as follows:

- Owner : SAPRR (Société des Autoroutes Paris-Rhin-Rhône)
- Supervision : SCETAURROUTE
  SETRA
- Design (alternative) : BOUYGUES Company
- Contractor : BOUYGUES Company
- Subcontractors :  
  - FREYSSINET STUP for external tendon anchorages
  - CIPEC for Diagonal internal prestressing (BBR parallel wires)
Texas SDHPT Experience with External Tendons on Segmental Bridges

by A. Matejowsky

Synopsis: The State of Texas is involved in two projects which use precast concrete segmental erection methods with external post-tension tendons. Design and construction features of these projects, along with construction problems related to external tendons, are described. The future of segmental construction and use of external tendons in Texas are discussed.

Keywords: box beams; bridges (structures); cable-supported structures; cantilever beams; concrete construction; epoxy resins; post-tensioning; precast concrete; prestressing; prestressing steels; segmental construction; unbonded prestressing
Alan Matejowsky is a Bridge Design Engineer with the Texas Department of Highways and Public Transportation. He has been involved with the design and construction of concrete segmental bridges built by the State in Texas.

Introduction

The State of Texas has been a forerunner in the development and use of concrete segmental bridge construction in North America. When plans were being made for expanding the intersection of IH 10 and IH 35 in downtown San Antonio, aesthetics was a prime consideration. The desire for an open, uncluttered bridge structure for the elevated portions of the project led to the selection of a winged box girder shape which is being built by segmental methods.

The Neches River Bridge, near Port Arthur, Texas, was designed by the State as a concrete cast-in-place segmental bridge built by the balanced cantilever method, with alternate designs allowed in the construction specification. The low-bid contractor proposed an alternate concrete cable-stay structure that incorporates segmental construction methods.

San Antonio Downtown Y Project

Portions of IH 10 and IH 35 in downtown San Antonio were built in the early 1950's as some of the first expressways in the State. Commercial development along the right-of-way makes expanding on grade impractical, so elevated expressway with on-grade frontage roads is the solution selected for increasing capacity to ten and twelve main traffic lanes. The need to maintain traffic without decreased capacity throughout construction was a factor in selecting a span-by-span method of segmental construction.

Figure 1 shows the overall layout of the project. IH 35 enters the downtown sector from Austin to the northeast. IH 10 enters from El Paso from the northwest. The two interstates share the south leg before splitting, with IH 35 continuing to south Texas and IH 10 continuing east to Houston. The sequence of work calls for the outbound lanes of each leg to be constructed first beginning at the outer end of each leg and working toward downtown; followed by the inbound lanes beginning at the center of the project and working away from downtown. This sequence will aid in traffic flow during construction by limiting the amount of construction work zone that a vehicle will have to go through on any given trip. This order is illustrated on Fig. 2.

The first construction project is Number I-A and includes 366,615 square feet of finished bridge area. Segments are 5'-10" deep, approximately 8' long and vary from 26' to 42' wide. Boxes are 6' and 8' wide. The segments, Figure 3, have solid wings with
transverse pretensioning. Both internal and external tendons are used, but external tendons are the main longitudinal post-tensioning reinforcement.

Construction project II-B, with 370,520 square feet of deck area, Figure 4, uses precast panels to form the bottom of the wings. The panels are set in the match-casting form with void forms attached and concrete is placed around the panel. Match-cast surfaces are obtained on the entire cross-section. Segments are up to 58' wide with 10' wide boxes. Transverse pretensioning is utilized in this project as well.

The third project, III-A&B, with 611,790 square feet of finished bridge area, was designed with precast panels in the wings. As Figure 5 shows, the wing has an open face at the match-cast joint. The contractor has chosen to cast the segments without the panel but still with the same wing shape as shown. We were concerned with the ability to place concrete in the thin (4") bottom slab of the wing, but there has been no problem with getting good concrete placement using high-range water reducer concrete.

A typical erection sequence for a span is outlined in Figure 6. All joints between match-cast segments are glued with epoxy. The epoxy joint is clamped and allowed to cure before final post-tensioning is applied to the span. All spans have at least one transverse concrete closure joint. Project I-A has two joints as shown; Projects II-B and III-A&B have one closure joint at the center line of the pier. Closure joints are sometimes made as one pour, while some other joints are made in two pours. While the design takes advantage of the two pour joint to help decrease secondary moments, it does require more time and labor to construct.

Project I-C, with a finished bridge area of 766,055 square feet, is a companion to project I-A with similar details. Major differences are: (1) the boxes are 8' and 10' wide; (2) main longitudinal draped tendons are a combination of internal and external tendons; and (3) interior diaphragms are used only at segments that have a deviation of external tendons. Typical segments, Figure 7, have a rib under the top slab. This project is under contract with casting of first segments expected early in 1989.

Projects II-C, with 401,530 square feet, and III-C&D, with 581,390 square feet of bridge area, will be let to construction late in 1989. Both projects will be similar to their companions with differences of: (1) wider boxes, up to 16'; (2) longitudinal draped tendons will be a combination of internal and external tendons; (3) interior diaphragms at external tendon deviation segments only; (4) solid wings. A difference from Project I-C is that the top slab is thickened without a rib.
We think that all of the changes being made from one project to the next will speed construction by simplifying details.

All of the project will be overlayed to obtain a more uniform riding surface. Units are either 2, 4 or 6 spans with average span length of 100'. Bearings are laminated elastomeric (with and without sliding surface), laminated fabric (with and without sliding surface) or pot/disc.

Neches River Bridge

The Neches River Bridge is a 9440' long structure providing 143' vertical clearance over a shipping waterway. The approach spans to the main unit are typical pretensioned girder and slab design. The contractor chose to build the main unit of precast segments erected by the span-by-span and cantilevered cable stay methods. The 140' spans (Figure 9) are built by the span-by-span method. The 640' main span will be erected by cantilevering segments and supporting them with stay cables anchored in the back approach spans.

The segments, cross-section shown in Figure 10, are 10' long. Each segment has a post at the center of the segment to act as a strut between top and bottom slabs. Cable stays anchor at alternate segments, Figure 11.

The segments are post-tensioned transversely in both the top and bottom slabs. The tendons anchor at the juncture of the top slab and the sloped bottom slab/web.

The spans are post-tensioned longitudinally to carry construction loads during the span-by-span erection of the 140' side spans and positive live load moments in all spans. Bottom slab internal tendons are 6-.5" strands in round ducts anchored at various points along the span in anchor blocks at the base of the post in the segment. The major tendons are external draped tendons of up to 27-.5" strands. The external tendons anchor at pier segments in blocks at the juncture of the top and bottom slabs of the cross-section and deviate near midspan in blocks at the bottom of the post in the segment.

Piers and pylons of the main unit are also segmental construction and, along with superstructure segments, are precast at a casting yard approximately 250 miles from the bridge site. Segments are barged to the site on the Intracoastal Waterway.

Once the side spans have been erected to the pylons on both sides of the river, the main span will be cantilevered, Figure 12. A sequence of erecting two span segments, one pylon segment and a cable stay completes a cycle that is repeated fourteen times to reach midspan. A cast-in-place closure completes the erection.
The temporary pier is removed and cable stay forces are adjusted to complete the bridge.

To date, the three span-by-span approaches on one side of the river have been erected.

Construction Problems

Many small problems have arisen during construction of both projects. These problems can be attributed to design considerations, contractor inexperience and State inspectors inexperience. It can be expected that using new construction techniques on projects of this magnitude will lead to problems related to the learning process. Very few problems can be directly blamed on the use of external tendons.

The 6' wide boxes have a beam across the bottom of the box instead of full segment diaphragms to serve as deviation blocks. These beams are also used as jacking points during epoxy clamping and curing. Figure 13 shows that some post tensioning tendons have fouled beams adjacent to deviation points. This increases the friction losses in the tendon and applies downward forces on the bottom slab.

External tendons are installed in bent pipe in the diaphragm of pier segments. The pipe should be tangent to the path of the tendon as the tendon emerges from the diaphragm. This requirement was not met on some segments and surface spalling cracks resulted on the face of the diaphragm, Figure 14. Conventional reinforcing steel contained the cracking, but concrete patching was required.

Other pier segment diaphragms have shown some surface cracking, Figure 15. These cracks are different from the previous example in that the duct is much further from the opening in the diaphragm. Some of these cracks are about .02" wide. We have observed cracks of the same pattern on some segments prior to tendon stressing and think these are caused by shrinkage. We will be monitoring these cracks to determine changes in width during the loading history of the segment.

Future Outlook for Segmental Construction and External Tendons

Our overall experience with segmental construction with external tendons is good. We anticipate using these methods on other structures where aesthetics and construction access are considerations. The open, clean appearance of our box girders are an asset to the highway system (Figure 16). We have more projects in design phase that use external tendons, both segmental and cast-in-place continuous boxes.
FIGURE 1

DOWNTOWN "Y" IMPROVEMENT PROJECT
SAN ANTONIO, TEXAS
TYPICAL SECTION
PHASE 1

TYPICAL SECTION
PHASE 2

TYPICAL SECTION
PHASE 3

Fig. 2
I-A AT PIER
Fig. 3

II-B IN SPAN
Fig. 4

III-A & B IN SPAN
Fig. 5
Place segments on truss, apply and cure epoxy joints.

Place bottom slab closure joint.

Install and stress bottom slab internal tendons.

Fig. 6
Place remainder of closure joint.

Install and stress draped external tendons.

Span complete. Move erection girder to next span.

Next span complete. Draped tendons overlap at pier segment to provide continuity.

Fig. 6 (Cont'd.)
Internal Tendons

Transverse Pretension Strands

SECTION

Internal Tendon

External Tendons

Rib

Draped Internal Tendon

I-C IN SPAN

Fig. 7

Internal Tendons

External Tendons

SECTION

Transverse Pretension Strands

II-C & III-C & D IN SPAN

Fig. 8
HALF-ELEVATION  
Fig. 9

TRANSVERSE SECTION  
Fig. 10

TRANSVERSE SECTION  
AT STAY ANCHOR BLOCKS  
Fig. 11
Temporary Post-Tensioning

Three deck segments cantilevered.
Two pylon segments stacked.

Add deck segment and install cable stay.

Cantilever one deck segment.
Add one pylon segment.

Add deck segment and install cable stay.
Fig. 12
LONGITUDINAL SECTION

Fig. 13

PIER SEGMENT

Fig. 14

PIER SEGMENT

Fig. 15
Re Island Bridge External Prestressing

by G. Causse

SYNOPSIS
The bridge that bears the link between Re Island and the main land was completed in April 1988. It crosses a 3000 meters wide sea channel that separates Re Island from the town of La Rochelle, on the west coast of France.

The deck of the bridge is a concrete box girder built by the balanced cantilever method with precast segments. The prestressing tendons are partly inside the concrete and partly external.

Keywords: box beams; bridges (structures); cantilever bridges; grouting; joints (junctions); post-tensioning; precast concrete; prestressing; prestressing steels; segmental construction; structural design; unbonded prestressing
G. CAUSSE is a graduate of the Ecole Polytechnique and Ecole Nationale des Ponts et Chaussées of Paris.

Until recently he was working in the bridge design department of SETRA, during which time he was responsible for the control of the design work for the RE Island bridge project.

During 1988, he joined SCETAROUTE where he is currently Assistant Director of the bridge design division.

1. CONCEPTUAL DESIGN

The proximity of La Rochelle harbour and the local navigation necessitated taking into account the risk of ship collision on the piers.

Thus, rather large span lengths had to be chosen, so that the foundations were not over dimensioned by ship collision calculations.

The platform of the deck is 15.50 meters wide (50'10"). It carries a 9.00 meters wide carriageway and two 2.50 meters wide cycle lanes.

The bridge is slightly curved with a constant radius of 4000 meters. The highest point of the platform is 40.00 meters (132') above average sea level. The slopes are 3.5 % near Re Island and 1.85% near La Rochelle, the navigation channels being nearer from the island.

In the basic project, a prestressed concrete bridge was designed to be built with the balanced cantilever method with precast segments. The typical span length was 120 meters (394'). The bridge was made of six successive elementary viaducts, respectively 551.80 meters, four times 480 meters and 487.00 meters long. The two approach viaducts had a certain number of shorter spans with a constant deck depth for aesthetic reasons, and to raise the bearing devices as far as possible from sea level.
The bridge had thus 29 spans. Each elementary viaduct was linked to the neighbouring one with a 24 meters long statically determinate span resting on the extreme segments of two 48 meters long cantilevers.

The call for bids allowed major alternative designs. But the most economical alternatives appeared to be very similar to the basic project.

Bouygues contractor's proposition was adopted. The main modifications in the conceptual design were:

- a reduction of the typical span length to 110 meters (361').
- a smaller number of approach spans so that the total number of spans remained 29.
- the removal of the statically determinate spans between the elementary viaducts, and its replacement by a single hinge placed near the center of a span.

2. DETAILED DESIGN OF THE DECK

The deck is a prestressed concrete, two webbed, box girder, with a parabolic shaped intrados. The girder depth was 7.00 meters at pier and 4.00 meters at midspan. In the approach, spans the depth of the girder remained constant and equal to 4.00 meters.
The main features of the deck are summed up in the following table:

### DECK FEATURES

<table>
<thead>
<tr>
<th>Feature</th>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Deck width</td>
<td>15.50 m</td>
<td></td>
</tr>
<tr>
<td>Deck depth over the piers of the 110 meters spans</td>
<td>7.00 m</td>
<td>23'</td>
</tr>
<tr>
<td>Deck depth at the keystones and in the constant depth zones</td>
<td>4.00 m</td>
<td>13'</td>
</tr>
<tr>
<td>Webs thickness: at the top</td>
<td>0.46 m</td>
<td>18&quot;</td>
</tr>
<tr>
<td>Webs thickness: at the bottom</td>
<td>0.36 m</td>
<td>14&quot;</td>
</tr>
<tr>
<td>Upper slab thickness</td>
<td>0.25 m</td>
<td>10&quot;</td>
</tr>
<tr>
<td>Lower slab thickness: over piers</td>
<td>0.85 m</td>
<td>33&quot;1/2</td>
</tr>
<tr>
<td>Lower slab thickness: at keystones</td>
<td>0.19 m</td>
<td>7&quot;1/2</td>
</tr>
<tr>
<td>Lower slab width: over piers</td>
<td>4.00 m</td>
<td>13'</td>
</tr>
<tr>
<td>Lower slab width: at midspan</td>
<td>5.86 m</td>
<td>19&quot;3/4</td>
</tr>
<tr>
<td>Half pier-segments length</td>
<td>2.80 m</td>
<td>9&quot;2&quot;</td>
</tr>
<tr>
<td>Segments 1 to 12 length</td>
<td>3.80 m</td>
<td>12&quot;6&quot;</td>
</tr>
<tr>
<td>Segments 13 and 14 length</td>
<td>3.55 m</td>
<td>11&quot;8&quot;</td>
</tr>
</tbody>
</table>

The segments were precast on the site with the short cell precasting method. Eight casting machines were used, six of which were for current segments, one for pier segments and one for special segments such as hinge segments.

The segments were placed by a cable stayed launching girder. Both balanced segments could be placed at a time, and post-tensioned. The segments match cast joints were epoxy glued.

After the completion of the prestressing of a span, the launching girder could move by rolling on the deck with tire under-carriage.
3. POST-TENSIONING DESIGN

The deck was post-tensioned by three families of tendons:

cantilever construction tendons (a, in figure): those were traditional post-tensioning tendons made of twelve 0.6" strands. The ducts of those tendons were inside the concrete.

On each segment, two tendons were anchored in the upper gussets. They were stressed just after placing the segment and could carry its dead weight. About eighty percent of the segments had another pair of cantilever tendons anchored on anchorage blocks placed near the upper corners of the box girder. Those were stressed after the placement of three to four complementary segments in order to avoid overstresses and opening of bottom slab joints.
All those cantilever construction tendons were placed in the upper gussets so that the webs were totally free of tendons.

External post-tensioning (b): those tendons are made of nineteen 0.6" strands, situated in grouted ducts that are placed outside the concrete.

The tendons anchored in hinged spans have a particular shape. The other ones are two spans long, their geometry is described as follows:

They are anchored our pier segment diaphragm N, then go straight to an intermediate diaphragm located at the quarter of the span length where they are deviated to take an horizontal direction, at midspan they are just ten millimeters above the bottom stab, are deviated again on an intermediate diaphragm and reach pier segment N + 1, near the upper slab. Their shape in the next span is symmetrical to the previous one, and they are finally anchored on pier segment N + 2.

In the hinged spans there is no intermediate diaphragms, the cables go downwards from the pier segment diaphragm with a fan shape and are anchored individually on bossages situated near the lower corner of the box girder. One of the cables goes to the hinge segment were it is anchored.

The technology of these external cables is the following:

In the current part of the cable, it is placed in a high density 6 mm thick polyéthylène pipe. The pipe is grouted after tensioning the cable, with a delayed cement grout. The grouting is performed from a point situated between the intermediate diaphragms and towards the pier segments. Openings situated just after the pier segments deviation allow the control of the correct filling of the duct.
Whenever the cable passes through the concrete for deviation or anchorage, the polyethylene duct is continuous and goes through a steel pipe reservation, so that the cable can be dismounted and replaced. The extremities of the steel pipe are bell-mouthed so that the cable cannot be born by a corner of concrete when going out of it. The minimum radius of steel pipes is 3.00 meters, and the bell-mouth shape has a radius of one meter.

The anchorages are special Freyssinet International anchorages designed for external prestressing. The system allows the replacement of the cable by avoiding the grout to be in contact with the metallic parts of anchorage that are bond to concrete.

The third family of tendons (c) are: traditional twelve 0.6" inch strands tendons, situated inside the concrete. Their purpose is to increase the post-tensioning force near the center of the spans, and to modify its eccentricity when necessary. The so called "continuity tendons", can be stressed a few hours after closing the spans, to provide capacity for thermal gradient effects.

They are placed in the gussets of the box girder (upper and lower ones) and are anchored on anchorage blocks located near the corner of the box.

Some of these "continuity tendons" were temporary and were used to lock the hinges during erection.

The following table indicates the number of cables of each family, according to the location in the different spans of a viaduct:

<table>
<thead>
<tr>
<th></th>
<th>Central Span</th>
<th>Hinged Span</th>
<th>Intermediate Span</th>
</tr>
</thead>
<tbody>
<tr>
<td>a) Cantilever construction</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Tendons (twelve 0.6&quot;)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>anchored on a block</td>
<td>10 pairs</td>
<td>12 pairs</td>
<td>10-12 pairs</td>
</tr>
<tr>
<td>anchored on joint area</td>
<td>13 pairs</td>
<td>13 pairs</td>
<td>13 pairs</td>
</tr>
<tr>
<td>b) Continuity tendons</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(twelve 0.6&quot;)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>upper gusset</td>
<td>1 pair</td>
<td>none</td>
<td>4 pairs</td>
</tr>
<tr>
<td>lower gusset</td>
<td>6 pairs</td>
<td>none</td>
<td>2 pairs</td>
</tr>
<tr>
<td>c) External tendons</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(nineteen 0.6&quot;)</td>
<td>6 pairs</td>
<td>6 pairs</td>
<td>6 pairs</td>
</tr>
<tr>
<td>parallels</td>
<td>fan shaped</td>
<td>parallels</td>
<td></td>
</tr>
</tbody>
</table>
4. CONCLUSION

We shall finish up this paper by quoting some of the main participants in the construction of the bridge and by providing the major features of the bridge construction.

Owner: District of Charente Maritime (France).

Engineer: Direction Departementale de l'Equipement de Charente Maritime.

Design of Basic project and Control of the final design and shop drawings: SETRA and SOGELERG.

Architect: Charles LAVIGNE

Final Design: BOUYGUES

Contractor: BOUYGUES and BOUYGUES OFFSHORE

Contractual execution time limit: 20 months

Time spent from first bored pile to last placed segment: 12 months

Bridge length: 2926.5 m (9601'5")

Number of segments: 798

Decks quantities:

Concrete: 31 700 m³ (41 500 cubic yards)

Reinforcing steel: 4165 mtons (4591 US short tons)

Traditional longitudinal post-tensioning steel: 1052 mtons (1160 US short tons).


Transverse post-tensioning steel: 10 mtons (11 US short tons).
Photo 1: General view of the bridge

Photo 2: Detail of external prestressing: pier segment
Photo 3: Lay out of external prestressing in hinged span

Photo 4: Cable stayed launching girder
External Prestressing in Two-Chord Large Span Structural Systems

by M. Ivkovic and Z. Perisic

Synopsis: Design considerations of externally prestressed large span reinforced concrete girders with tendons completely outside the concrete cross section are dealt with.

The analyzed systems are two-chord structural systems. The lower, downward convex tensioned chord usually consists only of prestressing tendons, while the upper, compressed chord is a reinforced concrete straight-line or upward convex polygonal girder. The desired configuration of tendons is achieved by compressed elements interconnecting the two chords at suitable distances. In such a way, the rise of tendons can be several times larger than the height of the reinforced concrete section, thus greatly increasing their efficiency compared to the classical internally or externally prestressed girders.

An important characteristic of such structural systems is that adding a very small prestressing force reduces the deformation. Therefore, the dead load deflection can be easily controlled by the suitable choice of prestressing force. The time-dependent deflection is not considerably greater than the elastic one, even for a very high creep and shrinkage, as it is also primarily governed by the shape and deformation of tendons. Because of such properties, these structural systems are exceptionally favorable for roof structures of medium and very large spans but can also be successfully used for highway bridges.

Due to the significant reduction of the chords cross-section areas and the bending stiffness, of such structural systems, the design has to be done using the Second Order Theory. The criteria for cases when it is not necessary have been discussed.

Beside the theoretical analysis, some experiences in design and construction of the new hangar at the Belgrade International Airport in Yugoslavia, whose 135.80 m (445 ft) span main roof reinforced concrete girders are externally prestressed with tendons free in space, outside the concrete cross section, have also been presented.

Keywords: box beams; creep properties; girders; hangars; long span; precast concrete; prestressed concrete; prestressing; roofs; serviceability; structural design; unbonded prestressing
INTRODUCTION

The two-chord reinforced and prestressed concrete structural systems have aroused great interest among designers all over the world in the last few years, owing to a number of advantages over the classically prestressed girders.

The upper, compressed chord is usually the reinforced concrete straight-line or polygonal upward convex girder, while the lower, downward convex tensioned chord may be of rigid steel elements, of prestressed concrete or only of prestressing tendons. Both chords are interconnected with axially rigid elements, whose spacing along the span provides equal vertical deformations of both chords, that is, the continuity of chord deformations. Generally, both chords possess bending stiffness and, in the analysis, concrete time-dependent deformations have to be considered.

Lower chord designed only of prestressing tendons is particularly suitable for and frequently used in large span systems. In such a case the lower chord has only the axial stiffness but not the bending one. This case can also be understood as a reinforced concrete girder on elastic supports, where the stiffness of supports depends on both the tensile capacity and the configuration of the lower chord prestressing tendons.

The general case of continuous solution, when both chords possess the bending stiffness, takes into account the effects of creep and shrinkage in the form of integral–differential equation and is analyzed first in this paper. The simplifications for cases when the lower chord does not have bending stiffness, using the Age-Adjusted Effective Modulus Method (AAEMM) [1] instead of integral relationship between stress and strain in concrete are
presented next.

When the chords are interconnected only by a finite number of rigid elements at finite distances, the equality of vertical deformations and the continuity of deformations of both chords are not provided; the solution in continuous form can no longer be used. Then the numerical solutions, based on discretization of the system into finite number of bars, and the force or displacement methods, taking into consideration time-dependent properties of reinforced concrete elements, have to be used.

Finally, as an illustration, some experiences gained in the design and construction of the new prestressed concrete hangar at the Belgrade International Airport in Yugoslavia, are presented. The two-chord prestressed main roof girders, with a span of 135.80 m (445 ft) and the lower chord prestressing tendons free in space and completely outside the concrete cross section, have been used in the design.

**GENERAL CONTINUOUS SOLUTION**

Fig. 1 shows the most frequently used configuration of the two-chord systems, especially for prestressed girders with external prestressing tendons outside the compressed chord cross section.

It is evident from Fig. 1 that the structural system consists of two chords interconnected by axially rigid vertical members. The spacing of these members in the X direction has to be small enough to provide equal vertical deformations of both chords.

Fig. 2 shows the differential element of the structural system with external actions and internal forces.

The problem of determination of forces and deformations within the serviceability limit states is solved using the theory of bending and the linear theory of creep for reinforced or prestressed concrete members. For the general solution, the stress strain relationship for concrete is taken in the well-known form of Volterra's second order integral equation [5], while the AAEMM formulation is used for the simplified practical calculation.

\[
\varepsilon_c(t) = \frac{f_c(t)}{E_c(t)} + \int_{t_0}^{t} K(t,\tau) \frac{f_c(\tau)}{E_c(\tau)} d\tau + \varepsilon_{sh}(t) + \alpha_t \theta(t) \tag{1}
\]

\[
f_c(t) = E_c(t) [\varepsilon_c(t) - \varepsilon_{sh}(t) - \alpha_t \theta(t)] - \int_{t_0}^{t} E_c(\tau) R(t,\tau) [\varepsilon_c(\tau) - \varepsilon_{sh}(\tau) - \alpha_t \theta(\tau)] d\tau \tag{2}
\]

Equations (1) and (2) present two forms of stress-strain relationship and strain-stress relationship where \(K(t,\tau)\) is the
kernel of the integral equation (1) and $R(t,\tau)$ its resolvent, with the relationship between them in the form:

$$K(t,\tau) = R(t,\tau) - \int_{t_0}^{t} K(t,\eta) \cdot R(\eta,t) \, d\eta,$$

(3)

where $\varepsilon_{sh}(t)$ - shrinkage,

$\alpha(t)$ - linear thermal coefficient,

$\Theta(t)$ - temperature change with time, and

$E_c(\tau)$ - modulus of elasticity of concrete as a function of age.

Taking into consideration equations (1) and (2), for the technical theory of bending:

$$\frac{1}{\rho(x,t)} = \mathcal{K}(x,t) = \frac{M(x,t)}{E_c(t)I(x,t)} + \int_{t_0}^{t} K(t,\tau) \frac{M(x,\tau)}{E_c(\tau)I(x,\tau)} \, d\tau$$

(4)

$$M(x,t) = E(t)I(x,t)[\mathcal{K}(x,t)] -$$

$$- \int_{t_0}^{t} \mathcal{K}(x,\tau)E_c(\tau)I(x,\tau)R(t,\tau)d\tau,$$

(5)

where $I(x,t)$ - moment of inertia of the cross section area, and

$\mathcal{K}(x,t)$ - curvature of the girder under permanent actions.

From the equilibrium conditions for the differential element, with the assumption of equal vertical displacements of chords, the following is obtained for the vertical displacement $w$:

$$\frac{d^2 M_1}{dx^2} + \frac{d^2 M_2}{dx^2} - (H_1-H_2) \frac{d^2 w}{dx^2} + H_1 \frac{d^2 y_1^o}{dx^2} + H_2 \frac{d^2 y_2^o}{dx^2}$$

$$+ g_1 \frac{ds_1}{dx} + g_2 \frac{ds_2}{dx} + p_1(x) + p_2(x) = 0,$$

(6)

where $y_1 = y_1^o + w$, and $y_2 = y_2^o + w$.

If $\mathcal{K}(x,t) = \frac{d^2 w(x,t)}{dx^2} = - \frac{M(x,t)}{E(t)I(x,t)}$, and if the assumptions $E_c(t) = E_c(\tau)$ and $I(x,t) = I(x,\tau)$, which have no significant influence on the accuracy of the solution, are introduced, the equation for the displacement $w(x,t)$ is obtained:
\[
\frac{d^4 w(x,t)}{dx^4} + \int_{t_0}^{t} \frac{d^4 w(x,\tau)}{dx^4} R(t,\tau) \, d\tau + \\
+ \left( K_1^2 - K_2^2 \right) \frac{d^2 w(x,t)}{dx^2} - \varphi(x) = 0,
\]  

(7)

where \( K_1^2 = \frac{H_1}{R_1+R_2} \), \( K_2^2 = \frac{H_2}{R_1+R_2} \), \( R_1 = E_1I_1 \), \( R_2 = E_2I_2 \),

and \( \varphi(x) = \frac{d^2 y_1^o}{dx^2} + \frac{d^2 y_2^o}{dx^2} + g_1 \frac{ds_1}{dx} + g_2 \frac{ds_2}{dx} + p_1(x) + p_2(x) \).

For the case \( (t=t_0) \) (instantaneous, elastic solution), integral-differential equation (7) becomes a non-homogeneous bi-square total differential equation, whose solution is relatively easy to obtain. The function \( \varphi(x) \), depends on the initial geometry of system, as well as on the intensity and distribution of the external loads acting upon the girder.

Integral-differential equation (7) with the corresponding boundary conditions provides the possibility to determine the displacement \( w(x,t) \) as a function of parameters \( H_1 \) and \( H_2 \), which are also time-dependent. However, in order to determine the state of stress and strain, it is necessary to consider the deformation conditions for the solution of \( H_1 \) and \( H_2 \). In this case, the change of length of the chords under the action of external forces, frequently called the "length" condition, provides the necessary and sufficient deformation condition.

If

\[
L_{1o} + \Delta L_{1o} = L_1,
\]

then \( L_{1o} = \ell + \frac{1}{2} \int_{0}^{\ell} \left( \frac{dy_1^o}{dx} \right)^2 \, dx, \)

\[
L_1 = \ell + \frac{1}{2} \int_{0}^{\ell} \left[ \left( \frac{dy_1^o}{dx} \right)^2 + 2 \frac{dy_1^o}{dx} \frac{dw(x,t)}{dx} + \left( \frac{dw(x,t)}{dx} \right)^2 \right] \, dx,
\]

\[
\Delta L_{1o} = \left[ H_1 + \int_{t_0}^{t} H_1 K(t,t_o) \, dt \right] \frac{3L_{1o} - 2\ell}{E_1A_1},
\]

where \( i = 1,2 \) for both chords and \( \int \frac{1}{\cos^3 \varphi} \, dx = 3L_{1o} - 2\ell \),

and \( \varphi \) is the angle between the system line and the positive direction of X-axis, \( A_1 \) is the area of the cross section of the chords, and \( \ell \) is the span of the system. The condition (8) is finally obtained in the form:
Integral-differential equation (7) and conditions (9) provide the necessary and sufficient conditions for the solution of stresses and strains of a two-chord system under arbitrary vertical load.

For engineering practice, however, the solution of equation (7) and conditions (9) is a very complex problem. The simplified solution of the problem can be obtained, with the required and sufficient accuracy, by introducing the algebraic stress-strain relationship in concrete instead of the Volterra's integral equation.

In the algebraic form the equations (4) and (5) may be expressed as:

\[ \mathcal{X}_1(x,t) = \frac{1}{E_1 I_1} \left[ b_t M_1(x,t) + (a_t-b_t)M_1(x,t_0) \right] \]  \tag{10}

or

\[ M_1(x,t) = E_1 I_1 \left[ \mathcal{X}(x,t) \frac{1}{b_t} - \left( \frac{a_t}{b_t} - 1 \right) \mathcal{X}(x,t_0) \right], \]  \tag{11}

and the equation (7) becomes

\[ \frac{1}{b_t} \frac{d^4 w(x,t)}{dx^4} - \left( \frac{a_t}{b_t} - 1 \right) \frac{d^4 w(x,t_0)}{dx^4} + \\
+ \left( K_{1t}^2 - K_{2t}^2 \right) \frac{d^2 w(x,t)}{dx^2} - \varphi(x) = 0 \]  \tag{12}

where

\[ K_{1t}^2 = \frac{H_1(t)}{R_1+R_2}, \quad K_{2t}^2 = \frac{H_2(t)}{R_1+R_2}, \quad R_1 = E_1 I_1, \quad R_2 = E_2 I_2, \]

\[ a_t = 1 + \varphi_t, \quad b_t = 1 + \chi_t \varphi_t, \]

\( \varphi_t \) stands for the creep coefficient \( \varphi(t,t_0) \), and \( \chi_t \) is Bazant's ageing coefficient \( \chi(t,t_0) \).

Now the equation (12), with algebraic relationship (11), is transformed into a total non-homogeneous differential equation of the fourth order with quasi-constant coefficients.

The stress-strain state may be now determined in two or more steps, depending on the history of permanent loads. The elastic problem must be solved first. If the load is applied at \( t=t_0 \), the coefficients \( a_t \) and \( b_t \) in equation (12) are equal to one, and the differential equation is obtained in the form:
\[
\frac{d^4w(x,t_o)}{dx^4} + (K_{1o}^2 - K_{2o}^2) \frac{d^2w(x,t_o)}{dx^2} - \varphi(x) = 0. \quad (13)
\]

The general solution for this case depends on the values of parameters \(K_1\) and \(K_2\).

a) If \(K_1 > K_2\), the general solution is obtained in the form:

\[
w(x,t_o) = C_{1a} \sin K_{1o} x + C_{2a} \cos K_{1o} x + C_{3a} x + \\
+ C_{4a} + \int \left[ \int \bar{\varphi}(x) \, dx \right] \, dx,
\]

where \(K_{1o}^2 = \frac{H_1(t_o) - H_2(t_o)}{R_1 + R_2}\),

and \(\bar{\varphi}(x)\) - particular integral of non-homogeneous differential equation (13).

b) If \(K_1 < K_2\):

\[
w(x,t_o) = C_{1b} \sinh K_{1o} x + C_{2b} \cosh K_{1o} x + C_{3b} x + \\
+ C_{4b} + \int \left[ \int \bar{\varphi}(x) \, dx \right] \, dx.
\]

c) If, however, \(K_1 = K_2\), (the case of vertical load only):

\[
\frac{d^4w(x,t_o)}{dx^4} - \varphi(x) = 0, \quad (16)
\]

and the unknown function \(w(x,t_o)\) is obtained by the direct successive integration of the known function \(\varphi(x)\).

In all those cases, integration constants are obtained from the boundary conditions at the ends of the girder.

When \(w(x,t_o)\) is known, horizontal forces \(H_1(t_o)\) and \(H_2(t_o)\) can be determined from the conditions of "length" (9) which, are now:

\[
[b_t H_1(t) + (a_t - b_t) H_1(t_o)] \frac{3L_{1o} - 2l}{E_1 A_1} - \int_o^l \frac{dy_i^0}{dx} \frac{dw(x,t)}{dx} \, dx - \\
- \frac{1}{2} \int_o^l \left(\frac{dw(x,t)}{dx}\right)^2 \, dx = 0; \quad i = 1, 2. \quad (17)
\]

For the instantaneous elastic solution \(t=t_o\) and \(a_t=b_t=1\), the length conditions (17) acquire the form:
With solution (14), (15) or (16) and conditions (18), the state of stress and strain for instantaneous elastic solution of the two-chord system is completely determined.

In the second step, with \( H_1(t_0), H_2(t_0) \) and \( w(x,t_0) \) already known, it is possible to determine \( H_1(t), H_2(t) \) and \( w(x,t) \) for any time \( t \).

In this case equation (12) can be written in the form:

\[
\frac{1}{b_t} \frac{d^4w(x,t)}{dx^4} + K^2_t \frac{d^2w(x,t)}{dx^2} - \psi(x) + \left( \frac{a_t}{b_t} - 1 \right) \psi(x,t_0) = 0, \tag{19}
\]

where \( \psi(x,t_0) = \frac{d^2w(x,t_0)}{dx^2} \) and \( K^2_t = \frac{H_1(t) - H_2(t)}{R_1 + R_2} \).

The general solution then has the following form, depending on the value of \( K^2_t \):

a) for \( K^2_t > 0 \)

\[
w^*(x,t) = C^a_{1t} \sin K_t x + C^a_{2t} \cos K_t x + C^a_{3t} x + C^a_{4t} + \left[ \int \int \psi(x) dx \right] dx - \left( \frac{a_t}{b_t} - 1 \right) \int \int \psi(x,t_0) dx dx \tag{20}
\]

b) for \( K^2_t < 0 \)

\[
w^*(x,t) = C^b_{1t} \sinh K_t x + C^b_{2t} \cosh K_t x + C^b_{3t} x + C^b_{4t} + \left[ \int \int \psi(x) dx \right] dx - \left( \frac{a_t}{b_t} - 1 \right) \int \int \psi(x,t_0) dx dx \tag{21}
\]

c) for \( K^2_t = 0 \)

the function \( w^*(x,t) \) may be obtained by direct integration of the equation:
\[
\frac{d^4 w^*(x,t)}{dx^4} - \phi(x) + \frac{a_t}{b_t} - 1) \psi(x,t_o) = 0,
\]

where \(\bar{\psi}(x)\) - particular integral of the differential equation (19), and
\(w^*(x,t) = \frac{1}{b_t} w(x,t)\).

Now, when \(w(x,t)\) is obtained from equations (20), (21) or (22), horizontal forces may be obtained directly from conditions (17).

**PRESTRESSED TWO-CHORD SYSTEMS WITH PRESTRESSING TENDONS OUTSIDE THE CONCRETE CROSS SECTION**

A prestressed two-chord system with an upper compressed reinforced concrete chord with considerable bending stiffness, usually straight, or polygonal upward convex, and a lower tensioned chord upward concave, consisting only of prestressing steel or a combination of rigid steel members and prestressing steel, with a negligible bending stiffness, will be analyzed next. Fig. 3 shows such a structural system with a tensioned chord made only of prestressing tendons.

In such a case, the determination of the state of stress and strain is considerably simplified compared to the already shown general solution and is reduced to the solution of differential equation (12), taking into consideration that:

\[
K_1^2 = -\frac{H_1(t)}{R_1}, \quad K_2^2 = -\frac{H_2(t)}{R_1}, \quad R_1 = E_1I_1, \quad K_\pi = -\frac{H_1(t) - H_2(t)}{R_1}
\]

and using conditions (17), where the lower chord does not possess the property of creep.

Here too, the problem is solved in two steps. First, the stress-strain state for \(t=t_0\) (instantaneous elastic problem) has to be solved. With \(w(x,t_0), H_1(t_0), H_2(t_0)\) obtained, \(w(x,t)\) and \(H_1(t)\) and \(H_2(t)\) are defined by means of slightly modified equations (14), (15) or (16); that is, (20), (21) or (22), with "length" conditions (17) and (18).

