Design for Crack Control in Reinforced and Prestressed Concrete Beams, Two-Way Slabs and Circular Tanks — A State-of-the-Art

by E. G. Nawy

Synopsis: This paper presents the state-of-the art in the evaluation of the flexural crack width development and crack control of flexural cracks in reinforced and prestressed concrete structures. It is based on extensive research over the past five decades in the United States and overseas in the area of macro-cracking in reinforced and prestressed concrete elements.

Mitigation and control of cracking has become essential in order to maintain the integrity and aesthetics of concrete structures and their long-term durability performance. The trend is stronger than ever towards better utilization of concrete strength, use of higher strength concretes in the range of 12,000-20,000 psi and higher compressive strength, more prestressed concretes and increased uses of limit failure theories - all these trends require closer control of serviceability requirements of cracking and deflection behavior.

The paper discusses and presents common expressions for the mitigation and control of cracking in reinforced concrete beams and thick one-way slabs, prestressed, pretensioned and post-tensioned flanged beams, reinforced concrete two-way action structural floor slabs and plates, and large diameter circular tanks. In addition, recommendations are given for the maximum tolerable flexural crack widths in concrete elements based on the cumulative experience of many investigators over the past five decades. The expressions include the ACI 318-99 crack control provisions in reinforced concrete beams and one-way slabs, and the Concrete Euro Code 1999 for the design of concrete buildings.

Keywords: beams; concrete; concrete strength; crack control; cracking; crack width; environment; equations for reinforced and prestressed beams; Eurocode; flexural crack width; long-term cracking; tanks; tolerable crack widths; two-way action structural slabs
Edward G. Nawy, FACI, is Professor of Civil Engineering, Rutgers University, and holds the distinguished professor rank. Active in ACI since 1949, Professor Nawy is the founding chairman and a current member of ACI Committee 224 on Cracking; past chairman of ACI Committee 435 on Deflection of Concrete Building Structures; member of ACI Committee 340, Design Aids for ACI 318 Building Code; member of Joint ASCE-ACI Committee 421 on Design of Reinforced Concrete Slabs. Professor Nawy has published in excess of 160 papers and is the author four major textbooks and one handbook: SIMPLIFIED REINFORCED CONCRETE (1987); REINFORCED CONCRETE - A fundamental Approach (4th Ed., 2000) and translated into several languages; PRESTRESSED CONCRETE - A Fundamental Approach (3rd Ed., 2000); FUNDAMENTALS OF HIGH PERFORMANCE CONCRETE (2nd Ed., 2001); and CONCRETE CONSTRUCTION ENGINEERING HANDBOOK (1998), as well as chapters in several handbooks. He holds several honors including the ACI Chapter Activities Award, the Henry L. Kennedy Award and the Concrete Research Council's Robert Philleo Award, was twice president of the ACI New Jersey Chapter, served two terms on the Rutgers University Board of Governors, and is Honorary professor, Nanjing Institute of Technology, China. He is a registered professional engineer in New York, New Jersey, Pennsylvania, California and Florida, and a chartered civil engineer overseas. He has also served as program evaluator for the national Accreditation Board for Engineering and Technology (ABET).

INTRODUCTION

Presently, the trend is stronger than ever in better utilization of concrete strength, use of higher strength high-performance concretes of 20,000 psi (138 MPa) compressive strength and higher, use of high-strength reinforcement, more prestressed concretes and increased use of limit failure theories - all these trends require closer control of serviceability requirements in cracking and deflection behavior. Hence, knowledge of the cracking behavior of concrete elements, and how to mitigate cracking, become essential.

Concrete cracks early in its loading history. Most cracks are a result of the following actions to which concrete can be subjected:

1. Volumetric change caused by drying shrinkage, creep under sustained load, thermal stresses including elevated temperatures, and chemical incompatibility of concrete components.

2. Direct stress due to applied loads or reactions or internal stress due to continuity, reversible fatigue load, long-term deflection, camber in prestressed systems, or environmental effects including differential movement in structural systems.

3. Flexural stress due to bending.

While the net result of these three actions can be the formation of cracks, the
mechanisms of their development cannot be considered identical. Volumetric change generates internal micro-cracking which may develop into full cracking, while direct internal or external stress or applied loads and reactions could either generate internal macro-cracking, such as in the case of fatigue due to reversible load, or flexural micro-cracking leading to fully developed cracking.

This paper will concentrate on the macro-cracking aspect of cracking behavior. Yet it is important to briefly discuss micro-cracking.

MICRO-CRACKING

Micro-cracking may be mainly classified into two principal categories: a) bond cracks at the aggregate-mortar interface, b) paste cracks within the mortar matrix. Interfacial bond cracks are caused by interfacial shear and tensile stresses due to early volumetric change without the presence of external load. Volume change caused by hydration and shrinkage can create tensile and bond stresses of sufficient magnitude as to cause failure at the aggregate-mortar interface.\textsuperscript{1-6} As the external load is applied, mortar cracks develop due to increase in compressive stress, propagating continuously through the cement matrix up to failure.

It appears that the damage to cement paste seems to play a significant role in controlling the stress-strain relationship in concrete. The coarse aggregate particles act as stress-raisers that decrease the strength of the cement paste. As a result, micro-cracks develop that can only be detected by large magnification. The importance of additional research lies not only in the evaluation of the micro-cracks, but also in their significance on the development of macro-cracks which generate from those micro-cracked centers of plasticity.

FLEXURAL CRACKING AND CRACKING MITIGATION

External load causes in direct and bending stresses potentially resulting in flexural, bond and diagonal tension cracks. Once the tensile stress in the concrete exceeds its tensile strength, internal micro-cracks form. These cracks develop into macro-
cracks propagating towards the external fiber zones of the element.

Immediately after the full development of the first crack in a conventionally reinforced concrete element, the stress in the concrete at the cracking zone is reduced to zero and is assumed by the reinforcement\(^5\). The distribution of ultimate bond stress, longitudinal tensile stress in the concrete and longitudinal tensile stress in the steel can be schematically represented in Fig. 1.

Crack width is a primary function of the bond characteristics and deformation of reinforcement between the two adjacent cracks 1 and 2 in Fig. 1, if the small concrete strain along the crack interval \(\varepsilon_{c}\) is neglected. Hence, the crack width would be a function of the crack spacing and vice versa up to the level of stabilization of crack spacing (Fig. 2).

The major parameters affecting the development and characteristics of the cracks are: cross-sectional percentage of reinforcement, bond characteristics and size of bar, concrete cover, concrete strength, and the concrete stretched area, namely the concrete area in tension. On this basis, one can propose the following mathematical model:

\[
w = \alpha \varepsilon_{s}^{\beta} \varepsilon_{c}^{\gamma}
\]

where \(w\) = maximum crack width, and \(\alpha\), \(\beta\) and \(\gamma\) are nonlinearity constants. Crack spacing \(a_{c}\) is a function of the factors enumerated previously, being inversely proportional to bond strength and active steel ratio (steel percentage in terms of the concrete cross-sectional area in tension). \(\varepsilon_{s}\) is the reinforcement strain induced by external load.

The basic mathematical modal in equation (1) with the appropriate experimental values of the constants \(\alpha\), \(\beta\) and \(\gamma\) can be derived for a particular type of structural member. Such a member can be a one-dimensional element such as beam, a two-dimensional structure such as a two-way slab, or a three-dimensional member such as a shell or circular tank wall. Hence, it is expected that different forms or expressions apply for the evaluation of the macro-cracking behavior of different structural elements consistent with their fundamental structural behavior.\(^2\text{-}^4\)
FLEXURAL CRACKING AND CRACK CONTROL IN REINFORCED CONCRETE BEAMS AND THICK ONE-WAY SLABS

Requirements for crack control in beams and thick one-way slabs (span/thickness ratio about 15) in the ACI 318 Building Code are based on the statistical analysis of maximum crack width data from a number of sources. On the basis of this analysis and the vast amount of data available, the following general conclusions were reached:

1. The steel stress is the most important variable.
2. The thickness of the concrete cover is an important variable, but not the only geometric consideration.
3. The area of concrete surrounding each reinforcing bar is also an important geometric variable.
4. The bar diameter is not a major variable.
5. The size of the bottom crack width is influenced by the amount of strain gradient from the level of the steel to the tension face of the beam.

The simplified expression relating crack width to steel stress is given in Eq. 2 as follows:

\[ w_{\text{max}} = 0.076 \beta f_s \sqrt{d_c A} 10^{-3} \]  

(2)

where \( f_s \) = reinforcing steel stress, ksi

\( A = \) area of concrete symmetric with reinforcing steel divided by number of bars, in.\(^2\)

\( d_c = \) thickness of concrete cover measured from extreme tension fiber to center of bar or wire closest thereto, in.

\( \beta = h_2/h_1 \) where \( h_1 = \) distance from neutral axis to the reinforcing steel, in.

\( h_2 = \) distance from neutral axis to extreme concrete tensile surface.

When the strain, \( \varepsilon \), in the steel reinforcement is used instead of stress, \( f_s \), Eq. (2) becomes...
$w_{\text{max}} = 2.2 \beta \varepsilon_s \frac{1}{d_c A} 10^{-3}$

and is valid in any system of measurement.

The cracking behavior in thick one-way slabs, namely those with clear cover exceeding 1-1/2 in., is similar to that in shallow beams. For such one-way slabs (Eq. 3) can be adequately applied if $\beta$ ranging from 1.25 to 1.35 is used.

ACI 224 Report and ACI 340 Report give recommendations for tolerable crack widths under various environmental conditions, as given in Table 1.

**ACI 318-99 Code Provisions**

The ACI 318-99 mitigates cracking through control of the spacing of the reinforcing bars, as an indirect measure of crack control. The expression that the code requires to be used is:

$$s \text{ (in.)} = \left(\frac{540}{f_s}\right) - 2.5 c_c$$

but not greater than $12 \left(\frac{36}{f_s}\right)$, where

$f_s$ = computed stress in the reinforcement at service load = unfactored moment divided by the steel area and the internal arm moment. Alternatively, $f_s$ can be taken as $0.60 f_y$.

$c_c$ = clear cover from the nearest surface in tension to the flexural tension reinforcement, inches.

$s$ = center-to-center spacing of flexural tension reinforcement, inches, closest to the tension face of the section.

From these provisions, the maximum spacing for 60,000 psi (414 MPa) reinforcement = $12 \left[\frac{36}{(0.6 \times 60)}\right] = 12$ in. (305 mm). The maximum allowable spacing of 12 inches is in conformity with the extensive testing performed by the author on over excess 100 two-way action slabs, discussed in subsequent sections. Hence this limitation on the distribution of flexural reinforcement in one-way slabs and wide-web reinforced concrete beams is appropriate. However, in beams of normal web width in usual buildings, these provisions might not be as workable. In
the opinion of the author, it is more advisable and perhaps safer to use the applicable crack width equation in designing for the tolerable crack width that the particular environmental condition requires.

The SI expression for the value of reinforcement spacing in Eq. 4 and $f_s$ in MPa units is,

$$s (\text{mm}) = \left( \frac{95,000}{540 f_s} \right) - 2.5 c_c$$

but not to exceed $300(252/f_s)$. For the usual case of beams with grade 420 reinforcement and 50 mm clear cover to the main reinforcement, with $f_s = 252$ MPa, the maximum bar spacing is 250 mm.

It should be noted that if the concrete structural member is in severe environment, where the ACI bar spacing provisions can result in crack widths of 0.0137 in. or more, satisfying the ACI 318-99 requirements for bar spacing is not adequate for sustaining the long-term structural integrity of the member. This is why, use of the Gergely-Lutz equation in conjunction with Table 1, or the Euro EC2 Code expressions presented in the next section, is the advisable design route to follow for crack control in beams and thick one-way slabs.

**Euro EC2 Code Provisions**

The Euro Code EC2 requires that cracking should be limited to a level that does not impair the serviceability of the structure or cause its appearance to be unacceptable. It limits the maximum design crack width to 0.3 mm (0.012 in.) under normal environmental conditions and under quasai permanent combination of loads. This ceiling is expected to be satisfactory with respect to appearance and durability. Stricter requirements of tolerable crack width are stipulated for more severe environmental conditions.

The EC2 Code stipulates that the design crack width be evaluated from the following expression:

$$w_k = \beta \ s_{mn} \ e_{sm}$$

(6)
where

\[ W_k = \text{design crack width} \]

\[ s_{rm} = \text{average stabilized crack spacing} \]

\[ \varepsilon_{sm} = \text{mean strain under relevant combination of loads and allowing for the effect of tension stiffening, shrinkage, etc.} \]

\[ \beta = \text{coefficient relating the average crack width to the design value} \]

\[ = 1.7 \text{ for load-induced cracking and for restraint cracking in sections with minimum dimension in excess of 800 mm (32 in.)} \]

The strain, \( \varepsilon_{sm} \), in the section is obtained from the following expression:

\[ \varepsilon_{sm} = \frac{\sigma_t}{E_s} \left[ 1 - \beta_1 \beta_2 \left( \frac{\sigma_{sr}}{\sigma_t} \right)^2 \right] \tag{7} \]

where

\[ \sigma_t = \text{stress in the tension reinforcement computed on the basis of a cracked section.} \]

\[ \sigma_{sr} = \text{stress in the tension reinforcement computed on the basis of a cracked section under loading conditions that cause the first crack.} \]

\[ \beta_1 = \text{coefficient accounting for bar bond characteristics} \]

\[ = 1.0 \text{ for deformed bars and 0.5 for plain bars.} \]

\[ \beta_2 = \text{coefficient accounting for load duration} \]

\[ = 1.0 \text{ for single short-term loading and 0.5 for sustained or cyclic loading} \]

\[ E_s = \text{Modulus of elasticity of the reinforcement} \]

The average stabilized mean crack spacing, \( s_{rm} \), is evaluated from the following expression:

\[ s_{rm} = 50 + 0.25 k_1 k_2 d_b / p_1, \text{ mm} \tag{8} \]
Design and Construction Practices to Mitigate Cracking

where

\( d_b = \) bar diameter, mm.

\( p_t = \) effective reinforcement ratio = \( A_s / A_t \); the effective concrete area in tension, \( A_t \), is generally the concrete area surrounding the tension reinforcement, of depth equal to 2.5 times the distance from the tensile face of the concrete section to the centroid of the reinforcement. For slabs where the depth of the tension zone may be small, the height of the effective area should not be taken less than \( (c - d_b) / 3 \), where \( c = \) clear cover to the reinforcement.

\( k_1 = 0.8 \) for deformed bars and 1.6 for plain bars.

\( k_2 = 0.5 \) for bending and 1.0 for pure tension.

In cases of eccentric tension or for local areas, an average value of \( k_2 = (\varepsilon_1 + \varepsilon_2) / 2 \) can be used, where \( \varepsilon_1 \) is the greater and \( \varepsilon_2 \) the lesser tensile strain at the section boundaries, determined on the basis of cracked section.

In the absence of rigorous computations as described thus far, choice of minimum area of reinforcement, \( A_s \), for crack control is stipulated such that

\[
A_s = k_c k f_{ct,eff} A_{ct} / \sigma_s
\]

where

\( A_s = \) reinforcement area within the tensile zone.

\( A_{ct} = \) Effective area of concrete in tension

\( \sigma_s = \) maximum stress permitted in the reinforcement after the formation of the crack. The yield strength may be taken in lieu of \( \sigma_s \), although lower values may be needed to satisfy crack width limits.

\( f_{ct,eff} = \) tensile strength of the concrete effective at the formation of the first crack. A value of 3 N/mm\(^2\) (435 psi) can be used.
k_e = coefficient representing the nature of stress distribution, 
= 1.0 for direct tension and 0.4 for bending
k = coefficient accounting for non-uniform stresses due to restraint resulting from intrinsic or extrinsic deformation. It varies between 0.5 and 1.0.

The EC2 Code also stipulates that for cracks dominantly caused by flexure, their width will not usually exceed the standard 0.30 mm (0.012 in.), if the size and spacing of the reinforcing bars are within the range of values in Tables 2 and 3 for bar size and spacing. Evidently, for severe exposure conditions, such as those listed in Table 1 for tolerable crack widths beyond the 0.012 in. crack width level, crack width computations become mandatory.

**The Australian Code Provisions on Flexural Crack Control**
The Australian Code does not recommend any formula for the calculation of crack widths. Crack control for flexure in reinforced concrete beams is achieved if the center-to-center spacing of bars near the tension face of the beam does not exceed 8 in. (200 mm) and the distance from the side or soffit to the center of the nearest longitudinal bar is not greater than 4 in. (100 mm). In the case of fully prestressed concrete beams, the maximum tensile stress in the concrete due to short-term service loads should not exceed $3 \sqrt{f_e'}$. To control flexural cracking in partially prestressed concrete beams, the increment in steel stress near the tension face is limited to 29Ksi (200 MPa), as the load increases from its value when the extreme concrete tensile fiber is at zero stress to the short-term service load value; and the center-to-center spacing of reinforcement, including bonded tendons, is limited to 8 in. (200 mm).

Flexural cracking in reinforced concrete slabs is controlled by limiting the center-to-center spacing of bars in each direction to the lesser of 2.5 times the thickness of slab or 20 in (500 mm). In fully prestressed slabs, similar to beams, the maximum tensile stress in the concrete due to short-term service loads is limited to $3 \sqrt{f_e'}$. For partially prestressed slabs, the incremental steel stress should not exceed 22 ksi (150
MPa) and the center-to-center spacing of reinforcement including bonded tendons, is not to exceed 20 in. (500 mm). It should be noted that the extensive Nawy work\textsuperscript{5-7} demonstrated that the maximum crack spacing in two-way reinforced concrete slabs should not exceed 12 in. (300 mm), otherwise yield line wide cracks would be prematurely generated. Hence, the ACI 318 Code limits the maximum spacing to twice the slab thickness.

**FLEXURAL CRACKING AND CRACK CONTROL IN PRESTRESSED PRETENSIONED AND POST-TENSIONED BEAMS**

The increased use of partial prestressing, allowing limited tensile stresses in the concrete under service and overload conditions while allowing non-prestressed steel to carry the tensile stresses, is becoming prevalent due to practicality and economy. Consequently, an evaluation of the flexural crack widths and spacing and control of their development become essential. Work in this area is relatively limited because of the various factors affecting crack width development in prestressed concrete. However, experimental investigations support the hypothesis that the major controlling parameter is the reinforcement stress change beyond the decompression stage. Nawy, et al, have undertaken extensive research since the 1960’s on the cracking behavior of prestressed pretensioned and post-tensioned beams and slabs because of the great vulnerability of the highly stressed prestressing steel to corrosion and other environmental effects and the resulting premature loss of prestress.\textsuperscript{10,11} Serviceability behavior under service and overload conditions can be controlled by the design engineer through the application of the criteria presented in this section.

**A. Mathematical Model Formulation for Serviceability Evaluation**

1. **Crack Spacing**

Primary cracks form in the region of maximum bending moment when the external load reaches the cracking load. As loading is increased, additional cracks will form and the number of cracks will be stabilized when the stress in the concrete no longer
exceeds its tensile strength at further locations regardless of load increase. This condition is important as it essentially produces the absolute minimum crack spacing which can occur at high steel stresses, to be termed the stabilized minimum crack spacing. The maximum possible crack spacing under this stabilized condition is twice the minimum, to be termed the stabilized maximum crack spacing. Hence, the stabilized mean crack spacing, \( a_{cs} \), is evaluated as the mean value of the two extremes.

The total tensile force \( T \) in Fig. 3 transferred from the steel to the concrete over the stabilized mean crack spacing can be defined as

\[
T = \gamma a_{cs} \mu \Sigma o
\]  

(9a)

where

\( \gamma = \) a factor reflecting the distribution of bond stress

\( \mu = \) maximum bond stress which is a function of \( \sqrt{f_c} \)

\( \Sigma o = \) sum of reinforcing elements’ circumferences

The resistance \( R \) of the concrete area in tension \( A_t \) can be defined as

\[
R = A_t f_t^t
\]  

(9b)

where

\( f_t^t = \) tensile splitting strength of the concrete. By equating Eqs. 9a and 9b, the following expression for \( a_{cs} \) is obtained, where \( c \) is a constant to be developed from the tests:

\[
a_{cs} = c \frac{A_t f_t^t}{\Sigma o \sqrt{f_c}}
\]  

(10a)

The concrete stretched area, namely the concrete area, \( A_t \), in tension for both the evenly distributed and non-evenly distributed reinforcing elements, is illustrated in Fig. 4. With a mean value of \( f_t^t / \sqrt{f_c} = 7.95 \) in this investigation, a regression
Design and Construction Practices to Mitigate Cracking

analysis of the test data resulted in the following expression for the mean stabilized crack spacing

\[ a_{cs} = 1.20 \frac{A_d}{\Sigma \sigma} \] (10b)

2. Crack Width

If \( \Delta f_x \) is the net stress in the prestressed tendon or the magnitude of the tensile stress in the normal steel at any crack width load level in which the decompression load (decompression here means \( f_c = 0 \) at the level of the reinforcing steel) is taken as the reference point, then for the prestressed tendon

\[ \Delta f_x = f_{nt} - f_d \] (11)

where

- \( f_{nt} \) = stress in the prestressing steel at any load beyond the decompression load
- \( f_d \) = stress in the prestressing steel corresponding to the decompression load

The unit strain \( \varepsilon_s = \frac{\Delta f_x}{E_s} \). It is logical to disregard as insignificant the unit strains in the concrete due to the effects of temperature, shrinkage and elastic shortening. The maximum crack width as defined in Eq. 1 can be taken as

\[ w_{max} = k \, a_{cs} \varepsilon_s \] (12a)

where \( k \) and \( a \) are constants to be established by tests, or

\[ w_{max} = k' \, a_{cs} (\Delta f_x)^{\alpha} \] (12b)

and \( k' \) is a constant in terms of constant \( k \).

B. Expressions for Pretensioned Beams

Eq. 12a is rewritten in terms of \( \Delta f_x \) so that analysis of the test data of all the simply supported test beams in this work leads to the following expression at the reinforcement level:
Linearizing Eq. 13 for easier use by the design engineer leads to the following simplified expression of the maximum crack width at the reinforcing steel level closest to the tensile face of the beam:

\[ w_{\text{max}} \text{ (in.)} = 5.85 \times 10^{-5} \frac{A_t}{\sum o} (\Delta f_s) \]  \hspace{1cm} (14a)

In SI units, the expression in Eq. 14 (a) becomes

\[ w_{\text{max}} \text{ (mm)} = 8.48 \times 10^{-5} \frac{A_t}{\sum o} (\Delta f_s) \]  \hspace{1cm} (14b)

where \( A_t = \text{cm}^2 \), \( \sum o = \text{cm} \) and \( \Delta f_s = \text{MPa} \).

The maximum crack width (in.) at the tensile face of the concrete is

\[ w_{\text{max}} \text{ (in.)} = 5.85 \times 10^{-5} R_i \frac{A_t}{\sum o} (\Delta f_s) \]  \hspace{1cm} (14c)

where \( R_i \) is the distance ratio as defined in the notations.

A plot of the data and the best fit expression for Eq. 14a is given in Fig. 5 with a 40 percent spread, which is reasonable in view of the randomness of crack development and the linearization of the original expression of Eq. 13.

### C. Expressions for Post-Tensioned Beams

The expression developed for the crack width at the reinforcement level closest to the tensile face in post-tensioned bonded beams which contain mild steel reinforcement

\[ w_{\text{max}} \text{ (in.)} = 6.51 \times 10^{-5} \frac{A_t}{\sum o} (\Delta f_s) \]  \hspace{1cm} (15a)

In SI units, the expression in Eq. 15(a) becomes

\[ w_{\text{max}} \text{ (mm)} = 9.44 \times 10^{-5} \frac{A_t}{\sum o} (\Delta f_s) \]  \hspace{1cm} (15b)
For the crack width at the tensile face of the concrete lower fibers:

\[ w_{\text{max}} \text{ (in.)} = 6.51 \times 10^{-3} R_i \frac{A_i}{\sum A_i} (\Delta f_s) \]  \hspace{1cm} (15c)

For non-bonded beams, the factor 6.51 in Eqs. 15a and 15b becomes 6.83.

A plot of the data and the best fit expression for Eq. 15a is given in Fig. 6.

A typical plot of the effect of the various steel percentages on the crack spacing at the various stress levels \( \Delta f_s \) is given in Fig. 7. It is seen from this plot that crack spacing stabilizes at a net stress level of 36 to ksi.

**D. Cracking of high-strength high performance prestressed beams.**

More recent work at Rutgers University on the cracking behavior of high-strength pretensioned and non-bonded post-tensioned beams having cylinder compressive strengths, \( f'_c \), in the range of 10,200 psi to 14,200 psi (70.3 to 97.9 MPa), have resulted in the following expression for the crack width at the reinforcement level of the pretensioned member

\[ w_{\text{max}} \text{ (in.)} = 2.74 \times 10^{-5} \frac{A_i}{\sum A_i} (\Delta f_s) \]  \hspace{1cm} (16a)

or, in SI units:

\[ w_{\text{max}} \text{ (mm)} = 4.0 \times 10^{-5} \frac{A_i}{\sum A_i} (\Delta f_s) \]  \hspace{1cm} (16b)

The factor 2.74 in Eq. (16a) is an average of values from the statistical expression that resulted in the expressions given in Eqs. 17 and 18. The reduced crack width due to the use of high-strength high performance concrete, is expected in view of the increased bond interlock between the reinforcement and the concrete.
For post-tensioned beams, the reduction multiplier $\lambda_o$ is

$$\lambda_o = \frac{1}{0.75 + 0.06\sqrt{f'_c}}$$

where $f'_c$ and the reinforcement stress are in ksi.

### E. Cracking in Circular Prestressed Concrete Tanks

Circular prestressed tanks are cylindrical shell elements of very large diameter in relation to their height. Hence, it is possible to treat the wall with respect to flexural cracking in a manner similar to the behavior of two-way action plates. Vessay and Preston $^{18}$ modified the Nawy expressions developed for two-way action slabs and plates so that the maximum crack width can be defined as:

$$w_{\text{max}} (\text{in}) = 4.1 \times 10^{-6} \varepsilon_{ct} E_{ps} \sqrt{G_i}$$

In SI units, the expression becomes

$$w_{\text{max}} (\text{mm}) = 0.6 \times 10^{-6} \varepsilon_{ct} E_{ps} \sqrt{G_i}$$

where $E_{ps}$ is in MPa and the dimensions of all the parameters of the grid index $G_i$ are in millimeters.
Design and Construction Practices to Mitigate Cracking

\[ \varepsilon_{ct} = \text{tensile surface strain in the concrete} = \frac{(\lambda_s f_p)}{(E_p)} \]

\[ f_p = \text{actual stress in the steel} \]

\[ f_{p0} = \text{initial prestress before losses} \]

\[ \lambda_s = \frac{f_p}{f_{p0}} \]

\[ G_I = \text{grid index} = \frac{s_1 s_2 d_c}{d_{b1} \pi} \frac{8}{\pi} \]

\[ d_{b1} = \text{diameter of steel in direction 1} \]

\[ s_1 = \text{spacing of the reinforcement in direction 1 closest to the tensile face} \]

\[ s_2 = \text{spacing of the reinforcement in direction 2} \]

\[ d_c = \text{concrete cover to center of steel, in.} \]

Note that \( w_{\text{max}} = 0.004 \text{ in.} \) (0.10 mm) should be the maximum crack width for liquid-retaining tanks and sanitary structures.

CRACKING AND CRACK CONTROL IN TWO-WAY ACTION SLABS AND PLATES

Flexural crack control is essential in structural floors where cracks at service load and overload conditions can have serious performance effect in such structures as office buildings, schools, parking garages, industrial buildings and other floors, where the design service load levels exceed those in normal size apartment building panels and also in all cases of adverse exposure conditions.

A. Flexural Cracking Mechanism and Fracture Hypothesis

Flexural cracking behavior in concrete structural floors under two-way action is significantly different from that in one-way members. Crack-control equations for beams underestimate the crack widths developed in two-way slabs and plates, and do not tell the designer how to space the reinforcement. Cracking in two-way slabs and plates is controlled primarily by the steel stress level and the spacing of the reinforcement in the two perpendicular directions. In addition, the clear concrete cover in two-way slabs and plates is nearly constant [3/4 in. (19 mm) for interior exposure], whereas it is a major variable in the crack-control equations for beams. The
results from extensive tests on slabs and plates by Nawy et al. demonstrate this
difference in behavior in a fracture hypothesis on crack development and propagation
in two-way plate action. As seen in Fig. 7 stress concentration develops initially at the
points of intersection of the reinforcement in the reinforcing bars and at the welded
joints of the wire mesh, that is, at grid nodal points, thereby dynamically generating
fracture lines along the paths of least resistance, namely, along $A_1B_1$, $A_1A_2$, $A_2B_2$, and
$B_2B_1$. The resulting fracture pattern is a total repetitive cracking grid, provided that
the spacing of the nodal points $A_1, B_1, A_2$, and $B_2$ is close enough to generate this
preferred initial fracture grid of orthogonal cracks narrow in width, as a preferred
fracture mechanism.

If the spacing of the reinforcing grid intersections is too large, the magnitude of
stress concentration and the energy absorbed per unit grid is too low to generate
 cracks along the reinforcing wires or bars. As a result, the principal cracks follow
diagonal yield-line cracking in the plain concrete field away from the reinforcing bars
early in the loading history. These cracks are wide and few.

This hypothesis also leads to the conclusion that surface deformations of the
individual reinforcing elements have little effect in arresting the generation of the
 cracks or controlling their type or width in a two-way-action slab or plate. In a similar
manner, one may conclude that the scale effect on two-way-action cracking behavior is
insignificant, since the cracking grid would be a reflection of the reinforcement grid if
the preferred orthogonal narrow cracking widths develop. Therefore, to control
 cracking in two-way-action floors, the major parameter to be considered is the
reinforcement spacing in the two perpendicular directions. Concrete cover has only a
minor effect, since it is usually a small, constant in value of 0.75 in. (20 mm).

For a constant area of steel determined for bending in one direction, that is, for
energy absorption per unit slab area, the smaller the spacing of the transverse bars or
wires, the smaller should be the diameter of the longitudinal bars. The reason is that
less energy has to be absorbed by the individual longitudinal bars. If one considers
that the magnitude of fracture is determined by the energy imposed per specific
volume of reinforcement acting on a finite element of the slab, a proper choice of the reinforcement grid size and bar size can control cracking into preferred orthogonal grids.

It must be emphasized that this hypothesis is important for serviceability and reasonable overload conditions. In relating orthogonal cracks to yield-line cracks, the failure of a slab ultimately follows the generally accepted rigid-plastic yield-line criteria.

B. Crack Control Equation

The basic Eq. 1 for relating crack width to strain in the reinforcement is

\[ W = \alpha a^\beta \varepsilon^\gamma \]  \hspace{1cm} (20)

The effect of the tensile strain in the concrete between the cracks is neglected as insignificant, \( a_c \) is the crack spacing, \( \varepsilon \), the unit strain in the reinforcement, and \( \alpha, \beta, \gamma \) are constants. As a result of the fracture hypothesis presented, the mathematical model in Eq. 17 and the statistical analysis of the data of 95 slabs tested to failure \(^5,6\), the following crack-control equation emerged:

\[ W = K\beta f_i \sqrt{\frac{d_{bs} s_2}{Q_t}} = K\beta f_i \sqrt{G_i} \]  \hspace{1cm} (21)

where the quantity under the radical, \( G_i = \frac{d_{bs} s_2}{Q_t} \), is termed the grid index, and can be transferred into

\[ G_i = \frac{s_1 s_2 d_c}{d_{bl}} \frac{8}{\pi} \]  \hspace{1cm} (22a)

where

\( K \) = fracture coefficient, having a value of \( 2.8 \times 10^{-5} \) for uniformly loaded restrained two-way-action square slabs and plates. For concentrated loads or reactions, or when the ratio of short to long span is less than 0.75 but
larger than 0.5, a value of \( K = 2.1 \times 10^{-5} \) is applicable. For a span aspect ratio of 0.5, \( K = 1.6 \times 10^{-5} \). Units of coefficient \( K \) are in square inch per lb.

\[ \beta = \frac{\text{distance from the neutral axis to the tensile face of the slab}}{\text{distance from the neutral axis to the centroid of the reinforcement}} \]

Grid (to simplify the calculations use \( \beta = 1.25 \), although it varies between 1.20 and 1.35).

\( f_s \) = actual average service load stress level, or 50% of the design yield strength, \( f_y \) (ksi).

\( d_{bi} \) = diameter of the reinforcement in direction 1 closest to the concrete outer tensile fibers (in.).

\( s_1 \) = spacing of the reinforcement in direction 1.

\( s_2 \) = spacing of the reinforcement in perpendicular direction 2.

\( l \) = direction of the reinforcement closest to the outer concrete tensile fibers; this is the direction for which crack control check is made.

Reinforcement area \( A_s \) per unit width

\[ Q_{ii} = \frac{\text{active steel ratio}}{12(d_{bi} + 2c_i)} \quad (22b) \]

where \( c_i \) is clear concrete cover measured from the tensile face of the concrete to the nearest edge of the reinforcing bar in direction 1.

\( w \) = crack width at face of concrete caused by flexural load at the service level (in.)

Subscripts 1 and 2 pertain to the directions of reinforcement. Detailed values of the fracture coefficients for various boundary conditions are given in Table 1.

Using SI units, the expression in Eq. 21 becomes

\[ w_{max} (mm) = 0.145K\beta f_s \sqrt{G_l} \quad (23) \]
where \( f_s \) is in MPa and all the terms for the grid index \( G_i \) in Eq. 21 are in mm.

A graphical solution of Eq. 2 is given in Fig. 8 for rapid determination of the reinforcement size and spacing needed for crack control where \( f_y = 60,000 \) psi (414 MPa) and \( f_s = 40\% \) \( f_y = 24,000 \) psi (165.5 MPa).

C. Tolerable Crack Widths in Concrete Structures

The maximum reasonable crack width that can be tolerated in a structural element without distress depends on the particular function of the element and the environmental conditions to which the structure is liable to be subjected. Table 1 from the ACI Committee 224 Report on cracking serves as a reasonable guide on the tolerable crack widths in concrete structures under the various environmental conditions that are normally encountered.

The crack control equation and guidelines presented are important not only for the control of corrosion in the reinforcement but also for deflection control. The reduction of the stiffness EI of the two-way slab or plate due to orthogonal cracking when the limits of tolerable crack widths in Table 1 are exceeded, can lead to excessive deflection both short-term and long-term. Deflection values several times those anticipated in the design, including deflection due to construction loading, can be reasonably controlled through camber, and control of the flexural crack width in the slab or plate. Proper selection of the reinforcement spacing \( s_1 \) and \( s_2 \) in the perpendicular directions, as discussed in this section, and not exceeding 12 in. center to center, can maintain good serviceability performance of a slab system under normal and reasonable overload conditions.

Long-term Effects on Cracking

In most cases, the magnitude of crack widths increases in long-term exposure and long-term loading. The increase in crack width can vary considerably in cases of cyclic loading, such as in bridges, but the width increases at a decreasing rate with
time. In most cases, a doubling of crack width or even a larger increase after several years under sustained loading can be expected.

**Crack control in deep beams**

Several cases have been reported, where wide cracks have developed on the side faces of beams between main flexural reinforcement and the neutral axis. Although the measured crack widths at the main reinforcement level were within acceptable code limits, the side face crack widths near mid-depth were as much as three times as wide.

Based experimental and analytical investigations of cracking in deep beams, Frantz and Breen developed recommendations for side face crack control in beams in which the depth, d, exceeds 36 in. (915 mm). ACI 318 requires now skin reinforcement to be uniformly distributed along both faces of the member for a distance d/2 nearest the flexural tension reinforcement.

**Anchorage zone cracking in prestressed concrete**

Longitudinal cracks develop in the anchorage zones of prestressed concrete members, particularly in post-tensioned beams, due to transverse tensile stresses set up by the concentrated forces at the end zones. To prevent such cracks from developing, which can lead to failure of the member, transverse reinforcement has to be used applying proper design procedures, such as the strut-and-tie method for proportioning the reinforcement. Two types of cracks may develop: spalling cracks which begin at the end face and propagate parallel to the prestressing force, and bursting cracks which can develop along the line of the force or forces, but away from the end face.

For many years stirrups were designed to take the entire calculated tensile force based on the analysis of the uncracked section. Classical and finite element analyses by Nawy and Breen et al. show similar stress distributions for which confining closed stirrups have to be provided. Also, use of spiral reinforcement indicates more beneficial effect in arresting the end anchorage zone cracks when used as transverse reinforcement. It is advisable that more closely spaced closed stirrups be provided at the end anchorage zones of post-tensioned members than what the theoretical analysis indicates. AASHTO Specifications give detailed guidelines on the
Design and Construction Practices to Mitigate Cracking

Design of anchorage zone reinforcement to prevent bursting and spalling cracks, primarily using the strut-and-tie approach for the proportioning and location of the reinforcement.\textsuperscript{11,20}

**Tolerable crack widths in prestressed members**

The Euro EC2 Code and some authors state that corrosion is a greater problem in prestressed concrete members because of the smaller area of steel used and because of the possible consequences of corrosion on high stressed steel.\textsuperscript{20,21,22} The CEB Model Code,\textsuperscript{3,8} does not allow any tension in the concrete for severe exposure conditions, namely, no cracking is permitted. Poston et al\textsuperscript{3,21,22} found that chloride ion concentration at the level of reinforcement due to penetration of chlorides from external sources to be basically proportional to crack width. In general, a crack width not exceeding 0.005 in. (13 mm) is the upper tolerable limit for prestressed members in aggressive environment.

**CONCLUSIONS**

With the aid of the expressions presented in this paper summarized in the following, the design engineer and the constructor can limit the flexural macro-crack widths that develop in concrete systems. By limiting the width to remain within the tolerable levels given in Table 1 for the prevailing environmental conditions, it would be possible to prevent or considerably minimize long-term corrosion deterioration and deflection increase, and also maintain the aesthetic characterisitics of the various elements of the system.

1. **Reinforced Concrete Beams and Thick One-Way Slabs:**

   \[ w_{\text{max}} = 0.076 \beta f_s \sqrt{f_y A} \times 10^{-3} \text{ (see Eq.2), or} \]

   \[ s \text{ (in.)} = \left( \frac{540}{f_s} \right) - 2.5 c_c, \text{ but not greater than 12(36/f_s)}, \text{ where} \]
   \[ f_s \text{ is in ksi (see Eq.4).} \]

The Euro EC2 Code requires crack width computation for cracking mitigation, using
2. Prestressed Pretensioned Beams

(a) Steel reinforcement level:

\[ s_{\text{max}} = 5.85 \times 10^{-5} \frac{A_i}{\Sigma_o} (\Delta f_s) \]  

(see Eq. 14a)

(b) Tensile face of concrete:

\[ w'_{\text{max}} = 5.85 \times 10^{-5} R \frac{A_i}{\Sigma_o} (\Delta f_s) \]  

(see Eq. 14b)

3. Prestressed Post-Tensioned Beams

(a) Steel reinforcement level

\[ w_{\text{max}} = 6.51 \times 10^{-5} \frac{A_i}{\Sigma_o} (\Delta f_s) \]  

(see Eq. 15a)

(b) Tensile face of concrete

\[ w_{\text{max}} = 6.51 \times 10^{-5} R \frac{A_i}{\Sigma_o} (\Delta f_s) \]  

(see Eq. 15b)

For non-bonded beams, the factor 6.51 become 6.83.

For high-strength high-performance prestressed concrete beams, with cylinder compressive strengths in the range of 10,000 – 15,000 psi, the maximum crack width is reduced to

\[ w_{\text{max}} (\text{in.}) = 2.74 \times 10^{-3} \frac{A_i}{\Sigma_o} (\Delta f_s) \]  

(see Eq. 16a)
In SI units,

\[ w_{\text{max}}(\text{mm}) = 4.0 \times 10^{-5} \sum A_i \left( \Delta f_i \right) \]  
(see Eq. 16b)

4. Prestressed Concrete Circular Tanks

\[ w_{\text{max}}(\text{in.}) = 4.1 \times 10^{-6} \sigma_{\text{f}} E_{\text{ps}} \sqrt{G_i} \]  
(see Eq. 19a)

5. Two-way Action Structural Slabs and Plates

\[ w_{\text{max}} = K \beta f_s \sqrt{G_i}, \quad \text{where} \quad G_i = \frac{s + s_e d_e}{d_e} \times \frac{8}{\pi} \]  
(see Eqs. 21,22)

6. Deep Beams and End-Anchorage blocks

For deep beams whose effective depth, \( d \), exceeds 36 in. (915 mm), side face crack control is exercised by having skin reinforcement uniformly distributed on both vertical faces at a distance \( d/2 \) nearest the flexural tension reinforcement. The high concentrated prestressing forces at the end anchorage blocks cause splitting and bursting cracks that can result in the failure of such beams, particularly those that are post-tensioned. It is imperative to provide closely-spaced closed confining rectangular hoops at the end zones as required by analysis. AASHTO Specifications give detailed guidelines on the design of anchorage zone reinforcement to prevent bursting and spalling cracks, primarily using the strut-and-tie approach for the proportioning of the reinforcement.

APPENDIX

Example 1 - Reinforced Concrete Beam in Severe Environment (Reproduced from the author’s textbook, Ref. 2)

A reinforced concrete beam, 12 in. wide and a clear cover of 1.5 in. A-1 is part of a structure subjected to severe environment. Check its crack control performance by the
following two methods given that \( f'_c = 5.0 \text{ ksi, } f_y = 75 \text{ ksi, } A_s = 3 \text{ No. 8 bars.}
\)

(a) Gergely-Lutz crack control expression for the maximum crack width, \( w \), in beams and one-way slabs:

\[
w = 0.076 \beta f_c \sqrt[3]{d_c A} \times 10^{-3} \text{ in.}
\]

where \( \beta = 1.20 \), \( A = \) concrete area in tension/ number of bars, \( f_c = 0.6 f_y \)

(b) Reinforcement spacing approach by the ACI 318-99.

**Solution:**

(a) *Equation approach*

\[
f_c = 0.6 f_y = 0.6 \times 75 = 45 \text{ ksi, } d_c = 2.5 \text{ in.}
\]

\[
A = \frac{(2 \times 2.5)12}{3} = 20 \text{ in.}^2, \text{ hence the crack width is}
\]

\[
w = 0.076 \times 1.2 \times 45 \sqrt[3]{2.5 \times 20} \times 10^{-3} = 0.0151 \text{ in. (0.38 mm).}
\]

The allowable crack width according to Euro EC2, or CEB-FIP = 0.30 mm for normal environment and less for severe environment.

The ACI 224 tolerable width = 0.012 in. (0.30 mm). Hence, the 3 No. 8 bars are non-effective in crack control according to both ACI 224 and Euro EC2.

(b) *Proposed ACI 318-99 Approach*

Max. allowable spacing \( s = \frac{540}{f_c} - 2.5 c_c = \frac{540}{45} - 2.5 \times 2 = 7 \text{ in. (179 mm)} \)

But not to exceed \( 12(36/f_y) = 12 \times 36 / 45 = 9.6 \text{ in. (244 mm)} > 7 \text{ in.} \), hence a maximum allowable spacing of 7 in. controls.

Actual spacing \( s = \frac{1}{2} (12 - 2 \times 2.0 \text{ in. side cover} - 2 \times 0.5 \text{ in. for stirrups} - 2 \times 0.5 \text{ in. to center of outside bars}) \), or \( s = 3 \text{ in. center to center, < 7 in. allowable; hence the beam section is O.K. for crack control by the currant ACI 318 provisions, although it fails when applying the tolerable crack width criteria of Table 1, and also fails by the Euro EC2 requirements. One solution would be to use a larger number of smaller diameter bars in two layers to reduce the value of "A", leading to smaller computed crack width.}
Example 2 – Post-Tensioned Prestressed Concrete Flanged Section Containing Non-Prestressing Reinforcement (Reproduced from the author’s textbook in Ref. 11): A post-tensioned prestressed concrete T-beam is prestressed with twelve 7/16-in. dia. 7-wire strands of 270-K grade and additionally reinforced with four #6 non-prestressed mild steel reinforcement. The geometry of the section is as shown in Figure 9. Given that \( f'_c = 5000 \) psi, \( E_c = 57000 \sqrt{f'_c} \) psi, and \( E_s = 28,000 \) ksi, find the mean stabilized crack spacing and the crack widths at the reinforcement level, as well as at the tensile face of the beam at \( \Delta f_s = 30,000 \) psi, assuming there is no failure in shear or bond. Then determine whether the beam satisfies the serviceability criteria for crack control for humidity and moist air.

