ACI Code Requirements for Deflection Control: A Critical Review

by A. Scanlon, D. R. Cagley Orsak, and D. R. Buettner

Synopsis: ACI Building Code requirements for deflection control are critically reviewed. Provisions for minimum thickness, deflection computations, and permissible computed deflections are reviewed. Differences in the approaches to deflection control for one-way and two-way construction are identified. Limitations in the application of the prescribed deflection calculation method are discussed. Results of a survey of consulting firms concerning deflection control in design offices are presented. The paper concludes by suggesting possible directions for future changes in building code requirements for deflection control.

Keywords: beams; building code; cracking; deflection; minimum thickness; slabs
Deflection control is an important serviceability consideration in the structural design of concrete buildings. While provision of an adequate level of safety against collapse is the primary design consideration, the structural engineer must take into account possible adverse effects of excessive deflections on the performance of the structure at service load levels. Potential problems associated with excessive deflections are well known and include damage to nonstructural elements including partitions and windows, jamming of doors and windows, gaps between partitions and floors, and between columns and floors, improper operation of equipment, visual perception of sagging floors and ceilings, and the need to provide expensive floor leveling materials.

Some guidance is provided in Section 9.5 of the ACI Code (ACI 318-99) on design for deflection control of one-way and two-way nonprestressed construction, prestressed concrete construction, and composite construction. The general approach to deflection control in the code has remained essentially unchanged since 1971. This paper presents a review of the current provisions with some suggestions for improvement of these provisions.
DEFLECTION CONTROL AND THE BUILDING CODE

The introduction to the 1999 ACI Code contains the statement: “A building code states only the minimum requirements necessary to provide for public health and safety. The code is based on this principle.”

Because deflection control is an issue which for the most part has no impact on public health and safety the question is sometimes raised as to why deflection control should be part of the building code. Some engineers have expressed the opinion that deflection control is an issue between the design engineer and client and should not be dealt with in the building code. A structure may have unacceptably large deflections and yet have an adequate margin of safety against collapse. Deflection control is an economic issue involving a balance between first cost and potential costs associated with maintenance, repair and other costs that might be incurred as a result of a problem related to deflection. There is also the question of the engineer’s professional reputation and potential professional liability issues. Deflection control has traditionally been mentioned in the code even though it is not a safety issue. Engineers will continue to look to the code both for guidance and criteria for some measure of acceptability in deflection levels.

Because serviceability criteria are not as clearly defined as safety criteria and vary much more depending on requirements of particular projects, the question of establishing minimum requirements becomes quite difficult. The engineer may well have to exceed the minimum requirements given in the code to meet the requirements of the client. In many cases, the client will rely on the professional judgment of the engineer as to the needs for a particular project.

ACI 318 REQUIREMENTS

The ACI Code provides a two-tier approach to deflection control. Under certain circumstances deflection control requirements will be deemed to have been satisfied if the flexural member depth is greater than a minimum value expressed as a fraction of the span length. Otherwise the code requires that deflections be computed and compared with specified permissible values. Details of these provisions are outlined in the following sections.

Minimum Thickness Requirements

Minimum thicknesses for one-way slabs and beams are given in Table 9.5(a) of ACI 318-99. In this table span length is defined as clear span plus depth of member but need not exceed distance between centers of supports. The table is reproduced in Appendix A.

These minimum thicknesses can be used for members NOT supporting or attached to partitions or other construction likely to be damaged by large deflections. Members that do support or are attached to partitions or other
construction likely to be damaged by large deflections are not covered by
Table 9.5 (a) and consequently, deflections should be computed for these
members and compared to specified permissible values.

Minimum thicknesses for two-way slab systems are presented in Table 9.5
(c) and Eqs. 9-11 and 9-12 of ACI 318-99. In this case the span length to be
used is defined as length of the clear span in long direction, measured face­
to-face of supports. In contrast to one-way construction, no restrictions are
placed on use of these minimum thickness values for members supporting
partitions. However the commentary does note that “The minimum
thicknesses in Table 9.5 (c) are those that have been developed through the
years.” and “These limits apply to only the domain of previous experience
in loads, environment, materials, boundary conditions, and spans.” This
cautions note in the commentary suggests that care should be taken in
selecting the slab thickness for arrangements that fall outside the realm of
previous experience.

The minimum thickness requirements for one-way and two-way
construction appear to have developed independently and as a result a
number of inconsistencies can be identified in comparing the two sets of
requirements:

a) Different definitions of span length: There appears to be no reason
to use different definitions of span length. For members built
integra1ly with supports, whether one-way or two-way, clear span
seems to be the logical choice while for members not built integra1ly
with supports, distance between edge of bearing seems to be a
reasonable definition. The question also comes up in determining
permissible computed deflections as discussed later.

b) Different levels of conservatism for one-way and two-way slabs: If
a two-way flat plate is designed based on minimum thickness and
subsequently changed to a one-way system by addition of stuff
beams or walls along column lines in the short span direction, an
increase in minimum thickness or deflection calculations would be
required. It is not logical to increase the slab thickness when
stiffening elements have been added. Scanlon and Choi (1999)
showed that the one-way slab minimum thickness values are
generally conservative for typical building spans and recommended
an alternative approach to minimum thickness. Proposals for
revising minimum thickness of one-way or two-way construction
have also been presented by Rangan (1982), Grossman (1981),
Thompson and Scanlon (1988), and Gardner and Zhang (1995)
among others.
(c) Different treatment of partition damage: The minimum thickness for two-way construction is applicable even if the slab is supporting partitions, which is not the case for one-way slabs.