For vertical load only, when \(K_{1t}^2 = K_{2t}^2\), the solution of the stress strain state is reduced to the integration of equations (16) and (22), in order to define \(w(x,t_0)\) and \(w(x,t)\) with considerably simplified "length" conditions (17) and (18).

For the case shown in Fig. 3:

\[
y_1^o = 0, \quad y_2^o = \frac{4f_2}{l^2} x(l-x).
\]
As the load is only vertical, $H_1 = H_2$. For the action of dead load and prestressing force (case $t = t_o$), the solution of equation (16) gives:

$$w_g(t_o) = \frac{1}{24R_1} \left[ g - 8 \frac{H_g(t_o)}{R_2} \right] \left[ x^4 - 2x^3 + x^2 \right].$$  \hspace{1cm} (23)

The length conditions, for $t = t_o$, that is, $a_t = b_t = 1$, using expression (17), are:

$$3L_{1o} - 2\ell \left[ \frac{H_g(t_o) E_1 A_1 R_1}{E_2 A_2} + \frac{3L_{2o} - 2\ell}{E_2 A_2} \right] - \int_0^\ell \frac{dy_2}{dx} \frac{dw}{dx} \frac{g_0}{dx} \left( \frac{d\bar{w}}{dx} \right)^2 \left( \frac{d\bar{w}}{dx} \right) \left( \frac{d\bar{w}}{dx} \right) \left. \right|_0^\ell = 0.$$  \hspace{1cm} (24)

In many practical cases for typical shallow catenaries for $f/\ell < 1/10$, it is possible to neglect the third term of equation (24), compared to the first two terms. The length condition is then:

$$3L_{1o} - 2\ell \left[ \frac{H_g(t_o) E_1 A_1 R_1}{E_2 A_2} + \frac{3L_{2o} - 2\ell}{E_2 A_2} \right] - \int_0^\ell \frac{dy_2}{dx} \frac{dw}{dx} \frac{g_0}{dx} \left( \frac{d\bar{w}}{dx} \right)^2 \left( \frac{d\bar{w}}{dx} \right) \left. \right|_0^\ell = 0,$$  \hspace{1cm} (25)

and

$$H_g^{(1)}(t_o) = g \left[ \frac{1/15 f_2^2 g_0^3}{8/15 f_2^2 g_0^3 + \left( \frac{3L_{1o} - 2\ell}{E_1 A_1} R_1 + \frac{3L_{2o} - 2\ell}{E_2 A_2} R_1 \right)} \right].$$  \hspace{1cm} (26)

However, if the third term of equation (24) cannot be neglected, the value for $H_g^{(2)}(t_o)$ is obtained as one of the roots of the square algebraic equation in the form:

$$H_g^{(2)}(t_o) - a_t H_g^{(2)}(t_o) + b_t = 0$$  \hspace{1cm} (27)

where

$$a_t = \left[ \frac{C_1}{K_{N1}} + \frac{C_2}{K_{N2}} \right] R_1^2 + \frac{8}{15} f_2^2 R_1 \ell + 16D \ell^5 f_2 g \left[ \frac{1}{64Df_2^2} \right],$$

$$b_t = \frac{g_0}{64Df_2^2} \left[ \frac{1}{15} R_1 F_2 \ell^3 \right] + Dg \ell^7 \right].$$

and, in this case, $D = 8.4325 \cdot 10^{-4}$,

$$C_1 = 3L_{1o} - 2\ell, \hspace{0.5cm} C_2 = 3L_{2o} - 2\ell.$$

It is easy to show that $H_g^{(1)}(t_o)$ with $w_g(t_o)$ from expression (23) presents the solution to the problem according to the First Order Theory, when the influence of deformations of the system upon the equilibrium of forces acting on the girder is neglected. The solution $H_g^{(2)}(t_o)$, from equation (27), with $w_g(t_o)$ from (23), presents the solution of the problem according to the Second
Order Theory. The comparison of the \( H_{g}^{(1)}(t_0) \) and \( H_{g}^{(2)}(t_0) \) values shows whether the solution according to the First Order Theory is sufficiently accurate.

When the problem is solved for \( t=t_0 \), it is easy to obtain \( w_{g}(t) \) for any \( t>t_0 \) and corresponding \( a_t \) and \( b_t \), using the equation (22) and the value \( H_{g}(t) \) from modified conditions (17) for the Theory of First, as well as for the Theory of Second Order. Then:

\[
\begin{align*}
  w_{g}(t) &= \frac{1}{24R_1} \left\{ b_t \left[ g - \frac{8f_2}{\xi^2} H_{g}(t) \right] + \right. \\
  &\quad \left. + (a_t - b_t) \left[ g - \frac{8f_2}{\xi^2} H_{g}(t_0) \right] \right\} (x^4 - 2x^3 + x^2). \\
\end{align*}
\]

(28)

According to the conditions (25), that is, modified condition (17), with \( w_{g}(t) \) from (28) the following expression is obtained:

\[
H_{g}^{(1)}(t) = \frac{a_t g \frac{15}{15^2} f_2^3 - (a_t - b_t) H_{g}(t_0) \left[ \frac{8f_2}{15^2} + \frac{R_1}{E_1A_1} C_1 \right]}{b_t \left[ \frac{8f_2}{15^2} + \frac{R_1}{E_1A_1} C_1 \right] + \frac{R_1}{E_2A_2} C_2}. \quad (29)
\]

The equation (27) and condition (24) give:

\[
H_{g}^{(2)}(t) = \left[ g - P_t \frac{1}{b_t} \right] \frac{\xi^2}{8f_2}, \quad (30)
\]

where

\[
P_t = b_t \left[ g - \frac{8f_2}{\xi^2} H_{g}(t) \right].
\]

**EXAMPLE OF APPLICATION: THE NEW PRESTRESSED CONCRETE HANGAR AT THE BELGRADE INTERNATIONAL AIRPORT IN YUGOSLAVIA**

The new prestressed concrete hangar at the Belgrade International Airport was designed for the simultaneous maintenance of two Boeing 747 jets. The authors of this paper are the main designers of the hangar structure together with A. Pakvord and M. Adi, professors of Civil Engineering at the Department of Civil Engineering at the University of Belgrade. The hangar was finished in 16 months, in 1986.

Fig. 4 shows the general plan of the hangar which is of a rectangular base, 135.80 by 70.05 m. The door of the hangar provides 21.90 m clear height. It is situated along the entire longer side of the base, which is without any intermediate columns. The door has six wings moving along the three rails, so that two
thirds of the door may be simultaneously opened. It is foreseen that Boeing 747 jets enter the hangar with their front parts. In order to save the necessary space, two 16.80 by 22.40 m recesses have been designed in the rear hangar wall, increasing the overall hangar depth to 92.45 m.

The main structural members of the hangar are the three prestressed reinforced concrete box girders, of 135.80 m span, parallel to the longer side of the base, spaced at 22.40 m. The main girders are laid on reinforced neoprene bearings on the top of the reinforced concrete main columns in the side walls of the hangar. The height of the main columns in the portal plane is 36.35 m, while the other two pairs of main columns are lower, for 0.90 m each. The highest level of the hangar structure is 37.35 m and was governed by the flying safety regulations in the Airport zone.

The complete roof structure of the hangar is hung on the main girders. In this way the minimum surface of the facade walls and the minimum effective volume have been achieved, thus reducing the service costs for the winter heating of the hangar. The roof structure consists of reinforced concrete secondary girders spaced at 8.40 m and purlins at 3.75 m. Light-weight concrete plates together with thermo- and hydro-insulation and plastic lanterns are placed over the purlins.

The hangers carrying the roof structure are 50 mm diameter steel rods. Two pairs of such hangers are placed at 8.40 m along the girders span. They transmit the roof weight to the special supports inside the main box girders. The hangers have threads with nuts at their ends by which the level of the complete roof can be easily adjusted from inside the reinforced concrete girder, when, and if necessary.

The hangar door is a steel structure. At its upper edge the door rests horizontally on the roof structure, over special steel guides. The wind and seismic forces perpendicular to the door of the hangar are transmitted to the main columns and reinforced concrete walls in the rear wall recesses by means of a horizontal steel brace in the roof plane. The horizontal forces perpendicular to the side hangar walls are accepted by the frame system, consisting of the main girders and columns, together with the structure of the rear wall of the hangar.

The foundations of the hangar have been designed for construction directly on the loose soil and are rather shallow. The foundation level of the main columns is only 3.25 m under the hangar floor and the foundations surface is 11.00x7.50 m.

The main prestressed reinforced concrete box girders shown in Fig. 5, are two-chord structural systems, with a span of 135.80 m and a maximum rise of 9.70 m. The upper compressed chord is made of the $f'_c = 45$ MPa reinforced concrete. The cross-
section of the box girder is constant along the whole span, having 4.00 m external width and 2.80 m external height. The upper slab is 35 cm thick, the side walls are 25 cm and the thickness of the bottom slab is 20 cm. The percentage of the mild reinforcement in the box girder along the whole span is about one percent.

The lower chord is made of 27 tendons consisting of 11 strands 15.2 mm in diameter each. The applied prestressing system is the Yugoslav IMS (Institute for Testing of Materials of Serbia, Belgrade) system which has been successfully used for over 35 years in Yugoslavia and some other countries.

The admissible force in a strand is 1,786 kN. Three tendons of 11 Ø 15.2 mm strands are placed together in a single polyethylene protective tube, 142 mm in diameter, so that there are 9 such composite tendons along the whole span. However, near the ends of the girders the tendons are split up and separately anchored using 27 standard permanent anchors.

The designed polygonal tendon configuration is provided by 7 pyramidal "chairs" made of steel pipes, oriented in the direction of the line of symmetry of intersecting angles of the lower chord. The tendons are led across the chairs over special rotational bearings which reduce the friction coefficient to about 0.04. This resulted in minimal frictional losses.

The general view of the prestressing tendons and hangers carrying precast roof elements is shown on Fig. 6, taken during the hangar construction.

The structural system of the main girders, shown in Fig. 5, particularly for such large spans, may be understood as a two-chord catenary system, discussed in the first part of this paper. The upper reinforced concrete chord has axial and bending stiffness, while the lower steel chord can carry only tension. The chords are interconnected at suitable distances by compressed elements. For shorter spans, such a structural system may be simply understood as a reinforced concrete girder on elastic supports. The stiffness of supports depends on the tensile capacity and the configuration of the lower prestressing steel chord.

By prestressing such a structural system, the compressive force is introduced into the upper reinforced concrete chord, but at the same time "lifting" forces appear at the points of the supporting chairs.

By designing the external prestressing tendons outside the basic reinforced concrete cross section of the main girders, a very rational and economical structural system has been obtained [2]. The efficient corrosion protection in the contemporary external prestressing, allows for tendons to be left free in space. The main characteristic of such structural systems is that a relatively high rise may be given to the tendons, increasing very much their efficiency. In such two-chord structural systems, the rein-
forced concrete chord is exposed to relatively high normal compressive forces and small bending moments, so that it is in the stress state free of cracks. For dead loads such structural systems are not subjected to tensile stresses at all. Therefore, the most appropriate task is fulfilled by each of the materials - the concrete withstands only compressive stresses while the steel is exposed only to tensile stresses.

Due to the considerable decrease of dimensions, that is, the area of the cross section, such girders are relatively lightweight. They are exceptionally favorable for significant dead loads and relatively small, mostly symmetrical live loads, as it is usually the case with roof structures of medium and very large spans [3], but can also be successfully used for highway bridges.

The analysis of the main girders was done according to the First and Second Order Theories, using a computer program which provided the possibility of considering the creep, shrinkage and relaxation effects. In spite of the large span and a considerable reduction of the stiffness of the reinforced concrete part of the system, the results obtained by both theories did not differ by more than eight percent. The analysis was performed using the algebraic stress-strain relationship for concrete, and the Age-Adjusted Effective Modulus Method (AAEMM). The influence of creep and shrinkage of concrete on time-dependent behaviour of the girders was analysed for the supposed load histories, depending mostly on the construction method. The minimum and maximum possible values of the expected creep and shrinkage of concrete were introduced into the design.

The characteristic cross sections of the main girders were dimensioned according to the limit states design. The limit state of deformations was analyzed very carefully, considering that due to the significant reduction of stiffness, such systems are relatively deformable. Nevertheless, the calculated midspan deflection caused by the snow, is only 15 cm, due to the low live load to the total dead load ratio. Shear design did not cause any problem due to the high ratio of dead to live load, and mostly symmetrical total load.

A very significant characteristic of such structural system is that adding a very small prestressing force can significantly affect the deformation while, at the same time, a small change of internal forces takes place in the system. By prestressing the lower chord, the upper reinforced concrete chord is elastically supported. The stiffness of the supports depends on both the tensile capacity and the configuration of tendons. Therefore, the deflection of the main girders can be easily controlled by the suitable choice of prestressing force. As for the time-dependent deflection of such systems, it is not considerably greater than the elastic one, even for very high creep and shrinkage, as it is also primarily governed by the shape and the deformation of the lower chord tendons [4]. In the case of this hangar, the expected average creep factor is about 2, due to the construction technology.
The analysis has shown that, due to the creep and shrinkage of concrete, forces in such systems change very little with time. This explains why the loss of prestressing force with time, due to the time-dependent deformations of concrete, is incomparably lower than in classically prestressed concrete girder with tendons inside the cross-section.

The main girders have been cast on scaffolds, right under the final position, about 9 m above the floor of the hangar, which was the height necessary to provide the tendons rise. The prestressing of the main girders was designed to be carried out in two stages, simultaneously from both ends. The first stage of prestressing was performed while the girders were still on scaffolds and resulted in 15 cm camber. At that moment, the girders, were loaded only by the self-weight and the load of suspended short secondary girders and purlins which were designed to be lifted together with the main girder, as shown in Fig. 6 and 7. The prestressing force of the first stage was 27,200 kN, making 56% of the allowed value. It produced 15 cm camber at the midspan.

After the first stage of prestressing was completed, the main girders were lifted for about 24 m to the designed positions on the top of the main columns. The lifting weight of the main girders, together with the hanged short secondary girders and purlins, was 1,700 t each.

Fig. 7 shows the lifting of the main girders. Two main girders are already in place on the top of the columns, while the last one is lifted halfway.

The second and final stage of prestressing was performed after the application of the total dead load, with the exception of thermo- and hydro-insulation. The final prestressing force of 37,500 kN, making 78% of the allowed value, was determined to provide the 15 cm camber of the main girders at the midspan, with respect to the final designed position. As the loss of the prestressing force in already prestressed tendons due to the elastic deformations in such structural systems can exceed 50%, two or even more cycles of tensioning were necessary to provide the designed forces in all tendons. The designed extreme total force in all 9 tendons amounts to 41,800 kN, or about 87% of the admissible value. After the second stage of prestressing, the tendons were injected by simple cement emulsion.

The secondary mountable roof girders, shown in Fig. 8, are also two-chord structural systems, with a span of 15.65 m and rise of 1.50 m. The upper, compressed chord is of reinforced concrete while the lower chord and the two vertical chairs are made of rigid steel box sections. Like the main girders, the secondary roof girders are very rational, economical and relatively light-weight.
CONCLUSIONS

The design and construction of the new prestressed concrete hangar at the Belgrade International Airport, using the described structural ideas and systems, have confirmed that composite two-chord systems with prestressing steel elements outside the compressed reinforced concrete chord are very suitable and very economical structures for roofs of large and exceptionally large spans. Contrary to the former experience, at least in Yugoslavia, this structure showed significant economical and technical advantages compared with the corresponding steel or classically prestressed concrete design alternatives. Using the numerous advantages of such structural systems, a relatively light, mountable and economical roof structure, and especially a very short term of construction have been obtained for this hangar. With the structural system of the main girders outside the covered part of the building, free in space but requiring no special maintenance during the service life, and with the roof structure hanged on the main girders, the minimum effective volume and the minimum facade surfaces, as well as the minimum service and maintenance costs have also been achieved.

REFERENCES

Fig. 1--Elements of the two-chord structural system
Fig. 2--Differential element of the two-chord system
Fig. 3--Example of the two-chord structural system with the prestressing tendons outside the concrete cross section
Fig. 4--The general plan of the new prestressed concrete hangar at the Belgrade International Airport in Yugoslavia
Fig. 5--The view and characteristic cross-sections of the main girder

Fig. 6--The main girder under construction
Fig. 7 -- Lifting of the main girders: first two girders are already lifted while the third one is half-way up.

Fig. 8 -- The secondary roof girders
Strengthening of Existing Bridges
(Simple and Continuous Span)
by Post-Tensioning

by F.W. Klaiber, K.F. Dunker, and W.W. Sanders, Jr.

Synopsis: Approximately 40% of the bridges in the United States are classified as deficient and in need of rehabilitation or replacement. Of these bridges, many are classified as deficient because their load-carrying capacity is inadequate for today's increased traffic. This insufficient load-carrying capacity has resulted from poor maintenance, increases in legal load limits, deck overlays, changes in design specifications, and other factors. In response to the need for a simple, efficient procedure for strengthening existing bridges, the authors have been investigating the use of post-tensioning. The authors have investigated the use of post-tensioning on simple span bridges as well as on continuous span bridges. Various post-tensioning schemes have been tested on laboratory models as well as on existing bridges. This paper will briefly review the post-tensioning research that has been completed by the authors in the past few years. This work indicates that post-tensioning is a viable, economical technique for flexural strengthening of steel-beam composite-concrete deck bridges.

Keywords: bridges (structures); flexural strength; loads (forces); post-tensioning; renovating; research; spans; strengthening; stresses; structural design; tests; unbonded prestressing
INTRODUCTION

About one-half of the approximately 600,000 highway bridges in the United States were built before 1940, and many have not been adequately maintained. Most of these bridges were designed for lower traffic volumes, smaller vehicles, slower speeds, and lighter loads than are common today. In addition, deterioration due to environmental factors is a growing problem. According to the Federal Highway Administration (FHWA), almost 40% of the nation's bridges are classified as deficient and in need of rehabilitation or replacement. Many of these bridges are deficient because their load-carrying capacity is inadequate for today's traffic. Strengthening can often be used as a cost-effective alternative to replacement or posting for restricted loads.

For the past eight years, the authors have been investigating the use of external post-tensioning for strengthening existing bridges. Although this work involves numerous projects, for discussion purposes it will be presented here as follows:

Project 1. Strengthening of Existing Single-Span Steel-Beam Concrete Deck Bridges: Phases I, II, and III.

Project 2. Strengthening of Existing Continuous Composite Bridges: Phases I and II.

This paper will review the purpose and results of these research projects. Project 1 and Phase I of Project 2 are completed, so final results can be reported. Because Phase II of Project 2 is still in progress, only preliminary results can be presented.
As previously noted, Project 1 consisted of three separate phases. Portions of Project 1 have been documented in the literature (1,2); thus, it will be reported here only briefly. Phase I of the research program (3) was to determine a method of strengthening single-span, composite steel-beam concrete deck bridges. Primarily it was the exterior beams of these bridges which required strengthening.

After reviewing several strengthening techniques, we found that post-tensioning the exterior beams with high-strength tendons was the most promising technique. The feasibility of this strengthening technique was verified analytically, by using orthotropic plate theory, and experimentally, through testing of various post-tensioning schemes on a half-scale model bridge (L = 26'-0", W = 15'-8 3/8") shown in Fig. 1. Although various post-tensioning schemes were investigated (post-tensioning of all beams, post-tensioning of exterior beams only, post-tensioning of one exterior beam, etc.) to obtain data on the lateral distribution in the bridge, only the exterior beams actually required post-tensioning to reduce the overstress. However, when the exterior beams were subjected to post-tensioning, due to lateral distribution, significant stress reduction resulted in the interior beams. The midspan bottom flange strains that resulted from subjecting the external beams to a post-tensioning force of 20 kips are shown in Fig. 2. Excellent agreement can be seen between the measured strains and the theoretical curve.

As a result of the success of the Phase I laboratory investigation, Phase II (4,5) of the investigation was undertaken. Phase II consisted of field strengthening and monitoring two existing bridges. One of the bridges, located in northwestern Iowa (Bridge 1: L = 52', W = 31'-10 1/2"), was a prototype of the model bridge tested during Phase I. The other bridge, located in central Iowa (Bridge 2: L = 72', W = 31'-10 1/2"), was considerably longer and had a 45° skew.

Photographs of the post-tensioning brackets used on the two bridges are shown in Fig. 3. The brackets used on Bridge 1 (Fig. 3a) were similar to the brackets used on the model bridge. Figure 3 shows two tendons (1 1/4" in diameter) used on Bridge 1, while four tendons (2 - 1/2" in diameter and 2 - 1" in diameter) were used on Bridge 2. Although only the exterior beams of both bridges were post-tensioned, because of lateral distribution significant stress reduction was also obtained in the interior beams.

For composite action, both bridges had angle-plus-bar type shear connectors. Analysis indicated that for the bridges to carry additional live load, the exterior beams of Bridge 1 and all the beams of Bridge 2 needed additional shear connectors. An assumption—that the type of steel in some bridges requiring strengthening might be unknown—prompted the decision to add shear
connectors by bolting rather than by welding. High-strength bolts tested as shear connectors in the laboratory were found to be somewhat stronger than shear studs of equivalent size (6). In the field, cores (4" in diameter) were drilled in the bridge deck above the beams in the two bridges, and high-strength bolts were double-nutted to the beams. Later the cores were filled with nonshrink grout.

Both bridges were instrumented to determine strains and deflections. To determine the bridges' response to post-tension strengthening, they were both load tested with overloaded trucks (see Fig. 4 for details) before and after the exterior beams were post-tensioned. The two bridges were initially strengthened and tested during summer 1982 and were retested during summer 1984. During the intervening time period, the bridges were periodically inspected; no behavior changes were noted in either bridge.

**Bridge 1:** During the original post-tensioning of Bridge 1, each exterior beam was subjected to an average force of 182.0 kips, to produce a strain reduction of 218 με in the lower flange coverplates of the exterior beams. However, because of the presence of end restraint, a strain reduction of only 145 με was obtained. When the post-tensioning force was removed from the bridge during summer 1984, a magnitude of 172.0 kips was measured. This decrease in force (5.5%) is primarily the result of reduction in the amount of end restraint present, however, a small percentage of the decrease is due to relaxation of the post-tensioning tendons. A post-tensioning force of 196.8 kips, which produced an average upward displacement of 0.18 in. in the exterior beams, was reapplied to each exterior beam during summer, 1984. The midspan bottom flange coverplate strains resulting from this post-tensioning force are shown in Fig. 5. Also shown in the figure are theoretical curves, obtained from finite element analysis, for both simple supports and fixed ends. The measured strains are essentially midway between the two theoretical curves, thus indicating the presence of significant end restraint. Additional verification of the presence of end restraint is the sizable magnitude of strains measured at instrumented sections 9 in. from the centerline of support. Although the desired strain reduction was not obtained, end restraint was also found when live load was placed on the bridge. Thus, end restraint reduced the effectiveness of the post-tensioning and also reduced the strains produced by live load.

**Bridge 2:** Initially, each exterior beam was subjected to an average post-tensioning force of 305.6 kips that, based on the assumption of simple supports, was to produce a strain reduction in the lower flange of 212 με. However, when this force was applied, a reduction of slightly less than half that amount (102 με) was obtained because of the skew effect and the presence of end restraint. During summer, 1983, this bridge was resurfaced and had major portions of the curb sections removed and replaced. Because the post-tensioning force was still applied to the bridge, large upward displacements were visible upon removal of the dead
load and reduction in the effective section. When the post-tensioning force was removed from the bridge during summer, 1984, a magnitude of 271.5 kips was measured. This decrease of 11.2% is thought to be primarily the result of losses incurred during rehabilitation of the bridge which took place the previous summer. A post-tensioning force of 371.2 kips was then reapplied to each exterior beam, resulting in an upward displacement of 0.33 inches. Shown in Fig. 6, along with the theoretical curves assuming simple supports and fixed ends, are the midspan bottom flange coverplate strains resulting from this post-tensioning force. The measured strains on the exterior beams are closer to the fixed end theoretical curves than to the simple span theoretical curves. Because of the skew, the end restraint present in this bridge was greater than that observed in Bridge 1. As in Bridge 1, significant strains were measured in the vicinity of the supports; these also verify the presence of end restraint. End restraint again reduced the effects of live load. In effect, the live load tension strains, which were lower-than-expected, compensated for the lower-than-expected post-tensioning compression strains.

The third phase of the investigation (7) involved the development of a simplified design methodology for practicing engineers so that they could avoid use of orthotropic plate theory and/or finite-element analysis.

By analyzing an under-capacity bridge in which the exterior beams are smaller than the interior beams, an engineer can determine the overstress in the exterior beams. This overstress is based on the procedure of isolating each bridge beam from the total structure. The cross-sectional properties of the individual beams are based on the rules given in the AASHTO Standard Specifications for Highway Bridges (8). The amount of post-tensioning required to reduce the stress in the exterior beams can then be determined if the amount of post-tensioning force remaining on the exterior beams is known. The amount of force remaining on the exterior beams can be quantified with force and moment fractions. A force fraction, FF, is the ratio of the axial force that remains on a post-tensioned beam at midspan to the sum of the axial forces for all bridge beams at midspan, while a moment fraction, MF, is simply the moment remaining on the post-tensioned beam divided by the sum of mid-span moments for all bridge beams. Knowing these fractions, the required post-tensioning force may be determined by utilizing the following relationship:

$$f = FF \left(\frac{P}{A}\right) + MF \left(\frac{P\cdot e}{I}\right)$$  \hspace{1cm} (1)

where

- $f$ = desired stress reduction in lower beam flange.
- $P$ = post-tensioning force required on each exterior beam.
- $A$ = cross-sectional area of exterior beams.
- $e$ = eccentricity of post-tensioning force measured from a bridge's neutral axis.
c = distance from neutral axis of beam to lower flange.
I = moment of inertia of exterior beam at section being analyzed.

Distribution factors were determined by a finite element analysis; the accuracy of the finite element model was determined through use of the data obtained in the laboratory and field testing. A series of experiments was conducted with the finite element model to determine the sensitivity of the distribution factors to the various bridge variables. The possibility of having one distribution factor for axial force and moment was investigated; however, it was found that a much greater amount of axial force than moment remains on the exterior beams at midspan. Thus for design purposes, force fractions, FF, and moment fractions, MF, should be kept separate as shown in Eqn. 1. These experiments demonstrated that variables such as the stiffness of shear connectors, cover plate length, bridge skew, and deck crown had no significant effects. Those variables having significant effects on the distribution factors are discussed briefly in the following paragraphs.

End Restraint: As previously noted, a considerable amount of end restraint was found in the two actual bridges strengthened. Through analysis it was found that end restraint increased the amount of post-tensioning force and moment retained by the exterior beams. However, because of the difficulty in quantifying end restraint existing in actual bridges, partial end restraint was neglected in the design methodology. This is conservative in that simple support distribution factors are smaller than those for actual support conditions (i.e., partially restrained ends).

Span and Anchorage Location: Span length was found to be the most significant variable in the moment fractions; exterior beams were found to retain less moment as the span length was increased. When the span length was held constant and anchorages were moved toward the supports, more moment was distributed away from the exterior beams. The distribution is thus dependent on the length of the beam subjected to post-tensioning (i.e., distance between anchorages).

Relative Beam Stiffness: Moment distribution was found to be dependent on relative beam stiffness; stiffer exterior beams retained more post-tensioning moment. Thus, if the exterior beams were smaller than the interior beams, the exterior beams retained considerably less of the post-tensioning moment than if the beams were all the same size.

Utilizing the finite element model, bridges having various combinations of the involved variables were analyzed to determine the midspan force and moment fractions. By using a multiple linear regression procedure, the calculated fractions were analyzed to identify the variables that had the most effect on the distribution fractions. Relatively simple equations with high
degrees of accuracy were derived for the distribution fractions. For the distribution fractions at midspan and at other locations within the span, the reader is referred to Refs. 2 and 7. Reference 7 presents a complete design methodology in which other considerations such as gain in post-tensioning force due to bridge deflections resulting from live load, loss of post-tensioning force due to temperature differentials, and the like are considered.

The strengthening procedure and design methodology just described have since been used on several bridges in the states of Iowa, Florida, and South Dakota. In all instances, the procedure was employed by local contractors without any significant difficulties.

PROJECT 2

Phase I: As previously noted, only Phase I (9,10,11) of this project is completed while Phase II is currently in progress. Thus, final results of Phase I are presented, while only preliminary findings of Phase II are given. In addition to the problems with single-span bridges that were addressed in Project 1, a large inventory of continuous composite bridges are also structurally inadequate. Some of these bridges are overstressed in the negative moment regions, some in the positive moment region, while a large number are overstressed in both regions. On the basis of Project 1 results, it appeared that post-tensioning could also be used to strengthen continuous bridges. Thus, the primary objective of this phase of the investigation was to determine the feasibility of strengthening continuous composite bridges by post-tensioning. More specific objectives of the study were to determine:

- the distribution of post-tensioning forces and moments in the positive and negative moment regions
- the best tendon arrangement (straight, inclined, continuous, etc.) for post-tensioning the positive and negative moment regions
- the contribution of the deck to composite action in negative moment regions.

The objectives of this project were fulfilled through testing of a one-third-scale model bridge, testing of a full-scale mockup of the negative moment region of a bridge beam, and the development of a finite element model. The model bridge, mockup, testing program, and some representative results will be discussed in the following paragraphs. The finite element model developed was calibrated and verified by using results from the laboratory model tests. This program was used to investigate the effect of the various bridge parameters on post-tensioning force and moment distribution.
Because of space limitations in the Iowa State University (ISU) Structural Engineering Laboratory, it was necessary to limit the bridge model in number of spans as well as scale. After consideration of various combinations, we decided to construct a four-beam, three-span continuous bridge. An overview of the model bridge is given in Fig. 7. The overall dimensions of the bridge are total length = 41'-11", width = 8'-8", deck thickness = 2.25", and individual span lengths = 12'-8", 16'-4", 12'-8". To solve the problem of dead load on the model and to prevent uplift when the model was subjected to certain loading patterns, each beam was vertically restrained at each support. The system employed (see Fig. 6) prevented vertical movement; however, it did not restrict horizontal movements. To model accurately the one-third-scale bridge beams, the beam sections were fabricated from steel sheet and plate. The exterior and interior beams were 6" and 7" in depth, respectively.

As may be seen in Fig. 7, the post-tensioning in the negative moment regions was applied on top of the deck surface. This was done for convenience; obviously, in the field, in the negative moment region the post-tensioning would have to be done under the deck, or a portion of the deck would have to be removed to accommodate the post-tensioning systems and then be replaced later.

The full-scale mockup of the negative moment region previously noted was constructed so that the strength and behavior of post-tensioning arrangements correctly positioned under the deck (which obviously was not possible on the bridge model) could be determined. The mockup was strengthened by using both straight and harped post-tensioning tendon profiles. Various post-tensioning schemes were tested on the mockup with and without the presence of vertical loading. Space limitations preclude additional discussion of the mockup tests herein. However, those desiring additional information are referred to Ref. 11.

A total of 66 strain gages were mounted at critical sections of the model bridge beams; each tendon was also instrumented with strain gages so that post-tensioning forces could be accurately measured. Deflections at the midspan of each beam (twelve displacements) were measured by using direct current differential transformers. To facilitate rapid collection of data, all strains and displacements were measured and recorded with a data acquisition system.

The test program employed consisted of determining the bridge's response, strains, and deflections to the following loading conditions:

- vertical concentrated load at various locations on the middle and end spans
- various arrangements of post-tensioning (i.e., negative moment regions of exterior beams,
negative moment regions of all beams, positive moment regions of end spans only, etc.)

- combination of the various post-tensioning arrangements plus vertical loading.

The following paragraphs discuss briefly some of the results from this investigation. Post-tensioning continuous bridges is considerably more complex than post-tensioning simple-span bridges in that longitudinal and transverse distribution effects both must be determined. Twenty-two different post-tensioning schemes (symmetrical and asymmetrical) were tested on the bridge model.

Shown in Fig. 8 are the measured strains for an asymmetrical post-tensioning scheme (positive moment regions of exterior Beam 4 subjected to a nominal post-tensioning force of 20 kips). This scheme of post-tensioning could be utilized to strengthen an exterior beam that has been damaged by an overheight vehicle or weakened by localized corrosion. In the beam post-tensioned, Beam 4, the strains are opposite of those resulting from dead load. Strains in the bottom flange are tensile at midspans and compressive at the interior supports. Thus, these strains reduce dead load strains at critical locations, thereby increasing the live load capacity of the beam. Transverse distribution effects, similar to those in single-span bridges, transfer beneficial strain reductions to the adjacent beam, Beam 3 (See Fig. 8b). Post-tensioning Beam 4 has a very small effect on Beams 1 and 2 (see Fig. 8c and 8d); however, the effect on Beam 1 is adverse in that the strains produced in Beam 1 by post-tensioning Beam 4 are additive to the dead load strains in Beam 1. Other asymmetrical tests produced similar results; post-tensioning a specific region of a beam created beneficial strains at that region; however, it also produced adverse strains in other regions. Therefore, asymmetrical post-tensioning, due to distribution effects, should be thoroughly investigated before being employed to strengthen a particular region of a bridge.

Shown in Fig. 9 are the experimental and theoretical bottom flange strains for two symmetrical load cases. The theoretical curves shown in this figure were obtained by using the finite element analysis previously discussed. In Case 1 (see Fig. 9a) the positive moment regions of all exterior beams are post-tensioned, while in Case 2 (see Fig. 9b) the negative moment regions of all exterior beams are post-tensioned. In both cases a nominal force of 20 kips was applied to each beam. In both of these cases there is excellent agreement between the measured strains and the theoretical curves.

As previously noted in both cases, only the exterior beams were post-tensioned. Thus, the strains that are shown in the interior beams are the result of lateral load distribution. As this figure indicates, distributions are approximately the same, in terms of distribution fractions, for both positive moment region post-tensioning and negative moment post-tensioning.
Also shown in Fig. 9 are the longitudinal strain distributions. Both post-tensioning schemes produced beneficial strains (i.e., opposite to those produced by dead load) both at midspan and at interior supports. In Case 1, the positive moment regions were post-tensioned, producing compressive strains in the bottom flange near midspan and tensile strains in bottom flange in the vicinity of the interior supports. Similar results (see Fig. 9b) were obtained when the negative moment regions of the exterior beams were post-tensioned. Thus, depending on the bridge, it may be possible to strengthen it (both in positive and negative moment regions) by post-tensioning only the positive moment regions of the exterior beams or the positive moment regions of all beams. Post-tensioning the positive moment regions is obviously simpler than post-tensioning the negative moment regions of a given bridge because of the presence of the deck.

In the strengthening of a continuous span bridge by post-tensioning, the engineer has so many variables (beams to be post-tensioned, region(s) of beam to be post-tensioned, magnitude of post-tensioning force, post-tensioning length, etc.) that in essentially all cases it is possible to "fine-tune" the bridge (i.e., reduce strains in the desired locations by the desired amount).

Phase II: Phase II of this investigation was undertaken as a result of the successful laboratory investigation (Phase I). The objectives of this phase of the investigation were

- to design and install a post-tensioning strengthening system for a selected continuous span, steel beam-composite concrete-deck bridge
- to instrument and test the bridge for determination of its behavior before and after post-tensioning
- to monitor the bridge's behavior for an extended period of time.

The bridge selected for strengthening (see Fig. 10) is located in northwest Iowa, near the town of Fonda. The four-beam bridge consists of three spans (45'-9", 58'-6", 45'-9"); the exterior beams (W21x62) and the interior beams (W24x76) bear at approximately the same elevation. The difference in beam heights provides a crown in the bridge deck (nominal deck thickness 6 3/4"). The similarity between this bridge and the laboratory model will permit comparisons of field and laboratory data.

On the basis of an analysis of the bridge, it was determined that the desired stress reduction in the bridge (in both the positive and negative moment regions) could be accomplished by post-tensioning the positive moment regions of all beams in the bridge (12 different regions). Shown in Fig. 11 (upper number in box) are the theoretical post-tensioning forces required on each beam to obtain the desired stress reduction at the various
overstressed regions in the bridge. The lower number in the box indicates the magnitude of force that was actually applied to each beam. In all but two beams slightly more post-tensioning force was applied than was theoretically required.

The bridge was instrumented with 66 weldable strain gages positioned at critical sections of the beams; each of the 24 tendons (two required per beam) were instrumented so that post-tensioning forces could be accurately determined. Deflection at the midspan of each beam in one end span, each beam in the middle span, and one exterior beam and one interior beam of the other end span (10 displacements) were measured by using direct current differential transformers. A data acquisition system facilitated rapid data collection.

The testing program utilized on this bridge was essentially the same as that used on the single span bridges (Bridges 1 and 2). Data were recorded when the bridge was subjected to an overloaded truck (total weight = 57.92 kips) positioned at various predetermined locations before and after post-tensioning, and at various stages of the post-tensioning sequence.

As the testing program previously described was only recently completed, analysis of the data obtained is still in progress. A preliminary review of the data indicates that the bridge's behavior is essentially the same as the laboratory model's except for the presence of end restraint found in the field.

This bridge will be inspected approximately every three months for one year to monitor its behavior. At the end of that period the bridge will again be service-load tested to determine any behavioral changes from its initial strengthening.

**SUMMARY**

The results of the research outlined in this paper indicate that post-tensioning is a viable, economical technique for flexurally strengthening simple span and continuous span steel-beam and concrete-deck composite bridges. The post-tensioning systems employed on the single span bridges and the continuous bridges essentially reduce dead load strains (stresses) in the bridges, thus making it possible for the bridges to carry additional live loads. Post-tensioning will slightly increase the ultimate strength of the bridge and in most cases induce a small upward camber. It does not, however, significantly reduce live load deflections or significantly influence live load distribution.

**ACKNOWLEDGMENTS**

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State University and funded by the Iowa Highway Research Board and the Highway Division, Iowa Department of Transportation, Ames, Iowa. The authors thank the engineers at the Iowa DOT, especially William A. Lundquist, Bridge Engineer, and John P. Harkin, Bridge Rating Engineer, for their support, cooperation and counsel. Special thanks are accorded the numerous graduate students who assisted with the various phases of the two projects. The opinions, findings, and conclusions expressed herein are those of the authors and not necessarily those of the Iowa Department of Transportation or the Iowa Highway Research Board.