Solution:

\[ \Delta f_s = 30,000 \text{ psi} = 30 \text{ ksi} \]

**Mean Stabilized Crack Spacing:**

\[ \Delta = 8 \times 12 = 96 \text{ in.}^2 \]

\[ \Sigma o = 12 \times \Pi \times \frac{7}{16} + 4 \times 2.36 = 25.93 \text{ in.} \]

\[ a_{cr} = 1.54 \frac{A_t}{\Sigma o} = 1.54 \times \frac{96}{25.93} = 5.70 \text{ in. (145 mm)} \]

**Maximum Crack Width at Reinforcement Level:**

\[ w_{max} = 6.51 \times 10^{-5} \cdot \frac{A_t}{\Sigma o} (\Delta f_s) = 6.51 \times 10^{-5} \times \frac{96}{25.93} \times 30 = 0.0072 \text{ in. (0.18 mm)} \]

**Maximum Crack Width at Tensile Face of Beam:**

\[ R_s = \frac{22 - 9.31}{22 - 9.31 - 4} = 1.46 \]
Maximum Tolerable Crack Width for Humidity:

From Table 1, the maximum tolerable crack width for the stated humidity conditions is 0.012 in. (0.3 mm) > 0.0102 in. (0.26 mm). O.K.

Example 3 - Crack-control Evaluation for Serviceability in a Rectangular Two-way Floor Panel Subjected to Severe Exposure Conditions. (Reproduced from the author's textbook, Ref. 2): Select the bar size and spacing necessary for crack control at the column reaction region of the 7-in.-thick slab that is uniformly loaded. The ratio of the short span to the long span is 0.8. Select the bar size for two conditions:

Condition A: Floor is subjected to severe exposure of humidity and moist air.
Condition B: Floor sustains an aggressive chemical environment where the design working stress level in the reinforcement is limited to 15 ksi (15,000 psi), given:

\[ \beta = 1.20, \quad \frac{l_y}{l_I} = 0.8, \quad f_s = 60 \text{ ksi (414 MPa)} \]

Solution:

Condition A: Humidity and moist air

Tolerable \( w_{\max} = 0.012 \text{ in. (0.3 mm)} \) (Table 1). Try No. 4 bars \( d_b = 0.5, d_C = 0.75 + 0.25 = 1.0 \text{ in.} \). Assume that \( s_1 = s_2 = s \) for the given panel.

The aspect ratio, \( l_y/l_I = 0.8 \), \( K = 2.1 \times 10^{-5} \) for concentrated reaction at the column support (Table 4), to give \( G_I = 394 \text{ in.}^2 \). Therefore,

\[ 394 = \frac{s^2 d_b}{d_{bl}} \times 8 = \frac{s^2 \times 1.0}{0.5} \times \frac{8}{8.8} \text{ in.} \]

Hence use No. 4 bars at 8Qw in. center to center each way for crack control.
**Condition B: Aggressive chemical environment**

Tolerable $w_{\text{max}} = 0.007$ in. (0.18 mm) (Table 1). $f_s = 15$ ksi to be used as a low stress level for sanitary or water-retaining structures instead of $0.4f_y$.

Try No. 5 bars ($d_b = 0.625$ in.), to give a grid index $G_f = 343$ in.$^2$

$$d_{s1} = 0.75 + 0.312 = 1.06 \text{ in.}$$

$$G_1 = 343 = \frac{s^2 \times 1.06 \times 8}{0.625 \times \Pi}$$

to get $s = 8.9$ in.

Use No. 5 bars at 9-in. (229-mm) center-to-center spacing each way for crack control.

**Reinforcement summary:**

- **Condition A:** No. 4 bars at 8 in. c-c (12.7-mm diameter at 216 mm c/c)
- **Condition B:** No. 5 bars at 9 in. c-c (15.9-mm diameter at 229 mm c/c)

**REFERENCES**


4. Gergely, Peter, and Lutz, Leroy A., "Maximum Crack Width in Reinforced Concrete Flexural Members," Causes, Mechanism, and Control of Cracking in Concrete, SP-20, E.G. Nawy, Editor, American Concrete Institute, Farmington Hills, MI, 1968, pp. 87-117.


13. Frosch, R. J., “Another Look at Cracking and Crack Control in Reinforced Concrete,” ACI Structural Journal, V. 96, No. 3, American Concrete Institute, Farmington Hills, MI, May-June, 1999, pp. 437-442.

14. Nawy, E. G., “Discussion to Section 10.6, ACI 318-99 Code,” Concrete International V. 21 No. 5, American Concrete Institute, Farmington Hills, MI, May 1999, pp. 318:10-12


**METRIC (SI) UNIT EQUIVALENTS**

<table>
<thead>
<tr>
<th>Unit</th>
<th>Equivalent</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 inch</td>
<td>25.4 mm</td>
</tr>
<tr>
<td>1 foot</td>
<td>0.305 m</td>
</tr>
<tr>
<td>1 in.$^2$</td>
<td>645.16 mm$^2$</td>
</tr>
<tr>
<td>1 in.$^3$</td>
<td>16.387 mm$^3$</td>
</tr>
<tr>
<td>1 in.$^4$</td>
<td>416.231 mm$^4$</td>
</tr>
<tr>
<td>1 psi</td>
<td>6.895 kPa</td>
</tr>
<tr>
<td>$f_c$ psi</td>
<td>0.083036 MPa</td>
</tr>
</tbody>
</table>

1 lb = 4.448 N
1 kip = 4448 N
1 lb/ft = 14.594 N/m
1 kip/ft = 14.594 kN/m
1 kip-in. = 113 N-m
Table 1. Maximum tolerable flexural crack widths

<table>
<thead>
<tr>
<th>Exposure condition</th>
<th>Crack width</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dry air or protective membrane</td>
<td>0.016 in.</td>
</tr>
<tr>
<td>Humidity, moist air, soil</td>
<td>0.012 in.</td>
</tr>
<tr>
<td>De-icing chemicals</td>
<td>0.007 in.</td>
</tr>
<tr>
<td>Seawater and seawater spray; Wetting and drying</td>
<td>0.006 in.</td>
</tr>
<tr>
<td>Water-retaining structures (excluding nonpressure pipes)</td>
<td>0.004 in.</td>
</tr>
</tbody>
</table>

Table 2. Maximum bar diameters for high-bond bars

<table>
<thead>
<tr>
<th>Steel Stress (MPa)</th>
<th>Maximum bar size (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>160</td>
<td>32</td>
</tr>
<tr>
<td>200</td>
<td>25</td>
</tr>
<tr>
<td>240</td>
<td>20</td>
</tr>
<tr>
<td>280</td>
<td>16</td>
</tr>
<tr>
<td>320</td>
<td>12</td>
</tr>
<tr>
<td>360</td>
<td>10</td>
</tr>
<tr>
<td>400</td>
<td>8</td>
</tr>
<tr>
<td>450</td>
<td>6</td>
</tr>
</tbody>
</table>
Table 3. Maximum bar spacing for high-bond bars

<table>
<thead>
<tr>
<th>Steel Stress (MPa)</th>
<th>Maximum Bar Spacing (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Pure Flexure</td>
</tr>
<tr>
<td>160</td>
<td>300</td>
</tr>
<tr>
<td>200</td>
<td>250</td>
</tr>
<tr>
<td>240</td>
<td>200</td>
</tr>
<tr>
<td>280</td>
<td>150</td>
</tr>
<tr>
<td>320</td>
<td>100</td>
</tr>
<tr>
<td>360</td>
<td>50</td>
</tr>
</tbody>
</table>

Table 4. Fracture coefficients for slabs and plates

<table>
<thead>
<tr>
<th>Loading type(^a)</th>
<th>Slab shape</th>
<th>Boundary condition(^b)</th>
<th>Span ratio(^c)</th>
<th>Fracture coefficient (10^{-5} K)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>Square</td>
<td>4 edges r</td>
<td>1.0</td>
<td>2.1</td>
</tr>
<tr>
<td>A</td>
<td>Square</td>
<td>4 edges s</td>
<td>1.0</td>
<td>2.1</td>
</tr>
<tr>
<td>B</td>
<td>Rectangular</td>
<td>4 edges r</td>
<td>0.5</td>
<td>1.6</td>
</tr>
<tr>
<td>B</td>
<td>Rectangular</td>
<td>4 edges r</td>
<td>0.7</td>
<td>2.2</td>
</tr>
<tr>
<td>B</td>
<td>Rectangular</td>
<td>3 edges r, 1 edge h</td>
<td>0.7</td>
<td>2.3</td>
</tr>
<tr>
<td>B</td>
<td>Rectangular</td>
<td>2 edges r, 2 edges r</td>
<td>0.7</td>
<td>2.7</td>
</tr>
<tr>
<td>B</td>
<td>Square</td>
<td>4 edges r</td>
<td>1.0</td>
<td>2.8</td>
</tr>
<tr>
<td>B</td>
<td>Square</td>
<td>3 edges r, 1 edge h</td>
<td>1.0</td>
<td>2.9</td>
</tr>
<tr>
<td>B</td>
<td>Square</td>
<td>2 edges r, 2 edges h</td>
<td>1.0</td>
<td>4.2</td>
</tr>
</tbody>
</table>

\(^a\)Loading type: A, concentrated; B, uniformly distributed.
\(^b\)Boundary condition: r, restrained; s, simply supported; h, hinges.
\(^c\)Span ratio: S, clear short span; L, long span.
Figure 1. Schematic stress distribution between two flexural cracks: (a) Section between two principal cracks; (b) Bond Stress distribution; (c) Longitudinal tensile stress in the concrete; (d) Longitudinal tensile stress in the reinforcement.
Figure 2. Schematic variation of crack width with crack spacing.
Figure 3. Forces and stress distribution in prestressed beam stabilized cracks: (a) longitudinal forces, (b) bending and bond stress distribution.

- \( \Delta a_o \) = crack spacing
- \( l_d \) = development length
- \( f_{s2} \) = reinforcement stress at crack 2 > \( f_{s1} \)
- Bending steel stress, \( f_s \)
- Bond stress, \( \mu \)
Figure 4. Effective concrete area in tension: (a) even reinforcement distribution, (b) non-even reinforcement distribution.
Figure 5. Linearized Maximum Crack Width vs. $\frac{A_f}{\Sigma_0} \Delta f_s$ (KIP/IN) in Pretension Beams
Figure 6. Linearized Maximum Crack Width vs. $\left( \frac{A_t}{x_o} \right) f_r$ In Post-tensioned Beams
Figure 7. Grid unit in two-way action reinforcement.
Figure 8. Crack control reinforcement distribution in two-way action slabs and plates for all exposure conditions: $f_y = 60$ ksi, $f_s = 0.40$, $f_y = 24$ ksi.
Figure 9. Geometrical dimensions of prestressed beam in example 2.
Early-Age Thermal Cracking in Laser-Screeded Concrete Slabs

by H. Haynes

Synopsis: A case history is presented in which a laser-screeded slab showed more cracks than would be expected in conventional strip-cast slabs. It was determined that laser-screeded slabs are more sensitive to early-age, thermal cracking than strip-cast slabs because of extra restraint provided by fixed-edge boundary conditions. Among the possible solutions is closer spacing of contraction joints and fog curing during the first day.

Keywords: case history; concrete slabs; cracks; curling; early age; fog curing; laser screed; slab-on-grade; temperature and shrinkage; thermal strain; thermal stress
INTRODUCTION

Laser-guided self-propelled screeds (Figure 1) for slab-on-grade construction were introduced in 1986. One machine can screed 40,000 ft$^2$, (3,700 m$^2$) in one day, about four times the production rate of conventional strip-cast construction. The cost saving of using laser-screed equipment is significant, and its use for construction of large slabs is already an industry standard.

New developments can bring new problems. This paper addresses a field problem with laser-screeded slabs. The author, a consulting concrete engineer in the San Francisco Bay area of California, has come across two projects where laser-screeded slabs showed more cracking than expected. The cracks seemed to be related to thermal contraction and drying shrinkage. Few details of these projects were known.

The author later had an opportunity to investigate a similar problem that is discussed in detail here. Evaluation of the findings suggests that laser-screeded slabs may be more sensitive to cracking due to early-age thermal behavior than conventional strip-cast slabs.

CASE HISTORY

A tilt-up warehouse located in Fremont, California, was built in 1998 having a slab-on-grade approximately 209,000 ft$^2$, (19,400 m$^2$). Laser-screed equipment was used to place the slab during six operating days. The average area of slab cast was about 35,000 ft$^2$, (3200 m$^2$) per day. In order to assure a continuous supply of concrete, placement operations were conducted from 1 AM to 8 AM.

The slab was 5 in. (127mm) thick and reinforced with #3 bars at 18 in. (460 mm) each way. Contraction joints spaced at 16-ft. (4.9 m) intervals in one direction and 20-ft. (6.1 m) intervals in the other were saw-cut to a depth of 1 in. (25 mm) immediately after the final finishing operation. The sub-base conditions were 1 in. (25 mm) of sand over 5 in. (127 mm) of Class II base rock. The concrete used Type II portland cement with a total cementitious material content of 526 lb/cy (31 0 kg/m$^3$) of which 15% by mass was Type F fly ash. The water to cementitious material ratio was 0.53 and the specified slump was 3 in. (75 mm). A water-reducing admixture was used. A quartzitic sand was used as fine aggregate, and limestone 3/4-in. (9 mm) nominal maximum size as coarse aggregate. The compressive strength of the concrete at 28 days averaged 4320 psi. (29.8 Mpa)

The slab showed cracks between contraction joints (some with multiple cracks) within days of placing the concrete. After six months of fork-lift traffic, cracks in the aisles about 0.030-in. (0.8 mm) and wider showed raveling along the edges. Vertical movement of the slab could be felt as a fork-lift drove across one of the cracks. Additional cracks appeared in the traffic aisles.

The following common causes of cracking were ruled out:

5. Core samples from the slab showed that the thickness was uniformly 5 in. (127 mm) and the compressive strength was 5,060 psi. (34.9 Mpa)
6. The depth of saw cut contraction joints measured 1 in. (25mm)
7. During placement of concrete, the slump was consistently between 4 to 4.5 in. (100 to 114 mm) The increase in slump from that of the proposed mixture was not a
significant contributor to cracking; the added water would have increased the w/cm ratio from 0.53 to about 0.57.

8. The quality of the aggregate was premium. Local contractors have noticed that extra cracking can result when "normal" river-run sand or coarse aggregates are used. The tilt-up walls used a concrete mixture similar to that of the slab, and the walls had minimal cracks.

9. Contraction joints were sawed in a timely manner.

10. Although additional cracks appeared in the aisles, the majority of cracks existed before traffic was applied to the slab.

Spacing of contraction joints at 16 and 20 ft. (4.9 and 6.1 m) did not follow Portland Cement Association's suggested joint spacing of 13 to 15 ft. (4.0 to 4.6 m); however, the spacing was closer than typically used in the local area for strip-cast slabs, which was 20 to 24 ft. (6.1 to 7.3 m).

It appeared weather was one factor that was a main contributor to cracking. Temperatures were moderately hot during the days of concrete placement in the month of May. NOAA Weather records for neighboring cities showed that the average maximum temperature was about 85°F (29°C) at 3 PM and the minimum was about 60°F (16°C) at 6 AM. One month later, when the tilt-up wall panels were cast, the weather conditions were almost identical. The author suggests that lack of adverse cracking in the wall panels was due to wall panels having four edges free from restraint during their early age.

The structural engineer on the project said that no cracking had appeared on another building of similar design, which was constructed several months earlier in a nearby city. The weather was overcast at that time.

The author believes that early-age thermal behavior contributed to the cracking of the slab. In addition, there is a feature to laser-screeded slabs that is different than that of conventional strip-cast slabs. With strip-casting, the long, narrow slabs are divided into shorter sections by transverse saw cutting or other methods. The saw-cut sections have two sides restrained before cracking and two sides free of restraint. With laser-screeded slabs, large areas are monolithically cast and then divided by saw cuts into smaller sections, leaving all four edges restrained before cracking. In the author’s opinion this edge restraint promotes cracking in laser-screeded concrete, which does not occur in strip-cast slabs.

**EARLY-AGE THERMAL BEHAVIOR**

From the author’s experience, cracks observed the day after concrete placement are influenced by early-age thermal behavior. This type of crack is more common than generally known. The appearance of these cracks is similar to that of drying-shrinkage cracks.

With drying shrinkage, it is well known that concrete becomes drier at the top surface than at the bottom. This moisture gradient produces differential shrinkage strains, the highest tension occurring at the top. The moisture gradient is a source of curling in the slab.

Less well known is that concrete slabs curl from thermal differentials that are directly related to first-day thermal conditions of the concrete slab. Concrete placed on moderately hot and sunny days can be significantly hotter at the top surface, than at the bottom, due to solar radiation. At some point as the concrete hardens, the thermal gradient through the thickness of the slab becomes the "locked-in" condition of the concrete. In other words, the concrete hardens with the material at the top surface having a bulk volume defined by a higher temperature than that of the material at the bottom. When cool ambient temperatures cause a decrease in concrete temperature, the top surface contracts more than
the bottom, which causes bending stresses for curling. If ambient temperatures at night are substantially lower than the locked-in concrete temperatures, thermal stresses from curling and axial contraction can exceed the tensile strength of the young concrete and cause cracks.

**Test Slabs Show Early-Age Thermal Behavior:** Rhodes and Evans demonstrated early-age thermal behavior of test slabs by exposing freshly cast concrete to simulated sunshine. The purpose of their tests was to evaluate membrane curing compounds. They tested slab sections 8 in. deep, 12 in. wide, and 84 in. long (203 x 305 x 2134 mm). Concrete was placed at 8 AM, coated with a curing compound or other method, and exposed to simulated solar radiation from a bank of incandescent lights, which heated the top surface to a temperature between 120 and 130°F (49 and 54°C). At 6 PM the lights were turned off, and at 8 AM the lights were turned on again. This cycle was repeated for at least 7 days. The temperature at the top, middle and bottom of the slab was measured, along with the amount of curl deflection. The data were presented in strip chart form, a sample of which is shown in Figure 2. Curling began when the lights were turned off at 6 PM.

Figure 3 shows thermal gradients through the slab on which the clear curing compound was applied (data from Figure 2) at various times after casting at 8 AM. Concrete temperature at placement was 70°F (21°C). At noon the temperatures on the top and bottom surfaces were 110 and 85°F (43 and 29°C), respectively, and by 6 PM they were 121 and 96°F (49 and 36°C). The temperatures rapidly decreased once the lights were turned off at 6 PM. At 9 PM the temperatures on the top and bottom surfaces were 98 and 98°F (37°C) (103°F or 41°C at mid-depth), and by 8 AM they were 82 and 85°F (28 and 29°C).

Assuming that 3 PM was the “lock-in” time for the concrete, the temperature differentials, \( \Delta T \), were calculated. The maximum gradient of temperature differentials was found at 8 AM of day 2 (Figure 3b). This gradient was very close to linear. The total strain was calculated by

\[
e = \Delta T \gamma,
\]

where \( \gamma \) is the linear coefficient of thermal expansion at 5 millionths/°F (9 millionths/°C). Using superposition, the average axial strain across the slab was calculated as +102 millionths in tension, and the bending strain was ±72 millionths.

Accurately converting strain to stress was difficult because of uncertainty with the effective modulus of elasticity. Tests by Kasei and Okamura on determination of stress and strain in concrete at early ages showed an elastic modulus of about 3 x 10^6 psi (20.7 GPa) at 24 hours. Early-age concrete shows high creep properties, and the amount of creep relaxation for concrete in tension at early ages is a topic little researched. Assuming creep relaxation was 50 percent, the effective modulus was about 0.5(3 x 10^6) = 1.5 x 10^6 psi. Hence, calculated total stress at the top and bottom surfaces was approximately +263 and +45 psi (1.8 and 0.3 MPa), respectively. Axial stress was +145 psi (1.0 MPa) and bending stress ±109 psi (0.8 Mpa). In order to develop these stresses in the slab, the edge conditions would have had to be fully restrained. This aspect of the analysis will be discussed in the next section.

Rhodes and Evans provided flexural strength data for their concrete at early ages. At 24 hours the flexural strength was about 250 psi (1.7 Mpa) for concrete cured in water at 75°F (21°C), and 290 psi (2.0 Mpa) at 100°F (38°C). Hence, cracking may or may not have occurred for this slab, assuming the edges were fully restrained.

The calculated thermal stresses would be higher if the evening temperatures were lower. A hypothetical case will illustrate this situation. Using the above example and assuming that the concrete temperature decreased to a uniform 65°F (18°C) by 8 AM of the second day, the following results were calculated: the total strain at the top surface was +260 millionths tension, and at the bottom +130 millionths tension. This equated to tensile stresses at the top and bottom surfaces of +390 and +195 psi (2.7 and 1.3 Mpa), respectively.
Interestingly, the bending stress stayed about the same as the previous example, but the axial stress increased by about +150 psi. Cracking most likely would have occurred in this circumstance.

The calculated stresses are influenced by the "lock-in" conditions of the first day. If final set of concrete were the time of lock-in, this would suggest that noon was a better lock-in time than the assumed 3 PM. Or perhaps 6 PM would be the best time of lock-in, because of high creep relaxation. For the example in Figure 3, a change of lock-in time from noon to 3 PM to 6 PM significantly increases the magnitude of the differential axial thermal strains, but does not significantly change the differential bending strain gradient. Determination of lock-in conditions is an important topic that requires research.

**Curling Behavior:** Response of the Rhodes and Evans' slabs to thermal and shrinkage behavior was observed by studying curl deflection data. Figure 4 shows the maximum and minimum curl deflections of slabs for seven days of thermal cycles. The average results of seven slabs with curing compound are shown. These slabs were cured with either clear or white curing compounds, and the sub-grade conditions were saturated sand, dry sand, and waterproofed. The slab with no curing had a saturated sand sub-grade. The results for wet-burlap curing were from an average of three slabs in which the sub-grade conditions were saturated sand, dry sand, and waterproofed.

Maximum curl occurred at 8 AM, when the differential thermal gradient was greatest after cool evening temperatures. Minimum curl was at 6 PM, when the thermal gradient was similar to that of the lock-in conditions of the first day. The difference between the maximum and minimum curl is due to thermal effects. The difference between zero and minimum curl is due to drying shrinkage. Curl due to drying shrinkage increased each additional day, while curl due to thermal behavior remained relatively constant.

Figure 4 shows that curing compound did not effectively control curl due to drying shrinkage. ASTM C-1315 on Liquid Membrane-Forming Compounds Having Special Properties for Curing and Sealing Concrete specifies that the rate of moisture loss at 100°F (38°C) and 32% relative humidity should not exceed 0.40 kg/m². This rate equates to 27 lb/1000sf/day (0.13 kg/m²/day), which is a fair amount of moisture loss. For comparison, water evaporating from a pond at 73°F (23°C) with zero wind is about 150 lb/1000sf/day (0.73 kg/m²/day). At the other end of the spectrum is the rate of moisture emission permitted from concrete slabs prior to covering with vinyl sheeting, which is 3 lb/1000sf/day (0.015 kg/m²/day).

After three to four days, curl due to drying shrinkage was equal to that of curl from thermal effects. Development of a moisture gradient through the test slabs occurred rapidly under the severe test conditions of incandescent lights from 8 AM to 6 PM. Actual ambient conditions have fewer hours of strong sun exposure, so curl from drying shrinkage would not occur as quickly, but it will occur.

Water curing was most effective in minimizing the amount of curl from thermal behavior and eliminating curl from shrinkage. Curl from shrinkage will begin when the concrete starts to dry.

**Effect of Slab Edge Restraint:** Edge restraint is particularly important in this discussion of laser-screeded slabs. Edge restraint conditions determine whether effective strains and stresses occur due to thermal and drying-shrinkage behavior. For example, a slab with no edge restraint is free to curl from differential thermal and shrinkage behavior. Free slabs are those with wide cracks at contraction joints or random cracks. The actual width of a crack to qualify for free condition is not known, but the author would suggest 1/32 in. (1 mm).
If the differential temperature gradient is linear for a slab with free edges, as shown in Figure 5a, then the effective strain and stress are zero (assuming no restraint by subgrade drag). When each element of concrete can freely respond to a change in temperature, the strain and stress in the x, y, and z directions cancel out. It is intuitively confusing that the slab has deflected by curling, yet the effective strain and stress are zero. This is the situation, however, because the forces acting on the slab are from internal stresses, as opposed to externally applied loads. (Gravity loads are not included in this discussion.)

If the differential temperature gradient is non-linear (Figure 5b), then some internal restraint occurs and effective stresses develop.

Full restraint, which is the edge condition for a slab-section with uncracked concrete, is shown in Figure 5c and 5d. For linear and non-linear differential temperature gradients, the thermal strains are fully restrained and therefore develop effective stresses.

**Edge Restraint of Laser-Screeded and Strip-Cast Slabs:** The difference in edge restraint between laser-screeded and strip-cast slabs is important in explaining unexpected cracks in laser-screeded slabs. Otherwise, the extra cracks could be attributed solely to high early-age thermal stresses.

For laser-screeded slabs, sections of concrete defined by contraction joints are still uncracked on the first day as cool evening temperatures approach. The concrete slab sections have fixed edges on four sides, as shown in Figure 6a.

For conventional strip-cast slabs, there are two stages of construction: placement of initial strips followed on a different day by casting in-fill strips, Figure 6b. The longitudinal edges of the initial strips are free, while in the transverse direction the edges are fixed. The in-fill strips have the longitudinal edges cast against the initial strips, and these edges experience minor partial restraint. The reason for minor restraint relates to the behavior of the initial strips, whose longitudinal edges will curl as cool evening temperatures advance. When the initial strip curls, the edges to the in-fill strip will also curl. In the transverse direction, the edges are fixed.

For simplicity, it can be assumed that the laser-screeded slab has four edges fixed, while the strip-cast slab has two edges fixed and two free. Elastic analysis of flat plates exposed to a linear thermal gradient across the thickness has the maximum bending stress calculated by the equation ±ΔTYE/2(1-υ), where υ is Poisson’s ratio. A two-dimensional element such as a beam having fixed ends (its sides are free) has bending stress calculated by the equation ±ΔTYE/2. The difference in stress between the plate and beam is the factor 1/(1-υ) applied to the plate. The laser-screeded slab is equivalent to the plate, and the strip-cast slab is approximately equivalent to the beam. Poisson’s ratio for concrete is about 0.2, so the factor 1/(1-υ) is equal to 1.25. This implies that the maximum bending stress in laser-screeded slabs due to thermal gradients is about 25 percent greater than that in strip-cast slabs.

Slabs cast in the field will not follow elastic analysis behavior. Also, strip cast slabs behave in a more complex manner than that of a two-dimensional beam. However, the theory indicates that laser-screeded slabs will develop higher bending stresses due to thermal behavior than strip-cast slabs. This indicates that laser-screeded slabs are more sensitive to cracking by early-age thermal behavior than that of strip-cast slabs.

**SOLUTIONS**

This section discusses two possible solutions to the problem of thermal cracking in laser-screeded slabs: proper contraction joint spacing, and fog curing during daytime.
Construction. Other possible approaches, such as continuously reinforced concrete and steel fiber reinforced concrete, will not be discussed.

**Contraction Joint Spacing:** A close spacing of contraction joints should minimize early-age thermal cracks, in a manner similar to that for drying-shrinkage cracks. PCA's suggested spacing of 13 to 15 ft. (4.0 to 4.6 m) for a slab of 5-in. (127 mm) thickness may have solved the cracking problem for the warehouse slab in the case history. Joint spacing may need to be even closer for laser-screeded slabs cast in hot weather. Experience will provide the correct answer.

The disadvantage of such close spacing of joints is increased installation and maintenance costs. Basically, from an operations standpoint, the fewer joints in a warehouse slab the better.

**Fog Curing:** The advantage of fog curing is maintaining lower temperatures in the freshly placed concrete on the first day by evaporative cooling effects. The average temperature and gradient need to be as low as possible at the time of lock-in, so when cool evening temperatures arrive, $\Delta T$ is too small to cause cracking.

The data on wet burlap curing in Figure 4 show the benefit of evaporative cooling. These slabs had a curl deflection due to $\Delta T$ of 0.007 in. (0.18 mm), as opposed to 0.010 in. (0.25 mm) for curing compound and 0.014 in. (0.36 mm) for no curing. The reduction in curl deflection of wet burlap is moderately significant, but the present author believes that this reduction can be more significant for field concrete.

In a laboratory the relative humidity can be high when curing with wet burlap. At high relative humidity, evaporative cooling effects are small. For example, when air temperature is 90°F (32°C) and relative humidity 80%, evaporative cooling reduces the temperature to about 85°F (29°C). However, at 90°F and relative humidity of 50%, evaporative cooling reduces the temperature to 75°F (21°C).

For a laser-screeded slab, the benefits of evaporative cooling can be obtained by water or fog curing. Difficulties arise, however, in attempting to implement these curing procedures. Timing and large surface areas are obstacles. To avoid any unnecessary increase in concrete temperature, the concrete should experience the effects of evaporative cooling as soon as possible. Fog curing can begin during concrete placement and continue during finishing operations. Water curing always begins after the final finishing operation, which is too late for addressing the problem discussed here.

The large surface area of a laser-screeded slab requires a significant fogging operation to keep the air saturated with moisture. Water-misting heads connected to hoses at city water pressure of about 50 psi (0.3 Mpa) will not likely be sufficient, because the water droplets are relatively large and heavy. Fine droplets of water which will remain suspended in air as in true fog can be created by fogging heads that atomize water at pressures of about 1000 psi. (7 Mpa). Off-the-shelf equipment exists which integrates fans and fogging heads to blow fog across the slab. To the author's knowledge, this type of fogging operation has not been applied to field slabs, but the method appears feasible. As a cautionary note, the fogging operation should be suspended at night, because it is undesirable to create concrete temperatures lower than the ambient nighttime temperatures.

**SUMMARY**

Field experience indicates that laser-screeded slabs are more sensitive to cracking when cast during moderately hot, sunny weather than are strip-cast slabs because of early-age thermal behavior in conjunction with extra restraint due to fixed-edge boundary conditions.
The temperature of the concrete on the first day defines the "lock-in" conditions of the concrete from which thermal differentials are calculated. Cool temperatures on the first night produce a differential thermal gradient that causes curling as well as uniform axial shortening. Restraint of these thermal movements produces stresses that can exceed the tensile strength of the young concrete.

Laser-screeded slabs are restrained on four sides rather than two, as in strip-cast slabs. Under similar weather conditions, this extra restraint produces higher stresses that may cause laser-screeded slabs to crack, while strip-cast slabs may not crack.

Two solutions to the cracking problem of laser-screeded slabs are discussed. A closer spacing of contraction joints is obviously one of the solutions. However, to avoid installation of extra contraction joints in warehouse slabs, an alternate approach is to fog-cure laser-screeded slabs starting at the time of concrete placement. Evaporative cooling effects will yield lower concrete temperatures on the first day.

REFERENCES


Figure 1. Laser-guided, self-propelled screed for construction of large area slabs-on-grade.
Figure 2. Data after Rhodes and Evans on concrete slabs 8 inches deep, 12 inches wide, and 84 inches long having a saturated subgrade: (a) curling of slabs, and (b) Temperature gradient through slabs.
Assume 3pm is time of “lock-in” condition for concrete. Maximum thermal differential: ΔT = T_{3pm} - T_{8am} of 2nd day

Actual ΔT Gradient Assumed Linear ΔT Gradient

Assume slab is fully restrained and thermal gradient is linear
\[ e = \gamma \Delta T, \text{ where } \gamma = 5 \times 10^{-6} \text{ in/in°F} \]
\[ \delta = (E_{eff.})(e) \text{ where } E_{eff.} = 0.5E_{elastic} \]

Figure 3. Data after Rhodes and Evans from slab coated with clear curling compound and having a saturated sand subgrade: (a) temperature gradients through slab on first day, (b) temperature differential between 3pm lock-in conditions and 8am of second day, (c) Calculated strain and stress gradient across slab.
Figure 4. Maximum and minimum curling of slab for seven days of thermal cycling where lamps were turned on at 8am and off at 6pm.
No restraint, or free edge condition

Linear thermal gradient

\[ \text{effective } e = 0 \text{ in/in} \]
\[ \text{effective } \delta = 0 \text{ psi} \]

Nonlinear thermal gradient

\[ \text{effective } e \neq 0 \]
\[ \text{effective } \delta \neq 0 \]

Full restraint, or fixed edge condition

Linear thermal gradient

\[ e = \text{max} \]
\[ \delta = \text{max} \]

Bending Only

Axial component will move zero axis to left or right.

Nonlinear thermal gradient

Figure 5. Effective stress and strain for slabs: (a) with no restraint and linear thermal gradient, (b) with no restraint and nonlinear thermal gradient, (c) with full restraint and linear thermal gradient, and (d) with full and nonlinear thermal gradient.
Figure 6. Edge restraint for uncracked concrete in:
(a) laser-screeded slabs and (b) strip-cast slabs.
Synopsis:

This paper provides an outline of the provisions for design for cracking given in the current version of Eurocode 2; the Eurocode for the design of concrete structures. The basic theory underlying the clauses is derived, the content of the clauses themselves are outlined and the development of simplified detailing rules for the control of cracking is considered.

Keywords: bar size; beams; crack control; crack width; design; examples; Eurocode; structures; theory
Andrew Beeby is Professor of Structural Design in the School of Civil Engineering at the University of Leeds, England. He is a member of the UK code committee for concrete structures and of CEN TC 250 SC2, the committee of the European Standards Organisation (CEN) concerned with Eurocode 2.

INTRODUCTION

The purpose of this paper is to provide an outline of the current thinking within the European code drafting community on the treatment of cracking in the design of reinforced and prestressed concrete structures. It must be said at the outset that the provisions have not been finalised and there are still areas of debate. This paper therefore provides only an interim picture of the state of affairs.

Though it is likely to be well known to many readers of this paper, it may be useful at the start of the paper to have an outline of the Eurocode project generally and the current state of development.

The Treaty of Rome has, as one of its aims the removal of barriers to the free movement of goods and services across the national boundaries within the European Union. The great range of different design codes within the EU was seen to constitute a barrier to the free movement of services and so a decision was made (in around 1980) to develop a uniform and co-ordinated set of design codes for structural design. This has turned out to be a far greater, and more ambitious, task than the European Commission initially envisaged and it is still far from complete, despite 20 years of effort. Early work was carried out by committees appointed directly by the European Commission but they eventually handed the work over the the European Standards Organisation, CEN

The Eurocodes currently under preparation are the following:

- Eurocode 0  Basis of design
- Eurocode 1  Actions on structures
- Eurocode 2  Design of concrete structures
- Eurocode 3  Design of steel structures
- Eurocode 4  Design of composite steel and concrete structures
- Eurocode 5  Design of timber structures
- Eurocode 6  Design of masonry structures
- Eurocode 7  Geotechnical design
- Eurocode 8  Earthquake resistant design of structures
Eurocode 2, the code of practice for the design of concrete structures, is one of the most advanced. A complete version was published as a Pre-Standard (ENV) in 1991 (1). In this form it may be used for design in any of the member countries of CEN but is not mandatory. After having been published for 3 years, an ENV must be the subject of an international enquiry and vote on its future. This has taken place for Eurocode 2 and the result was that it should be converted to a full European Standard, taking account of comments made by the member countries. These comments were extensive and a new project group was set up to produce a revised draft. This group is currently about half way through developing a final draft, which is expected to be the subject of a final vote in 2002. If accepted, the code will then replace competing documents in all CEN member countries, though there will be an overlap permitted for some years between the Eurocode and the national codes it replaces.

The problem of developing an international consensus on design procedures has been made easier for concrete design by the work of the Comite Euro-International du Beton (CEB) and the Federation International de la Precontrainte (FIP). These two international bodies, working together, drafted a number of Model Codes for concrete design. The 1978 Model Code (2) formed the basis for early drafts of the Eurocode and the current revision will be influenced to a major degree by the more recent 1990 Model Code (3). The CEB and the FIP have recently merged to form the Federation International du Beton (fib).

The general level of stresses under service loads has increased over the years in all countries and this has led to the deformations in service (crack widths and deflections) to increase. The treatment of serviceability in design has therefore become of increasing importance. Eurocode 2 contains detailed provisions for three specific conditions:

1. The limitation of stresses in steel and concrete under service loads
2. The limitation of cracking
3. The limitation of deflections.

A recent paper (4) has given details of the provisions for deflections and so this paper will concentrate on the cracking provisions as they appear in the prestandard version of Eurocode 2. These provisions may change significantly before publication of the final Standard.

**OUTLINE OF CRACK CONTROL PROVISIONS**

There are three principal elements in the provisions for crack control. These are:

- The provision of minimum areas of reinforcement.
- A method for calculating design crack widths.
• Simplified rules which will avoid the necessity for explicit calculation of crack widths in most normal situations.

Most design codes do not recognise that there are two forms of cracking which need to be covered and that the prediction of the cracking in the two cases is different. These forms are: cracking caused by the restraint provided by the structure to volume change in the member considered and cracking caused by loading. Most codes only deal with cracking resulting from loading. While it is not immediately obvious that the Eurocode is different to others in this respect, cracking due to restraint was very much in the minds of the drafting committee who believed it to be the more important practical problem. The differences are only significant at low levels of strain (typically below 700 microstrain) and arise because of differences between strain controlled and load controlled behaviour during the formation of the first cracks. These levels of strain are, however, typical of many practically important structural situations.

Cracking due to restraint to a reduction in volume arises from two main causes:

(a) The restraint by reinforcement or by members connected to the element considered to drying shrinkage. This can result in the development of tensions in the concrete which may be sufficient to cause cracking. This cracking may appear a considerable time after construction.

(b) The restraint provided by attached members to reductions in volume resulting from cooling of the fresh concrete where the temperature has been raised by the hydration reaction. In this instance, the temperature has risen during the first few hours after mixing while the concrete is still very young and weak but cools after the concrete has gained strength. Cracking due to restraint of early thermal movements will obviously occur soon after construction.

There has been argument about the relative importance of these two mechanisms and, clearly, this will depend upon the circumstances. For example, in the UK the average external relative humidity is in the region of 90% and we are fortunate to have good aggregates with generally low shrinkage tendencies; the effects of drying shrinkage in external concrete is thus generally small. On the other hand, pressures to construct fast has tended to encourage the use of rapid hardening cements which release heat more rapidly, resulting in greater early heating. Early thermal cracking is therefore seen as the more important problem in the UK. Indeed, it is considered that early thermal cracking is probably the principal cause of cracking problems in the UK and probably within Europe. The opposite situation arises in countries
having a very dry environment combined with lower quality, more shrinkable aggregates.

Despite the recognition that cracking due to restraint is a major cause of cracking problems in practice, the great majority of research into cracking has been concerned with cracking due to loading and, as a consequence, it is this which most crack prediction formulae and code provisions are concerned with. This appears to be true for the Eurocode but, nevertheless, the drafters of the Eurocode were well aware of the importance of cracking due to restraint and it has had a significant influence on the form of the clauses, as will be seen.

Understanding the Eurocode provisions may be assisted by a brief introduction to cracking behaviour.

THE CRACKING PHENOMENA.

The approach almost universally used to explain the basic cracking behaviour of reinforced concrete is to consider the cracking of a concrete prism reinforced with a central bar which is subjected to pure tension. Bending does influence the phenomena but this is dealt with in the Eurocode by an empirical adjustment of the coefficients. The theoretical analysis given in this section of the paper will not be found published in exactly this form but it appears to the author to give a rational basis for the code provisions.

Figure 1 illustrates the conditions in such a prism as it is subjected to increases in strain. It is important to note that the argument presented initially considers increases in strain or extension and not increases in stress or load. The difference between stress controlled and strain controlled behaviour will be considered later.

As the strain is increased, the stress in the concrete and steel increases until the tensile strength of the concrete is reached (Figure 1(b)). At this stage, the stress in the concrete is just equal to the tensile strength of the concrete, $f_t$, and the stress in the steel is $f_{st}E_s/E_c$ or $\alpha f_{st}$.

At this point the concrete will crack and there will be a considerable redistribution of forces. The stress in the concrete at, or immediately adjacent to the crack, must drop to zero. However, with increasing distance from the crack, bond stresses at the bar-concrete interface will transfer force from the bar to the concrete until, at some distance, $S_0$, from the crack, the stress reaches a uniform value. The effect of the crack is thus to shed load from a portion of the concrete in the region of the crack. This means that less concrete is resisting the forces induced by the strain and the stiffness of the prism reduces. If the strain remains constant, this reduction in stiffness leads to a reduction in the force and hence the stresses in the prism. As a consequence, the uniform stress away from the crack actually decreases to below the tensile strength of
the concrete. This situation immediately after the formation of the first crack is shown in Figure 1(c). The overall extension, and hence the average strain is the same in Figures 1(b) and (c).

Now consider some further increase in applied extension. As the average strain increases, the stresses in the steel and concrete increase until the tensile strength of the concrete is again reached. This situation is illustrated in Figures 1(d), which shows the distribution of the average concrete stress and (e) which shows the distribution of steel stress. These illustrate the state at the instant before the formation of the second crack. From considerations of equilibrium and compatibility we can define the stress and strains at the crack and at a section distant from the crack where the stresses are uniform. This is done as follows.

Away from the crack we know the stress in the concrete is equal to the tensile strength, \( f_{ct} \) and that the stress in the steel is \( f_{st} \). The force carried by the prism is thus:

\[
T = f_{ct} A_c + \alpha f_{st} A_s
\]  

[1]

where \( T \) is the tensile force applied to the prism.

If the reinforcement ratio, \( \rho \), is defined as \( A_s/A_c \) then the equation for the force can be rewritten as:

\[
T = f_{st} (1 + \alpha \rho ) A_c
\]  

[2]

The force in the bar at the crack must be equal to \( T \) since all sections of the prism must carry the same force. Consequently, the stress in the reinforcement at the crack must be:

\[
\sigma_{sr} = T/A_s = \alpha f_{st} (1 + 1/\alpha \rho)
\]  

[3]

By considering the deformations, an expression can now be derived for the crack width. Since both the steel and the concrete are assumed to be elastic, the strains in the steel and concrete will be proportional to the stresses shown in Figures 1(d) and (e). The width of the first crack just before the formation of the second crack will be equal to the extension of the steel caused by the crack which will be proportional to the shaded area in Figure 1(e) plus the shortening of the concrete surface. This will be proportional to the shaded area in Figure 1(f). The surface strains shown in Figure 1(f) differ from the average stresses in Figure 1(d) due to the effects of 'shear lag'. Figure 2 tries to illustrate what is happening. If a local force, \( \delta_p \), is transferred to the concrete by bond at a point on the bar-concrete interface, the force disperses into the element as illustrated by the stress trajectories until, at some distance from the loaded face, the stress is uniform over the section. Elastic theory shows
that the distance from the loaded face to the section where the stress can be considered to be uniform will be proportional to the distance C on the figure. It is commonly assumed that there is a 45° dispersion of forces so that the effect of the applied force will affect the surface stresses and strains a distance C from the loaded face (see Figure 2(b)). In the situation of a bar transferring force to the concrete, this effect can be considered to cause the distribution of the surface stress to be similar to the distribution of average stress but offset away from the crack by a distance \( K_0 \cdot C \) where \( C \) is the cover and \( K_0 \) is a constant.

Any area can be expressed as the base times the perpendicular height times a constant of integration depending on the shape of the curve considered. Assuming that the shaded areas marked A in Figure 1(f) have the same form as the shaded area in Figure 1(e), we can take the same constant of integration for both. The shaded rectangle B can easily be calculated. The crack width can now be written as:

\[
\begin{align*}
\ w &= \int_{-S_0}^{S_0} \left( \sigma_s - \alpha f_c \right) / E_s + \int_{-S_0}^{S_0} f_c / E_c \\
&= 2S_0 \beta \left[ (\sigma_s - \alpha f_c) / E_s + f_c / E_c \right] + 2k_0 C f_c / E_c
\end{align*}
\]

Substituting for \( \sigma_s \) from [3] and writing \( E_c = E_s / \alpha \) gives:

\[
\begin{align*}
\ w &= 2\alpha f_c \left[ S_0 \beta \left[ 1 + \frac{1}{\alpha \rho} \right] + K_0 C \right] \\
&= 41.5
\]

\( \beta \) is a constant of integration and \( K_0 \) is a stress lag coefficient.

\( S_0 \) will now be considered.

Stress is transferred from the steel to the concrete by bond over the length \( S_0 \). Assuming that the average bond stress is \( \tau \) then, since the stress in the concrete is zero at the crack and the total force transferred to the concrete over the distance \( S_0 \) is \( f_c A_c \), we can write:

\[
f_c A_c = \tau \phi S_0
\]

where \( \phi \) is the diameter of the bar and \( \pi \phi \) the perimeter over which the bond stress acts.