Computed and Permissible Deflections

The code prescribes a simplified methodology for computing deflections although more comprehensive methods are permitted as long as effects of cracking and reinforcement on member stiffness are considered for immediate application of load, and creep and shrinkage are taken into account when considering long-term deflection. The effective moment of inertia concept is used to allow for a gradual transition from uncracked to fully cracked stiffness as loading increases. Effective moment of inertia is given by,

\[ I_e = \left( \frac{M_{cr}}{M_a} \right)^3 I_x + \left[ 1 - \left( \frac{M_{cr}}{M_a} \right)^3 \right] I_{cr} \]

(1)

Where,

\[ M_{cr} = \frac{f_c I_x}{y} \]

(2)

and

\[ f_r = 7.5 \sqrt{f'_c} \]

(3)

A simple long-term multiplier is provided for the computing of long-term deflections. The multiplier is applied to the computed immediate deflection corresponding to the sustained load level considered. The multiplier is given by,
\[ \lambda = \frac{\zeta}{1 + 50 \rho} \] (4)

Where \( \zeta \) takes the following values

- 5 years or more ................ 2.0
- 12 months ...................... 1.4
- 6 months ....................... 1.2
- 3 months ....................... 1.0

The compressive reinforcement ratio is included in Eq. 4 to account for the effect of reinforcement in the compressive zone which resists creep deformation. The commentary points out that long-term deflections are affected by many factors including temperature, humidity, curing, and age at time of loading. Many references are available for computing creep and shrinkage effects separately including those listed in the commentary and the ACI 435 report on Control of Deflection in Concrete Structures (ACI 435-95R). The simple multiplier is considered by ACI 318 to be satisfactory for use with the code procedures and the limits specified in Table 9.5 (b).

If minimum thickness cannot be used to satisfy deflection control requirements, deflections must be computed and compared to specified permissible deflections listed in Table 9.5 (b) of ACI 318-99 (See Appendix). The code requires two separate deflection calculations, one for immediate deflection due to live load, and one for that part of the total deflection occurring after attachment of non-structural elements (sum of the long-term deflection due to all sustained loads and the immediate deflection due to any additional live load). The latter will be referred to as the "incremental long-term deflection".

The calculation of live load deflection depends on whether the live load is assumed to occur during first-cycle loading or subsequent loading. If first cycle loading is assumed and cracking occurs during loading, the live load deflection is taken as the difference between the total load deflection and the dead load deflection. Since the stiffness gradually decreases as loading increases due to progressive cracking, the stiffness under dead load is greater than the stiffness under total load. On the other hand, if it is assumed that the member has already experienced an application of live load, it should be recognized that the member cannot become uncracked after removal of the live load. In this case the stiffness for both dead load and total load is that corresponding to application of total load. This is illustrated in Figure 1. The definition of \( M_a \) in ACI 318-99 implies that first cycle loading should be considered since \( M_a \) is defined as the maximum
moment in the member at stage deflection is computed. Dead load
deflection would then be calculated based on the dead load moment. In
many concrete structures the highest loading is experienced during
construction as a result of shoring, reshoring, and other construction loads.
In such cases, the stiffness for all service load checks should be that
associated with dead plus live load. This has the effect of decreasing the
computed live load deflection but increasing the computed dead load
deflection and consequently the incremental long-term deflection. In the
event that restraint built into the structural system would produce shrinkage
cracking then that should be taken into account in the design.

The effective moment of inertia procedure has now been a part of the code
for almost 30 years and has generally been found to provide satisfactory
results for members with medium to high reinforcement ratios. However
some difficulties have been experienced in applying the method to members
with low reinforcement ratios, particularly lightly reinforced slabs for which
flexural stiffness is sensitive to cracking. The following factors contribute
to poor correlation between computed and measured deflections in these
cases.

1. The modulus of rupture specified as $7.5\sqrt{f_c}$ represents a low
estimate of the material property as determined from laboratory tests
on small samples. ACI 209 reports that the typical range is about 6
to $12\sqrt{f_c}$. On the surface it appears that a conservatively low
estimate of the cracking moment is being used to compute
deflections. However, cracking is affected not only by applied loads
used to compute $M_3$, but also by restraint of shrinkage and
temperature deformations. In some cases cracking can be detected
before forms are removed and the member is subjected to dead load.
The effect of restraint cracking should therefore be considered in
deflection computations.

2. If incremental long-term deflection due to sustained load is
computed based on first cycle loading, the stiffness will often be
based on an uncracked section because the dead load moments may
be less than the computed cracking moment. Recognizing that loads
approaching the specified live load may be experienced at early age,
the sustained load deflection should be computed based on the
stiffness associated with dead plus live load. The problem may be
exacerbated by the tendency for unanticipated overloading of slabs
due to shoring and reshoring.

3. A further complicating factor comes into play for flat plates and flat
slabs supported on columns. The elastic distributions of moments
adjacent to columns produce locally high intensities that invariably
initiate cracking in the negative moment regions around columns.
The cracking allows redistribution of these high moments but the stiffness has been reduced.

4. For members with low reinforcement ratios the rate at which the effective moment of inertia approaches the fully cracked moment of inertia is too low.

To account for the factors mentioned above, Scanlon and Murray (1982) recommended that a reduced effective modulus of rupture be used in computing deflections of two-way slab systems. A value of $4\sqrt{f_c}$ has been found to produce satisfactory results in several case studies involving field measured deflections. A similar approach has recently been suggested by Gilbert (1999). Because of the many uncertainties and variabilities associated with estimating the factors that affect deflections a more precise estimate may not be worthwhile.

The historical development of deflection limits has been reported by Warwaruk (1979). It appears that deflection limits in use today have developed over the years based on experience with rules of thumb dating back at least to the 19th century. In the past thirty years or so a great deal of effort has been put into developing more refined algorithms for computing deflections. At the same time relatively little effort has gone into defining appropriate criteria for deflection limits (Scanlon and Pinheiro, 1992). Four deflection limits are specified in Table 9.5 (b), two for live load deflection, and two for incremental long-term deflection, depending on whether or not the deflection is likely to cause damage. These limits are specified as fractions of span length, $l/180$ and $l/360$ for live load deflection, $l/240$ and $l/480$ for incremental long-term deflection. The limit for incremental long-term deflection, $l/480$, which applies when damage to non-structural elements is likely, can be exceeded if special precautions are taken to prevent damage. The ACI code does not place a limit on total deflection as some codes do, however the other limits indirectly limit the permissible total deflection, immediate and long-term. While these deflection limits appear to cover a broad spectrum of design situations quite satisfactorily, it is recognized that there may be situations requiring more stringent limits. For example a limit of $l/1000$ has been suggested for members supporting brittle partitions such as unreinforced masonry. ACI Committee 435 (1963) provides suggested limits to cover some applications requiring more stringent limits than given in the code.

In computing the incremental long-term deflection the engineer needs to make an assumption regarding the time of installation of non-structural elements. In most cases this will be unknown at the design stage. It is probably not unreasonable to make the conservative assumption that installation will take place immediately in which case the full value of the multiplier would be applied. It is also necessary to make an assumption
regarding the portion of live load to be considered as sustained. Live load surveys for office and similar buildings have shown that the actual live load at any time is typically quite low compared to the design live load.