REFERENCES


Fig. 1. Single Span Composite Bridge Model (Bridge Model No. 1).

Fig. 2. Post-Tensioning, Midspan, Bottom Flange Beam Strains (Bridge Model No. 1).
Fig. 3. Post-Tensioning Brackets and Tendon Arrangements.

(a) Bridge No. 1.

(b) Bridge No. 2.
Fig. 4. Field Test Vehicles Used on Bridges 1 and 2.
Fig. 5. Post-Tensioning, Midspan, Bottom Flange Beam Strains (Bridge No. 1).

Fig. 6. Post-Tensioning, Midspan, Bottom Flange Beam Strains (Bridge No. 2).
Fig. 7. Continuous Span Composite Bridge Model (Bridge Model No. 2).
Fig. 8. Bottom Flange Beam Strains from Post-Tensioning Positive Moment Regions of Beam 4 (Bridge Model No. 2).


Fig. 9. Bottom Flange Beams Strains (Bridge Model No. 2).
Fig. 10. Fielding Testing of Continuous Bridge.
Fig. 11. Post-Tensioning Forces Applied to Continuous Bridge.
Post-Tensioned Repair and Field Testing of a Prestressed Concrete Box Beam Bridge

by A.E.N. Osborn and H.K. Preston

ABSTRACT

Over 600 bridges composed of adjacent prestressed concrete box beams were built in the early 1950’s in Pennsylvania. The box beams were placed side-by-side and had an asphalt wearing course on top. Span lengths ranged from 30 to 60 feet. Their design was very conservative by today’s standards.

In these bridges, the concretes have high chloride contents; water leaks down through the joint between box units and the strands often have inadequate cover. Thus, it is not surprising that many of the box beams are deteriorating due to corrosion of their prestressing strands.

This project was directed toward developing economical repair schemes for these bridges. The literature survey did not reveal any schemes specifically applicable to adjacent box beams.

Two external reinforcement repair schemes were developed, and trial installations were made on a bridge near York, Pennsylvania. Both schemes included the removal of deteriorated concrete, placement of external reinforcement beneath the beam and application of gunite to the soffit of the beam.

In Scheme 1, the external reinforcement consisted of epoxy-coated reinforcing bars. This repair method restored ultimate flexural capacity, but did not restore lost prestress. It was the least costly of the two methods.

In Scheme 2, post-tensioned, epoxy-coated strand were used. This restored the full ultimate flexural capacity and most of the lost prestress. Difficulties were encountered in installing anchors for the post-tensioned system, but its performance was good.

The bridge was tested after repair. The external reinforcements were found to be fully composite with the original beams. The tests also revealed the lateral distribution of wheel loads. In spite of the poor condition of the bridge, the wheel loads were well distributed laterally, leading to a structure which was stronger and stiffer than expected.

Keywords: box beams; bridges (structures); coatings; deterioration; epoxy resins; lateral pressure; loads (forces); load tests (structural); post-tensioning; prestressing steels; repairs; shotcrete
INTRODUCTION

Numerous prestressed concrete bridge structures were constructed in Pennsylvania in the infancy of the prestress industry. Several fabrication practices used at that time have proven to lack durability. Consequently, many of these bridges are now distressed and repair methods are needed to restore them. This paper presents the results of a research project, funded by the Pennsylvania Department of Transportation (PADOT) and the Federal Highway Administration (FHWA) for the purpose of exploring methods to restore deteriorated bridges composed of prestressed, concrete, adjacent box beams.

Background of Problem

Over 600 adjacent box beam bridges have been built in Pennsylvania. Many were constructed before 1955. A typical bridge has nine beams, each 3 feet wide and may have several simply supported spans. Mortar filled shear keys and lateral tie rods help to transfer concentrated vertical loads to adjacent beams.

Two fabrication practices used before 1955 have led to the present state of deterioration of this bridge type. First, calcium chloride was often added to the concrete mix to accelerate curing in cold weather. Calcium chloride, in the presence of air and water, causes corrosion of steel items embedded in concrete. The corrosion causes pitting of the prestressing strand and loss of cross-sectional area. The corrosion product, rust, exerts bursting forces on the surrounding concrete leading to cracking and spalling.

Second, forming practices were uneven. The prestressing strands were not provided with sufficient cover, particularly on the beam soffits. Adequate cover of high quality concrete will normally protect the embedded steel from corroding by blocking the passage of water. Water is able to reach the beam soffits because of leakage through joints between the box units.

An extensive literature search (References 1-13) was made to discover previously tried repair methods. Nothing applicable to adjacent box beam bridges was found.

Two methods of repair were developed and installed on a test bridge near York, Pennsylvania. One method used deformed reinforcing bars and the other, which is the subject of this paper, used post-tensioned strands. The repaired bridge was tested to determine the overall behavior of the bridge and to test the efficacy of the repairs.
DESCRIPTION OF TEST BRIDGE

The test bridge was chosen by PADOT and was considered to be representative of this bridge type. The bridge carries Locust Point Road (L.R. 66038) over the Little Conewago Creek in York County, Pennsylvania. The bridge consists of three approximately 52 foot spans and is skewed 30 degrees. Each span has nine adjacent box beams, each 36 inch wide and 33 inch deep. A 24.75 inch diameter circular void was at the center of each box. Edge beams have 10 inch by 10 inch curbs and steel guard rails. An asphalt wearing course provides the driving surface. Buried expansion joints are used over piers and abutments. Staggered tie rods join the boxes at span third points. Figure 1 shows plan and cross-sectional views of the test bridge.

Bridge bottom reinforcement, according to state drawings, consisted of two rows of 1/4-inch diameter strands, with 40 strands in each row. Top reinforcement consisted of four #6 bars. Welded wire fabric was used for shear and temperature reinforcement. Reinforcement details, as designed, are shown in Figure 2.

The condition of the bridge was evaluated by visual inspection, optical survey and core inspection and testing. Many spalls were observed on beams soffits. Within the spalls, corroded strand was seen. Often the spalls extended for some considerable length. In these cases many strands were corroded through and were hanging down from the soffit. In the most severely deteriorated beam, less than 20 strands were exposed. Most spalls were quite shallow, generally less than .4 inches. The deepest spalls, up to about .8 inches deep, were located adjacent to joints. Numerous damp areas were also seen, indicating the leakage of water through joints. Gaps at joints ranged from .1 to .8 inches wide. On the top surface of the bridge, longitudinal cracks occurred above joints between box beams and over buried expansion joints.

Of the three spans, the middle span was considered to be in the worst condition. However, test repairs were made to the south span since this was the only span completely above dry land. A reflected soffit plan showing the observed condition of the south span is presented in Figure 3.

Cores were taken at a joint and at an end diaphragm. The joint core showed that mortar in shear keys was of poor quality. The top of the key was filled with asphalt. The bottom of the key was filled with gravel. The keys had an unusual profile as shown in Figure 2. The core also showed that the asphalt wear course consisted of many layers with a total thickness of 5 inches. Beneath the top layer, the asphalt was fragmented.
The diaphragm core (see Figure 3 for location) was for the full depth of the member. Chloride ion contents and concrete compressive strengths were measured from the core. Chloride ion contents ranged from 6.5 lbs to 11.2 lbs per cubic yard of concrete. These levels are 5 to 8 times those needed to support corrosion of embedded steel. Compressive strengths measured on two samples were 7700 psi and 8900 psi. Based on the compressive strengths, it is estimated that the flexural tensile strength is over 800 psi and the modulus of elasticity is about 4,700,000 psi. Concrete of this strength is relatively impermeable. Two layers of strands were found at the bottom of the core. No corrosion was visible.

An optical survey of the soffit of the south span showed the general bridge profile. The bridge had a longitudinal slope of about 1.3 percent, a cross slope of .7 percent and a camber of about .05 inches.

A sample of corroded strand was taken for tension testing. It broke at a tensile load of 5800 lbs. The original strength of the strand was probably about 9000 lbs. The area of the strand at the break was about .023 square inches compared to the original diameter of about .035 square inches. Thus the strength of the corroded strand was consistent with its reduced cross-sectional area. However, the corroded strand was quite brittle with an ultimate elongation of only .65 percent and a reduction of area of 0 percent. The original strand, which was not stress-relieved at the time these bridges were built, could be expected to have an elongation of 2.0 percent and a reduction of area of 30 percent.

REPAIRS TO TEST BRIDGE

The stated objective of the project called for the development of several repair concepts which would restore the original ultimate strength of the bridge beams. Although having merit, approaches which did not restore ultimate strength of the individual beams were not to be considered. This condition precluded the use of concepts which employed new analytical tools, relaxed code requirements or repairs to redistribute moments and shears.

Two repair methods were chosen for implementation. The selection process was based on viability and cost. Project funds were only sufficient for the repair of two beams from one span. Each beam was to receive one of the chosen repair methods.
Alternate Repair Concepts

Four repair concepts were developed. Two used external post-tensioned reinforcement and two used external non-post-tensioned reinforcement. The four approaches are listed below.

External Post-tensioning Approaches:

1. Four .5 inch diameter straight strands, tensioned against end anchorages bolted to beam soffits, then gunited after tensioning for composite action.

2. Two .6 inch diameter strands draped from the top, through beam joints, which followed the soffit in the middle third of the beam. Also gunited after tensioning.

External Non-post-tensioning Approaches:

3. Six #6 reinforcing bars, suspended below the soffit and gunited to provide composite action.

4. One 18 inch wide by .125 inch thick steel plate, bonded to the soffit with epoxy and further anchored with bolts.

Approaches 2 and 4 were rejected primarily because of anticipated construction difficulties with associated higher cost. References 17 and 18 contain more complete explanations of the selection process. This paper discusses the installation and testing of approach 1. Installation and testing of approach 3 was similar and is described in detail in Reference 18.

Design of Post-tensioned Repair

Original design calculations were not available. It is likely that the original design was based on AASHO-1949 Specifications and "Specifications for Prestressed, Precast Reinforced Bridge Deck " dated May 1, 1952, by PADOT. Design parameters for a similar bridge built at the same time were listed in Reference 2, as follows:

- Design loads per beam = 80 percent of the wheel load from an HS20-S16-44 truck plus impact
- Allowable tensile stress = 0 psi
- Area of 1/4 inch strands = 0.0352 square inches each
- Initial strand tension = 135,000 psi
- Strand tension after losses was assumed to be 80 percent of initial tension
These parameters established the strength requirements for the repaired beam. A prototype deteriorated beam was assumed to have the following deficiencies:

- Bottom row of 40 strands broken
- Bottom 1.25 inch of concrete spalled off
- Top row of uncorroded strands fully effective and located 3-1/4 inches above original bottom

Based on the test bridge, these assumptions of deterioration are very conservative.

The difference between the original design strength and the assumed deteriorated strength dictates the additional strength which must be provided by the repair. The repair design was based on the addition of .5 inch diameter strands positioned 2 inches below the bottom.

The final design of the post-tensioned repair called for the placement of four .5 inch diameter strands, each tensioned to a load of 18 kips. A lesser number of strands would have been possible, except that a symmetric pattern would have been difficult to place. A simple welded plate assembly (Figures 4 and 5) was designed as an end anchorage for the strands. Each assembly was to be bolted to the beam soffit with six Hilti HAS Super 78-10 polymer grouted anchor bolts. Uncertainties in the shear strength of these embedments dictated a conservative design. Also, the anchorage design was predicated on preventing the anchorage from being the weak link in the system. A schematic of the anchorage is presented in Figure 4.

Flexural capacity and midspan bottom fiber stresses for the original beam, deteriorated beam and repaired beam are listed in Table 1. Also listed are design requirements according to 1949 AASHO and 1983 AASHTO specifications. The "assumed" conditions are based on the loss of the bottom row of strands and their cover as mentioned previously. The "actual" conditions are based on the average condition observed on the south span of the test bridge which indicated an average loss of less than 10 strands per beam.

<table>
<thead>
<tr>
<th>TABLE 1 - FLEXURAL MOMENTS AND BOTTOM FIBER STRESSES</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ultimate Flexural Capacity at Midspan (foot kips)</td>
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<tr>
<td>Original Beam (As designed)</td>
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</table>
Table 1 - Flexural Moments and Bottom Fiber Stresses  
(Continued)

<table>
<thead>
<tr>
<th></th>
<th>Ultimate Flexural Capacity at Midspan (foot kips)</th>
<th>Midspan Bottom Stress under Dead and Live Loads (psi)*</th>
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</thead>
<tbody>
<tr>
<td>Deteriorated Beam (Assumed)</td>
<td>757</td>
<td>-763**</td>
</tr>
<tr>
<td>Repaired Beam (Assumed)</td>
<td>1122</td>
<td>-307**</td>
</tr>
<tr>
<td>Deteriorated Beam (Actual)</td>
<td>1295</td>
<td>+24***</td>
</tr>
<tr>
<td>Repaired Beam (Actual)</td>
<td>1718</td>
<td>+266***</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Code Requirements</th>
<th>Required Ultimate Flexural Strength (foot kips)</th>
<th>Required Midspan Bottom Fiber Stress (psi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>AASHTO-1949 Specification</td>
<td>1056</td>
<td>0</td>
</tr>
<tr>
<td>AASHTO-1983 Specification</td>
<td>777</td>
<td>-424</td>
</tr>
</tbody>
</table>

* + = compressive stress
- = tensile stress
** stress calculation based on AASHTO-1949 truck load distribution
*** stress calculation based on measured truck load distribution similar to AASHTO-1983

The durability of repairs was an important design consideration. For this reason all exposed steel hardware was epoxy coated. The gunite is expected to protect uncoated steel hardware. The gunite also eliminates the hazard from a strand should it break in service. On this project, this hazard is minimal since there is no traffic under the bridge.
Construction of Repair

The Post-tensioned repair was made in 9 stages as follows:

1. Loose and delaminated concrete was removed and loose prestressing strands were cut off.

2. The beam soffit was sandblasted to clean off contaminants and improve bond of the gunite. Exposed strands were sandblasted to near white metal.

3. "L" shaped anchors were set into drilled holes. These were used to hold up the mesh reinforcement and help provide composite action between the original beam and the gunite.

4. The post-tensioning anchorage frames were installed. A problem drilling the bolt holes for the frame was encountered during this stage. The frames had been fabricated and epoxy coated in a factory.

5. Four high bond epoxy coated prestressing strands were threaded through the frames and tensioned to about 18 kips each. The ends of the strand were cut off and encased in sealant filled caps. Bolt heads were painted.

6. Galvanized welded wire mesh was tied to the "L" anchors.

7. Form boards were positioned on either side of the beam soffit.

8. The 3-1/2 inch thick gunite was sprayed on in 3 layers using the dry process technique.

9. The soffit of the gunite was screeded and troweled. A curing compound was sprayed on the gunite.

Features of post-tensioning system

Strands used were .6 inch diameter since wedge anchors for epoxy coated, .5 inch diameter strands were not available. The post-tensioning loads, however, remained the same as designated for .5 inch strands.

Sufficient clearance must be maintained between the anchorage and the pier wall to accommodate the ram. The Dywidag system used on this project required 36 inches.

An experienced supervisor was retained for the post-tensioning operation.

The wedges were set and slack was removed by first tensioning each strand to 1.8 kips. Then each strand was tensioned to 18 kips.
Load in the strand was measured by the calibrated pressure gage in the hydraulic line and verified by the elongation of the strand. The strand elongated about 2 inches over its 42 foot length under full tension of 18 kips. The measured elongation was used after release of the jack to determine the final force in the strand. A photograph of the strand tensioning in progress is shown in Figure 5.

Problems Encountered During Construction

The repairs for a single beam took about one week. They would have been done sooner except that the contractor had much difficulty in drilling holes for the anchor frame bolts. The holes were wider than the spacing between prestressing strands in the original beam. It was found that the percussion type masonry bits used by the contractor were inappropriate. The strands were very hard, which caused the drill bit to become dull. A water lubricated diamond core bit would have been superior, if one of the proper diameter was available.

Two of the anchor bolts were found to be loose after installation. The anchorage strength was downgraded accordingly. If this problem was more common, required repairs would have been difficult to install. This illustrated the need for quality control for critical items. A revised design which allowed for a percentage of unusable anchor bolts would be prudent.

Another potential problem was in the use of the "L" anchors. The standard form of these anchors have lead expansion bushings. The combination of lead, steel, zinc coating and dissimilar concretes can create a galvanic cell which will corrode the anchors. Other inert anchor materials should be considered for future projects.

A crack developed between the gunite and the original beam soffit. However, it did not appear to affect the composite behavior of the repair. Before attempting an actual construction project, other methods of applying the gunite should be explored. Cores should be taken through the finished work to verify proper bond.

Cost of Repair

Since this was a research project and involved a relatively small area, construction unit costs were high. The cost to the project amounted to about $100. per square foot of soffit surface (1985).

The contractor provided an estimate of $78. per square foot to do a larger area. This price also seems high and may reflect the contractor's difficulty in installing the post-tensioned repair.
LOAD TESTS ON BRIDGE

The primary purpose of the tests was to verify that the gunite encased external reinforcement was acting compositely with the original beam under service loads. The tests were also designed to yield information about the lateral distribution of loads across the bridge, the distribution of strain through the cross-section of a beam, and the distribution of strain along the length of a beam.

Instrumentation and Data Acquisition

Devices used: Instrumentation devices consisted of 2 inch long bondable strain gages and 1 inch (span) x .001 inch (resolution) dial gages. These devices were concentrated on repaired beams 3 and 7, as shown on Figure 6. They were also distributed to the other beams so that useful information about lateral distribution could be obtained. The instrumentation layout is presented in Figure 6.

Strain gages were put on top surfaces of Beams 1 and 3 to measure strain distribution through their cross section. Four gages were placed near the end of Beam 5 to pick up negative moment effects. To simplify measurement of strains at the level of the epoxy-coated strands, short reinforcing bars called sister bars, with strain gages attached, were embedded in the gunite.

Strain gages were monitored with a Vishay Model P3500 Strain Indicator coupled with two Model SB-10 Switch and Balance units.

Installation: Weights suspended from the soffits by Invar wire provided a stable reference for the dial gages. Dial gage stands were mounted on steel plates bonded to concrete blocks which were partially buried in the soil.

The soffits were prepared for strain gages by grinding, applying a layer of epoxy and sanding. The epoxy layer provided a smooth, continuous bonding surface for the gages. All gages were waterproofed for long term stability so that the test could be repeated in later years, if desired. Three-wire cables connected the gages to the data acquisition system. Both dial gage and strain gage readings were recorded manually.

Tests Conducted and Magnitude of Applied Loads

Tests consisted of 1) a pre-repair load test using a single ten-wheel dump truck, 2) measurement of strain and deflection due to the application of post-tensioning, 3) a post-repair load test using one and two ten-wheel dump trucks, and 4) a post-repair load test using one and two twelve-wheel dump trucks. The ten- and
twelve-wheel truck patterns and axle loads are shown in Figure 7. The ten-wheel trucks weighed about 50 kips. Two such trucks, back to back, simulate the effects of HS-20-44 loading plus impact. Each twelve-wheel truck weighed about 75 kips and simulated a ML-80 load without impact. All tests were conducted between July 18, 1986 and October 23, 1986.

**Application of Load and Measurement of Response**

External post-tensioning forces were applied in 4 stages for each of the four strands. Strain and deflection readings were taken at each load stage.

Vertical load was applied by centering the truck wheels over individual beams across the bridge at midspan. Patterns for ten-wheel trucks centered over an edge beam and over the middle beam are shown in Figure 8. All loads were applied with the trucks stationary to simulate static conditions.

The strain and dial gage readings were taken with the truck(s) in each position and under zero live load, between each position, with the truck(s) off the bridge. Strains were converted to stress by multiplying by the estimated modulus of elasticity, 4,700,000 psi.

**Measurement of Response from Post-tensioning**

Figures 9a and 9b show the strains and deflections measured at the midspan soffits after application of all post-tensioning forces. Figure 9c shows the relative distribution of strains and deflections to each beam. Total force, after seating of the strands and immediate losses, was about 70 kips. This force created about 20 microstrain at the bottom of the repaired beam and caused an upward deflection of about .05 inches. Due to transfer of loads laterally to adjacent beams, only about 25 percent of the force was resisted by the repaired beam. In an actual repair of the entire structure, the relative distribution to each beam would be practically uniform.

**Pre-Repair and Post-Repair Test Results**

Results are presented for just two load positions: one with the truck(s) along an edge beam and one with the truck(s) centered on the bridge. Results for the remaining positions can be found in Reference 18.

**Demonstration of Composite Behavior:** The post repair tests revealed the composite behavior between the original beam and the gunite encased repair. Figure 10 shows the distribution of strain
through the cross section of the repaired beam under single and
double truck loading. The strain distribution is nearly linear
which indicates that the reinforcement within the gunite is acting
compositely.

Lateral Load Distribution, Ten-Wheel Dump Truck Loadings (HS 20-
44): The pre-repair and post-repair test results were essentially
the same with respect to lateral distribution of loads. Figures
11a and 11b show deflection and strain across the bridge with the
two trucks positioned as close to the edge beam as possible. The
highest deflection is .14 inches and the highest strain is 70
microstrain for an estimated stress of about 330 psi. Both occur
in the edge beam under load. Figure 11c shows that the edge beam
carries about 33 percent of the total truck load. (This is
equivalent to 66 percent of the total wheel load on one side of
the truck.) With the trucks in the middle of the bridge, the
deflections and strains in the beam under load are much less, as
shown in Figures 12a and 12b. The distribution is much wider and
the loaded beam is only resisting about 30 percent of the truck
wheel load.

Twelve-wheel Dump Truck Loadings (ML-80): Deflections and strains
are shown in Figures 13a and 13b which reflect the loading of
standard AASHTO lane positions, with the lanes as close to one
edge as allowed. Figure 13c shows that the edge beam is only
carrying about 14 percent of the weight of both trucks. Since
there are four sets of wheels in the two trucks, this beam resists
56 percent of a wheel load for this load pattern. This is the
highest measured percentage for any combination of AASHTO load
patterns.

Measurements of stresses along the length of Beam 5 are presented
in Figure 14. The negative bending of the beam near its support
is indicated by the stress reversal in the figure.

Visual Inspection of Structure During Load Test

No new cracking of the original beams was observed during the
test. This is not surprising since analysis indicates that dead
and live load stresses plus the original prestress are
compressive. In the repaired beam additional compressive stresses
are generated after post-tensioning. However, the gunite is
unstressed by the post-tensioning. Some transverse cracking did
appear in the gunite. This is attributed to shrinkage and
flexural tension.
COMPARISON OF TEST RESULTS WITH ANALYTICAL STUDIES

Stresses and deflections were computed for simple span beam members of the type used in the test bridge. Computed values were about 20 percent higher than measured values. This differential is higher than the uncertainty in the modulus of elasticity. A more rigorous analysis was conducted using the "Transfer Matrix Method" described in Reference 16. This analysis showed that negative moments are developed at beam ends due to the skewed design of the bridge. The magnitude of negative moments is influenced by the position of load. This finding is consistent with the measured behavior of beam 5. The lateral distribution of loads to beams was also accurately predicted by the Transfer Matrix Method.

DISCUSSION AND CONCLUSIONS

The external post-tensioning of these box beams was shown to be a viable method of repair. Load tests conducted after repair showed that gunite can be used to develop composite interaction between the original beam and the external strands under design load levels. This does not prove that the composite action can be assured under ultimate load conditions. The long term bond of the gunite to the beam soffit has also not been established. Additional studies are needed to assure that composite action is maintained both under ultimate load conditions and over the long term.

The problems encountered in drilling through prestressing strand point to the need for different methods of attaching post-tensioning anchorages. It is possible that a larger number of smaller diameter anchors could have been used to avoid these problems. Also, there may be drill bits, unknown to the authors, which can more easily penetrate the existing strands. In addition, costs would be lower if only two anchorages per beam were needed instead of four. Research is needed to establish the shear strength of multiple bolts embedded in concrete where the bolts are aligned with the load path, where the bolts are embedded into a relatively thin bottom flange and where the bolts are positioned close to a transverse edge.

One of the most enlightening results of the test program was the measured lateral distribution of loads to beams adjacent to the loaded beam. In spite of the poor condition of the shear keys, lateral distribution was nearly equal to the distribution designated in current AASHTO specifications and that determined from analysis. This means that even bridges which appear to be severely distressed may have substantial reserve capacity since they were designed using conservative assumptions with respect to lateral load distribution and allowable tensile stresses. The
original design of the test bridge was found to exceed current design requirements by 57 percent as a result of these factors. In spite of the apparent poor condition of the bridge, a complete evaluation revealed only a 15 percent reduction in its original strength.

ACKNOWLEDGEMENTS

The authors wish to thank PADOT for their support of the research project which is the basis of this paper. Many individuals at Wiss, Janney, Elstner Associates participated in the research. However, the authors wish particularly to thank Carol Roach for her substantial efforts on this project.

REFERENCES


8. Irwin, C.A.K., The Strengthening of Concrete Beams by Bonded Steel Plates, Transport and Road Research Laboratory, Department of the Environment, Old Workingham Road, Crowthorne RG11 6AU Berkshire, England.

10. Koretzky, H.P., *What Has Been Learned from the First Prestressed Concrete Bridges - Repair of Such Bridges*. Transportation Research Record 664, Bridge Engineering, V. 1, Transportation Research Board.


NOTE: MEASURED WIDTH WAS 27'-4" DUE TO GAPS BETWEEN BOX BEAMS.

BRIDGE CROSS-SECTION
(LOOKING NORTH)

BRIDGE PLAN

Fig. 1 - Plan and cross section views of test bridge.
Outside dimensions are based on actual measurements. Reinforcement and prestressing strand layouts are based on standard drawings.

Fig. 2 - Typical box beam.
Fig. 3 - Condition survey of bridge
soffit - south span.
Fig. 4 - Post-tension repair details.

Fig. 5 - Jacking unit tensioning Strand No. 2 at end of Beam No. 3.
Fig. 6 - Layout of instrumentation on south span of test bridge, looking north.

Beam 3: Post-tensioned repair.
Beam 7: Reinforcing bars repair.
a. 10-wheel dump truck used in Test 1 (one-half HS20-44 loading).

c. 12-wheel dump truck used in Test 3 (ML-80 loading).

b. Two 10-wheel dump trucks used in Test 2 (HS20-44 loading).

**Fig. 7** - Wheel patterns and loads for trucks used in Tests 1, 2 and 3.
Fig. 8 - Wheel pattern for 10 truck positions, 1W and 4W, for second part of Test 3.
Fig. 9 - Measured deflection and strain due to post-tensioning of Beam No. 3.
Fig. 10 - Distribution of strain through the cross section at Beam No. 3.
Fig. 11 - Distribution of deflection and strain for two truck load patterns 1W in Test 2.
Fig. 12 - Distribution of deflection and strain for two truck load patterns 4W in Test 2.
Fig. 13 - Predicted deflection and strain due to two lanes of HS20-44 loading lanes placed on the west side of the bridge.
Fig. 14 - Strain measured on soffit of Beam No. 5 due to load pattern 5W.
Deviator Behavior and Design for Externally Post-Tensioned Bridges

by R.J. Beaupre, L.C. Powell, J.E. Breen, and M.E. Kreger

Synopsis: A laboratory investigation was performed to study deviation saddles, a type of tendon deviator used in externally post-tensioned precast segmental box girder bridges. Ten reduced-scale models of deviation saddles were fabricated and loaded to ultimate using a specially designed testing apparatus that applied load to each deviator just as would be applied to a deviator in a bridge. The objectives of the study were to:

(1) experimentally investigate the strength and ductility of deviators;
(2) evaluate deviator details in light of observed performance;
(3) define behavioral models for deviators;
(4) determine the effects of using epoxy-coated reinforcement; and
(5) establish design criteria. Data from the test series are presented, two analysis techniques are formulated and design recommendations are made for design of tendon deviators.

Keywords: box beams; bridges (structures); models; post-tensioning; precast concrete; prestressing steels; segmental construction; structural design; tests; unbonded prestressing
INTRODUCTION

One of the latest and most dramatic developments in segmental technology has been the use of external tendons which are defined as tendons in ducts not encased in the concrete of the webs or flanges of the box girder bridge and attached to the concrete boxes at only a few discrete points (see Fig. 1). This innovative type of construction has been shown to provide substantial economic savings, as well as savings in construction time. External post-tensioning differs from internal post-tensioning because the tendons are removed from the webs and flanges and placed in the cell-void. The tendon deviators maintain the draped profile of the external tendons and provide the only positive attachment of the tendons to the structure other than at the anchorage zones. This makes the deviator a key element of this bridge system.

Three basic kinds of tendon deviators have been utilized in externally post-tensioned segmental box girder bridges; the diaphragm (Fig. 2a), the rib or stiffener (Fig. 2b), and the saddle or block (Fig. 2c). The deviators are usually monolithically cast in the bridge segments to accommodate the required tendon duct configurations. The diaphragm and rib or stiffener are usually full web-height deviators. The diaphragms usually extend the entire width of the box section and are provided with an access opening for passage along the span. The rib or stiffener extends out only a small distance from the web wall. The advantage of using the diaphragm or rib type deviators is that compressive strut action in the concrete may be utilized to resist the tendon deviation forces. Compression struts can develop from immediately above the tendon duct to the top flange which provides these deviator types with greater inherent resistance than the saddle or block. However,
many disadvantages also exist with these types. They create added dead load, sometimes offsetting the savings from the efficient web thickness. Other disadvantages are construction related. The formwork for the diaphragm and rib and the geometry for the tendon pass-throughs becomes very complicated, especially for a curved span because the bridge is curving while the tendons remain on straight paths. In contrast, the block or saddle is usually a relatively small block of concrete located at the intersection of the web and bottom flange. Advantages of utilizing this type of deviator in a bridge are that there is relatively insignificant additional weight for the structure, the formwork is less complicated than that required for a diaphragm or a rib, and geometry complications are minimized because non-deviated tendon pass-throughs are generally not required. However, the disadvantage is that the deviator capacity may be greatly reduced as compared to the diaphragm or rib because there is no major direct compression strut formed after cracking. Therefore, the deviator force must be tied back into the box by reinforcement. This may require greater attention to detailing and may lead to more congestion of reinforcing for a saddle-type deviator than for a diaphragm or rib.

RESEARCH SIGNIFICANCE

Uncertainties exist concerning the behavior and proper design criteria for the tendon deviator details. In order to provide careful verification of behavior and develop efficient details, the study documented herein encompassed an experimental investigation of the tendon deviator details and suggested a design methodology for the deviators.

DEVIATOR MODELING AND SPECIMEN DESCRIPTION

The laboratory investigation of tendon deviators focused on deviation saddles since they are the inherently weakest of the three basic types. It was also felt that if the safety of deviation saddles could be confirmed by this investigation, the wider use of this type would offer the most advantages through reducing the structure weight, facilitating the fabrication of segments, and minimizing geometry complications.

The experimental program included fabrication and testing to ultimate of ten reduced-scale models of deviation saddles. The scale factors utilized were 1/3 for the first six models and 1/5 for the last four models (reduced since failure forces were approaching test setup capacity). Two deviator test specimens were fabricated inside a single cell box section. They were located on opposite sides of the box section at the intersection of the bottom flange and web wall. The box section was a typical single cell box girder bridge section except that the cantilevering wings were omitted. The basic box section design was based on the AASHTO Bridge Specification, and remained the same for all specimens. The box has little influence on the behavior of the immediate deviation saddle zone. Supplementary box reinforcement was placed in the bottom flange and web at the deviation saddle for local load distribution.

Deformed microreinforcing bars were used to model the usual #4 and #5 prototype reinforcing. Microreinforcing bars range in sizes #1.25, #1.5, and #2 which refer to their nominal diameter expressed in eighths of an inch. Reinforcement test results are presented in Table 1. Epoxy coated reinforcement which was utilized in several of the test specimens had a measured coating thickness of 7 mils. A 6000 psi concrete mix with a maximum aggregate size of 3/8 inch was utilized. The average 28-day compressive strength of the concrete for each test specimen is presented in Table 2. The rigid tendon duct of the prototype deviators was modeled with a 1-1/2 inch nominal diameter electrical conduit.
which was bent to required deviation angles using a hydraulic pipe bender. Multiple 3/8-inch diameter strands were used for tendons in loading the specimens.

Specimens 1A and 1B modeled a typical prototype deviation saddle detail used in a straight-span bridge (see Fig. 3). The reinforcement scheme consisted of primary link bars supplemented with two types of stirrups (designated as open stirrup and closed stirrup). For specimen 1A, three tendons were deviated which represented a tendon configuration of a deviation saddle located closest to the center of a span. The corner tendon had both a vertical deviation and a slight horizontal deviation directed away from the web, while the other tendon had only horizontal deviation directed towards the web. The specimen 1B reinforcement scheme was identical to that of specimen 1A. However, the tendon configuration was different. This deviation saddle only deviated two tendons which represented a tendon configuration typical of a deviation saddle located somewhere closer to the piers. The corner tendon had both a vertical deviation and a horizontal deviation directed away from the web, while the other tendon had only a slight horizontal deviation directed away from the web. The objective for specimens 2A and 2B was to isolate the behavior of the individual reinforcement patterns of specimens 1A and 1B. These specimens were not intended to be properly detailed deviation saddles, and they were expected to have an abnormally low factor of safety at ultimate. Reinforcement details for specimen 2A provided the link bar alone. Specimen 2B reinforcement details provided the two types of stirrups (open and closed stirrups) without the link bars. The tendon pattern for specimens 2A and 2B was identical to specimen 1B. The objective of specimens 3A and 3B was to determine if the frequently used epoxy coated reinforcement has any effect on the behavior and strength of a deviation saddle. Specimens 3A and 3B were companion specimens to specimens 1A and 1B. The only planned difference between these specimens was that the reinforcement was to be epoxy coated, but some very minor differences in the bar chairs and strain gage lead wires developed that were not discovered until after fabrication. In spite of this, it was felt possible to make direct comparisons between the epoxy coated specimens (3A and 3B) and the uncoated specimens (1A and 1B).

The objective of specimens 4A and 4B was to evaluate the modified reinforcement scheme and deviation saddle geometry shown in Fig. 4. This was an attempt to simplify and standardize reinforcement patterns and deviation saddle geometry for typical deviation saddle details. Because of reinforcement congestion, the link bars previously anchored at the intersection of the web and flange centerlines were replaced by loops anchored into the expanded nodes and outer stirrups tying the nodes to the web and flange. The actual reinforcement patterns utilized were based on small rectangular closed loops which enclosed each tendon, and outer closed stirrups which enclosed the entire deviation saddle. These bars were anchored under the top mat of reinforcement of the bottom flange. The deviation saddle geometry was changed to a horizontal top surface with vertical sides. The tendon configuration for specimen 4A was representative of a deviation saddle on the outside of a small radius curve. The tendon configuration for specimen 4B was representative of a deviation saddle on the inside of a small radius curve. Both specimens had two tendons which had both vertical and horizontal deviations. The objective of specimens 5A and 5B was to further evaluate the effect of epoxy coated reinforcement. Specimens 5A and 5B were companion specimens to specimens 4A and 4B with the only difference being that the reinforcement was epoxy coated.
TESTING PROCEDURE

A specially designed testing apparatus shown in Fig. 5 applied load to the deviator just as it would be loaded in a bridge. This load was applied incrementally. The generalized test setup could accommodate a variety of specimen sizes, tendon layouts and loading schemes. Specimens were usually loaded in two cycles. The first load cycle generally continued until yield of the reinforcement, and the second load cycle continued to failure of the specimen. Strain gages were placed internally on the reinforcement of the deviation saddle to determine contributions of individual reinforcement bars.

TEST RESULTS

Physical behavior of each specimen was observed and noted for the full range of loadings. This included general observation of the deviation saddle, their cracking pattern, reinforcement fracture locations, and strain data. Detailed results for each test are provided in References 2 and 4. Typical test results and photographs before and after failure are shown in Figs. 6-11.

The symbol D and Θ respectively correspond to the magnitude and direction of the deviation force on the deviator. The positive horizontal axis is directed towards the center of the box, and the vector direction is measured clockwise from this axis. The symbol $D_0$ is the nominal design jacketing force for the specimen $(0.8f_{pu}A_p)$ based on the total allowable force of the prototype tendons. The symbol $\Theta_0$ is the nominal design jacking load vector force direction. Since external tendons are basically an unbonded type of post-tensioned construction, it is likely that the maximum jacking force is the highest tendon load that would be exerted on the deviators. The ratio of $D/D_0$ is equal to the deviation saddle factor of safety. The strain gage plots typically indicated that at early load stages the maximum strains occurred in reinforcement located directly above the tendon with the highest deviation force, while at later load stages after a considerable amount of cracking, the highest strains were in the link bar or loop bar reinforcement legs which were acting in direct tension. The fracture locations in the legs of the reinforcement acting in direct tension confirm this (see Fig. 10). For the modified specimens (4A, 4B, 5A, and 5B), it is significant to note that all the strain gages indicated yield of reinforcement which revealed efficient use of the reinforcement. Final failures were generally explosive and dramatic with many of the deviator bars fracturing and splices opening (see Figs. 10 and 11).

For making strength comparisons, the critical load stages were denoted as microcracking, visible cracking, first yield of the reinforcement, and ultimate. Microcracking was assumed to be indicated by the first apparent jump in strain indicated by the strain gages (see Fig. 8). Visible cracking was noted when the first surface crack appeared. Yield of the reinforcement was noted when any of the strain gages reached the yield strain, and ultimate load stage was apparent due to the explosive nature of the failure. Magnitudes of these critical load stages and the nominal design load $D_0$ are given in Table 3. All specimens which were intended to be properly detailed deviation saddles (all specimens except 2A and 2B) had an acceptable factor of safety for ultimate load (values ranged from 2.24 to 3.16). The factor of safety was adequate for the yield load stage for the properly detailed deviation saddles 1A, 1B, 3A, 3B, 4B, and 5B (values range from 1.6 to 2.08). However, for tests 4A and 5A, the factor of safety at the yield load stage was unacceptably low, 1.33 and 1.06 respectively. The factor of safety against visible cracking was adequate for specimens 1A, 1B, 3B, 4B, and 5B (1.3 to 2.03). However, it was marginal for specimens 3A, 4A, and 5A (0.78 to 1.03).
Recommendations will be made which should remedy these deficiencies at the visible cracking stage and at the yield load stage. Average factors of safety for the critical load stages of all the tests except 2A and 2B are shown in Fig. 12.