Rearranging and writing \( \rho = A_s / A_c \) which, for the single bar can be written as \( \pi \phi^2 / 4A_c \) gives:

\[
S_0 = \frac{A_s \phi}{4\tau \rho}
\]
It is found experimentally that the bond stress is directly proportional to the
tensile strength of the concrete so that \( f_{ct}/\tau \) is a constant which will be
a function of the bond characteristics of the bar and the expression
simplifies to:

\[
S_o = K\phi/\rho
\]

[6]
where \( K \) is a coefficient depending on the bond characteristics of the bar.
Substituting for \( S_o \) in Equation [4] and combining the various constants
gives:

\[
w = f_{ct} [K_2(1+1/\alpha\rho)\phi/\rho + \alpha K_1 C]/E_s
\]

[7]
where \( K_1 \) and \( K_2 \) are constants which can be obtained empirically.
A series of 126 tests of centrally reinforced prisms tested recently at the
Ecole Polytechnique Federale de Lausanne (5) permit best fit values to
be obtained for \( K_1 \) and \( K_2 \) as 16 and 0.40 respectively. Figure 3 shows
the calculated results for the width of the cracks measured at a low strain
plotted against the measured width. The average error in the prediction
is 1% and the coefficient of variation is 14.9%.

Equation [7] predicts the width of the first crack to form in a prism
subjected to tension at the moment when the second crack is about to
form. What now happens if the strain in increased further? Provided the
second crack forms well away from the first crack, then conditions just
before the formation of the third crack will be identical to those just
before formation of the second crack and the width of the second crack
will therefore also be given by the above formula. Similarly for the third
and fourth cracks etc. However, this cannot go on for ever since, after a
few cracks have formed, the lengths of member affected by cracks will
start to overlap. In fact, there is a maximum number of cracks that can
form. It will be seen from Figure 1(f) that the concrete surface stress is
reduced below the tensile strength at all points within a distance \( S_o + K_o C \)
of the crack where \( C \) is now the cover to the reinforcement. It
follows that the next surface crack cannot form within this zone and so
the closest that a crack can form to a pre-existing crack is \( S_o + K_o C \). If
two cracks are more than \( 2(S_o + K_o C) \) apart, then there will be an area
between them where the surface stress will reach the tensile strength and
another crack can form. Anything less than \( 2(S_o + K_o C) \) and the stress
between the cracks will nowhere reach the tensile strength. Thus, when
the maximum number of cracks have formed, the distances between the
cracks will all be within the range:
Design and Construction Practices to Mitigate Cracking

\[(S_0 + K_0 C) \leq S \leq 2(S_0 + K_0 C)\]

[8]

Once all possible cracks have formed, extra strain cannot be accommodated by forming new cracks and can only be accommodated by increasing the widths of the existing cracks. If it is assumed that, in this stage, all the extension is accommodated in the cracks then the average crack width must be equal to the average strain multiplied by the average crack spacing. Thus:

\[w_m = S_m \varepsilon_m\]

[9]

Since \(S_m\) will be proportional to \(S_0 + K_0 C\), this can be rewritten by substituting from Equation [6] as:

\[w_m = \beta(K_0 C + K_0 \rho)\varepsilon_m\]

[10]

where \(\beta\) is a coefficient relating the average crack spacing to the minimum spacing.

The analysis given above assumes a strain controlled environment, as would be expected to occur where cracking was caused by the restraint of, say, early thermal shortening. To gain a more general appreciation of cracking behaviour, it will first help to clarify just how crack widths are predicted to vary as strains are increased. This is illustrated schematically in Figure 4.

When the first crack forms, the stresses immediately reduce due to the reduction in stiffness caused by the crack. Since the crack width is proportional to the stress in the concrete, the crack width is relatively small. As the strain is increased, the crack width increases until the state shown in Figures 1(d)-(f) are reached. At this point the crack width is as given by Equation [7]. As soon as the second crack forms, the stresses again reduce due to the reduction in stiffness caused by the second crack. Increase in strain causes an increase in the width of both the first and second crack until they again reach the width given by Equation [7] just before the third crack forms. This process is repeated until all possible cracks have formed. beyond this, the cracks increase in proportion to the applied strain according to Equation [9]. The crack width - strain response may thus be seen to have two phases:

(i) the crack formation phase

and (ii) the stable cracking phase
This is an idealised picture of events. One factor which has not been taken into account is the inherent variability in the concrete from section to section.

The first crack must form at the weakest section (assuming perfectly applied pure tension to the prism). As a consequence, all other sections will be stronger and the second crack will occur at a higher stress than the first. Similarly, the third crack will form at a higher stress than the second and so on. The crack widths just before the formation of each new crack will therefore be slightly larger than the crack widths at the formation of the previous crack. The variation in the concrete strength will also lead to variations in $S_0$ from crack to crack so that the initial cracks will not all be the same size but will vary somewhat randomly. More variations will be introduced as more cracks form and an increasing number of the spacings are reduced to below $S_0 + K_0 C$, giving some cracks smaller widths than given by Equation [7]. By the time the stabilised cracking phase is reached, the crack widths and crack spacings have a frequency distribution which is usually close to a Normal or Gaussian form with a coefficient of variation of about 40%.

If a prism is tested by increasing the load in stages rather than the strain, the behaviour in the crack formation stage will be different. When the first crack forms, there is a reduction in stiffness. However, since the load remains constant, the forces in the member cannot reduce so a sudden increase in strain must occur until the strain is reached corresponding to the load under the new stiffness. If the idealised situation shown in Figure 4 existed, it can be seen that there would not be a crack formation stage; as soon as the first crack formed, the strain would jump to the start of the stabilised cracking phase. In reality, due to the variability of the concrete, the response seen in tests tends to appear as shown in Figure 5. Thus, though there is a crack formation phase, only a relatively small increment in load is necessary to move from first cracking to the stabilised cracking phase. In most experimental work, the load is increased until there are a reasonable number of cracks to measure and the crack formation stage is missed. From the practical point of view, however, it is important because the strains in situations where cracking is due to restrained volume change generally result in the member being in the crack formation stage.

MINIMUM REINFORCEMENT

The assumption has been made in the derivation of the equations for crack width prediction under conditions of strain control that the reinforcement remains elastic. If, when the first crack forms, the
Design and Construction Practices to Mitigate Cracking

reinforcement yields then the stress in the concrete cannot again be raised to its tensile strength and therefore no further cracks can form. The total extension will be accommodated in this first crack and the formulae derived above will be useless. It is therefore essential that sufficient reinforcement is provided to ensure that yield does not occur when the first crack forms. Due to the considerable uncertainty about the actual minimum value of the concrete strength in a member, a highly sophisticated analysis to establish this minimum amount of reinforcement is not justified and the Eurocode adopts the following simplified approach.

Considering a member subjected to pure tension, it will be seen that, in order to avoid yield of the reinforcement on the formation of the first crack, the force required to yield the reinforcement must be greater than the force required to exceed the tensile capacity of the concrete. Thus:

\[ A_f f_r > A_r f_y \]  

In Eurocode 2, a series of empirical adjustments have been made to this equation in order to make it more generally applicable and the equation is modified to the statement that \( A_s \) should be not less than the value given by Equation [11] below.

\[ A_s = k f_{cr} A_t / \sigma_s \]  

In order to generalise the applicability of the equation in a convenient way, it has been found convenient, instead of the overall area of the member, to use the area of that part of the cross section which is in tension immediately before formation of the first crack. Thus, \( A_{t,cr} \) is used rather than \( A_t \). \( k \) is a factor which takes account of the form of the stress distribution. It takes a value of 1.0 for pure tension but reduces below this for other circumstances. For example, a value of 0.4 is taken for bending of reinforced concrete members and a value between 0.4 and 0 may be taken for prestressed members. These values can be calculated and shown to be reasonable when combined with the definition of \( A_{t,cr} \).

Where cracking is due to restraint of volume changes, it is inevitable that the tensile stresses developed at the concrete surface are greater than those in the body of the member. This is because the surface will dry out, or cool down, more rapidly than the core, resulting in greater shrinkage or lower temperature. To allow for this, the constant \( k \) is introduced which takes a value between 0.8 and 0.5 depending on the size of the section.
Generally, the stress in the steel, $\sigma_s$, may be taken as the yield strength, $f_{yk}$, though there may be circumstances where a lower value is appropriate.

Since cracking may occur earlier than 28 days, it may be excessively conservative to take the 28 day tensile strength of the concrete and it is permitted to take the tensile strength at the time when cracking is expected to occur: $f_{ctm}$.

It will be seen that Equation [11] has been derived from consideration of cracking due to restrained volume change, though this point is not stated in the code. The CEB group which drafted the original proposal felt that there were very few situations where some degree of restraint was not present and that it was therefore prudent to apply the formula universally.

The Eurocode formulae for crack width calculation.

The history of the Eurocode rules is complex. CEB Model Code 78 (2) contains a formula which is basically Equation [10]. However, in order to enable the equation to be applied to members subjected to bending rather than only to pure tension members, the definition of the reinforcement ratio was changed to an effective reinforcement ratio based on the area of concrete surrounding the bars and having the same centroid as the reinforcement. In order to calculate the average strain, the same model of behaviour was proposed as was proposed for the calculation of deflections. This is described more fully in (3) and resulted in the equation:

$$\varepsilon_{cm} = \sigma_e(1 - \beta_1 \beta_2 (\sigma_e/\sigma)^2)/E_e$$

[12]

During the 1980s a group was set up within the CEB serviceability Commission to consider a revised version for the proposed CEB/FIP Model Code 90 (3). They also accepted Equation [10] but with modifications. Firstly, the definition of the effective area of concrete surrounding the reinforcement was amended slightly. Its definition can be seen from Figure 6. Research had shown that the balance between the constants $K_o$ and $K$ in Equation [10] vary depending upon whether the member is subjected to bending or tension and also the depth of the tension zone. This is taken into account by adjusting $K$ and by limiting the depth of the effective area of concrete surrounding the reinforcement to not more than $(h - x)/3$.

There was considerable debate within the group about the importance of the cover in defining the crack width. The durability of reinforced concrete members is very sensitive to the amount and quality of the cover. It was felt that the explicit introduction of cover in the formulae
Design and Construction Practices to Mitigate Cracking

encouraged designers to minimise the cover since low cover results in smaller crack widths. The cover is also, to a degree, introduced into the formula indirectly since the reinforcement ratio is a function of the cover. For this reason, the term $K_C$ was simplified to $50\,\text{mm}$. This is probably not unreasonable in many practical circumstances.

In order to calculate the crack width during the crack formation stage, an alternative, very simple, proposal was included for the calculation of the average strain. This was to assume that $\sigma_s$ was equal to $\sigma_{ut}$. Assuming that the loading considered was long term results in a value for $\beta_2$ in Equation [12] of 0.5, resulting in a value for the average strain of:

$$\varepsilon_{sm} = \sigma_{ut} / 2E_s$$  \[13\]

In drafting the associated Eurocode for the design of liquid retaining and containment structures, a slightly more rigorous formulation has been adopted such that:

$$\varepsilon_{sm} = 0.6k_f k_{1\times}(A_{ct}/E_s A_s + 1/E_c)$$ \[14\]

It can be shown that Equations [13] and [14] give very similar answers.

The proposals from this group were used as the basis for the rules in the Eurocode. In summary, these are:

Crack widths may be calculated from the formula:

$$w_k = \beta s_m \varepsilon_{sm}$$ \[15\]

where $\varepsilon_{sm}$ is given by Equation 12 and $s_m$ is obtained from Equation 16 below.

$$s_m = 50 + 0.25k_f k_s \phi / \rho$$ \[16\]

These proposals are very similar to the equations [9] and [10] derived above for crack prediction for the stabilised cracking state and, using the definitions for the average strain given in Equations [13] or [14] for the crack formation stage, very similar in effect to Equation [7].

Interestingly, the CEB/FIP Model Code 90 did not accept the proposals of the working group who derived the proposals included in the Eurocode but developed alternative and less rigorous proposals. There is currently pressure to revise the Eurocode to incorporate the Model Code proposals.
Clearly, it is necessary to define permissible crack widths and the Eurocode gives the values set out in Table 1.

SIMPLIFIED METHODS OF CRACK CONTROL

Having developed design formulae for crack control, these may be used to carry out parameter studies which, in turn, may be used to develop simplified detailing rules.

Two approaches were used. The Eurocode formulae for the prediction of crack width (Equations 12, 15 and 16) may be written in a single equation as:

\[ w_k = (50 + 0.25k_c \psi \rho_{cut})(1 - \beta_2 \beta_3(\sigma_s/\sigma_c)^2) \sigma_s / E_s \]  

[17]

Assuming that ribbed bars are being considered, that the long term crack width is required and that the design crack width is 0.3 mm then, for bending, Equation [17] may be rewritten as:

\[ 141176 = (50 + 0.1 \psi \rho_{cut})(1 - 0.5(\sigma_s/\sigma_c)^2) \sigma_s \]

Assuming a tensile strength for the concrete of 2.5 N/mm² and that \( \rho_{cut} \) is 4 times the overall reinforcement ratio, \( \rho \), it is possible to obtain an approximate value for \( \sigma_s \) as a function of the reinforcement ratio and hence further simplify the formula to:

\[ \phi = \frac{1411760 \rho}{\sigma_s(1 - 0.08/(\rho \sigma_c)^2)} -2000 \rho \]

If this equation is used to plot graphs of bar diameter against reinforcement ratio for various levels of steel stress, results such as those shown in Figure 7 will be obtained. The minimum bar diameters were abstracted from these curves and used to produce a table of maximum bar diameter to limit the crack width to 0.3 mm as a function of steel stress. The table from the Eurocode is reproduced as Table 2.

The alternative approach adopted was to study the variation in allowable bar spacing as a function of steel stress. The relationships for bar spacing cannot be developed as simply as that for bar diameter but the same basic principle that a minimum value of spacing could be obtained for each level of steel stress, though more assumptions needed to be made. This study resulted in the values of bar spacing given in Table 3.
It should be noted that the designer may use either of these tables so he can use whichever is the most advantageous in the particular circumstances of the design.

**COMPARISON OF ACI AND EUROCODE METHODS OF CRACK CONTROL**

It is not straightforward to compare the ACI and Eurocode approaches to crack control but the following brief study may assist. Consider a slab 200 mm deep reinforced with 6 No. 16 mm diameter bars per metre with a cover of 20 mm. The stress in the reinforcement is 250 N/mm². The Gergely-Lutz equation, from which the ACI crack control relationship has been derived, may be used to calculate a crack width. This equation gives a crack width of 0.23 mm in this situation. Use of the Eurocode 2 equations gives 0.15 mm. It appears that, in this case, the ACI approach is significantly more conservative than the Eurocode. This conservatism can be explained by two factors: firstly, the Eurocode allows for a reduction in the strain due to ‘tension stiffening’ (add bit from Equation 17) and, secondly, it is found that shallow slabs have significantly smaller crack widths than do deeper members. This is allowed for in Eurocode 2 in the definition of $\rho_{cr}$ but there is no such allowance in the ACI approach. If these two factors were ignored, then, in this case, the Eurocode formula would give a larger crack width than the Gergely-Lutz equation. This is a single comparison and, unfortunately, no general conclusion can be drawn. This may be illustrated for the case of a deep slab in which $2.5(c + \phi/2) < (h - x)/3$. In this circumstance, for a single layer of bars, the ACI relationship for demonstrating adequacy of crack control for exterior exposure may be written as:

$$145 \geq f_s[(C + \phi/2)^2 S_e]^1/3$$

Conversion to SI units, assuming a stress in the reinforcement of 250 N/mm² and rearranging gives:

$$262200 \geq S_e(C + \phi/2)^2$$

Ignoring any reduction in strain due to tension stiffening, the Eurocode 2 formula may be written for a slab as:

$$0.3 \geq 1.7[50 + S_e(C + \phi/2)/\pi\phi]f_s(h - x)/(d - x)E_s$$

Taking $E_s$ as 200000 N/mm², the stress in the reinforcement as 250 N/mm² making a reasonable assumption for $(h - x)/(d - x)$ and rearranging into a format similar to the ACI equation gives:

$$250 \geq S_e(C + \phi/2)/\phi$$
Comparison of these two formulae show that, while $S_b$, $C$ and $\phi$ are considered by both formulae to influence the cracking, the way these variables interact is predicted to be very different.

CONCLUDING REMARKS.

This paper has tried to summarise and explain the basis for the crack control provisions in Eurocode 2, has outlined the provisions themselves and has tried to show how the formulae for the prediction of crack widths have been used to develop simple detailing rules so that calculation of crack widths should only be required in special cases.

It has been pointed out that work on the revision of the code is currently taking place prior to the submission of the code to the member countries of CEN for final acceptance as a full European standard and that therefore the provisions described here may change.

REFERENCES


### NOTATION

- **$A_s$** = Area of reinforcement  
- **$A_c$** = Area of concrete  
- **$A_{et}$** = The area of the tension zone of the member immediately before formation of the first crack.  
- **$C$** = The cover  
- **$E_c$** = The modulus of elasticity of concrete  
- **$E_{c,eff}$** = The effective elastic modulus including allowance for creep  
- **$E_s$** = The modulus of elasticity of steel  
- **$K$, $K_o$, $K_1$, $K_2$** = Coefficients used in the derivation of crack prediction formulae.  
- **$S_o$** = The distance from a crack within which the stress distribution is affected by the crack  
- **$S_b$** = The spacing of bars in a slab.  
- **$b$** = The breadth of a section  
- **$d$** = The effective depth of a section  
- **$f_{ck}$** = The characteristic compressive strength of the concrete  
- **$f_{ct}$** = The tensile strength of concrete  
- **$f_{ct,eff}$** = The tensile strength of the concrete at the time cracking is expected to occur. This should not generally be taken as less than 3 N/mm².  
- **$f_{yk}$** = The characteristic yield strength of the reinforcement  
- **$h$** = The overall depth of a section  
- **$k_c$** = A coefficient which takes account of the nature of the stress distribution; $k_c = 1$ for pure tension and 0.4 for pure bending.  
- **$k$** = A coefficient which takes account of the effect of non-uniform self-equilibrating stresses; $k = 0.8$ for members less than 300 mm deep and 0.5 for members greater than 800 mm deep. Linear interpolation may be used for intermediate values.  
- **$k_1$** = A coefficient which takes account of the bond properties of the bars; $k_1 = 0.8$ for plain bars and 0.4 for deformed bars.  
- **$k_2$** = A coefficient which takes account of the nature of the stress distribution across the section; $k_2 = 1$ for pure tension and 0.5 for bending.  
- **$s_{rm}$** = The average final crack spacing  
- **$w_k$** = The characteristic crack width  
- **$w_m$** = The mean crack width  
- **$\alpha$** = The modular ratio $= E_s/E_c$
\[ \beta = \text{A constant relating the characteristic crack width to the average value. It usually takes a value of 1.7} \]
\[ \beta_1 = \text{A coefficient which takes account of the bond properties of the bars.} \beta_1 = 0.5 \text{ for plain bars and 1.0 for deformed bars} \]
\[ \beta_2 = \text{A coefficient which takes account of the duration of the loading;} \beta_2 = 1.0 \text{ for short term loading and 0.5 for long term or repeated loading.} \]
\[ \varepsilon_{em} = \text{The average strain in the reinforcement allowing for tension stiffening.} \]
\[ \phi = \text{Bar diameter} \]
\[ \rho_{ef} = \text{Effective reinforcement ratio} \]
\[ \sigma_{st} = \text{The stress in the reinforcement at a crack under the conditions which will just cause a crack to form.} \]
\[ \sigma_s = \text{The stress in the reinforcement calculated on the basis of a fully cracked section} \]
\[ \tau = \text{the average bond stress} \]
\[ \zeta = \text{A coefficient which defined the proportion of the element which can be considered to be fully cracked} \]

**APPENDIX.**

**EXAMPLES OF CRACK CONTROL CALCULATIONS USING THE EUROCODE 2 FORMULAE**

There are two basic types of situation where calculations for crack widths might be needed: situations where the cracking is dominantly due to loading and situations where the cracking is due to the restraint of imposed deformations. An example of each type will be given.

(1) Cracking caused by loading.

Check that the cracking in the slab shown in Figure A1 will not exceed 0.3 mm. The design, as shown, has been arrived at from consideration of the Ultimate Limit State where the design ultimate moment was found to be 120 kNm per metre. The moment under the design service load is 70 kNm per metre.

(a) **Using the bar size or bar spacing tables.**

It is necessary first to make an estimate of the stress in the reinforcement under the design service loading. This could be calculated on the basis of a cracked section but a more approximate method is usually adequate. In this case, the characteristic yield strength of the reinforcement was
Design and Construction Practices to Mitigate Cracking

500 N/mm² and the partial safety factor applied to the reinforcement is 1.15. Consequently the stress under the ultimate load should be \( \frac{500}{1.15} = 435 \text{ N/mm}^2 \). Assuming a linear relationship between stress and load gives a stress under the service load of \( \frac{435 \times 70}{120} = 254 \text{ N/mm}^2 \). From Table 2 in the paper, it will be seen that a bar diameter up to just below 20 mm would be satisfactory. Since the actual design uses 16 mm bars, the cracking will be satisfactory. A similar exercise could be carried out using Table 3. Here a stress of 254 N/mm² corresponds to a maximum spacing of about 180 mm. Again, the actual design is satisfactory since it uses the lesser spacing of 150 mm. It should noted that Eurocode 2 only requires that either Table 2 or Table 3 be checked; it is not necessary to check both.

(b) By direct calculation of the design crack width.

The first step is to calculate the steel stress and hence the average strain. The approximation used above could be employed but the full calculation will be presented.

Assuming that the modular ratio \( \left( \frac{E_s}{E_c} \right) \) is 15, the neutral axis depth of a fully cracked section with the reinforcement shown in Figure A1 can be calculated to be 76 mm, giving a lever arm, \( z \), of 220 - 76/3 = 195 mm. The stress in the reinforcement can now be calculated to be:

\[
\sigma_s = \frac{M}{A_s z} = \frac{70 \times 10^6}{(1340 \times 195)} = 268 \text{ N/mm}^2
\]

This will be seen to be slightly larger than the value obtained approximately in (a) above.

In order to calculate the average strain it is necessary to know the cracking moment. Assuming that the tensile strength of the concrete is 3 N/mm², this may be calculated to be 31.25 kNm. Since the relationship between the load and the reinforcement stress calculated assuming a fully cracked section is linear, the stress in the reinforcement immediately on formation of the first crack, \( \sigma_{sr} \), may be calculated to be 268 x 31.25/70 = 120 N/mm². The average strain may now be calculated from Equation 12 as:

\[
\varepsilon_{am} = \frac{268(1 - 0.5(120/268)^2)}{200000} = 0.00121
\]

The depth of the effective tensile zone is now required. This may be established from Figure 6(c) as the lesser of 2.5 x 30 = 75 or \( (250 - 74)/3 = 58 \).

The effective reinforcement ratio is thus \( \frac{1340}{(58 \times 1000)} = 0.0231 \)

The average crack spacing may now be calculated from Equation 12 as:

\[
s_{rm} = 50 + 0.25 \times 0.8 \times 0.5 \times 16 / 0.0231 = 119 \text{ mm}
\]
The design crack width may now be obtained from Equation 15 as:

\[ w_k = 1.7 \times 119 \times 0.00121 = 0.245 \text{ mm} \]

This is less than 0.3 mm therefore the design is satisfactory. The calculated result may be seen to be consistent with Tables 2 and 3.

(2) Cracking resulting from restraint to early thermal shortening.

The horizontal reinforcement will be designed for a 300 mm thick cantilever wall for a tank. The structure is configured so that the loads applied to the structure do not produce any horizontal forces or moments in the wall. Reinforcement is therefore only required to control the cracking. It is assumed that contraction joints are widely spaced, that the temperature drop in cooling from the peak temperature generated by hydration to ambient is sufficiently large that cracking is unavoidable. This may not be an economical way to design and has only been chosen to demonstrate the use of the equations in a conveniently simple situation. The yield strength of the reinforcement is 500 N/mm\(^2\).

For controlled cracking to be a possibility, Equation 11 must be satisfied. As written, Equation 11 is not as useful as might be thought. In this case, \( k_c = 1 \), \( k = 0.8 \), \( f_{ \text{teff} } \) should be taken as 3 N/mm\(^2\) and it is stated that, normally, the steel stress may be taken as the yield strength. Substituting these values gives a minimum steel area for crack control of 1440 mm\(^2\) per metre; assuming a symmetrical arrangement of reinforcement, this gives 720 mm\(^2\)/m in each face. Tables 2 or 3 may now be used to pick a bar size or bar spacing. Unfortunately, it has been assumed that the stress in the reinforcement will be 500 N/mm\(^2\) on first cracking and it will be found that it is not possible to find a practical reinforcing detail which will result in a 0.3 mm design crack width if this stress is used. Inspection of Tables 2 and 3 suggests that a practical arrangement might be obtained if the stress in the reinforcement was around 300 N/mm\(^2\). Equation 11 may now be used to find the area of reinforcement necessary to give a stress of 300 N/mm\(^2\). This turns out to be 1200 mm\(^2\)/m in each face. This is nearly provided by 12 mm diameter bars at 100 mm centres in both faces (giving 1130 mm\(^2\)/m in each face). Inspection of Tables 2 or 3 will show that both are approximately satisfied by this selection of reinforcement.

This completes the design for most cases but it will be interesting to calculate the design crack width corresponding to the use of 12 mm bars at 100 mm centres in both faces.
Design and Construction Practices to Mitigate Cracking

\( \sigma_{cr} \), may be calculated approximately from Equation 11 if the actual area of reinforcement provided is inserted for \( A_s \) and the equation rearranged to give the stress in the reinforcement. In this case the stress is given by:

\[
\sigma_{cr} = 1 \times 0.8 \times 3 \times 150 \times 1000 / 1130 = 318 \text{N/mm}^2
\]

Equation 13 now gives the average strain as:

\[
\varepsilon_{sm} = 0.5\sigma_{cr}/E_s = 0.000795
\]

Equation 14 would have given the slightly higher value of 0.001.

From Figure 6(b) the thickness of the effective area is the lesser of half the wall thickness or 2.5 \((c + \phi)/2\). Assuming a cover of 35 mm, this makes the thickness the lesser of 150 or 102 mm which gives the effective reinforcement ratio as 0.011. Equation 16 now gives the average crack spacing as:

\[
S_{nm} = 50 + 0.25 \times 0.8 \times 1 \times 12 / 0.011 = 268 \text{ mm}
\]

Eurocode 2 states that a value of 1.3 may be taken for \( \beta \) in Equation 15 for sections 300 mm thick or less where restrained cracking is being considered rather than 1.7. Equation 15 now gives the design crack width as:

\[
w_k = 1.3 \times 268 \times 0.000795 = 0.28 \text{ mm}
\]

Had the higher strain given by Equation 14 been used, the calculated crack width would have been 0.35 mm.

Other reinforcement arrangements could have been chosen; for example, almost the same crack widths would have been obtained if 16 mm diameter bars at 150 mm centres in both faces had been used.
Table 1. Crack width limits (in mm)

<table>
<thead>
<tr>
<th>Exposure condition</th>
<th>Reinforced concrete</th>
<th>Post-tensioned members</th>
<th>Pre-tensioned members</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dry</td>
<td>*</td>
<td>0.2</td>
<td>0.2</td>
</tr>
<tr>
<td>Humid</td>
<td>0.3</td>
<td>0.2</td>
<td>Decompression</td>
</tr>
<tr>
<td>Humid with frost &amp;</td>
<td>0.3</td>
<td>Decompression</td>
<td>Decompression</td>
</tr>
<tr>
<td>salt</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Seawater</td>
<td>0.3</td>
<td>Decompression</td>
<td>Decompression</td>
</tr>
</tbody>
</table>

* Crack width will depend upon appearance and widths greater than 0.3 mm may be acceptable.

‘Decompression’ requires that the prestressing tendons are at least 25 mm within compressed concrete.

Table 2. Maximum bar diameters for high bond bars

<table>
<thead>
<tr>
<th>Steel stress N/mm²</th>
<th>Maximum bar diameter (mm)</th>
<th>reinforced sections</th>
<th>Prestressed sections</th>
</tr>
</thead>
<tbody>
<tr>
<td>160</td>
<td>32</td>
<td>25</td>
<td>25</td>
</tr>
<tr>
<td>200</td>
<td>25</td>
<td>16</td>
<td>16</td>
</tr>
<tr>
<td>240</td>
<td>20</td>
<td>12</td>
<td>12</td>
</tr>
<tr>
<td>280</td>
<td>16</td>
<td>8</td>
<td>8</td>
</tr>
<tr>
<td>320</td>
<td>12</td>
<td>6</td>
<td>6</td>
</tr>
<tr>
<td>360</td>
<td>10</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>400</td>
<td>8</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>450</td>
<td>6</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 3. Maximum bar spacings for high bond bars

<table>
<thead>
<tr>
<th>Steel stress N/mm²</th>
<th>Maximum bar spacing (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>160</td>
<td>300</td>
</tr>
<tr>
<td>200</td>
<td>250</td>
</tr>
<tr>
<td>240</td>
<td>200</td>
</tr>
<tr>
<td>280</td>
<td>150</td>
</tr>
<tr>
<td>320</td>
<td>100</td>
</tr>
<tr>
<td>360</td>
<td>50</td>
</tr>
</tbody>
</table>
Design and Construction Practices to Mitigate Cracking

Figure 1. Stress conditions in a prism during the early stages of cracking (strain control).
Figure 2. Dispersion of stress within a prism.
Figure 3. Comparison of calculated and experimental crack widths during the crack formation stage (from Reference 3).
Figure 4. Idealised development of effective area surrounding the reinforcement.
Design and Construction Practices to Mitigate Cracking

Figure 5. Development of cracking under load control.

Figure 6. Definition of effective area surrounding the reinforcement.
Figure 7. Relationship between bar diameter and reinforcement ratio for different levels of steel stress.

A1. Slab details used in example.
Diagonal Cracking and Diagonal Crack Control in Structural Concrete

by P. Adebar

Synopsis: A number of fundamental concepts relevant to all types of cracking are examined. A tension stiffening relationship derived from first principles indicates that traditional empirical relationships include significant residual tension stresses from uncracked concrete. Service load crack strains should not be estimated using an empirical tension stiffening expression. While primary cracks continue to form up to strains of 0.0010, due to deformation of concrete between visible cracks, the minimum strain that should be used with the stable crack spacing is 0.0005. A magnification factor must be applied to crack spacings at smaller strains, or a minimum strain of 0.0005 used to estimate crack width. Test results indicate that the 95th percentile crack width is 2.0 times the average crack width. Procedures for diagonal crack inclination, spacing and width are reviewed, and a simplified expression for estimating diagonal crack widths is presented. Diagonal crack widths are generally larger than flexural crack widths in members with orthogonal reinforcement due to diagonal strains being larger than reinforcing bar strains. Current code requirements for side-face reinforcement were developed to control flexural cracking, and may not be adequate to control diagonal cracking in certain exposure conditions. The simplified expression for diagonal cracking was used to develop an expression for the maximum spacing of side face reinforcing bars to control flexural and diagonal cracking in large members. A design example illustrates the proposal. Finally, it is shown how the proposed methodology can be used to extend the current ACI expression for spacing of reinforcement near a surface in tension to treat the case of diagonal cracking.

Keywords: concrete; cracking; diagonal cracking
ACI member Perry Adebar is Associate Dean of the Faculty of Applied Science at the University of British Columbia, Vancouver, Canada. He is a member of joint ACI-ASCE Committee 441, Reinforced Concrete Columns, ACI Committee 445, Shear and Torsion, and the seismic design subcommittee of the Canadian concrete code CSA A23.3. Dr. Adebar was co-recipient of the 1998 ACI Structural Research Award.

INTRODUCTION

Considerable research has been conducted to investigate the problem of cracking and crack control of structural concrete. The vast majority of this effort has been on structural concrete members subjected to direct tension or bending (see for example Ref. 1). Comparatively little work has been done on the issue of diagonal cracking and diagonal crack control in structural concrete. Yet diagonal cracking is often more critical and more difficult to control than cracking due to axial tension or bending. Diagonal cracks are generally wider than axial tension cracks or bending cracks because the reinforcement is at an angle to diagonal cracks. As an example, recent tests on bridge bent caps (2) have demonstrated that diagonal cracks are more critical than flexural cracks.

An extensive research program has recently been completed on diagonal cracking and diagonal crack control. Phase I of the research program (3),(4) involved a laboratory study on a 17.1 m (56 ft) long bridge girder with composite precast concrete deck panels [Fig. 1(a)]. The girder was a hybrid element with a reinforced concrete web and steel flanges. Fig. 1(b) shows the observed crack pattern over half the girder after the dead load was applied. The vertical cracks were due to a combination of bending stresses and restrained longitudinal shrinkage due to the steel flanges. The crack pattern at the service load level of \( P = 250 \text{ kN} \) [Fig. 1(c)] indicates that the diagonal crack widths were wider than the flexural crack widths even though the member was subjected to relatively low (1.4 MPa) shear stress at the service load level.

To avoid having to test additional full-length girders as shown in Fig. 1, an element testing scheme (shown in Fig. 2) was devised to permit the testing of 2.5 m (8 ft) long elements with similar cross sectional dimensions as the member in Fig. 1. Twenty-one elements with web depths of 1200 mm (47 in.) were tested in Phase II to investigate the efficiency of different amounts and arrangements of side face reinforcement in controlling vertical and diagonal cracking (5),(6). Phase III of the study involved the field testing of an actual bridge (Fig. 3) to evaluate service load cracking. The results from this test confirmed that diagonal cracking [see Fig. 3(b)] is more critical than flexural cracking in members with orthogonal (longitudinal and transverse) reinforcement.
FUNDAMENTAL CONCEPTS: UNIAXIAL TENSION

Prior to examining the more complex case of diagonal cracking, which generally involves reinforcement in two directions and cracks inclined at a third direction, it is useful to examine a number of fundamental concepts with regard to the simplest case of a reinforced concrete member in uniaxial tension.

A concrete element with a single reinforcing bar surrounded by a uniform layer of concrete is pre-cracked as shown in Fig. 4(a). The cracks are uniformly spaced at the stable crack spacing $s$, which depends on the thickness of concrete cover over the bar and the size of the bar. If the reinforcing bar can slip relatively freely through the concrete, significant crack widths would result from the elastic strain $\varepsilon$ of the reinforcing bar. If the concrete between cracks remained essentially rigid, the resulting crack width would be

$$w = \varepsilon \cdot s$$

In reality, slip of the reinforcing bar through the concrete induces bond stresses which transfer force from the reinforcing bar to the concrete between the cracks.

Based on the results of a number of different experimental studies, Fronteddu and Adebar (7) suggested the following relationship between bond stress and bar slip

$$\tau = 0.9 \sqrt{f'_c} \left( \frac{u}{0.04} \right)^{0.4} \leq 0.9 \sqrt{f'_c}$$

where $\tau$ is the bond stress (in MPa), $f'_c$ is the cylinder compressive strength of concrete (in MPa), and $u$ is the slip (in mm). To account for degradation of bond near the crack surface due to loss of confinement, the bond stress is reduced to zero at the crack over a distance of about 0.15 $s$.

The bar slip will be maximum at the crack locations, and will be zero midway between the cracks. A simplified solution that yields reasonable results is to assume that the slip varies linearly (7). Thus, for any value of maximum bar slip at the crack, the slip is known at every point along the reinforcing bar, and using Eq. (2), the complete bond stress distribution can be determined along the reinforcing bar. As the reinforcing bar will slip on both sides of a crack, the width of a crack is equal to twice the maximum bar slip.

For the pre-cracked element shown in Fig. 4(a), the concrete tensile stress is zero at the cracks and is maximum midway between cracks. The concrete stress at any point at a distance $x$ from a crack is given by
As the concrete between the cracks resists a significant portion of the axial tension force, the reinforcement force is correspondingly reduced. The reduction in reinforcement stress and the associated reduction in reinforcement strain and crack width, is known as tension stiffening.

Rather than deal with the complex variation of concrete tensile stress between cracks, it is often convenient to quantify tension stiffening using an equivalent average concrete tensile stress that is assumed to be uniform. Fig. 4(b) shows the predicted relationship between average concrete tension stress and crack width. It is interesting to note that the average tension stress in concrete increases with increasing crack width. When the bond stress - slip relationship given by Eq. (2) is used, the average concrete tension stress reaches a constant value of

\[ f_c = 0.66 \sqrt{f_c} \frac{P}{d_b} \]

at large crack widths.

The tension stiffening relationship shown in Fig. 4(b) has a very different appearance than many relationships that have been previously proposed \((8),(9),(10)\). These other relationships have all had reducing average concrete tension stress with increasing crack width. The reason for the difference comes from the fact that Fig. 4(b) is for concrete that is pre-cracked, while these previous “tension stiffening” relationships were determined empirically from concrete specimens that were initially uncracked. Fig. 5(a) shows the typical average concrete tension stress measured during the formation of a crack in a plain concrete specimen \((11)\). When the predicted average concrete tension stress due to tension stiffening and the residual concrete tension stress in plain concrete are combined together \[\text{see Fig. 4(c)}\], the resulting relationship involves a total concrete tension stress that initially reduces with increasing crack width.

A typical measured concrete tension stress – average strain relationship is shown in Fig. 5(b). In this test, the deformations were measured over a gauge length that was about 5 times the stable crack spacing for the specimen. The experimentally measured total concrete tension stress reduces much more gradually than predicted. The reason is that the predicted relationship assumes that all cracks form simultaneously at the stable crack spacing. In reality, the cracks form gradually and as a result, there is a gradual reduction in average concrete tension stress, or in other words, a gradual increase in deformation due to the applied load. The visible steps in Fig. 5(b) are due to the formation of the cracks. While the average deformation measured over several crack spacings is reduced by the delay in crack formation, the relationship between
the applied load and the width of a particular crack is not affected. For this reason, it is not appropriate to use an empirically determined concrete tension stress relationship and the stable cracking spacing to estimate crack widths.

Fig. 6 illustrates the relationship between longitudinal strain and crack spacing measured in the Phase II element tests (Fig. 2). The elements were subjected to bending, however the sections were large and the service load strain gradient was small. That is, the longitudinal strain varied gradually over the depth, and the cracking is more closely related to direct tension cracking than to flexural cracking. Two types of crack spacings are shown in Fig. 6. The observed crack spacings were determined by physically measuring the distance between visible cracks. Each hollow data point shown in Fig. 6 is the average spacing of all cracks crossing a longitudinal grid line on an element. Typically, there were between 5 and 20 cracks crossing a grid line depending on the location of the grid line and the level of loading.

The solid data points shown in Fig. 6 are the average crack spacings calculated from Eq. (1) using the measured average crack width $w$ along a grid line and the associated measured average strain $\varepsilon$ of the cracked concrete, which is equal to the average strain of the reinforcing steel. That is, the solid data points represent the average width of the 5 to 20 cracks crossing a grid line divided by the average strain along the same grid line.

For each element, the stable crack spacing was determined as the mean crack spacing over the range of strains between 0.0005 and 0.0015. When the crack spacings were normalized using the stable crack spacings, the resulting crack spacing ratios (crack width at a particular strain level divided by the stable crack spacing) for the different specimens showed very similar trends. Therefore the data was combined together in two plots, one for lightly reinforced members [Fig. 6(a)] and one for more heavily reinforced members [Fig. 6(b)].

A number of important phenomena are illustrated in Fig. 6. The first is that the average crack spacings determined from measured crack widths and measured strains (solid dots) are consistently less than the observed crack spacings (hollow dots), particularly for the more heavily reinforced elements. The reason is that the deformation of the concrete between visible (primary) cracks contributes significantly to the average strain of cracked concrete. A second important observation is that observed crack spacings (hollow dots) continue to decrease (visible cracks continue to form) up to strains of about .001, while the average crack spacings determined from average crack widths divided by strain (solid dots), stabilize earlier at a strain of about .0005.

Fig. 6(b) indicates that Eq. (1) and the stable crack spacings can be used to make a reasonable estimate of crack widths for strains greater than .0005. At strains less than .0005, the crack spacings that should be used to calculate the crack widths from strain may be up to four times larger than the stable crack
Adebar

spacing. Two different approaches can be used to correct Eq. (1) for strains less than .0005. The simplest approach is to not use a strain less than .0005 in Eq. (1). That is, the crack width shall not be taken less than .0005 s. The alternate approach is to correct the estimate of crack spacings. At strains less than .0005, the crack spacing shall be taken as the stable crack spacing

\[ l + \alpha \left( 1 - \frac{\varepsilon}{0.0005} \right) \geq 1.0 \]

multiplied by the factor

where \( \alpha = 2 \) for lightly reinforced elements (\( \rho \leq 0.7\% \)) and \( \alpha = 1 \) for more heavily reinforced elements (\( \rho \geq 0.7\% \)). The trends predicted using this factor are shown as dashed lines in Figs. 6(a) and 6(b).

The crack spacings that should be used to calculate crack width from strain continue to decrease at very high strain levels, however, it is conservative to use a larger than actual crack spacing, hence it is conservative to use the stable crack spacings to estimate crack widths at large strains.

The discussion thus far has been with regard to average crack widths. Half of all the cracks will be larger than the average crack width, so an important question is how much larger can these cracks get. Based on the work of Broms (12), Frosch (13) has recently suggested that the maximum crack spacing, and correspondingly the maximum crack width, be taken as 2.0/1.5 = 1.33 times the average value.

To examine the relationship between average and maximum crack widths at a particular strain level, the width of each crack crossing a grid line of uniform strain in an element was divided by the average width of the 5 to 20 cracks crossing the same grid line. This process was repeated for a number of grid lines on the specimens, as well as for a number of levels of loading. In total, about 3500 crack width measurements were converted to crack width ratios using this procedure. Fig. 7 summarizes the results in terms of a histogram as well as the best fit frequency distribution. It was found that the distribution of crack width ratios (crack width divided by average crack width along the same grid line) conforms to a Weibull distribution. Some crack widths were as large as 3.0 times the average crack width, however the number of these was very limited. It has been suggested (14) that a useful definition of maximum crack width is the 95th percentile value. It was found that the 95th percentile crack width is 2.0 times the average crack width, i.e., 5% of the cracks are larger than 2.0 times the average crack width.

**DIAGONAL CRACKING**

Using a generalization of Eq. (1) for cracks due to uniaxial tension, the width of diagonal cracks \( w_d \) can be estimated by multiplying the normal strain perpendicular to the diagonal cracks \( \varepsilon_d \) by the diagonal crack spacing \( s_d \)
Vecchio and Collins (9) presented the following transformation equation for estimating the spacing of diagonal cracks:

\[ s_d = \frac{l}{\sin \theta + \cos \theta} \]

where \( \theta \) is the inclination of the diagonal cracks measured from the longitudinal axis of the member, \( s_l \) is the longitudinal crack spacing and \( s_t \) is the transverse crack spacing. \( s_l \) and \( s_t \) can be estimated from any method used to determine the crack spacing for a member subjected to uniaxial tension using the characteristics of the longitudinal and transverse reinforcement, respectively. Fig. 8 provides a graphical representation of the transformation given by Eq. (7).

Different approaches can be used to estimate the inclination of diagonal cracks. One approach is to assume that diagonal cracks are fixed in the direction of the initial cracks, while another approach is to assume that diagonal cracks continuously rotate and are normal to the principal tension strain direction. As the principal tension strain is the largest normal strain component, it is more conservative to use the latter approach when estimating diagonal crack widths.

Estimating the principal tensile strain of a structural concrete element can be accomplished by more rigorous methods or using the simplified procedure presented in this paper. To understand the different approaches, it is helpful to make the analogy with methods for calculating longitudinal strain in a reinforced concrete beam as these calculations are well known. The most general approach for determining the longitudinal strain is to conduct a plane sections strain compatibility analysis by computer, where tension stiffening and the contribution from distributed longitudinal reinforcement in the web can be rigorously accounted for. A second, more simplified approach for estimating the strain of the longitudinal reinforcement on the flexural tension side of a member subjected to bending is given by

\[ \varepsilon_l = \frac{M}{j_d} \left( \frac{1}{E_s A_s} \right) \]

where \( j_d \) is the internal flexural lever arm and \( A_s \) is the area of longitudinal reinforcement on the flexural tension side.

A general strain compatibility procedure for estimating principal diagonal strain is the modified compression field theory (9) which makes use of empirically developed stress-strain relationships for diagonal cracked concrete. The stress-strain relationships account for compression softening of concrete.
due to transverse tensile straining, and tension stiffening of diagonally cracked concrete. The method was used to predict the diagonal crack widths during the girder test shown in Fig. 1.

Computer program Response (15) was used to perform the modified compression field theory predictions. The longitudinal crack spacing \( s_1 = 195 \text{ mm} \) and the transverse crack spacing \( s_2 = 131 \text{ mm} \) were determined from the expression suggested in the 1978 CEB-FIP Model Code (14) which for a member with deformed reinforcing bars subjected to uniform strain simplifies to

\[
s = 2c + 0.2 s_b + \frac{0.1 d_b}{\rho_{ef}}
\]

where \( c \) is the clear cover to the reinforcing bars, \( s_b \) is the spacing of the reinforcing bars, \( d_b \) is the bar diameter and \( \rho_{ef} \) is the ratio of steel area to concrete area within \( 7.5d_b \).