The span length to be used in applying Table 9.5 (b) to calculate the permissible deflection limit for two-way slab systems is not clearly defined. If the mid-panel deflection is computed it appears to be most logical to take the span length measured along the diagonal whereas if the column strip deflection is being calculated, the span length should be measured between the two columns on the column strip. It would also seem logical to use clear span for calculating permissible deflection limits.

In addition to the detailed treatment of deflection of one-way and two-way nonprestressed construction given in the code some general guidance is provided for deflection of prestressed and composite construction.

**Construction Requirements**

Deflection control is not simply a design issue. At least as important is the proper attention to construction procedures. Section 6.2 of ACI 318-99 addresses the question of removal of forms, shores, and reshoring, and includes the general statement: “Forms shall be removed in such a manner as not to impair safety and serviceability of the structure.”

**DEFLECTION CALCULATION PRACTICES IN THE U.S.**

Consulting engineers typically design structures (and particularly concrete structures) using purchased software. There are a number of relatively inexpensive and readily available software packages for design of concrete beams, slabs and two-way systems. One can argue that the design practice for deflections is whatever procedures are built into commonly used commercial software.

ACI Committee 435 conducted a survey of consulting firms in 1999. The results of that survey are most interesting. All respondents use one or more commercial software packages. That software in turn incorporates the basic code prescribed procedures for deflections. Essentially the \( L_{\text{effective}} \) concept is used along with all code prescribed procedures for long term deflection calculation. The deflection is compared with permissible limits (e.g. \( L/360 \)) and member sizes are increased until the deflection criteria is satisfied.

The software does not include options to enable the designer to account for restraint (and the effect this would produce on cracking and, in turn, on
stiffness). Software incorporates \( 7.5\sqrt{fc} \) as the modulus of rupture and designer input on this parameter is not available.

According to the survey, engineers are generally comfortable with purchased software and feel it adequately predicts deflections. If any calculation problems are perceived to exist, engineers feel that the deflection predictions for two-way systems (flat slabs and flat plates) are the most unreliable and that the limited deflection related problems that have occurred in structures have occurred in two-way systems.

**FUTURE CODE DEVELOPMENTS**

The deflection control provisions given in the ACI code appear to have resulted in satisfactory performance of concrete structures over the years. While deflection at service load levels is understood to have no impact on structural safety, it is expected that deflection control will continue to be an integral part of the code, to provide guidance to engineers in design for serviceability. This paper has pointed out a number of areas where improvement and clarification of the code provisions are possible. These include a more unified treatment of one-way and two-way construction, and additional guidance on deflection calculations for two-way systems. Computers are becoming increasingly used in structural design practice and future developments of the code provisions should keep this in mind.

As new developments are made through research involving different reinforcing materials, the calculations for deflection of members using these materials will have to be reviewed. The new materials have different properties than the Grade 60 reinforcement on which the current deflection calculations have been based.

The code should continue to provide guidance to designers on deflection limits, but perhaps some of the limiting conditions could be better defined.

**REFERENCES**

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ACI Committee 435 (1995). Control of Deflection in Concrete Structures (ACI 435-95R). American Concrete Institute, Detroit, 77 pp.

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Warwaruk, J. (1979). Deflection Requirements - History and Background Related to Vibrations. *ACI Special Publication SP 60*, American Concrete Institute, Detroit, pp 13-41.
### TABLE 9.5(a) — MINIMUM THICKNESS OF NONPRE-STRESSED BEAMS OR ONE-WAY SLABS UNLESS DEFLECTIONS ARE COMPUTED

<table>
<thead>
<tr>
<th>Member</th>
<th>Simply supported</th>
<th>One end continuous</th>
<th>Both ends continuous</th>
<th>Cantilever</th>
</tr>
</thead>
<tbody>
<tr>
<td>Solid one-way slabs</td>
<td>/'20</td>
<td>/'24</td>
<td>/'28</td>
<td>/'10</td>
</tr>
<tr>
<td>Beams or ribbed one-way slabs</td>
<td>/'16</td>
<td>/'18.5</td>
<td>/'21</td>
<td>/'8</td>
</tr>
</tbody>
</table>

### TABLE 9.5(b) — MAXIMUM PERMISSIBLE COMPUTED DEFLECTIONS

<table>
<thead>
<tr>
<th>Type of member</th>
<th>Deflection to be considered</th>
<th>Deflection limitation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flat roofs not supporting or attached to non-structural elements likely to be damaged by large deflections</td>
<td>Immediate deflection due to live load L</td>
<td>/'180</td>
</tr>
<tr>
<td>Floors not supporting or attached to non-structural elements likely to be damaged by large deflections</td>
<td>Immediate deflection due to live load L</td>
<td>/'360</td>
</tr>
<tr>
<td>Roof or floor construction supporting or attached to nonstructural elements likely to be damaged by large deflections</td>
<td>That part of the total deflection occurring after attachment of nonstructural elements (sum of the long-term deflection due to all sustained loads and the immediate deflection due to any additional live load)</td>
<td>/'420</td>
</tr>
</tbody>
</table>

### TABLE 9.5(c) — MINIMUM THICKNESS OF SLABS WITHOUT INTERIOR BEAMS

<table>
<thead>
<tr>
<th>Yield strength, f_y (psi)</th>
<th>Without drop panels†</th>
<th>With drop panels†</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Exterior panels</td>
<td>Interior panels</td>
</tr>
<tr>
<td></td>
<td>Without edge beams</td>
<td>With edge beams†</td>
</tr>
<tr>
<td>40,000</td>
<td>33</td>
<td>36</td>
</tr>
<tr>
<td>60,000</td>
<td>35</td>
<td>33</td>
</tr>
<tr>
<td>75,000</td>
<td>28</td>
<td>31</td>
</tr>
</tbody>
</table>

Note: Span length, l (center-to-center span) is defined in ACI 318, Section 8.7. Span length, l (clear span) is defined in ACI 318, Section 9.0.
Figure 1. Effect on Loading Cycle on Dead and Live Load Deflections.
Serviceability Provisions in the New Eurocode for the Design of Concrete Structures

by A. W. Beeby

Synopsis:

This paper provides an outline of the provisions for design for serviceability given in the current version of Eurocode 2; the Eurocode for the design of concrete structures. The provisions for stress limitation and crack control are outlined briefly together with some of the reasoning behind them. A more detailed description of the background and content of the clauses dealing with the control of deflection are given.