The epoxy coated reinforcement had adverse effects on the behavior of the deviation saddle at microcracking and visible cracking stages. The average reduction in strength at the serviceability levels of microcracking and visible cracking was 17% and 24% respectively. However, at the critical strength stage of yielding, coated reinforcement had little effect on the behavior. At this stage, all the load is basically transferred to the reinforcement which if well anchored is not particularly dependent on the local bond characteristics of the reinforcement. The well anchored coated reinforcement favorably affected the behavior of the deviation saddle at the ultimate load stage with an average increase in strength of 18% since it allowed for the complete mobilization of all the reinforcement within the deviation saddle. It can be concluded from these comparisons that the use of epoxy coated reinforcement resulted in an increased redistribution of force within the deviation saddle before fracture of the direct tension reinforcement. The comparison between uncoated and coated reinforcement types is summarized in the bar graph of Fig. 13.

ANALYSIS METHODS

Two separate analysis methods were investigated for each test. The first method utilized simplified analysis models (direct tension model, shear friction model, and beam model), and the second method utilized strut-tie analysis models (tie model for direct tension reinforcement, and strut-tie model for top surface reinforcement). These analyses models were formulated based on the physical behavior of the specimens. The $\phi$ factor used in comparisons with test results for both analyses had a value of one since the material strengths and specimen dimensions were known accurately. The analysis of the direct tension reinforcement was the same for both methods.

In the simplified analysis method illustrated in Fig. 14, the direct tension model is used for the analysis of the direct tension reinforcement in the deviation saddle. The shear friction model explains the actions of the shear friction reinforcement which transfers the shear across a crack interface which may form below the tendon ducts. The beam model explains the action of the top surface reinforcement which provides added strength to the deviation saddle to resist pull-out forces. This reinforcement is stressed like tensile reinforcement in a beam, and it also distributes surface cracks.

For the second method referred to, two separate strut-tie models, shown in Fig. 15, were developed. One models the contribution of the primary direct tension reinforcement and the other the contributions of the top surface reinforcement. The strut-tie model is based on the premise that reinforced concrete structures carry load through a set of compressive struts which are distributed and interconnected by tensile ties. The reinforcing bars utilized in direct tension in the deviation saddles are simply tension ties linking the deviation force to the box reinforcement. The strut-tie model for the top surface reinforcement resistance is a combination of compressive struts branching from the average location of the tendon duct to the tension tie which is the top surface reinforcement. Complete details and results of the analyses are presented in Ref. 4. Comparisons of test results with analytical predictions are included in Table 3. The values shown are at ultimate and the analytical results are based on the measured ultimate strength of the reinforcement in order to show the accuracy of the analysis method. Design would be based on reinforcement yield points and introduce further conservatism.
CONCLUSIONS

The following conclusions can be drawn based on the ten deviation saddle tests of this investigation.

1) The safety of deviation saddles has been verified in this investigation. Properly detailed deviation saddles will perform adequately under service load conditions and have a sufficient factor of safety at ultimate.

2) Excluding the purposefully misdesigned specimens 2A and 2B, all specimens except specimens 1B and 3B exhibited adequate ductility and thus gave sufficient warning of the impending explosive failure. The specimens which resisted the pull-out force mainly by direct tension reinforcement (specimens 4A, 4B, 5A, and 5B) always displayed adequate ductility because the reinforcement had to strain substantially to fail.

3) The epoxy coated reinforcement had adverse effects on the behavior of the deviation saddle at microcracking and visible cracking stages. However, at the critical strength stage of yielding, coated reinforcement has little effect on the behavior. The well anchored coated reinforcement favorably affected the behavior of the deviation saddle at the ultimate load stage.

4) From the cracking patterns and the strain data, three behavioral mechanisms were evident in the deviation saddle. They were the pull-out resistance of the direct tension reinforcement, the flexural beam type resistance of the top surface reinforcement, and the shear friction resistance of the specimens across the critical cracked plane which was observed to be directly below the tendon ducts. The pull-out resistance of the direct tension reinforcement and the effective bending of the top surface reinforcement are additive. Some uncertainty exists concerning the effectiveness of this top surface reinforcement at the yield load stage of the deviation saddle. The top surface reinforcement is beneficial in controlling and distributing cracks on the top surface of the deviation saddle.

5) All final failures (except with special specimen 2B) were the result of the fracture of the direct tension reinforcement. Test observations and analysis indicated that in all of these tests shear friction did not appear to be critical to the failure of the deviation saddle.

6) The capacity of the specimens could be determined by either the simplified analysis models or the strut-tie analysis models. Both analysis methods appear to produce reasonable agreement with the test results, although both methods rely on subjective assumptions for the analysis of the top surface reinforcement.

7) In comparing the two types of reinforcement which are utilized to resist pull-out force in this study (direct tension reinforcement and top surface reinforcement), it is obvious that the direct tension reinforcement (the link bar in specimens 1A, 1B, 2A, 3A, 3B and the loop bar in specimens 4A, 4B, 5A, 5B) is significantly more efficient than the top surface reinforcement in resisting the deviated force.

8) The critical force which acts on the deviation saddle is the deviation force contributed by the tendon with the maximum deviation component. In this test series, this is the force that was closely confined by the direct tension reinforcement. In the early configuration (specimens 1A, 1B, 3A, and 3B), this was the
deviated force of the corner tendon which had both vertical deviation and horizontal deviation. The other tendons of the early specimens did not greatly influence the deviator capacity of the specimens because they were not enclosed within the critical reinforcement (direct tension reinforcement). In the revised configuration (specimens 4A, 4B, 5A, and 5B), both tendons were enclosed in separate direct tension reinforcement, but the corner tendon was more critical since it had greater vertical deviation than the other tendon. The basic direct tension reinforcement around the critical tendon should be proportioned for this maximum tendon deviation force. The other tendons could be provided with the same reinforcement to simplify detailing, or some lesser amount determined by a similar analysis based on the individual tendon deviated force.

RECOMMENDATIONS

The following recommendations are made based on the ten deviation saddle tests of this investigation. Recommendations are focused on the deviation saddle since it is the weakest of the three basic types of tendon deviators. However, these recommendations can be conservatively applied to the diaphragm and rib or stiffener since these type of deviators generally have added strength contribution from the concrete.

1) Total service load design forces for the deviator should be the sum of the vertical and horizontal components of the deviated force from each tendon. These can be calculated as the maximum allowable initial jacking force multiplied by the size of the angle change for the vertical and horizontal planes of the tendon. Under AASHTO Specification\(^5\), the maximum allowable initial jacking force is limited to 80% of the ultimate strength of the tendon (0.8(f\(_{pu}\)(A\(_{pt}\))).

2) At service load levels, reinforcement stresses should be limited to the specified allowable stresses in AASHTO Sec. 8.15.2.

3) For load factor design, neither AASHTO\(^5\) nor ACI\(^1\) clearly specify an appropriate load factor for the prestress tendon force. In view of the explosive nature of failure, and in order to guarantee a reasonable factor of safety commensurate with other load and resistance factor combinations, it is suggested that for this specific application the load factor on prestress force should be at least 1.7. Conventional reinforcement should be assumed at the yield point. The \(\phi\) factor that should be used in the design of the direct tension reinforcement should be 0.90 since the primary acting force is tension. The \(\phi\) factor for the shear friction calculation should be 0.85.

4) The recommended design detail is very similar to the modified deviation saddle detail (specimens 4A, 4B, 5A, and 5B) of the test series shown in Fig. 4. The general approach to the design of the deviation saddle should be to rely only on the very efficiently utilized direct tension reinforcement for the pull-out force resistance of the deviation saddle. Any contribution to the pull-out resistance from the concrete is ignored as is any additional resistance from any beam type element above the tendon ducts.

For the direct tension reinforcement, utilize small closed rectangular stirrups (labeled as loop bars) along the entire deviator tendon axis which loop around each individual tendon duct of the deviation saddle and are well anchored in the box-deviator corner node under the top mat of reinforcement of the bottom flange. Additionally, provide closed tie reinforcement which encircles the entire corner
node, ties the node into the web and flange, and provides reinforcement along the deviator top surface for controlling and distributing surface cracks (this reinforcement should be neglected in the calculation of the pull-out capacity). An amount of closed stirrups of the same area and spacing as the loop bar reinforcement should produce very satisfactory results.

Each individual loop bar group should be designed with the specified load factors and $\phi$ factors to resist the full pull-out force of the tendon with the largest vertical deviation. It will be more economical and will minimize fabrication errors to provide the other tendons with the same reinforcement regardless of their tendon deviation forces. The dimensions of the loop bars and the outer closed stirrup must be based on the tendon duct curvatures and outer diameters, reinforcement clearances, and development lengths. For the loop bars, the minimum clearance at the highest point of the tendon duct should be approximately 1". The vertical inside dimension of the outer closed stirrup should be at least 2" larger than the loop bars. The maximum bar size utilized for the loop bars should be limited to a deformed #5 size so as to be able to fully develop the $90^\circ$ hook.

5) Deviation saddle geometry utilized should be a horizontal top surface with vertical sides. This makes fabrication of the segments easier and provides the lowest height deviation saddle possible, which is critical in shallow highway bridge structures for clearance of deviated tendons from adjacent deviation saddles.

The concrete dimensions of the deviation saddle are controlled by the requirements of the tendon duct curvatures and outer diameter, reinforcement clearances, and cover requirements. The lowest point of the tendon duct above the top of the bottom flange should be based on required clearance (1" to 2") for constructability (protective sheathing placed on extension of tendon duct is generally used for external post-tensioning). The transverse location of the ducts should be as close as possible to the web wall since it is desirable to have as small of an eccentricity to the web as possible to minimize bending moments in the bottom flange. The width of the deviation saddle in the longitudinal direction of the bridge is dependent on the spacing and amount of reinforcement (4" to 6" center to center reinforcement spacing is recommended to allow constructability). Also, it is dependent on the minimum radius that the tendon duct can be bent.

6) Provide a full bottom flange width deviation saddle as shown in Fig. 16 for curved spans with small radii when all the tendons in a deviation saddle have large horizontal deviations. For straight spans, when the horizontal tendon angles are small (less than 3°) and the horizontal components are directed either into the web or away from the web, it should only be necessary to use a deviation saddle which is similar to those tested. It may also be advisable to provide the full bottom flange width deviation saddle no matter what the tendon deviations are when epoxy coated reinforcement is being utilized because it was observed in this test series that the specimens with epoxy coating reinforcement cracked at a much lower load than that of the conventionally reinforced specimens (averaged 24% lower). Since the reinforcement is being epoxy coated because of severe corrosion conditions, it would be advantageous to go one step further and provide the full bottom flange width deviation saddle which will substantially increase the factor of safety against visible cracking.
A reduction in concrete volume for the full bottom flange width deviation saddle could be made by reducing the longitudinal dimension of the deviation saddle by one-half in the center of the bottom flange at a certain distance from the tendon ducts. Near the webs the deviation saddle would be the same as the models tested, but would be joined to the opposite deviation saddle by a concrete strut half the dimension of the deviation saddle.

7) In cases where a full bottom flange width deviation saddle is not provided, a shear friction calculation should be made. In most cases, extra shear friction reinforcement will not have to be provided since there is usually an excess of direct tension reinforcement at the critical shear plane. Also, the outer closed stirrups contribute to the shear friction reinforcement. The shear friction equation that should be used for this check is Equation 8-10 of AASHTO Sec. 8.15.4.3. Any net tension across the shear plane is taken into account by subtracting it from the total capacity of the reinforcement crossing the shear plane. The maximum allowable shear strength provided by this equation is the lesser of the two values, 0.09f'cAv or 360Acv, where Acv is the area of concrete section resisting shear transfer. This area is assumed to be the area below the tendon ducts from the centerline of tendon closest to the web wall to the front face of the deviation saddle. The μ factor is taken as 1.4 since the deviation saddle is monolithically cast.

DESIGN EXAMPLES

Two design examples are presented to help clarify the recommendations discussed above. The first one is shown in Fig. 17a and b for a deviation saddle from a straight span. Since the horizontal deviations are less than 3°, a deviation saddle similar to those tested is assumed. The second design example is shown in Fig. 18a and b for a deviation saddle from a curved span. A full bottom flange width deviation saddle is assumed because the horizontal angles are quite significant.

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REFERENCES


11. American Concrete Institute, Building Code Requirements for Reinforced Concrete (ACI 318-83), Detroit, 1983.


Table 1  Deviator reinforcement properties

<table>
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<th>Size</th>
<th>Yield Strength (ksi)</th>
<th>Ultimate Strength (ksi)</th>
<th>Yield Strain (micro in./in.)</th>
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Table 2  Average 28-day compressive strength

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<td>Test 4B</td>
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<tr>
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Table 3  Test results

<table>
<thead>
<tr>
<th>Specimen</th>
<th>Nominal Design (D_o)</th>
<th>Deviation Saddle Force (kips)</th>
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<th>Simplified Analysis*</th>
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* Simplified analysis assuming beam element unrestrained at ends.
** Based on capacity of critical tendon 2 which assumes no redistribution.
Figure 1. External Post-Tensioning in Long Key Bridge (From Ref. 1)
Figure 2. Deviator Types

a) Diaphragm

b) Rib

c) Saddle
Figure 3. Specimens 1A, 1B, 3A, and 3B
Figure 4. Specimens 4A, 4B, 5A, and 5B
Testing Concept (From Ref. 1)

Figure 5. Testing Apparatus
Figure 6. Test 4A Loading History
Figure 7. Test 4A Crack Patterns
TEST 4A PHASE 1

DEVIATION SADDLE FORCE VS. REINFORCEMENT STRAIN

Figure 8. Test 4A Reinforcement Strain Data - Phase 1
Figure 9. Test 4A Reinforcement Strain Data - Phase 2
Figure 10. Test 4A - Reinforcement Failure Locations
Figure 11. Test 4A - Before and After Failure
FACTORS OF SAFETY AT CRITICAL STAGES

RATIO $D/D_o$ FOR PROPERLY DETAILED DEVIATION SADDLES

Figure 12. Average Factor of Safety for Critical Stages
COMPARISON OF REINFORCEMENT TYPES
RATIO D/D₀ FOR PROPERLY DETAILED DEVIATION SADDLES

Figure 13. Comparison of Factors of Safety for Reinforcement Types

*Ratio of epoxy coated reinforcement specimen strength to the uncoated reinforcement specimen strength
Figure 14. Simplified Analysis Models
Tension Ties Linking Deviation Force to Box Reinforcement

Strut-Tie Model for Top-Surface-Reinforcement Resistance

Figure 15. Strut-Tie Models
Figure 16. Plan and Elevation of Full Bottom Flange Deviation Saddle
Example Design Calculation- Straight Span - Two tendons

Tendon 2-19-0.5"Ø 270 ksi strands Aps=2.91 sq. in. (closest to web wall)
Tendon 1-12-0.5"Ø 270 ksi strands Aps=1.84 sq. in.

fc=6000 psi  Grade 60 reinforcement

Maximum Allowable Jacking Force=0.8(FPS)(Aps)
Tendon 2 =\(0.8\times(270 \text{ ksi})(2.91 \text{ sq. in.})=628.6 \text{ k}\)
Tendon 1 =\(0.8\times(270 \text{ ksi})(1.84 \text{ sq. in.})=397.4 \text{ k}\)

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<td>2</td>
<td>+2.93°</td>
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<td>1</td>
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<td>Total</td>
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Load Factor=1.7
\(\bar{\sigma}=0.9\)  (Tension)
\(\bar{\sigma}=0.85\)  (Shear)

Design tendon loops based on 89.9 k
\(F_u=\bar{\sigma}(A_s)(f_y)\)

\(A_s=F_u/(\bar{\sigma}f_y)=(89.9 \text{ k})(1.7)/(0.90)(60.0)=2.83 \text{ sq. in.}\)

Assume #4 bars \(A_s=0.20 \text{ sq. in.}\)

# of loops=2.83 sq. in./(2(0.20 sq. in.))=7.1  use 7-#4 loops each tendon
use 7-#4 closed outer stirrups

Shear friction check
\(V_u=(\bar{\sigma}A_s f_y-N_u)\mu\)  AASHTO Sec. 8.15.4.3

Area equal to two legs of 7 loops for each tendon and one leg of 7 outer closed stirrups

\(A_s=7(5)(0.20 \text{ sq. in.})=7.00 \text{ sq. in.}\)
\(N_u=(89.9 \text{ k})(1.7)=152.8 \text{ k}\)
\(\mu=1.4\)  (monolithically cast)
\(f_y=60 \text{ ksi}\)
\(\bar{\sigma}=0.85\)

\(V_u(\text{req'd})=(44.9 \text{ k})(1.7)=76.3 \text{ k}\)

\(V_u(\text{provided})=[(0.85)(7.0 \text{ sq. in.})/(60 \text{ ksi})-152.8 \text{ k}][1.4=286.9 \text{ k} > 76.3 \text{ k} \ O.K.\)
\(V_u(\text{max provided})=360A_{cv}=360(15.5 \text{ in.})(28 \text{ in.})/1000=156.2 \text{ k} > 76.3 \text{ k} \ O.K.\)

Figure 17a.  Design Example for Straight Span
Link bar vertical dimension = 3.75" + 3.56" + 2" + 2\((\tan 8.22\degree)\) + 1" + 2\((1/2\")) = 12.5" (out-to-out dimension) use 13"

Link bar horizontal dimension = 3.56" + (12") + (\tan 2.93\degree) + 1" + 2\((1/2\")) = 6.2" (out-to-out dimension) use 6"

Outer stirrup vertical dimension (out-to-out) = 13" + 3" = 16"

Development-90\degree hook

\[ l_{hb} = 1200(db)/(f'c)^{1/2} = 7.7" \]

ACI 12.5.3.2 Factor=0.7 (loops only)

\[ l_{dh} = 7.7\times(0.7) = 5.4" \] use 6" O.K.

**Figure 17b.** Design Example for Straight Span
Example Design Calculation - Curved Span - Two tendons

Provide full bottom flange width deviation saddle since small radius curved span

Tendon 2-19-0.6\(^{\text{°}}\)Ø 270 ksi strands \(\text{Aps}=4.09\) sq. in. (closest to web wall)
Tendon 1-19-0.6\(^{\text{°}}\)Ø 270 ksi strands \(\text{Aps}=4.09\) sq. in.
\(f'c=6000\) psi  Grade 60 reinforcement

Maximum Allowable Jacking Force = 0.8(fps)(Aps)
Tendon 2  = (0.8)(270 ksi)(4.09 sq. in.) = 883.4 k
Tendon 1  = (0.8)(270 ksi)(4.09 sq. in.) = 883.4 k

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Load Factor = 1.7
\(\phi=0.9\) (Tension)
\(\phi=0.85\) (Shear)

Design tendon loops based on 122.9 k

\(\text{Fu}=\phi(\text{As})(\text{fy})\)

\(\text{As}=\text{Fu}/(\phi\text{fy})=(122.9\text{ k})1.7/((0.90)(60.0))=3.87\) sq. in.

Assume #5 bars \(\text{As}=0.31\) sq. in.

# of loops = 3.87 sq. in. / (2(0.31 sq. in.)) = 6.2  use 7- #5 loops each tendon
use 7-#5 closed outer stirrups

No shear friction check required since full bottom flange deviation saddle provided

Figure 18a. Design Example for Curved Span
Section A-A-Maximum Deviated Tendon

Link bar vertical dimension = 4.5" + 4.31"/2 + (12")/(tan 6.0°) + 2" + 2(5/8") + 1" + 2(5/8") = 13.8" (out-to-out dimension) use 14"

Link bar horizontal dimension = 4.31" + (12")/(tan 4.0°) + 1" + 2(5/8") = 7.4" (out-to-out dimension) use 8"

Outer stirrup vertical dimension (out-to-out) = 14" + 3" = 17"

Development-90° hook

\[ l_{bh} = \frac{1200(d_b)(f'c)^{1/2}}{f_{cb}} = 9.7" \]

ACI 12.5.3.2 Factor = 0.7 (loops only)

\[ l_{ch} = 9.7"(0.7) = 6.8" \text{ use 8" O.K.} \]

Strut across bottom flange half the width of deviation saddle

Reinforce strut with nominal surface reinforcement in both directions

Deviation Saddle Elevation

Figure 18b. Design Example for Curved Span
Externally Prestressed Concrete Slab Bridges: Model Test Results

by C. Menn and P. Gauvreau

Synopsis: A research project is currently underway at the Swiss Federal Institute of Technology, Zurich, to establish the feasibility of an alternative structural system for short-span highway bridges. Concerns over the long-term durability of structural systems currently used in the 25 to 40 m span range provided the primary motivation for the study. The proposed system consists of a solid concrete slab which is externally prestressed. The external tendons are deviated at the third points of each span using struts. A 1:3-scale model bridge has been constructed and is currently being tested to verify the behaviour of the bridge under permanent, service and ultimate static loads, as well as dynamic and fatigue loads. The favourable results obtained thus far have confirmed the feasibility of the proposed structural system, and will serve as a basis for extending the concept to spans greater than 40 m in length.

Keywords: bridges (structures); composite construction (concrete and steel); concrete slabs; durability; models; prestressed concrete; scale (ratio); serviceability; tests; unbonded prestressing
Christian Menn has designed over eighty bridges in Switzerland, including some of the country's longest-spanning works. He holds a doctorate from the Swiss Federal Institute of Technology in Zurich, where he has been a professor since 1971.

Paul Gauvreau is a research associate at the Swiss Federal Institute of Technology in Zurich. He received his Master's degree in civil engineering from Princeton University in 1983. He worked as a bridge design engineer in the U.S. and Canada from 1983 to 1987.

DURABILITY AND THE STRUCTURAL SYSTEM

In recent years, the long-term durability of bridges has become a focus of concern for engineers throughout the world. An alarming number of bridges has fallen into severe disrepair as a result not only of inadequate inspection and maintenance, but also of shortcomings in design and detailing. In addition to the obvious task of repairing deterioration in existing structures, it is also imperative that engineers develop better ways of ensuring durability in the design of new structures.

Adequate protection against deterioration results from both careful detailing of individual bridge components as well as proper overall design of the structural system. The structural system can significantly improve a bridge's resistance to decay where the protection provided by detailing alone cannot be relied on. For example, fully sealed expansion joints in a multiple-span bridge help to prevent damage in the concrete and bearings located underneath. Leaking is possible, however, even in the best detailed expansion joint. A higher degree of protection is provided by eliminating as many of the joints as possible through the use of a continuous superstructure. A multiple-span bridge which is fully continuous between abutments can therefore be regarded as more intrinsically durable than a similar bridge composed of simply-supported spans.

In a similar manner, proper design and careful detailing of the superstructure cross-section can also significantly increase the bridge's resistance to decay. In the particular context of concrete
structures, the ideal cross-section with regard to long-term durability is one in which

- the surface area exposed to harmful environmental conditions is minimized,
- the proper placement, vibration and curing of the concrete are in no way hindered,
- adequate cover for the reinforcement is guaranteed, and
- every concrete surface is accessible for inspection and maintenance.

By evaluating the overall design of the section according to the above criteria, it is possible to establish the level of intrinsic durability of a given cross-section type.

Solid slabs have the highest intrinsic durability of any section type. For a given width of roadway, the total exposed surface area of the section is minimized. The arrangement of reinforcing and prestressing steel in a slab bridge permits problem-free placement and vibration of concrete. Specified cover can likewise be easily achieved at all locations. Access to every concrete surface is guaranteed due to the absence of interior surfaces. Solid slabs are, however, not normally economical for spans greater than 25 m in length.

On the other hand, the intrinsic durability of box girders, T-girders and other similar types of sections depends to a large extent on the dimensions of the section, and, consequently, on span length. The most critical factors in this regard are web thickness and, in the case of hollow sections, available interior clearance. Thin webs lead to a congested arrangement of reinforcing and prestressing steel which hinders the proper placement and vibration of the concrete. Insufficient interior clearance further complicates the concreting operations. Good workmanship, which is crucial to high-quality concrete, is very difficult to achieve when workers are unable to stand erect. Lost forms, which have been used in hollow sections of very limited clearance, permanently prevent access to concrete surfaces. Webs of sufficient thickness and adequate interior clearance will normally only be found in spans greater than 40 m in length.

The 25 to 40 m span range is therefore problematic with regard to long-term durability. These spans are too long for the economical use of solid slabs, yet
too short for the durable construction of box girders. The cross-sections traditionally used in this range of spans, i.e. box girders, T-girders (including precast girders) and voided slabs, all suffer from low intrinsic durability as a result of thin webs and, in the case of boxes and voided slabs, insufficient interior clearance.

AN ALTERNATIVE STRUCTURAL SYSTEM

The Swiss Federal Institute of Technology in Zurich is currently investigating the use of innovative composite structural systems for highway bridges. As a result of the low intrinsic durability of conventional concrete structural systems in the 25 to 40 m span range, a major phase of the study has focused on developing an alternative composite structural system which would guarantee both economy and durability in this range of spans.

External prestressing, in which unbonded tendons are located outside of the concrete, can improve the inherent durability of box and T-girders by relieving the congestion of reinforcing and prestressing steel in the webs. The benefits of external prestressing are rather dubious, however, in the particular case of box girders, since removing the tendons from the webs in no way helps to solve the problem of concreting and formwork removal operations under conditions of insufficient interior clearance. The external prestressing of boxes and T-girders was therefore not given further consideration.

A more promising solution resulted from the use of external prestressing in conjunction with solid slabs. External prestressing offers an efficient means of extending the economical range of slab bridges to spans of up to 40 m. The high intrinsic durability of conventional slabs, moreover, is in no way diminished by external prestressing.

Figure 1 shows the proposed implementation of the concept. The arrangement of slab, cables and struts is admittedly not new; bridges of similar appearance have been built as far back as the early nineteenth century.

Clearly visible in Figure 1 are the concrete slab, external tendons, and short structural steel posts located at the 1/3 and 2/3 points of each span. The posts deviate the external tendons and transfer the vertical component of their force to the slab. The
depth to span ratio of the slab, the dominant visual element, gives an overall impression of great slenderness.

The system is intended for multiple-span continuous bridges and viaducts. Its design is therefore adapted to a span-by-span construction sequence. Of primary importance in this regard is the prestressing concept, which, apart from ensuring good behaviour at both serviceability and ultimate limit states, reduces falsework requirements to a minimum. The tendons have been divided into two distinct groups: the external tendons, visible in Figure 1, and the internal tendons, located within the slab.

The internal tendons are continuous for the entire length of the bridge and are arranged in two parallel groups. According to the proposed construction sequence, they would be stressed immediately after hardening of the concrete, in order to enable prompt removal of falsework and formwork. The required area of steel is thus designed to resist dead load during the construction stage. After the bridge has been completed, these tendons contribute primarily to the slab's live load resistance in the secondary spans between the piers and the short steel posts.

The external tendons are coupled to short tendons which are cast into the slab over the supports. The external tendons below the slab are divided into three segments, visible in Figure 1: (1) a segment sloping down from the slab to the bottom of the steel struts in the left third of the span, (2) a horizontal segment between the struts in the middle third of the span, and (3) a segment sloping up from the struts to the slab in the right third of the span. Each segment is anchored at the base of the posts, thus eliminating the need for deviation saddles and allowing the entire tendon to be stressed at the base of the strut. After completion of the bridge, dead load is resisted primarily by the external tendons. The area of steel is calculated to ensure safety of the completed structure at ultimate limit state. The prestressing force is chosen to provide a nominal upward camber under permanent load, which results in an effective prestress of about 45 percent of the yield stress. An appropriately chosen combination of duct, grout, grease or wax will protect these tendons from corrosion. They are at all times accessible for inspection and maintenance. They can also be easily replaced, which further enhances the inherent durability of the system.
This prestressing concept makes possible a rational span-by-span construction sequence, a typical cycle of which is as follows:

1. Cast central portion of the superstructure on falsework, excluding the cantilevers.
2. Stress internal tendons.
3. Remove falsework and formwork.
4. Stress external tendons over the supports.
5. Erect vertical struts, install and stress external tendons (first stage).
6. Cast cantilever portions of the section on travelling forms, and stress transverse tendons.
7. Grout external tendons over the supports and internal tendons.
8. Install waterproofing and wearing surface.
9. Stress external tendons (second stage) and adjust bridge profile.

**SCALE MODEL TEST PROGRAM**

The structural behaviour of the proposed system has been thoroughly investigated. A major component of this work has been the testing of a large-scale model bridge, to confirm the results of the analytical models used, and to reveal further insights into the bridge's behaviour. Because the system is made up of relatively flexible elements, dynamic testing was also required. In addition, the model was used to study the behaviour of the structure in fatigue, with special attention paid to the external tendon anchors at the base of the steel posts.

The model is based on a prototype bridge of two 30 m spans, continuous over the intermediate support. The scale is 1:3, which results in two 10 m spans (Figure 2). The overall deck width of the model is 4 m. The typical cross-section, shown in Figure 3, consists of a concrete slab which is thickened to accommodate the internal tendons. The prestressing tendons are made up of 7 mm diameter wires with standard buttonhead anchors. The internal tendons, located entirely inside the slab, consist of two 9-wire tendons in each
web. The external tendons consist of 2 tendons of 14 wires each.

The model was constructed following the span by span sequence proposed above. Similitude of stresses between model and prototype was therefore ensured. For convenience, however, the entire concrete slab was cast simultaneously on falsework; travelling forms and transverse post-tensioning were not used. The sequence of the dead loads of the central portion of the slab, the cantilevers and superimposed dead load was taken into account by loading the model with concrete blocks at the appropriate times and locations. These blocks, visible in Figure 4, compensate for the reduced self-weight of the model, making possible a 1:1 correspondence between stresses in model and prototype.

Five principal aspects of the bridge's behaviour were investigated:

1. Behaviour Under Permanent Load

The bridge's behaviour under the effects of dead load, internal prestressing, and external prestressing was investigated during the first phase of the test program, to determine the long-term effects of creep and shrinkage. During a ninety day monitoring period following the completion of the bridge, regular measurements were taken of the vertical deflections in the slab and the forces in the external tendons.

The results of this phase are shown in Figures 5 and 6. The stress in the external tendons at the end of the ninety day period was equal to about 86 percent of the jacking stress. This value is consistent with those obtained from conventional post-tensioned structures. The net vertical slab deflection after ninety days was equal to about 1 mm, or $\frac{1}{10\,000}$ of the span length.

It can therefore be concluded that the effects of creep and shrinkage on the proposed structural system are not severe, and that the system's long-term behaviour poses no extraordinary design problems.

2. Dynamic Behaviour

This phase of the test program was undertaken to determine appropriate impact factors for live load.
It was conducted in two separate stages, whereby natural frequencies and damping were measured both before and after the application of service live load. In this way, it was possible to determine to what extent cracking in the slab influenced the dynamic behaviour of the system.

The natural frequency of the bridge was obtained from measurements of its amplitude in forced vertical vibration as a function of forcing frequency. A 100 kg oscillating mass was mounted under the slab at the midpoint of one of the spans. The amplitude of the mass was held constant at 0.01 m, while the forcing frequency was varied. The natural frequency of the cracked system was approximately 2.58 Hz. The response curve of the uncracked system was almost identical to that of the cracked system; the uncracked natural frequency was approximately 2.72 Hz. The effect of cracking on the dynamic behaviour of the bridge can therefore be neglected.

The natural frequencies measured for the model corresponded to a frequency of about 1.5 Hz for the prototype. This corresponds to a dynamic amplification factor of 1.48 for a single truck (1). For the more severe case of full live load on the bridge, however, the impact factor is approximately equal to 20 percent, which is consistent with impact factors for conventional structural systems in this span range.

3. Behaviour at Serviceability Limit State

This phase of the test program considered the combined effects of dead load, internal and external prestressing, and service live load. Its purpose was to gain insight into the structural system's behaviour at serviceability limit state. The live load was simulated using hydraulic jacks at six locations per span. The maximum jack force, $Q_{\text{max}}$, was equal to 29 kN.

This was most extensive phase of the entire experimental program. Each test consisted of loading a given combination of the jacks up to full design load, to simulate a desired live loading pattern. Forces in the external tendons and vertical deflections of the slab were recorded for all tests. The principal live load cases are as follows:

- full live load on both spans,
• full live load on one span only,

• partial live load applied to one side of the bridge centreline, to produce torsion,

• partial live load applied to one half of one span only (critical for longitudinal bending in the slab).

Although not strictly a live load case, an additional test was conducted to simulate the removal of one of the external tendons, to confirm that the tendons are indeed easily replaceable.

Figures 7 and 8 show the increase in external tendon force and slab deflections for live load symmetrically applied to both spans. It is evident from these figures that the behaviour of the system is linear up to maximum service load levels. The maximum increase in external tendon stress is equal to about 8 percent of the tendon's yield stress. (The tendon stress lies well within the upper and lower bounds given by the analytical lower and upper limits, respectively, of slab stiffness). The maximum measured increase in tendon stress, occurring when live load is applied to one span only, is equal to only 12 percent of yield stress. The external tendon forces due to live load are therefore relatively small.

4. Fatigue Behaviour

Of primary importance in this phase of the test program was the behaviour of the external tendon anchors, located at the base of the structural steel struts. The design fatigue live load, equal to about 33 percent of full service live load, was simulated dynamically by means of the same oscillating mass used for the dynamic tests.

The model underwent a full 2 million cycles of fatigue live load with no signs of distress anywhere in the structure. It can thus be concluded that conventional buttonhead prestressing anchors will be adequate for the proposed structural system. The reason for this favourable fatigue behaviour is due to the low stress range in the external tendons, equal to about 4 percent of the yield stress under design fatigue load.

5. Behaviour at Ultimate Limit State
The only test phase not completed at the time of writing is the investigation of the model's behaviour at ultimate limit state. The magnitude of the live load will be increased beyond service load until failure of the structure occurs. The purpose of this test is to determine the ultimate load of the system and the mode of failure. From this test, the inherent safety factor against collapse will be determined. Of particular interest will be the amount of plastic hinging required in the slab before yielding of the external tendons can occur.

FUTURE PROSPECTS

The model test program's excellent results have not only confirmed the bridge's feasibility, they have also laid the groundwork for externally prestressed concrete slab bridges of greater span lengths. The basic concept is by no means restricted to the 25 to 40 m span range; the intrinsic durability, good structural behaviour and rational construction sequence of the proposed structural system could also be made available to bridges spanning up to 60 m.

The main challenge in adapting the concept to longer spans will be to ensure adequate stiffness. This will be accomplished by replacing at least part of the prestressing steel in the external tendons by structural steel, whose axial stiffness is roughly five times greater than that of prestressing steel for a constant yield force. In the case of a non-prestressed lower support system, it would be necessary to control long-term deformations by means of fabrication camber and possible jacking of the structural steel during erection. Stiffness requirements during the construction stage will require some modification of the proposed construction sequence. It will be necessary to erect at least a portion of the lower support system before removal of the falsework.

Large shear deformations of the unbraced central panel were effectively prevented in the 25 to 40 m span range by the slab, which was sufficiently stiff over the relatively short interior spans. These deformations will be much larger for interior spans approaching 20 m. The use of X-bracing in the central panel will eliminate these undesirable deformations, while still retaining the slender slab, a definite visual asset.
Another modification to the original structural system which is being considered is the addition of compression members between the bottom of the struts and the intermediate piers, substantially increasing the lever arm for resisting negative moment. Preliminary studies have shown that this measure also contributes somewhat to increasing the overall stiffness of the system.

These enhancements to the original concept will be investigated in further detail using the scale model bridge, modified accordingly. We believe that this system holds great promise for a new generation of efficient, economical, and inherently durable structures.

REFERENCES


Fig. 1--Externally prestressed concrete slab bridge
Fig. 2--Scale model: longitudinal section and plan
Fig. 3—Scale model: typical section in span
Fig. 4--Scale model in laboratory
Fig. 5--External tendon stresses under permanent load

Fig. 6--Slab deflections at midspan under permanent load
Fig. 7--External tendon stresses under live load—both spans loaded

Fig. 8--Slab deflections under live load—both spans loaded
Behavior of 1/5 Scale Segmental Concrete Girders with External and Internal Tendons

by B.G. Rabbat and K. Sowlat

Synopsis. The behavior of two segmental concrete girders incorporating external tendons was compared to that of a similar girder with internal tendons. The girders were 31-ft-long and consisted of 11 match-cast segments. Test variable was the location of the tendon ducts. In the first girder, the ducts were embedded in the girder cross section. The ducts of the second girder were external to the concrete cross section except at pier segments and intermediate deviation diaphragms. The third girder was similar to the second except that portions of the external ducts were embedded in a second-stage concrete cast. The segments included multiple shear keys and were dry jointed. All ducts were grouted. Each girder was simply supported over a 30-ft span and loaded statically to destruction under a two point load. The first and third girders attained their respective flexural strengths predicted by the classic bending theory for monolithic girders with bonded tendons. The second girder exceeded the flexural strength predicted by the provisions of the AASHTO Specifications for members with unbonded tendons.

Keywords: bridges (structures); girders; post-tensioning; precast concrete; prestressed concrete; prestressing steels; research; scale (ratio); segmental construction; structural design; tests; unbonded prestressing
INTRODUCTION

This paper was prepared for the International Symposium on External Prestressing to summarize an experimental investigation conducted at the Construction Technology Laboratories, Inc. on girders with external tendons. More details about this program are available in Reference 1.

During the last decade, use of external tendons has emerged as an economic technique to post-tension segmental concrete bridge girders in the United States and Europe. External prestressing has also been used to rehabilitate and strengthen bridge girders. Advantages of external prestressing have been discussed by Virlogeux.2

OBJECTIVE

The objective of this investigation was to compare the behavior of dry-jointed precast segmental girders with external tendons with that of a similar girder with internal bonded tendons. The effects of potential damage to anchorages at girder ends due to a severe earthquake were to be evaluated through releasing the anchor wedges of selected tendons prior to loading the girders to destruction.