Figure 9 compares the predicted and observed diagonal cracking at 4.60 m from the support for two load levels. The diagonal cracking at \( P = 600 \text{ kN} \), which is immediately prior to yielding of the tension flange (close to failure), is reasonably well predicted by program Response. On the other hand, the diagonal cracking at the service load level of \( P = 250 \text{ kN} \) is not particularly well predicted. The diagonal crack inclination and principal diagonal strain are reasonably well predicted, while the spacing of the diagonal cracks is not. The reason is that the program uses the same diagonal crack spacing at the service load level as at ultimate load level. As was discussed earlier, it is not appropriate to use the stable diagonal crack spacing at the early stages of crack formation. At \( P = 250 \text{ kN} \) (total shear of 295 kN including dead load), the applied shear stress is only about 1.4 MPa, which is not sufficient to cause the stable diagonal crack pattern to form. As the actual crack spacings are larger than predicted, the actual crack widths are correspondingly larger than predicted. It is interesting to note how the initial vertical cracking due to restrained shrinkage influenced the diagonal crack pattern shown in Fig. 9.

A simplified approach for estimating the principal tensile strain of a cracked linear element is given by

\[
\varepsilon_1 = \varepsilon_l + \frac{\nu \tan \theta}{\rho_v E_s} + \frac{2 \nu}{E_c}
\]

where \( \varepsilon_l \) is the longitudinal strain, \( \varepsilon_c \) is the principal strain, \( \nu \) is the Poisson's ratio, \( \theta \) is the inclination of the principal tension strain, \( \rho_v \) is the ratio of transverse reinforcement, \( E_s \) is the modulus of elasticity of reinforcing steel, and \( E_c \) is the modulus of elasticity of concrete. Note that the magnitude of principal tensile strain, \( \varepsilon_1 \), depends on the magnitude of longitudinal strain \( \varepsilon_1 \), the shear stress \( \nu = V/b_n j_d \), the quantity of transverse reinforcement expressed as an area ratio \( \rho_v \), the modulus of elasticity of reinforcing steel and of concrete \( E_s \) and \( E_c \), and the inclination of the principal tension strain \( \theta \) which can be estimated by solving the following
Design and Construction Practices to Mitigate Cracking

Design and Construction Practices to Mitigate Cracking

\[ \tan \theta = \left( \frac{E_c \varepsilon_i + 2}{\tan \theta E_c + 2} \right)^2 \]

This angle is needed to determine the principal tensile strain from Eq. (10), and to determine the crack spacing from Eq. (7).

To account for diagonal cracking in a simplified way, the spacing of the diagonal cracks can be taken equal to the spacing of vertical cracks, \( s_d = s_l \) [see Fig. 8(b)], and the principal tensile strain can be approximated from a further simplified version of Eq. (10)

\[ \varepsilon_i = \varepsilon_i + \frac{\nu}{\rho_v E_s} \]

in which the angle \( \theta \) is assumed to be 45 deg, and the small contribution from the principal compressive strain is neglected. Thus the width of diagonal cracks can be estimated from the following simplified expression

\[ w_d = \left( \varepsilon_i + \frac{\nu}{\rho_v E_s} \right) s_l \]

The parameter \( \nu/\rho_v E_s \), which is approximately equal to the stirrup strain at the service load level, captures the influence of the applied shear stress \( \nu \) and quantity of stirrups \( \rho_v \) on the diagonal crack width. Note that according to Eq. (13), the width of diagonal cracks will always be larger than the width of longitudinal cracks due to the diagonal strain always being larger than the longitudinal strain.

SIDE FACE REINFORCEMENT IN LARGE MEMBERS

Distributed longitudinal reinforcement is required along the side faces of large concrete members to control cracking. The side face reinforcement requirements of four commonly used North American codes are summarized in Fig. 10. The ACI 318 building code (16) and AASHTO bridge code (17) require side face reinforcement in all members that are deeper than 914 mm (36 in.). In the current provisions of these codes, the required amount of side face reinforcement depends only on the member depth (except that the amount of reinforcement need not exceed one-half of the flexural tension reinforcement). Frantz and Breen (18) proposed that the amount of side face
reinforcement depends also on the clear concrete cover to the side face reinforcement, \( c \), and the diameter of the side face reinforcing bars, \( d_b \) (see Fig. 10 insert) in addition to the member depth. The current ACI/AASHTO requirements are similar to the Frantz and Breen proposal for \( 2c + d_b = 106 \) mm (4.2 in.), see Fig. 10.

The Canadian concrete code (CSA A23.3) (19) and Canadian highway bridge design code (CHBDC) (20) require side face skin reinforcement in members with overall depths greater than 750 mm (30 in.). For exterior exposures, a reinforcement ratio (steel area to concrete area ratio) of 1.0% is required in the outer skin which is assumed to be \( 2c + d_b \) thick on each side of the web. The Canadian highway bridge code requires reinforcement with an area equal to 1% of total web area, distributed over 70% of the web depth, which results in a minimum of 1.4% longitudinal reinforcement in the effective zone. In calculating the required area of reinforcement, the width of the web need not be taken greater than 250 mm (10 in.) as the side face reinforcement is assumed to act as skin reinforcement in wider members.

Fig. 10 indicates that there are significant differences regarding the required amount of side face reinforcement. Previous tests to evaluate the required amount of side face reinforcement have involved only flexural cracking. It is believed that the more stringent requirements of the Canadian codes may be needed in certain exposure conditions to control diagonal cracking in members subjected to significant service level shear stresses. In order to assess the different code requirements and to develop a general design method for side face reinforcement, additional large scale tests involving diagonal cracking were conducted using the element testing scheme shown in Fig. 2.

Table 1 summarizes the longitudinal side face reinforcement provided in the 21 test specimens, which had a web thickness of 180 mm. Table 2 summarizes the transverse reinforcement. The "F" series specimens were subjected to pure flexure, while the "FS" series specimens were subjected to a combination of flexure and shear. Fig. 11 shows typical crack patterns observed in the two types of tests. Additional information about the experimental study is given in Ref. 5.

Fig. 12 summarizes the observed crack widths at different levels of cracking. The 11,000 crack widths that were measured are summarized in terms of the 95\(^{th}\) percentile crack width, which was determined by fitting a Weibull distribution to the crack widths measured at each level of loading. The flexure test specimens [Fig. 12(a)] are shown at different levels of maximum longitudinal strain on the tension face, while the combined flexure and shear specimens [Fig. 12(b)] are shown at different levels of shear stress.

Fig. 13 compares the amount of side face reinforcement with the 95\(^{th}\) percentile vertical crack widths [Fig. 13(a)] and the 95\(^{th}\) percentile diagonal crack widths [Fig. 13(b)] over the service load range. In the specimens subjected to pure flexure, the crack widths are compared at the same strain
level, thus the added benefit of additional reinforcement in reducing the strains is not included. In order to decide how much side face reinforcement is appropriate, the issue of what is an acceptable crack width must be considered. The current ACI building code suggests limiting the maximum crack widths for interior and exterior exposures to 0.41 mm and 0.33 mm, respectively, while the AASHTO bridge code maximum crack width limits for moderate and severe exposures are 0.40 and 0.30 mm.

The results from this study indicate that the current ACI / AASHTO code requirements for side face reinforcement are adequate to limit the vertical (flexural) crack widths to within the 0.30 mm limit [Fig. 13(a)]. Controlling diagonal cracks is a more complicated phenomenon that depends not only on the amount and arrangement of longitudinal reinforcement, but also the amount of transverse reinforcement and the shear stress level. The ACI/AASHTO side face reinforcement requirements were found to limit the diagonal crack widths to about 0.40 mm depending on the shear stress level and amount of transverse reinforcement [Fig. 13(b)]. Thus additional side face reinforcement may be required in certain exposure conditions to control diagonal cracking, and a general procedure is needed for determining the appropriate amount and arrangement of side face reinforcement to control diagonal cracking.

A general expression that accounts for all important parameters can be developed by combining Eq. (9) and (13). Assuming that the maximum crack width is twice the average crack width and that the longitudinal strain at the level of critical side face cracking is about half the strain at the extreme tension face (this was observed in the tests), results in the following expression for the spacing of the side face reinforcement

\[
s_b = \frac{5}{1 + \frac{b_w d_b}{8 A_b}} \left( \frac{w_{\text{max}}}{\varepsilon_i} + \frac{\nu}{\rho, E_s} - 2c \right)
\]

where the width of the web \(b_w\) need not be taken greater than \(16d_b + 2c\) as the side face reinforcement will act as skin reinforcement in wider beams. \(A_b\) is the cross sectional area of a side face reinforcing bar, and \(\varepsilon_i\) is the strain of the longitudinal reinforcement on the flexural tension side at the service load level, which can be estimated from Eq. (8) or in lieu of such computations can be taken as \(0.6f_y/E_s\). The maximum tolerable crack width \(w_{\text{max}}\) depends on the particular exposure condition. Recall that the ACI / AASHTO codes limit the maximum crack widths for interior/moderate and exterior/severe exposures to about 0.40 mm and 0.30 mm, respectively.

To demonstrate the use of the proposed expression, the required amount of side face reinforcement required in the 1200 mm deep bridge girder shown in
Fig. 1 will be designed to limit the maximum crack widths to 0.30 mm. In the central portion of the girder, the shear stresses are negligible. The maximum service load bending moment near midspan is 1788 kNm. Accounting for both the 5700 mm$^2$ of the flexural reinforcement concentrated at the bottom of the member ($j_d = 1275$ mm) and the 1800 mm$^2$ of reinforcing bars distributed in the web ($j_d = 675$ mm), the strain of the longitudinal reinforcement on the flexural tension side can be estimated from Eq. (8) as $\varepsilon_t = \frac{1788 \times 10^6}{[200,000 \times (5700 \times 1275 + 1800 \times 675)]} = 0.00105$. Using 10M reinforcing bars (100 mm$^2$) with a clear cover of 40 mm, the required spacing of the side face reinforcement in the 180 mm wide girder can be calculated from Eq. (14)

$$s_b = \frac{5}{1 + \frac{180 \text{ mm} \times 10 \text{ mm}}{8 (100 \text{ mm}^2)}} \left( \frac{0.30 \text{ mm}}{0.00105 + 0} - 2 (40 \text{ mm}) \right) = 316 \text{ mm}$$

Thus provide 10M@305 mm (12 in.) on both side faces ($\rho = 0.36\%$) to control flexural cracking in the central region of the girder.

In the end regions of the girder, the bending moment and hence the longitudinal strain is reduced; but there are significant shear stresses. Due to the uniform live load shear (point loads were applied), the critical section is at $d$ from where the point load is applied. At that section, the bending moment is 1369 kNm and the applied shear force is 265 kN. The strain of the tension chord at that section is $1369/1788 \times 0.00105 = 0.00080$, and the shear stress $\nu = 265 \times 10^3 \text{ N} / (180 \text{ mm} \times 0.8 \times 1200 \text{ mm}) = 1.54 \text{ MPa}$. The stirrups in the girder consist of 10M double legged stirrups at 100 mm, thus $\rho_v = 2 \times 100 \text{ mm}^2 / (100 \text{ mm} \times 180 \text{ mm}) = 0.011$. Again using 10M side face reinforcing bars, the required spacing of the side face reinforcement is

$$s_b = 1.538 \left( \frac{0.30 \text{ mm}}{0.00080 + \frac{1.54 \text{ MPa}}{0.011 \times 200,000 \text{ MPa}}} - 2 (40 \text{ mm}) \right) = 185 \text{ mm}$$

Thus provide 10M@180 mm (7 in.) on both side faces ($\rho = 0.60\%$) to control diagonal cracking in the end regions of the girder. Note the considerable increase in side face reinforcement required near the support to control diagonal cracking.

**SUMMARY AND CONCLUDING REMARKS**

An assumed bond stress – slip relationship and the assumption of linear slip were used to derive a tension stiffening relationship for precracked concrete subjected to uniaxial tension. The resulting relationship between bond slip and average concrete tension stress was found to have a very different shape than
Design and Construction Practices to Mitigate Cracking

Empirically developed relationships, i.e., an increasing or uniform average concrete stress with increasing deformation. The reason for the difference is that empirical relationships combine the residual tension stresses in uncracked concrete with the tension stiffening effect of cracked concrete.

Empirically determined tension stiffening relationships have a higher than predicted average concrete tension (residual plus tension stiffening) stress due to the delay in formation of cracks. That is, due to the uncracked portions of concrete, the deformations are reduced. While the average stiffness of cracked concrete is increased by the delay in crack formation, the widths of individual cracks are not. As a result, it is not appropriate to estimate crack widths using an empirical tension stiffening expression and the stable crack spacing. Tests indicate that primary cracks continue to form up to strains of 0.001; however, the concrete between visible primary cracks deforms significantly and hence reduces the average crack widths (or reduces the effective crack spacing) for a given strain level. Thus the limiting strain level at which it is unconservative to estimate crack widths using the stable crack spacing was found to be 0.0005.

In order to estimate crack widths at smaller strain levels, a magnification factor [Eq. (5)] must be applied to the crack spacing. Alternatively, a minimum strain of 0.0005 must be used when estimating crack widths.

Crack width limits are usually expressed in terms of the maximum crack widths. The 95\textsuperscript{th} percentile value is believed to be an appropriate definition of "maximum" for crack width. Analysis of over 3500 crack width measurements indicates that crack widths follow a Weibull distribution, and that the 95\textsuperscript{th} percentile crack width is 2.0 times the mean (average) crack width.

The main focus of the current paper was to present procedures for estimating diagonal crack widths, and then to apply these in order to develop an expression that can be used to determine the required spacing of side face reinforcement to control diagonal cracking in large members. Diagonal crack widths can be predicted by computer using the modified compression field theory. An example of such an analysis was presented for the large scale laboratory test specimen shown in Fig. 1. It was found that the crack pattern near ultimate is well predicted; however the crack widths were under predicted at the service load level. The explanation is that at the low shear stresses levels in the serviceability range, the diagonal crack spacing is much wider than the stable crack spacing used in the analysis. The inclination of the diagonal cracks were well predicted in both the serviceability and ultimate conditions.

While a computer solution is useful, it does not lead to the development of a simplified design procedure. A simplified expression was derived in this paper for estimating diagonal crack widths. The simplifications used to derive this expression include: neglecting the contribution of the principal compression strain (this is usually small in the serviceability range), neglecting the effect of tension stiffening on reducing crack widths, assuming the critical crack
direction will be at 45 degrees, and assuming that the diagonal crack spacing is equal to the longitudinal crack spacing (this was shown to be a reasonable assumption for typical cases). The result is Eq. (13), which clearly indicates that the diagonal crack width is always greater than the flexural crack width due to the higher diagonal strains in a member with diagonal cracks. The longitudinal crack width is equal to $e_1 \times s_1$. The increase in diagonal crack widths over flexural crack widths is a result of the second term within the brackets which is approximately equal to the stirrup strain.

The current ACI / AASHTO code requirements for side face reinforcement were developed to control flexural cracking. Experimental results reviewed briefly in this paper indicate that the provisions do accomplish their objective, at least for the reinforcing bar spacings investigated. The experimental results also indicate that the current requirements may not be sufficient to control diagonal cracking for the "exterior" exposure condition according to ACI, or the "severe" exposure condition according to AASHTO. Eq. (13) was combined with a simplified version of the 1978 CEB-FIP Model Code expression for crack spacings to develop Eq. (14), which can be used to determine the required spacing of side face reinforcing bars in large members to control both flexural and diagonal cracking. A design example was presented in order to demonstrate the use of Eq. (14).

In addition to accounting for diagonal strains, Eq. (14) accounts for the clear cover to the side face reinforcement, the percentage of side face reinforcement (which is why $A_b$ and $b_w$ appear in the expression), and the bar diameter $d_b$. It is possible to develop a simplified version of this expression by combining Eq. (13) with a simplified crack spacing equation. For example, if the crack spacing is assumed to depend only on the clear concrete cover and bar spacing, a simplified expression results in which the term in front of the brackets in Eq. (14) is a constant.

The current ACI building code contains the following expression for the spacing of reinforcement closest to a surface in tension

$$s_b = \frac{540}{f_s^*} \cdot 2.5c$$

but not greater than $432 / f_s^*$, where the spacing of the bar and the clear concrete cover are in inches and the reinforcement stress $f_s^*$ is in ksi. This expression assumes that the cracks are normal to the reinforcement. The methodology used to develop Eq. (13) can be used to extend this expression for the case of diagonal cracking. Consistent with the summation of orthogonal strains in Eq. (13), the reinforcement stress in Eq. (15) should be taken as the sum of the reinforcement stress in the two orthogonal directions for the case of diagonal cracking $f_s^* = f_{sx} + f_{sy}$ and the clear concrete cover should be to the layer of reinforcement that will be spaced at $s_b$. As assumed
Design and Construction Practices to Mitigate Cracking

in the derivation of Eq. (13), only the reinforcement in one direction of an orthogonal grid of reinforcement needs to satisfy Eq. (15) in order to provide diagonal crack control.

ACKNOWLEDGMENT

The hybrid bridge girders shown in Fig. 1 and Fig. 3 are the patented COMPO-GIRDER™ developed by IOTA Construction Ltd. of Abbotsford, BC, Canada.

NOTATION

\( A_b \) = area of a single side face reinforcing bar;
\( A_s \) = area of longitudinal reinforcement on flexural tension side;
\( b_w \) = web width;
\( c \) = clear cover to reinforcement;
\( d \) = effective depth of member;
\( d_b \) = bar diameter;
\( E_c \) = modulus of elasticity of concrete;
\( E_s \) = modulus of elasticity of steel;
\( f_c \) = concrete stress;
\( f_{c'} \) = cylinder compression strength of concrete;
\( f_s \) = reinforcing steel stress;
\( f_y \) = yield strength of longitudinal reinforcement;
\( j_d \) = internal flexural lever arm;
\( M \) = bending moment at section of interest;
\( P \) = applied point load in test;
\( s \) = spacing of cracks;
\( s_b \) = spacing of side face reinforcing bars;
\( s_{1t} \) = spacing of cracks perpendicular to the longitudinal reinforcement;
\( s_t \) = spacing of cracks parallel to the longitudinal reinforcement;
\( s_d \) = spacing of diagonal cracks;
\( u \) = bond slip;
\( w \) = crack width;
\( w_d \) = width of diagonal cracks;
\( w_{max} \) = maximum crack width;
\( v \) = average shear stress;
\( V \) = shear force at section of interest;
\( \alpha \) = factor in Eq. (5) to account for amount of reinforcement;
\( \varepsilon \) = average normal strain;
\( \varepsilon_{d} \) = average normal strain perpendicular to diagonal cracks;
\( \varepsilon_{t} \) = average normal strain at level of longitudinal reinforcement on the flexural tension side;
\( \varepsilon_{1} \) = principal tensile average strain;
\( \tau \) = bond stress;
\( \theta \) = crack inclination from horizontal axis;
\( \rho \) = area of reinforcing steel to surrounding concrete;
\[ \rho_{ef} = \text{ratio of steel area to area of concrete within 7.5} \delta_b; \]
\[ \rho_v = \text{transverse reinforcement ratio}; \]

REFERENCES

1. ACI Committee 224, "Cracking of Concrete Members in Direct Tension (ACI 224.2R-92)," American Concrete Institute, Farmington Hills, Mich. 1992, 12 pp.


13. Frosch, R.J., "Another Look at Cracking and Crack Control in Reinforced Concrete," ACI Structural Journal, V. 96, No. 3, May-June 1999, pp. 437-442.

Design and Construction Practices to Mitigate Cracking


16. ACI Committee 318, "Building Code Requirements for Structural Concrete (ACI 318-95) and Commentary (ACI 318 R-95)," American Concrete Institute, Farmington Hills, Mich. 1995, 369 pp.


Table 1—Summary of longitudinal side face reinforcement in test specimens

<table>
<thead>
<tr>
<th>Specimen</th>
<th>$d_b$ (mm)</th>
<th>$s_b$ (mm)</th>
<th>$c$ (mm)</th>
<th>$\rho_t$ (%)</th>
<th>$\rho_{st}$ (%)</th>
<th>$A_{st}$ (mm$^2$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>F1, FS1</td>
<td>10</td>
<td>450</td>
<td>40</td>
<td>0.3</td>
<td>0.3</td>
<td>267</td>
</tr>
<tr>
<td>F2, FS2</td>
<td>10</td>
<td>300</td>
<td>40</td>
<td>0.4</td>
<td>0.4</td>
<td>400</td>
</tr>
<tr>
<td>F3, FS3, FS3X</td>
<td>10</td>
<td>160</td>
<td>40</td>
<td>0.7</td>
<td>0.7</td>
<td>750</td>
</tr>
<tr>
<td>F4, FS4</td>
<td>10</td>
<td>160</td>
<td>30</td>
<td>0.7</td>
<td>0.9</td>
<td>750</td>
</tr>
<tr>
<td>F5, FS5</td>
<td>10</td>
<td>110</td>
<td>30</td>
<td>1</td>
<td>1.3</td>
<td>1090</td>
</tr>
<tr>
<td>F6, FS6</td>
<td>10</td>
<td>80</td>
<td>30</td>
<td>1.4</td>
<td>1.8</td>
<td>1500</td>
</tr>
<tr>
<td>F7, FS7</td>
<td>6.4§</td>
<td>51</td>
<td>35</td>
<td>0.7</td>
<td>0.8</td>
<td>750</td>
</tr>
<tr>
<td>F8, FS8</td>
<td>6.4§</td>
<td>5130</td>
<td>3530</td>
<td>1.1</td>
<td>1.3</td>
<td>1150</td>
</tr>
<tr>
<td>F9, FS9</td>
<td>6.4§</td>
<td>5116</td>
<td>3530</td>
<td>1.4</td>
<td>1.6</td>
<td>1500</td>
</tr>
<tr>
<td>F10, FS10</td>
<td>10</td>
<td>300</td>
<td>30</td>
<td>0.4</td>
<td>0.5</td>
<td>400</td>
</tr>
</tbody>
</table>

plus 0.9% volume 30 mm hooked steel fiber

§ welded wire fabric
† percentage reinforcement within exterior skin ($2c + d_b$)
‡ total area of side face reinforcement in half of girder depth
Table 2—Summary of transverse reinforcement in test specimens

<table>
<thead>
<tr>
<th>Specimen</th>
<th>$d_b$ (mm)</th>
<th>$s_b$ (mm)</th>
<th>$c$ (mm)</th>
<th>$\rho_v$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>F1→F3</td>
<td>10</td>
<td>200</td>
<td>30</td>
<td>0.56</td>
</tr>
<tr>
<td>F4→F6</td>
<td>10</td>
<td>200</td>
<td>40</td>
<td>0.56</td>
</tr>
<tr>
<td>F7→F9</td>
<td>10</td>
<td>200\textsuperscript{6}</td>
<td>40</td>
<td>1.26</td>
</tr>
<tr>
<td>F10</td>
<td>10</td>
<td>200</td>
<td>40</td>
<td>0.56</td>
</tr>
<tr>
<td>FS1→FS</td>
<td>10</td>
<td>100</td>
<td>30</td>
<td>1.11</td>
</tr>
<tr>
<td>FS3X</td>
<td>10</td>
<td>200</td>
<td>30</td>
<td>0.56</td>
</tr>
<tr>
<td>FS4→FS</td>
<td>10</td>
<td>100</td>
<td>40</td>
<td>1.11</td>
</tr>
<tr>
<td>FS7→FS</td>
<td>10\textsuperscript{6}</td>
<td>100\textsuperscript{5}</td>
<td>40</td>
<td>1.81</td>
</tr>
<tr>
<td>FS10</td>
<td>10</td>
<td>100</td>
<td>40</td>
<td>1.11</td>
</tr>
</tbody>
</table>

\textsuperscript{6} welded wire fabric
Fig. 1 – Full-scale laboratory test on a concrete–steel hybrid girder to observe flexural and diagonal side face cracking: (a) test-setup; (b) initial cracking due to dead load and restrained shrinkage; and (c) cracking due to service live load.
Fig. 2 - Large-scale element testing scheme used to apply combined flexure and shear to 21 specimens with varying side face reinforcement.
Fig. 3 - Field testing of a hybrid girder bridge to measure service load cracking: (a) over-view of bridge under test; (b) measuring critical diagonal crack widths (22).
Fig. 4 – Predicted concrete stresses: (a) pre-cracked element subjected to uniaxial tension; (b) average concrete stresses due tension stiffening; (c) total concrete stresses due to tension stiffening and residual tension in uncracked concrete.
Fig. 5 – Experimentally measured concrete tension stresses: (a) residual stresses in plain concrete (11); (b) total concrete tension stresses (7).
Elements F1 to F3
\( \rho_f \leq 0.7\% \)
- observed crack spacing
- crack spacing determined from average crack width

Fig. 6 - Variation of crack spacings with longitudinal strain for: (a) lightly reinforced elements; and (b) more heavily reinforced elements.
Fig. 7 - Variability of crack width ratios (crack width normalized by average crack width for given strain).
Fig. 8 – Diagonal crack spacings (a) transformation in polar co-ordinates; and (b) comparison of actual and assumed transition.
Fig. 9 - Comparison of predicted and observed diagonal cracking at 4.6 m from the support of the girder shown in Fig. 1: (a) and (b) $P = 250$ kN; (c) and (d) $P = 600$ kN.
Fig. 10 – Comparison of different North American requirements for side-face reinforcement.
Fig. 11 - Typical crack patterns in: (a) elements subjected to pure flexure; and (b) elements subjected to combined flexure and shear.
Fig. 12 – Comparison of maximum crack widths: (a) vertical cracking in specimens subjected to flexure; and (b) diagonal cracking in specimens subjected to combined flexure and shear.
Fig. 13 - Influence of percentage of side-face reinforcement on control of service load crack widths: (a) vertical cracking; and (b) diagonal cracking.
Positive Moment Cracking in Diaphragms of Simple-Span Prestressed Girders Made Continuous

by A. Mirmiran, S. Kulkarni, R. Miller, M. Hastak, B. Shahrooz, and R. Castrodale

Synopsis:
Precast prestressed girder bridges can be made continuous for live load if the deck and diaphragm are cast with sufficient positive and negative moment reinforcements. The continuity eliminates costly joints and enhances seismic performance, structural integrity and overall durability of the structure. If diaphragm is poured with sufficient negative moment reinforcement before the deck is cast, continuity may also apply to the dead load of the slab. Although, connection of the girders at the diaphragm varies from state to state, it generally consists of bent bars or bent strands. Also, a short length of the girder may be embedded into the diaphragm. The continuity connection is a doubly reinforced section, which requires a time-dependent analysis including differential shrinkage, creep due to prestressing and dead loads, and temperature effects. These time-dependent effects can result in considerable positive restraining moments at the supports, which can in turn crack the diaphragm or pull the girder out of the diaphragm. These positive moment cracks are not only unsightly, but may also result in durability issues for the bridge. Furthermore, it questions the integrity of the continuity connection. The paper examines the extent of positive moment cracking based on field observations, time-dependent analysis, and previous studies.

Keywords: composite; concrete; continuous; cracking; diaphragm; girder; prestressed
ACI member Amir Mirmiran is an associate professor in the Department of Civil and Environmental Engineering at the University of Cincinnati. He is a member of ACI-ASCE Committees 343, Concrete Bridges; and 440, FRP Reinforcement. His research interests include concrete bridges and application of fiber composites.

Siddharth Kulkarni is a structural engineer with Bechtel Corporation in Houston, Texas. He received his master’s degree from the Department of Civil and Environmental Engineering at the University of Cincinnati.

Richard A. Miller is an associate professor in the Department of Civil and Environmental Engineering at the University of Cincinnati. His research interests include bridges, prestressed concrete, and concrete material properties.

Makarand Hastak is an assistant professor in the Department of Civil and Environmental Engineering at the University of Cincinnati. He received his PhD from Purdue University. His research interests include construction engineering and management, construction project cost control, and risk assessment.

ACI member Bahram M. Shahrooz is an associate professor in the Department of Civil and Environmental Engineering at the University of Cincinnati. He is a member of Committees 335, Composite and Hybrid Structures; and 442, Response to Lateral Forces.

ACI Member Reid W. Castrodale is an associate and senior engineer with Ralph Whitehead Associates, Inc., in Charlotte, North Carolina, where he provides specialized design services for RC and PC bridges. He received his PhD from University of Texas at Austin. He is active on several ACI and PCI committees.

INTRODUCTION

Positive moment cracking is a serviceability problem in the diaphragms of simple-span prestressed precast girders made continuous, due to time-dependent effects such as creep and shrinkage. Prestressed, precast girders were originally used in simple-span bridges. Studies of the early 1960’s demonstrated that precast girders could be made continuous for live load, if the deck and the closure diaphragm are cast with sufficient positive and negative moment reinforcement (1-7). Continuity has many advantages in bridge construction, as it eliminates costly joints, and enhances seismic performance, structural integrity and durability of the bridge. Continuity in
Design and Construction Practices to Mitigate Cracking

Precast concrete is usually achieved by a closure diaphragm at the girder ends, cast monolithically with the deck. The girders carry their own dead load as simple spans, and the live load as continuous spans. Although, connection of the girders at the diaphragm varies from state to state, it generally consists of negative reinforcement in the deck, and bent bars or bent strands as positive reinforcement in the diaphragm. Also, a short length of the girder is often embedded in the diaphragm. The continuity connection is a doubly reinforced section, which is subjected to live loads as well as time-dependent effects such as temperature, differential shrinkage, and creep due to prestressing and dead loads. Positive moments may potentially crack the diaphragm or pull the girders out of the diaphragm. The cracks are not only unsightly, but may also raise durability concerns for the bridge. More importantly, the cracks question the integrity of the connection in providing continuity for the bridge.

Figure 1 shows the sequence of construction for continuity connections. If the deck and the diaphragm are cast together, continuity and composite action are developed at the same time. The prestress, weight of the girder and the deck are then carried by the girder alone as a simple span structure. The superimposed dead loads, future wearing surface, live load and impact and the time-dependent effects of creep, shrinkage and temperature are resisted by the composite section as a continuous structure. If the diaphragm is constructed with negative moment reinforcement prior to casting of the deck, continuity of the girders is established before developing the composite action. In this case, dead load of the deck is carried by the continuous system, thus reducing stresses in the positive moment region. This method of construction has been suggested, but has not been used in many bridges. It has also been recommended that partial pour of the diaphragm may further help reduce the positive moment cracking. This paper examines the extent of positive moment cracking based on field observations, time-dependent analysis, and previous studies.

**RESEARCH SIGNIFICANCE**

Despite 40 years of research into the use of precast girders in continuous bridges, the practice is not yet widespread among many states. Some states that use the continuity connections design the girders as simple spans, neglecting the benefits of continuity altogether. This study provides some insight into the extent of positive moment cracking based on field observations, time-dependent analysis, and previous studies.

**PREVIOUS STUDIES**

In the 1960's, the Portland Cement Association (PCA) carried out laboratory investigations to determine whether precast girders could be made continuous with a cast-in-place deck (1-6). Two types of positive moment
connections were tested: embedded straight bars with their free ends welded to a structural angle, and embedded hooks. The welded connections were found adequate, while hooked connections fractured, mainly due to the tight radius of the bend. Effects of creep and shrinkage were evaluated by testing two specimens, one with no positive moment connection, and the other with bent bars. The specimen with no positive moment connection cracked, and the live load continuity was reduced. However, cracking had no effect on the ultimate capacity of the negative moment connection. The specimen with bent bars had no cracking. However, it is possible that cracks would have developed if test duration had been long enough. The PCA studies culminated in a design method for continuity connections (7). It recommended limiting the stress in bent bars to 60% of the yield strength under live loads and time-dependent effects.

Although the PCA studies encouraged the use of precast girders in continuous bridges, the AASHTO Specifications did not provide any design guidelines. Therefore, there are widespread differences in connection details, materials and construction sequences. The National Cooperative Highway Research Program Project 12-29 was launched to develop guidelines on the continuity connections. The studies by CTL developed new procedures of predicting restraint moments due to creep and shrinkage (8). It concluded that construction of a positive moment connection is difficult, time consuming and costly, while there is no structural benefit for such connections. When there is no positive reinforcement, the positive moments from creep and shrinkage crack the connection. When positive reinforcement is provided, the connection is more resistant to cracking, but restraining moments will be much larger. Thus, reinforced and unreinforced connections both eventually crack, and the parametric study found that mid-span moments are virtually independent of the positive reinforcement provided in the diaphragm.

The Missouri Cooperative Highway Research Program commissioned a number of studies on the feasibility of extending strands into the diaphragm to develop positive moment continuity (9-12). Three strand configurations of straight, frayed and 90° bent were tested. It was found that bent strand provided the best anchorage, with half the slip of the straight or frayed strand. Straight strand was found to perform marginally better than frayed strand. Tests on full-scale girders culminated in a design method for positive moment connections using bent strands. The studies recommended that the stress in the strand be limited to 15% of the ultimate strength of the strand to avoid fatigue failure. It was also recommended that the diaphragm be cast before the slab.

Abdalla et al. (13) tested three two-span continuous Type-I girders and a 686 mm (27") box girder, all with debonded strands. For positive moment continuity, four strands were bent at 90° angles and embedded into the diaphragm. It was found that the CTL method did a better job of
predicting time dependent moments if the effect of deck steel was accounted for; however, there were still some significant differences between the measured and predicted values. Tadros and co-workers (14,15) explored continuity in Nebraska NU type girders. They concluded that the construction sequence greatly affects the development of positive moments. It was recommended that if the diaphragm is cast first, the slab should be cast within 230 days to prevent cracking. It was also recommended that if the diaphragm is cast first, negative moment connections be supplied between the beams to prevent cracking and spalling at the joint between the diaphragm and the deck. The study further recommends that when the diaphragm and deck are cast together, an unbonded joint be used between the diaphragm and beam to allow the beam to rotate under the deck weight. Recently, Clark and Sugie (16) studied positive and negative moment connections of precast girders. They suggested that instead of calculating the effect of creep and shrinkage, the connection be designed for a positive moment of 750 kN-m (550 ft-kip) for spans in the 20-36 m (65-120 ft) range where the beams are 1100 mm (42 in) deep or greater. For smaller beams, they suggested designing for a positive moment of 600 kN-m (440 ft-kip).

NATIONAL SURVEY AND FIELD OBSERVATIONS

To establish the current state-of-practice, the authors carried out a detailed survey of a large number of state, county, city, township, consulting, and construction engineers, contractors and fabricators in the U.S. and abroad. About half of respondents design the girders as simple span, even though the deck is continuous over the supports. Of the states that provide positive moment connection, almost all use either bent strands or bent bars with an equal split between the two methods. Most states embed the girders into the diaphragm, but the embedment length varies widely with the majority in the range of 0-50 mm (0-2 in) or greater than 254 mm (10 in). The most common problem reported with the connections was cracking of the diaphragm. The authors visited a number of continuous bridges in Tennessee to examine the severity of such cracks. Figure 2 shows conditions of a number of diaphragms. In most cases, there were either no visible cracks or minor shallow cracks. Deeper cracks were attributed to the construction sequence that casts the deck while the girders are still creeping at an early age. This observation showed the importance of design and construction parameters on the positive restraining moments, as will be discussed in the next section.

ANALYTICAL WORK

Time-dependent effects include differential shrinkage, creep, and thermal gradient. Differential shrinkage mainly arises from the difference in the age or shrinkage properties of the girder and the deck concrete. Creep is
due to sustained stresses applied by prestressing and dead load. Creep due to prestress, when restrained by the continuity connection typically causes a negative bending curvature in each span and a positive restraining moment at the support. The weight of the structure counteracts the creep from prestress, and reduces the positive restraining moment at the support similar to the effect of differential shrinkage. Contrary to the differential shrinkage, creep effects are more critical when continuity is established early on in the age of the girder. The creep and shrinkage effects are more complicated, if girders of different age are used in the same bridge. There are two methods available for calculating time-dependent effects; the PCA method (7) and the CTL method (8) (i.e., NCHRP 322). The main difference between the two methods is the finite length diaphragm that is considered by the CTL method.

In the present study, a spreadsheet program is developed to automate the time step analysis including the creep and shrinkage effects following the latest edition of the ACI 209 (17). The prestress losses are based on the procedure suggested by the PCI Committee on Prestress Losses (18), with correction factors of the ACI 209 for loading age, relative humidity and volume-to-surface ratio (17). The program allows the user to input the girder age when continuity is achieved, i.e., casting of the diaphragm, independently from the girder age at which the deck is cast. The implications of such construction sequence are discussed later in this section. A lookup table is provided for all AASHTO girders. The program allows unequal span lengths for up to 5-span bridges. However, the span configuration must be symmetric. The program also considers the restraining effect of the slab reinforcement on the shrinkage of the slab. This is termed as “Dischinger” effect by the PCA. Figure 3 shows a comparison between the present study and test results of Abdalla et al. (13). In the same figure, predictions from the PCA method (7) and the CTL method (8) are shown with the restraining effect of slab reinforcement on shrinkage starting at 3 days or 30 days after casting of the slab. The predictions for the present study are also shown with the same age for restraining effects. Good agreement with the test results is obtained. However the difference of the present study with the PCA and CTL methods can be attributed to several factors including changes in the ACI 209 for creep and shrinkage properties. Another important consideration is the time intervals. This point is demonstrated in Figures 4 and 5 for a Type III girder with two 19.8 m (65 ft) spans. The difference between the results of the present study and the CTL method in Figure 4 is due the large time steps used in the CTL method (8). Once the same time steps are applied to the present study, a much better agreement is obtained as noted in Figure 5. A major difference of the present study with the CTL method is in the assumption of finite length of diaphragm by the CTL method, which is more pronounced in multi-span bridges.
A parametric study was conducted using the above model to investigate the effect of girder type or size, girder spacing, span length, number and pattern of strands, strand type (stress relieved or low relaxation), age at transfer of prestress, age at continuity, age at which the deck is cast, and creep and shrinkage properties on the restraining moments. Figure 6 shows the restraining moment for the center support of a two-span Type III or IV girders, with the diaphragm and deck both cast when the girder is 14 days old. The span length for both types of girders is 19.8 m (65 ft). The girders are designed to carry the dead loads as simple span, and the live loads and impact as continuous composite sections. It is clear that for the same span lengths (and the same strand type and size), smaller girders require more prestressing force, and therefore, have larger positive restraining moments. This comparison is based on the same girder spacing of 2.4 m (8 ft) for both types of girders. Figure 7 shows the restraining moments for the center support of a two-span Type III girder with 2.4 m (8 ft) girder spacing, and girder age of 28 days at casting both the diaphragm and the deck. The three graphs shown in the figure represent 16.76 m (55 ft), 19.81 m (65 ft), and 22.86 m (75 ft) span lengths. Therefore, for the same girder size and girder spacing, and the same strand type and size, longer spans require more prestressing force, and as such result in larger restraining moments.

Figure 8 shows the restraining moments for the center support of a bridge with two 19.8 m (65 ft) spans. Type III girders are used with the diaphragm and deck both cast at the same time. The three graphs shown in the figure represent casting at girder ages of 14, 28, and 90 days. Clearly, age of girder at the time of continuity and casting of the deck has a pronounced effect on the restraining moments. The older the girders, the lower the positive moments. Figure 9 shows the effect of sequence of construction on the restraining moments at the center support of a bridge with two 19.8 m (65 ft) spans using Type III girders with 2.4 m (8 ft) spacing. The three graphs relate to either casting the diaphragm and the deck at the same time or with a phased construction. Case 1 and 2 represent casting the diaphragm and the deck at the same time when the girders are 14 days and 28 days old, respectively. Case 3 represents casting the diaphragm when the girder is 14 days old, and casting the deck when the girder is 28 days old. If the deck is cast after continuity is established (i.e., Case 3), there will be an early surge in positive restraining moment. This is because, at this time, (a) the opposing effect of differential shrinkage does not exist, and (b) the opposing effect of the weight of the deck and superimposed dead load does not exist. As the age difference between casting of the diaphragm and casting of the deck increases the initial surge also increases, but the ultimate positive restraining moment is substantially reduced. Once the deck is cast, there is a sudden drop in the positive moments due to the weight of the deck. It is clear that positive restraining moments in Case 3 are lower than those in Case 1, but still larger than those in Case 2.
Therefore, it is generally better to cast both the deck and the diaphragm at a later age. Note that in Case 3, negative reinforcement must be present in the diaphragm to provide for continuity.

CONCLUSIONS

Continuity connections are subject to large positive restraining moments due to time dependent effects. National survey indicates that cracking of these connections is of great concern to many state DOTs due to serviceability problems that they cause. Since severe cracks and pullout of girders have been experienced in some bridges, eliminating positive moment reinforcement altogether is not appropriate. However, simplified methods of designing for a certain moment, perhaps a multiple of cracking moment, may be developed, as some studies suggest. Field observations showed that minor cracks do not impose significant serviceability problem. A time-dependent analysis is suggested to evaluate the effect of various design parameters on the positive moment cracking of the diaphragm. In order to avoid severe cracks, early age girders must be avoided in continuous bridges.

ACKNOWLEDGEMENTS

This study is supported by the National Cooperative Highway Research Program (NCHRP Project 12-53). The opinions and findings expressed here, however, are those of the authors alone, and not necessarily the views of the NCHRP or the NCHRP Panel.

REFERENCES


17. ACI Committee 209, “Prediction of Creep, Shrinkage and Temperature Effects in Concrete Structures,” *Designing for Creep and Shrinkage in Concrete Structures*, Publication SP-76, American Concrete Institute, Detroit, MI, 1989, pp. 193-300.

Design and Construction Practices to Mitigate Cracking

(a) Erect Simple Span Girders

(b) Cast Diaphragm and Deck

Figure 1. Sequence of Construction for Continuity Connection

Figure 2. Field Observations: (a) No Visible Cracks at Interior Beams, (b) No Visible Cracks at Exterior Beam, (c) Minor Shallow Cracks, and (d) Deep Cracks
Figure 3. Restraining Moments for Specimen 3 (13)

* Effect of slab reinforcement on shrinkage after 30 days
** Effect of slab reinforcement on shrinkage after 3 days
Figure 4. Comparison of the CTL Method with the Present Study Using Smaller Time Steps

- Actual girder age at which the effect of slab reinforcement on shrinkage starts. (Age of deck = 30 days)
- Type III Girder
- Two 19.8 m (65 ft) Spans
- Continuity at 28 Days
- Deck Cast at 28 Days
Girder age at which the effect of slab reinforcement starts is assumed to be the same for both methods.

Figure 5. Comparison of the Present Study with the CTL Method Using Same Time Steps
Figure 6. Restraining Moments for Two-Span Continuous Type III and Type IV Girders with Continuity at Girder Age of 14 Days
Figure 7. Restraining Moments for Two-Span Continuous Type III Girders with Various Span Lengths with Continuity at Girder Age of 28 Days
Figure 8. Restraining Moments for Two-Span Continuous Type III Girders with Continuity Established at Different Girder Age
Figure 9. Restraining Moments for Two-Span Continuous Type III Girders with Various Sequences of Construction.
Flexural Crack Control in Reinforced Concrete

by R. J. Frosch

Synopsis:
The ACI building code has adopted a new design method for the control of flexural cracking. This design method is intended for structures containing steel reinforcement and not requiring specialized crack control procedures. It is the objective of this paper to explore the background for this method, highlight the assumptions, and develop design tools that can be applied for special design cases. This paper presents a summary of a physical model for cracking that was the basis for the new design method, illustrates the development and limitations of the design method, and develops design tools that are applicable for the control of cracking in structures requiring increased levels of crack control and in structures incorporating alternative reinforcement materials. Examples illustrating the use of the new design method as well as tools extending its applicability are presented.

Keywords: concrete durability; crack control; crack spacing; crack width
INTRODUCTION

Crack control is an important issue for primarily two reasons, aesthetics and durability (1,2,3). First, wide cracks detract from a structure visually as well as may unduly alarm the public that there are structural problems. Second, wide cracks may cause durability related problems. Cracks provide a rapid route for oxygen, water, and possible chlorides to reach the reinforcement, which may lead to corrosion and structural deterioration. To combat corrosion, many engineers have been specifying thicker concrete covers. Both research and experience have indicated that thicker covers can increase durability. In designing with thicker covers, however, engineers have found that the common design method for the control of cracking, often referred to as the z-factor method, becomes unworkable.

Research was conducted to investigate the role of concrete cover on cracking and to provide tools for the control of cracking in structures containing thicker covers. This research, presented in Ref. 4, developed a procedure for the calculation of crack widths that was based on the physical phenomenon. Additionally, a design recommendation was presented that ultimately resulted in changes to the ACI building code.

RESEARCH SIGNIFICANCE

For proper application of this new design procedure, it is important to understand the background for its development as well as the limitations imposed by that development. As an example, the building code states that these provisions are not intended for “structures subject to very aggressive exposure or designed to be watertight” (5). This paper explores the limitations and provides tools for the application of this new design approach for specialized structures. In addition, the control of cracking in structures utilizing new reinforcing materials is explored.
BACKGROUND

To understand the limitations of the current design method, it is useful to review the background of its development. As mentioned, research presented in Ref. 4 developed a calculation procedure for the determination of crack widths based on the physical phenomenon. A summary of the physical model is presented here (complete details are available in Ref. 4).