Keywords: beams; code provisions; deflection control; Eurocode; serviceability; stress checks
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INTRODUCTION

The purpose of this paper is to provide an outline of the current thinking within the European code drafting community on the design of reinforced and prestressed concrete structures for serviceability. It must be said at the outset that the provisions for serviceability design in the Eurocode have not been finalised and there are still areas of debate. This paper therefore provides only an interim picture of the state of affairs.

Though it is likely to be well known to many readers of this paper, it may be useful at the start of the paper to have an outline of the Eurocode project generally and the current state of development.

The Treaty of Rome has, as one of its aims the removal of barriers to the free movement of goods and services across the national boundaries within the European Union. The great range of different design codes within the EU was seen to constitute a barrier to the free movement of services and so a decision was made (in around 1980) to develop a uniform and co-ordinated set of design codes for structural design. This has turned out to be a far greater, and more ambitious, task than the European Commission initially envisaged and it is still far from complete, despite 20 years of effort. Early work was carried out by committees appointed directly by the European Commission but they eventually handed the work over to the European Standards Organisation, CEN

There have probably been two major difficulties in the development of the Eurocodes. The first has arisen from the attempt to produce a uniform basis for structural design applicable to all structural design in any material. The second is the inevitable difficulties of obtaining agreement between different countries on the detailed design rules for any particular material. It is doubtful if a completely coherent and generally accepted set of theories exist describing the behaviour of any structural material. As a result, the design rules for any material in any particular country have developed from the theories accepted in that country, moderated by many years of experience. The designers in any country have developed design methods which experience has shown to work. These methods are enshrined in that countries design codes but are likely to be different to those in other countries. In the absence of a generally accepted and complete theory of behaviour, how can one decide which countries procedures are the best?
The Eurocodes currently under preparation are the following:

- Eurocode 0: Basis of design
- Eurocode 1: Actions on structures
- Eurocode 2: Design of concrete structures
- Eurocode 3: Design of steel structures
- Eurocode 4: Design of composite steel and concrete structures
- Eurocode 5: Design of timber structures
- Eurocode 6: Design of masonry structures
- Eurocode 7: Geotechnical design
- Eurocode 8: Earthquake resistant design of structures

Eurocode 2, the code of practice for the design of concrete structures, is one of the most advanced. A complete version was published at a Pre-Standard (ENV) in 1991 (1). In this form it may be used for design in any of the member countries of CEN but is not mandatory. After having been published for 3 years, an ENV must be the subject of an international enquiry and vote on its future. This has taken place for Eurocode 2 and the result was that it should be converted to a full European Standard, taking account of comments made by the member countries. These comments were extensive and a new project group was set up to produce a revised draft. This group is currently about half way through developing a final draft, which is expected to be the subject of a final vote in 2002. If accepted, the code will then replace competing documents in all CEN member countries, though there will be an overlap permitted for some years between the Eurocode and the national codes it replaces.

The problem of developing an international consensus on design procedures has been made easier for concrete design by the work of the Comite Euro-International du Beton (CEB) and the Federation International de l’Precontrainte (FIP). These two international bodies, working together, drafted a number of Model Codes for concrete design. The 1978 Model Code (2) formed the basis for early drafts of the Eurocode and the current revision will be influenced to a major degree by the more recent 1990 Model Code (3). The CEB and the FIP have recently merged to form the Federation International du Beton (fib).

An increasingly important aspect of concrete design is design for serviceability and this paper will outline the serviceability provisions as they appear in the prestandard version of Eurocode 2. These provisions may change significantly before publication of the final Standard and the debates on these issues currently going on will be considered.

**GENERAL INTRODUCTION TO THE SERVICEABILITY PROVISIONS**

Eurocode 2 contains detailed provisions for three specific conditions:

1. The limitation of stresses in steel and concrete under service loads
2. The limitation of cracking
3. The limitation of deflections.

For each of these, the code has to define:

- Criteria defining the limits to satisfactory performance,
- A definition of the design loading under which the criteria should be checked
- A definition of the assumptions that should be made about the material properties
- The specification of an appropriate behavioural model which may be used to predict the behaviour.

The definition of criteria is recognised to be probably impossible in any general way. The objective of design for serviceability is to ensure that the behaviour of the structure is not such as to impair the proper functioning or appearance of the structure under the expected service conditions. Designers wish to have clear rules specified which will allow them to demonstrate clearly that their design meets the requirements of the code. Code drafters are well aware that this is impossible and that what will impair the proper functioning of a structure will depend upon the precise purpose of the structure and can only be established by the client and the designer coming to an understanding of the exact nature of the required function of the structure and the economic consequences of varying the criteria. The code makes clear that the designer should agree the service criteria with the client but recognises that it cannot avoid specifying limits that are believed to be generally acceptable. Furthermore, the general principle adopted in the drafting is that there should be simple methods which permit serviceability to be checked without detailed calculation, backed up by more rigorous procedures for use in special cases. Simple methods, such as permissible span/depth ratios for the control of deflection, can only be derived on the basis of defined criteria.

Three levels of service load are defined, the use of which depends upon what is being checked:

1. The Characteristic or Rare combination of loads. This corresponds to a loading which may be expected to occur sometime during the life of a structure, but only rarely.
2. The Frequent combination.
3. The Quasi-Permanent combination of loads, which is used to establish long term effects.

Each of the aspects of behaviour listed above will now be considered in turn.
STRESS CHECKS.

The Eurocode proposes limits to the stress under service load in both reinforcement and concrete.

The limiting stress in ordinary reinforcement is $0.8f_{yk}$ where $f_{yk}$ is the characteristic strength of the reinforcement. Some limit on the service stress in the reinforcement seems reasonable since any inelastic deformation of the reinforcement is likely to lead to large, permanently open cracks or possibly excessive deflections. Cold worked steels will deform inelastically at stresses below the specified characteristic proof stress so a limit below the expected yield seems appropriate. Whether, however, this limit should be $0.8f_{yk}$ or, say, $0.9f_{yk}$, remains a matter for debate. This is checked under the Rare combination of loads.