TEST PROGRAM

Test Specimens

The following three segmental girders were tested:

1. Bonded Tendon Girder with internal tendons
2. Unbonded Tendon Girder with external tendons
3. Modified Unbonded Tendon Girder similar to the second girder except that a second stage concrete cast was placed on the top of the bottom flange covering portions of the external tendon ducts for bond development.

Girder geometry and midspan cross sections are shown in Fig. 1. Profile of tendons is given in Fig. 2. At test time, concrete compressive strength of segments ranged between 5810 and 7660 psi (40 and 53 MPa). All tendons were grade 270 low
relaxation seven wire strands. More details about the test specimens are given in Reference 1. The test setup is shown in Fig. 3.

Test Procedure

Each girder was subjected to two loading cycles. In the first cycle, each girder was loaded in small increments until an inelastic behavior was observed and a midspan deflection of approximately 3 in. (76 mm) was reached. The girder was then completely unloaded. Anchor wedges for the top two strands were burned at both ends to simulate anchorage loss due to a severe earthquake. Burning of anchorages was a last minute addition to the test program. During the second loading cycle, each girder was loaded in increments up to destruction.

BEHAVIOR OF SPECIMENS

A plot of the midspan applied moment versus deflection for the first and second loading cycles for each of the three girders is given in Fig. 4. Peak moments (including effects of girder dead load and loading hardware) and corresponding midspan deflections for first and second loading cycles are summarized in Table 1. After unloading the girders following the first loading cycle, the anchor wedges of the top two strands were burned. Strand strains were monitored during this process. For the two girders with external tendons, strand strains indicated a loss of stress in the top two strands immediately after burning the anchorages. Loss of strand stress was attributed to slip of strands at the anchorages. With a loss of bond for the top two strands at the girder end regions, the effectiveness of these strands became questionable. This explained the loss of strength during the second loading cycle for the Unbonded Tendon and Modified Unbonded Tendon Girders as depicted in Fig. 4.

Bonded Tendon Girder

After burning the anchor wedges following the first loading cycle, the top two strands remained bonded. Tendon ducts were inside the section and were grouted. Transfer of prestress of the top two strands occurred in the end regions of the girder. The bonded tendon girder failed in a flexural mode. Distress started with a localized crushing of concrete along the top surface of the concrete at midspan. It was followed by strands breaking which led to a loss of load carrying capacity.

Unbonded Tendon Girder

During the first loading cycle, after exceeding the decompression moment, the dry joints in the constant moment region started to open. Diagonal cracks occurred at the first joints of the shear spans. These are the joints next to the loading points. Upon unloading the girder following the first
loading cycle, the joints closed completely. As noted earlier, after burning the anchorages of the top two strands, loss of prestress was observed for these strands. The short length of strand bonded within the pier segment (end segment) was inadequate for transfer of prestress. Loss of prestress resulted in decrease of the effective axial compression in the member. Further, the shear strength of the girder was reduced due to loss of the vertical component of prestress in the shear span. The Unbonded Tendon Girder failed in a shear compression mode.

**Modified Unbonded Tendon Girder**

Because of the secondary concrete cast placed on the bottom flange of this girder there was a delay in opening of the dry joints in the constant moment region. Diagonal shear cracks occurred at the first joints of the shear spans as observed with the Unbonded Tendon Girder. Following the first loading cycle and burning of the anchorages, the top two strands slipped at the girder ends. The failure mode during the second loading cycle was similar to that of the Unbonded Tendon Girder. A shear compression failure occurred.

**FLEXURAL STRENGTH ANALYSIS**

Flexural strength of prestressed sections can be calculated from equations given in the AASHTO Specifications\(^{(3)}\) or the ACI Code.\(^{(4)}\) Alternatively, principles of classic bending theory can be applied to compute the flexural strength of girders with bonded tendons. This theory states that plane sections remain plane after bending. A computer-aided analysis was performed to determine the flexural strength based on strain compatibility and equilibrium of internal forces. For the Bonded Tendon Girder, the calculated flexural strength was 7260 kip-in. (820 kN m). This compares with a flexural strength of 7320 kip-in. (827 kN m) computed according to the AASHTO Specifications\(^{(3)}\) and ACI Code\(^{(4)}\) provisions for bonded tendons. For the Unbonded Tendon Girder, if all tendons were assumed bonded for analysis purposes, the calculated flexural strength according to the classic bending theory was 7210 kip-in. (815 kN m). The difference in calculated strength is because of the difference in strand locations in the Bonded tendon versus Unbonded Tendon Girders. During the second loading cycle, the Bonded Tendon Girder exceeded the calculated strength by 5 percent. The Modified Unbonded Tendon Girder reached the calculated strength for bonded tendons during the first loading cycle.

The AASHTO Specifications\(^{(3)}\) provide the following equation to compute the average stress at ultimate in unbonded tendons.

\[
fsu^* = fse + 15,000 \text{ psi} \\
(fsu^* = fse + 103.4 \text{ MPa})
\]
where
\[ f_{su}^* = \text{average stress in prestressing steel at ultimate load, psi} \]
\[ f_{se} = \text{effective steel prestress after losses, psi}. \]

Based on this equation, the flexural strength for the Unbonded Tendon Girder was calculated at 4380 kip-in. (495 kN m) assuming six effective strands. Therefore the Unbonded Tendon Girder exceeded the calculated strength during the first loading cycle by 29 percent.

CONCLUSIONS

Based on the test results the following conclusions were drawn.

1. The Bonded Tendon Girder and the Modified Unbonded Tendon Girder reached the flexural strength predicted by the classic bending theory for bonded tendons.

2. The strength attained by the Unbonded Tendon Girder exceeded the flexural strength calculated according to the provisions of the AASHTO Specifications for unbonded tendons.

ACKNOWLEDGMENTS

The experimental investigation reported in this paper was sponsored by the South Carolina Department of Highways and Public Transportation. The test specimens were designed by Figg and Muller Engineers, Inc. of Tallahassee, Florida. The test specimens were manufactured and tested at the Construction Technology Laboratories, Inc., Skokie, Illinois.

REFERENCES


4. ACI Committee 318, "Building Code Requirements for Reinforced Concrete (ACI 318-83)," American Concrete Institute, Detroit, Michigan, 1983, 111 pp.
### Table 1 - Data Summary

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*Includes moment due to girder dead load and loading hardware
+Upon unloading

Metric Equivalents: 1000 kip-in. = 113 kN m; 1 in. = 25.4 mm
Fig. 1 - Girder Geometry and Tendon Locations at Midspan
Fig. 2 - Tendon Profiles
Fig. 3 - Test Setup
Fig. 4 - Applied Moment versus Midspan Deflection
Strength and Ductility of a Three-Span Externally Post-Tensioned Segmental Box Girder Bridge Model

by R.J.G. MacGregor, M.E. Kreger, and J.E. Breen

Synopsis. An experimental investigation was conducted to examine the service and ultimate-load behavior of segmentally precast box-girder bridges with external post-tensioning tendons. A primary interest of this study was to examine the effect of joint type (dry versus epoxied joints) on the stiffness, strength, and ductility of the structure. A three-span reduced-scale segmental box girder model was constructed, then tested in three stages. Flexural behavior was examined first, then shear tests were conducted on the damaged structure. Test results and observations are presented.

Keywords: box beams; bridges (structures); ductility; models; post-tensioning; precast concrete; prestressing steels; segmental construction; strength; structural design; tests; unbonded prestressing
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John E. Breen, FACI, holds the Nasser I. Al-Rashid Chair in Civil Engineering at The University of Texas at Austin. He is the past Chairman of ACI Committee 318, Standard Building Code, and is a former chairman of ACI-ASCE Committee 441, Reinforced Concrete Columns, and of ACI’s Technical Activities Committee.

INTRODUCTION

A significant number of precast segmental box girder bridges with external pre-stressing tendons have been constructed in the United States and Europe. Substantial economic savings have been indicated for this type of construction. However, questions have been raised as to how these bridges will behave when subjected to loads greater than service-level loads. It is not known to what degree the behavior of these bridges will resemble the behavior of monolithic, fully bonded or unbonded prestressed concrete girders. In addition, segmental box-girder bridges have been constructed with epoxy between the match-cast segments (epoxied joints) and also without any type of joinery material in the joints (dry joints). It is not known whether epoxied or dry joints have any influence on the stiffness, strength, or ductility of the structure.

A one-quarter scale three-span externally post-tensioned box girder bridge model was constructed in the Ferguson Structural Engineering Laboratory at The University of Texas at Austin. The model was load-tested to determine the level of strength and ductility that may be expected for precast segmental bridges with external tendons, current tendon anchorage and deviation details, and alternate joinery details. A description of the experimental program, evaluation of selected test results, and conclusions follow.

TEST PROGRAM

Description of Model Structure

The model bridge, shown in Fig. 1, was a three-span segmental box girder with geometric symmetry about the center of the bridge. Figure 2 shows a plan view and elevation of the structure. Each span contained ten typical segments, and over each support was a pier segment containing the anchorages for the prestressing tendons. Because the typical segments were match cast separately from the pier segments, a cast-in-place closure
A prestressing strip was provided between the typical segments and the pier segments at the ends of each span. Joints between typical segments were either epoxy filled or dry (no epoxy). Both the south and center spans contained epoxied joints.

Cross sections for typical segments and pier segments are presented in Fig. 3. The typical segment shape was chosen to give a span-to-depth ratio (18.75) and efficiency rating (0.60) typical of contemporary construction. A modified box section was developed for use in the model structure to provide access to the external tendons. Webs were shifted towards the center of the box to facilitate moving the draped external tendons outside the box. At midlength of each typical segment was a full-height diaphragm through which external tendons passed freely or were deviated as required.

Shear keys were used on segment faces to transfer shear across segment joints and to provide an interlock between match-cast segments. Keys were also provided in the flange regions, with their primary purpose being to assist in aligning the segments during erection. Details for the shear keys are shown in Fig. 4.

Concrete strength for typical segments was nominally 6000 psi at 28 days. In order to adequately resist anchorage forces in the pier segments, a concrete mix design with a minimum 28-day compressive strength of 10,000 psi was used.

A schematic of the post-tensioning tendons is shown in Fig. 5. Tendons were draped downward from high over the supports to just above the bottom flange near midspan. Theoretical tendon locations are shown for sections at the exterior support, midspan, and interior support. The structure was erected on traveling falsework using the span-by-span method of construction. Single-span tendons (1A, 1B, 2, 4A, and 4B) which contained 5-3/8" diameter strands on each side of the box, were used during erection of individual spans. Additional multi-span tendons (3 and 5) containing 2-3/8" diameter strands on each side of the box, were added after erecting span 2 and span 3. External tendons were pressure grouted in their plastic and metal pipe sheaths at deviation points.

In addition to the external tendons, internal tendons were provided in the top corners of the box and at the ends of the thin top flange. The additional tendons were provided to augment flexural and torsional capacity, and to control shear lag. All internal tendons had a straight profile and were anchored at the extreme ends of the structure. Each internal tendon consisted of two 3/8" diameter strands. All prestressing strand used in the model structure was Grade 270 low-relaxation strand.

**Instrumentation**

The reduced-scale bridge model was instrumented to measure reactions, deflections, and local deformations such as tendon strains and joint openings.

Reactions were measured at three of the four supporting piers. At each location, two load cells were used to measure the reaction beneath each of the webs. Measured reactions were used to provide a check of static equilibrium, and to provide information about load distribution in the continuous structure.

Vertical deflections were measured at each support and at three locations in each span using displacement transducers and dial gages. Horizontal displacements were also monitored at supports and at midspan of the span being tested.

External tendon strains were measured using resistance-type strain gages attached to individual strand wires at nine locations on each side of the three-span structure.
Strains were measured in each tendon in the inclined portions near the supports and also in the horizontal region at midspan.

Distortions along the height of critical joints were measured using displacement transducers and mechanical crack-width gages. Displacement transducers were attached to the tension flange of the box, and spanned across the joint. The crack monitors placed across bottom-opening joints were located at the top and bottom of the vertical portion of one of the webs and on one end of the bottom flange. For top-opening joints, the inclined tendons did not allow access to the web regions, so a single crack monitor was attached to the top flange.

**Load Tests**

A series of weights to compensate for model effects were applied during construction to bring the model to the correct dead load configuration.

The three-span model was then loaded to examine the complete range of flexural behavior for the dry and epoxy-jointed exterior spans. The model was also subjected to very low-level torsional loads and to loadings that were intended to mobilize the shear strength of the dry and epoxy-jointed spans. The test program consisted of three distinct load phases:

- structural characterization
- factored loads
- ultimate strength

The first phase of testing involved loading the structure to the design service live loads and then increasing loads to higher levels to establish the cracking loads at critical locations, or the decompression loads at critical joints or existing flexural cracks throughout the structure. In the second phase of testing, factored design loads were applied to the structure. In the final phase, the structure was loaded until the ultimate strength was attained. Failure of both the north and south span (dry vs. epoxy joints) was dominated by flexural behavior. Exploratory tests were then carried out on the damaged structure to investigate shear behavior at opening joints.

Each possible load configuration was analyzed using a limit analysis to determine the location of critical joints. The flexural and shear-test load locations were chosen so that the same joints were critical for both tests. If two different joints were critical for the flexure and shear tests, the desired critical shear mechanism may not have developed properly in the cracked epoxy-jointed span.

So that comparisons could be made with the AASHTO H11-20 truck load, it was necessary to determine the quantity of applied load that was equivalent to the reduced-scale service live loads. Because the tests were dominated by flexural behavior, moments at joint locations were used to convert between service loads and test loads. The applied "equivalent live load with impact" ($LL+I$) was chosen to provide the same moment at the critical joint as the maximum service-load moment calculated at any location. Maximum service-load moments were determined using influence diagrams developed for each joint.

A schematic illustrating the positions of loads during flexure and shear tests on exterior spans is shown in Fig. 6.
TEST RESULTS

Service Load Behavior

The measured deflected shape of the three-span structure is shown in Fig. 7 for a typical service live load application on the dry-jointed north span. The measured maximum service live load deflections, as determined in different load cycles, were \( L/5660 \) for the dry-jointed exterior span, \( L/6250 \) for the epoxy-jointed exterior span, and \( L/7500 \) for the epoxy-jointed interior span. The deflection in the dry-jointed exterior span was approximately 10 percent more than for the epoxy-jointed exterior span. This difference may be caused by a slightly smaller effective cross-section in the dry joints caused by differential shrinkage in the thin flanges of the precast segments.

The live-load tendon stress increases in the midspan region of the load span were measured to be less than 2 ksi in all spans. The stress response remained constant for five consecutive live load cycles indicating that the tendons did not slip at the deviators at service level loads.

Although the live-load stress range was small and slip was not apparent during live load cycles, there is need for research to assess the fatigue properties of external tendons at the deviation locations. The change in tendon force between two adjacent segments of an external tendon occurs by friction while under-going a concentrated angle change at the deviators (Fig. 8). The force transfer occurs over a short length under high lateral deviation pressures. The friction force combines with the lateral pressure to induce a high surface shear on the strand wires that are in contact with the deviation hardware. Also, repeated load cycles or the occurrence of a previous overload may increase the potential for slip at the deviators. The high lateral pressures and surface shears at the deviators and the potential for tendon slip indicates a possible fretting fatigue problem (1).

Cracking Load vs Decompression Load

A primary purpose for using epoxy at segment joints is to provide reserve capacity against joint opening for over-load conditions. In the epoxy jointed exterior span of the model, cracking occurred through the concrete adjacent to a midspan match-cast joint at an applied load of 5.4 multiples of live load including impact (5.4*(LL+I)). For subsequent applied load cycles, the load required to decompress the flexural tension fiber, and cause the crack adjacent to the epoxy joint to begin to open, was measured at an applied load of 2.6*(LL+I). This measured decompression load for the cracked south span was somewhat higher than the measured decompression load of 1.9*(LL+I) for the dry jointed north span. However, this was affected more by difference in effective prestress than joint type. If zero tension is used as the limit for service behavior, then the epoxy joints provided a potential factor of safety against joint opening of approximately 2.0. Similar behavior was also noticed in the epoxy jointed interior span. In setting design criteria, however, it should be realized that the true factor of safety might be less than this because of traffic overloads, calculation inaccuracies, actual in situ epoxy behavior, and fatigue behavior of the concrete/epoxy joint. It would therefore be prudent to specify a small residual compressive stress in the extreme flexural tension fiber for epoxy-jointed segments without bonded reinforcement crossing the joint. In dry joints, without bonded reinforcement crossing the joint, the beneficial tensile capacity offered by the epoxy is not present, so higher design residual compressive stresses are recommended.
Factored Loads

After completing the service load tests the three-span structure was loaded with additional weight to simulate the factored dead load condition of 1.3*DL. Each of the exterior spans of the structure was then individually loaded with the factored design live load with impact, 2.86*(LL+1). Factored load tests were not conducted on the interior span. The structure behaved linearly throughout the load cycle with a slight reduction in stiffness when the midspan joint decompressed. At these higher load levels the measured maximum factored live load deflections, as determined in different load cycles, were L/1764 for the dry jointed exterior span and L/2310 for the epoxy jointed exterior span. In this case the deflections in the dry jointed span are approximately 25 percent more than in the epoxy-jointed exterior span, with the difference caused by the reduced effective cross-section in the dry joints and the tensile capacity in the uncracked regions of the epoxied joints.

The factored-load tendon stress increases in the midspan region of the loaded span were measured to be less than 5 ksi in all spans. The tendons did not appear to slip at the deviators for any of the factored load cycles.

Mechanism Behavior

The applied load is plotted against the resultant midspan deflection for the ultimate load test of the dry jointed north span, in Fig. 9. The deflections represent the net deflection of the structure, at the location shown on the schematic, after adjustment for support deflections. The deflections increase linearly with applied load up to the decompression load, Pd. As the midspan joints begin to open, stiffness reduces, and deflections increase at an escalating rate. The stiffness continues to decrease until the support joint opens and a mechanism forms. For load levels higher than the “mechanism load”, Pm, the stiffness remains relatively constant with slight decreases as the ultimate strength is approached.

The measured deflected shapes of the three-span structure with factored dead load (1.3*DL) and increasing levels of applied load are shown in Fig. 10. At the applied service live load, 1.0*(LL+1), and the applied factored design load, 2.9*(LL+1), the deflections are small and the deflected shape appears as a smooth curve. The deflected shape remains smooth until the midspan joints open widely at 3.0*(LL+1). Beyond this load, “hinging” occurs at the midspan joints, and the midspan deflections increase considerably. When the support joint opens at 4.8*(LL+1) the mechanism forms and deflections begin to increase very rapidly. Due to reduced flexural requirements, the center span has less post-tensioning than the exterior spans. The support joint therefore opened on the interior side of the interior pier segment. The final deflected shape of the structure clearly illustrates the mechanism behavior of the structure at ultimate load levels.

Because the external tendons are bonded to the concrete section only at discrete locations along the span, large concentrated rotations must occur at opening joints to develop the large tendon elongations required for increased tendon stresses. These rotations allow the internal forces to redistribute to stiffer uncracked regions. This is apparent from the reaction and joint-moment data for the flexural test of the north span, shown in Fig. 11. If no redistribution had taken place, these plots would be linear. The changes in slope indicate redistribution of moments and the related reaction is occurring. The initial slopes reflect the expected behavior from an elastic analysis. As the midspan joints begin to open at the decompression load, the resultant loss in stiffness causes a larger portion of
the additional load to be carried at the interior support. As loading is further increased, the support joint opens causing a reduction in stiffness at the support as reflected by the increased reaction rate. The internal forces then redistribute back towards the midspan region with the ultimate distribution of internal forces being controlled by the relative stiffness of the support and midspan regions.

This redistribution of internal forces, caused by "hinging" at the critical joint locations, will also cause redistribution of the secondary prestress forces near ultimate load levels. The secondary prestress forces are caused by geometric constraints on the entire structure when the tendons are initially stressed (2). To develop the required tensile forces with external tendons, large rotations must occur at the segment joints. As the joints "hinge" and the mechanism forms, the forces from the initial geometric constraints dissipate. If the segments are detailed to allow large rotations to occur at the segment joints, then the geometric constraints will no longer be valid. The geometric constraints, and the corresponding secondary forces therefore affect the service load behavior, with the ultimate condition approaching the plastic mechanism behavior.

As loads are increased beyond service levels, the tendon stresses exhibit several stages of behavior, shown in Fig. 12a. The concrete stress profile at the critical opening joint is shown in Fig. 12b for important stages of tendon stress development. Initially, before the joints begin to open, the tendon-stress increases are linear with the applied load. The tendon stresses remain linear until the neutral axis at the opening joint reaches the level of the tendon, Point B, at an applied load that is slightly greater than the decompression load, $P_d$. Beyond this load, the tendon stresses increase slowly at first as the increased moments are resisted primarily by an increased internal-force lever arm. When the resultant concrete compressive stresses are concentrated in the top flange of the section, Point C, then additional moments must be resisted by increased tendon forces. To develop the required tensile forces with external tendons, large rotations must occur at opening joints resulting in increased deflections and joint openings.

The applied-load stresses for a typical tendon during the flexural strength load cycle are shown in Fig. 13. Tendon strain measurements were made at the exterior and interior ends of the span, joints (1,2) and (9,10) respectively, and at midspan, joint (5,6). The midspan tendon stresses remained linear with applied load up to approximately $1.8*(L+1)$ when the concrete section had decompressed to the level of the external tendons. This load is slightly higher than the measured decompression load of $1.4*(L+1)$. The tendon stresses increased slowly at first until the midspan joints opened at $3.0*(L+1)$. At this load level the resultant compressive stress had concentrated in the top flange and additional moments were resisted by a direct increase in tendon stress. Subsequently, as the support joint opened, at approximately $4.8*(L+1)$, midspan moments increased and the rate of tendon stress development also increased.

At an applied load of $5.0*(L+1)$ the tendon began to slip from the interior end towards the midspan region. Slip also occurred from the exterior end at approximately $6.2*(L+1)$. The tendon slipped through the deviator when the change in tendon force exceeded the maximum friction capacity. Substantial tendon slip was noticed in all tendons at all locations for ultimate load levels.

The increase in midspan tendon stresses corresponding with flexural strength ranged from 35 to 60 ksi in the midspan region and from 15 to 20 ksi at the critical support joint. The ACI formula for unbonded post-tensioning tendons (3) was used to predict the ultimate tendon stresses at midspan and at the critical support joint. The
average measured tendon stress increases in the midspan regions were accurately predicted by the ACI formula with measured-to-predicted ratios of 0.98 in the north dry span and 1.03 in the south epoxied span. This result is reasonable if it is remembered that the tests used to develop the ACI formula (5) were conducted on specimens with short span lengths approximately equal to that used in the model bridge. The AASHTO formula (4) for unbonded tendons predicts lower stress increases, but implicitly assumes much longer spans. The slightly higher stress increases in the epoxied span may be due to the concentration of rotation at a single joint causing larger induced deformations in the tendon.

The average measured tendon stress increase at the critical support joint was overestimated by the ACI formula with measured to predicted ratios of 0.61 at the north interior support and 0.69 at the south interior support. At these locations the effective depth of the external tendons is reduced because of the drape from the support. An increase in the ratio of the free length of tendon segments to the effective depth of tendons leads to reduced stress development under applied loads.

The maximum midspan stress that was achieved in the model tendons was affected by the load level at which slip began. If tendon slip began at a low load level then the ultimate midspan tendon stress was low. Conversely, if slip did not occur until higher load levels then the ultimate midspan tendon stress was increased. Therefore, before prototype extrapolation can be made, additional information is required to determine the force transfer mechanism at deviators.

A final comment can be made about the overall performance of the structural system. The maximum applied-load moment at the center of the north span, for an applied-load equal to the design service live load (DL+(LL+I)), was approximately 50 ft-kips. When the ultimate flexural strength was reached in the north span, at an applied load of 1.3*DL+6.8*(LL+I), the maximum applied-load moment at the same midspan location was approximately 250 ft-kips. With the additional factored dead load, 0.3*(DL), causing a midspan moment of approximately 30 ft-kips, the overall factor of safety above the service load condition, with respect to the midspan moment, is approximately (250+30)/50 or 5.6.

**Joint Behavior**

The local behavior of the segments near an opening joint was affected by the amount of shear that was being transferred across the joints. In the flexural tests, with the load applied as a series of forces along the longitudinal axis of the structure, small shears were transferred across the critical opening joints. In this case the concentrated rotations occurred either at the joints in the dry span or at a crack adjacent to the precast joint in the epoxied spans. At ultimate load levels the joint/crack had opened into the top flange of the girder in both the dry-jointed and epoxy-jointed spans.

The local force transfer mechanism in the segments adjacent to the opening joints when the flexural strength was reached is shown schematically in Fig. 14a. The joint/crack had opened into the top flange causing the load to arch across the segment joint. The small shears that are transferred across the open joints at this stage are carried by the vertical component of the "arch force" at the joint. The segment reinforcement transfers the shears from the load point to the edge of the segment, and then the arch action transfers the force across the joint.

In the shear tests, a concentrated force was applied to the structure so that significant shear would be transferred across opening joints. In this case, after the joint
had opened up through the bottom flange, an inclined crack formed from the load point to the bottom of the web at the edge of the segment, as shown in Fig. 14b. As load was increased to ultimate levels, the concentrated rotations occurred at the inclined crack leaving the joint region in firm contact. This was true for both the dry-jointed and the epoxy-jointed spans.

The local force transfer mechanism in the segments adjacent to the opening joints when capacity was reached is shown schematically in Fig. 14b. A compressive strut formed from the load point to the lower corner of the segment. The segment web reinforcement transmitted this force across the inclined crack to the top of the segment. The shear force was then transferred across the joint utilizing much of the depth of the webs.

The reinforcement for the concrete segments near opening joints must be properly detailed to allow the large rotations required for tendon stress increases. Local truss mechanisms, such as shown in Fig. 14, should be developed for the critical segments to ensure that force transfer can be made across the joints. The bottom longitudinal reinforcement must be anchored close to the opening joint and must resist the horizontal component from the transient shears (Fig. 14c) plus an inclined strut running from the load point to the bottom corner of the segment. The web reinforcement must be able to resist the transient shear from global loads plus the vertical component of the inclined strut. The web reinforcement must be anchored under the bottom longitudinal reinforcement and high in the section so that anchorage is maintained when the neutral axis shifts to the top flange of the segment.

CONCLUSIONS

Several important observations have been made concerning the full range of behavior of segmental box-girder bridges with external post-tensioning tendons.

1. The structure was extremely stiff for service load conditions, with live-load deflections of about $L/6000$ in the exterior spans and $L/7500$ for the interior span. Tendon slip was not noticed during service load cycles. Additional information is required to assess the problem of fretting fatigue at the deviators of external tendons.

2. The cracking load in the epoxy joints was approximately twice the load required to decompress the flexural tension fiber and begin to open a previously cracked joint. Cracking occurred through the concrete adjacent to the epoxied joint.

3. The structure exhibited linear behavior to load levels higher than the factored design ultimate load.

4. The maximum applied-load moment in the north span, when flexural capacity was reached, was approximately 5.6 times the maximum applied service-load moment in the north span. This indicates that the applied midspan moment has an overall factor of safety above the service load condition of approximately 5.6.

5. Large concentrated rotations are required at opening joints to cause tendon stresses to increase with the applied load. These rotations allow the internal forces to redistribute to stiffer regions. The secondary prestress forces also redistribute as ultimate load levels are reached.

6. The applied-load tendon stresses at midspan of the loaded span, corresponding to the flexural capacity of the girder, were accurately predicted by the ACI formula
for unbonded prestressing tendons. The AASHTO formula for unbonded tendons underestimated the ultimate tendon stresses at midspan. The applied-load tendon stresses at the critical support joint, corresponding to the flexural capacity of the girder, were overestimated by the ACI formula and underestimated by the AASHTO formula. Additional information is necessary, however, before extrapolations can be made to the prototype structure.

7. Tendon slip was noticed at the deviators in all the tendons at all locations during the ultimate strength cycles. The tendons also slipped during cracking and joint-opening cycles.

8. The local transfer of forces across opening joints depended on the level of shear being transmitted across the joint. For opening joints with small shear transfer, the joint/crack opened in a flexural mode into the top flange of the structure with the concentrated rotations occurring at the joint. For opening joints with large shear transfer, an inclined crack formed from the load point to the lower corner of the segment adjacent to the joint. The concentrated rotations occurred at the inclined crack.

A more complete discussion of the test setup, procedures, observations, and conclusions are available in Reference 6.

ACKNOWLEDGEMENTS

This paper is based on research sponsored by the Texas State Department of Highways and Public Transportation and the Federal Highway Administration. In addition, substantial financial support for the research program was provided by the National Science Foundation under Grant ECE-8419430. All opinions and conclusions expressed are those of the authors and do not necessarily represent the views of the sponsors.

The authors would like to particularly acknowledge the contributions from Mr. Alan Matejowsky of the TSDHPT who provided valuable suggestions and practical insight throughout all phases of the research project. In addition, the authors would like to acknowledge the guidance and assistance provided by local industry in development and construction of the model bridge structure. In particular, the assistance and cooperation of Prescon Corporation of San Antonio and Ivy Wire and Steel of Houston were especially appreciated. Finally, the hard work and personal contributions by Mr. Elie Ilomsi of Prescon Corporation were greatly appreciated.
REFERENCES


3. American Concrete Institute Committee 318, "Building Code Requirements for Reinforced Concrete (ACI 318-83), American Concrete Institute, Detroit, Michigan.


FIG. 1 - BOX-GIRDER BRIDGE MODEL
FIG. 2 - MODEL DIMENSIONS
NOTES:
SPAN to DEPTH RATIO = 18.75
EFFICIENCY FACTOR = .60

a. TYPICAL SPAN SEGMENT

b. PIER SEGMENT

FIG.3 - MODEL CROSS-SECTIONS
FIG. 4 - SHEAR KEY DETAILS
FIG. 5 - SCHEMATIC POST-TENSIONING LAYOUT
FLEXURAL TESTS

SHEAR TEST

FIG. 6 - TEST-LOAD SCHEMATICS
FIG. 7 - TYPICAL SERVICE LOAD DEFLECTIONS
FIG. 8 - DEVIATOR SCHEMATIC
APPLIED LOAD vs DEFLECTION

Flexural Strength Tests of North Span - Ultimate Cycle

SUPPORT JOINT OPENS
-MECHANISM FORMS

MIDSPAN JOINTS OPEN

FIG. 9 - APPLIED-LOAD vs DEFLECTION
DEFLECTION PROFILE

Flexural Strength Tests of North Span – Ultimate Cycle

1.3*DL+(LL+I) 1.3*DL+2.86*(LL+I)

-4 en cp

1.3*DL+3.0*(LL+I)

0

MIDSPAN JOINTS OPEN

-8

1.3*DL+4.8*(LL+I)

-12

SUPPORT JOINT OPENS

-16

1.3*DL+6.8*(LL+I)

FLEXURAL STRENGTH

0 25 50 75

Location (ft from N.E.)

FIG. 10 - ULTIMATE LOAD DEFLECTIONS
FIG. 11 - REACTIONS AND JOINT-MOMENTS
FIG. 12 - TENDON STRESS RESPONSE TO APPLIED LOAD

a. TENDON STRESS RESPONSE
b. KEY POINTS FOR TENDON STRESSES
CHANGE IN TENDON STRESS vs APPLIED LOAD

Flexural Strength Tests of North Span - Tendon 1b - Ultimate Cycle

FIG. 13 - TYPICAL TENDON STRESS RESPONSE
FIG. 14 - SHEAR MECHANISMS AT OPENING JOINTS
A New Methodology for the Analysis of Beams Prestressed with External or Unbonded Tendons

by A.E. Naaman

Synopsis: A simple methodology for the solution of beams prestressed or partially prestressed with external or unbonded tendons in the linear elastic cracked and uncracked range of behavior is described. It leads to equations allowing the computation of stresses in the concrete section, the tensile reinforcing steel, the compression reinforcing steel, and the prestressing steel. In particular, it is shown that the stress in unbonded tendons is a function of the applied loading, the steel profile, and the ratio of the crack width (or crack band width) to the span. These factors can all be accounted for through the use of a strain reduction coefficient $\Omega$ for the uncracked range of behavior and a similar coefficient $\Omega_C$ for the cracked range of behavior. It is shown that, when the strain reduction coefficients $\Omega$ and $\Omega_C$ are taken equal to unity, the solutions developed here revert to the solutions developed earlier for partially prestressed beams with bonded tendons.

Keywords: beams (supports); cracking (fracturing); flexural strength; partial prestressing; prestressed concrete; prestressing; serviceability; structural analysis; structural design; unbonded prestressing
External prestressing implies the use of prestressing tendons outside the concrete section of a structural concrete member. These tendons are primarily designed to ensure horizontal prestressing of the structure and generally represent only a portion of its total reinforcement. The remaining reinforcement may consist of reinforcing steel, prestressing steel, or a combination of them. External prestressing is being increasingly considered in the construction of new concrete structures, particularly bridges (1,2), and is a primary method for the rehabilitation and strengthening of old structures (3).

Externally prestressed tendons are not bonded to the concrete. Thus, from an analysis and design viewpoint, they can be treated as "unbonded tendons", assuming that secondary effects are negligible. The use of unbonded tendons may not only imply ungrouted steel tendons, but may also refer to the use of unbonded polymeric tendons. Therefore from the analysis, design and constructional aspects, external prestressing, unbonded tendons and polymeric tendons are part of the same family of problems and could be addressed by the same general solution.

The use of external prestressing to rehabilitate a reinforced concrete bridge leads to a structural system referred to as partial prestressing. Although recent years have seen extensive advances in the analysis and design of partially prestressed members(4 to 7), little information exists on how to accommodate in the analysis unbonded or external tendons.

As the use of external or unbonded tendons poses some uncommon problems in analysis or design, rational methods must be developed to deal with such problems. Rational analytic solutions would allow the accurate determination of stresses and strains at any section along the member and verification of serviceability and strength limit states such as cracking, fatigue, long term deflections and the like. Recognizing the unusual nature of the behavior of beams prestressed with unbonded tendons, several studies have attempted in the past to provide clarifications to such behavior and to address various aspects of their analysis and design under service and ultimate loads (8 to 18).

The main objective of this paper is to present a rational, simplified methodology for the analysis of beams prestressed with external or unbonded tendons throughout their linear elastic cracked and uncracked range of behavior. Particular emphasis is given to determining stresses and strains in the constituent materials, steel and
concrete, for commonly applied loadings and tendon profiles. It is pointed out that the main feature of the proposed method is to reduce the analysis of beams with unbonded tendons to that of beams with bonded tendons through the use of simple predetermined coefficients.

FLEXURAL ANALYSIS OF BEAMS

A schematic representation of the moment deflection relationship of a beam prestressed or partially prestressed with external or unbonded tendons is shown in Fig. 1. This curve could also represent the moment curvature relation of a beam section. The curve can be conceptually divided into several parts: part AB corresponds to the linear elastic uncracked state of behavior, part BC illustrates the onset of cracking, part CD represents the linear elastic cracked range, part DE corresponds to the cracked nonlinear range of behavior, point E represents the nominal or ultimate resistance, and point F the failure point. The curve of Fig. 1 describes the range of possible behavior. In theory a sudden change in deflection occurs at the onset of cracking (segment BC); however the real behavior shows a more gradual change in slope from AB to DE.

Actual design of beams with external or unbonded tendons may or may not allow cracking under service loads. However, as loads are random in nature and overloads are quite common, the cracking state should be considered in any comprehensive evaluation. Moreover, cracking is allowed by the ACI Code in partially prestressed members using bonded tendons, and in slabs with unbonded tendons provided sufficient amount of non-prestressed conventional steel is added. In U.S. practice, both service load design (i.e. working stress design or allowable stress design) and ultimate strength design are required for prestressed concrete structures. Service load design implies linear elastic behavior in the uncracked and cracked range. A summary of the analysis procedure suggested here for the linear elastic response is given below.

Linear Elastic Analysis in the Uncracked State

This state is represented by segment AB in Fig. 1. The material components, steel and concrete, are assumed to work in their linear elastic range of behavior. However, no bond exists between prestressing tendons and concrete.

It is commonly assumed that, in this range of behavior, the change in the prestressing force F due to applied loading is negligible. Thus F is assumed to remain constant and the analysis of the section for stresses and strains is similar to that with bonded tendons.

If for the purpose of exact analysis or accuracy, the change in the prestressing force with applied loading is needed, then the analysis of a prestressed beam under load can be much more involved than that assuming constant F particularly if the tendons are unbonded. For the case of bonded tendons a complete solution assuming a variable value of F has been already described in an earlier publication (19). For unbonded tendons the methodology described below, consists of reducing the
problem to the case of bonded tendons through the use of a strain reduction coefficient $\Omega$. It can be shown that the coefficient $\Omega$ depends only on the steel profile and type of loading, and needs to be determined only once for common loading and tendon configurations.

The following assumptions are made for the analysis:

- The materials are linear elastic in the range of behavior considered,
- linear strain distribution is assumed along the concrete section, and
- second order effects, if any, for external tendons are negligible.

It is also assumed that the critical design section is the midspan section at which the maximum eccentricity of the tendons is assigned. Simply supported beams are considered first.

The following equations provide a methodology to determine the stress change in the prestressing steel for any moment larger than the dead load moment, $M_D$, and smaller than the cracking moment, $M_{cr}$.

$M_D < M < M_{cr}$ (segment AB of Fig. 1)

The reference state is defined as the state of loading corresponding to the combined action of the effective prestressing force (after losses) and the dead load moment. The corresponding stress diagram along the section is shown in Fig. 3. Thus:

For $M = M_D$

$$f_{ps} = f_{pe} \quad (1)$$

and

$$F_e = A_{ps} f_{pe} \quad (2)$$

where $f_{ps}$ is the stress in the prestressing steel at any loading state, $f_{pe}$ is the effective stress in the prestressing steel, $F_e$ is the effective prestressing force, and $A_{ps}$ is the cross sectional area of the steel.