As shown in Figure 1, the crack width at the level of the reinforcement can be calculated as follows:

\[
  w_c = \varepsilon_s S_c
\]

(1)

where:

- \( w_c \) = crack width
- \( \varepsilon_s \) = reinforcement strain = \( \frac{f_s}{E_s} \)
- \( S_c \) = crack spacing
- \( f_s \) = reinforcement stress
- \( E_s \) = reinforcement modulus of elasticity

**Strain Profile**

To determine the crack width at the beam surface, it is necessary to account for the strain gradient. The strain gradient is illustrated in Figure 2, which assumes that plane sections remain plane. The crack width computed above can be multiplied by an amplification factor \( \beta \) that accounts for the strain gradient. The factor, \( \beta \), is computed as follows:

\[
  \beta = \frac{\varepsilon_{z}}{\varepsilon_{y}} = \frac{h - c}{d - c}
\]

(2)

**Crack Spacing**

Based on the work of Broms (6), it was found that the crack spacing depends primarily on the maximum concrete cover. Specifically, the
minimum theoretical crack spacing will be equal to the distance from the point at which the crack spacing is considered to the center of the reinforcing bar located closest to that point. In addition, the maximum spacing is equal to twice this distance. As illustrated in Figure 3, the critical distance for the maximum crack spacing can occur at two locations, and the crack spacing can be calculated as follows:

\[ S_c = \Psi_s d' \]  

where:
- \( S_c \) = crack spacing
- \( d' \) = controlling cover distance
- \( \Psi_s \) = crack spacing factor
  - 1.0 for minimum crack spacing
  - 1.5 for average crack spacing
  - 2.0 for maximum crack spacing

**Crack Control**

Based on the physical model, the equation for the calculation of maximum crack width is as follows:

\[ w_c = 2 \frac{f_*}{E_s} \beta \sqrt{d_c^2 + \left( \frac{s}{2} \right)^2} \]  

This equation can be rearranged to solve for the maximum permissible bar spacing, \( s \).

\[ s = 2 \sqrt{\left( \frac{w_c E_s}{2 f_* \beta} \right)^2 - d_c^2} \]  

where:
- \( s \) = maximum permissible bar spacing, in.
- \( w_c \) = limiting crack width, in.
- \( E_s \) = reinforcement modulus of elasticity, ksi
- \( f_* \) = reinforcing bar stress, ksi
- \( d_c \) = bottom cover measured from the center of lowest bar, in.

For a given limiting crack width and bar stress, the bar spacing can be plotted versus the concrete cover. The reinforcement stress used in Eq. (5) corresponds with the actual bar stress considered which is typically the service load stress. Alternately, a reinforcement stress of 60 percent of
yield may be used to account for service levels. The factor, \( \beta \), varies as the cover increases. Therefore, based on a review of sections with varying cover, \( \beta = 1.0 + 0.08d_c \) was found to provide a reasonable estimate.

Figure 4 is plotted for Grade 60 reinforcement \( (f_s = 36 \text{ ksi}, E_s = 29,000 \text{ ksi}) \). In this figure, curves are shown for two different limiting crack widths, 0.016 in. and 0.021 in. The crack width of 0.016 in. corresponds to the ACI 318-95 (7) design recommendations for interior exposure conditions while 0.021 in. corresponds to a 1/3 increase in the recommended crack widths. A 1/3 increase in crack widths was considered acceptable due to the large-scatter inherent in crack widths and since Eq. (5) considers the maximum crack width.

**DESIGN RECOMMENDATION**

Based on consideration of the results from the physical model, a simplified design curve for the maximum spacing of reinforcement was proposed in Ref. 4 as given by Eq. (6). This curve is plotted in Figure 4.

\[
s = 12\alpha_s \left[ 2 - \frac{d_c}{3\alpha_s} \right] \leq 12\alpha_s, \tag{6}
\]

where:

- \( \alpha_s = \frac{36}{f_s} \gamma_c \)
- \( d_c \) = thickness of concrete cover measured from extreme tension fiber to center of bar or wire located closest thereto, in.
- \( s \) = maximum spacing of reinforcement, in.
- \( \alpha_s \) = reinforcement factor
- \( \gamma_c \) = reinforcement coating factor
  - 1.0 for uncoated reinforcement
  - 1.5 for epoxy-coated reinforcement, unless test data can justify a higher value.
- \( f_s \) = Calculated stress in reinforcement at service load, kips, in. Shall be computed as the moment divided by the product of steel area and internal moment arm. It shall be permitted to take \( f_s \) as 60 percent of the specified yield strength \( f_y \).
ACI 318-99 DESIGN METHOD

The proposal presented was modified from its original form and resulted in the design equation presented in ACI 318-99 (5) under code section 10.6.4.

\[ s = \frac{540}{f_s} - 2.5c_c \leq 12\left(\frac{36}{f_s}\right) \]  

(7)

where:

- \( s \) = center-to-center spacing of flexural tension reinforcement nearest to the extreme tension face, in. (where there is only one bar or wire nearest to the extreme tension face, \( s \) is the width of the extreme tension face.)
- \( f_s \) = calculated stress in reinforcement at service loads, ksi. It shall be permitted to take \( f_s \) as 60 percent of specified yield strength.
- \( c_c \) = clear cover from the nearest surface in tension to the surface of the flexural tension reinforcement, in.

The ACI 318-99 equation considers the clear cover (\( c_c \)) directly rather than using the cover to the center of the bar (\( d_c \)). It was felt that the clear cover would be simpler to apply in design. In addition, this modified form of the equation is slightly more conservative than the original proposal. The ACI design curve is also plotted in Figure 4 where \( c_c \) is converted to the dimension \( d_c \) considering an average bar size of \#8 (\( d_b = 1.0 \) in.) to provide comparison. It can be seen that this design equation reasonably describes the reinforcement spacing for a range of concrete covers while maintaining crack widths within the ranges previously discussed.

Two primary assumptions, however, were used in the derivation of the original design recommendation and are inherent in the ACI design method. These assumptions may prove to be a limitation for some design applications. First, the crack widths controlled are based on a crack width of approximately 0.016 in. at the beam bottom face. Considering the scatter inherent in cracking (it has been often noted that scatter in crack widths is in the range of 50%), crack widths both below and above this value should be expected in service. Therefore, as indicated in the building code, these provisions are not applicable for structures subject to very aggressive exposure conditions or for structures designed to be watertight.
Second, the maximum spacing was based on a reinforcement modulus of elasticity of 29,000 ksi that corresponds with the use of steel reinforcement. Therefore, the ACI design provisions are not applicable for structures using reinforcement containing a different modulus of elasticity. In fact, all test results used to ascertain the accuracy and applicability of the crack width equation (Eq. (4)) were for steel reinforcement.

**SPECIFIED CRACK WIDTH CONTROL**

For the design of specialized structures that require tighter control of the crack width, it is important to develop design tools that are applicable. Using the physical model, it is possible to consider any limiting crack width that the designer chooses appropriate. This feature allows for versatility especially for varying structural exposure conditions. In the same manner that the physical model was used to develop the simplified design curves presented by Eq. (6), simplified design curves can be developed for any specified crack width.

The maximum bar spacing can be determined directly using Eq. (5) once the desired limiting crack width is selected. Figure 5 presents the maximum bar spacing versus concrete cover, $d_c$, for a range of limiting crack widths. The graph was developed for Grade 60 reinforcement stressed at 36 ksi ($0.6f_y$). The range of crack widths presented is based on recommendations for various exposure conditions as provided by ACI Committee 224 (8).

As the limiting crack width is decreased, the spacing of the reinforcement must be decreased. For a structure containing a concrete cover of 1.5 in. and stressed at 36 ksi, it is evident that it is not possible to control the crack width to 0.004 in. With a maximum reinforcement spacing of 3 in., however, it is possible to control to approximately 0.006 in. Again, it must be noted that these crack widths are measured at the beam surface. Smaller crack widths are expected at the reinforcement.

As illustrated, there may be cases where crack width control to a specified crack width may not be possible by reducing only the reinforcement spacing. In these cases, it may also be necessary to reduce the design service level stress in the reinforcement. Figure 6 illustrates the effect of changing the reinforcement stress for a design limiting crack width of 0.006 in. As noted, various reinforcement spacings can be used to achieve the same limiting crack width through control of service load stress.

The designer can directly utilize Eq. (5) to control crack widths to any desired level. Alternately, simplified design curves can be developed as presented in Figure 7. These curves were developed based on the original design recommendation (Eq. (6)) with a factor added in the reinforcement factor, $\alpha$, to account for varying crack control limits. The modified reinforcement factor is presented as follows:
142 Frosch

\[ \alpha_s = \frac{36}{f_s} \gamma_s \gamma_{w_s} \gamma_E \]  

(8)

where:

\[ \gamma_{w_s} = \text{crack width factor} = \frac{w_c}{0.016 \text{ in.}} \]

\[ w_c = \text{desired crack width limit, in.} \]

As noted, the original design recommendation easily permits modification to allow for the control of various crack widths. Since the ACI design equations were based on the same format, they also can be modified to account for various desired crack control levels. The ACI design equation (Eq (7)) can be adjusted to account for varying crack control limits by multiplying \( f_s \) by \( 1/\gamma_{w_s} \).

**REINFORCING MATERIALS**

As new materials are being considered for use in reinforced concrete design, crack widths will remain of importance. Even though many materials do not have the potential for corrosion, the control of crack widths for aesthetic reasons will continue. Since Eq. (5) is based on the physical phenomenon, it remains applicable for materials with different moduli of elasticity. It must be noted, however, that bond is essential for the development of cracks and a regular crack spacing as computed by Eq. (3). Similar to epoxy coated reinforcement, lower bond strengths of alternative reinforcement can directly affect crack spacing and crack width. Testing of these reinforcement bars is essential to verify adequate development of cracks and the applicability of the crack width calculation.

Assuming adequate bond strength, the modulus of elasticity can be accounted directly in Eq. (5). Alternately, design curves were developed based on the original design recommendation (Eq. (6)) as follows:

\[ \alpha_s = \frac{36}{f_s} \gamma_s \gamma_{w_s} \gamma_E \]  

(9)

where:

\[ \gamma_E = \text{modulus of elasticity factor} = \frac{E}{E_s} \]

\( E = \text{modulus of elasticity of reinforcement, ksi} \)

\( E_s = \text{modulus of elasticity of steel, 29,000 ksi} \)
This modified form of the reinforcement factor, \( \alpha_s \), contains all multipliers previously presented and is a general form that permits adjustments for epoxy coating, crack width limits, and the reinforcement modulus of elasticity. Similarly, the ACI design equation (Eq. (7)) can also be modified to account for the modulus of elasticity of various reinforcement materials by multiplying \( f_s \) by \( 1/\gamma_E \).

**DESIGN EXAMPLES**

Design examples are presented to illustrate the use of both the original design proposal as well as the ACI design method. Results from both procedures are provided to allow comparison. The examples also illustrate the incorporation and use of the modification factors presented here. It should be noted that in the typical design case of a structure containing steel reinforcement and not requiring special crack control procedures, all modification factors are 1.0. Therefore the equations simplify back to the basic equations presented by Eq. (6) for the original proposal and Eq. (7) for the ACI method. Therefore, modifications are required to be considered only in special instances.

**Example 1**

In the first example (Figure 8), it is desired to determine the adequacy of the reinforcement layout for crack control. The beam contains uncoated steel reinforcement and is contained in a structure that does not require special crack control procedures.

**Original Design Proposal**

As the structure contains uncoated steel and does not require special crack control procedures, no modifications are required (\( \gamma_c, \gamma_w, \) and \( \gamma_E = 1.0 \)).

\[
\begin{align*}
    f_s &= 0.6 f_y = 0.6(60 \text{ ksi}) = 36 \text{ ksi} \\
    \alpha_s &= \frac{36}{f_s} \gamma_c \gamma_w \gamma_E = \frac{36}{36} (1)(1)(1) = 1 \\
    d_c &= 1.5 + 0.375 + 1.128 \frac{2}{2} = 2.44 \text{ in.} \\
    s &= 12 \alpha_s \left[ 2 - \frac{d_c}{3 \alpha_s} \right] \leq 12 \alpha_s,
\end{align*}
\]
\[ s = 12(1) \left[ 2 - \frac{2.44}{3(1)} \right] = 14.24 \text{ in.} > 12(1) = 12 \text{ in.} \quad \therefore \quad s = 12 \text{ in.} \]

Spacing as Designed:

\[ s = \left( 16 - 2(1.5 + 0.375 + \frac{1.128}{2}) \right) / 3 = 3.7 \text{ in.} < 12 \text{ in.} \quad \text{OK} \]

**ACI Design Method**

As the structure contains uncoated steel and does not require specialized crack control procedures, no modifications are required.

\[ f_r = 0.6 f_y = 0.6(60 \text{ ksi}) = 36 \text{ ksi} \]
\[ c_c = 1.5 + 0.375 = 1.875 \text{ in.} \]

\[ s = \frac{540}{f_r} - 2.5c_c \leq 12 \left( \frac{36}{f_r} \right) \]
\[ s = \frac{540}{36} - 2.5(1.875) = 10.3 \text{ in.} \leq 12 \left( \frac{36}{36} \right) = 12 \text{ in.} \quad \therefore \quad s = 10.3 \text{ in.} \]

Spacing as Designed:

\[ s = 3.7 \text{ in.} < 10.3 \text{ in.} \quad \text{OK} \]

Both methods indicate that the spacing provided is adequate for crack control. It should be noted that the original design proposal uses the cover to the center of the bar, \( d_e \), while the ACI design method uses the clear cover, \( c_c \). The results provided by the ACI design method are slightly more conservative.

The beam in this example contains two layers of reinforcement. However, as noted in the calculations, only the bottom layer of reinforcement is considered through either the parameter \( d_e \) or \( c_c \). Only the bottom layer of reinforcement influences the crack width at the bottom face because this reinforcement is located closest to the surface. Also, in this example, all bars considered are of the same size. The procedure is the same if bar sizes are mixed. For the original design method, the cover to the center of the bar should conservatively consider the largest bar diameter. For the ACI design method, mixed bar sizes do not affect the results as only the clear cover is considered.
Example 2

The second example is provided for the design of a slab (Figure 9). The slab contains uncoated reinforcement spaced at 6 in. on-center. For this structure, it is determined that special crack control procedures are required and the crack width should be limited to approximately 0.006 in. at the slab bottom face.

**Original Design Proposal**

As the structure contains uncoated steel, $\gamma_c$ and $\gamma_E = 1.0$. However, a modification factor, $\gamma_w$, is required to account for the increased level of crack control.

\[
\begin{align*}
  f_s &= 0.6 f_y = 0.6(60 \text{ ksi}) = 36 \text{ ksi} \\
  \gamma_w &= \frac{w}{0.016 \text{ in.}} = \frac{0.006}{0.016} = 0.375 \\
  \alpha_s &= \frac{36}{f_s} \gamma_c \gamma_w \gamma_E = \frac{36}{36} (1)(0.375)(1) = 0.375 \\
  d_c &= 0.75 + \frac{0.5}{2} = 1.0 \text{ in.} \\
  s &= 12\alpha_s \left[ 2 - \frac{d_c}{3\alpha_s} \right] \leq 12\alpha_s \\
  s &= 12(0.375) \left[ 2 - \frac{1.0}{3(0.375)} \right] = 5.0 \text{ in.} > 12(0.375) = 4.5 \text{ in.} \quad \therefore \quad s = 4.5 \text{ in.}
\end{align*}
\]

Spacing as Designed:

\[
s = 6 \text{ in.} > 4.5 \text{ in.} \quad \text{NG}
\]

The design spacing is too large to provide the desired level of crack control at a service stress of 36 ksi. Therefore, either the spacing needs to be decreased to 4.5 in. or the service load stress should be reduced. A service stress of 27 ksi will be checked.

\[
\begin{align*}
  \alpha_s &= \frac{36}{f_s} \gamma_c \gamma_w \gamma_E = \frac{36}{27} (1)(0.375) = 0.5 \\
  s &= 12(0.5) \left[ 2 - \frac{1.0}{3(0.5)} \right] = 8 \text{ in.} > 12(0.5) = 6 \text{ in.} \quad \therefore \quad s = 6 \text{ in.}
\end{align*}
\]
Spacing as Designed:

\[ s = 6 \text{ in.} = 6 \text{ in.} \quad OK \]

As noted, a service load stress of 27 ksi can be used to provide crack control to approximately 0.006 in. with a center-to-center bar spacing of 6 in.

**ACI Design Method**

The ACI design method is based on a structure containing uncoated steel reinforcement; therefore, modifications are not required in this regard. However, a modification is required to account for the increased level of crack control of 0.006 in. This modification is accounted by an adjustment in the service stress.

\[
\begin{align*}
    f_s &= 0.6f_y = 0.6(60 \text{ ksi}) = 36 \text{ ksi} \\
    \gamma_{w_i} &= \frac{w_i}{0.016 \text{ in.}} = \frac{0.006}{0.016} = 0.375 \\
    f_s &= 36 \left( \frac{1}{\gamma_{w_i}} \right) = 36 \left( \frac{1}{0.375} \right) = 96 \text{ ksi} \\
    c &= 0.75 \text{ in.} \\
    s &= \frac{540}{f_s} - 2.5c \leq 12 \left( \frac{36}{96} \right) \\
    s &= \frac{540}{96} - 2.5(0.75) = 3.75 \text{ in.} \leq 12 \left( \frac{36}{96} \right) = 4.5 \text{ in.} \quad \therefore \quad s = 3.75 \text{ in.}
\end{align*}
\]

Spacing as Designed:

\[ s = 6 \text{ in.} > 3.75 \text{ in.} \quad NG \]

According to the modified ACI Method, the reinforcement is also too large. This spacing must be reduced or the service stress decreased. It can be shown that a maximum service stress of 25.7 ksi is required to maintain a bar spacing of 6 in. It must be noted that by accounting for the increased level of crack control, the stress, \( f_s \), is computed to be greater than the yield stress of the reinforcement. This is an artificial value of the stress that is used to account for the smaller crack width and does not represent a real value.
Results Comparison

These examples illustrate the simplicity that is provided by both design methods. In fact, the procedures are practically identical. The original proposal was designed to accommodate modification factors through the variable, $\alpha$. Therefore, this design method easily accommodates the modifications presented here. However, the ACI design method can also be easily modified to accommodate other design requirements. As noted in both examples, the ACI method is slightly more conservative than the original design proposal. For structures not requiring special crack control procedures, the end result of both methods is typically the same. Bar spacings commonly provided in beams will typically be spaced less than the computed maximum. For slabs with typical covers (0.75 in. clear), the original design proposal and the ACI method produce the same result ($s \leq 12$ in.).

CONCLUSIONS

The current ACI design method for the control of flexural cracking was designed and intended for structures containing steel reinforcement not requiring special crack control procedures. This paper presents a review of the background for this design method and develops methods for the control of cracking in specialized structures. Design methods were developed based on the original design proposal that resulted in the current ACI design method. The control of cracking is addressed for structures requiring specified crack width limits as well as for structures incorporating alternative reinforcement materials.

Design Recommendation

The following design recommendation is presented to extend the applicability of the current ACI design method (Eq. (7)):

For structures requiring specialized crack control procedures and/or containing alternative reinforcement, the calculated stress in reinforcement at service loads, $f_s$, shall be multiplied by $1/\gamma$.

where:

$$\gamma = \gamma_w \gamma_e$$
$\gamma = \text{reinforcement modification factor}$

$\gamma_w = \text{crack width factor} = \frac{w_c}{0.016 \text{ in.}}$

$\gamma_E = \text{modulus of elasticity factor} = \frac{E}{E_s}$

$w_c = \text{desired crack width limit, in.}$

$E = \text{modulus of elasticity of reinforcement, ksi}$

$E_s = \text{modulus of elasticity of steel, 29,000 ksi}$

CONVERSION FACTORS

1 in. = 54.4 mm
1 kip = 4.448 kN
1 ksi = 6.895 Mpa

REFERENCES

1. Gergely, P. “Role of Cover and Bar Spacing in Reinforced Concrete,” Significant Developments in Engineering Practice and Research: A Tribute to Chester P. Siess. SP-75, American Concrete Institute, Detroit, 1981, pp. 133-147.


4. Frosch, R.J., “Another Look at Cracking and Crack Control in Reinforced Concrete,” ACI Structural Journal, V. 96, No.3, May-June 1999, pp. 437-442.

5. ACI Committee 318, Building Code Requirements for Structural Concrete (ACI 318-99) and Commentary (ACI 318R-99), American Concrete Institute, Farmington Hills, MI, 1999.

7. ACI Committee 318, Building Code Requirements for Structural Concrete (ACI 318-95) and Commentary (ACI 318R-95), American Concrete Institute, Detroit, 1995.

8. ACI Committee 224, "Control of Cracking in Concrete Structures," (ACI 224R-90), American Concrete Institute, Detroit, 1990, 22 pp. Also ACI Manual of Concrete Practice, Part 3.

Figure 1: Crack Width Model

Figure 2: Strain Profile
Figure 3: Critical Crack Spacing Distance

Figure 4: Design Approach (Grade 60 Steel)
Design and Construction Practices to Mitigate Cracking

Figure 5: Varying Crack Width (Grade 60 Steel)

Figure 6: Varying Reinforcement Stress
Figure 7: Design Curves (Grade 60 Steel, $f_s = 36$ ksi)
Figure 8: Example 1

Figure 9: Example 2
Crack Mitigation Effects of Shrinkage Reducing Admixtures

by A. Bentur, N. S. Berke, M. P. Dallaire, and T. A. Durning

Synopsis: Shrinkage reducing admixtures (SRA’s) are a new type of admixtures which is effective in reducing the drying shrinkage of concrete. SRA performance has typically been evaluated on the basis of unrestrained drying shrinkage tests. However, it is usually the cracking performance of concrete when shrinkage is restrained that is of primary interest to the marketplace. The current paper presents an evaluation of SRA’s based on several parameters: free shrinkage, tensile stresses which develop in a uniaxially restrained rig, and the sensitivity to cracking in such conditions. The positive influence of SRA’s on all of these three parameters is demonstrated. A comparison is made between the effect of SRA and of low-volume, polypropylene fiber reinforcement. The latter is known to be effective in controlling early age plastic shrinkage cracking. The present data show that in the case of hardened concrete, after one day of curing, low volumes of fibers do not give any advantage, and it is in this range where the SRA is effective. Thus, the two types of additives can complement each other: the fibers are efficient in controlling plastic shrinkage cracking while the SRA can take over the role of crack control in the hardened concrete, where low volume-low modulus fibers are not effective.

Keywords: cracking; creep; drying shrinkage; fibers; restrained shrinkage; shrinkage reducing admixture
Arnon Bentur, Fellow of ACI, is Professor of Civil Engineering, and Director of the S. Neaman Institute for Advanced Studies in Science and Technology at Technion, Israel Institute of Technology. His areas of research are concrete technology, high performance concretes and cement composites. He co-authored several books and is co-editor of the Modern Concrete Technology Book Series, and serves on the editorial board of several international journals. He is the chair of the RILEM technical committee on Early Age Shrinkage Stresses and Cracking of Cementitious Materials.

Neal S. Berke is a Principal Scientist at Grace Construction Products. He is responsible for all durability research including calcium nitrite, microsilica, shrinkage reducing, and fiber products. Dr. Berke is a member of ACI, ASTM, NACE International, TRB, and several other technical organizations. He received his Ph.D. in Metallurgical Engineering from the University of Illinois at Urbana-Champaign. He has authored over 70 technical papers.

ACI member Michael P. Dallaire, P.E. is a Concrete Materials Engineer with AAT onsite at Turner-Fairbank Highway Research Center, and oversees concrete laboratory operations. He is a former Research Engineering Associate at Grace Construction Products. He has a Masters of Civil Engineering from the University of New Hampshire. His interests include the durability of concrete with specific interests in cracking and shrinkage.

Timothy A. Durning, P.E., member of ACI, is an International Product Group Manager in the Concrete Products Group of W.R. Grace. He is responsible for all Grace durability admixtures, including DCI® Corrosion Inhibitor, Eclipse® Shrinkage Reducing Admixture, and the new Grace Structural Fibers™. He has a Bachelors degree in Civil and Urban Engineering from the University of Pennsylvania, and a MBA from Boston University.

INTRODUCTION

Shrinkage of concrete is an inherent characteristic and is addressed in the design of the concrete mixture and structure to minimize cracking. The detrimental influence of shrinkage cracking is receiving greater attention in recent years since there is greater awareness of the long-term performance of structures, which are affected by the occurrence of cracks.
Furthermore, the shift towards low water-to-binder ratio (w/b) concretes, which is more sensitive to shrinkage cracking, contributes to the increased awareness of drying shrinkage cracking.

Usually, the shrinkage cracking performance of concretes is evaluated indirectly by determining their shrinkage, with the implied assumption that higher shrinkage will correlate with greater cracking tendency. This assumption may be valid for conventional concrete mix designs, but may not hold true for concretes in which special means were taken in its mix composition to reduce cracking, such as the addition of fibers. Therefore, the more relevant way to assess the performance of means taken to reduce shrinkage cracking is by restrained shrinkage tests in which cracking can be induced. A comprehensive evaluation of shrinkage cracking performance requires the characterization of shrinkage and shrinkage stresses which are induced in the restrained conditions and the cracking which may result.

Various tests have been developed to characterize restrained shrinkage cracking, such as ring tests (1-5), plate tests (6,7) and longitudinal tests (8-15). The latter are particularly effective since they facilitate the determination of stresses which develop (in addition to the observations of cracks), and the ones which are fully instrumented can determine the viscoelastic properties of the concrete which may play an important role especially at early ages (16-19). Several systems of this kind have been used to assess the performance of concrete, studying characteristics such as the influence of low w/b ratio compositions and shrinkage reducing admixtures (9,13,16).

This paper presents an evaluation of the effectiveness of various means, which can be applied to control shrinkage and cracking by modifying the composition of the concrete. A comprehensive approach is taken here, based on testing to determine the stresses which develop under restrained conditions, in addition to evaluations of free shrinkage and the occurrence of cracks. The parameters evaluated in this study are shrinkage reducing admixtures (SRA’s), addition of fibers and reduction in water-to-cement ratio (w/c), by use of superplasticizers.

The results in this study are compared to results of large scale restrained shrinkage tests representing conditions closer to what would be observed in the field.
EXPERIMENTAL

Concrete Composition

Two test series were carried out.

Series I was intended to evaluate the effect of SRA admixtures in concretes with a cement content of 340 kg/m$^3$ (658 lb/yd$^3$) and w/c of about 0.40. The combination of SRA and superplasticizer was also studied in this series. In one set of tests (Set 1) a control concrete with 0.40 w/c was prepared and compared with the performance of a concrete with similar workability containing 2% of SRA (actives SRA per mass of cement), labeled SRA-A (t-butyl alcohol). The improved workability, which accompanied the use of this admixture, resulted in concrete with a lower water content and a w/c of 0.36. Evaluation of the effect of fibers was also included, where a mix with 2.4 kg/m$^3$ (4lb/yd$^3$) of fiber-C was evaluated.

In the second set of tests (Set 2) the w/c of the reference concrete was 0.425. It was compared with a mix having the same w/c and 2% of SRA-B (a commercially available glycol ether blend).

Series II was carried out to evaluate the effect of adding a low-volume, polymer fiber in concrete with 258 kg/m$^3$ (500 lbs/yd$^3$) cement and 0.57 w/c. The fibers added were 0.6 kg/m$^3$ (1lb/yd$^3$) of fiber labeled fiber-A and 0.9 kg/m$^3$ (1.5lb/yd$^3$) of fiber-B. These concretes did not contain SRA’s.

Tests

Specimens for compressive and tensile strength tests, free shrinkage and restrained shrinkage were prepared. The specimens for shrinkage and restrained shrinkage were demolded after one day of sealed curing, and thereafter exposed to drying conditions of 23°C (73°F) and 50% RH. The free shrinkage specimens were usually beams of 150 mm x 150mm x 600 mm (6 in. x 6 in. x 24 in.) and in some cases comparison was made with smaller beams of 75 mm x 75 mm x 280 mm (3 in. x 3 in, x 11 in.) tested according ASTM C 157. The larger specimens were measured for volume change by the use of a Whittemore gage and cast-in-place gage inserts. The restrained shrinkage test, which also commenced at one day, was carried out in special rigs (Fig. 1).

The stress development curves in the restrained shrinkage test were obtained, and two parameters were used to characterize the behavior of the concrete: time to cracking and the stress at the time of cracking. These were compared with the free shrinkage curves and the free shrinkage at the time of cracking.
RESULTS

Series I

The compressive strength values for the first set of concretes in Series I were 52 MPa (7380 psi) for the reference with 0.40 w/c and 45.5 MPa (6465 psi) for the concrete with SRA-A (0.36 w/c). The free shrinkage curves and restrained shrinkage curves are presented in Figs. 2 and 3. The positive influence of the SRA is clearly evident. It is interesting to note the significantly lower shrinkage rates at earlier times with the SRA concrete as seen by the lower slope in the curves.

Table 1 shows the effects of the different combinations of the admixtures on the two sets of tests in this series.

Series II

The effect of the presence of low volumes of fibers is presented in Table 2. It can be clearly seen that their presence did not reduce the cracking sensitivity. This is consistent with the restraining stress curves in Fig. 3, showing that a higher fiber addition rate of 2.4 kg/m$^3$ (4 lb./yd$^3$) had a relatively small benefit. This is not surprising since the volume fraction of fibers is still low, relative to what is typically used for enhancing structural properties.

DISCUSSION

The presence of SRA clearly shows its effectiveness in reducing shrinkage as well as shrinkage cracking. The latter can be readily related to the slower build up of tensile stresses (Fig. 3).

It should also be noted that within each series, the cracking stress is similar, regardless of the admixture used. The value of this stress is relatively low compared to the expected tensile strength at the age of cracking (Table 1, Set 1). The stress at cracking was in the range of 50 to 60% of the splitting tensile strength. This indicates that the onset of cracking can not be predicted by simple stress analysis and that other influences have to be considered, in particular the viscoelastic response of the concrete. Stress relaxation have been shown to be a factor mitigating stress development and this may account for the fact that there is not always a direct relation between the reduction in shrinkage and the reduced cracking sensitivity (16). Brooks and Jiang (13) showed that the presence of an SRA had reduced the creep of the concrete, but time to cracking was still increased.
The low-volume, polymer fiber did not have any positive influence on reduction in cracking sensitivity. This is not surprising since the cracking evaluations were carried out for hardened concrete, whereas the influence of low-volume, polymer fibers is primarily in the plastic stage, to control plastic shrinkage cracking. The data here can be readily explained on the basis of reports showing that in hardened concrete the low volume polymer reinforcement has hardly any effect on shrinkage (20), and its content is too low to provide a significant reinforcing effect. Thus, the restraining stress curves for the control and reinforced concrete in Fig. 3 are practically identical.

The test conditions in this study provided 100% restraint, a condition that is unlikely to exist for most field concrete. Most field concrete is restrained from shrinkage by its supporting elements. The relative restraint can be high in many structures, but rarely is 100%. One way to characterize the interaction between the restraining element and the shrinking element is as two springs in parallel. The stiffness of the restraining member is designated as $K_r$, and the stiffness of the shrinking element is designated as $K_s$.

As the shrinking element tries to shrink it is partially restrained. The actual deformation that takes place will be determined by the relative stiffness of $K_r$ and $K_s$. If $K_r = K_s$, the actual deformation will be one half of the expected shrinkage. This is a situation where the level of restraint would be described as 50%. If $K_r = 2K_s$ ($K_s = 2$, $K_r = 1$), then the actual deformation is one third of the desired shrinkage. This represents 67% restraint.

$$\% \text{ restraint} = \left[1 - \frac{K_r}{(K_r + K_s)}\right] \times 100 \quad \text{or} \quad \left[\frac{K_s}{(K_s + K_r)}\right] \times 100$$

What is a representative amount of restraint encountered in actual structures? Although this can vary widely, one example can be drawn from reference 21, the ACI detailing manual for a typical bridge deck supported on rolled steel sections, shown on drawings H-4 and H-4A, B, and C on pages 152-159. Here the steel girders as well as the longitudinal reinforcing steel in the deck represent the restraint. On page 159, the total quantities of steel and concrete are listed for this 296-foot long (4 span) bridge. (Note that foot-pound units are being used for this discussion to be consistent with reference 21.) The total structural steel is 237,000 lbs. The total deck reinforcing steel is 102,000 lbs, of which approximately 40%, or 40,000 lbs, is longitudinal steel. The total quantity of deck concrete is 330 cy (excludes concrete in the edge barriers) of 4500 psi specified design strength concrete. Based on these quantities, the axial stiffness of both the concrete deck and the restraining steel can be determined using $K = E \times A / L$ (where $E=$ Young’s modulus, $A =$ cross sectional area, and $L =$ length; $E$ for the concrete is determined using ACI 318-89, section 8.5.1). This calculation shows that the restraint for this bridge deck is 33%.
For longer span bridges, with more massive girders, the relative restraint increases, but this example clearly shows that structures that are known to crack at least partially due to restrained shrinkage are far from 100% restrained.

To understand the impact of SRA on cracking of concrete with less than 100% restraint, two previously reported studies are referenced (15,22). Figure 4 shows the experimental setup for the large scale restrained shrinkage tests from reference 15. In this study the restraining member had compressive strength of roughly 70 MPa, and was several months old before test specimens were cast, so it had undergone much of its shrinkage. The strengths of the reference and SRA-B containing shrinkage specimens were 55 MPa and 48 MPa, respectively, as reported in Table 3. The actual modulus was not measured, so ACI guidance that the concrete modulus is proportional to the square root of the compressive strength will be used (ACI 363-R). The cross sectional area of the restraining member was 1.2m*0.2 m = 0.24m². The area for the shrinkage specimen was 0.1m*0.9 m =0.09m², so the area of the restraining member was 0.24/0.09=2.67 times that of the shrinkage specimen. The % restraint can then be determined:

% restraint, reference concrete = (\sqrt{70}*2.667)/( \sqrt{70}*2.667+\sqrt{55}*1) = 75%

% restraint, SRA concrete = (\sqrt{70}*2.667)/( \sqrt{70}*2.667+\sqrt{48}*1) = 76%

With this high level of restraint, a high dosage of the SRA (2.2% by wt of cement) was sufficient to keep the concrete from cracking (now in place for over 5 years), whereas the reference slab cracked at 64 days. The crack on the reference slab was recently measured at 3 mm in width.

Reference 22 reports the results of a smaller study using the ring test. In this study, the concrete was 50 mm in thickness, 150 mm in height. The steel restraining ring was 25 mm in thickness, and also 150 mm in height. In this case, just considering the portland cement mixes both with and without SRA-B, the restraint conditions based on 28 day properties were (reference concrete = 64MPa= 9300psi, SRA concrete= 59 MPa = 8500 psi. Area of concrete =7484 mm²= 11.6 in², Area of steel =3742 mm²= 5.8 in²):

% restraint, reference concrete =((29,000,000*5.8)/(29,000,000*5.8+57,000* \sqrt{9300}*11.62) = 72%
% restraint, SRA concrete =((29,000,000*5.8)/(29,000,000*5.8+57,000* \sqrt{8500}*11.62) = 73%
With this level of restraint, the average time to cracking (average of two specimens) for the reference was 44 days, whereas neither of the SRA specimens had cracked when the test was suspended at 120 days. Recent work by J. Weiss gave similar results and indicated that creep can play a significant role in reducing stresses (23).

CONCLUSIONS

1. The addition of SRA leads to a marked reduction in drying shrinkage, especially at early ages. This results in a slower rate of development of restraining stress and significantly prolonged time to cracking for concrete that is 100% restrained.

2. The results in this study suggest that low volume polypropylene fibers and shrinkage reducing admixtures may be complementing each other: the first is an effective means for controlling plastic shrinkage cracking whereas the second is efficient in mitigating shrinkage cracking in hardened concrete, a range in which the low-volume, low-modulus fibers are not effective.

3. Larger scale and laboratory experiments indicate that at less than 100% restraint (which is more typical of actual construction) use of a SRA can result in the elimination of cracking due to restrained drying shrinkage.

4. Further work is needed to better understand the role of creep, restraint and shrinkage interaction on cracking performance.

REFERENCES


Table 1: Results of restrained shrinkage tests of series I Cement Factor = 340 kg/m³

<table>
<thead>
<tr>
<th>mix set of series I</th>
<th>Mix type</th>
<th>Time to failure, hrs.</th>
<th>Tensile stress at failure, MPa (psi)</th>
<th>Splitting tensile strength at failure, MPa (psi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Set 1</td>
<td>Reference (w/c=0.40)</td>
<td>163</td>
<td>2.11 (300)</td>
<td>3.42 (485)</td>
</tr>
<tr>
<td>+ SRA-A (w/c=0.36)</td>
<td>670</td>
<td>1.88 (267)</td>
<td>3.92 (556)</td>
<td></td>
</tr>
<tr>
<td>2.4 kg/m³ Fiber C</td>
<td>233</td>
<td>1.61 (233)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(w/c=0.4)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Set 2</td>
<td>Reference (w/c=0.425)</td>
<td>332</td>
<td>1.37 (194)</td>
<td></td>
</tr>
<tr>
<td>+ SRA-B (w/c=0.425)</td>
<td>672</td>
<td>1.06 (151)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 2: Effect of addition of low volume of polymer fibers on cracking performance compositions tested in series III. Cement Factor = 258 kg/m³, w/c = 0.57

<table>
<thead>
<tr>
<th>Mix type</th>
<th>Time to failure, hrs.</th>
<th>Tensile stress at failure, MPa (psi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reference</td>
<td>286</td>
<td>1.64 (233)</td>
</tr>
<tr>
<td>0.6 kg/m³ Fiber A</td>
<td>285</td>
<td>1.56 (222)</td>
</tr>
<tr>
<td>0.9 kg/m3 Fiber B</td>
<td>310</td>
<td>1.70 (242)</td>
</tr>
</tbody>
</table>
Table 3  Restrained shrinkage slab - plastic and hardened concrete data (ref. 15)

<table>
<thead>
<tr>
<th>Admixtures</th>
<th>Cement Content (kg/m³)</th>
<th>w/c ratio</th>
<th>Air (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reference 195 ml/100kg ASTM TYPE B Retarder</td>
<td>396</td>
<td>0.47</td>
<td>2.0</td>
</tr>
<tr>
<td>2.2% s/s by cement SRA-B 195 ml/100kg ASTM TYPE B Retarder</td>
<td>393</td>
<td>0.47</td>
<td>1.7</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>SRA by Cement (s/s)</th>
<th>SRA by Mix Water (s/s)</th>
<th>28 Days f_c (MPa)</th>
<th>Time to Through Crack (Days)</th>
<th>% Drying Shrinkage (64 Days)</th>
<th>% Drying Shrinkage (250 Days)</th>
<th>% Drying Shrinkage (500 Days)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reference 195 ml/100kg ASTM TYPE B Retarder</td>
<td>NA</td>
<td>NA</td>
<td>54.8</td>
<td>64</td>
<td>0.061</td>
<td>0.091</td>
</tr>
<tr>
<td>2.2% s/s by cement SRA 195 ml/100kg ASTM TYPE B Retarder</td>
<td>2.2</td>
<td>4.7</td>
<td>48.2</td>
<td>&gt;2000 Has not cracked</td>
<td>0.031</td>
<td>0.056</td>
</tr>
</tbody>
</table>
Figure 1: Schematic description of the restrained shrinkage test.
Figure 2: Free shrinkage of specimens of Series I, Set 1.
Figure 3: Development of restraining stress in concretes of series I set 1, including a curve of the control mix with 2.4\,kg/m^3 (4\,lb/yd^3) of polymer fiber.
Figure 4: Schematic of large restrained slabs used in evaluation of SRA in reference 15.
Use of Fibers for Plastic Shrinkage Crack Reduction in Concrete

by P. Balaguru

Synopsis:

Use of discrete fibers to reduce plastic shrinkage cracking of concrete is discussed. The results presented cover a wide range of fibers in terms of their material properties such as modulus of elasticity, diameter, lengths, and surface characteristics. Fiber contents used ranges from 0.45 kg/m³ to 60 kg/m³ and the matrix composition evaluated vary from mortar to concrete with normal and low density aggregates. The influence of fiber properties, fiber geometries, volume fractions, and matrix compositions were evaluated for the crack reduction of concrete during the initial and final setting period. These cracks eventually influence the long-term durability of concrete. The results indicate that fibers provide a definite contribution to crack reduction and the major parameters that influence the crack reduction are: fiber count, geometry of the fiber, modulus of elasticity of the fiber, and fiber volume fraction. The fiber volume fraction needed for effective crack reduction ranges from 0.1 to 5 percent.

Keywords: concrete; cracking; lightweight concrete; metallic fibers; mineral fibers; plastic shrinkage; polymeric fibers
ACI Fellow P. Balaguru is a Professor of Civil Engineering at Rutgers, The State University of New Jersey. He is the chairman of ACI Committee 215 Fatigue of Concrete. He served as Chairman of Committee 549 Ferrocement and Other Thin Sheet Products and Secretary of ACI Committee 214, Evaluation of Results of Tests Used to Determine the Strength of Concrete, and is a member of ACI Committees 544, Fiber Reinforced Concrete; and 440, FRP Bar and Tendon Reinforcement. Dr. Balaguru is a co-author of the book, "Fiber Reinforced Cement Composites." In addition, he has authored more than 200 publications on concrete structures and construction management. His research interests are reinforced and prestressed concrete and the development of new construction materials. He served as a board-of-director for the New Jersey ACI for six years.

INTRODUCTION

Fibers have been used for thousands of years to enhance the properties of brittle matrices. The modern use of fibers started in the early 1960's. In the beginning, straight steel fibers were used to improve toughness and fatigue resistance of plain concrete. Deformed fibers introduced in the 1970's improved the performance by an order of magnitude in the areas of workability and toughness. The fiber volume fraction needed for the desired mechanical properties dropped to about 0.4 volume percent. The reduced fiber volume was extremely helpful for the mixing, placing and finishing operations.

Polymeric fibers, also called synthetic fibers, came into use in the 1980's. These fibers had a very low elastic modulus as compared to steel fibers and their diameters were in the order of 20 micro meters. The small diameter in combination with low density of about 0.9 g/cm$^3$ resulted in a large fiber count for a given fiber volume. This aspect was utilized to incorporate a small volume fraction of about 0.1 percent to reduce the plastic shrinkage cracking which occurs during the initial and final setting period. The cracking can be expected to be completed in about 10 hours after casting. The fibers not only reduce the total crack area but also reduce the maximum crack width. This reduction in crack area was expected to improve the overall durability by reducing ingress of water resulting in freezing damage and ingress of chemicals. Since the volume fraction fibers is small and the modulus of elasticity is low, only limited improvement in mechanical properties can be expected.

Larger diameter polymeric fibers introduced in the 1990's could be used at larger volume fractions, thus improving the mechanical properties
such as toughness. Even though these fibers have lower modulus as compared to steel fibers, they provide improvements comparable to steel fibers.

In addition to the popular aforementioned fibers, specialized fibers are available in the market. These include: (i) non-corroding flexible steel fibers, (ii) polymeric fibers that are very short (pulp), (iii) cellulose fibers, (iv) polymeric fibers with very low modulus such as Polyurathene urea fibers, (v) asphalt fibers, (vi) glass fibers, and (vii) basalt fibers.

Almost all the fibers available in the market have been evaluated at the Rutgers University over the period of twenty years. Performance of flexible metallic fibers, and polymeric fibers with low modulus, are presented in this paper. The results are also compared with information available in the published literature [1-3]. In addition, this paper also provides a summary of the findings and the major variables that influence the crack area reduction.

**PRIMARY VARIABLES**

The primary variables that influence the plastic shrinkage and the resulting plastic shrinkage cracking that occur due to restraints are: cement content, water-cement ratio, and the surface drying conditions. Rich mixes with larger cement contents and concrete exposed to hot-dry conditions tend to crack more. Wind velocity and the protection of exposed surface also plays a major role.

In the case of fibers, the major factors are: (i) volume fraction, (ii) fiber count for a given volume fraction, (iii) modulus of elasticity, (iv) length/diameter (aspect) ratio, (v) cross section geometry (rectangular vs. circular), (vi) variation in diameters and lengths, (vii) deformations along the length such as hooks at the ends, (viii) fibrillation of fibers, and (ix) surface characteristics. The matrix composition also plays an important role. For example, short thin fibers (pulp) perform well in rich cement mortars whereas long, larger diameter fibers perform better in concrete with coarse aggregates.

Evaluating the fiber contribution to crack reduction is a real challenge. The test methods used consist of evaluating slabs made of cement rich mixes and fibers, subjected to rapid drying conditions. This procedure, suitable for comparing different fibers, should be evaluated against actual field applications, but it is difficult and expensive to cast large slabs. Therefore, large scale tests have been conducted only once or twice. These limited tests do support the results obtained using smaller slabs [1].
Information on asphalt, polyurethene, and flexible metallic fibers, obtained in the current investigation and published results for other fibers are discussed in this paper. The test variables are presented in the next section, under experimental program.