There are two limits to stress in the concrete in compression. These are;

- $0.6f_{ck}$, where $f_{ck}$ is the characteristic compressive strength of the concrete, under the rare combination of loads where the member may be exposed to aggressive environments. This limit is specified to avoid the possible formation of cracking parallel with the applied compression. It is suggested that this cracking could lead to a reduction in durability.

- $0.45f_{ck}$ under the Quasi-Permanent combination of loads in circumstances where it is considered that excessive creep could result in unacceptable deformations.

These limits to the compressive stresses in the concrete have been, and still are, the subject of much debate. A considerable proportion of the member countries of CEN do not currently carry out any checks for stresses under the service loads and see no reason introduce them. Other countries, however, have traditionally designed to meet limiting permissible stresses under service loads rather than using ultimate load approaches and do not wish to see their traditional design approaches abandoned.

The stresses in steel and concrete may be checked on the assumption that the concrete and steel are both elastic and that concrete carries no tension. There is a let-out clause which says that, for reinforced concrete, the stresses can be assumed to be within the limits provided that design for the ultimate loads has been carried out according to the code. Unfortunately, this is not strictly true and direct calculation of the stresses will show that the compressive stresses will control the design of columns.

CRACK CONTROL

The Eurocode proposes the limits to crack widths given in Table 1.
Two methods are given to enable these limits to be satisfied in design: direct calculation of crack widths or a check on bar arrangements. For either of these two methods to work, it is essential that at least a minimum amount of reinforcement is provided sufficient to ensure that the reinforcement does not yield when the first crack forms. If insufficient reinforcement is provided then when the first crack forms the steel will yield and this will limit the tensile force that can develop to a level below that necessary to form further cracks. All the subsequent deformation will be concentrated in this first crack. This issue is not generally a problem where a member is stressed exclusively by applied loading since the reinforcement will have been designed to carry the design ultimate load and, if this is less than the minimum, it simply means that the cracking strength is greater than the design ultimate load and cracking will not occur under service conditions. It is, however, a problem if the member is restrained against contraction due to shrinkage or early thermal contraction. In these cases, if the stresses induced by restrained shortening exceed the tensile strength of the concrete then cracking will occur independently of any applied load. It is often difficult to predict the degree of restraint provided or the possible shortening and so it is generally prudent to provide at least the proposed minimum reinforcement. This minimum is given by the formula:

\[ A_s \geq k_c k_{fc,eff} A_{ct} / \sigma_t \]

This requirement is not found to be particularly onerous for bending but can require considerable areas of reinforcement in members subjected to pure tension.

Crack widths may be calculated from the formula:

\[ w_k = \beta s_{rm} \varepsilon_{tm} \]

where \( s_{rm} \) is the average final crack spacing and is given by the formula:

\[ s_{rm} = 50 + 0.25 k_1 k_2 \phi / \rho_t \]

The effective reinforcement ratio is the ratio of the area of the reinforcement to the area of concrete immediately surrounding the bars. For beams this area will be given by the breadth of the tension zone multiplied by 2.5(h - d) where h is the overall depth of the section and d is the effective depth.

This formula is empirical and has been shown from extensive tests to give a reasonable prediction in most normal situations. The prediction of crack widths is, however, well known to be highly uncertain and it was felt by the drafting committee that the use of simple detailing rules should normally be an adequate method of complying with the crack width provisions. Parameter studies showed that practical limitation of
crack widths could be achieved by controlling either the bar diameter or the bar spacing as a function of the steel stress. Tables are given for both these possibilities.

CONTROL OF DEFLECTIONS

The code states that, in normal circumstances, deflections should not impair the functioning of the structure if the overall deflection is limited to span/250 and, where brittle finishes or partitions are used, the deflection occurring after installation of the finishes and partitions is limited to span/500. Generally, it is satisfactory to check the deflection under the quasi-permanent load. The code makes clear, however, that the designer should satisfy himself that these limits are appropriate for the particular structure considered and its function.

The calculation of the deformations in a cracked reinforced member is done applying a simple conceptual model of behaviour. This is illustrated in Figure 1 which shows a short element of a cracked member subjected to a uniform moment. The stress in the reinforcement at each crack will be that calculated if the concrete is assumed to carry no tension (a fully cracked section). At increasing distance from the crack, the stress in the reinforcement decreases and tension is transferred to the concrete until, if the cracks are fairly widely spaced, the stress distribution will become that in an uncracked section where the concrete is fully mobilised. When calculating the deformations, what is required is the integral of the local deformations over the length considered. This can be conveniently done by establishing an average deformation. This average deformation could be obtained by either integrating the strains over the length between cracks or the same result could be obtained by considering part of the beam to be fully cracked and part to be uncracked, as shown by the broken line in Figure 1. Using this idealisation, it can be seen that the average steel stress (say) is given by the relation:

\[ \sigma_{sn} = \xi \sigma_{st} + (1 - \xi) \sigma_{si} \]

It should be able to be seen that this relationship can be used for any stress or deformation parameter. For example, the concrete strain at the compression face could be calculated by the equation simply by substitution concrete strain for steel stress. Similarly for curvature. Nor is the expression necessarily limited to beams; for example, it could be applied equally validly to tension members.

The remaining question is the definition of the coefficient \( \xi \). Clearly, a function is required which gives a value for \( \xi \) of zero when the moment is equal to the cracking moment and increases towards 1.0 as the moment increases. The following empirical equation is given in the code for \( \xi \):

\[ \xi = 1 - \beta_1 \beta_2 \left( \frac{\sigma_{sn}}{\sigma_{st}} \right)^2 \]
Long term deformation are calculated by using an effective modulus of elasticity which allows for the effects of creep and the long term value of $\beta_2$. The use of the steel stress in the formula is done to make the formula as general as possible. $\sigma_{st}$ and $\sigma_{s2}$ can be calculated for a reinforced beam, a tension member, a column or a prestressed beam. The formulae thus provide a completely general approach to the estimation of the conditions of stress and deformation in a cracked section. For a reinforced concrete beam, it will be seen that the equation for $\zeta$ for members using deformed bars and short term loading may be rewritten as:

$$\zeta = 1 - \left(\frac{M_{cr}}{M}\right)^2$$

It may be seen that there are distinct similarities with the ACI 318 method for calculating deflections. Both methods give a deflection equal to the uncracked deflection at the cracking moment and approach the fully cracked deflection as the applied moment is increased above the cracking moment. Figure 2 compares the two methods for a beam where the uncracked second moment of area is five times the cracked second moment. It will be seen that at moments up to about three times the cracking moment, the Eurocode approach will lead to deflections slightly greater than the ACI approach but that at higher moments the two methods are indistinguishable. The differences between the two approaches are probably too small for it to be possible to establish which method gave the closer agreement with experimental data. It might be reasonable to conclude that, while the Eurocode method was more general, the ACI method was slightly easier to use.