For a moment larger than the dead load moment and smaller than the cracking moment, the following equation can be written (Fig. 3):

$$f_{ps} = f_{pe} + \Delta f_{ps} \quad (3)$$

where $\Delta f_{ps}$ represents a change in stress in the prestressing tendons. For bonded tendons:

$$\Delta f_{ps} \text{ bonded} = E_{ps} \Delta \varepsilon_{ps} \text{ bonded} \quad (4)$$

where $\Delta \varepsilon_{ps}$ represent the strain change in the prestressing steel in the section considered due to an increment in bending moment $\Delta M = (M - M_D)$. It is also equal to the strain change in the concrete at the level of the steel at the section considered, that is:

$$\Delta \varepsilon_{ps} \text{ bonded} = \Delta \varepsilon_{cps} \text{ bonded} \quad (5)$$
If the section of maximum bending moment is being analyzed, then $\Delta \varepsilon_{cps}$ is also the strain change in the critical design section (here the midspan section), $(\Delta \varepsilon_{cps})_m$.

For external or unbonded tendons the change in stress in the tendons is assumed given by:

$$\Delta \varepsilon_{ps}^{unbonded} = \frac{\Delta \varepsilon_{ps}}{\text{average}}\quad (6)$$

where $(\Delta \varepsilon_{ps})_{\text{average}}$ is the average strain increase in the external tendon over the length of the member. Let us define the strain reduction coefficient $\Omega$ as:

$$\Omega = \frac{(\Delta \varepsilon_{ps})_{m\,\text{unbonded}}}{(\Delta \varepsilon_{ps})_{m\,\text{bonded}}} = \frac{(\Delta \varepsilon_{cps})_{\text{average}}}{(\Delta \varepsilon_{cps})_{m\,\text{bonded}}}\quad (7)$$

where the subscript $m$ implies the midspan section or the section of maximum moment.

Note that for bonded tendons, $\Omega$ equals 1. For unbonded tendons $\Omega$ can be calculated in the most general form from the following equation:

$$\Omega = \frac{2}{M_{\text{max}} (e_o)_{\text{max}}} \int_0^{\frac{l}{2}} M(x) e_o (x) \, dx\quad (8)$$

Applying the above equation to the loading cases and profiles shown in Fig. 2 leads to the values of $\Omega$ listed in Table 1.

The use of the coefficient $\Omega$ can be integrated in the analysis of the uncracked section to generate expressions for the stresses and strains in the steel and concrete at the critical section (i.e. section of maximum moment and maximum eccentricity). The solution equations are given in Table 2 and apply to any state of loading between the reference state and the cracking state (Fig. 3) assuming linear elastic uncracked section behavior. It can be observed that for $\Omega = 1$, these equations revert to the equations given in (19) for bonded tendons.

**Linear Elastic Analysis in the Cracked State**

This part of the analysis covers segment CD of the moment deflection curve described in Fig. 1.

If the applied moment exceeds the cracking moment, a crack will appear in the midspan region. For the purpose of analysis, assume that only one crack will form at the section of maximum moment and let us analyze the beam assuming that it is divided into two parts, one uncracked portion with moment of inertia $I_g$ and one cracked portion with moment of inertia $I_{cr}$ (Fig. 4). The cracked portion has a length $l_c$ while the uncracked portion has a length $(l-l_c)$. In a manner similar to what was
done in the previous section, one can define a bond coefficient \( \Omega_c \) which represents the ratio of average strain change in the unbonded tendon to the strain change in the concrete at the level of the steel at the section of maximum moment. This last item is also equal to the strain change in the equivalent bonded tendon assuming a cracked section. It can be shown that, for symmetrical loading and tendon profile, the value of \( \Omega_c \) can be obtained in the most general manner from the following equation:

\[
\Omega_c = \frac{\Omega_{cr}}{I_g} + \frac{2}{l} (1 - \frac{\Omega_{cr}}{I_g}) \int_{0}^{l/2} M(x) e_0(x) \, dx \frac{M(x)}{M_{max}(e_0)_{max}}
\]

where \( M_{max} \) and \( (e_0)_{max} \) represent the applied external moment and the eccentricity of the tendons at the section of maximum moment (here the midspan section).

The value of \( \Omega_c \) has been calculated for the various combinations of loadings and tendon profiles described in Fig. 2. The solutions obtained are summarized in Table 3. It can be observed that the value of \( \Omega_c \) depends on:

1) the value of \( \Omega \) for the uncracked state,
2) the ratio of cracked moment of inertia at the section of maximum moment to the gross moment of inertia, and
3) the ratio of length of the cracked zone to the span length.

To illustrate how \( \Omega_c \) varies with these variable, a plot of \( \Omega_c \) for a typical loading and tendon profile is shown in Fig. 5. It can be observed that, while \( \Omega_c \) is greatly influenced by the ratio \( I_{cr}/I_g \), it is not as sensitive to the value of \( I_c/l \) at small values of \( I_c/l \). Indeed one can assume that as a first approximation, the value of \( \Omega_c \) is equal to the value of \( \Omega I_{cr}/I_g \). Going back to Table 2 where the values of \( \Omega_c \) are given, it can be observed that \( \Omega_c \) can be approximated in all cases by \( \Omega I_{cr}/I_g \) provided \( I_c/l \) is small. This is indeed the case, since \( I_c \) can be seen as the width of the crack at midspan or the width of a plasticized region with smeared cracking in the midspan region. Thus, for practical design purposes:

\[
\Omega_c = \Omega \frac{I_{cr}}{I_g}
\]

The mathematical equations expressing equilibrium, compatibility and stress-strain relationships assuming that the constituent materials remain in the linear elastic range of behavior, can be set similarly to the case of the analysis of a cracked section containing bonded prestressing steel and conventional reinforcing bars. The strain compatibility equations will contain the coefficient \( \Omega_c \). These equations can then be solved. The general solution equations are given in Appendix I with some notation explained in Fig. 6. The figure shows the strain diagrams along the section for the reference state and the current state of
loading. Also shown are the corresponding strain in the prestressing steel.

The first equation of Appendix I is a cubic equation in c where c is the depth to the neutral axis (zero stress point) of the elastic cracked section. As $\Omega_c$ must be estimated from Eq. 10, and as $l_{cr}/l_g$ depends on c, some iteration in the solution of Equation 1 may be needed. The following computational steps can be followed: assume a value of $l_{cr}/l_g$, determine c from Equation 1, compute corresponding $l_{cr}$, and check if the assumed $l_{cr}/l_g$ is acceptable. Otherwise, iterate. Once an acceptable value of c is obtained, the corresponding stresses in the constituent materials, steel and concrete, can be determined from the equations of Appendix I. In particular the stress in the unbonded prestressing steel can be calculated for any applied moment larger than the cracking moment ans smaller than the moment leading to inelastic behavior of one of the components materials. Note that the solution equations given in Appendix I cover prestressed and partially prestressed rectangular and T sections with and without compressive reinforcement. When the value of $\Omega_c$ is taken equal to 1, these equations revert to the case of beams prestressed with bonded tendons and become identical to the solution equations derived earlier in (19).

CONCLUDING REMARKS

The methodology described in this paper allows for a rational yet simplified analysis of beams prestressed with external or unbonded tendons in the linear elastic cracked or uncracked range of behavior. It provides a link between the case of bonded tendons and the case of unbonded tendons through the use of strain reduction coefficients $\Omega$ and $\Omega_c$ which can be computed only once for common combinations of loadings and tendon profiles. Although the solution described above allow for the analysis of several serviceability limit states, it is conceivable that the concept of strain reduction coefficient $\Omega$ can be extended to cover the ultimate flexural strength limit state (20). However, substantial additional research is needed to ascertain the validity of application of such methodology to the ultimate strength limit state.

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REFERENCES


8. ACI-ASCE Committee 423, "Recommendations for Concrete Members Prestressed with Unbonded Tendons," American Concrete Institute, ACI 423.3R-83, 1983.


The solution equations for the cracked section, assuming linear elastic behavior, are given below. For a given moment \( M \), the following equations lead to the stresses in the component materials.

\[
\begin{align*}
\left[ \frac{A_{ps}}{3M} \left( \epsilon_{pe} + \Omega_{c} \epsilon_{ce} \right) b_w \right] c^3 + \left[ \frac{b_w - A_{ps} E_{ps}}{M} \left( \epsilon_{pe} + \Omega_{c} \epsilon_{ce} \right) b_w d_p \right] c^2 \\
+ \left[ 2 \left( b - b_w \right) h_f + \frac{2A_s E_s}{E_c} + \frac{2A_s' E_s'}{E_c} + \frac{2A_{ps} E_{ps}}{E_c} \epsilon_{ps} + \Omega_{c} \epsilon_{ce} \right] c - \left[ \left( b - b_w \right) h_f^2 \right] \\
+ \left[ \frac{2A_s E_s}{E_c} \left( d_s - d_p \right) - \left( b - b_w \right) h_f^2 \right] \\
+ \frac{2A_s E_s}{E_c} \left( d_s - d_p \right) - \left( b - b_w \right) h_f^2 \right] = 0
\end{align*}
\]

Solve above equation for the value of \( c \) then compute:

\[
\begin{align*}
\left( b - b_w \right) c^2 &= \frac{A_{ps} E_{ps} (\epsilon_{pe} + \Omega_{c} \epsilon_{ce})}{2bc} \\
E_s f_s &= \frac{E_{ps} f_{ps} \left( \frac{d_s - c}{c} \right)}{E_c} \\
\epsilon_{ps} &= \frac{f_{ps}}{E_{ps}} \\
E_s &= \frac{f_s}{E_s} \\
\epsilon_s &= \frac{E_s'}{E_s} f_c \left( \frac{c - d_s}{c} \right) \\
\epsilon_{s} &= \frac{f_s}{E_s} \\
\end{align*}
\]

NOTE:

a) \( F = A_{ps} f_{ps} \) (thus \( F \)) varies with the applied moment \( M \).

b) For rectangular sections \( b = b_w \); if no compressive reinforcement is used then \( A_s \) = 0.

c) Sign convention; for concrete (compression +; tension -); for steel (tension +; compression -).
<table>
<thead>
<tr>
<th>Loading Type and Tendon Profile</th>
<th>Strain Reduction Coefficient: Uncracked State</th>
</tr>
</thead>
<tbody>
<tr>
<td>Uniform Load and Straight Tendons</td>
<td>( \Omega = \frac{2}{3} )</td>
</tr>
<tr>
<td>Uniform Load and Single Draping Point</td>
<td>( \Omega = \frac{5}{12} + \frac{1}{4} \frac{e_s}{e_m} )</td>
</tr>
<tr>
<td>Uniform Load and Parabolic Tendon</td>
<td>( \Omega = \frac{8}{15} + \frac{2}{15} \frac{e_s}{e_m} )</td>
</tr>
<tr>
<td>Concentrated Midspan Load and Straight Tendons</td>
<td>( \Omega = \frac{1}{2} )</td>
</tr>
<tr>
<td>Concentrated Midspan Load and Single Draping Point</td>
<td>( \Omega = \frac{1}{3} + \frac{1}{6} \frac{e_s}{e_m} )</td>
</tr>
<tr>
<td>Concentrated Midspan Load and Parabolic Tendon</td>
<td>( \Omega = \frac{5}{12} + \frac{1}{12} \frac{e_s}{e_m} )</td>
</tr>
<tr>
<td>Third Point Loads and Straight Tendons</td>
<td>( \Omega = 1 - \alpha = \frac{2}{3} )</td>
</tr>
<tr>
<td>Third Point Loads and Single Draping Point</td>
<td>( \Omega = \frac{23}{54} + \frac{1}{54} \frac{e_s}{e_m} )</td>
</tr>
<tr>
<td>Third Point Loads and Parabolic Tendons</td>
<td>( \Omega = \frac{44}{81} + \frac{1}{81} \frac{e_s}{e_m} )</td>
</tr>
</tbody>
</table>
### TABLE 2
Solution Equations for the Uncracked State

<table>
<thead>
<tr>
<th>Loading Case</th>
<th>Solution Equations</th>
<th>Equation Number</th>
</tr>
</thead>
<tbody>
<tr>
<td>(F₀ + M₀)</td>
<td></td>
<td>11</td>
</tr>
<tr>
<td></td>
<td>( f_{ps} = f_{pe} )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( \varepsilon_{pe} = \frac{f_{ps}}{E_{ps}} )</td>
<td>12</td>
</tr>
<tr>
<td></td>
<td>( \varepsilon_{ce} = \frac{1}{E_{c}} \left[ \frac{A_{ps} f_{pe}}{l} \left( r^2 + e^2 \right) - \frac{M_0 \varepsilon_0}{l} \right] )</td>
<td>13</td>
</tr>
<tr>
<td>(F₀ + M₀)</td>
<td>( f_{ps} = f_{pe} + \frac{\Omega (M-M_0) \varepsilon_0}{\frac{E_0}{E_{ps}} + A_{ps} \left( r^2 + e^2 \right) \Omega} )</td>
<td>14</td>
</tr>
<tr>
<td>M₀ &lt; M &lt; M_{cr}</td>
<td>( f_{cs} = \frac{f_{ps} \varepsilon_0}{A_c} \left[ 1 + \frac{\varepsilon_0 \left( \frac{d_s - y_1}{r^2} \right)}{l} \right] - \frac{M \left( d_s - y_1 \right)}{l} )</td>
<td>15</td>
</tr>
<tr>
<td>M₀ &lt; M &lt; M_{cr}</td>
<td>( f_s = \frac{E_s}{E_c} f_{cs} )</td>
<td>16</td>
</tr>
<tr>
<td>M₀ &lt; M &lt; M_{cr}</td>
<td>( f_{cl} = \frac{f_{ps} \varepsilon_0}{A_c} \left[ 1 - \frac{\varepsilon_0}{k_b} \right] + \frac{M y_1}{l} )</td>
<td>17</td>
</tr>
<tr>
<td>(F₀ + M₀)</td>
<td>( M_{cr} = (M_{cr})<em>0 + \Delta M</em>{cr} )</td>
<td>18</td>
</tr>
<tr>
<td>(F₀ + M₀)</td>
<td>( (M_{cr})<em>0 = A</em>{ps} f_{pe} \left( \frac{\varepsilon_0 + Z_b}{A_c} \right) + 7.5 Z_b \sqrt{f_{pc}} )</td>
<td>19</td>
</tr>
<tr>
<td>(F₀ + M₀)</td>
<td>( \Delta M_{cr} = \frac{A_{ps} \varepsilon_0 \left( \frac{\varepsilon_0 + Z_b}{A_c} \right) [ (M_{cr})<em>0 - M_0 ]}{\frac{1}{\Omega} \left( \frac{E_0}{E</em>{ps}} + A_{ps} \left( r^2 - \frac{Z_b}{A_c} \right) \right)} )</td>
<td>20</td>
</tr>
</tbody>
</table>

**NOTE:**

a) \( F = A_{ps} f_{ps} \); \( f_{ps} \) (thus \( F \)) varies with the applied moment \( M \).

b) Sign convention: for concrete (compression +; tension -); for steel (tension +; compression -).
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<td>( \Omega_c = \Omega \frac{l_{cT}}{I_g} + \left( 1 - \frac{l_{cT}}{I_g} \right) \left[ \frac{1}{2} \frac{e_a}{e_m} \frac{l_{cT}}{l^2} \right] )</td>
</tr>
<tr>
<td>Uniform Load and Single Draping Point</td>
<td>( \Omega_c = \Omega \frac{l_{cT}}{I_g} + \left( 1 - \frac{l_{cT}}{I_g} \right) \left[ \frac{1}{4} \frac{e_a}{e_m} \frac{l_{cT}}{l^4} + \frac{1}{2} - \frac{1}{2} \frac{l_{cT}}{l^2} \right] ) OR ( \frac{1}{3} \frac{l_{cT}}{l^3} + \frac{1}{4} \frac{l_{cT}}{l^4} )</td>
</tr>
<tr>
<td>Concentrated Midspan Load and Parabolic Tendon</td>
<td>( \Omega_c = \Omega \frac{l_{cT}}{I_g} + \left( 1 - \frac{l_{cT}}{I_g} \right) \left[ \frac{1}{5} \frac{l_{cT}}{l^5} \right] + \frac{1}{3} - \frac{2}{3} \frac{l_{cT}}{l^3} + \frac{1}{5} \frac{l_{cT}}{l^5} )</td>
</tr>
<tr>
<td>Uniform Load and Parabolic Tendon</td>
<td>( \Omega_c = \Omega \frac{l_{cT}}{I_g} + \left( 1 - \frac{l_{cT}}{I_g} \right) \left[ \frac{1}{3} \frac{l_{cT}}{l^3} \right] )</td>
</tr>
<tr>
<td>Concentrated Midspan Load and Straight Tendons</td>
<td>( \Omega_c = \Omega \frac{l_{cT}}{I_g} + \left( 1 - \frac{l_{cT}}{I_g} \right) \left[ \frac{1}{2} - \frac{1}{2} \frac{l_{cT}}{l^2} \right] )</td>
</tr>
<tr>
<td>Concentrated Midspan Load and Single Draping Point</td>
<td>( \Omega_c = \Omega \frac{l_{cT}}{I_g} + \left( 1 - \frac{l_{cT}}{I_g} \right) \left[ 1 - \frac{l_{cT}}{l} \right] \left[ 1 - \frac{l_{cT}}{l} \right] ) ( \frac{1}{2} \frac{e_a}{e_m} - \frac{1}{3} \frac{l_{cT}}{l^2} )</td>
</tr>
</tbody>
</table>

* Refer to Figures 2 and 4 for Notation.
Figure 1
Schematic moment deflection curve of an externally prestressed beam

Figure 2
Typical loadings and tendon profiles considered in the computation of the strain reduction coefficients \( \Omega \) and \( \Omega_c \)
Figure 3
Strain differential between bonded and Unbonded tendons in bending - Uncracked state

Figure 4
Typical representation of elastic uncracked, elastic cracked, and idealized elastic cracked beam
Figure 5
Typical illustration of the variation of $\Omega_c$

Figure 6
Strain diagrams from the uncracked to the cracked state
Ultimate Behavior of Precast Segmental Box Girders with External Tendons

by J. Muller and Y. Gauthier

Synopsis: The concept of precast segmental construction with external tendons has been developed extensively since 1978, starting with the construction of the Long Key Bridge (Florida, U.S.A.). Since this first experience, many other structures (more than 5,500,000 SF of deck) have been designed and successfully built using the same method.

The performance of all bridges now in operation has been excellent. However, some questions were raised in the minds of Engineers, unfamiliar with the method, as to the behaviour of structures prestressed with external tendons beyond the range of design loads (serviceability limit state). Because continuous reinforcement is not usually provided across the match cast joints between segments, concern was expressed that adequate ultimate behaviour and sufficient strength could not be obtained.

In order to provide a satisfactory answer to these legitimate questions, a special computer program (DEFLECT) has been developed to analyse accurately the response of the structures prestressed by external tendons. Moreover, several tests are available to confirm the behaviour of such structures while verifying the validity of the DEFLECT computer program. This design tool has been used to predict the structural behaviour of simply supported and continuous structures beyond joint opening, up to ultimate capacity of the girders with and without thermal loads. Several different prestressing methods have been analyzed with different bonding conditions at the point of deviation of the external tendons. It was found systematically that structures prestressed with either internal or external tendons behave essentially in the same way at all loading stages up to ultimate.

Keywords: bending moments; box beams; bridges (structures); computer programs; moment-curvature relationship; post-tensioning; precast concrete; prestressing steels; segmental construction; serviceability; stress-strain relationships; structural design; unbonded prestressing
Honorary ACI member, Jean M. Muller is presently Technical Director of Jean Muller International Inc. of Paris, France and of Jean Muller International Inc. of San Diego, California, USA. He has been involved actively in the design of innovative concrete bridges for the past 40 years.

ACI member, Yves Gauthier is presently Assistant Technical Director of Jean Muller International Inc. of San Diego, California, USA. He was actively involved in the structural analysis of concrete bridges with external prestressing which forms the basis of this article.

1. Scope

The concept of precast segmental construction with external tendons has been developed extensively since 1978, starting with the construction of the Long Key bridge.

Since the experience of the four segmental bridges in the Florida Keys, representing a deck area of 2,200,000 SF, many other structures have been designed and successfully built using the same method.

The following list of structures also includes projects still under construction:
The performance of all bridges now in operation has been excellent. However, some questions were raised in the minds of Engineers, unfamiliar with the method, as to the behaviour of structures prestressed with external tendons beyond the range of design loads (serviceability limit state). Because continuous reinforcement is not usually provided across the match cast joints between segments, concern was expressed that adequate ultimate behaviour and sufficient strength could not be obtained.

This report provides a satisfactory answer to these legitimate questions.

Several tests are available to confirm the behaviour of girders prestressed with external tendons. They were used to verify the validity of a computer program developed to analyze accurately the behaviour of these structures.

It was found systematically that structures prestressed with either internal or external tendons behave essentially in the same way at all loading stages up to ultimate.

A design tool now exists to predict accurately the ultimate strength of a structure with external prestressing.
2. Presentation of the DEFLECT computer program

The "DEFLECT" computer program allows analysis of structures beyond the decompression of the match cast joints. The computation is based upon the relationship between bending moment and beam curvature.

2.1. Methodology

In the precast segmental bridges, when the loads increase beyond the service loads, usually there is no crack occurring between joints of the segment, and all the cracks are concentrated at the joint location, thus inducing opening of the joints (Fig 1).

![Crack pattern in segmental bridge](image)

This fact is especially true for precast segmental bridges with dry joints (no epoxy glue in the joints).

When the joint opens, the beam rigidity is modified. The rigidity of the structure depends on the deformability of the precast segment between two adjacent joints.

To predict the beam rigidity, it is necessary to know the relationship between bending moment and curvature or bending moment and rotation at joints.

Consequently, the problem will be solved if it is possible to calculate the rotation $\theta$ occurring at a joint for a given pair of actions, $N$ (axial force)
and $M$ (bending moment), which induces a specified opening of the segment joint as shown on Fig 2.

![Diagram of M and N forces](image)

**FIG 2**

**NOTATION AT JOINTS**

The deformability of a typical segment can be obtained by means of a finite element analysis in order to account for the shape of the section, the length of the precast segments and the height of the section in compression at the joints.

The segment is divided into volume elements as shown in Fig 3.:

![Finite Element Model](image)

**FIG 3**

**FINITE ELEMENT MODEL**

All the calculations are carried out assuming that the stress distribution at joint is linear (contact...
zone is plane) and the concrete in tension between
two opened joints is not cracked.

Step 1

For a given neutral axis, a unit translation and then
a unit rotation are applied to all the nodes of the
joints which are above the neutral axis. The results
of both calculations are combined in such a way that
the resulting displacements of all the nodes above
the neutral axis are a rotation $\delta w$ around the neutral
axis.

Step 2

The computer program can calculate for all nodes $P_i$
the axial force $N_i$ resulting from the applied
displacements. Assuming $Z_g$ to be the altitude of the
center of gravity, one can write:

$$N = \sum N_i$$
$$M_g = \sum N_i (Z_i - Z_g)$$

The relationship between the rotation $\delta w$ and the
applied load characterized by the pair of values, $N$
(axial) and $M$ (moment), can be obtained by changing
the depth of the neutral axis.

For convenience, this is expressed between the two
parameters $M/N$ and $Edw/N$ ($E$ = Modulus of
elasticity).

In figure 5, the moment-curvature relationship is
given for a typical segment of the Sunshine Skyway
bridge approaches.

If the resultant force stays within the central core,
the response is linear and the slope of the diagram
is equal to $I$ ($I$ = bending inertia).

If the eccentricity increases, the curvature
increases much faster but is always perfectly
$e = \frac{M}{N}$

SUNSHINE SKYWAY BRIDGE APPROACHES

Fig 5  MOMENT / CURVATURE RELATIONSHIP FOR TYPICAL SEGMENT

$S = \text{Section Area}$

$X = \text{Length of structure along longitudinal axis}$
reversible up to a point very close to ultimate state.

At ultimate state, depending on the sign of the bending moment, the curvature is 450 or 80 times greater than for the section totally under compression.

It must be pointed out that the analysis is conducted assuming that the materials remain in the elastic range.

Starting with the bending moment-curvature relationship, the "DEFLECT" program determines the equilibrium of the segments along with the post-tensioning steel over-stress under the loads applied to the structure. The prestressing steel over-stress depends upon the geometry of the tendons and the bonding conditions prevailing at the deviation blocks.

The stress strain diagram for post-tensioning steel is shown in fig 6.

The stress distribution in the structure is calculated by iteration until satisfactory accuracy is obtained. The methodology also allows the analysis of loads combined with temperature gradients (linear or non-linear).

The validity of the DEFLECT program has been checked by comparing it with available test data for similar structures.

2.2. Comparison with available test data

2.2.1. CEBTP Test

A test was conducted at Centre Experimental de Bâtiments et Travaux Publics (C.E.B.T.P.) in France
TESTING OF SEGMENTAL GIRDER
WITH EXTERNAL TENDONS

C.E.B.T.P. TEST

FIG 7
on a precast segmental beam with dry joints and external post-tensioning tendons.

Figure 7 shows the test set-up and results. The plotted deformations obtained from the test have been compared to the theoretical "DEFLECT" calculations. Both of the curves are consistent except at failure since the computer program uses the elasticity theory and cannot account for the concrete plasticity at the ultimate stage. However, the "DEFLECT" analysis underestimates the ultimate capacity of the beam.

It should be mentioned that at ultimate loading there was a 46 ksi overstress in the prestressing steel as compared with the 15 ksi overstress allowed by the existing AASHTO code.

Another computer run was made for a beam with exactly the same geometry and prestress tendons as the test beam with external tendons, but it was assumed that the tendons were fully bonded on the whole length.

The differences between the three curves are insignificant at all loading stages up to and including ultimate capacity.

2.2.2. CTL (Construction Technology Laboratories) Test

To ascertain the suitability of the concepts of precast segmental construction with external tendons to sustain seismic loads, a comprehensive testing program was implemented with particular attention focused on redundancy and ductility of the structure. For purposes of comparison, three identical beams were tested and instrumented first up to design load and in the elastic range and then loaded to ultimate capacity.

This test has already been described in previous articles. (P.C.I. Journal March/April 1987 page 86: "Testing of segmental concrete girders with External Tendons" by K. Sowlat and B.G. Rabbat).

The following conclusions may be drawn from the test results:

- Beyond the design stage, the unbonded beam is more flexible than the bonded one.

- A complete elastic recovery was experienced after loading the beams up to 1.7 times the design load in spite of the substantial joint opening of the critical joints (0.14 in).

- The unbonded beams develop an ultimate moment, which was the same as that of the bonded beam.
C.T.L. TEST
FIG 8
These experimental studies are confirmed by computer analysis.

Figure 8 shows the test set up for the unbonded beams and the comparison between measured beam deflection and theoretical deflection, calculated by means of the "DEFLECT" program for two tests corresponding to two different post-tensioning patterns.

Again, the plotted curves are quite consistent, confirming the "DEFLECT" computer program to be a satisfactory tool to predict the ultimate capacity of precast segmental structures.

3. Analysis of Precast Segmental Box Girders at Service Load Design

The superstructure of the Sunshine Skyway Bridge high level approaches has been selected as representing an excellent current example of box girder bridges with moderate spans. A typical unit consists of 8 spans of 135 ft.

Figure 9 gives the dimensions and the properties of the typical cross sections and the prestressing layout for the first and second spans. The other spans are identical to Span 2, except that the 1 3/8" P/T bars do not exist in the intermediate spans (Nos. 3 thru 6).

The bridge has been designed for a 18 °F linear temperature gradient which induces, in continuous bridges, similar tensile stresses in the bottom flange as the complex gradients and relatively high temperature differentials outlined in appendix A of National Cooperative Highway Report 276.

The joint decompression has been checked for the following load combinations (no tension allowed):

- DL + (LL + I) (Dead Load + Live Load + Impact)
- DL + Gradient (18 °F)
- DL + (LL + I) + Gradient (9 °F)

The superstructure was also checked for two additional combinations:

- DL + Gradient (18 °F) + 1/2 (LL + I)
- DL + Gradient (-9 °F) + (LL + I)

4. Analysis of Precast Segmental Box Girders with External Tendons at Load Factor Design

The superstructure of the Sunshine Skyway approaches has also been analyzed at load factor design by means of the "DEFLECT" computer program.
TYPICAL CROSS SECTION

SPAN 1
PT. LAYOUT

SPAN 2
PT. LAYOUT

SUNSHINE SKYWAY BRIDGE APPROACHES

FIG 9
The first span which is the most critical under ultimate loads has been analyzed for the following load combinations.

4.1. 1.3 DL + 2.17 (LL + I) See Figure 10

Under ultimate loads, two joints are open at mid-span. The materials remain in the elastic range with:

\[
fs = 188 \text{ ksi (70\% G.U.T.S.)} \\
fc = 319 \text{ ksf (40\% f'c)}
\]

Two other joints are slightly open on either side of the pier segment.

4.2. 1.3 DL + 0.5 GRADI + 2.17 (LL + I) See Figure 10

At ultimate load, we have also analyzed the effect of a temperature gradient.

The comparison of the forces and of the deflections with 4.1.1 shows that the effect of the temperature gradient does not modify significantly the behaviour of the superstructure under ultimate loads.

\[
fs = 190 \text{ ksi (70\% G.U.T.S.)} \\
fc = 355 \text{ ksf (45\% f'c)}
\]

The maximum overstress in the external tendons is: 21 ksi.

4.3. 1.3 DL + 3.49 (LL + I) See Figure 10

This load combination corresponds to the ultimate capacity of Span 1. The plot of the structure shows that the ultimate capacity of Span 1 is obtained when joints at mid-span and at the pier are simultaneously fully opened.

The stress in the concrete is in the neighbourhood of 85\% f'c but the P/T steel stress stays smaller than 217 ksi (80\% G.U.T.S.) with a maximum 50 ksi overstress.

Based upon the comparison with available test data, "DEFLECT" underestimates slightly the ultimate capacity of the beam, since the calculations assume an elastic response.

4.4. 1.3 DL + 2.17 (LL + I), with no Bond in Deviation Blocks See Figure 11

The P/T tendons are encased in steelpipe ducts at the deviation block location. Usually the bond in the duct allows the transfer of overstress. In order to
SUNSHINE SKYWAY BRIDGE APPROACHES

ULTIMATE BEHAVIOR

FIG 10
show the safety factor of the superstructure, we have also analyzed the superstructure, assuming that there was no bond in deviation saddles.

Figure 11 shows the state of the first span under ultimate loads. Obviously the structure is more flexible but the stresses are still within the elastic range.

\[ fs = 172 \text{ ksi} (66\% \text{ G.U.T.S.}) \text{ (overstress = 11.6Ksi)} \]
\[ fc = 454 \text{ ksf} \ (57\% f'c) \]

Fig: II  **SUNSHINE SKYWAY BRIDGE APPROACHES**

**ULTIMATE BEHAVIOR**
5. Structural Behaviour of Precast Segmental Bridges with External Tendons beyond the Joint Opening

The aforementioned load combinations have demonstrated the satisfactory flexural behaviour of precast segmental bridges at load factor design (per AASHTO) and at ultimate capacity.

It is important to understand the mechanism of load transfer after opening of the joints.

Figure 12 shows the trajectory of the pressure lines corresponding to different load combinations for Span 1. In fact it is obvious that an arch effect is developed in the structures, thus allowing the shear forces to be directly transferred to the bearings.
6. Comparison of Different Prestressing Methods for a 100 ft Simple Beam

Another interesting experience reported in this paragraph was to consider a 100 ft simple beam with the same cross section and prestress as the outside span of the Skyway approaches. Cross section and properties are shown in Figure 9. Prestress consists of \(2 \times 76 = 152\) STRANDS 0.5" diameter giving a steel area of \(23.2\) sq = 0.161 sf.

The same beam was successively assumed to have the following characteristics:

- \(P_0\): External prestress with 4 deviation saddles and 10' long segments.
- \(P_1\): External prestress with deviation saddles at 10' intervals
- \(P_2\): External prestress with deviation saddles at 5' intervals
- \(P_3\): Internal prestress modeled with deviation saddles at 1' intervals.

The results are very interesting and somewhat surprising (see Fig 13). The so-called bonded beam \(P_3\), so often taken as a reference of maximum performance, does not show the largest deflection near ultimate. A beam with a finite number of deviation and/or anchor blocks allows more flexibility to develop at ultimate.

As a conclusion, all beams behave much alike.

Fig 13
Comparison of several prestressing methods
7. Second order deflection

Special care must be taken in the analysis of the second order deflections since, when the beam is close to failure, a slight increase of the eccentricity magnifies the beam deflection. The increased deflection induces a loss of post tensioning tendon efficiency if the tendon does not follow the concrete deflection.

FIG 14 shows the superiority of a beam with properly distributed deviation blocks.

8. Conclusion

Prestressing with external tendons has proved to be a very practical and efficient tool in the field of concrete bridge particularly with precast segmental construction. It is not intended to replace internal prestressing in all structures, but rather complement the more conventional prestressing methods with bonded tendons.

In terms of structural behaviour and design methods, the experience gained during a period of now more than ten years has proven the method of external prestressing to be essentially equivalent to the method of internal prestressing. Design tools are now available to analyze and predict the behaviour of structures pretressed with external tendons at all loading stages from serviceability limit state up to ultimate limit state and even actual ultimate capacity.

It is anticipated that external prestressing will be used more extensively in the future in concrete construction, but also in steel or composite concrete and steel structures.
Externally Prestressed Bridges

by J. Eibl

Synopsis: Experience gained in the design of several externally prestressed bridges is reported. New cables especially developed by contractors are discussed. Also the main experimental results gained at the author's institute for reinforced concrete are given.

Keywords: bridges (structures); prestressed concrete; prestressing; prestressing steels; structural design; tests; unbonded prestressing
1. INTRODUCTION

Experience has shown that the grouting of prestressing cables needs great care if later damages are to be avoided. As the latter cannot safely be guaranteed in every case the question has been raised in Germany whether the inspection of cables and their replacement, if necessary, could be provided in future bridge design. Such an intention leads to cables without bond. In principle they could be arranged within the webs of T-beams or hollow box cross sections as in the case of bonded cables. Striving however for more robust bridges it seems reasonable to place the cables outside the webs, so that an easier and safer concreting of this structural member can be achieved. As we know, bundles of prestressing cables within the webs have led to considerable damage in the past.

Because the strain increase in the ultimate limit state is smaller with unbonded cables than with grouted ones, one usually needs more mild reinforcement within the cross section. This however can also help to gain more durable structures. In the past it has been demonstrated that even in cases of high levels of prestressing a reasonable amount of mild reinforcement has to be used to achieve controlled cracking.

This type of reasoning, which has already led to a remarkable number of external prestressed bridges in France, is among others greatly supported by BRUGGELING [4,6,7,8] (Fig. 1a), who more or less demands a reinforced bridge with an additional favourable stress state generated by prestressing.

Starting from the same basis of consideration the author has designed, in close cooperation with the German Autobahn Authorities and the German Railways, five bridges which are under construction now. In the case of a Swiss bridge, which has
already been completed, the author acted as a scientific consultant to the Swiss Authorities.

2. DESIGN ASPECTS

Starting from this basis as a first alternative one could arrange the cables as shown in Fig. 1b, which allows easy placement, easy replacement if necessary and also the possibility of using the same cables at different places and for different purposes during the construction process without high labour costs.

Fig. 1
A structurally more effective arrangement of cables is shown in Fig. 1c where vertical forces are applied to the concrete section at the bends of the cables.

In both cases a number of anchoring devices has to be placed within the cross section. These are necessary to change the direction of cables in horizontal or vertical direction and for permanently or temporarily anchoring in case of special erection procedures such as incremental launching e.g. The transfer of such sometimes great anchorage forces to the cross section has been carefully analyzed by means of threedimensional F.E.

Fig. 2
investigations in order to avoid great tensile stresses, which might occur in the webs.

Fig. 2 d demonstrates bending which results from anchoring in a vertical beam which was only fixed at the neighbouring webs as shown in Fig. 2 c. In Fig. 2 e the anchoring beam was only fixed at the top and bottom slab, while Fig. 2 f gives the best solution with regard to minimal bending stresses. In this case the vertical beam with a rather big dimension in the bridge's longitudinal direction is fixed at top and bottom and has an additional rather thin shear connection to the neighbouring web.

The increase of cable forces due to applied live loads or to increased ultimate loads as well as the relative displacements at the saddle are small (Fig. 3). According to:

\[
\int_0^L (\varepsilon_c(s) - \varepsilon_s) \, ds = 0 \quad \varepsilon_s = \frac{1}{L} \int_0^L \varepsilon_c(s) \, ds
\]

(1)

\[
\Delta u_A = \int_{s=0}^{s=s_A} [\varepsilon_c(s) - \varepsilon_s] \, ds
\]

the strain difference \(\varepsilon_c(s) - \varepsilon_s\) cancels nearly out along the path of integration.

Of great importance is the type of prestressing cables, which are to be used. In the case of external prestressing a homogeneous distribution of prestressing steel over the concrete area is not relevant and therefore one will prefer rather heavy cable units. The upper limit is dictated by the handling process at erection. A force of 2-3 MN seems to be reasonable.
In Germany three cable types have been especially developed for this purpose (Fig. 4). Others will be tested in the future.
Type 1 and 2 of BBRV rsp. SUSPA cables consist of 7 mm dia. wires with $f_y/f_{ult} = 1470/1670$ N/mm$^2$ and are prestressed to 0.7 $f_{ult}$. This is different from bonded cables which can only be stressed up to 0.55 $f_{ult}$ according to German standards. The reason is that unbonded cables with their small stress variation under traffic loads may be treated as statically stressed.

The wires are arranged in a plastic tube surrounded by grease or mortar. Type 2 contains an additional metal cover inside the plastic tube which does not hinder cable transport on coils. This is necessary to avoid heavy deformation of the plastic tube at saddle points as our experiments have shown. The anchorage system is the BBRV rsp. SUSPA system. When cables are filled by grease the complete cable is manufactured and sealed in the plant thus guaranteeing high quality at the site.