EXPERIMENTAL PROGRAM

The variables investigated can be broadly grouped as: (i) matrix composition, (ii) fiber type, (iii) fiber volume fraction, and (iv) size of test slabs and drying conditions. In the area of matrix composition the mixture proportion varied from rich cement mortar mix to concrete with coarse aggregates. Limited number of tests were also conducted using lightweight aggregates. Water-cement ratio was always kept high to induce cracking in the control slabs.

The fiber composition consisted of: (i) asphalt, (ii) cellulose, (iii) glass, (iv) nylon, (v) polyethylene, (vi) polyolephene, (vii) polypropylene, (viii) polyurethene, (ix) polyester, (x) steel, and (xi) corrosion resistant steel. The fiber cross sections were either circular or rectangular. Most of the fibers were made of single filaments. Some of the polypropylene fibers were fibrillated. Equivalent fiber diameters ranged from 3 micrometers to 0.8 mm. Fiber lengths varied from fraction of a mm to 60 mm. The fiber contents ranged from 0.45 to 60 kg/m³.

Most of the tests were conducted using 600 x 900 x 19 mm slabs. The dimensions for other geometries were: 900 x 900 x 19 mm, 600 x 900 x 50 mm, and 560 x 356 x 100 mm. Different types of end or perimeter restraints were provided for each case. Details can be found in Reference 2. The test variables are summarized in Table 1.

Test Method

Four specimen configurations were used in this study. The first configuration consisted of a 19 mm thick slab. The mold was constructed using a plywood base. A title board was glued to the top of the plywood to obtain a smooth non-absorbing surface. The rims were made of plexiglas. A thin polyethylene sheet was placed on top of the title board to eliminate
friction (or adhesion) between the concrete (mortar) and the tile board. A strip of 12.5 x 25 mm hardware cloth was nailed to the base along the perimeter of the specimen to provide restraint along the perimeter. The restraint minimizes the movement of the slab from the edges, thus creating a potential for developing cracks in the slab. The molds for 900 x 900 x 19 mm slabs were similar to the 600 x 900 mm slabs except for the plan dimensions.

The 50 mm thick slabs were cast using molds similar to the molds used for the 19 mm slabs, except for the perimeter restraint. The restraint consisted of a 25 x 50 mm hardware cloth placed at a depth of 22 mm.

The mold used for the 100 mm thick slab had a plan dimension of 550 x 356 mm and two restraints, provided by using 32 mm risers along the 356 mm width. A 63 mm riser was placed in the middle, along the 356 mm width, to induce a crack. This resulted in a concrete thickness of only 38 mm along the riser.

Right after casting, most of the slabs were placed on a flat surface and subjected to a wind velocity of 10 to 22 km/h using high velocity fans. The 50 mm thick plain concrete slabs designated as CON 8 and CON 9, and CON 10 were placed in a specially constructed wind tunnel which was capable of generating 51 km/h wind. The wind tunnel was needed in order to maintain the wind velocity. The higher wind velocity did not improve the cracking to a significant level and hence, the other slabs were not placed in the tunnel.

The cracks started to develop in 3 to 3 1/2 hours after casting. The mechanism for the development of cracks is a complex process. Conceptually, it can be assumed that the concrete shrinks as it hardens and develops cracks when restraining forces exceed tension capacity. The primary factors are: amount of shrinkage, type of restraint and the tensile strength of the concrete during the hardening process. In most cases, the cracking was complete in about 8 hours. The crack widths and lengths were measured after 24 hours. The longer duration was chosen to make sure that all cracks developed and stabilized. The cracks were grouped into 4 categories designated as large, medium, small, and hairline with average crack widths of 3, 2, 1, and 0.5 mm respectively. The length of the cracks were measured for each category and multiplied by the average width. Crack areas of all four categories were added to obtain the total crack area for a given slab. Since the average crack widths were used for the computation, the total area of cracking should be considered only approximate since the primary purpose of the test was the comparative evaluation of the plain and fiber reinforced matrix. The approximation did not affect the final analysis. The control slab (no fibers) crack value was taken as 100%. The crack area of the other panels was expressed as a percentage of the control.
More details, including figures of test set-up can be found in Reference 1.

Materials

The constituent materials used consisted of: ASTM Type I or Type III cement, natural sand, crushed stone or lightweight aggregate, tap water, nitrate admixture for accelerated curing, and fibers. The maximum coarse aggregate size was 9 mm for both normal and lightweight aggregates. The lightweight aggregate was made of expanded shale. An accelerating admixture was used for some tests to increase the heat of hydration thereby facilitating the development of cracks.

The steel fibers were made of low carbon steel and had hooked ends. These fibers were 30, 50, and 60 mm long with respective diameters of 0.5, 0.5 and 0.8 mm. The corrosion resistant flexible fibers were made of amorphous steel and had rectangular cross sections.

The synthetic fibers made of nylon, polypropylene, and polyester were 19 mm long except in one case in which a mix of three fiber lengths were used, Table 1. Polypropylene fibers were fibrillated and all the other fibers were in single filament form. Polypropylene fibers were also evaluated in the pulp form (micro fibers), PP 1, PP 2 and PC 1 in Table 1. PC 1 fibers had larger diameters than PP 1 and PP 2. Polyethylene and cellulose fibers were in the pulp form (micro fibers). Cellulose fibers had the highest modulus of elasticity in the non-metallic fiber group. Polyester fibers had a higher modulus than nylon, polyethylene, and polypropylene but much lower than cellulose fibers. Polyurethane fibers had mixed lengths.

The matrix consisted of either mortar with cement and sand, or concrete made with lightweight aggregate, or normal weight concrete. The cement:sand ratios for the various mix proportions were 1:1, 1:1.5, 1:2, and 1:3 (Table 1). The water/cement ratios were 0.4 and 0.5 for cement:sand ratios of 1:1 and 1:1.5. For the cement:sand ratios of 1:2 and 1:3, the water/cement ratios were 0.5 and 0.7 respectively.

Two mix proportions were used for concrete. The first mix had 312, 720, and 1080 kg/m³ of cement, sand, and coarse aggregate respectively: CON 8, CON 9, and CON 10 in Table 1. The water/cement ratio was 0.6. The second group designated as CON 12, CON 13, FM 8, and NN 7 in Table 1 had 600, 600, and 900 kg/m³ of cement, sand, and coarse aggregate respectively. The water/cement ratio was again 0.6.
The fiber content for hooked-end steel fibers was either 45 or 60 kg/m³. For non-metallic fibers, the fiber content ranged from 0.45 to 4.75 kg/m³. The mixes with 4.75 kg/m³ are not listed in Table 1 because they did not develop cracks.

RESULTS AND DISCUSSION

For the fiber volume fractions investigated, there was no problem in mixing, placing or screeding. The fibers distributed well and the mix was uniform. Examination of the hardened slabs also indicated uniform fiber distribution and hence variation in fiber distribution and matrix quality can be considered as insignificant.

The results presented in Tables 3 and 4 encompass a large number of variables. They were subdivided into groups representing very short fibers (pulp), fibers that are few mm long, metallic fibers, matrix and test slab variations. The lengths of large, medium, small and hairline cracks and their total area are presented in the tables. The total area is used for most of the comparisons. However, the maximum crack width plays an important role in long-term durability because liquids permeate through larger cracks easily.

Metallic Fibers

Results for two groups of fibers are shown in Table 3 C. Tests were also conducted for various fiber lengths and fiber volume fractions of flexible metallic fibers. Results are not presented for the sake of brevity. Both types of fibers had the same modulus of elasticity. Fiber strength does not influence the results. Flexible fibers had a much larger surface area as compared to hooked-end fibers. Typical crack area reductions are shown in Figs. 1 and 2. A careful review of the results presented in Table 3 C and the results of tests, lead to the following observations.

- A fiber content 15 kg/m³ results in crack free surface for flexible fibers. In the case of hooked-end fibers, a fiber content of at least 60 kg/m³ is needed to eliminate cracking. Surface area of fibers plays an important role in the load transfer mechanism across the cracks. This aspect seems to be magnified when the concrete is in the plastic stage. Fibers with large surface areas seem to provide a better load transfer along micro cracks, thus preventing the widening of cracks. Tests conducted using fine fibers such as steel wood confirms this hypothesis.
The amount of flexible fibers needed to eliminate cracking decreases if the surfaces of the fibers are rough. Fibers with split ends were also more effective.

- Increase in fiber content, fibers with larger aspect (length/diameter) ratio provide better crack reduction.

- Addition of fibers eliminates large cracks, even at very low volume fractions.

**Micro Fibers**

Results for the micro (polymeric) fibers are presented in Table 3 A and Fig. 3. The compositions were asphalt, cellulose, polyethylene, polypropylene, or refined polyethylene. Graphical comparison is provided for only asphalt fibers, Fig. 3.

- Micro fibers are more effective in rich cement mortars. As the aggregate contents increase, the fibers become less and less effective. Aggregate size also plays an important role. Micro fibers are not as effective in concrete with coarse aggregates as compared to mortar with fine sand. The author believes that the fibers should be able to develop a critical force to restrict the crack growth. Rich cement mortars provide sufficient adhesion in a short length as compared to typical concrete and therefore these fibers are more efficient in cement rich mixes.

- The second major parameter is the modulus of elasticity of the fibers. When the concrete is in the plastic stage, the bond strength is very low and hence if the fiber elongates at a very low force, its contribution is better utilized. It can be seen that cellulose fibers which are much stiffer than polyethylene fibers did not perform well. Micro steel fibers that are short, are also not efficient to reduce plastic shrinkage cracking.

- Asphalt fibers had a very low aspect ratio and did not perform as well as the other fibers. Even for the very fine fibers, aspect ratio plays a role in crack reduction.

**Discrete Polymeric Fibers**

Discrete polymeric fibers are the most commonly used commercial fibers for plastic shrinkage crack reduction. The fiber lengths typically vary from 5 to 40 mm. These fibers made of single circular filaments or fibrillated fibers with rectangular cross sections are very effective even at low dosages, Table 3 b.
• Fiber content as low as 0.45 kg/m\(^3\) provide crack reduction. A fiber content of 0.9 kg/m\(^3\) provide significant crack reduction for all fibers.

• Fiber count is the most important parameter. For the same fiber content by volume, finer fibers provide better results.

• Aspect ratio plays a role in the crack reduction. But longer fibers result in lower fiber count resulting in inferior performance. A critical length is needed for load transfer across the crack. This length seems to be about 19 mm.

• Modulus of elasticity of the fibers also influences the crack reduction. Lower modulus provides better results. Polyurethane urea fibers have a very low modulus even as compared to nylon or polypropylene and they are still effective, as shown in Fig. 4.
  • Fibers are also effective in light-weight concrete.

• Longer fibers provide better crack reductions in concrete with coarse aggregates as compared to mortar.

• Fibers not only reduce total crack area, but also reduce or eliminate large cracks (wider than 3 mm).

Test Conditions

Based on the various plan dimensions tested by a number of investigators, 600 x 900 mm rectangular slabs seem to provide the repeatable results. The restraints provided along the perimeter seem to be effective. Larger dimensions reduce the effectiveness of the restraint. The rectangular section also provides a two-dimensional effect.

Thin slabs (19 mm) should be used only for comparative purposes. The author recommends 50 mm thick concrete slabs with coarse aggregate for a more realistic evaluation. If the performance of mortar is needed for special applications, then thinner slabs with no coarse aggregates can be used.
Increase in wind velocity beyond 20 km/h does not seem to influence the drying process. Therefore wind tunnels with high velocity fans are not needed.

CONCLUSIONS

• Fibers are useful to reduce cracking during the initial and final setting stage of concrete. The fiber volume fractions needed are low. A fiber content of 0.9 kg/m$^3$ is sufficient for most cases.

• Micro fibers are not as effective as longer fibers in concrete. But they are effective in mortars, specially in mortars with higher cement content.

• Fibers are also effective in light-weight concrete.

• The most influential parameters are: fiber count, fiber content, modulus of elasticity and aspect ratio.

• Flexible metallic fibers are more effective than standard steel fibers.

REFERENCES


### TABLE 1A. Test Variables; Micro Fibers (pulp)

<table>
<thead>
<tr>
<th>Mix Design</th>
<th>C:S Ratio</th>
<th>W/C Content</th>
<th>Fiber Content ( \text{kg/m}^3 )</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>CON 1 1:1</td>
<td>0.5</td>
<td>4.82</td>
<td>C = cement, S = sand, W = water</td>
<td></td>
</tr>
<tr>
<td>PE 1</td>
<td>1:1</td>
<td>0.5</td>
<td>0.45</td>
<td>CE = cellulose</td>
</tr>
<tr>
<td>PE 2</td>
<td>1:1</td>
<td>0.5</td>
<td>0.90</td>
<td>PE = polyethylene</td>
</tr>
<tr>
<td>PE 3</td>
<td>1:1</td>
<td>0.5</td>
<td>1.80</td>
<td>PP = polypropylene</td>
</tr>
<tr>
<td>PP 1</td>
<td>1:1</td>
<td>0.5</td>
<td>0.90</td>
<td>PN = refined polyethylene</td>
</tr>
<tr>
<td>pulp</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>PP 2</td>
<td>1:1</td>
<td>0.5</td>
<td>1.80</td>
<td>CON = control</td>
</tr>
<tr>
<td>PC 1</td>
<td>1:1</td>
<td>0.5</td>
<td>1.80</td>
<td></td>
</tr>
<tr>
<td>CE</td>
<td>1:1</td>
<td>0.5</td>
<td>1.80</td>
<td></td>
</tr>
<tr>
<td>CON 2 1:1</td>
<td>0.4</td>
<td>0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>PE 4</td>
<td>1:1</td>
<td>0.4</td>
<td>0.90</td>
<td></td>
</tr>
<tr>
<td>PN 1</td>
<td>1:1</td>
<td>0.4</td>
<td>0.90</td>
<td></td>
</tr>
<tr>
<td>CON 3 1:1.5</td>
<td>0.4</td>
<td>0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>PE 5</td>
<td>1:1.5</td>
<td>0.4</td>
<td>0.90</td>
<td></td>
</tr>
<tr>
<td>PN 2</td>
<td>1:1.5</td>
<td>0.4</td>
<td>0.90</td>
<td></td>
</tr>
<tr>
<td>CON 4 1:2</td>
<td>0.5</td>
<td>0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>PE 6</td>
<td>1:2</td>
<td>0.5</td>
<td>0.90</td>
<td></td>
</tr>
<tr>
<td>PN 3</td>
<td>1:2</td>
<td>0.5</td>
<td>0.90</td>
<td></td>
</tr>
<tr>
<td>CON 5 1:3</td>
<td>0.70</td>
<td>0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>CON 6 1:1.5</td>
<td>0.5</td>
<td>0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>AS 1</td>
<td>1:1.5</td>
<td>0.5</td>
<td>1.8</td>
<td></td>
</tr>
<tr>
<td>AS 2</td>
<td>1:1.5</td>
<td>0.5</td>
<td>3.0</td>
<td></td>
</tr>
<tr>
<td>AS 3</td>
<td>1:1.5</td>
<td>0.5</td>
<td>6.0</td>
<td></td>
</tr>
<tr>
<td>AS 4</td>
<td>1:1.5</td>
<td>0.5</td>
<td>9.0</td>
<td></td>
</tr>
</tbody>
</table>
### TABLE 1 B. Test Variables; Discrete Polymeric Fibers

(5 to 40 mm long and < 50 micro meters in diameter)

<table>
<thead>
<tr>
<th>Mix Design</th>
<th>C:S</th>
<th>W/C</th>
<th>Fiber Content kg/m³</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>CON 8 1:1.5</td>
<td>0.5</td>
<td>0</td>
<td>0.60</td>
<td></td>
</tr>
<tr>
<td>A 11</td>
<td>1:1.5</td>
<td>0.5</td>
<td>0.90</td>
<td></td>
</tr>
<tr>
<td>A 12</td>
<td>1:1.5</td>
<td>0.5</td>
<td>1.20</td>
<td></td>
</tr>
<tr>
<td>A 13</td>
<td>1:1.5</td>
<td>0.5</td>
<td>1.50</td>
<td></td>
</tr>
<tr>
<td>A 14</td>
<td>1:1.5</td>
<td>0.5</td>
<td>1.80</td>
<td></td>
</tr>
<tr>
<td>CON 9 1:1.5</td>
<td>0.5</td>
<td>0</td>
<td>0.6</td>
<td></td>
</tr>
<tr>
<td>A 21</td>
<td>1:1.5</td>
<td>0.5</td>
<td>0.9</td>
<td></td>
</tr>
<tr>
<td>A 22</td>
<td>1:1.5</td>
<td>0.5</td>
<td>0.9</td>
<td></td>
</tr>
<tr>
<td>A 23</td>
<td>1:1.5</td>
<td>0.5</td>
<td>1.20</td>
<td></td>
</tr>
<tr>
<td>A 24</td>
<td>1:1.5</td>
<td>0.5</td>
<td>1.50</td>
<td></td>
</tr>
<tr>
<td>A 25</td>
<td>1:1.5</td>
<td>0.5</td>
<td>1.80</td>
<td></td>
</tr>
<tr>
<td>PP 3</td>
<td>1:1</td>
<td>0.5</td>
<td>0.9</td>
<td></td>
</tr>
<tr>
<td>PP 4</td>
<td>1:1</td>
<td>0.5</td>
<td>1.8</td>
<td></td>
</tr>
<tr>
<td>CON 10</td>
<td>1:1.5</td>
<td>0.5</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>PP 5</td>
<td>1:1.5</td>
<td>0.5</td>
<td>0.9</td>
<td></td>
</tr>
<tr>
<td>PP 6</td>
<td>1:1.5</td>
<td>0.5</td>
<td>0.9</td>
<td></td>
</tr>
<tr>
<td>PP 7</td>
<td>1:1.5</td>
<td>0.4</td>
<td>0.9</td>
<td></td>
</tr>
<tr>
<td>PP 8</td>
<td>1:2.0</td>
<td>0.5</td>
<td>0.9</td>
<td></td>
</tr>
<tr>
<td>PY 1</td>
<td>1:1.5</td>
<td>0.5</td>
<td>0.6</td>
<td></td>
</tr>
<tr>
<td>CON 11</td>
<td>1:1.5</td>
<td>0.5</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>PU 1</td>
<td>1:1.5</td>
<td>0.5</td>
<td>0.6</td>
<td>PU 1 to PU 2 - 9 mm long</td>
</tr>
<tr>
<td>PU 2</td>
<td>1:1.5</td>
<td>0.5</td>
<td>0.9</td>
<td>PU 4 to PU 6 - 12 mm long</td>
</tr>
<tr>
<td>PU 3</td>
<td>1:1.5</td>
<td>0.5</td>
<td>1.2</td>
<td></td>
</tr>
<tr>
<td>CON 12</td>
<td>1:1.5</td>
<td>0.5</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>PU 4</td>
<td>1:1.5</td>
<td>0.5</td>
<td>0.6</td>
<td></td>
</tr>
<tr>
<td>PU 5</td>
<td>1:1.5</td>
<td>0.5</td>
<td>0.9</td>
<td></td>
</tr>
<tr>
<td>PU 6</td>
<td>1:1.5</td>
<td>0.5</td>
<td>1.2</td>
<td></td>
</tr>
<tr>
<td>NN 1</td>
<td>1:1.5</td>
<td>0.5</td>
<td>0.45</td>
<td>NN 2 to NN 3 - 19 mm long</td>
</tr>
<tr>
<td>NN 2</td>
<td>1:1.5</td>
<td>0.5</td>
<td>0.6</td>
<td>NN 4 mix to 19, 25 and 38 mm</td>
</tr>
<tr>
<td>NN 3</td>
<td>1:1.5</td>
<td>0.5</td>
<td>0.90</td>
<td>long fibers</td>
</tr>
<tr>
<td>NN 4</td>
<td>1:1.5</td>
<td>0.5</td>
<td>0.90</td>
<td></td>
</tr>
<tr>
<td>Mix Desig.</td>
<td>C:S Ratio</td>
<td>W/C</td>
<td>Fiber Content</td>
<td>Remarks</td>
</tr>
<tr>
<td>------------</td>
<td>-----------</td>
<td>-----</td>
<td>---------------</td>
<td>--------------------------</td>
</tr>
<tr>
<td>SCON 1:1.5</td>
<td>0.5</td>
<td>0</td>
<td>45</td>
<td>SF 1, SF 4 - 30 mm long</td>
</tr>
<tr>
<td>SF 1 fibers</td>
<td>1:1.5</td>
<td>0.5</td>
<td>45</td>
<td>SF 2, SF 5 - 50 mm long</td>
</tr>
<tr>
<td>SF 2 fibers</td>
<td>1:1.5</td>
<td>0.5</td>
<td>45</td>
<td>SF 3, SF 6 - 60 mm long</td>
</tr>
<tr>
<td>SF 3 fibers</td>
<td>1:1.5</td>
<td>0.5</td>
<td>60</td>
<td>all fibers with hooked ends</td>
</tr>
<tr>
<td>SF 4</td>
<td>1:1.5</td>
<td>0.5</td>
<td>60</td>
<td></td>
</tr>
<tr>
<td>SF 5</td>
<td>1:1.5</td>
<td>0.5</td>
<td>60</td>
<td></td>
</tr>
<tr>
<td>SF 6</td>
<td>1:1.5</td>
<td>0.5</td>
<td>60</td>
<td></td>
</tr>
<tr>
<td>FM 0</td>
<td>1:1.5</td>
<td>0.5</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>FM 1 fibers</td>
<td>1:1.5</td>
<td>0.5</td>
<td>3.0</td>
<td>FM - flexible metallic</td>
</tr>
<tr>
<td>FM 2</td>
<td>1:1.5</td>
<td>0.5</td>
<td>6.0</td>
<td>These fibers are corrosion resistant</td>
</tr>
<tr>
<td>FM 3</td>
<td>1:1.5</td>
<td>0.5</td>
<td>15.0</td>
<td></td>
</tr>
<tr>
<td>FM 4</td>
<td>1:1.5</td>
<td>0.5</td>
<td>30.0</td>
<td></td>
</tr>
</tbody>
</table>
### TABLE 2. Test Variable: Various Panel Sizes

<table>
<thead>
<tr>
<th>Mix Desig.</th>
<th>Matrix</th>
<th>W/C Ratio</th>
<th>Fiber Content kg/m$^3$</th>
<th>Panel Size mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>CON 13</td>
<td>Mortar C:S=1:2</td>
<td>0.50</td>
<td>0</td>
<td>900 x 900 x 19</td>
</tr>
<tr>
<td>PE 7</td>
<td>Mortar C:S=1:2</td>
<td>0.50</td>
<td>0.90</td>
<td>900 x 900 x 19</td>
</tr>
<tr>
<td>PP 9</td>
<td>Mortar C:S=1:2</td>
<td>0.50</td>
<td>0.90</td>
<td>900 x 900 x 19</td>
</tr>
<tr>
<td>CON 14</td>
<td>Concrete</td>
<td>0.60</td>
<td>0</td>
<td>900 x 600 x 50</td>
</tr>
<tr>
<td>CON 15</td>
<td>Concrete</td>
<td>0.60</td>
<td>0</td>
<td>900 x 600 x 50</td>
</tr>
<tr>
<td>CON 16</td>
<td>Concrete</td>
<td>0.60</td>
<td>0</td>
<td>900 x 600 x 50</td>
</tr>
<tr>
<td>CON 17</td>
<td>Light wt.</td>
<td>0</td>
<td>0.45</td>
<td>900 x 600 x 19</td>
</tr>
<tr>
<td>NN 5</td>
<td>Concrete</td>
<td>0.60</td>
<td>0.60</td>
<td>900 x 600 x 19</td>
</tr>
<tr>
<td>NN 6</td>
<td>Concrete</td>
<td>0.60</td>
<td>0.90</td>
<td>900 x 600 x 50</td>
</tr>
<tr>
<td>CON 18</td>
<td>Concrete</td>
<td>0.60</td>
<td>0</td>
<td>900 x 600 x 50</td>
</tr>
<tr>
<td>NN 7</td>
<td>Concrete</td>
<td>0.60</td>
<td>0.90</td>
<td>900 x 600 x 50</td>
</tr>
<tr>
<td>CON 19</td>
<td>Concrete</td>
<td>0.60</td>
<td>0</td>
<td>550 x 356 x 100</td>
</tr>
<tr>
<td>PP 10</td>
<td>Concrete</td>
<td>0.60</td>
<td>0.90</td>
<td>550 x 356 x 100</td>
</tr>
<tr>
<td>NN 8</td>
<td>Concrete</td>
<td>0.60</td>
<td>0.90</td>
<td>550 x 356 x 100</td>
</tr>
</tbody>
</table>

Note: CON 14, 15, 16 were using Type I, II and III cements respectively.
Table 3 B. Details of Cracking, Discrete Polymeric Fibers

<table>
<thead>
<tr>
<th>Mix</th>
<th>Length of cracks, mm</th>
<th>Crack area mm² of control</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>L</td>
<td>M</td>
</tr>
<tr>
<td>Desig.</td>
<td>L</td>
<td>M</td>
</tr>
<tr>
<td>CON 8 240</td>
<td>900</td>
<td>540</td>
</tr>
<tr>
<td>A 11</td>
<td>-</td>
<td>1080</td>
</tr>
<tr>
<td>A 12</td>
<td>1020</td>
<td>615</td>
</tr>
<tr>
<td>A 13</td>
<td>90</td>
<td>900</td>
</tr>
<tr>
<td>A 14</td>
<td>105</td>
<td>840</td>
</tr>
<tr>
<td>A 15</td>
<td>60</td>
<td>780</td>
</tr>
<tr>
<td>CON 9</td>
<td>180</td>
<td>900</td>
</tr>
<tr>
<td>A 21</td>
<td>90</td>
<td>300</td>
</tr>
<tr>
<td>A 22</td>
<td>45</td>
<td>270</td>
</tr>
<tr>
<td>A 23</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>A 24</td>
<td>-</td>
<td>30</td>
</tr>
<tr>
<td>A 25</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>PP 3</td>
<td>915</td>
<td>305</td>
</tr>
<tr>
<td>PP 4</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>CON 10</td>
<td>-</td>
<td>875</td>
</tr>
<tr>
<td>PP 5</td>
<td>-</td>
<td>250</td>
</tr>
<tr>
<td>PP 6</td>
<td>-</td>
<td>350</td>
</tr>
<tr>
<td>PP 7</td>
<td>-</td>
<td>483</td>
</tr>
<tr>
<td>PP 8</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>PY 1</td>
<td>500</td>
<td>975</td>
</tr>
<tr>
<td>CON 11</td>
<td>-</td>
<td>950</td>
</tr>
<tr>
<td>PU 1</td>
<td>-</td>
<td>700</td>
</tr>
<tr>
<td>PU 2</td>
<td>-</td>
<td>360</td>
</tr>
<tr>
<td>PU 3</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

PP 3 and PP 4 compared to CON 1; PP 5, PPO 6 compared to CON 10
PP 7 and PP 8 compared to CON 3 and 4
PY 1 compared to CON 3
Table 3 A. Details of Cracking, Micro Fibers (pulp)

<table>
<thead>
<tr>
<th>Mix</th>
<th>Length of cracks, mm</th>
<th>Crack area mm²</th>
<th>Percentage of control</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>L</td>
<td>M</td>
<td>S</td>
</tr>
<tr>
<td>CON 1 1730</td>
<td>508</td>
<td>406</td>
<td>711</td>
</tr>
<tr>
<td>PE 1</td>
<td>1803</td>
<td>254</td>
<td>635</td>
</tr>
<tr>
<td>PE 2</td>
<td>460</td>
<td>152</td>
<td>610</td>
</tr>
<tr>
<td>PE 3</td>
<td>-</td>
<td>-</td>
<td>152</td>
</tr>
<tr>
<td>PP 1</td>
<td>889</td>
<td>1422</td>
<td>762</td>
</tr>
<tr>
<td>PP 2</td>
<td>483</td>
<td>533</td>
<td>330</td>
</tr>
<tr>
<td>PC 1</td>
<td>305</td>
<td>940</td>
<td>914</td>
</tr>
<tr>
<td>CE</td>
<td>1118</td>
<td>1489</td>
<td>381</td>
</tr>
<tr>
<td>CON 2 -</td>
<td>508</td>
<td>686</td>
<td>1905</td>
</tr>
<tr>
<td>PE 4</td>
<td>-</td>
<td>229</td>
<td>1016</td>
</tr>
<tr>
<td>PN 1</td>
<td>-</td>
<td>102</td>
<td>533</td>
</tr>
<tr>
<td>CON 3 -</td>
<td>787</td>
<td>1397</td>
<td>508</td>
</tr>
<tr>
<td>PE 5</td>
<td>-</td>
<td>-</td>
<td>660</td>
</tr>
<tr>
<td>PN 2</td>
<td>-</td>
<td>-</td>
<td>520</td>
</tr>
<tr>
<td>CON 4 -</td>
<td>940</td>
<td>762</td>
<td>305</td>
</tr>
<tr>
<td>PE 6</td>
<td>-</td>
<td>711</td>
<td>305</td>
</tr>
<tr>
<td>PN 3</td>
<td>-</td>
<td>508</td>
<td>229</td>
</tr>
<tr>
<td>CON 5</td>
<td>No cracks</td>
<td></td>
<td></td>
</tr>
<tr>
<td>CON 6 102</td>
<td>914</td>
<td>254</td>
<td>356</td>
</tr>
<tr>
<td>AS 1</td>
<td>51</td>
<td>610</td>
<td>381</td>
</tr>
<tr>
<td>AS 2</td>
<td>-</td>
<td>305</td>
<td>610</td>
</tr>
<tr>
<td>AS 3</td>
<td>25</td>
<td>508</td>
<td>407</td>
</tr>
<tr>
<td>AS 4</td>
<td>26</td>
<td>381</td>
<td>508</td>
</tr>
</tbody>
</table>

L = Large; M = Medium; S = Small; H = Hairline
Table 3B. Details of Cracking, Discrete Polymeric Fibers
(Continued)

<table>
<thead>
<tr>
<th></th>
<th>L</th>
<th>M</th>
<th>S</th>
<th>H</th>
</tr>
</thead>
<tbody>
<tr>
<td>CON 12</td>
<td></td>
<td>970</td>
<td>650</td>
<td>1400</td>
</tr>
<tr>
<td>PU 4</td>
<td></td>
<td>225</td>
<td>514</td>
<td>972</td>
</tr>
<tr>
<td>PU 5</td>
<td></td>
<td>210</td>
<td>470</td>
<td>790</td>
</tr>
<tr>
<td>PU 6</td>
<td></td>
<td></td>
<td>24</td>
<td>874</td>
</tr>
<tr>
<td>NN 1</td>
<td></td>
<td>125</td>
<td>1350</td>
<td>878</td>
</tr>
<tr>
<td>NN 2</td>
<td></td>
<td>100</td>
<td>1000</td>
<td>250</td>
</tr>
<tr>
<td>NN 3</td>
<td></td>
<td></td>
<td>125</td>
<td>1250</td>
</tr>
<tr>
<td>NN 4</td>
<td></td>
<td></td>
<td>675</td>
<td>1625</td>
</tr>
</tbody>
</table>

L = Large; M = Medium; S = Small; H = Hairline
NN 1, 2, 3, 4 compared to CON 10
Table 3 C. Details of Cracking, Metallic Fibers

<table>
<thead>
<tr>
<th>Mix Design</th>
<th>Crack length, mm</th>
<th>Crack Percentage of control</th>
</tr>
</thead>
<tbody>
<tr>
<td>SCON 300</td>
<td>700 500 300</td>
<td>2950 100</td>
</tr>
<tr>
<td>SF 1</td>
<td>- 550 400 200</td>
<td>1600 54</td>
</tr>
<tr>
<td>SF 2</td>
<td>- - 300 400</td>
<td>500 17</td>
</tr>
<tr>
<td>SF 3</td>
<td>- 500 400 600</td>
<td>1720 58</td>
</tr>
<tr>
<td>SF 4</td>
<td>- 100 300 300</td>
<td>650 22</td>
</tr>
<tr>
<td>SF 5</td>
<td>- - - 100</td>
<td>50 2</td>
</tr>
<tr>
<td>SF 6</td>
<td>- - 800 200</td>
<td>900 31</td>
</tr>
<tr>
<td>FM 0</td>
<td>100 300</td>
<td>1700 1000 3100 100</td>
</tr>
<tr>
<td>FM 1</td>
<td>- - 750 1100</td>
<td>1300 42</td>
</tr>
<tr>
<td>FM 2</td>
<td>- - - 1000</td>
<td>500 16</td>
</tr>
<tr>
<td>FM 3</td>
<td>No Cracks</td>
<td>0 -</td>
</tr>
<tr>
<td>FM 4</td>
<td>No Cracks</td>
<td>0 -</td>
</tr>
</tbody>
</table>

L = Large; M = Medium; S = Small; H = Hairline
Table 4. Details of Cracking, Various Panel Sizes with Concrete

<table>
<thead>
<tr>
<th>Mix Design</th>
<th>L</th>
<th>M</th>
<th>S</th>
<th>H</th>
<th>Area mm² of control</th>
<th>Percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>CON 13</td>
<td>-</td>
<td>508</td>
<td>660</td>
<td>711</td>
<td>2032</td>
<td>100</td>
</tr>
<tr>
<td>PE 7</td>
<td>-</td>
<td>-</td>
<td>660</td>
<td>813</td>
<td>1067</td>
<td>53</td>
</tr>
<tr>
<td>PP 9</td>
<td>-</td>
<td>-</td>
<td>229</td>
<td>1067</td>
<td>762</td>
<td>83</td>
</tr>
<tr>
<td>CON 17</td>
<td>250</td>
<td>150</td>
<td>100</td>
<td>1125</td>
<td>1713</td>
<td>100</td>
</tr>
<tr>
<td>NN 5</td>
<td>-</td>
<td>125</td>
<td>125</td>
<td>750</td>
<td>750</td>
<td>44</td>
</tr>
<tr>
<td>NN 6</td>
<td>-</td>
<td>125</td>
<td>125</td>
<td>250</td>
<td>500</td>
<td>29</td>
</tr>
</tbody>
</table>

L = Large; M = Medium; S = Small; H = Hairline
Mixes: CON 14, Con 15, CON 16, and CON 19 had a few hairline cracks.
Mixes: NN 7 and PP 10 did not develop any cracks.
Figure 1. Comparison of Crack Area; Steel Fibers.
Figure 2. Weighted Crack Area; Flexible Metallic Fibers.
Figure 3. Weighted Crack Area; Comparison of Asphalt Fibers.
Figure 4. Comparison of Crack Areas; Polyurea Fibers.
Mitigation of Seismic Induced Diagonal Cracks in Concrete Columns by External Prestressing

by M. Saatcioglu

Synopsis

A new technology was developed for controlling diagonal shear cracking in earthquake resistant concrete bridge columns. The technology consists of external prestressing columns in transverse direction by means of individual hoops consisting of prestressing strands and specially designed anchors. It is especially suitable for seismic retrofitting circular bridge columns against shear. The technology was verified through experimental investigation of shear deficient columns. Three full-scale columns were tested under constant axial compression and incrementally increasing lateral deformation reversals. The results, presented and discussed in the paper, indicate that the technique can control diagonal cracking, thereby suppressing shear failure and promoting flexural behavior. A design procedure is also presented for seismic retrofit of shear-deficient columns.

Keywords: bridge columns; bridges; concrete columns; crack control; cracking; earthquake engineering; prestressed concrete; prestressing; retrofitting; seismic retrofit; shear design
INTRODUCTION

Performance of reinforced concrete columns during previous earthquakes has demonstrated that diagonal tension cracks caused by seismic shear force reversals lead to premature and brittle failures. Shear damage observed in short and stubby columns was responsible for a large number of structural collapses. Figure 1 illustrates examples of typical shear damage observed in bridge columns during previous earthquakes.

The shear damage may be attributed to the deficiencies in design provisions of older codes which may have resulted in lack of sufficient transverse reinforcement. The same deficiency may be found in some of the more recently designed columns, where design flaws and or the interference of non-structural elements result in increased shear stresses that may not have been accounted for in design. Indeed, unintended supports provided by masonry walls and other non-structural elements often reduce the design shear span and create “short columns” with very high shear stress reversals.

Seismic risk caused by lack of sufficient transverse reinforcement can be mitigated through a newly developed retrofit methodology that involves external prestressing of circular and
rectangular concrete columns. The external prestressing provides control for inclined shear cracking, thereby maintaining concrete shear resistance, while also enhancing reinforcement contribution to shear resistance, resulting in significant improvements in seismic shear capacity of otherwise inadequate members. The new retrofit methodology was developed through experimental research, conducted at the University of Ottawa. The results of full-scale column tests and the improvements in mechanism of shear resistance are discussed in the following sections.

**MECHANISM OF SHEAR RESISTANCE**

One of the primary functions of transverse reinforcement in concrete columns is to provide shear resistance. Short and stubby columns attract shear stresses that may exceed the diagonal tension capacity of concrete. This results in inclined shear cracks. Shear cracks are usually well controlled by reinforcement crossing them, thereby maintaining the ability of concrete to transfer shear across the crack. The excess shear is resisted by the reinforcement, until the transverse strains perpendicular to inclined cracks increase significantly, in which case the transfer of shear across the crack diminishes quickly and ceases to exist entirely beyond a certain strain level. Shear reinforcement is usually designed by following the 45-degree truss analogy employed in the ACI design code (ACI 318, 1999). Eq. 1 gives the nominal shear strength of a reinforced concrete column.

\[
V_n = V_c + V_s
\]  

(1)

Where, \(V_c\) and \(V_s\) are components of shear resistance provided by concrete and steel, respectively. The concrete shear resistance includes contributions from uncracked and cracked concrete, where the latter is activated by aggregate interlock. As diagonal tension cracks widen due to the yielding
of insufficient transverse reinforcement, the ability of concrete to carry shear across a crack diminishes. This eventually leads to a diagonal tension failure. The deterioration of concrete becomes faster under seismic induced inelastic shear force reversals which tend to produce wider cracks crossing each other. Therefore, the shear carried by concrete may not be relied on for seismic resistance (ACI 318, 1999), leaving shear reinforcement as the only element of resistance.

Prestressing overcomes the tendency of diagonal cracks to form and widen, improving concrete shear resistance. This improvement is directly proportional to the amount of prestressing that has to be overcome before the concrete can develop diagonal tension. Figure 2 illustrates the mechanism of shear resistance in a laterally prestressed concrete column. The contribution of transverse prestressing to concrete shear resistance is shown below.

\[ V_{pc} = 2A_{ps} f_{pe} \frac{h}{s_p} \]  

(2)

The benefits realized by prestressing are not limited to the enhancement in concrete shear resistance. Prestressing strands also act as additional shear reinforcement, providing extra enhancement for column shear resistance. This is expressed in Eq. 3.

\[ V_{ps} = 2A_{ps} (f_{py} - f_{pe}) \frac{h}{s_p} \]  

(3)

Where, \( V_{ps} \) is the additional resistance provided by prestressing steel and is limited by the remaining tension capacity in the strand \( (f_{py} - f_{pe}) \). The total shear resistance provided by prestressing can then be written as;

\[ V_p = V_{pc} + V_{ps} = 2A_{ps} f_{py} \frac{h}{s_p} \]  

(4)
The nominal shear capacity of a reinforced concrete column, retrofitted by external prestressing, consists of contributions from concrete, internal shear reinforcement, and external prestressing, as indicated by Eq. 5.

\[ V_n = V_c + V_s + V_p \]  

(5)

EXPERIMENTAL PROGRAM

Properties of Column Specimens

Three full-scale reinforced concrete columns were tested under constant axial compression and incrementally increasing lateral deformation reversals. The columns had a circular section with 610 mm diameter. They were designed to represent pre-1970's design practice. One column was tested without any retrofit and represented "as built" conditions. Two others were prestressed externally to control diagonal shear cracking.

The specimens were representatives of a bridge column between the footing and the point of inflection. Shear span was 1485 mm, resulting in a shear span-to-depth ratio of 2.43. This shear-span-to-depth ratio corresponds to columns that show predominantly shear response. The columns were cast in two stages, using ready mix concrete. The footings were cast first, followed by the columns. The average cylinder strength for column concrete was determined to be 45 MPa at the time of column tests. The reinforcement arrangement consisted of 12 No. 25 (25.2 mm diameter) Grade 400 MPa longitudinal reinforcement, placed uniformly along the perimeter of the section. No. 10 (11.3 mm diameter) grade 400 MPa ties were used at 300 mm spacing, with the first tie placed at 75 mm above the footing. The ties had overlapping ends. Coupon tests were performed to establish the stress-strain characteristics of reinforcement. Figure 3 illustrates the overall geometry
of test specimens.

Size 9 seven-wire strands (9.53 mm nominal diameter and 54.8 mm² nominal area) were used for retrofitting. This size provided the required prestressing, while allowing sufficient flexibility for handling purposes. The strands were of Grade 1860 MPa, exhibiting the stress-strain relationship shown in Fig. 4. All the steel used had elastic modulus of approximately 200,000 MPa.

Retrofitting

Retrofitting was done by placing the strands around the columns as hoops, and stressing them individually. The strands were anchored using twisted ring anchors developed by Dywidag-Systems International, which had been developed for circular tanks. Therefore, the anchors were modified to fit the column sizes considered. Figure 5 shows a schematic view of the anchors used. The ends of the strands were locked in to the anchors using standard wedges. Figure 6 illustrates the hardware used in retrofitting a circular column.

Test Set-up and Instrumentation

The tests were conducted using three computer servo-controlled MTS hydraulic actuators with 1000 kN capacity. The columns were secured on the laboratory strong floor by means of 4 high-strength bolts. Two actuators were mounted vertically, one on either side of the column, to apply 1800 kN of constant axial compression during testing. This corresponded to approximately 13% of the computed concentric capacity of columns. The third actuator was mounted horizontally and supported by a reaction frame. The lateral force was applied slowly in the displacement control mode, consisting of incrementally increasing lateral drift cycles at 0.5%, 1.0%, 2.0%, etc., until the load resistance dropped by at least 50%. Figure 7 illustrates the test set-up.
Design and Construction Practices to Mitigate Cracking

The columns were well instrumented for displacement and strain measurements. Linear Variable Differential Transducers (LVDT) were used to measure displacements. Four LVDTs were attached vertically on two opposite sides, perpendicular to the direction of loading near the base to measure rotations within 300 mm and 600 mm from the footing. Additional two Temposonic LVDTs were attached to the column section near the base to measure the extension of longitudinal column reinforcement within the footing. Another Temposonic LVDT was placed horizontally at 1485 mm from the footing to measure the tip deflection. Strain gauges were installed on reinforcement as well as the prestressing strands. The latter group of gauges were particularly important since the initial prestress, as well as the incremental change in stress during testing would provide valuable information in assessing the effectiveness of external prestressing. Two data acquisition systems were used to collect data.

TEST RESULTS

The reference column was labeled as BR-C1 and was reinforced with 2% longitudinal reinforcement and 11.3 mm diameter perimeter ties at 300 mm spacing. This amount of reinforcement produced nominal shear capacity that was approximately equal to the shear force associated with flexural yielding when full concrete shear resistance was fully utilized. Therefore, the columns did not have adequate shear capacity to resist inelastic force reversals at high deformation levels, beyond a significant level of cracking.

Observations during testing indicated that the first set of flexural cracks formed during the third cycle of loading at 0.5% drift ratio. Shear cracks initiated when the column was forced to experience 1.0% drift. These cracks subsequently became wider during the second and third cycles at the same deformation level as new cracks formed and the spalling of concrete cover began near
the base. The first cycle at 2.0% drift resulted in rapid widening of previously formed diagonal cracks which caused a significant drop in load resistance. Subsequent cycles at this level of deformation resulted in the spalling of cover concrete within the bottom 1/3 of the column. The longitudinal bars started to buckle as the load resistance diminished severely. The column continued to sustain approximately one half of its maximum resistance until the buckling of longitudinal reinforcement occurred. Figure 8 depicts the hysteretic force displacement relationship recorded during testing. The relationship indicates that the column survived inelastic deformation reversals at 1% drift, but failed due to diagonal tension when forced to sustain 2% drift cycles. The failure also triggered the buckling of longitudinal bars when additional deformation cycles were imposed. Figures 9 shows the damage sustained at the end of testing.