Accuracy of deflection calculations.

There has been a considerable amount of discussion on the usefulness of calculating deflections. This has arisen because it was felt that calculation of deflections was inherently very inaccurate and that the results were therefore misleading rather than useful. Proponents of this view believe that some simple method, such as span/depth ratios provide as good a guidance on deflection performance as it is reasonable to make use of. The basis for this argument can be illustrated using an example. Deflections have been calculated for a 200 mm deep simply supported slab of 6 m span reinforced with 12 mm bars at 150 mm centres. A concrete compressive strength of 40 N/mm² has been assumed and the quasi-permanent id span moment is 25 kNm per metre. The code gives an elastic modulus for the concrete of 35 kN/mm². The major source of variability in the deflection of lightly reinforced members such as slabs is the tensile strength of the concrete. What is specified for a structure is normally the compressive strength. The tensile strength can be estimated from the compressive strength but only inaccurately. Furthermore, a stronger concrete than specified may be used, possibly to enable shorter formwork striking times or tensile stresses may develop due to restraint of shrinkage or thermal
movements. The tensile strength cannot therefore be known at the
design stage with any precision. The Eurocode gives a range of tensile
strengths for any given value of compressive strength. A mean strength
is given together with a lower and upper characteristic value. The
characteristic values vary by $\pm 30\%$ from the mean value. Figures 3 and
4 show the calculated short term and long term deflections respectively
calculated for the three specified tensile strengths. A creep coefficient
of 2.5 has been assumed. It will be seen that the variation in tensile
strength leads to a much greater variation in the deflection. This
variation is likely to be an underestimate of the actual variability since
no account has been taken of variability in the elastic modulus of the
concrete or the creep coefficient or inherent inaccuracies in the
prediction method. Table 2 below summarises the calculated deflections
under the quasi-permanent load.

It can be seen that the ratio of the largest to the smallest short term
deflection is 4.25 while the ratio for the long term deflection is 1.7 and
that, for the long term increment, it is 1.6. All these are large variations
and, as mentioned above, are likely to be an underestimate of the actual
variations. The view that a calculated deflection can give a highly
misleading impression of what may actually occur can thus be
understood. This led to the deflection calculation method not appearing
in the main body of the code but being included in an appendix. This
problem of accuracy of deflection calculation is not peculiar to the
Eurocode but is inherent in all attempts to calculate deflections at the
design stage, whatever method is used.

Span/ depth ratios.

Codes recognise that it would be an unnecessary waste of design effort
to require calculations for all members and provide alternative simple
checks for the control of deflection. By far the most usual approach is
to define permissible ratios of span to overall depth or effective depth
which, provided they are not exceeded can be deemed to satisfy the
deflection criteria. The Eurocode follows this approach and uses ratios
of span to effective depth. In many cases such limits have been derived
from considerations of experience but, in the case of the Eurocode, the
specified values were obtained from a parameter study using the
calculation method described above. The procedure used can be
expressed in simplified terms as follows.

The deflection is given by:

$$a = k_d L^2 (1/r)$$

The curvature can be calculated from:

$$\left( \frac{1}{r} \right) = \frac{M_{ser}}{E_{c,eff} I_{eff}}$$
It is convenient to work in terms of non-dimensional parameters such that:

\[ \alpha = \frac{M_{se}}{bd^2} \quad \text{and} \quad \beta = \frac{I_{eff}}{bd^3} \]

This results in the expression for \(1/r\) as:

\[ (1/r) = bd^2\frac{\alpha}{E_cbd^3}\beta \quad \text{or} \quad (d/r) = \alpha/E_c\beta \]

Substituting this into the expression for deflection gives:

\[ a = kL^2(d/r)E_{c,eff}\beta/\alpha \]

Assuming a constant value for the modulus of elasticity, \(E_{c,eff}\), combining all the various constants into a single constant, \(K\) and rearranging gives:

\[ a/L = K(L/d)(d/r) \quad \text{or} \quad (L/d) = (a/L)/K(d/r) \]

\(a/L\) can be calculated from the deflection limit (for example, if the limiting deflection is span/250, then \(a/L = 1/250\)). \(K\) and \((d/r)\) can be calculated for the particular beam considered and hence \(L/d\) can be calculated. If \(L/d\) is calculated in this way for beam sections with a range of reinforcement ratios, a result is obtained as shown in Figure 5. This is for simply supported rectangular section beams with a concrete strength of 40 N/mm² and with a maximum stress in the reinforcement calculated on the basis of a cracked section of 200 N/mm².

It is found more convenient, rather than to produce separate charts for beams with different support conditions, to produce a set of basic ratios for beams with different support conditions and a further table of multipliers which are a function of reinforcement ratio and steel stress. This was done for the UK code, BS8110 (4) except that it was found more convenient operationally to produce the table in terms of \(M/bd^2\) instead of reinforcement ratio. Such a set of tables was proposed for the Eurocode but was discarded as being too complicated. A simpler approach was therefore included where there are span/effective depth ratios given for two situations: members with 0.5% and 1.0% of tension reinforcement. This table is reproduced below. Interpolation between 0.5 and 1% of reinforcement is permitted. It is also permitted to assume that "concrete lightly stressed" covers slabs and "concrete highly stressed" covers beams.

FUTURE DEVELOPMENTS

It has been explained that, at present, a project group is working on producing a revised draft of the Eurocode taking account of national comments on the prestandard version. It is not at present clear what the final form of this document will take but there will certainly be some
changes to the serviceability provisions. There is still argument about the provisions for stress checks. Also, the provisions for crack control will require some revision as a result of experience in drafting the Eurocodes for the design of concrete bridges and for the design of liquid retaining and containment structures. The change which is almost certain to be made is to the span/effective depth provisions. Contrary to the opinions expressed at the time that the clauses were originally drafted, it is now the view from the national comments that the current span/effective depth ratio provisions are too simplistic and that something similar to the original proposal of a continuous function between reinforcement ratio and span/effective depth ratio appears likely to be included. Furthermore, the Eurocodes will in future take account of concrete strengths up to 100 N/mm². High strength concrete can give significant advantages for deflection control because of their higher cracking moment and it is likely that this will be included in the forthcoming revisions.