Type 3 has been developed by Dyckerhoff & Widmann and consists of the usual monostrands surrounded by mortar in a plastic tube. At saddle points specially designed holders guarantee the correct position of the strands thus avoiding high transverse pressure that otherwise may damage the surrounding plastic tube. An effective anchorage system allows to replace every single monostrand.

3. EXPERIMENTAL RESULTS

All cable types have been tested experimentally to check whether they can endure $2.5 \times 10^6$ load cycles under a live load amplitude of 35 N/mm$^2$ and a relative displacement at the saddle of 0.6 mm (see Fig. 5). The cable was supported at the saddle on torus-shaped metal half-shells in the case of Type 1 and 2 tendons. Type 3 was only supported by a special mortar bed moulded by formwork on top of the concrete saddle.
The radius of curvature at the saddle was 4 m. The change of inclination varied from 1 to 6 degrees. In the case of the BBRV cable used for the bridge in Switzerland temperature variations between -15°C and +35°C have also been applied during tests.

Fig. 5

All relative displacement between the different elements of the cross section have been recorded.

The main results gained by these tests may be summarized as follows:

- All cables showed satisfactory behaviour
- At angles > 1 degree the saddle acted as a fix-point for the cable
- Friction was about 1% at first prestressing
- The grease filled Type 1 cables showed a noticeable deformation on the inner surface of the plastic tube due to the transverse pressure

4. APPLICATIONS

In Fig. 6, a sketch of the Bridge at Preonzo Claro which has a maximum span of 62.5 m, is given. It was originally designed by the consulting office of Guzzi in Zürich. In this case BBRV cables with a capacity of 2.35 MN, 3.7 MN, and 4.6 MN were used.
The bridge has nearly been completed. Experience showed that 4.6 MN cables should be an upper limit with regard to handling inside the hollow box cross section.

Bridge of Preonzo Claro (Autostrada Chiasso - San Gottardo)

Fig. 6

Longitudinal Section

Cable-Layout

Fig. 7

In Fig. 7 the Wintroper Talbrücke, an Autobahn bridge with two parallel decks is shown. It will be built by the incremental launching method as developed by LEONHARDT. In this sketch the cable layout is also drawn.
A similar Autobahn bridge - Berbke - with maximum spans of 45 m and a length of 298.50 m is also under construction and is being built using the incremental launching system. This bridge is curved in its ground plan with a radius of 2350 m.

Fig. 8 shows two different systems which will be applied to three single-span bridges for the German Railways. Their erection will start by the end of this year.

Fig. 8

In the case of the Autobahn bridges, the necessary prestressing force was determined by the demand that $\sigma_c$ at the boundaries should be zero under dead load. One third of live load, plus the usual forces associated with $\pm 5^\circ$C between the cables and the neighbouring concrete had to be considered, a deviation from the usual regulations for bridges with bonded cables. Crack width was to be checked by means of a given specification demanding a calculated crack width of 0.15 mm under increased live loads. This was to control service behaviour.

Also the bridges must be able to support half of the traffic load associated with the full service conditions without two cables so that easy replacement of cables can take place with only small traffic restrictions.
At the ultimate state an increase of the prestressing force was allowed only if detailed analysis of the cracked state was done.

In the case of the railway bridges, for the first time, after a thorough investigation decompression already under half of the live load was allowed. Until now full prestress was always been demanded by the German Railways.

Fig. 9 finally shows a new design of the author for a composite externally prestressed railway bridge, which is still in the bidding process and consists of a top and bottom concrete slab and steel truss webs. It is in so far an extension of external
prestressing as the cable sag is now increased by anchoring the cables outside the cross section at vertical members of the steel structure. This allows the webs to be resolved to steel truss girders which may also serve as scaffolding girders during the erection process when the upper and lower concrete deck is poured.

5. ADVANTAGES AND DISADVANTAGES

Regarding the little experience the author could gather, the following advantages and disadvantages are seen.

If the cables cannot be applied external to the cross section, usually only a lower lever arm for prestressing is available than in the case of bonded prestressing and with additional loads in the ultimate state more mild reinforcement is necessary. This however led to only half of the usual calculated crack width. Intensive comparison calculations show that with less prestressing force and more mild reinforcement, the costs for the entire reinforcement calculated on the basis of a cost relation of prestressed steel/mild steel = 4.5 have an increase of 10%.

An advantage is that cable manufacturing especially with grease filled cables manufactured off the site guarantee greater quality. External cables allowed better concreting of the webs and thus thinner webs. In addition new types of steel webs may be used. The possibility of easy inspection and replacement will certainly prevent some of the past bad experiences. With a higher amount of mild steel partial prestressing will lead to better crack control. Restrictions in present regulations that do not allow cracks to cross prestressing cables are no longer necessary. De-icing salt is practically of no influence to the cables. The cost increase of about 3% for the whole bridge is negligible. These developments may also lead to new structural types, as is shown with the last example.
REFERENCES


Synopsis: A nonlinear finite element analysis was conducted to examine the full range of behavior of segmentally precast box-girder construction with external post-tensioning tendons. A primary objective of the study was to examine the effect of dry joints (without epoxy) on the strength and ductility of box girder construction. A secondary consideration was the influence of supplemental bonded internal tendons on the behavior of the structure.

Keywords: box beams; bridges (structures); finite element method; post-tensioning; precast concrete; prestressing; prestressing steels; segmental construction; structural analysis; unbonded prestressing
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Gregory L. Fenves is an assistant professor of civil engineering at the University of California at Berkeley. He obtained his Ph.D. in 1984 from the University of California at Berkeley. His research interests include nonlinear finite element analysis, particularly for earthquake response of concrete dams.

ACI member Kamal C. El-Habr is a structural designer for Whitman, Requardt and Associates, Baltimore, Maryland. He received his BS and MS degrees from The University of Texas at Austin in 1984 and 1988.

INTRODUCTION

Since the completion of segmental bridges in the 1950's and 1960's in Europe, and in the 1970's in the United States, segmental prestressed concrete box-girder bridges have become a predominant form of medium and long-span bridge construction (100 to 650 foot spans) that has been characterized by rapid evolution. Economic factors have continually suggested refinements in construction practices to increase productivity and minimize construction time. Each of these developments in construction methods has required innovative design procedures. Principal among these are the relocation of tendon anchorage zones, the use of multiple shear keys, the omission of epoxy from the joints, and the use of tendons external to the webs and flanges.

A significant number of segmental prestressed concrete box girder bridges with external tendons and dry joints have already been constructed. Substantial economic and construction time savings have been indicated for this type of construction. However, relatively little analytical investigation has been undertaken to evaluate the behavior of segmental bridges, incorporating the new developments, for all ranges of loads. The analytical evaluation of their behavior is desirable before additional construction proceeds.

FINITE ELEMENT FORMULATION

A segmental girder with external tendons is modeled with three types of finite elements: a one-dimensional beam element for the segments, a tendon element connected to the segments by rigid diaphragms, and a joint element that transmits forces between two segments. Figure 1 illustrates a simple-span girder of two segments modeled with four beam elements, three tendon elements, and one joint element.
Beam Element

Segments are modeled by six degree-of-freedom beam elements. Transverse loading in a vertical line of symmetry is assumed. The nonlinear behavior of the concrete and passive reinforcement is approximately modeled by a series of fibers in the cross section [1,2,3]. A uniaxial constitutive relationship is specified for each fiber (such as in Ref. 4 for concrete and a bilinear relationship for steel). Although a fiber model requires more computation than a model of the beam section, moment-curvature-axial force relationships do not have to be defined [3].

The strain in the element is

$$\epsilon = \Delta \epsilon + \epsilon_0$$  \hspace{1cm} (1)

where $\Delta \epsilon$ is the increment of strain from the current strain state, $\epsilon_0$, and all strains (and stresses) are positive in compression. Neglecting shear deformations, the assumption of plane sections gives

$$\Delta \epsilon = \Delta \epsilon_a + y \Delta \phi$$  \hspace{1cm} (2)

where $\Delta \epsilon_a$ and $\Delta \phi$ are increments of axial strain and curvature, respectively. In the finite element approximations these increments are expressed as

$$\begin{bmatrix} \Delta \epsilon_a \\ \Delta \phi \end{bmatrix} = B \Delta u$$  \hspace{1cm} (3)

where $\Delta u = \Delta u_1, ..., \Delta u_6$ are the increments of nodal displacements and $B$ contains derivatives of $N$, the standard linear and cubic shape functions for a beam element [5].

The force-incremental displacement relationship can be expressed as:

$$\begin{bmatrix} P \\ M \end{bmatrix} = \begin{bmatrix} D_{11} & D_{12} \\ D_{21} & D_{22} \end{bmatrix} \begin{bmatrix} \Delta \epsilon \\ \Delta \phi \end{bmatrix} + \begin{bmatrix} P_o \\ M_o \end{bmatrix}$$  \hspace{1cm} (4)

where the following summations over the fibers in the cross-section replace the integrals of stress:

$$D_{11} = \Sigma E_i A_i, \; D_{12} = D_{21} = \Sigma E_i A_i y_i, \; D_{22} = \Sigma E_i A_i y_i^2$$

$$P_o = \Sigma \sigma_{i} A_i, \; M_o = \Sigma \sigma_{i} A_i y_i$$

In the summations, $E_i$, $A_i$, $y_i$, and $\sigma_i$ are the tangent modulus, area, distance from the reference axis, and current stress of fiber $i$, respectively.

Upon use of Eqs. (3) and (4), the principle of virtual displacements gives the equilibrium equations

$$K_t \Delta u = \Delta F - (P_o - M_o)$$  \hspace{1cm} (5a)

where

$$K_t = \int B^T D B dz$$  \hspace{1cm} (5b)
\[ \Delta P = \int N^T \Delta p \, dz \]  
\[ E_\circ = \int N^T p_\circ \, dz \]  
\[ R_\circ = \int B^T \{ \frac{P_\circ}{M_\circ} \} \, dz \]  

in which \( \Delta P \) is the load increment from the current load \( p_\circ \). The quantity \( (P_\circ - R_\circ) \) represents the unbalanced forces from previous load increments; it is zero if each load increment is brought to equilibrium.

After solution for \( \Delta u \), the strain in each fiber is computed from Eqs. (1) through (3), the stresses are computed from the constitutive relationships, and summed to give \( P_\circ \) and \( M_\circ \). Equation (5e) then gives the current internal resisting forces.

**External Tendon Element**

Neglecting slippage at the attachment points, the tendons are modeled by axial elements connected through rigid links to the girder segments. The rigid links represent the points of tendon attachment at diaphragms (or deviators) and anchorages, which, in this analytical procedure, are assumed much stiffer than the box girder. Figure 2 shows the geometry of the tendon element.

As with the beam element, the strain in the tendon can be represented by Eq. (1). Assuming small displacements, the increment in strain is

\[ \Delta \epsilon = \beta \Delta u' \]  

where \( \beta = \frac{1}{L_p} < -1 \) and \( \Delta u' = < \Delta u_1', \Delta u_2' > \) in the local coordinates of the tendon, and \( L_p \) is the length of the tendon. The force displacement relationship is obtained by the principle of virtual displacements:

\[ p' = \beta^T D \beta \Delta u' + \beta^T F_\circ \]  

where \( D = \frac{A_p E_p}{L_p} \), \( F_\circ = A_p \sigma_\circ \), \( A_p \) is the cross-sectional area of the tendon, \( D_p \) is the tangent modulus of the tendon, and \( \sigma_\circ \) is the current stress in the tendon (which includes prestress).

The displacement increment, \( \Delta u' \), at the ends of the tendon can be expressed in terms of the displacement increments at the ends of the rigid links by the compatibility relationship \( \Delta u' = t \Delta u \), where \( \Delta u = < \Delta u_1, \ldots, \Delta u_6 >^T \) and

\[ t = \begin{bmatrix} C & S & -\beta_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & C & S & -\beta_2 \end{bmatrix} \]  

in which \( C = \cos \Theta, S = \sin \Theta, \beta_1 = e_1 C + g_1 S \) and \( \beta_2 = e_2 C + g_2 S \) (in this study \( g_1 = g_2 = 0 \)). Consequently, the force displacement relationship for the tendon element is:

\[ p = (\beta t)^T D (\beta t) \Delta u + (\beta t)^T F_\circ \]  

The current strain in the tendon is given by \( \Delta \epsilon = \beta t \Delta u \) and Eq. (1), which can be used to evaluate the axial force in the tendon.
Joint Element

Externally post-tensioned bridges have been constructed with no epoxy in joints between precast segments. Under load, the joints may open, causing a redistribution of internal resisting forces in the girders. The joint opening is modeled by assuming that deflections are small, sections remain plane, and there is no slippage between segments across the joint.

The element is shown in Fig. 3. The element has two rotational degrees and two horizontal translational degrees of freedom to represent the opening of the joint. The ends of adjacent beam elements (segments) are connected by a distributed spring that only resists closing of the joint (Fig. 4a). The assumed kinematic relationship at the joint is shown in Fig. 4b, where the gap is given by

\[ g = \varepsilon u \]  

where \( \varepsilon = < 1 - y - 1, y > \), \( u = < u_1, ..., u_4 >^T \), and negative \( g \) indicates opening of the joint. The location, \( y_c \), of the gap opening is defined by \( g = 0 \), or

\[ y_c = \frac{u_1 - u_3}{u_2 - u_4} \]  

The value of \( y_c \), when compared to the values of \( g \) at the top and bottom, determines the four states of the joint: fully closed, open at top, open at bottom, and fully open.

The force in the joint spring is \( f = kg \) over the depth of contact, \( s \leq y_c \leq r \), where \( s \) and \( r \) depend on the state of the joint opening, and \( f = 0 \) over the open portion of the joint. Representing the force in the spring as \( f = \Delta f + f_o \) and the gap as \( g = \Delta g + g_o \), the principle of virtual displacements gives the following force displacement relationship:

\[ p = k_t \Delta u + f_o \]  

where \( p = < p_1, ..., p_4 >^T \), \( \Delta u = < \Delta u_1, ..., \Delta u_4 >^T \) and

\[ k_t = \int_s^r \varepsilon^T k \varepsilon dy \] \[ f_o = \int_s^r \varepsilon^T f_o dy \]

Using the definition of \( \varepsilon \) and \( f_o \), closed-form expressions for \( k_t \) and \( f_o \) can be formed in terms of the current state of opening of the joint [7]. The state of the joint and internal resisting forces can be updated after solution for \( \Delta u \).

The joint stiffness, \( k \), used in the model was 100,000 ksi. This was large in comparison to the axial stiffness of the girder, but was not so large as to result in an ill-conditioned stiffness matrix or oscillatory solution as determined from a parameter study. Additional details on the formulation of the model and solution procedure can be found in Ref. 6 and 7.
APPLICATION OF MODEL

The nonlinear finite element model described in the previous section was used to analyze a three-span, post-tensioned segmental box-girder bridge with external tendons. Comparisons are made between computed behavior of a girder with dry joints between precast segments and behavior of a similar monolithic structure. In addition, behavior of the segmental box girder with additional bonded internal tendons is examined.

During development of the finite element model and the course of this investigation a reduced-scale, three-span, post-tensioned segmental box-girder bridge model with external tendons was designed and fabricated at The University of Texas at Austin. As a matter of convenience the basic details of the experimental model were used as input for the finite element model. A plan view and elevation of the model are presented in Fig. 5, and the fiber models for typical segments and pier segments are shown in Fig. 6. In the finite element model each segment is divided into two elements, and closure strips at the end of each span are considered as separate elements with the same cross section as typical segments, but without reinforcement. The area of each fiber, distance from the fiber centroid to the reference axis, and material types are given in Tables 1a and 1b. Dead load for the box-girder model is 1.80 kips/ft, and is applied as concentrated forces at nodes in the finite element model.

Material behavior is idealized by a second-degree nonlinear stress-strain relationship for concrete [4], and a bilinear curve for each type of reinforcing steel.

Table 2 lists the properties for the different material types. The external tendons are divided into elements that span between anchorage locations and deviation points at the diaphragms, and have a bilinear stress-strain relationship (Fig. 7).

The external tendon geometry and prestressing forces used in the finite element analyses are presented in Fig. 8. Data for the tendons are tabulated below each span. Two lines of data are presented for each tendon. Each entry of data in the top line corresponds with the vertical distance of the tendon below the reference axis (Fig. 6) at deviator locations. Data in the second line correspond with the tendon forces between deviators. Tendon force is constant between deviator locations because tendons are not attached to the girder between adjacent deviator locations or between anchorage and deviator locations.

Failure at a joint-opening location corresponds with crushing of concrete in the compression flange. Concrete crushing is assumed to occur when an average stress of 0.85$f_c$ acting over a rectangular area defined by the width of the compression flange and a depth equal to $\beta_1 c$ (as defined in ACI 318-83 [8]) is needed to satisfy equilibrium at a joint.

Computed Behavior of Segmental Versus Monolithic Construction

Behavior of the segmental girder is compared to the behavior of a similar monolithic girder where the reinforcement is assumed continuous except through the closure strips. Both structures are subjected to a concentrated applied load on the interior midspan. The load is incremented up to failure which is manifested by either the crushing of concrete at a joint (limiting concrete stress is reached) or by the inability of the computer solution to achieve equilibrium.

The segmental girder fails when the interior midspan joint opens to approximately 94% and the average concrete compressive stress exceeds its limiting value at the joint.
However, the monolithic girder first forms a hinge at midspan when the corresponding prestressed and non-prestressed reinforcement yields. Then, a mechanism develops when two more hinges form at the closure strips of the interior span because concrete reaches its limiting strength at the bottom fibers of the girder.

The applied load versus displacement response, and response of one of the tendons at interior midspan for both girders are plotted in Fig. 9 and 10. The maximum load capacity, deflection, and tendon stress are 56 kips, 0.67 in., and 194 ksi, respectively for the segmental girder. In contrast, the maximum load capacity, deflection, and tendon stress for the monolithic girder are 108 kips, 2.8 in., and 245 ksi.

The low strength and ductility of the externally prestressed segmental girder can be attributed to the unbonded tendons and dry joints. Because the tendons between deviation points are unbonded, the strain in the tendon is constant along its length, and the strain at the critical section is less than what it would be for internally bonded tendons under similar conditions. Hence, the stress in the tendons increases very little so that when the crushing strain has been reached in the concrete, stress in the prestressing steel is far below its ultimate strength (Fig. 10). Also, the existence of joints tends to concentrate compressive stresses in the concrete at the joints, resulting in “premature” failure.

In the monolithic girder, multiple cracks form in the interior midspan. The absence of the joints eliminates the problem of concentrated compressive stresses because the cracks are not as deep as the joint openings in the segmental girder. Also, the increased size of the compression zone allows stress in the tendons to increase to yield at an applied load of 100 kips. Furthermore, the improved section behavior afforded by distributed cracking also results in a large increase in ductility.

The degree of ductility reduction indicated for the segmental girder should be viewed with some amount of caution because of potential errors associated with the assumptions incorporated in the analysis of the opening joint near ultimate loads. When the joint-opening width becomes large, it is unlikely that the joint section can be considered plane any longer. Also, any overestimation of joint stiffness increases the fraction of the joint that opens at a given load, and results in a concentration of compressive stresses in the flange that leads to joint failure at a reduced load.

**Computed Behavior of Segmental Girder with Bonded Internal Tendons**

The segmental girder is considered here with bonded internal tendons in the top and bottom flanges of the segments. These tendons are composed of the same material as the external tendons, with a total cross-sectional area of 0.68 sq. in. in the top flange and 0.34 sq. in. in the bottom flange. The tendons are located at 1.3 in. from the top and bottom of the section, and are stressed to 185 ksi (before elastic shortening). In the analysis, the internal tendons are approximated by end forces and two additional steel fibers in each segment. The response of the girder with internal tendons is compared to the response of the segmental girder in Fig. 11 and 12. The addition of bonded internal tendons results in an increase in capacity (from 56 to 65 kips), and a slight reduction in maximum midspan deflection. The internal tendons increased the compression stress acting over the entire girder cross section which slightly reduced the initial stress in external tendons at midspan, and substantially reduced internal tendon stresses at ultimate (See Fig. 12). The added internal tendons also reduced the width of opened joints.
CONCLUSIONS

Finite element analysis of an externally post-tensioned segmental box girder yields the following conclusions:

1. The segmental box girder with external tendons and dry joints (no epoxy) between segments exhibits reduced strength and ductility when compared with a similar monolithic girder. The reduced strength and ductility are attributed to the unbonded tended lengths and dry joints.

2. The monolithic box girder develops multiple cracks in the midspan region. The deformation capacity available at each crack location permits tendon deformations between deviation points (location of tendon attachments) large enough to yield the tendons.

3. Supplemental bonded internal tendons improve flexural strength and slightly reduce ductility. Behavior of the girder with supplemental bonded tendons indicates the utility of keeping joints closed as long as possible.

ACKNOWLEDGEMENTS

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REFERENCES


8. American Concrete Institute Committee 318, "Building Code Requirements for Reinforced Concrete (ACI 318-83), American Concrete Institute, Detroit, Michigan.
Table 1 Fiber Data

(a) Typical Segments

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* See Table 2
C = Concrete; S = Steel

(b) Pier Segments

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* See Table 2
C = Concrete; S = Steel
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* See Table 1

$\epsilon_{co} = \text{ultimate concrete strain}$

$\epsilon_{su} = \text{ultimate steel strain}$
Figure 1. Finite element model for a simple-span, segmental girder with external tendons.

Figure 2. External tendon element
Figure 3. Joint element model

Figure 4a. Gap between segments

Figure 4b. Force-displacement relationship for the joint
Figure 5. Model dimensions
Figure 6a  Typical segment fiber model

Figure 6b  Pier segment fiber model
Figure 7. Stress-strain relationship for external tendons

Figure 8. Tendon geometry and prestressing forces
Span B

Tendon B: 10-3/8" dia., $A_{ps} = 0.85\text{ in.}^2$

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Span C

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<td>146</td>
<td>150</td>
<td>154</td>
<td>159</td>
<td>160</td>
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</table>

Tendon CB: 4-3/8" dia., $A_{ps} = 0.34\text{ in.}^2$

<table>
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<tr>
<th>y (in.)</th>
<th>-2.85</th>
<th>-1.91</th>
<th>5.75</th>
<th>5.75</th>
<th>5.75</th>
<th>-1.15</th>
<th>-2.0</th>
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<td>62.5</td>
<td>63.6</td>
<td>64.0</td>
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</table>

Figure 8. Tendon geometry and prestressing forces (continued)
Figure 9. Applied load-displacement curve at interior midspan

Figure 10. Applied load versus interior midspan stress of tendon B
Figure 11. Comparison of the load-deflection response for segmental girders with and without internal tendons

Figure 12. Comparison of tendon stress-applied load response for segmental girders with and without internal tendons
Computer Evaluation of the Effect of Prestressing on a Shell Structure

by J.M. Nagele, S.C. Das, and R.M. Bakeer

Synopsis: The purpose of this investigation is to examine the forces developing in the dome of a prestressed concrete nuclear containment structure due to prestressing of vertical tendons. A computer program was developed for personal computers to evaluate the effect of prestressing forces. A parametric study was conducted, using the program, to examine the effect of dimensions, prestressing force, properties of tendons, ..... etc. on the resulting stresses in the dome.

Keywords: computer programs; domes (structural forms); evaluation; jacking; nuclear reactor containment; prestressed concrete; prestressing; prestressing steels; shells (structural forms); structural analysis
INTRODUCTION

A program was developed for personal computers to determine the forces induced by the prestressing of vertical tendons of the dome of a concrete nuclear containment structure. The structure to be analyzed consists of a cylinder topped with a hemispherical dome. A total of 42 mutually perpendicular, two-way, U-shaped, vertical prestressing tendons are anchored below the foundation slab of the structure in tendon galleries, as shown in Fig. 1. These tendons run vertically through the cylindrical wall and continue through the dome down to the other side of the wall where these tendons are also anchored. The tendons form a series of adjacent semicircles over the dome with the largest semicircle near the apex of the dome. Fig. 1 shows a cross-section of the structure under consideration, and Fig. 2 shows the plan view of the arrangement of the 42 U-shaped vertical tendons. This particular arrangement is economical because it introduces the prestress in the dome and wall simultaneously and therefore reduces the number of required anchorages. Even though horizontal tendons were also used in the cylindrical part of the structure, the focus of this paper is on the forces in the dome part of the structure and only the effect of prestressing of vertical tendons is discussed herein. The computer program [2, 5] was developed to determine the forces in the vertical tendons while considering curvature and friction losses. Losses due to length and wobble effects are neglected in this study because these losses are relatively small in magnitude when compared with friction losses [2]. Therefore, the prestressing losses of the tendons in the cylinder are neglected, and only friction losses in the dome are considered.

THE COMPUTER PROGRAM

The computer program is written in FORTRAN 77 for use on IBM type personal computers with 640 KB memory. The program is compiled using the IBM Profes-
Prestressing

The prestressing of tendons in a prestressed concrete shell yields normal pressures acting toward the center of the dome and frictional forces acting along its curved surface. Fig. 3a shows the radial forces acting on the dome, whereas Fig. 3b shows the forces due to curvature and the change in direction of these forces at an angle $\alpha'$. The program computes the normal and frictional forces in terms of cylindrical coordinates $(r, \theta, z)$ and then translates these forces into spherical coordinates $(R, \phi, \theta)$. The three components of pressure in spherical coordinates are the radial, circumferential and meridional pressures. The radial pressures act normal to the curved surface, whereas the circumferential and meridional pressures act in the plane of the curved surface. However, the circumferential pressures are in the direction perpendicular to the tendons, and the meridional pressures are in the direction parallel to the tendons.

**ANALYSIS OF THE DOME FORCES**

The dome is divided into several regions defined by the user in the input for the computer program. Due to symmetrical vertical tendon layout in this particular structure, only one-fourth of the hemispherical dome needs to be analyzed. The section considered in the analysis, equivalent to a planar sector of circle with an angle of $\pi/2$, is divided into horizontal rings and vertical slices, as shown in Fig. 4. To facilitate direct numerical integration in the program, the segment length of a region is chosen to be less than $\sqrt{Rt}$, where $R$ and $t$ are the minimum average radius and the thickness of the shell, respectively. The quadrilaterals formed by these intersecting lines are defined as regions. These regions are numbered from the starting point, where the azimuth angle, $\theta$, equals 0.0, and the colatitude angle, $\phi$, is equal to $\pi/2$. The numbering proceeds to values of $\theta$ of $\pi/2$ and $\phi$ of $\pi/2$ and up to $\phi$, equal to 0.0. However, the regions within the two rings at the apex are considered as one because the area of these individual regions becomes infinitesimal. The program determines the angular boundaries of each region and calculates the corresponding areas [2].

The friction forces are calculated as well as the normal forces for increments along the curved vertical tendons. The radial, circumferential, and meridional pressures are then calculated and transformed into terms in a Fourier series. With the pressure on the
shell described as a Fourier series, a finite element program, such as Kalnin's "Static Analysis of Thin Elastic Shells of Revolution" [9], can be used for the computer analysis. Detailed description of the mathematical calculations involved in the program are available in the Ref. [2].

SELECTION OF THE PROGRAM INPUT

The computer program was originally developed to calculate the forces in a specific shell. The characteristics of that shell are used in this study as the control parameters. Several executions of the program were conducted to investigate the effect of various parameters on the resulting prestressing forces in the dome. In all cases, only one parameter was changed at a time with the remaining parameters defined as their control values. Table 1 shows the different values of the various parameters investigated in this study as well as their control values, (bold values in parentheses). The upper and lower bounds of the parameters shown in Table 1 are selected according to the guidelines provided in Ref. [1, 3, 4, 6, 7]. The number of Fourier harmonics was not varied, because this parameter has no effect on the calculation of the pressures due to prestressing.

RESULTS OF THE ANALYSIS

Selected regions in the dome are chosen at various locations on the grid to demonstrate the results of the analysis. These regions are numbers 11, 36, 50, 75, and 107 shown in Fig. 4. Variations in the values of radius, friction coefficient, ultimate strength, number of tendons, coefficient of jacking stress and coefficient of lock-off stress versus radial pressure were plotted on the semi-logarithmic graphs shown in Fig. 5 through 8. The colatitude and azimuth pressures on regions 11 and 107 are tabulated in Tables 2 and 3 for the various parameters under investigation. Tables 1, 2 and 3 are interrelated where Table 1 shows the values of the different parameters under investigation, whereas Tables 2 and 3 show the effect of varying one particular parameter at a time on the resulting pressures in the dome. Columns 2 through 5 in Tables 2 and 3 show the resulting pressures in the dome when using the four values of the indicated parameter in the first column along with the control values of the other parameters. For example, the first row of Table 2 shows the circumferential pressures in region 11 of the dome for radii of 50, 60, 72 and 85 feet, using 42 tendons with ultimate strength of 2230 ksf, a friction coefficient of 0.3, a coefficient of jacking stress of
0.85, a coefficient of lock-off stress of 0.7, 5 Fourier harmonics and a number of divisions of 9 and 14 in the azimuth and colatitude directions, respectively.

In all cases, the effect of the changing parameter on the radial pressure was similar to the effect on the circumferential and meridional pressures. The results shown in Fig. 5 and Tables 2 and 3 show that by increasing the radius of the dome, the radial pressures decrease. This is true because the increase in radius increases the areas of the individual regions and the force on each region is then distributed over a larger area resulting into lower pressures.

The friction coefficient has a greater effect on the regions near the apex than the other parts of the dome, as shown in Fig. 6. The lower regions, numbers 11 and 36, show a negligible variation in radial pressure with the variation in the friction coefficient. However, for the upper regions near the apex, the pressure decreases with increasing the friction coefficient. This decrease in pressure is due to the greater frictional losses in prestress forces along the curved surface of the dome.

Fig. 7 demonstrates that when using tendons with higher ultimate strength, the induced pressures increase because, for greater strengths, the tendons are stressed more.

The effect of the number of tendons on the radial pressures is presented in Fig. 8. Although there is some random scatter of the data, the best fit lines are positively sloping. The graph shows that by increasing the number of tendons, the induced force in a given region increases. For one stressed tendon passing through a particular region, a force is induced in that region. On the other hand, if two stressed tendons pass through a region, then the force induced in that region doubles. A direct comparison can be made from the resulting pressures for the same dome when using either 25 and 50 tendons. The radial pressure on region 11 for 25 tendons is 5.3 ksf while the radial pressure for 50 tendons is 10.6 ksf for a direct ratio of 1 to 2.

The coefficients of maximum jacking stress and lock-off stress have only a minor effect on the forces induced by prestressing. There is no more than a 15 percent increase in the pressures at the different regions due to changing the coefficient of jacking.
stress from 0.75 to 0.90. For the increase in coefficient of lock-off stress from 0.65 to 0.80, the increase in pressure is less than 14 percent.

The number of divisions in the azimuth angle greatly affects the pressures. When the dome is divided into 4 sections in the azimuth direction, region 18 encompasses a given area. Regions 35 and 36 of the same dome divided into 8 sections includes the same area as region 18 of the 4 sectioned dome. Likewise, regions 52, 53, and 54 of the same dome divided into 12 azimuth divisions also describe the same area of the dome surface. The resulting pressures in these regions are listed in Table 4. The pressures in regions 35 and 36 are, in general, greater than the pressure in region 18. Similarly, the pressures in regions 52, 53, and 54 are nearly triple the pressure in region 18. This is true because the same force is applied to the given area identified by these regions. The force is then divided into two or three regions and then divided by the individual areas. Thus, ideally the same pressure would be applied to this given area defined by multiple regions. Therefore, the pressure in a given area is directly proportional to the number of regions into which the area is broken.

As before, selected regions occupying the same area are chosen for comparison. Region 20 of the dome divided into 6 colatitude sections, regions 38 and 47 of the dome divided into 12 colatitude sections, and regions 56, 65, and 74 of the dome divided into 18 colatitude sections all describe the identical area of the dome surface. The azimuth direction is defined by angle $\theta$ and the colatitude direction is defined by the angle $\phi$, as shown in Fig. 4. Table 5 presents the pressures induced in these regions. A direct relationship cannot be identified as easily as with the variations in azimuth divisions. Dividing a given area by two increases the total pressure in that area. However, the ratio of azimuth divisions to pressure is not consistent due to the curvature of the dome and the resulting friction losses. Dividing the region into smaller areas describes the curvature of the dome more accurately as the smaller regions would more accurately define the pressures in that area.

CONCLUSIONS

The results of the parametric study highlight the relationship between the different input variables and the pressures produced by prestressing of the vertical tendons. For each region the resultant prestressing
force was identified by forces in the radial, circumferential, and meridional directions. Parameters which increased pressure on the regions include increasing ultimate tendon strength, number of tendons, coefficient of jacking stress, and coefficient of lock-off stress. A decrease in pressure resulted when the radius increased or the coefficient of friction increased. The results of the program for various azimuth and colatitude angle divisions showed that the divisions in the colatitude angle are more critical. In the analysis of a dome, it should be divided into a large number of colatitude rings to ensure a greater accuracy. The friction losses would then be more accurately defined, and thus, a more precise output of pressures would result. The results of the program are useful in predicting the outcome of the pressures of the tendons of a prestressed concrete containment shell. During design and construction phases of these structures, important changes can be made. The computer program provides a quick method of determining the pressures applied to a shell taking into account the various parameters such as the radius, friction coefficient, ultimate tendon strength, number of tendons, coefficient of jacking stress, coefficient of lock-off stress, divisions in colatitude angle, and divisions in azimuth angle.

ACKNOWLEDGMENT

Part of the work described in this paper in connection with the nuclear containment structure was performed by the second author in conjunction with other engineers at the firm of Brown & Root, Inc., Houston, Texas during the development stage of the South Texas Project in 1972.

REFERENCES


4. Murray, D.W., "Observations on Analysis, Testing and Failure of Prestressed Concrete


TABLE 1--PARAMETERS CONSIDERED IN THE STUDY

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
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<td>Radius of Dome (feet)</td>
<td>50 60 (72) 85</td>
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<tr>
<td>Friction Coeff.</td>
<td>0.08 0.15 (0.3) 0.35</td>
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<td>Ult. Tendon Strength (ksf)</td>
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<tr>
<td>No. of Tendons</td>
<td>25 (42) 50 60</td>
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<tr>
<td>Coeff. of Jacking Stress</td>
<td>0.75 0.80 (0.85) 0.90</td>
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<tr>
<td>Coeff. of Lock-off Stress</td>
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<tr>
<td>Div. in Colatitude Angle</td>
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<td>No. of Fourier Harmonics</td>
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(xxx) Control Value of Parameter

TABLE 2--PRESSURES IN REGION 11

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<th>CIRCUMFERENTIAL PRESSURES (ksf)</th>
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</thead>
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<td>Coeff. Lock-off</td>
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* Only Radius is changed and other parameters have their control values.
### TABLE 3--PRESSURES IN REGION 107

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* Only Radius is changed and other parameters have their control values.

### TABLE 4--PRESSURES FOR VARIATION IN AZIMUTH DIVISIONS

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<td>52</td>
<td>-7.38</td>
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<td>53</td>
<td>-11.57</td>
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<tr>
<td></td>
<td>54</td>
<td>-15.12</td>
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TABLE 5--PRESSURES FOR VARIATION IN COLATITUDE DIVISIONS

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<td>12</td>
<td>38</td>
<td>-6.69</td>
<td>2.36</td>
<td>1.22</td>
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<td>47</td>
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<td>5.66</td>
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<td>18</td>
<td>74</td>
<td>-12.26</td>
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Fig. 1--Cross-section of Prestressed Concrete Nuclear Containment Structure.
Fig. 2--Plan of Vertical U-Shaped Tendon Layout.

Figure 3. Variation in Tendon Stresses.
Fig. 4--Identification of Regions.

- Azimuth direction is defined by angle $\Theta$
- Colatitude direction is defined by angle $\phi$

Fig. 5--The Effect of Radius on Radial Pressure.
Fig. 6--Effect of Friction on Radial Pressure.

Fig. 7--Effect of Ultimate Tendon Strength on Radial Pressure.
Fig. 8--Effect of Number of Tendons on Radial Pressure.
Synopsis: In the near future prestressed concrete structures will in Europe be designed according to Eurocode 2 (EC2). The EC 2 principles governing the design of structures with bonded tendons and with external tendons are given in this paper, and a comparison is made between the structural behaviour of the two different types of structures. It is shown that the reliability of both is comparable if the characteristic strain due to prestressing \((\varepsilon_{\text{p}} + s)\) is introduced in the calculations.

Keywords: building codes; limit state design; prestressed concrete; prestressing; safety factor; serviceability; stress-strain relationships; structural design; unbonded prestressing
Eilhard Woelfel is Dr.-Ing. from the Technical University Aachen. He is Head of the Department of the "Institut fuer Bautechnik" in Berlin (FRG). This Institut has the role of coordination in the field of Civil Engineering in Germany and takes part in these efforts in Europe.

The Eurocodes

The Eurocodes are being drafted to serve in the European Community as the officially recognized means to prove the compliance of buildings and civil engineering work with the legal requirements for structural safety and serviceability. They shall promote the functioning of the Internal Market by removing obstacles rising from differing rules for planning and design. A total set of 9 volumes is envisaged for the Eurocodes.

Eurocode 1 - Common unified rules for different types of construction and material;

Eurocode 2 - Concrete structures;
Eurocode 3 - Steel structures;
Eurocode 4 - Composite steel and concrete structures;
Eurocode 5 - Timber structures;
Eurocode 6 - Masonry structures;
Eurocode 7 - Foundations;
Eurocode 8 - Structures in seismis zones;
Eurocode 9 - Actions (design loads) on structures.


Rules for application for bridges will be elaborated later on in part 2. This section will also contain the design rules for external tendons.

For the moment, the author can only give his personal view on how to proceed with the design of structures with external tendons on the basis of the Principles of EC 2. As a matter of the fact, some proposals will be amended during the ongoing discussion,
and some figures will change. This paper should be a basis for these discussions, in the ACI-meeting and later on in the EEC-Commissions.

**Safety concept in Eurocode 2**

The design equations in EC 2 are written in a form:

\[ \gamma_F \cdot S = R \left( \frac{f_s}{\gamma_s}, \frac{f_c}{\gamma_c} \right) \]

The partial safety factors \( \gamma_F \) and \( \gamma_M \) are derived from statistical considerations and are influenced from:

- the limit state considered
  (higher safety factors for Ultimate Limit State than for Serviceability Limit State)

- the scatter of the parameters
  (higher standard deviations lead to higher safety factors)

- the statistical definition of the parameters
  (mean value or p-percent fractile)

- compromises, which have to be made in order to have a relatively small set of different factors.