Column BR-C2 was the first circular column that was retrofitted prior to testing. The retrofitting was done by means of 7-wire strands that were placed around the column as exterior hoops. The hoop spacing was 150 mm, starting at 75 mm from the column base. The strands were prestressed to approximately 300 MPa, producing an average uniform lateral pressure of 0.34 MPa on concrete. The column survived the initial six cycles of loading without any sign of distress. The maximum load resistance was attained at 2.0% drift while some hairline flexural cracks and concrete crushing were observed near the base. The flexural cracks became more apparent at 3% drift while the cover concrete started to spall off near the base. However, the column maintained its load resistance without any strength decay. It was clear that the behavior was no longer governed by shear, as was the case in the companion non-retrofitted column (BR-C1). The prestressing strands not only controlled the diagonal cracking and functioned as shear reinforcement, but also confined the compression concrete and improved the flexural behavior. At 4.0% drift, the maximum lateral load dropped slightly as more cover concrete spalled off and the first two prestressing strands started
to sink into the concrete surface as concrete expanded sufficiently to bring the bearing pressure underneath the strands to critical levels. No major diagonal cracks developed during the entire test, with the exception of some minor hairline cracks that formed at 4% drift. The existing flexural cracks started to widen at 5% drift, while additional flexural cracks appeared around the third prestressing hoop from the base. The lateral load resistance started to drop gradually during each cycle of 5% drift, resulting in a strength decay of about 25%. The concrete cover was completely spalled off up to the second prestressing hoop level. One of the longitudinal bars on each side buckled at 6% drift cycles, causing the strength to drop by more than 50%, at which point the test was terminated. Figure 10 illustrates the hysteretic force-displacement relationship, demonstrating the significant improvement in column deformability attained relative to that of the companion Column BR-C1 without the retrofit. Figure 11 shows the column at 4% drift, without any noticeable damage.

The second retrofitted column was labeled BR-C4, and was tested to investigate the significance of the spacing of prestressing strands. Because of the favorable performance of BR-C2, the possibility of relaxing the external hoop spacing was investigated while keeping the level of initial prestressing at approximately 300 MPa, producing an average uniform lateral pressure of 0.17 MPa on concrete. The strands for this column were positioned with 300 mm spacing such that they were placed at mid-spacing of the existing circular ties inside the column. The observed behavior during testing was identical to that of column BR-C2 until the end of 1.0% drift cycles. During the first cycle at 2.0% lateral drift, diagonal cracks started to become visible and the cover concrete near the base started to crush as flexural cracks widened. By the end of the third cycle, the bottom cover concrete had been completely spalled off and diagonal cracks had become wider, accompanied by some strength decay. During the first cycle to 3.0% drift, one of the wires of the second strand from
the bottom ruptured near the anchor due to increased diagonal tension, coupled with the stress concentration effect at the edge of the anchor hole. This resulted in some relaxation in the remaining wires of the seven-wire strand. The strength decay continued as one of the diagonal crack started to widen excessively between the second and third strands. The cover concrete continued to crush near the base as the first prestressing wire started to sink in to the laterally expanding column concrete. The test was terminated during the third cycle of 3.0% drift when all the longitudinal bars buckled and the column failed abruptly. Figure 12 shows the hysteretic relationship recorded during testing, indicating that the wider spacing adopted for this column did not produce sufficient improvement in column deformability. The column was able to sustain deformation cycles at 2% drift with some strength decay, and failed rapidly thereafter. Figure 13 illustrates the observed damage at 3% drift.

**EFFECTIVENESS OF EXTERNAL PRESTRESSING**

The effectiveness of external prestressing to mitigate diagonal shear cracks was established experimentally by examining the variation of hoop strains in prestressing steel and by comparing the hysteretic relationships presented in the previous section. Figure 14 illustrates the strain profiles recorded in prestressing steel along the height of each column, at selected drift levels. The initial strains in strands were either computed from the initial stress values recorded during prestressing, as in BR-C2, or recorded prior to testing, as in BR-C4. The figure indicates that the highest strains were recorded within the bottom third of the column where the critical region under combined flexure and shear was located. When the strands were placed at 150 mm spacing, the maximum increase in prestressing strands was approximately 3500 micro strains and occurred at 6% drift ratio. At this level of transverse strain, the diagonal cracks were well controlled. Unlike the un-retrofitted
companion column BR-C1, the mode of behavior changed from diagonal tension to flexure, and the column failed in flexure. It experienced flexural crushing of concrete near the base, followed by the compression buckling and tension rupturing of longitudinal reinforcement at 6% lateral drift ratio.

Column BR-C2 was initially prestressed to approximately 300 MPa, producing transverse compression on concrete. The prestressing eliminated diagonal tension cracks almost completely until the end of testing, increasing concrete shear resistance significantly. The column sustained 4% lateral drift without any appreciable strength decay as indicated in Fig. 10. The maximum strain values recorded in prestressing strands at this stage of loading was 3100 micro strains with a corresponding increase of 1900 micro strains over the initial value. The incremental increase in steel strain may be taken as an indication of transverse tension in concrete. This value is below the value usually associated with yielding of column ties in concrete. The incremental strains were verified against the strain measurements recorded on column ties and showed very good correlation. Hence, it can be stated that the concrete in these columns remained intact, developing only hairline cracks, while providing full resistance to shear ($V_c$ in Eq. 5) until the end of testing.

When the external hoop spacing was increased to 300 mm, as in the case of column BR-C4, the effectiveness of prestressing hoops reduced significantly. The column maintained its strength at 2% drift, and failed gradually during 3% drift cycles. This was only a 1% improvement in drift capacity over that of column BR-C1, which was not retrofitted. The incremental strain increase in prestressing hoops was 2150 micro strain at 2% drift, signifying wide enough diagonal cracks to yield internal column ties. The incremental strain increase reached 3200 micro strain at 3% drift when the column failed through a diagonal tension crack that took place between the second and third hoops.
DESIGN OF PRESTRESSING FOR CONTROL OF DIAGONAL CRACKING

Designing prestressing for diagonal crack control to improve seismic shear resistance requires the determination of the amount and spacing of strands, as well as the level of prestress. Shear deficient columns often have widely spaced ties of insufficient area. These ties tend to yield during seismic response prematurely, leading to shear failure. Furthermore, concrete develops diagonal cracks that tend to widen significantly in the absence of adequate crack control. Under stress reversals, these wide cracks cross each other, leading to the deterioration of concrete. Transverse prestressing, when used as a seismic retrofit methodology, provides enhancements to both the concrete and reinforcement shear resistance.

It is desirable to retrofit concrete columns to sustain seismic induced shear force reversals without a significant strength decay while also minimizing or eliminating the need for post-earthquake repair work. If the integrity of concrete is maintained during inelastic deformation cycles then the need for post-earthquake repair is often eliminated or at the least, minimized. Since the deterioration of concrete in shear deficient columns often occurs due to the opening and closing of wide diagonal cracks, and criss crossing of cracks under stress reversals, mitigation of cracks is an effective means of maintaining structural strength and integrity of concrete. This is achieved by providing sufficiently high strength enhancement against diagonal tension so that the widths of diagonal cracks are controlled and the deterioration of concrete under stress reversals is not permitted. Elimination of diagonal cracking completely through prestressing may not be necessary, as some tension in concrete can be tolerated for an acceptable level of performance. The column tests reported in the current investigation indicated that the damage on concrete could be controlled if the transverse strain is limited to 0.15% to 0.20%, depending on the lateral drift demand. In most cases 0.20% transverse strain provided acceptable level of crack control up to 4% lateral drift. At
Design and Construction Practices to Mitigate Cracking

this strain level, both concrete and internal reinforcement can be utilized for shear resistance. Requirements that satisfy this level of performance are given below.

\[ V_c \leq \phi V_c + \phi V_s + \phi V_p \]  \hspace{1cm} (6)

where;

\[ V_c = 0.2\sqrt{f'_c} bd \]  \hspace{1cm} (7)

\[ V_s = A_v f_s \frac{d}{s} \]  \hspace{1cm} (8)

\[ f_s = 0.002 E_s \leq f_y \]  \hspace{1cm} (9)

\[ V_p = 2A_{ps} (f_{pe} + 0.002 E_p) \frac{h}{s_p} \]  \hspace{1cm} (10)

Substituting Eqs. 7, 8, and 10 into Eq. 6 and solving for the area of prestressing steel;

\[ A_{ps} = \left[ \frac{V_c - 0.2\sqrt{f'_c} bd - A_v f_s \frac{d}{s}}{2(f_{pe} + 0.002 E_p)} \right] \frac{s_p}{h} \]  \hspace{1cm} (11)

The minimum value of prestressing should not be less than 50 MPa, which merely provides a sung-tight condition with minimum efficiency in terms of crack control, so that the hoops remain intact during an earthquake. The maximum level of prestressing is set at 50% of the ultimate tensile capacity of strand so that some reserve capacity is provided in prestressing steel, which may extend further during the earthquake as transverse strains in concrete result in lateral expansion.
limiting conditions are shown below.

\[ 50 \text{MPa} \leq f_{pe} \leq 0.5f_{pu} \] (12)

The spacing of prestressing hoops is limited to that provided favorable results in column specimen BR-C2.

\[ s_p \leq \frac{h}{4} \] (13)

**SUMMARY AND CONCLUSIONS**

A new technique was developed for controlling diagonal cracks in earthquake resistant concrete columns. The technique involves external prestressing of circular columns with individual hoops that consist of prestressing tendons and specially designed anchors. The hoops are placed directly on the concrete surface and provide uniform active and passive lateral pressure. Experimental research was conducted to demonstrate the effectiveness of the technology in controlling diagonal cracking and suppressing shear failure during inelastic deformation reversals. The results indicate improvements in lateral drift capacity from 1% in shear-critical non-retrofitted columns to up to 5% in companion retrofitted columns.

**REFERENCES**


NOTATIONS

\( A_s \) : Total area of transverse shear reinforcement within spacing, \( s \), in the direction of shear force, in mm\(^2\).

\( A_{ps} \) : Area of strand used to prestress column in the transverse direction, in mm\(^2\).

\( b \) : Column cross-sectional dimension perpendicular to shear force (diameter of circular section), in mm.

\( d \) : Distance from the extreme compression fiber to the centroid of longitudinal tension reinforcement in mm (for circular sections the centroid of longitudinal reinforcement in the opposite half of the member shall be used).

\( E_p \) : Modulus of elasticity of prestressing steel, \( E_p = 200000 \) MPa.

\( f'_c \) : Concrete compressive strength in column, in MPa.

\( f_{pe} \) : Effective prestress in MPa.

\( f_{py} \) : Yield strength of prestressing strand, in MPa.

\( f_{pu} \) : Ultimate strength of prestressing strand, in MPa.

\( f_t \) : Stress in transverse shear reinforcement, MPa.

\( f_y \) : Yield strength of transverse shear reinforcement, in MPa.

\( h \) : Column cross-sectional dimension parallel to shear force, or diameter of circular section, in mm.

\( s \) : Spacing of transverse shear reinforcement, in mm.

\( s_p \) : Spacing of external prestressing hoops, in mm.

\( V_c \) : Shear force resistance provided by tensile stresses in the concrete, newtons.
210 Saatcioglu

\( V_c \) : Maximum seismic shear force that the column may be subjected to during earthquake, determined as the larger of the factored shear force and the shear force associated with the formation of flexural plastic hinges in the column, newtons.

\( V_n \) : Nominal shear capacity of column, newtons.

\( V_p \) : Shear strength enhancement provided by external prestressing in newtons.

\( V_{pc} \) : Shear strength enhancement in concrete, introduced by external prestressing, newtons.

\( V_{ps} \) : Enhancement in shear resistance provided by steel reinforcement, introduced by external prestressing, newtons.

\( V_s \) : Shear resistance provided by internal shear reinforcement, newtons.
Fig. 1 Examples of column shear failures during a) the 1995 Kobe Earthquake, b) the 1994 Northridge Earthquake, and c) 1999 Taiwan Earthquake.
Fig. 2 Mechanism of shear resistance in a laterally prestressed column

\[ V_{pc} = 2A_{ps} f_{pe} h/s_p \]

Fig. 3. Geometric details of test columns
Fig. 4 Stress-strain relationship of prestressing strands

Fig. 5 Schematic view of the anchors used for external prestressing
Fig. 6 Externally prestressed column

Fig. 7 Test Setup
Design and Construction Practices to Mitigate Cracking

Fig. 8 Hysteretic relationship for BR-C1

Fig. 9 BR-C1 at 2% drift

Fig. 10 Hysteretic relationship for BR-C2

Fig. 11 BR-C2 at 4% drift
Fig. 12 Hysteretic relationship for BR-C4
Fig. 13 BR-C4 at 3% drift

Fig. 14 Strain in prestressing strands along the height of columns
Improving Watertightness of Reinforced Concrete Structures with Shrinkage-Reducing Admixtures

by J. K. Buffenbarger, C. K. Nmai, and M. A. Miltenberger

Synopsis:

Drying shrinkage cracking can adversely affect the aesthetics, durability, and serviceability of reinforced concrete structures, thereby negating some of the benefits provided by high-performance concretes. Developed years ago but relatively new to the construction industry, shrinkage-reducing admixtures (SRAs) have been shown to provide significant reductions in concrete drying shrinkage and subsequent cracking. The potential benefits that SRAs provide have resulted in increased use of these products in the past few years.

In this paper, data from laboratory testing and field investigations of SRA-treated concrete mixtures and their use in a few projects where watertightness was desired are presented and discussed. The findings of visual inspections of the projects performed shortly after construction and after a year in service will also be presented. The information to be presented verify the drying shrinkage reduction characteristics of SRAs and show that these innovative admixtures can provide substantial benefits with regards to improving watertightness and overall serviceability of reinforced concrete structures.

Keywords: concrete; cracking; drying shrinkage; durability; polyoxyalkylene alkyl ether; shrinkage-reducing admixture
ACI member, Julie K. Buffenbarger is a product manager/scientist at Master Builders, Inc., Cleveland, Ohio. She has a M.Sc. degree in organic synthesis from Bowling Green State University, Ohio, and five years industrial experience in analytical chemistry and ceramic technology. She is also a member of the American Chemical Society.

Charles K. Nmai, FACI, is chief engineer in the Engineering Services Group at Master Builders, Inc., Cleveland, Ohio. He has a Ph.D. in civil engineering from Purdue University and is a licensed engineer in Ohio. He is chairman of ACI Committees E 701, Materials for Concrete Construction, and 222, Corrosion; and is a member of 201, Durability; and 363, High-strength Concrete.

ACI member, Matthew A. Miltenberger is a Senior Project Engineer at Master Builders, Inc., Cleveland, Ohio. He has a M.Sc. in structural engineering and B.S. in civil engineering from University of Maryland; and a B.B.A. in Construction Management from University of Miami, Florida. He is a member of ACI Committees 355, High-strength Concrete; 365, Service Life Prediction; and 222, Corrosion. He was co-recipient of the 2000 ACI Leonard C. Wason Award.

INTRODUCTION

Durability refers to the ability of concrete to maintain its integrity in service. "Corrosion of steel reinforcement, freeze/thaw damage, salt scaling, alkali aggregate reactions, and sulfate attack, all of which can result in cracking and spalling of the concrete cover, are the major problems" (1). If concrete is properly designed for the environment to which it is to be exposed, and is properly placed and cured; it should last for many decades without costly repairs.
Design and Construction Practices to Mitigate Cracking

Engineers are currently shifting towards durability-based designs in an effort to extend the useful service lives of reinforced concrete structures in aggressive environments. The effectiveness of measures that are typically implemented to improve the resistance of concrete to these deterioration forces can be reduced by cracks in the hardened concrete. As a result, drying shrinkage is beginning to receive more consideration in the design and construction of concrete structures.

In an effort to meet the demands of the concrete construction industry, manufacturers of specialty construction materials have developed and introduced shrinkage-reducing admixtures (SRAs) that can be used to produce low shrinkage, high-performance concretes. SRAs are being studied and gaining acceptance within the world-wide concrete industry, especially in the United States (2-7). In this paper, structures fabricated and repaired with SRA-treated concrete are reviewed for their watertightness and overall serviceability.

**DRYING SHRINKAGE**

The need for adequate workability to facilitate placement and consolidation of concrete often necessitates the use of a greater amount of mixing water than is needed for the hydration process of portland cement. The loss of some of this excess "water of convenience" from a concrete matrix as it hardens results in a volume reduction that is known as shrinkage. If the volume reduction occurs before the concrete hardens, it is called plastic shrinkage. The volume reduction that occurs due to moisture loss after the concrete has attained final set is known as drying shrinkage.

Drying shrinkage is the decrease in the volume of a concrete element when it loses moisture by evaporation. Drying shrinkage is inevitable unless the concrete is either completely submerged under water or is an environment that has 100 percent relative humidity. Therefore, drying shrinkage is a phenomenon that occurs routinely in concrete constructed works. With adequate restraint, drying shrinkage can cause cracking if the induced tensile stresses exceed the tensile strength of the concrete. Cracks provide
easy access for oxygen, moisture, chlorides and other aggressive chemicals and agents into the concrete matrix, and can therefore impact long-term durability of concrete. Differentials in drying shrinkage between the top and bottom surfaces of slabs cause curling and possible cracking.

SHRINKAGE-REDUCING ADMIXTURES

SRAs were first developed in Japan in 1982 in a partnership between Nihon Cement Co., Ltd., now Taiheiyo Cement Corporation, and Sanyo Chemical Industries, Ltd. (2, 3). On October 15, 1985, U.S. Patent number 4,547,223 was granted to Goto et al. for the invention, the main component being a polyoxalkylene alkyl ether, a lower alcohol alkylene oxide adduct (8). Since, this invention interest in this technology has grown (9-12) and on September 17, 1996, U.S. Patent Number 5,556,460 was granted to Berke et al. for an SRA with a similar base composition (13). Several low viscosity, water soluble SRAs have been developed by Taiheiyo Cement and Sanyo Chemical Industries. These admixtures function by reducing capillary tension and the tensile forces that develop within the concrete pores as it dries (2-7). They are primarily used as integral admixtures, but some can be applied topically to concrete surfaces (5).

DESIGN AND CONSTRUCTION PRACTICE

Parameters that most influence drying shrinkage include the amount and size of reinforcement, and the size and shape, as well as the surface area-to-volume ratio of the concrete member. Steel-reinforced concrete will shrink less than plain concrete and the relative difference is a function of the reinforcement percentage providing restraint. In the same ambient condition, a small concrete member will shrink more than a larger member because of its higher surface area-to-volume ratio. The greater the area of area of exposure, the greater the rate of moisture loss, and hence the potential for drying shrinkage.
Improper concreting practices, such as job-site re-tempering, will increase drying shrinkage because of the increase in water content of the concrete. Prolonged moist curing will delay the onset of drying shrinkage, but, in general, the length of curing is reported to have little effect on drying shrinkage (14). Steam curing will, however, reduce drying shrinkage.

Liquid containment is typically a design consideration in water-treatment plants, wastewater-treatment facilities, tanks, and reservoirs. The concrete used for these facilities should be of good quality to provide resistance to environmental effects such as freezing and thawing and must be watertight to minimize or eliminate leakage or groundwater contamination. ACI 350 recommends the following to minimize leakage: a) well proportioned and consolidated concrete; b) minimization of crack widths; c) proper joint spacing; d) impervious protective coatings or barriers where required; and e) provision of adequate reinforcement (15).

Although proper design, such as specifying control joints and reinforcing steel is important, the most effective way of minimizing water leakage is by reducing the permeability and the drying shrinkage of concrete. ACI 350 recommends a maximum water-to-cementitious materials ratio of 0.45 and minimum cement contents based upon coarse aggregate topsize (15). Drying shrinkage and hence drying shrinkage cracking can be reduced further through the use of SRAs.

In the case studies that follow, laboratory and field data are presented that show the effectiveness of an SRA in reducing drying shrinkage cracking thereby improving the watertightness of the structures.

**Case Study #1: The Burbank Water Treatment Facility, Burbank, California**

Three water tanks were erected to increase capacity of the Burbank Water Reclamation Plant in Burbank, California. The engineer specified typical containment structure concrete mixture parameters with 1.0-inch maximum
aggregate size for pumpability, a slump of 7 ± 1 inches, an air content of 4 ± 1%, a 28-day compressive strength of 4000 psi and a maximum drying shrinkage of 0.042% at 28 days. Four concrete mixtures were studied for use on the project, a reference concrete mixture and three SRA-treated mixtures. The concretes were tested in accordance with applicable ASTM Standards (16). The concrete mixture design, fresh concrete properties and hardened concrete performance data are included in Table I.

All three SRA treated concrete mixtures met the 28-day compressive strength and drying shrinkage requirements. For economic reasons “Mixture 2” was used in the construction of the water treatment tanks. The constructed tanks were filled with water and hydrostatically tested. Detailed visual examinations for leakage specifically associated with drying shrinkage cracking were also conducted. All three tanks passed hydrostatic testing on the first trial and visually did not exhibit any leakage.

Case Study #2: The Los Angeles Metro Rail Red Line Tunnels, Los Angeles, California

The Metro Rail Red Line tunnels are parallel twin portals that link Hollywood to Universal City. The tunnels are part of a 6 1/2-mile-long rail line that starts in North Hollywood and will end in downtown Los Angeles.

Most of the 10,000 lineal feet of railway grade had a final elevation in bedrock that was stable enough to use standard 4,000-psi concrete for the cast-in-place tunnel lining. However, 2500 feet of the tunnel’s path was located in an unstable stratum of friable sandstone with high permeability. Without structural support, the exposed sandstone would erode from groundwater infiltration. Additionally, local environmental concerns of potential groundwater contamination due to the porosity of the strata had to be addressed. With these design considerations a monolithic tunnel lining was proposed. The tunnel lining concrete mixture required a low water-to-cement ratio for permeability reduction and had a maximum shrinkage requirement of 0.040% at 28 days to be watertight and durable to meet long service-life requirements.
The mixture proportions and hardened properties for the Metro Red Line Tunnel are tabulated in Tables II and III below. These data were obtained from testing laboratories used on the project and can be found elsewhere (17). Visual examination of the concrete showed no cracking after one year.

**Case Study #3: Dupont Circle Parking Garage Full-Depth Repair, Washington, D.C.**

The drying shrinkage of repaired concrete is one of the important factors that influence the dimensional behavior. Bond failures and cracking generally result from dimensional incompatibility between the repair material and the existing substrate. The existing concrete substrate has already experienced most of its time-dependent volume change such as drying shrinkage and creep. However, the repair concrete must also undergo volume changes after placement. Consequently, it is important to identify and select a low shrinkage repair material. Additionally, the durability of the concrete repair must be considered so that it may resist structural loading and environmental conditions (i.e. deicing salts) without degradation and deterioration.

A SRA was evaluated in the full-depth repair of the Dupont Circle Parking Garage through the use of vibrating wire strain gauges to examine the dimensional stability of the in situ concrete. This repair was necessitated by corrosion of the reinforcing steel and electrical conduits within the 11-inch thick flat slab. The repair concrete mixture proportions are in Table IV. Companion length change test specimens were fabricated. Two sets of ASTM C157 specimens were cast with one set cured under job site conditions (Reference and SRA 4-6) and the other set was cured under standard laboratory conditions (Reference and SRA 1-3) for twenty-eight days, thereafter both sets were placed in the standard laboratory environment. This allowed for examination of the relationship between drying shrinkage measurements in situ and traditional test specimens used for ASTM C 157.
Figs. 1 and 2 show the laboratory and in situ drying shrinkage from the Dupont Circle Parking Garage case study. Both figures illustrate the significant reductions in drying shrinkage that can be obtained with SRA treated concrete.

The Dupont Circle Parking Garage case study provides insight into the differences between laboratory and in situ measurements. The data presented in Tables V and VI show that the SRA significantly reduced drying shrinkage in both the standard test conditions and in the structure. The data also show that drying shrinkage results produced from ASTM C 157 testing are significantly greater than that experienced in situ. However, the 28-day laboratory results were similar to the drying shrinkage strains experienced in situ after fifteen months in the heated, underground parking garage. This may not be true for the other structures due to environmental and other differences.

The data shown in Fig. 3 illustrate the reduction of drying shrinkage with SRA in comparison to reference concrete mixtures in laboratory and field conditions at both early and late ages. For example, drying shrinkage reductions of 37 and 27% were obtained at 28 and 474 days respectively for laboratory test specimens. Reductions of 53 and 47% were observed at the same ages in situ (Fig. 3).

The surface-to-volume ratio is a significant factor when estimating the volume change in real structures from laboratory drying shrinkage data. Fig. 4 presents the in situ drying shrinkage in Table VI as a ratio of the laboratory drying shrinkage in Table V. At 28 days, the structure experienced 22-30% of the shrinkage experienced in laboratory specimens and after fifteen months, this ratio increased to 47-65%. The difference in magnitude seen in these percentages can be attributed in part to the larger surface to volume ratio in the small, slender laboratory specimens and environmental differences such as relative humidity (Table VI). The larger surface-to-volume ratio accelerates moisture loss from the specimens. This is confirmed by comparing the slopes of the drying shrinkage curves in Figs. 1 and 2. Fig. 1 shows that most of the drying shrinkage of the ASTM C157 specimens occurred in the first 180 days, whereas the in situ
measurements have yet to reach equilibrium. Since drying shrinkage occurs in the structure at a slower rate, the effect of creep reduces the induced tensile stress and thereby the cracking potential. Therefore, by reducing the early age drying shrinkage with a SRA the overall cracking potential is reduced for the life cycle of the structure.

Fig. 5 illustrates a visual inspection of the repairs after fifteen months. The SRA treated repair was well bonded and showed no evidence of cracking within the body of the repair. In contrast, the reference concrete showed visible hairline cracking within the body of the repair and along one edge. The absence of cracking in the SRA-treated repair will improve the corrosion resistance of the concrete by minimizing the ingress of water, oxygen and chloride ion from deicing salts.

CONCLUSIONS

Drying shrinkage is an inherent and unavoidable property of concrete, including high-performance concrete, and can detrimentally impact the aesthetics, durability, and serviceability of reinforced concrete structures. However, with implementation of durability-based designs and good concrete construction practices, drying shrinkage and subsequent cracking can be minimized to extend the useful service lives of reinforced concrete structures.

The shrinkage-reducing admixture described in this paper has provided significant reductions in drying shrinkage and subsequent cracking in both laboratory and field investigations. This novel admixture has provided substantial benefits with regards to improved watertightness, aesthetics and overall serviceability of reinforced concrete structures. The inclusion of shrinkage-reducing admixtures can be used to great advantages in slabs, bridge decks, liquid containment structures and repair work where cracking can lead to steel reinforcement corrosion and decreased resistance to other aggressive species. Inherently, improving durability has "...perhaps the highest potential of all for achieving remarkable cost-saving benefits in the infrastructure" (18).
ACKNOWLEDGEMENTS

The authors would like to thank the following for their invaluable assistance in obtaining the data reported in this paper: the R&D Laboratory group at Master Builders Inc.; Don Fretz, Master Builders, Inc.; Twining Laboratories of Southern California, Inc.; Physical Laboratories at Smith-Emery Company; Rick Hyden Technical Services Department at Southdown Concrete Products, Inc.; and Mike Chambers of Associated Ready Mixed Concrete, Inc. (Sun Valley, California).

REFERENCES


Table 1. Concrete mixture proportions, Burbank Water Treatment Facility

<table>
<thead>
<tr>
<th>Materials</th>
<th>Reference</th>
<th>Mixture 2</th>
<th>Mixture 3</th>
<th>Mixture 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Type II Cement, lb/yd^3</td>
<td>649</td>
<td>649</td>
<td>649</td>
<td>649</td>
</tr>
<tr>
<td>Sand, lb/yd^3</td>
<td>1289</td>
<td>1289</td>
<td>1289</td>
<td>1289</td>
</tr>
<tr>
<td>3/8&quot; Aggregate, lb/yd^3</td>
<td>334</td>
<td>334</td>
<td>334</td>
<td>334</td>
</tr>
<tr>
<td>1&quot; Aggregate, lb/yd^3</td>
<td>1432</td>
<td>1432</td>
<td>1432</td>
<td>1432</td>
</tr>
<tr>
<td>Total Water, lb/yd^3</td>
<td>292</td>
<td>292</td>
<td>292</td>
<td>292</td>
</tr>
<tr>
<td>Water/Cement Ratio</td>
<td>0.45</td>
<td>0.45</td>
<td>0.45</td>
<td>0.45</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Admixtures</th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Superplasticizer, fl oz/cwt</td>
<td>5.0</td>
<td>5.0</td>
<td>5.0</td>
<td>5.0</td>
</tr>
<tr>
<td>Air Entrainer, fl oz/cwt</td>
<td>0.40</td>
<td>0.40</td>
<td>0.40</td>
<td>0.40</td>
</tr>
<tr>
<td>SRA, gal/yd^3</td>
<td>0.0</td>
<td>0.50</td>
<td>0.75</td>
<td>1.0</td>
</tr>
</tbody>
</table>

Plastic Properties of Concrete Mixtures

<table>
<thead>
<tr>
<th>Mixture</th>
<th>Slump (in)</th>
<th>Air Content (%)</th>
<th>Plastic Unit Weight (lb/ft^3)</th>
<th>Concrete Temperature °F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reference</td>
<td>9.75</td>
<td>4.9</td>
<td>148.9</td>
<td>66</td>
</tr>
<tr>
<td>Mixture 2</td>
<td>9.25</td>
<td>5.0</td>
<td>148.7</td>
<td>67</td>
</tr>
<tr>
<td>Mixture 3</td>
<td>9.75</td>
<td>5.0</td>
<td>153.5</td>
<td>69</td>
</tr>
<tr>
<td>Mixture 4</td>
<td>10.25</td>
<td>3.8</td>
<td>152.9</td>
<td>70</td>
</tr>
</tbody>
</table>

Average Compressive Strength (psi)

<table>
<thead>
<tr>
<th>Mixture</th>
<th>1-day</th>
<th>3-day</th>
<th>7-day</th>
<th>28-day</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reference</td>
<td>2340</td>
<td>4270</td>
<td>4740</td>
<td>4900</td>
</tr>
<tr>
<td>Mixture 2</td>
<td>2640</td>
<td>3770</td>
<td>5210</td>
<td>5860</td>
</tr>
<tr>
<td>Mixture 3</td>
<td>2670</td>
<td>4690</td>
<td>5100</td>
<td>6210</td>
</tr>
<tr>
<td>Mixture 4</td>
<td>2890</td>
<td>4650</td>
<td>5410</td>
<td>6450</td>
</tr>
</tbody>
</table>

Average Length Change, % (negative sign denotes shrinkage)

<table>
<thead>
<tr>
<th>Mixture</th>
<th>7-day</th>
<th>14-day</th>
<th>21-day</th>
<th>28-day</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reference</td>
<td>-0.023</td>
<td>-0.033</td>
<td>-0.044</td>
<td>-0.049</td>
</tr>
<tr>
<td>Mixture 2</td>
<td>-0.011</td>
<td>-0.018</td>
<td>-0.027</td>
<td>-0.034</td>
</tr>
<tr>
<td>Mixture 3</td>
<td>-0.009</td>
<td>-0.014</td>
<td>-0.024</td>
<td>-0.028</td>
</tr>
<tr>
<td>Mixture 4</td>
<td>-0.007</td>
<td>-0.012</td>
<td>-0.020</td>
<td>-0.023</td>
</tr>
</tbody>
</table>
Table 2. Concrete mixture proportions, Metro Rail Line Tunnels

<table>
<thead>
<tr>
<th>Materials</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Type II cement, lb/yd³</td>
<td>700</td>
</tr>
<tr>
<td>Class F pozzolan, lb/yd³</td>
<td>123</td>
</tr>
<tr>
<td>Sand, lb/yd³</td>
<td>1242</td>
</tr>
<tr>
<td>3/8&quot; gravel, lb/yd³</td>
<td>243</td>
</tr>
<tr>
<td>1&quot; gravel, lb/yd³</td>
<td>1544</td>
</tr>
<tr>
<td>Water, lb/yd³</td>
<td>267</td>
</tr>
<tr>
<td>Water, gal/yd³</td>
<td>32</td>
</tr>
<tr>
<td>Water/cement ratio</td>
<td>0.32</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Admixtures</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Superplasticizer, fl oz/cwt</td>
<td>5.0</td>
</tr>
<tr>
<td>SRA, gal/yd³</td>
<td>0.75</td>
</tr>
<tr>
<td>Hydration Control, fl oz/cwt</td>
<td>1.0 - 8.0</td>
</tr>
<tr>
<td>Water Reducer, fl oz/cwt</td>
<td>8.0</td>
</tr>
<tr>
<td>Slump, in</td>
<td>6.0 - 9.0</td>
</tr>
<tr>
<td>Air Content, %</td>
<td>2.0</td>
</tr>
</tbody>
</table>

Table 3. Hardened properties for the Metro Rail Red Line Tunnels

<table>
<thead>
<tr>
<th>ASTM C157 Shrinkage Test Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>Age of Sample after (air storage)</td>
</tr>
<tr>
<td>5 days</td>
</tr>
<tr>
<td>7 days</td>
</tr>
<tr>
<td>14 days</td>
</tr>
<tr>
<td>21 days</td>
</tr>
<tr>
<td>28 days</td>
</tr>
</tbody>
</table>

* Negative sign denotes shrinkage

<table>
<thead>
<tr>
<th>Compressive Strength, psi</th>
</tr>
</thead>
<tbody>
<tr>
<td>Age</td>
</tr>
<tr>
<td>7 days</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>Average</td>
</tr>
</tbody>
</table>
Table 4. Repair mixture proportions for Dupont Circle Parking Garage

<table>
<thead>
<tr>
<th></th>
<th>Reference</th>
<th>SRA Treated</th>
</tr>
</thead>
<tbody>
<tr>
<td>Type I cement, lb</td>
<td>705</td>
<td>705</td>
</tr>
<tr>
<td>Sand, lb</td>
<td>1428</td>
<td>1428</td>
</tr>
<tr>
<td>#8 aggregate, lb</td>
<td>1650</td>
<td>1650</td>
</tr>
<tr>
<td>Water, gal.</td>
<td>34</td>
<td>34</td>
</tr>
<tr>
<td>Water/cement ratio</td>
<td>0.40</td>
<td>0.40</td>
</tr>
<tr>
<td><strong>Admixtures</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Water Reducer, fl oz/cwt</td>
<td>3.0</td>
<td>3.0</td>
</tr>
<tr>
<td>Superplasticizer, fl oz/cwt</td>
<td>8.0</td>
<td>8.0</td>
</tr>
<tr>
<td>SRA, gal/yd$^3$</td>
<td>N/A</td>
<td>1.5</td>
</tr>
<tr>
<td>Air Entrainer, fl oz/cwt</td>
<td>0.70</td>
<td>0.70</td>
</tr>
<tr>
<td>Calcium Nitrite Corrosion Inhibitor, gal/yd$^3$</td>
<td>3.0</td>
<td>3.0</td>
</tr>
<tr>
<td>Slump, in</td>
<td>6-9</td>
<td>4</td>
</tr>
<tr>
<td>Unit Weight, lb/ft$^3$</td>
<td>145.2</td>
<td>148.4</td>
</tr>
</tbody>
</table>
Table 5. ASTM C157 laboratory results (μstrain)

<table>
<thead>
<tr>
<th>Date</th>
<th>9/28/98</th>
<th>10/2/98</th>
<th>10/9/98</th>
<th>10/15/98</th>
<th>10/23/98</th>
<th>01/18/99</th>
<th>08/11/99</th>
<th>01/12/00</th>
</tr>
</thead>
<tbody>
<tr>
<td>Age (days)</td>
<td>3</td>
<td>7</td>
<td>14</td>
<td>20</td>
<td>28</td>
<td>115</td>
<td>320</td>
<td>474</td>
</tr>
<tr>
<td>Reference 1</td>
<td>50</td>
<td>-280</td>
<td>-490</td>
<td>-590</td>
<td>-650</td>
<td>-920</td>
<td>-1020</td>
<td>-109</td>
</tr>
<tr>
<td>Reference 2</td>
<td>30</td>
<td>-280</td>
<td>-480</td>
<td>-590</td>
<td>-660</td>
<td>-850</td>
<td>-930</td>
<td>-98</td>
</tr>
<tr>
<td>Reference 3</td>
<td>40</td>
<td>-280</td>
<td>-500</td>
<td>-630</td>
<td>-680</td>
<td>-980</td>
<td>-1060</td>
<td>-112</td>
</tr>
<tr>
<td>Reference 4</td>
<td>50</td>
<td></td>
<td></td>
<td></td>
<td>-680</td>
<td>-890</td>
<td>-990</td>
<td>-107</td>
</tr>
<tr>
<td>Reference 5</td>
<td>30</td>
<td></td>
<td></td>
<td></td>
<td>-640</td>
<td>-880</td>
<td>-960</td>
<td>-103</td>
</tr>
<tr>
<td>Reference 6</td>
<td>40</td>
<td></td>
<td></td>
<td></td>
<td>-640</td>
<td>-840</td>
<td>-900</td>
<td>-97</td>
</tr>
<tr>
<td>Lab Avg. (1 - 3)</td>
<td>40</td>
<td>-280</td>
<td>-490</td>
<td>-603</td>
<td>-663</td>
<td>-917</td>
<td>-1003</td>
<td>-106</td>
</tr>
<tr>
<td>Field Avg. (4 - 6)</td>
<td>40</td>
<td></td>
<td></td>
<td></td>
<td>-653</td>
<td>-870</td>
<td>-950</td>
<td>-102</td>
</tr>
</tbody>
</table>

SRA 1
| 40 | -190 | -300 | -400 | -440 | -660 | -780 | -78 |
| SRA 2 | 20 | -190 | -290 | -360 | -410 | -650 | -730 | -80 |
| SRA 3 | 60 | -140 | -260 | -340 | -410 | -580 | -650 | -75 |
| SRA 4 | 40 |         |         |          | -370 | -580 | -670 | -72 |
| SRA 5 | 40 |         |         |          | -370 | -650 | -720 | -73 |
| SRA 6 | 40 |         |         |          | -370 | -630 | -710 | -79 |
| Lab Avg. (1 - 3) | 40 | -173 | -283 | -367 | -420 | -630 | -720 | -77 |
| Field Avg. (4 - 6) | 40 |         |         |          | -370 | -620 | -700 | -74 |
Table 6. In situ volume change measurements

<table>
<thead>
<tr>
<th>Date</th>
<th>Age (days)</th>
<th>Reference Repair Vertical (μstrain)</th>
<th>SRA Repair Vertical (μstrain)</th>
</tr>
</thead>
<tbody>
<tr>
<td>9/25/98</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>9/26/98</td>
<td>1</td>
<td>-91.35</td>
<td>-65.23</td>
</tr>
<tr>
<td>9/28/98</td>
<td>3</td>
<td>-98.68</td>
<td>-65.58</td>
</tr>
<tr>
<td>10/23/98</td>
<td>28</td>
<td>-196.43</td>
<td>-92.66</td>
</tr>
<tr>
<td>01/18/99</td>
<td>115</td>
<td>-348.57</td>
<td>-160.61</td>
</tr>
<tr>
<td>08/11/99</td>
<td>320</td>
<td>-586.24</td>
<td>-297.73</td>
</tr>
<tr>
<td>01/12/2000</td>
<td>474</td>
<td>-688.46</td>
<td>-361.82</td>
</tr>
</tbody>
</table>

Temperature and Relative Humidity Measurements in Garage

<table>
<thead>
<tr>
<th>Date</th>
<th>9/26/98</th>
<th>9/28/98</th>
<th>10/23/98</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dry Bulb (F)</td>
<td>78</td>
<td>78</td>
<td>55</td>
</tr>
<tr>
<td>Wet Bulb (F)</td>
<td>69</td>
<td>72</td>
<td>45</td>
</tr>
<tr>
<td>Relative humidity (%)</td>
<td>64</td>
<td>74</td>
<td>45</td>
</tr>
</tbody>
</table>
Figure 1. ASTM C 157 shrinkage data for Dupont Circle Full Depth Repair

Figure 1 is described in Case Study #3: Dupont Circle Garage Full-Depth Repair, Washington D.C., paragraph three, first sentence.
Figure 2. In situ shrinkage data for Dupont Circle Full Depth Repair

Figure 2 is described in Case Study #3: Dupont Circle Garage Full-Depth Repair, Washington D.C., paragraph five, first sentence.
Figure 3. Shrinkage reduction with SRA
Figure 3 is described in Case Study #3: Dupont Circle Garage Full-Depth Repair, Washington D.C., paragraph five, first sentence.

Figure 4. Drying shrinkage test method comparison
Figure 4 is described in Case Study #3: Dupont Circle Garage Full-Depth Repair, Washington D.C., paragraph six, second sentence.
Figure 5. Sketch of cracks found in repairs after fifteen months

Figure 5 is described in Case Study #3: Dupont Circle Garage Full-Depth Repair, Washington D.C., paragraph seven, first sentence.
What is the Crack Width in Concrete Structures to Prevent Leakage?

by L. G. Mrazek

Synopsis: ACI 318-99 no longer refers to Z factors or crack width formulae as in previous editions of the code. Instead, ACI 318-99 correlates bar spacing with clear concrete cover, indicating that following these guidelines will reduce crack widths at the concrete surface. Field investigations have found leakage at certain type cracks which exhibit widths of .23mm (0.009") or greater. Research and condition surveys completed by the author have found greater potential for concrete deterioration at cracks which extend to embedded reinforcement as compared with low slump, low water/cement ratio concrete having adequate cover over reinforcement. Current codes and standards present considerable variation with regard to recommended maximum crack widths to prevent leakage. Use of ACI 318-99 to design liquid or gas retaining structures could lead to designs that are not conservative, not durable and possibly unsafe, if preventing leakage is an important requirement for the particular facility.

Keywords: crack width; deterioration; field measurements; leakage
Larry G. Mrazek, P.E., S.E. is a member of ACI 350, ACI 121 and an associate member of ACI 546. He has completed condition surveys and has specified repairs for environmental engineering concrete structures and reinforced and prestressed concrete parking structures. He is also a fellow of ASCE and a member of PCI, CRSI and ICRI.

INTRODUCTION

This paper presents a review of crack width limitations for structures that are subject to leakage and compares these limitations with field measurements of cracks in environmental engineering concrete structures as well as concrete parking structures. In addition to leakage testing, the author has specified repairs for cracks with minimum widths for both environmental engineering concrete structures and parking structures. Durability issues, as related to cracking and crack widths, are most important to the serviceability of a structure or facility; however, this paper does not address durability issues in detail.

Present building codes, guides and standards specify or recommend maximum crack widths. For example, BOCA 1999 and UBC 1997 reference conformance to ACI 318-95. Whereas, IBC 2000 references conformance only to ACI 318; i.e., the current issue of ACI 318. This paper discourages the use of crack control provisions of ACI 318-99 when designing structures which are sensitive to leakage.

Types of Cracks

Figure 1 illustrates cracks due to direct tension. Where internal loadings of cylindrical, non-prestressed concrete liquid containing structures or storage silos govern the design, shrinkage cracking is not as prevalent, as compared to shrinkage in rectangular containment structures. When shrinkage occurs, a secondary compressive stress is introduced into the tank or silo walls. This stress tends to reduce the occurrence of wide, vertical cracks. For larger diameter structures, where wall concrete is placed in quadrants, the effects of drying shrinkage have been found to be more pronounced. In most of the cylindrical-type structures, vertical structural cracks are primarily the result of hoop tension stresses.

Figure 2 shows a rectangular tank with long walls where both vertical shrinkage cracks and corner restraint cracks are shown. When the length to wall height ratio exceeds 3, the primary flexural stress is vertical, or cantilever action. Restraint cracks may also be seen in both prestressed and non-prestressed concrete floors, primarily at corners, stairways and other floor penetrations.

Figure 3 shows a flexural wall crack with typical stress and strain diagrams. For most structural elements, very small and negligible crack widths occur in the compression face. In order to prevent leakage, Reference (1) recommends, as a minimum, limiting the compression zone, “a,” to a depth of 50 mm (2”), or two times the maximum aggregate size, whichever is greater. This same reference
suggests that structures which have "a" as small as 25 mm (1") are normally not prone to leaking cracks.

Structures which are exposed to ambient temperature variations are susceptible to length changes; i.e., expansion or contraction. If these movements are not accounted for in the design, cracking and or joint distress will most likely occur. Proper design and installation of water-stops are very important to the function of joints in liquid-containing structures.

**Literature Review**

Table 1 summarizes the status of recommended crack widths as indicated in codes, standards and guides. In some instances, the particular reference does not differentiate between durability and leakage issues. ACI 318-99 (3) appears to generalize the aspects of cracking and abandons previous code guidelines and equations, substituting an expression for reinforcement spacing which is intended to reduce crack widths at the concrete surface. BOCA 1999 (9) and UBC 1997 (10) reference ACI 318-95, and IBC 2000 (11) refers to the current issue of ACI 318 for crack control.

An article (12) by the late Joe W. Kelly indicated that visual tests have shown that cracks less than 0.05 mm (0.002") wide in relatively smooth and flat surfaces are rarely noticed. The article continues with a paragraph on leakage, and states that leaking cracks are usually objectionable.