CONCLUDING SUMMARY

The Eurocode for the design of concrete structures, Eurocode 2, is probably the most advanced of the family of Eurocodes and should appear as a full European Standard by about the end of 2002. The provisions for serviceability design cover checks for stresses in the concrete in compression and the reinforcement in tension under service loads; checks on crack widths and checks on the deflections. All these checks are drafted at two levels: a simplified approach requiring no or minimal calculation and a more rigorous approach requiring direct calculation of the behaviour of the structure and comparison with specified criteria defining the limits of acceptable performance. There is still debate about some aspects of these provisions and the ones described in this paper may not represent the final form of the document.

NOTATION

\[ A_s = \text{Area of reinforcement} \]
\[ A_{et} = \text{the area of the tension zone of the member immediately before formation of the first crack.} \]
\[ E_{c,\text{eff}} = \text{the effective elastic modulus including allowance for creep} \]
\[ I_{\text{eff}} = \text{the effective second moment of area of the section} \]
\[ L = \text{the span of a beam} \]
\[ M = \text{the moment considered} \]
\[ M_{cr} = \text{the cracking moment} \]
\[ M_{ser} = \text{the service load moment} \]
\[ a = \text{the deflection} \]
\[ b = \text{the breadth of a section} \]
\[ d = \text{the effective depth of a section} \]
\[ f_{ck} = \text{the characteristic compressive strength of the concrete} \]
\[ f_{ct,\text{eff}} = \text{the tensile strength of the concrete at the time} \]
cracking is expected to occur. This should not generally be taken as less than 3 N/mm².

\( f_{yk} \) = the characteristic yield strength of the reinforcement

\( h \) = the overall depth of a section

\( k_c \) = a coefficient which takes account of the nature of the stress distribution; \( k_c = 1 \) for pure tension and 0.4 for pure bending.

\( k \) = a coefficient which takes account of the effect of non-uniform self-equilibrating stresses; \( k = 0.8 \) for members less than 300 mm deep and 0.5 for members greater than 800 mm deep. Linear interpolation may be used for intermediate values.

\( k_1 \) = a coefficient taking account of the bond properties of the bars. \( k_1 = 0.8 \) for plain bars and 0.4 for deformed bars.

\( k_2 \) = a coefficient which takes account of the nature of the stress distribution across the section. \( k_2 = 1 \) for pure tension and 0.5 for bending.

\( k_a \) = a constant depending on the form of the bending moment diagram.

\( 1/r \) = the curvature at the critical section

\( s_{en} \) = the average final crack spacing

\( \beta \) = a constant relating the characteristic crack width to the average value. It usually takes a value of 1.7

\( \beta_1 \) = a coefficient which takes account of the bond properties of the bars. \( \beta_1 = 0.5 \) for plain bars and 1.0 for deformed bars

\( \beta_2 \) = a coefficient which takes account of the duration of the loading. \( \beta_2 = 1.0 \) for short term loading and 0.5 for long term or repeated loading.

\( \varepsilon_{sm} \) = the average strain in the reinforcement allowing for tension stiffening.

\( \phi \) = the bar diameter

\( \rho_r \) = the effective reinforcement ratio

\( \sigma_{sm} \) = the average stress in the reinforcement

\( \sigma_s \) = the maximum stress permitted in the reinforcement immediately after formation of the first crack. This may normally be taken as the yield strength of the reinforcement.

\( \sigma_{s1} \) = the stress in the reinforcement in an uncracked member

\( \sigma_{s2} \) = the stress in the reinforcement calculated assuming a fully cracked section

\( \zeta \) = a coefficient which defined the proportion of the element which can be considered
REFERENCES


Table 1. Crack width limits (in mm)

<table>
<thead>
<tr>
<th>Load combination</th>
<th>Reinforced concrete</th>
<th>Post-tensioned members</th>
<th>Pre-tensioned members</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exposure condition</td>
<td>Quasi-Permanent</td>
<td>Frequent</td>
<td>Frequent</td>
</tr>
<tr>
<td>Dry</td>
<td>*</td>
<td>0.2</td>
<td>0.2</td>
</tr>
<tr>
<td>Humid</td>
<td>0.3</td>
<td>Decompression</td>
<td>Decompression</td>
</tr>
<tr>
<td>Humid with frost &amp; salt</td>
<td>0.3</td>
<td>Decompression</td>
<td>Decompression</td>
</tr>
<tr>
<td>Seawater</td>
<td>0.3</td>
<td>Decompression</td>
<td>Decompression</td>
</tr>
</tbody>
</table>

* Crack width will depend upon appearance and widths greater than 0.3 mm may be acceptable.

‘Decompression’ requires that the prestressing tendons are at least 25 mm within compressed concrete.
Table 2. Calculated deflections under the quasi-permanent load

<table>
<thead>
<tr>
<th>Tensile strength N/mm²</th>
<th>Short term deflection mm</th>
<th>Long term deflection mm</th>
<th>Long term deflection as fraction of span</th>
<th>Long term deflection increment mm</th>
<th>Increment as fraction of span</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.5 (lower characteristic)</td>
<td>17</td>
<td>27</td>
<td>span/219</td>
<td>10</td>
<td>span/581</td>
</tr>
<tr>
<td>3.5 (mean value)</td>
<td>7</td>
<td>23</td>
<td>span/265</td>
<td>16</td>
<td>span/384</td>
</tr>
<tr>
<td>4.6 (upper characteristic)</td>
<td>4</td>
<td>16</td>
<td>span/386</td>
<td>12</td>
<td>span/520</td>
</tr>
</tbody>
</table>