The following limit states are considered:

- **Serviceability Limit State (SLS)**, which comprises checking of:
  - crack width limitation
  - deflection
  - vibration
  - opening of joints of segmental construction
  - stresses in the prestressing steel

Concrete Stresses need not to be checked in SLS because serviceability is not affected by the stress in the concrete, provided that sufficient safety against failure in the ultimate limit state is provided for.

- **Ultimate Limit State (ULS)** with the checks of:
  - normal forces (including stability)
  - bending moments
  - shear, torsion, punching
  - fatigue (requires special safety considerations)

The parameters influencing the relevant limit
states are described by:

- their characteristic (reference) value which is the mean value of parameters whose scatter is relatively small (e.g. selfweight) or a certain fractile in the other cases (usually 5 % for material properties and 99 % for actions)

- their standard deviations respectively the variation coefficient \( v \).

Regarding prestressing effects, two characteristic values are mentioned in EC 2: \( P_{u} \) and \( P_{k1} \) which (according to the author's mind) are estimated as \( \pm 17 \% \)-fractiles \((P_{\text{mean}} \pm 1.0 \cdot \sigma\) for normal distributions). Reliability studies using this characteristic value of prestress show that sufficient reliability can be achieved by applying the safety factor for prestressing equal to unity \([4]\).

The set of safety factors for the SLS and the ULS is given in tables 1 and 2. The values taken from reliability studies \([4]\) are shown in brackets for demonstration. When comparing the safety factors for prestressing in ULS, one has to take into account that the reference value in EC 2 is the mean value whereas the author uses the 17 %-fractile as reference value. The factors \( \gamma_{F} \) (for self weight and live load) and \( \gamma_{M} \) contain a partial coefficient taking account of the uncertainty of the calculation which (in ULS) is chosen as 1.05.

The material safety factors (table 2) are identical regardless of the limit state.

Eurocode 2 part 1 deals with concrete structures only as far as they are prestressed by bonded tendons. Structures with unbonded tendons (internal and external) shall be treated in part 2 of EC 2 which will be prepared on the basis of the principles given in EC 2 part 1.

The question is whether the same safety elements should be chosen for both bonded and unbonded tendons. Before this question will be answered, the main differences in the design between bonded and unbonded tendons shall be discussed.

The treatment of prestress in the design calculation

In the following section, the effects of prestressing shall be treated first on the basis of
strains which are induced in the steel by tensioning of the tendons and which change locally due to friction and time due to time dependant strain variations (see fig. 1). At the time \( t = 0 \) and at the active end of the tendon \( x = 0 \), the tendon is strained to \( \varepsilon_{\text{pre}, \text{max}} \). The real value will not deviate very much from the calculated value, the standard deviation is small and can be disregarded.

At \( t = t \) and at any point \( x \) of the tendon, the steel strain will decrease by \( \Delta \varepsilon_p \) due to friction, creep, shrinkage and relaxation. These strain losses scatter considerably, and it is difficult to predict them. Estimated standard deviations of these strain losses are given in table 3 [2][3]. Taking account of the higher and lower fractile of the strain, the upper and lower fractile of the stress and thus of the tendon force in SLS can be read from the stress-strain-line of the prestressing steel.

Going from SLS to ULS, a strain increase due to the load increase will be observed in the tendon. Regarding a certain section of a structure with bonded tendons, the strain increase of the tendon \( \Delta \varepsilon \) crossing the crack is considerable (\( \approx 10 \% \) acc. to EC 2) and is to be added to the strain in SLS. Going from the upper and lower value of the strain into the stress-strain diagram, the corresponding stress is (for both strain values) \( f_k / \gamma_s \). The ultimate capacity of statically determinate structural elements is - as a rule for elements with bonded tendons - not influenced by the scatter of the prestress.

Regarding the whole span of a structural element (e.g. in order to calculate the statically indeterminate part of the prestressing action), the medium strain increase over the length \( \int \Delta \varepsilon dx / L \) is considerably smaller than the strain increase at a discrete section. The overall value of the strain will therefore not reach the yield strain of the steel; the statically indeterminate part of the prestressing action is therefore influenced by the scatter of the steel strain.

In structural elements with unbonded tendons, the strain increase of the tendons is due to the integrated deformation of the structure between the anchorages of the tendon and is therefore small. The overall strain in ULS will not reach the yield point. Thus, the stresses in unbonded tendons in ULS scatter in the same sense as in SLS.
The design of structures with unbonded tendons in ULS must take account of the uncertainties of the prestressing force whereas in structures with bonded tendons in ULS, only the statically indeterminate part of the prestressing action is influenced.

This difference in the mechanical behaviour of the structure has major consequences: whereas the level and the scatter of the prestress do not have a big influence on the reliability of a structure with bonded tendons in ULS, this influence for structures with unbonded or external tendons is very great. Thus, the engineer designing a structure with unbonded tendons must be interested in:

- reducing the prestressing losses,
- reducing the scatter of the losses respective to the increase reliability of their prediction
- designing rules such that these efforts lead to a reduction of costs.

Under these points of view, it shall be discussed whether the design rules in EC 2 are also adequate for structures with unbonded external tendons.

Possible variation of the tendon force

The variations of the tendon force will be influenced by:

- the type of the structural element (e.g. bridge deck or beam)
- the tendon profile
- the scatter of the mechanical parameters with influence on the prestressing losses (e.g. friction or creep coefficient)

For the comparison between internal bonded and external tendons, a special example of a beam will be regarded with adequate tendon layouts. For this example, it shall be shown whether fundamental differences between the stress variations of bonded and external unbonded tendons exist or not.

The main parameters of this example are:

| tendon length | 80 m |
distance between the section regarded and the active end of the tendon (one end stressing) 

50 m

sum of deviation angles 

35 °

unintentional angle of deviation (only internal tendons) 

50 • 0.005 = 15 °

mean concrete strain due to creep at the level of the tendon 

for an internal tendon 

0.7 • 10^{-3}

for an external tendon 

0.6 • 10^{-3}

The expected value of the strain losses are calculated on the basis of these assumptions (see table 3).

The comparison shows:

- The losses of prestrain of an internal tendon are about 1.5 times higher than for an external tendon. This is mainly due to higher friction losses caused by a higher friction coefficient and unintentional angles of deviation of the cable layout.

- The standard deviation of the losses is also about 1.5 times higher for internal bonded tendons than for external tendons.

The variation coefficient \( v \) of the prestrain related to the overall strain is the quotient:

\[
v (\varepsilon) = \frac{s}{\varepsilon_{p,\text{xt}}} = \frac{s}{\varepsilon_{p,\text{t}} - \Delta \varepsilon_p}
\]

With the figures of table 3, the results for the example are:

\( v = 8.9 \% \) for internal, bonded tendons

\( v = 3.8 \% \) for external tendons

Since the assumptions for the comparison have been chosen rather conservatively, it may be generally assumed:

\( v \ll 10 \% \) for internal, bonded tendons

\( v \ll 5 \% \) for external tendons.
Recommendations for the incorporation of the prestress in the design

In the Serviceability Limit State and in the Ultimate Limit State, the characteristic values of the tendon force shall be calculated from the stress-strain curve using an upper and lower fractile of the strain in the tendon. The safety factor on $\Delta \varepsilon_p$ is 1.0.

The fractiles of the tendon strain may be assumed to be:

$$\varepsilon_{pk} = \varepsilon_{pxt} \pm 1.0 \cdot s = 1.1 \quad 0.9 \quad \varepsilon_{pxt} \quad \text{for internal tendons}$$

$$= 1.05 \quad 0.95 \quad \varepsilon_{pxt} \quad \text{for external tendons}$$

The stress-strain curve is limited to a value $f_{yk}/\gamma_s$.

Literature


### Notation

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tbody>
<tr>
<td>A</td>
<td>area</td>
</tr>
<tr>
<td>L</td>
<td>span</td>
</tr>
<tr>
<td>P</td>
<td>tendon-force</td>
</tr>
<tr>
<td>R</td>
<td>resistance (bearing capacity)</td>
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<td>S</td>
<td>action effect</td>
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<td>f</td>
<td>strength</td>
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<td>p</td>
<td>fractiles</td>
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<td>s</td>
<td>standard deviation</td>
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<tr>
<td>v</td>
<td>variation coefficient</td>
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\[
\bar{x} \quad \{ \text{mean value of } x \}
\]

<table>
<thead>
<tr>
<th>Symbol</th>
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<tr>
<td>(\gamma)</td>
<td>safety-factor</td>
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<tr>
<td>(\varepsilon)</td>
<td>strain</td>
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<tr>
<td>(\Delta\varepsilon)</td>
<td>strain-variation</td>
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<td>(\sigma)</td>
<td>stress</td>
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### Indices:

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<thead>
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<tr>
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<td>k</td>
<td>characteristic</td>
</tr>
<tr>
<td>l</td>
<td>lower characteristic value</td>
</tr>
<tr>
<td>o</td>
<td>zero (eg time = 0)</td>
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<tr>
<td>p</td>
<td>prestressing, prestressing steel</td>
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<td>rel</td>
<td>relaxation</td>
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<tr>
<td>s</td>
<td>steel</td>
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<td>sh</td>
<td>shrinkage</td>
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<tr>
<td>t</td>
<td>time</td>
</tr>
<tr>
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<td>yield point</td>
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<td>force</td>
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<td>Q</td>
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Table 1: Safety factors $\gamma_F$ for actions

<table>
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<tr>
<th>acting</th>
<th>$S_L S$</th>
<th>$U_L S$</th>
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<tr>
<td></td>
<td>un-</td>
<td>favorable</td>
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<tr>
<td></td>
<td>unfavorable</td>
<td></td>
</tr>
<tr>
<td>self weight</td>
<td>1.0</td>
<td>1.0</td>
</tr>
<tr>
<td>($G_{\text{mean}}$)</td>
<td>(1.04)</td>
<td>(0.96)</td>
</tr>
<tr>
<td>prestressing</td>
<td>$1.1 P_{\text{mean}}$</td>
<td>$0.9 P_{\text{mean}}$</td>
</tr>
<tr>
<td></td>
<td>$(1.0 P_{k,s})$</td>
<td>$(1.0 P_{k,l})$</td>
</tr>
<tr>
<td>live load</td>
<td>1.0</td>
<td>-</td>
</tr>
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Table 2: Material-safety factors $\gamma_M$

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<th>prestressing steel</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma_M$</td>
<td>1.5</td>
<td>1.15</td>
<td>1.15</td>
</tr>
</tbody>
</table>
### Table 3: Losses of prestrain for internal and external tendons.

1) $s = \text{Standard deviation acc. [2], [3]}$

2) $s = \sqrt{s_1^2 + s_2^2 + \ldots + s_n^2}$
Fig 1: Strains $\varepsilon$ and stresses $\sigma$ in tendons
Reliability Analysis of Externally Prestressed Concrete Bridge Girders

by A.S. Nowak, A.E. Naaman, and S.-C. Ting

Synopsis: Evaluation of existing bridges is an important part of the strategy of dealing with the deteriorating infrastructure. Load and resistance parameters are random variables because of uncertainties in load components, material properties and dimensions. Therefore, the reliability of a structure is a convenient measure of its performance. The load and resistance models are first summarized. The major load components in bridges are dead load and live load. The live load model is based on the weigh-in-motion studies. Girder distribution factors were derived using special computer procedures for bridge analysis. Behavior of composite girders is considered using a nonlinear model. The basic characteristic of the section is the moment-curvature relationship. The reliability is measured in terms of the reliability index. The approach is demonstrated on evaluation of a prestressed concrete girder bridge. Three cases are considered: original design condition, damaged with corroded strands and repaired by external prestressing. The load components and load carrying capacities were evaluated and then the reliability indices were calculated for the three cases.

Keywords: bridges (structures); damage; deterioration; evaluation; girders; loads (forces); prestressed concrete; prestressing; reliability; repairs; serviceability; strengthening; structural analysis; unbonded prestressing
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A. E. Naaman is a Professor of Civil Engineering at the University of Michigan. His major areas of expertise include partially prestressed concrete, external prestressing and high performance fiber reinforced cement composites. He is an ACI Fellow and a member of committees 363, 446, 544, 549, 343 and 423.

S.-C. Ting is a doctoral candidate at the University of Michigan. He has an extensive experience in the design of concrete structures. The topic of his doctoral dissertation is evaluation of the effect of corrosion on the reliability of concrete girder bridges.

INTRODUCTION

Evaluation of existing bridges is an important part of the strategy to improve the U.S. infrastructure. There are nearly 600,000 highway bridges in the United States. About 40 percent are classified, according to the Federal Highway Administration's (FHWA) criteria, as deficient and in need of rehabilitation or replacement. More than 100,000 of these are judged to be structurally deficient because of deterioration or distress, and another 100,000 are considered functionally obsolete or inadequate for current requirements. The major factors which have contributed to the present situation are: the age, inadequate maintenance, increasing load spectra and environmental contamination. FHWA currently estimates the cost of a bridge replacement and rehabilitation program at about 50 billion dollars.

The disposition of the deficient bridges involves decisions ranging from closing to traffic for replacement or repair, through posting, to unrestricted use. Each decision involves clear economical and safety implications. There is a need for efficient evaluation methods as well as repair and strengthening techniques. However, bridge loads and resistance parameters (dimensions, material properties) are random variables and involve a considerable degree of uncertainty. Therefore, the structural reliability is a convenient measure of structural performance. The reliability methods can be used to calculate the probability of failure on the basis of the available statistical data. On the other hand, external prestressing may be considered as a viable alternative to repair and/or strengthen prestressed concrete girder bridges.

The analysis of a bridge involves the modeling of loads and structural response. The parameters include axle weights and truck configurations, truck position on the bridge, stiffness of structural members, load sharing or interaction of members in resisting the loads, redundancies, rates of de-
terioration, and others. The recent developments in the area of structural analysis, material behavior, bridge load modeling and probabilistic methods may serve as a basis for rational criteria to determine the load carrying capacity, reliability and remaining lifetime of the structure. There are powerful methods available to determine the distribution of forces in the structure, such as the finite element method (FEM) with many variants.

Significant improvements have been reported in bridge load modeling. Extensive live load data is available, as a result of truck surveys and weigh-in-motion (WIM) measurements (1). There are also new results of material and component tests. Statistical models were developed to describe the bridge behavior under truck loading (2, 3). Probability theory has developed in structural engineering especially for reliability-based design codes (4).

Current bridge evaluation methods used in the United States are based on deterministic approach. The American Association of State Highway and Transportation Officials (AASHTO) Manual for Maintenance Inspection of Bridges (5) provides the procedures for most bridge inspection and evaluation. As in the AASHTO Standard Specifications for Highway Bridges (6), the adequacy of the structure is determined by comparing design stresses with allowable values. Although the distribution of load to components is the same for existing and newly designed structures, increased allowable stresses are sometimes permitted for existing bridges.

The objective of this paper is to review the available reliability methods, and load and resistance models for bridges and illustrate their application to a prestressed bridge strengthened by external prestressing. The major parameters which determine the performance of prestressed concrete girders are identified. A procedure is developed for calculation of the reliability of bridge girders in various conditions of deterioration or damage and after their repair by external prestressing.

STRUCTURAL RELIABILITY

All quantities (except physical and mathematical constants) that enter into engineering calculations involve uncertainties due to randomness of design parameters (e.g. loads, material properties), imperfect models or human errors. Because of these uncertainties, absolute reliability is not an attainable goal. Selection of the optimum reliability level is an economical problem. Lower reliability results in frequent failures, while higher reliability requires higher initial costs (materials and labor). Therefore, structural reliability is a convenient acceptability criterion in the development of design and evaluation criteria.

Classical reliability theory was developed during the 1940's for military applications and has evolved as a result of increasing need in the area of electronic, electrical, mechanical, and aerospace engineering. The the-
ory is available for predicting the expected life, expected failure rate, or expected time between breakdown of elements (systems), given some test or failure data for the system and/or its components. In structural engineering the probabilistic analysis have been developed mostly in the last 20 years. It has been applied to safety evaluation of structural members in buildings and bridges. Little has been done, however, for the analysis of whole structural systems. The available methodology is presented in several recent publications, e.g. (4).

A structure fails when it cannot perform its function any longer, otherwise it is safe. Limit states are the boundaries between safety and failure. In bridge structures failure is defined as inability to carry traffic. Bridges can fail in many ways, or modes of failure, by cracking, corrosion, excessive deformations (non catastrophic failure), by exceeding carrying capacity for shear or bending moment (collapse, catastrophic failure), by local or overall buckling, and so on. Some members fail in brittle manner, some are more ductile. In the traditional approach each mode of failure is considered separately. Safety is provided by specifying the design capacity much larger than the expected loads. Probabilistic methods allow for quantification of the safety reserve.

There are two types of limit states. Ultimate limit states (ULS) are mostly related to the bending capacity, shear capacity and stability. Serviceability limit states (SLS) are related to gradual deterioration, user's comfort or maintenance costs. The serviceability limit states such as cracking, fatigue, deflection or vibration, often govern the bridge design. The main concern is accumulation of damage caused by repeated applications of load (trucks). Therefore, the model must include the load magnitude and frequency of occurrence, rather than just load magnitude as is the case in the ultimate limit states (3).

A traditional notion of the safety limit is associated with the ultimate limit states. For example, a beam fails if the moment due to loads exceeds the moment carrying capacity. Let \( R \) represent the resistance (moment carrying capacity) and \( Q \) represent the load effect (total moment applied to the considered beam). Then the corresponding limit state function, \( g \), can be written,

\[
g = R - Q
\]

If \( g > 0 \) the structure is safe, otherwise it fails.

The limit state function and the associated probability of failure are clearly defined in case of the ultimate limit states.

In general, due to uncertainties in material properties, dimensions, workmanship, truck weights and occurrence rate, both \( R \) and \( Q \) are random variables. Therefore, the probability of failure, \( P_F \), is equal to,

\[
P_F = \text{Prob}(R - Q \leq 0) = \text{Prob}(g \leq 0)
\]
Let the probability density function (PDF) of $R$ be $f_R$ and PDF of $Q$ be $f_Q$. Then, $R - Q$ is also a random variable and it represents the safety margin. Typical PDF's for $R$, $Q$ and $R - Q$ are shown in Fig. 1. The probability of failure is equal to the shaded area.

Direct calculation of $P_F$ requires integration of convolution functions which is usually very difficult, if not impossible. Therefore, it is convenient to measure structural safety in terms of a reliability index, $\beta$. The reliability index is defined as a function of $P_F$,

$$\beta = -\Phi^{-1}(P_F)$$

where $\Phi^{-1} = \text{inverse standard normal (Gaussian) distribution function.}$ For example, $\beta = 3$ corresponds to $P_F = 0.0015$, $\beta = 4$ corresponds to $P_F = 0.00003$ and $\beta = 5$ corresponds to $P_F = 0.000003$.

There are various procedures available for calculation of $\beta$. These procedures vary with regard to accuracy, required input data and computing costs. The simplest case involves a linear limit state function (Eq. 1). If both $R$ and $Q$ are independent (in the statistical sense), normal (Gaussian) random variables, then the reliability index is,

$$\beta = \frac{\mu_R - \mu_Q}{\sigma_R^2 + \sigma_Q^2}$$

where $\mu_R = \text{mean of } R$, $\mu_Q = \text{mean of } Q$, $\sigma_R = \text{standard deviation of } R$ and $\sigma_Q = \text{standard deviation of } Q$.

If the parameters $R$ and $Q$ are not both normal random variables then Eq. 4 gives only an approximate value of $\beta$. In such general cases, the reliability index can be calculated using more advanced procedures, sampling techniques or by Monte Carlo simulations (7). For example, an iterative procedure is available, based on normal approximations to non-normal distributions at the so called design point. Sampling techniques allow for a reduction of computational effort. The basic statistical parameters (means and standard deviations) are evaluated from a limited number of simulations.

**BRIDGE LOADS**

The major load components of highway bridges are dead load, live load with impact, environmental loads, earth pressure, and abnormal loads. Each load group includes several subcomponents. The statistical models for the various bridge loads are based on available data and on results from special studies.

Further studies are required to establish the site-specific live load model, distribution of truck weight, multiple presence in one lane and in adjacent lanes, and transverse position of truck on the bridge. An additional area in need of further research is dynamic load. Further field
measurements are required to determine the effect of structural type and surface condition. Additional studies must be carried out in order to estimate the growth in future live loads.

Dead load, $D$, is the gravity load due to the self weight of the structural and nonstructural elements permanently connected to the bridge. Because of different degrees of variation, it is convenient to consider separately the weight of factory made elements (steel, precast concrete members) and weight of cast-in-place concrete members. It is assumed that $D$ is normally distributed. The bias factor (mean-to-nominal ratio) varies from 1.03 (factory-made members) to 1.05 (cast-in-place members), and coefficient of variation varies from 0.04 to 0.08 (2, 7).

Live load covers a range of forces produced by vehicles moving on the bridge and it includes a static part, $L$, and dynamic part, $I$. The effect of live load depends on wheel force, wheel geometry (configuration), position of the vehicle on the bridge (transverse and longitudinal), number of vehicles on the bridge (multiple presence), stiffness of the deck (slab), and stiffness of the girders.

Ghosn and Moses (1) developed a live load model to predict the maximum 50 year load on a bridge, in terms of bending moment in the girders. The model is based on weigh-in-motion (WIM) data. By the use of strain measuring instrumentation, existing bridges were used as equivalent static scales to measure vehicle weights. Tape switches and timers were included to determine the vehicle speed, axle spacing, and arrival time in each lane. Because the system was inconspicuous, truck drivers did not avoid the "weigh station" and hence the sample can be considered unbiased.

The major factors affecting the dynamic load on a bridge include surface condition (bumps, potholes), natural frequency of the bridge (span length, stiffness, mass), and dynamics of the vehicle (suspension, shock absorbers). It is practically impossible to predict the percentage of dynamic load contributed by each of these three factors. In addition, the dynamic load effect may be changed by superposition of several trucks (multiple presence) or even by several axles of the same truck.

The bias factors (mean-to-nominal ratios) for live load vary depending on span length. Coefficients of variation vary from 0.2 to 0.3.

**RESISTANCE MODELS**

The capacity of a bridge depends on the resistance of its components and connections. The component resistance is determined mostly by material strength and dimensions. The variation of resistance has been modeled by tests, observations of existing structures and by engineering judgment. The information is available for the basic structural materials and components. However, bridge members are often made of several materials (composite members) which require special methods of analysis. Verifica-
tion of the analytical model may be very expensive because of the large size of bridge members. Therefore, the available data about resistance is still incomplete.

Typical stress-strain curves for concrete, reinforcing steel and prestressing steel are shown in Figs. 2, 3 and 4, respectively.

The ultimate flexural capacity of prestressed concrete bridge girders can be derived using a nonlinear model of performance. A computer program has been developed (7). The statistical data is taken from (8) and (9). Two phases of the performance are considered: uncracked section and cracked section. The strains are assumed to be linearly distributed. The section is divided into a number of horizontal strips of small depth. For given strains, stresses are calculated using material stress-strain curves. The bending moment is calculated as the resultant of the internal stresses. The moment-curvature is obtained by incremental increases in load (bending moment).

The moment changes under cyclic loading (trucks). If the total bending moment, \( M_Q \), exceeds the cracking moment, \( M_{cr} \), the section cracks and the tensile strength of concrete is reduced to zero. The crack stays open any time \( M_Q \) exceeds the decompression moment, \( M_d \) (if \( M_Q < M_d \), then all concrete is compressed, if \( M_Q > M_d \) then the crack opens).

Moment-curvature relationships have been derived for typical AASHTO sections, strengths of concrete and levels of prestress. The results show that the mean-to-nominal ratio of the ultimate moment is 1.04 and the coefficient of variation is about 0.035. The coefficient of variation is very small because all sections are under-reinforced and the ultimate moment is controlled by the prestressing tendons.

EXAMPLE OF RELIABILITY ANALYSIS OF A DAMAGED GIRDER

The reliability analysis is demonstrated on a typical prestressed concrete composite bridge girder with a span of 80 ft (24.4 m). The bridge cross section is shown in Fig. 5. Each girder is a standard AASHTO Type IV girder with a cast-in-place reinforced concrete slab. The slab thickness is 8 in (20.3 cm). The bridge was unshored during construction. Both the precast beam and the slab are made out of normal weight concrete with unit weight of 150 pcf (23.6 kN/m³). An asphalt topping was added on top of the slab leading to an additional dead load of 250 plf (3.65 kN/m). The following properties were assumed in the initial design: concrete compressive strength of the precast girder \( f'_c = 5000 \) psi (35 MPa); compressive strength of the cast-in-place slab= 4000 psi (28 MPa). The design followed the AASHTO specifications with an HS20 truck live loading. It led to a prestressing force at midspan consisting of 28 strands at an eccentricity of 20.16 in (51.2 cm) from the centroid of the precast section.
The strands were 1/2 inch (12.7 mm) in diameter and had a specified ultimate strength of 270 ksi (1862 MPa). The computed effective stress in the steel after losses was 144 ksi (993 MPa). Details of the strand layout for the midspan and the support sections are given in Fig. 6. The steel profile is selected to have two draping points each at 30 ft (9.14 m) from the support. Twelve strands are draped and sixteen strands are straight.

Various limit states and conditions were considered: ultimate moment, cracking, maximal crack width, fatigue in concrete, fatigue in steel and live load deflection. Reliability indices were calculated using Eq. 4, with definition of $R$ given in Table 1, and $Q$ defined as applied moment (dead load, live load and impact).

The moment-curvature relationship for the composite girder was derived using the available statistical data on dimensions and material properties as reported in Table 2. The resulting average curve is shown in Fig. 7 as a solid line while the dashed lines correspond to the average plus or minus one standard deviation. The girder midspan moments were calculated for dead load, $M_D$, superimposed dead load, and live load plus impact $M_{L+I}$. Observed nominal and mean values with bias, standard deviation, and coefficient of variation are reported in Table 2. Corresponding values of the reliability index $\beta$ for ultimate strength and various serviceability limit states are reported in Table 3. Note that the probability of failure or survival with respect to one limit state can be estimated from the values of the reliability index. Typical examples are given in Table 4. It can be observed for instance that, at time of completion, the probability of girder failure by fatigue of concrete under normal loading conditions is less than three in ten millions. It is noted that the nominal moment carrying capacity of the girder is 5180 k-ft (7023 kN-m) and the safety factor representing the ratio of nominal resistance in bending to the maximum service moment was calculated as 2.00.

In this example it is assumed that due to some severe environmental exposure to corrosion, 8 straight strands from the layer of reinforcement closest to the bottom fiber were damaged, and snapped off. The analysis of the bridge in this damaged condition indicated that: 1) the dead load moment exceeded slightly the cracking moment, 2) the girder would crack under live load, 3) the cracks will open at each application of the loading, 4) the stress range in the prestressing strands due to the full application of live loads is high enough to raise concern about fatigue.

The safety factor given by the ratio of the nominal moment resistance of the damaged girder to the maximum service moment is 1.59. Clearly however, due to the cracking condition and the possible fatigue in the reinforcement the bridge needed repair. Application of the reliability analysis to the damaged girder led to the moment curvature relationship shown in Fig. 8. The analysis also led to a mean resistance of 3830 k-ft (5193 kN-m) and a corresponding reliability index for the ultimate strength limit.
state \( \beta = 6.94 \) (see Table 3). This is equivalent to a minute probability of failure (collapse) (Table 4). However, analysis of other limit states indicated a sure probability of cracking and a value of reliability index of 0.16 for fatigue indicating a probability of failure by fatigue close to 50% at two million cycles of full live load. Variations of stress increments in the concrete and the prestressing steel due to the application of live load are shown if Fig. 9 and 10. It can be observed that stress increments in the prestressing steel of the damaged girder can be significant and should be accounted for in the evaluation of repair strategies.

Although a conventional analysis of the damaged girder may lead to the decision to conduct emergency repair and close the bridge during repair, the reliability analysis allows the bridge engineer to make a more rational decision. In this example, since the reliability index (for ultimate) of the damaged structure is still larger than 3.5 (the minimum index required for new members), the bridge does not need to be closed, and repair can be made at a more reasonable time and cost.

Repair consisted of removing the spalled concrete cover, cleaning the damaged area, patching with a latex modified mortar to initial shape, and replacing the damaged strands by external prestressing. External prestressing was achieved using two Dywidag prestressing bars having a nominal area of 1.485 in\(^2\) (9.58 cm\(^2\)) each, and a nominal strength of 150 ksi (1034 MPa). Figures 11 and 12 illustrate the method of anchoring the bars to cast-in-place reinforced concrete blocks attached to the lower portion of the precast beam. The surface of the beam over which the blocks were poured was roughened, and dowel bars were run through the web to improve horizontal shear transfer. The anchor blocks were placed at 15 ft (4.57 m) from each support to allow the dowel reinforcement to be placed through drilled holes in the web of the beam located between the draped and the straight tendons (Fig. 12).

The repaired girder provided the same cracking strength under service load as the original girder. Existing cracks which formed while the bridge was damaged will open slightly during application of full live load. However, the crack widths will be very small and the stress range in the reinforcement will be much smaller than the limit allowable for fatigue endurance. The moment curvature relationship of the repaired girder is shown in Fig. 8 and compared to that of the original and the damaged girder. Its nominal resistance is 5045 k-ft (6840 kN-m); values of the reliability index for the various limit states are given in Table 3. These values are comparable to those obtained for the original design and suggest that the repair procedure is acceptable.

**CONCLUDING REMARKS**

The reliability analysis of a structure being new, old, or damaged, for various strength and serviceability limit states provides not only convenient
measures of its performance (through the reliability index) but is also a rational and efficient procedure for its evaluation. When applied to the use of existing bridges, such analysis should allow bridge engineers to make rational decisions ranging from unrestricted use, to closing to traffic for replacement or repair. Once the methodology has been perfected at the research level and calibrated to reflect real life situations, its wide scale use in practice should be encouraged.

ACKNOWLEDGEMENT

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REFERENCES


7 Nowak, A. S. et al., 1988, “Risk Analysis for Evaluation of Bridges,” Report UMCE 88-7, Department of Civil Engineering, University of Michigan, Ann Arbor, November.


Table 1 — Definition of $R$ in the Reliability Analysis

<table>
<thead>
<tr>
<th>limit state</th>
<th>$R$</th>
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<tbody>
<tr>
<td>1. ultimate</td>
<td>moment carrying capacity</td>
</tr>
<tr>
<td>2. cracking</td>
<td>cracking moment</td>
</tr>
<tr>
<td>3. max. crack width</td>
<td>moment causing critical crack width (0.016&quot;)</td>
</tr>
<tr>
<td>4. fatigue in concrete</td>
<td>moment causing critical stress range $(0.4f'<em>c - f</em>{min}/2)$, where $f_{min} = \text{min. stress in concrete}$</td>
</tr>
<tr>
<td>5. fatigue in steel</td>
<td>moment causing critical stress range $(0.1f_{pu})$</td>
</tr>
<tr>
<td>6. max. live load</td>
<td>moment causing critical deflection $(L/180)$</td>
</tr>
<tr>
<td>deflection</td>
<td></td>
</tr>
</tbody>
</table>

Table 2 — Statistical Data for Basic Variables

<table>
<thead>
<tr>
<th>item</th>
<th>description</th>
<th>nominal</th>
<th>bias</th>
<th>mean</th>
<th>$V$</th>
<th>$\sigma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>girder dead load, kip-ft</td>
<td>657.5</td>
<td>1.030</td>
<td>677.2</td>
<td>.040</td>
<td>27.1</td>
</tr>
<tr>
<td>2</td>
<td>additional dead load, kip-ft</td>
<td>789.7</td>
<td>1.037</td>
<td>819.2</td>
<td>.083</td>
<td>67.6</td>
</tr>
<tr>
<td>3</td>
<td>live load plus impact, kip-ft</td>
<td>889.0</td>
<td>0.819</td>
<td>728.0</td>
<td>.240</td>
<td>174.7</td>
</tr>
<tr>
<td>4</td>
<td>internal strand strength, ksi</td>
<td>270</td>
<td>1.041</td>
<td>281</td>
<td>.025</td>
<td>7.0</td>
</tr>
<tr>
<td>5</td>
<td>external tendon strength, ksi</td>
<td>150</td>
<td>1.087</td>
<td>163</td>
<td>.042</td>
<td>6.8</td>
</tr>
<tr>
<td>6</td>
<td>girder concrete $f'_c = 5,000$ psi</td>
<td>4250</td>
<td>.948</td>
<td>4028</td>
<td>.150</td>
<td>604</td>
</tr>
<tr>
<td>7</td>
<td>slab concrete $f'_c = 4,000$ psi</td>
<td>3400</td>
<td>.997</td>
<td>3390</td>
<td>.180</td>
<td>610</td>
</tr>
<tr>
<td>8</td>
<td>in. strand effective depth, inch</td>
<td>49.43</td>
<td>.996</td>
<td>49.25</td>
<td>.011</td>
<td>0.52</td>
</tr>
<tr>
<td>9</td>
<td>ex. tendon effective depth, inch</td>
<td>37.00</td>
<td>.995</td>
<td>36.82</td>
<td>.014</td>
<td>0.52</td>
</tr>
<tr>
<td>10</td>
<td>slab thickness, inch</td>
<td>8.00</td>
<td>1.004</td>
<td>8.03</td>
<td>.059</td>
<td>0.47</td>
</tr>
</tbody>
</table>
Table 3 — Reliability Index $\beta$ for Various Limit States and Conditions

<table>
<thead>
<tr>
<th>item</th>
<th>limit state</th>
<th>original</th>
<th>damaged</th>
<th>repaired</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>ultimate</td>
<td>12.10</td>
<td>6.94</td>
<td>11.65</td>
</tr>
<tr>
<td>2</td>
<td>cracking</td>
<td>1.62</td>
<td>-1.71</td>
<td>1.76</td>
</tr>
<tr>
<td>3</td>
<td>maximal crack width</td>
<td>3.53</td>
<td>-1.34</td>
<td>3.05</td>
</tr>
<tr>
<td>4</td>
<td>fatigue in concrete</td>
<td>5.02</td>
<td>2.47</td>
<td>4.80</td>
</tr>
<tr>
<td>5</td>
<td>fatigue in steel</td>
<td>4.29</td>
<td>0.16</td>
<td>4.30</td>
</tr>
<tr>
<td>6</td>
<td>maximal live load deflection</td>
<td>9.04</td>
<td>4.19</td>
<td>8.92</td>
</tr>
</tbody>
</table>

Table 4 — Reliability Index $\beta$ versus Reliability $S$ and Probability of Failure $P_F$

<table>
<thead>
<tr>
<th>Reliability Index $\beta$</th>
<th>Reliability $S$ ($= 1 - P_F$) or Probability of Safety</th>
<th>Probability of Failure $P_F$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>0.500</td>
<td>$0.500 \times 10^{+0}$</td>
</tr>
<tr>
<td>0.5</td>
<td>0.691</td>
<td>$0.309 \times 10^{+0}$</td>
</tr>
<tr>
<td>1.0</td>
<td>0.841</td>
<td>$0.159 \times 10^{+0}$</td>
</tr>
<tr>
<td>1.5</td>
<td>0.933 2</td>
<td>$0.668 \times 10^{-1}$</td>
</tr>
<tr>
<td>2.0</td>
<td>0.977 2</td>
<td>$0.228 \times 10^{-1}$</td>
</tr>
<tr>
<td>2.5</td>
<td>0.993 79</td>
<td>$0.621 \times 10^{-2}$</td>
</tr>
<tr>
<td>3.0</td>
<td>0.998 65</td>
<td>$0.135 \times 10^{-2}$</td>
</tr>
<tr>
<td>3.5</td>
<td>0.999 767</td>
<td>$0.233 \times 10^{-3}$</td>
</tr>
<tr>
<td>4.0</td>
<td>0.999 968 3</td>
<td>$0.317 \times 10^{-4}$</td>
</tr>
<tr>
<td>4.5</td>
<td>0.999 996 60</td>
<td>$0.340 \times 10^{-5}$</td>
</tr>
<tr>
<td>5.0</td>
<td>0.999 999 713</td>
<td>$0.287 \times 10^{-6}$</td>
</tr>
<tr>
<td>5.5</td>
<td>0.999 999 981 0</td>
<td>$0.190 \times 10^{-7}$</td>
</tr>
<tr>
<td>6.0</td>
<td>0.999 999 999 013</td>
<td>$0.987 \times 10^{-9}$</td>
</tr>
<tr>
<td>6.5</td>
<td>0.999 999 999 959 8</td>
<td>$0.402 \times 10^{-10}$</td>
</tr>
<tr>
<td>7.0</td>
<td>0.999 999 999 998 72</td>
<td>$0.128 \times 10^{-11}$</td>
</tr>
<tr>
<td>7.5</td>
<td>0.999 999 999 999 68 1</td>
<td>$0.319 \times 10^{-13}$</td>
</tr>
<tr>
<td>8.0</td>
<td>0.999 999 999 999 999 389</td>
<td>$0.611 \times 10^{-15}$</td>
</tr>
</tbody>
</table>
Fig. 1 — Probability Density Functions for $R$, $Q$ and $R - Q$

Fig. 2 — Stress-Strain Curves for Concrete
Fig. 3 — Stress-Strain Curves for Reinforcing Steel

Fig. 4 — Stress-Strain Curves for Prestressing Steel
Fig. 5 — Cross Section of the Bridge Considered in the Example

Fig. 6 — Cross Section of the Girder with Layout of Prestressing Tendons
Fig. 7 — Moment-Curvature Relationship for the Girder

Fig. 8 — Moment-Curvature Curves for the Original, Damaged and Repaired Girder
Fig. 9 — Typical Stress Increment in Concrete Top Fiber with Live Load Moment

Fig. 10 — Typical Stress Increment in Prestressing Steel with Live Load Moment
Fig. 11 — Cross Section of the Repaired Girder

Fig. 12 — External Tendon Arrangement in the Repaired Girder