**Why Control Concrete Cracking?**

Although this paper is primarily concerned with leakage, there is a definite correlation between the corrosion of embedded reinforcement and cracking, resulting in durability issues. Leakage thru cracks exceeds leakage thru uncracked concrete by several orders of magnitude; i.e., leakage thru sound, low water/cement ratio concrete is minimal as compared to leakage thru cracks. The author has found that the ingress of oxygen and moisture, with or without contaminants such as chlorides, sulfides, etc., can initiate the corrosion of embedded metals if the pH of the protective concrete cover is reduced to an approximate value of 8 or less. During investigations of environmental engineering and parking structures, the author has found deterioration in the form of spalls and delaminations to be primarily the result of the expansion of corrosion products. On one particular project, after less than 12 months, cracks which exhibited efflorescence and rust developed into large delaminations, requiring expensive repairs.

Cracks in certain structures pose an aesthetic problem to the designer and owner, especially when cracks leach efflorescence and/or corrosion by-products.

Leaking cracks not only pose durability problems but, depending on the type of structure can lead to the contamination of potable water and the loss of liquids (which could be hazardous) from the containment structure into the environment. Moreover, this leakage could result in the staining of the interior of ceilings or
walls or the seepage of liquids to tunnel floors, basements or process building floors resulting in slipping hazards to occupants.

In addition to durability issues, the author has observed leaking cracks in parking structures, which resulted in costly staining and damage to automobile paint finishes as well as presenting slipping hazards to occupants using these facilities.

Leaking cracks in below grade rooms or tunnels have been known to cause damage to equipment or stored goods and, in some cases, have resulted in the temporary relocation of persons assigned to those areas.

It is important to note that leaking cracks may also expose inhabitants to highly toxic gases, such as ozone, methane and chlorine. Moreover, methane gas is highly combustible and susceptible to explosions.

**RESULTS OF FIELD TESTING**

Accurate measurements of crack widths are most important to the investigator as well as the specifier. In order to evaluate cracks in a structure and to determine if repairs are necessary, the engineer should have access to up-to-date crack measuring devices. The author has used several devices which are shown in Figure 4. Devices shown to the left and right of the crack comparator card provide magnification of the crack width, which increases the accuracy of the reading. When taking a reading, a bright light beam is focused at the interface of the device with the concrete surface. He found the crack comparator card to be the least accurate of the devices shown. When measuring a crack in the field, it is advisable to take several measurements along the crack and to measure widths at the surface and also at depths of approximately 3 mm to 6 mm (¼" to ¼").

The author has surveyed leaking cracks in walls of a primary settling tank at a waste water treatment plant and at both walls and ceilings in a settling tank for a water treatment plant. (13 and 14) In one particular ceiling, water was seen to leak thru a crack which measured approximately 0.23 mm (0.009"). There was no significant head of water on the top surface of the structure, which was less than five years old. Apparently, autogenous healing was ineffective in sealing this crack. This particular crack was due to a combination of drying shrinkage and restraint.

On a recent condition survey of a subsurface parking structure (15), the author noticed significant horizontal and vertical cracks in an exterior wall which was designed to span vertically and to resist lateral earth pressure. Crack widths measured 0.15 mm to 0.33 mm (0.006" to 0.013") and were seen to leak. The wall thickness was 305 mm (12").

**Bar Spacing and Crack Widths**

Figure 5 shows a plot of bar spacing versus clear concrete cover to tensile reinforcement, based on recommendations of ACI 318-99 and ACI 350R.
Assuming a reinforcing steel stress of 186 MPa (27 ksi.), the maximum bar spacing according to ACI 318-99 is 406 mm (16"). According to ACI 350R, the maximum bar spacing for any steel stress is 305 mm (12"). In order to compare bar spacings for both references (3) and (6), a steel stress of 186 MPa (27 ksi.) was assumed. In Table 2, crack widths are computed for different groups of bar sizes, Z-factors and steel stresses. Notes in Table 2 explain the assumptions for developing this table. Computed crack widths were found to be the same when assuming the same Z-factors and bar sizes, while assuming different steel stresses and bar spacings. ACI committee 350 developed guidelines for liquid-containing structures where their recommendations limited crack widths to approximately 0.25 mm (0.010") or less. Members of committee 350 have had many years of design and construction experience with environmental engineering concrete structures. Their recommendations, as presented in this report, intend to satisfy both durability and leakage issues.

In Figure 5, bar spacings according to ACI 318-99 were found to be considerably more liberal than those developed from ACI 350R. It should be noted that Z-factors are no longer included in ACI 318-99. ACI 318-99 refers the crack control design of liquid retaining structures to other standards or codes; e.g., ACI 318-95 and ACI 350R. When using these references, the selection of Z-factors requires the judgment of the design engineer.

CONCLUSIONS AND RECOMMENDATIONS

A review is presented of recommended crack widths in the present codes and standards for the prevention of leakage in liquid and gaseous retaining structures. Field measurements of cracks have shown that certain type cracks, as narrow as 0.23 mm (0.009") have generated leakage. Provisions of ACI 318-99 do not correlate crack widths with concrete cover, bar size or bar spacing. Designers and specifications writers of liquid containing concrete structures, using ACI 318-99 as a design reference may provide designs which will not meet the requirements of the owner with regard to leakage and durability.

Over time, autogenous healing of cracks has been shown to reduce or stop leakage. Many owners receive this explanation when a leaking problem develops; i.e., "just wait awhile for autogenous healing to stop the leak." Before leakage testing, the author recommends that all cracks in liquid- or gas-containing structures having widths greater than 0.23 mm (0.009") be repaired, using epoxy resin or polyurethane grout injection.

Although this paper does not cover durability issues in detail, the author reminds the designers and specification writers that cracks which leak and allow the ingress of moisture and oxygen, with or without contaminants, will usually result in the corrosion of reinforcement and embedded metals and subsequent concrete deterioration and failure. Thus, timely crack repairs, in most cases, will prevent costly future repairs and the probable temporary closure of a parking structure, tank or containment structure.
Further research and testing is needed on crack width development in liquid and gas-retaining structures that are subject to leakage or contamination by chlorides, sulfates and other aggressive environments.

REFERENCES


2. American Concrete Institute (1995) “Building Code Requirements for Structural Concrete,” (ACI 318-95) and Commentary (ACI 318R-95), Farmington Hills, MI.

3. American Concrete Institute (1999) “Building Code Requirements for Structural Concrete,” (ACI 318-99) and Commentary (ACI 318R-99), Farmington Hills, MI.


7. ACI Committee 207 Report “Effects of Restraint, Volume Change and Reinforcement on Cracking of Mass Concrete,” ACI 207.2R, Farmington Hills, MI.


13. Case Study, “Bissell Point Wastewater Treatment Plant, St. Louis, Missouri – Condition Survey and Repairs.”


Table 1: Maximum Recommended Crack Widths to Prevent Leakage

<table>
<thead>
<tr>
<th>REFERENCE</th>
<th>MAXIMUM CRACK WIDTH mm/(in)</th>
<th>COMMENTS</th>
</tr>
</thead>
<tbody>
<tr>
<td>ACI 318-95 (2)</td>
<td>0.33 (0.013)</td>
<td>Based on z = 145, exterior exposure, Flexural Cracks.</td>
</tr>
<tr>
<td>ACI 318-99 (3)</td>
<td>NONE (NONE)</td>
<td>Commentary explains that revised approach controls surface cracks to acceptable limits.</td>
</tr>
<tr>
<td>ACI 224R (4)</td>
<td>0.10 (0.004)</td>
<td>TABLE 4.1 of Publication.</td>
</tr>
<tr>
<td>ACI 224.2R (5)</td>
<td>0.20 (0.008)</td>
<td>Liquid Retaining Structures—Tension Cracks.</td>
</tr>
<tr>
<td>ACI 350R (6)</td>
<td>0.23 – 0.25 (0.009 – 0.010)</td>
<td>Flexural Cracks.</td>
</tr>
<tr>
<td>Corps of Engineers</td>
<td>0.15 (0.006)</td>
<td>Project requirements for lock repairs—there was no distinction between shrinkage, flexural or tension cracks.</td>
</tr>
<tr>
<td>ACI 207.2R (8)</td>
<td>0.28 (0.011)</td>
<td>Flexure</td>
</tr>
<tr>
<td></td>
<td>0.23 (0.009)</td>
<td>Direct Tension. “Cracks of this width will allow some leakage, although minimum &amp; controllable.”</td>
</tr>
<tr>
<td>ACI 313R (9)</td>
<td>0.25 (0.010)</td>
<td>Under initial filling of Silo.</td>
</tr>
</tbody>
</table>

Table 2: Computed Crack Widths Based on ACI 350R

<table>
<thead>
<tr>
<th>Z FACTOR</th>
<th>*Fs MPA/ksi</th>
<th>**BAR SIZES</th>
<th>+BAR SPACING mm/(in)</th>
<th>CRACK WIDTHS mm/(in)</th>
</tr>
</thead>
<tbody>
<tr>
<td>115</td>
<td>186 (27)</td>
<td>10M, 15M (#3,#4,#5)</td>
<td>300 (11.76)</td>
<td>0.26 (0.0103)</td>
</tr>
<tr>
<td>95</td>
<td>186 (27)</td>
<td>10M, 15M (#3,#4,#5)</td>
<td>168 (6.63)</td>
<td>0.22 (0.00846)</td>
</tr>
<tr>
<td>95</td>
<td>152 (22)</td>
<td>10M, 15M (#3,#4,#5)</td>
<td>311 (12.26)</td>
<td>0.22 (0.00846)</td>
</tr>
<tr>
<td>115</td>
<td>186 (27)</td>
<td>15M, 25M (#6,#7,#8)</td>
<td>157 (6.18)</td>
<td>0.26 (0.0103)</td>
</tr>
<tr>
<td>115</td>
<td>152 (22)</td>
<td>15M, 25M (#6,#7,#8)</td>
<td>290 (11.43)</td>
<td>0.26 (0.0103)</td>
</tr>
<tr>
<td>95</td>
<td>186 (27)</td>
<td>15M, 25M (#6,#7,#8)</td>
<td>88 (3.48)</td>
<td>0.22 (0.00852)</td>
</tr>
<tr>
<td>95</td>
<td>124 (18)</td>
<td>15M, 25M (#6,#7,#8)</td>
<td>300 (11.76)</td>
<td>0.22 (0.00852)</td>
</tr>
<tr>
<td>115</td>
<td>186 (27)</td>
<td>30M, 35M (#9,#10,#11)</td>
<td>134 (5.28)</td>
<td>0.26 (0.0103)</td>
</tr>
<tr>
<td>115</td>
<td>145 (21)</td>
<td>30M, 35M (#9,#10,#11)</td>
<td>285 (11.23)</td>
<td>0.26 (0.0103)</td>
</tr>
<tr>
<td>95</td>
<td>186 (27)</td>
<td>30M, 35M (#9,#10,#11)</td>
<td>76 (2.98)</td>
<td>0.22 (0.00850)</td>
</tr>
<tr>
<td>95</td>
<td>117 (17)</td>
<td>30M, 35M (#9,#10,#11)</td>
<td>303 (11.93)</td>
<td>0.22 (0.00850)</td>
</tr>
</tbody>
</table>

Fs = fy/1.75^1.5 or as shown in ACI 350R where 1.7 = Load Factor 1.5 = Environmental Durability Factor fy = 413.7 MPa (60,000 psi)

**Bar sizes are noted in Figures 2.6.7(a), 2.6.7(b) and 2.6.7(c) of ACI 350R.

• Bar Spacing and clear concrete cover are shown in ACI 350R Figures 2.6.7(a), 2.6.7(b), and 2.6.7(c). Refer also to Figure 6.
Figure 1. Tension cracks in cylindrical, non-prestressed concrete, liquid containing tank or storage silo.

Figure 2. Cracks in rectangular, concrete liquid containing tank.
NOTE:
1. Tension reinforcement, $A_s$ is shown horizontal, similar for cantilever walls or walls spanning vertically, where vertical reinforcement is located at outside layer.

Figure 3. Stress/strain diagram for flexural crack in wall
Figure 4. Instruments to measure crack widths.
FIGURE 5: Concrete Cover vs. Bar Spacing

NOTES: 1. Z = 95 Severe Environmental Exposure 2. Z = 115 Moderate Environmental Exposure 3. ACI350R - Fig. 2.6.7(a) 4. ACI350R - Fig. 2.6.7(b) 5. ACI350R - Fig. 2.6.7(c) 6. Refer to Table 2 for Corresponding Crack Widths in accordance with ACI 350R 7. fs in ksi

1 inch = 25.4 mm; 1 ksi = 6.895 MPa
Cracks—Concrete Repair's Life Threatening Wounds

By A. M. Vaysburd, R. W. Poston, and J. E. McDonald

Synopsis: Cracking in concrete repair systems is one of the truly critical phenomena of repair pathology responsible for corrosion, deterioration and failure. The problem of repair cracking has become widespread not only with respect to severe environments which are intensifying restrained volume change stresses but also with respect to repairs in relatively benign environments.

Cracking accelerates the penetration of aggressive substances into the concrete and repair material from the exterior environment which in turn aggravates any one or a number of various mechanisms of deterioration. Moisture transport mechanism in the repaired structures is a tool for transferring an outer standard environment into an inner environment, and from one inner environment (existing substrate) into another (repair material).

The crack resistance of concrete repair is bearing on three equally important "elephants": (1) design details and specifications; (2) repair materials; (3) in-situ workmanship and quality control. This study demonstrates that the properties of cementitious repair materials have to be engineered for dimensional compatibility with existing concrete to improve their resistance to cracking. How good should the cementitious composite material used for repair of existing concrete structures be? How good is good enough? The paper summarized the factors involved and approaches taken when selecting cementitious repair materials. Performance criteria is presented for the selection of dimensionally compatible repair materials and standard material data sheet protocol. The recommended approach can enable material quality improvement, more accurate service life prediction, and satisfactory performance of repaired concrete structures during their intended service life.

Keywords: compatibility; cracking; durability; performance criteria; restrained shrinkage; volume change
Alexander M. Vaysburd, FACI, is the director of research and development for Structural Preservation Systems, Inc., Hanover, MD. He is an internationally recognized expert in concrete and cementitious materials. Vaysburd is a member of ACI Committee 213, Lightweight Aggregate Concrete; 546, Repair Concrete; and 364, Rehabilitation. He has authored or co-authored numerous papers on various aspects of concrete technology. He is a co-recipient of the 1996 ACI Wason Medal for the Most Meritorious Paper, recipient of the 2000 Cedric Wilson Award, and co-recipient of the 2000 ACI Construction Practice Award.

Randall W. Poston, FACI, is a Principal of Whitlock Dalrymple Poston & Associates, Manassas, VA, and Austin, TX. He is a member on ACI’s board of Direction and chairman of the TAC Repair and Rehabilitation Committee (TRRC). He is also a member of the Technical Activities Committee, a member and past Chairman of ACI Committee 224, Cracking, and a member of ACI Committees 222, Corrosion of Metals in Concrete; 228, Nondestructive Testing of Concrete; and 318 Subcommittee E, Shear and Torsion. He is a co-recipient of the 1998 ACI Construction Practice Award.

James E. McDonald, FACI, is a research civil engineer at the U.S. Army Corps of Engineers Research and Development Center, Vicksburg, MS. He is world renowned as a concrete repair expert who has presented numerous reports and papers. McDonald is chairman of ACI Committee 210, Deterioration of Concrete in Hydraulic Structures, and a member of ACI Committee 364, Rehabilitation; and 546, Repair of Concrete. He is past president of the International Concrete Repair Institute and a co-recipient of the 1996 ACI Wason Medal for the Most Meritorious Paper.

Science is the creation of concepts and their exploration in the facts. It has no other test of the concept than its empirical truth to fact. Truth is the drive at the center of science; it must have the habit of truth, not as a dogma but as a process.
- Jacob Bronowski, “Science and Human Values”

INTRODUCTION

Concrete repair is a complex process that presents unique challenges different from those experienced in construction of new structures. A successful concrete repair integrates new materials with old materials to form a composite system that should have long-term durability in a variety of aggressive exposure conditions. The durability of a repaired concrete structure and, thus, its service life depend on the quality of the composite system formed by the repair material and the existing reinforced concrete substrate. The behavior and coexistence of these two components, their marriage or divorce, critically depends on three factors:

- Design details and specifications
- Selection of repair materials
- In situ workmanship and quality control

This paper focuses on materials and their characteristics related to repair. The focus on materials does not mean, however, that materials are the major problem with the premature failure of repairs. The term “material” means
any physical substance that is used by man to make things that he needs; materials are not an end product, materials per se do not perform. The value of a material lies in its ability to permit an engineering product to fulfill its function. In the production of a product—a durable repaired concrete structure—design and workmanship are equally important. Achieving satisfactory performance of a repair requires much more than having a “good” material. It takes all the necessary parameters that influence “high performance.” Poor design and shoddy workmanship combined with aggressive exposure conditions all too frequently lead to repair failure.

Deterioration/distress of repaired concrete structures is a result of a variety of physico-chemical processes, such as corrosion of embedded reinforcing steel, effects of freezing and thawing, alkali-aggregate reaction, etc. The most serious deterioration processes that lead to repair failures are caused by cracking of the repair. Restrained contraction of repair materials, the restraint being provided through bond to the existing concrete substrate, is a major factor leading to cracking and failure of the repair phase. In simple terms, the repair material cracks when the tensile strain exceeds the tensile strain capacity. While development of tensile cracks may be favorable from the point of view of stress distribution in the texture of concrete, the situation becomes very different when judged from the point of view of the permeability of concrete—its capacity to retard penetration of aggressive elements into the concrete.

If the conceptual model of a repair system represents the anatomy of concrete repair (Figure 1), by analogy, the relative volume changes taking place in the system represent the physiology of concrete repair (Figure 2). Cracking in the repair phase, caused by restrained volume changes, is one of the truly insidious phenomena of repair-system pathology.

The popular blank statements that corrosion causes cracking is a classic case of misunderstanding the cause and effect relationship. Cracks facilitate the transport of aggressive ions and gasses through the concrete cover to the embedded steel. Cracks initiate and promote corrosion, especially in severe environments, and corrosion, in turn, causes enlargement of cracks. Increased cracking aggravates any one or a number of other mechanisms of deterioration (Figure 3). For example, in repeated cycles of freezing and thawing in a wet environment, water will enter the cracks during the thawing portion of the cycle only to freeze again resulting in progressive deterioration with each cycle.

In many cases, several of the factors that cause tensile stresses beyond the resistance capability of a particular material operate simultaneously or sequentially, and thus make cracking worse than it would have been had only one factor been operating. These are situations that open the way for
chemicals to penetrate and react to both surfaces of these cracks and produce further cracking. The permeability of concrete repair materials (realcrete) has very little to do with laboratory test data (labcrete), or with results of in-service permeability tests performed between cracks (foolcrete). The permeability of real repair is governed by the amount of cracking in it.

Low permeability of a repair material is important, but a material’s sensitivity to cracking is judged to be more important in determining the durability of concrete repairs. About thirty years ago, Valenta [1] observed that “continuous cracks sometimes linking into wider cracks, originating from the concrete surface, play the biggest role in reducing impermeability.” By better understanding repair composite properties, adjusting material properties, and using new products, it may be possible to produce repair materials that are more resistant to the formation of cracks. Once a novice asked Rafael with what he mixed his paints. The master replied, “With brains” [2]. Let’s hope and help material manufacturers to mix the repair materials “with brains.”

REPAIR MATERIALS

The worst deficiency in an engineering construction material is not lack of strength or stiffness, as desirable as these properties are; rather, it is lack of resistance to cracking and to propagation of cracks (toughness). Compensation can be made for lack of strength or stiffness in design, but cracking causes problems that adversely affect durability.

The durability of repaired concrete structures is directly linked to the compatibility of repair materials with existing concrete substrates. The crack resistance of cementitious repair materials is dependent upon the degree of restraint, the magnitude of the shrinkage, the stress state, the amount of stress relief provided by creep, the tensile strength, and the tensile strain capacity of the material. To have good resistance to cracking, a material should have low values for drying shrinkage and modulus of elasticity and high values for tensile strength and creep.

Materials science and engineering have produced some exceptional materials: polymer fibers that are as strong as steel, carbon fibers that are stronger than steel, and very high compressive strength concrete. High-range water-reducing admixtures and silica fume have been used to drastically lower water-cement ratios. For example, pumpable concrete mixtures can be proportioned with water-cement ratios as low as 0.25. Nonetheless, there are limits to what can be achieved with cement-based materials. For example, a significant increase in the tensile capacity of concrete is unlikely. Therefore, a primary engineering objective should be
to minimize the tensile strains in a repair to minimize the potential for cracking in repair systems.

It is usually tacitly assumed that if there are no detrimental effects on the strength of a repair material, long-term durability of the repair will not be drastically affected. However, the use of additives or admixtures in a material can effect the durability and serviceability requirements for concrete structures. For example, the addition of silica fume to concrete can reduce creep and relaxation and, thus, make the material potentially more susceptible to cracking. Also, the addition of high-range water-reducing admixtures reportedly increase the tendency for cracking [3]. Driven by the high speed of construction, many repair materials today (so-called high-performance materials) tend to contain high amounts of normal and high-early strength cement blends. The crack resistance of such mixtures is low because of an increase in shrinkage and elastic modulus and a reduction of the creep and tensile strain capacity. Materials with high compressive strengths are desirable in some special applications, but in most cases, high strengths are unnecessary and can actually be harmful. High strengths are generally achieved with high cement content; consequently, these materials exhibit higher temperatures during hydration, high moduli of elasticity, lower creep, lower ductility, and possibly higher drying shrinkage. In order to take advantage of the opportunities offered by cementitious composites we often become preoccupied with the novelty of new things whilst disregarding some basic long standing needs—to minimize cracking. Perhaps we shall reconsider our perceptions on high strength and “high-performance” materials.

The potential impact of any changes in material should be evaluated for the given application and exposure conditions; yet, the effect of changes in materials on the long-term durability of repair composites is probably the least researched aspect of the material development process.

All applications do not require materials with identical properties; therefore, a universal material with all properties optimized is not a research goal. The goal for the present and the future should be to develop a variety of economical repair materials that will meet engineering design specifications and that will also have the necessary properties for a given application.

**PERFORMANCE CRITERIA AND STANDARD MATERIAL DATA SHEET**

Traditionally, data supplied by material manufacturers have been used in the selection of repair materials. Manufacturers provide test results for properties thought to be the most relevant; however, these data are
frequently inadequate for key material properties such as tensile strength, tensile strain capacity, modulus of elasticity, shrinkage, and creep.

Because of the variety of ways currently available to manufacturers for achieving a level of performance for a material, including durability, there is considerable pressure to develop and use performance criteria as a basis for evaluating a material. Dimensional compatibility between a repair material and the existing concrete substrate is a critical factor that affects durability. Unfortunately, development of criteria for dimensional compatibility has not kept pace with improvements in materials, primarily because of the lack of appropriate laboratory and field data. Consequently, research was initiated in 1993 by the U.S. Army Corps of Engineers to develop performance criteria for cement-based repair materials.

The properties of selected, commercially available concrete repair materials were evaluated in the laboratory investigation [4]. Each of the 12 materials (6 cementitious and 6 polymer-modified cementitious materials) were subjected to a series of standard and nonstandard laboratory tests to determine material properties which were perceived to be of interest in a repair context and to provide some basic information about the behavior of the materials. A concurrent field exposure study was initiated in which the same materials were installed in simulated repairs and exposed to different environmental conditions. This experimental field program was conducted to provide more information about the behavior of the repair materials, especially as related to their restrained volume changes and resulting cracking [5].

Three locations were selected for field exposure tests: South Florida (Boca Raton), Illinois (Chicago), and Arizona (Phoenix). The 12 repair materials selected for this study were evaluated at each site. Each material was used to repair the cavities in three slabs at each test site (Figure 4). Repairs were monitored for a minimum of 18 months to determine field performance for correlation with the results of laboratory tests.

The field testing revealed that, overall, the 12 repair materials exhibited more resistance to cracking than was originally anticipated. Six of the materials demonstrated satisfactory resistance to cracking under the range of service conditions studied. Two other materials did not crack when exposed in Florida and Illinois but did exhibit cracking when exposed to the high temperature, low humidity service environment in Arizona. The remaining four materials cracked in each of the service environments. No clear performance trends were found in comparison of material performance in Florida and Illinois. However, the materials were much more sensitive to cracking in Arizona.

Results of the laboratory and field investigations were correlated in an attempt to evaluate how individual material properties, or combinations of
properties, affect the potential for cracking of field repairs. This correlation provided the basis for development of performance criteria for the selection and specification of dimensionally compatible cement-based repair materials (Table 1). The proposed performance criteria should be considered as a general profile of desired material properties. The relative importance of individual properties will vary depending on the anticipated application and service conditions for a given repair. Therefore, the requirements should be modified as appropriate for a specific repair.

One of the important tasks of the study was to evaluate the effect of creep or creep relaxation of a material on its resistance to cracking. Although almost all of the materials exhibited some measure of tensile creep, this property alone does not appear sufficient to offset restrained cracking in materials that are prone to high drying shrinkage. It should be recognized, however, that a number of difficulties have been experienced in performing direct tensile and tensile creep tests. Additional comprehensive testing is needed to better understand the interrelationship between tensile creep and the potential for restrained shrinkage cracking.

Material data sheets from numerous manufacturers and suppliers in North America were evaluated during the selection of the materials studied in this project. The evaluation revealed that these data sheets provide engineers very limited, and sometimes misleading, information on which to base the selection of materials for a particular project. Typically, only data on those properties favorable to a particular material are reported, and test procedures used by manufacturers to determine material properties vary widely. This type of information does not provide user confidence in the given properties of a material, nor is it a credible basis for selection of materials that will result in durable repairs. Obviously, there is a pressing need for the development of a standardized data sheet for repair materials. The performance criteria developed and information gained through this research study have been used to create a proposed standard data sheet that includes requirements for data on basic material composition and technical advantages and limitations of the material under specific application and service conditions (Table 2).

**CONCLUDING REMARKS**

This paper discussed which properties of materials, measured by standard or nonstandard tests, can be used to accurately predict field performance of repairs and what are the acceptable ranges of these properties.

Since most types of deterioration in reinforced cementitious materials involve the ingress of water and other aggressive agents into the structure,
its transport characteristics are directly related to its durability. Both permeability and diffusivity are substantially increased once open pathways through the material are present. These pathways, of course, are present on micro (diffusivity) and macro (cracking) scales. "Macro" permeability is solely dependent on cracking—the biggest single factor in the overall permeability and durability of a repair which is exposed to a severe environment. Unfortunately, the majority of the permeability research is devoted to the "micro" aspects of permeability in the time when common sense dictates that only when deserved attention has been paid to the "macro" permeability aspects does it become worthwhile to turn the attention to the "micro" aspects.

The objective of the study summarized herein was to ensure that materials specified and used for concrete repairs are dimensionally compatible with the existing concrete substrate so that repairs will have minimal cracking if any. Adoption of the proposed performance criteria and material data sheet protocol will enhance the potential for compatibility of a repair material with the existing concrete substrate, and the resulting composite system will provide the desired long-term durability.

The authors are well aware that present repair problems cannot be resolved just by improving the repair materials. Evaluation of existing concrete conditions, design details and quality of workmanship on site are also of fundamental importance in ensuring durability of repaired structures. The environment in which the material is applied and cured can have a significant effect on material properties. However, adoption of the proposed performance criteria and material data sheet protocol with standardized test methods will allow clearer judgments of the relevance and reliability of performance testing data for given project conditions. Plenty of demanding and fruitful research is still to be done in the practical field of deformability and crack resistance of cementitious composites.

REFERENCES


Figure 1. Anatomy of concrete repair.
Figure 3. A holistic model of concrete repair failure.
Figure 4. Repair test slab.
Cracking in Concrete Structures During the August 17, 1999 Earthquake in Turkey

By M. Saatcioglu

Synopsis

A reconnaissance visit was conducted to Turkey shortly after the August 17, 1999 Earthquake to investigate the performance of concrete structures. The dominant form of construction in the area was reinforced concrete frames, infilled with masonry walls. Extensive cracking and damage was observed in most structures located in the disaster area. This paper presents an overview of the types of cracking that can be expected after a seismic activity, as well as those observed after the August 17, 1999 Earthquake in Turkey. Causes of seismic damage are discussed with examples. A brief review of the seismological aspects of the earthquake and the overall performance of reinforced concrete buildings are provided.

Keywords: concrete structures; cracking; earthquakes; earthquake engineering; Kocaeli earthquake; natural disasters; reinforced concrete design; seismic design
ACI Fellow Murat Saatcioglu is a professor of structural engineering at the University of Ottawa, Ottawa, Canada. He is a member of the Canadian National Committee on Earthquake Engineering, a director of the Canadian Association for Earthquake Engineering and associate director of the Ottawa-Carleton Earthquake Engineering Research Center. Dr. Saatcioglu is a member of ACI Committees 374 Performance-Based Seismic Design of Concrete Buildings, 441 Concrete Columns and the Chairman of 340 Design Aids for the ACI Building Code. He is the recipient of a number of awards, including the American Society of Civil Engineer’s (ASCE) 2000 Raymond C. Reese Research prize.

INTRODUCTION
An earthquake with a Richter magnitude of 7.4 struck north western Turkey on August 17, 1999, lasting about 50 seconds and resulting in massive destruction and loss of lives. It was one of the strongest earthquakes of the 20th century, and the largest seismic event recorded since the 1906 San Francisco and the 1923 Tokyo earthquakes that affected an industrialized region of the world. The epicenter was near the town of Golcuk, immediately to the south of Izmit and approximately 80 km south-east of Istanbul with over 10 million population. Figure 1 illustrates the location of the epicenter and the areas that sustained heavy damage. The official figure for casualties was about 17,000, though the unofficial estimates are as high as 40,000. A large number of buildings either collapsed or sustained heavy damage, leaving an estimated 750 thousand people in need of housing. The estimates of buildings severely damaged or collapsed was 77,000. A similar figure was given for buildings with moderate damage. An estimated 90,000 buildings were lightly damaged. The earthquake affected approximately 35% of the industrial base in Turkey, creating a total financial loss of approximately $ 15 to $ 20 billion.

A week long reconnaissance visit was conducted shortly after the earthquake. This paper highlights the observations made in terms of the extent and types of cracking observed in concrete frame and shear wall structures.

The earthquake was a result of the rupture of the North Anatolian Fault in northern Turkey, that runs parallel to the Black Sea coast. The North Anatolian fault resembles in many ways the well known San Andreas fault along the Californian coast. The tectonic movement is measured to be approximately 24 mm every year along the fault. It produces a right lateral slip, with significant horizontal offset every time it ruptures. The fault is approximately 750 km long, starting near the north-west shores of the Marmara Sea in the west, and extending to Erzincan in the east, where it joins the North-East Anatolian Fault. After the August 17, 1999 earthquake, the horizontal offset along the fault was measured to vary between 2.5 and 4.5 m. Figure 2 illustrates the horizontal offset observed along the fault. Although a vertical offset of 2.0 m was reported by others, no appreciable vertical offset was observed by the author in the segments inspected. The maximum horizontal peak acceleration was recorded to be 0.41g in the east-west direction, at a station in Adapazari. The vertical peak acceleration at the same station was 0.26g, though the maximum vertical acceleration recorded was in Duzce (east of Adapazari), with a magnitude of 0.49g.
NATURE OF CRACKING CAUSED BY SEISMIC EXCITATIONS

Extensive cracking of concrete is a natural phenomenon that is expected to occur during an extreme event like a strong earthquake. Therefore, the current seismic design practice is based on the dissipation of seismic induced energy by significant yielding of members in their critical regions. This implies that even properly designed concrete structures are expected to develop extensive cracking during strong earthquakes due to inelastic deformations. This however, should not necessarily be interpreted as widespread cracking everywhere in the structure, in the entire building stock of the area. Inelasticity is often limited to the critical regions of structural elements. Cracking outside these regions is often controlled by the presence of sufficient reinforcement, producing elastic (post-cracking but pre-yielding) behavior.

Flexure dominant elements may experience wide flexural cracks near the ends where maximum negative and positive moments result in the formation of plastic hinges, signifying yielding of flexural reinforcement. Longitudinal strains in these regions often exceed 0.2%, sometimes approaching 1% strain, entering into the strain hardening region. A wide crack often becomes visible at the interface of the adjoining member. This crack results from the penetration of yielding in reinforcement into the adjacent member. The amount of plastic strain and associated elongation of steel within the adjoining member governs the crack width at the end of the member. This phenomenon is often referred to as "anchorage slip," implying the relaxation of flexural fixity provided by the supporting element due to the yielding of anchoring reinforcement. Additional cracks form near the ends of members due to flexure. Cracks that are within a region approximately equal to the member depth may be wide enough to suggest yielding of reinforcement. This region is referred to as the plastic hinge region, and is caused by redistribution of stresses under progressively increasing plastic deformations. Both the interface crack and the cracks of the plastic hinge region are associated with plastic deformations of reinforcement that produce cracks in the surrounding concrete. Typical cracks caused by flexure are illustrated in Fig. 3(a). These cracks run perpendicular to the member axis, like all other flexural cracks do. However, seismic induced cracks are produced by stress reversals. Therefore, they propagate towards both extreme fibers, creating continuous cracks across the entire section depth. These cracks open and close during seismic response. Small crack widths of pre-yield sections do not result in any appreciable degradation of concrete under reversed cyclic loading. However, the interface crack, as well as those that are in post yield regions, do not close immediately when subjected to compression, as the previously yielded tension reinforcement has to be compressed sufficiently to overcome the extension of reinforcement that was caused by previous yielding in tension. In these sections, the closed crack surfaces do not necessarily mate perfectly, resulting in grinding of cracked surfaces and deterioration of concrete, causing strength degradation. Until this degradation occurs, however, the flexural response exhibits stable hysteresis loops with large energy dissipation
characteristics. Another common form of cracking encountered during seismic response is the shear cracking. Shear cracks are caused by diagonal tension and appear in the form of inclined cracks. They may or may not be combined with those caused by flexure. These cracks initiate when the diagonal tension capacity of concrete is exceeded. They become wider in the absence of sufficient reinforcement. Yielding of transverse shear reinforcement leads to the widening of diagonal cracks, which may trigger a shear failure. Because the shear failure is regarded as a brittle form of failure, buildings are designed to have higher shear capacities than those corresponding to flexural failure. However, observations during previous earthquakes indicate that a majority of structural failures can be attributed to poor behavior in shear. This may be explained by lack of transverse shear reinforcement, as well as degradation that occurs in the mechanism of shear resistance during seismic response. Indeed, criss crossing of inclined cracks under reversed cyclic loading, as illustrated in Fig. 3 (b), deteriorates concrete and diminishes the overall shear resistance significantly. Inclined shear cracks show pinching of the hysteresis loops, reducing energy dissipation of the system.

Diagonal cracks, covering the entire element may also form in infill walls, which develop diagonal struts and ties during load reversals, generating “X” cracking between the corners. This is illustrated in Fig. 3 (c).

Another form of shear cracking can be seen in beam-column connections of frame structures. The horizontal tension force generated by flexure in the longitudinal beam reinforcement, that is anchored in the column, and the flexural compression force that develops in beam concrete that pushes against the column, combined with the column shear, result in shear force reversals in beam-column joints. This type of joint shear cracking may be critical in frames of insufficient joint reinforcement. This is a widespread problem in the majority of existing buildings, as practical considerations during construction often results in either the complete omission or placement of insufficient transverse reinforcement in columns within the joint region. Many failures in the past have been attributed to poor behavior of joints, even if the individual members were properly designed. Typical joint cracks are schematically illustrated in Fig. 3 (a).

Aside from the perpendicular flexure and inclined shear cracks, longitudinal splitting cracks are often observed after strong earthquakes. Lack of proper development and/or splice of longitudinal reinforcement produce longitudinal splitting cracks that become visible along longitudinal reinforcement. Similar cracks seen on concrete cover may sometimes indicate the onset of spalling under excessive compression.

The following sections presents the types of seismic induced cracks observed after the 1999 Kocaeli Earthquake in Turkey.
OBSERVED DAMAGE AND CRACKING

The dominant structural system used in Turkey consists of reinforced concrete frames with masonry infills. Concrete, which is locally available, is generally preferred over other construction materials for economic reasons. The majority of concrete is used for cast-in-place construction, with increasingly larger percentage becoming ready-mix concrete. Precast construction is employed often for industrial buildings. Concrete shear walls appear to have gained popularity only in recent years. The structural damage was most intense in regions with soft soil. Therefore, it is important to recognize the amplifying role of soft soil on structural performance. A large portion of the area that showed significant structural damage has alluvial soil, including old river beds. Soil borings in the area consistently showed sand, silty sand, and clay at depths of up to and in excess of 20 m. Locals consistently indicated that the buildings could not have proper foundations because of the high water table only 0.5 m to 2.0 m below the surface. This provided suitable conditions for soil to liquefy. Figure 4 shows building failures caused by soil liquefaction.

The majority of collapses during the earthquake were attributed to poor performance of reinforced concrete frames and masonry infill walls. Buildings with 4 to 6 stories suffered the heaviest damage, inflicting most of the casualties. However, buildings that survived the earthquake also had the same framing system, including those that make up the large inventory of buildings in the city of Istanbul creating a great deal of controversy. It is important to note that, judging by the earthquake records, the structures in the entire earthquake stricken area were subjected to very high seismic demands, resulting in total collapses of thousands of buildings. Figure 5 illustrates the extent of destruction caused by the Earthquake.

Inspection of collapsed and damaged buildings revealed that very little or no seismic design had been implemented during design and construction of reinforced concrete frame systems. It was clear that the structural layouts used were susceptible to very high seismic deformation demands due to lack of proper bracing elements and extensive use of soft stories. The only mechanism of defense in such high demand structures would be the inelastic deformability of the structural systems. Unfortunately, all the buildings inspected lacked standard seismic design and detailing practices, which could have provided the inelastic deformability needed to save a great majority of structures. Proper design practices were missing in spite of the seismic design requirements of the Turkish Code.

Flexure Dominant Response and Resulting Cracking

The majority of buildings in the area were reinforced concrete frames, with unreinforced concrete or brick masonry infill walls. Lateral bracing in these buildings was provided by the masonry walls. The brick masonry used was often in the form of hollow architectural units. During the earthquake, these walls were able to participate in lateral load resistance to varying degrees, and were often damaged prematurely, developing diagonal tension and compression failures. The degree of lateral load resistance depended on the amount of masonry used, and the
framing system provided. In contrast to modern moment resisting frames of North American practice, the use of light partitions, such as dry walls, was not common in the earthquake stricken areas. Instead, masonry was used extensively for interior partitioning, as well as exterior enclosure of buildings, increasing wall-to-floor area ratios. Therefore, in spite of lower strength and expected brittleness of this type of masonry walls, the frames did benefit somewhat from such extensive use of masonry until the threshold of elastic behavior was exceeded. Beyond the failure of brittle masonry, there was no lateral bracing to control lateral drift, thereby resulting in high drift demands. Concrete frames, subjected to these high inelastic drift demands experienced inelastic flexural deformations and associated wide cracks. Figure 6 illustrates flexural cracking at the ends of beams in two different buildings, wide enough to suggest the onset of yielding in reinforcement.

Where the drift demand was very high, extensive plastification was observed in the hinging regions of beams. Figure 7 shows large inelastic deformations in the beams of a frame structure, which also shows wide enough cracks to suggest strain hardening in steel. The beams in Fig. 7 also indicates significant extension of beam reinforcement in the adjoining member, developing wide interface cracks. It is clear that these buildings were designed following the “strong-column weak-beam” concept which promoted the yielding of beams prior to column failure, though some spalling of the concrete at beam column joints were observed due to lack of joint reinforcement and buckling of column bars, pushing the cover outward, resulting in early spalling. Figure 8 illustrates flexural cracking of a beam, accompanied by column shear failure, prior to the development of full plastic hinge in the beam. This structure shows an example of a “weak-column strong-beam” design, examples of which were widespread in the disaster area.

**Diagonal Tension Cracks and Shear Failures**

During seismic response, the failure of brittle masonry walls placed heavy burden on the first story columns of multi-story buildings. The columns sustained heavy damage mostly because of lack of sufficient transverse reinforcement. The transverse steel consisted of 8.0 mm smooth reinforcement, generally placed at 300 mm or wider spacing. In some buildings some of the ties were left out as illustrated in Fig. 9. The ties did not appear to be sufficient either in terms of amount or detailing. This resulted in widespread diagonal tension cracks in columns and sometimes column shear failures, as illustrated in Fig. 10. In the majority of cases, the transverse reinforcement was limited to perimeter ties with 90 degree hooks. Columns that were subjected to heavy axial compression and flexural compression resulted in the crushing of concrete due to lack of confinement. The lack of transverse reinforcement was also observed in monolithic beam-column connections. Beam-column connections in the majority of buildings did not contain any transverse reinforcement, suggesting that joint shear design was never a consideration in these buildings. Figure 11 illustrates damage due to lack of confinement and joint reinforcement.
Deformation capacities of some structural elements were impaired because of unintended interference of non-structural elements with the structure. As masonry walls participated in lateral load resistance of the framing system, short column effects were created around window and other openings. Columns, not designed for the increased shear associated with reduced unsupported height, suffered brittle shear failures as depicted in Fig. 12(a). In some buildings, the landing slabs of staircases were connected to columns, and either applied unexpected lateral forces or caused short column effects as shown in Fig. 12(b). Figure 12(b) also illustrates joint shear cracks at beam-column joints.

Shear behavior of reinforced concrete shear walls were also investigated. The use of reinforced concrete shear walls in Turkey is limited, especially in older buildings. A number of buildings were found with narrow shear walls, which in some cases may be labeled as wide rectangular columns. These buildings performed reasonably well. Figure 13 illustrates an apartment complex under construction with light-weight concrete masonry units and narrow shear walls. These buildings survived the earthquake with minor damage to the structural framing system, although suffered extensive damage to masonry. Figure 13(b) shows a shear wall in this complex that developed diagonal shear cracks wide enough to suggest some yielding in reinforcement, but survived the earthquake, while also saving the entire structure. There were other shear wall buildings with older and significantly lower quality concrete. Although the concrete in these walls was damaged extensively, the shear walls did save the structures from collapsing. This is illustrated in Fig. 14.

Properly designed shear wall structures were found in the residential complex of Tupras Oil Refinery in Izmit. The entire complex consisted of frame-shear wall interactive systems. The building survived the earthquake without any sign of distress, though many frame buildings in the general area suffered extensive damage and complete collapses. Figure 15 illustrates the apartment complex and a typical shear wall building in this residential complex, with no damage and visible signs of cracking.

**SUMMARY AND CONCLUSIONS**

Reinforced concrete frame and shear wall buildings suffered extensive damage during the earthquake of August 17, 1999 in Turkey. The intensity of earthquake was very high, and imposed high inelastic deformation demands on structures. Many buildings collapsed entirely. A large number of buildings suffered irreparable damage caused by extensive inelasticity and lack of proper seismic design and detailing practices.

Two types of failure modes and associated crack patterns dominated building response. These were caused by flexure and shear. Wide flexural cracks, suggesting yielding and strain hardening of longitudinal reinforcement were observed mostly in the hinging regions of members. Wide interface cracks were also observed at member ends, signifying yield penetration into the adjacent members. Inclined cracks caused by diagonal tension was widespread among
concrete members. Lack of sufficient transverse reinforcement was evident in most damaged buildings. Wide shear cracks were observed in beam column connections, resulting in deterioration of concrete. Similarly, criss crossing of diagonal cracks in shear walls and columns were observed, sometimes leading to complete failures. Very little or no seismic design was employed in the buildings, in spite of fairly stringent earthquake resistant design requirements of the Turkish Code.

ACKNOWLEDGMENTS
The reconnaissance visit to Turkey was funded by the Natural Sciences and Engineering Research Council of Canada. The author is grateful to Bogazici University, Kandilli Observatory and Earthquake Research Institute, particularly to Drs. Mustafa Erdik and Ozal Yuzugullu for their cooperation and assistance.
Fig. 1—Epicenter and damaged areas

(a) Railroad offset  
(b) House offset

(c) Offset in tree line  
(d) Offset in water channel

Fig 2—Horizontal fault offsets observed
Fig. 3—Common types of cracking caused by seismic loading

Fig. 4—Examples of building failures due to soil liquefaction
Fig. 5—Sample failures illustrating the extent of damage and destruction

Fig. 6—Flexural cracks at beam ends
Fig. 7—Inelastic beam deformations and formation of plastic hinges
Fig. 8—Beam flexural cracking followed by premature column shear failure

Fig. 9—Lack of column ties

Fig. 10—Diagonal tension cracks in columns
Fig. 11—Lack of; (a) confinement and (b) joint shear reinforcement

Fig. 12—Diagonal tension cracks resulting from "short column" effect
Fig. 13—Apartment complex with light-weight masonry infills and narrow concrete shear walls

Fig. 14—Older concrete shear walls with extensive diagonal cracking
Fig. 15—Shear wall-frame interactive buildings of the Tupras Refinery