Table 3. Span/effective depth ratios from Eurocode 2

<table>
<thead>
<tr>
<th>Structural System</th>
<th>Concrete highly stressed (1% reinforcement)</th>
<th>Concrete lightly stressed (0.5% reinforcement)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Simply supported beam, one or two way spanning simply supported slab</td>
<td>18</td>
<td>25</td>
</tr>
<tr>
<td>End span of continuous beam or one way continuous slab or two way spanning slab continuous over one long side</td>
<td>23</td>
<td>32</td>
</tr>
<tr>
<td>Interior span of beam or one or two way spanning slab</td>
<td>25</td>
<td>35</td>
</tr>
<tr>
<td>Slab supported on columns without beams (Flat Slab)(based on longer span)</td>
<td>21</td>
<td>30</td>
</tr>
<tr>
<td>Cantilever</td>
<td>7</td>
<td>10</td>
</tr>
</tbody>
</table>
Figure 1. Conceptual model used in the derivation of the Eurocode deflection calculation method
Figure 2. Comparison of Eurocode and ACI formulae for deflection calculation

Figure 3. Short term deflection calculated for three levels of tensile strength
Figure 4. Long term deflections calculated for the same material properties as used in Figure 3

Figure 5. Influence of reinforcement ratio on the permissible span/effective depth ratios
UK Code Requirements for Deflection Control

by R. H. Scott

Synopsis: A brief history is given of UK Code requirements for deflection control since 1948. Current requirements of BS8110 are then presented followed by a summary of developments concerning high strength concrete. Finally, future developments involving Eurocode 2 are indicated.

Keywords: codes and standards; deflection control; structural concrete
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INTRODUCTION

The first formal Code of Practice in the UK concerned with the design of concrete structures was CP114, *The Structural Use of Reinforced Concrete in Buildings*, published in 1948 (1). This was followed by CP115, *The Structural Use of Prestressed Concrete in Buildings* (2), and CP116, *Structural Use of Precast Concrete* (3). These were all essentially permissible stress codes although CP114 later allowed resistance moments of beams and slabs to be calculated by the load-factor method.

CP114, CP115 and CP116 were all revised at regular intervals but in 1972 a major development occurred with the publication of CP110, *Structural Use of Concrete* (4). This covered reinforced, prestressed and precast concrete in a single document and introduced limit state design to UK design codes. It was, however, published in three parts, Part 1 presenting design rules supported by Parts 2 and 3 which contained design charts. CP110 received a major revision in 1985 when it was republished as BS8110, *Structural Use of Concrete* (5). This is also in three parts but arranged differently, Part 1 now being for "normal" design situations and Part 2 for "special circumstances". Design charts are only in Part 3. BS8110 has been amended at regular intervals since 1985 and Part 1 was revised and reprinted in 1997. It is this 1997 edition of Part 1, with the 1985 editions of Parts 2 and 3, which, together, form the current document for structural concrete design in the UK.

BRIEF HISTORY OF UK DEFLECTION CONTROL

CP114(1948) was succinct concerning deflection control. It said (Clause 306(a)) that:

"Reinforced concrete subject to bending action in a building should possess adequate stiffness to prevent such deflection or deformation as might impair
the strength or efficiency of the structure, or produce cracks in finishes or in superimposed partitions, etc."

This statement is, of course, as appropriate now as it was then, but CP114(1948) gave no further guidance as to how it should be implemented. In the 1957 edition, however, a straightforward table of Span/Effective Depth ratios was included, covering both beams and slabs, compliance with which was deemed to ensure that a member possessed sufficient stiffness for all normal cases. Modification factors were specified to reduce the permissible values in situations of high steel and/or concrete stress.

The basic approach of using Span/Effective Depth ratios (rather than the US practice of using minimum thicknesses), first adopted in CP114(1957), has persisted through CP110 and BS8110 (Part 1) although it has become considerably more sophisticated, as will be indicated shortly. Additionally, however, CP110 gave recommendations for the calculation of deflections which have been continued in BS8110 (Part 2). There are thus two approaches to deflection control currently permitted in the UK by BS8110. It is anticipated that in all normal circumstances deflection control will be achieved through the use of Span/Effective Depth ratios but there is also the alternative of calculating deflections when conditions are in any way unusual. Further details of both procedures are given below.

SPAN/EFFECTIVE DEPTH RATIOS

Beams

BS8110 anticipates that, in the majority of design situations, satisfaction of Span/Effective Depth ratios will ensure adequate deflection control, thus avoiding the more lengthy procedure of calculating actual deflection values. The intention is to limit TOTAL deflection to SPAN/250 which, it is anticipated, will normally ensure that the part of the deflection occurring after the construction of finishes and partitions will be limited to SPAN/500 or 20mm, whichever is the lesser, for spans up to 10m.

Basic Span/Effective Depth ratios are tabulated in Table 3.9 of BS8110 which is reproduced in this paper as Fig. 1 (see Acknowledgements for licence details). Values are given for cantilevers, simply supported beams and continuous beams. These values are then multiplied, if appropriate, by (250/NEW LIMIT) where deflection limits other than Span/250 are required, or by (NEW DEFLECTION/20) for deflection limits other than 20mm. For spans greater than 10m, the basic ratio is multiplied by (10/SPAN).
If beam behaviour was elastic then it would be satisfactory (and rigorous) to use ratios of span to overall depth. However, the actual stiffness of a reinforced concrete beam depends on the steel percentage and state of cracking and so some means of accommodating actual beam behaviour must be devised. This is partly achieved in BS8110 (and in all the British Codes which preceded it) by using ratios of span to effective depth rather than span to overall depth, as indicated above, and further achieved by the introduction of modifying factors, as described below.

BS8110 requires that the basic Span/Effective Depth ratios be modified to allow for the percentage of tension reinforcement and the percentage of compression reinforcement. The multiplication factors for tension reinforcement are tabulated in Table 3.10 of BS8110, reproduced here as Fig. 2, and are related to $M/bd^2$ (where $M$ is the design ultimate moment at the centre of the span) and the service stress in the tension steel. For more heavily loaded situations the coefficients are $< 1$ but for lightly loaded beams they are $> 1$ to reflect the beneficial contribution of tension stiffening. The multiplication factors for compression steel are tabulated in Table 3.11 of BS8110 (Fig. 3) and are all $> 1$ (i.e. beneficial). They are related to percentage of compression steel actually provided. Permissible Span/Effective Depth ratios obtained from this procedure take account of normal creep and shrinkage deflection.

The above procedure for determining the "final" Span/Effective Depth ratio is summarised in Fig. 4.

Solid Slabs

Normally Span/Effective Depth Ratios, as summarised above, are used for deflection control in both one-way and two-way spanning slabs (the shorter span being used in the latter case). As before, the appropriate ratio is obtained from Table 3.9 (Fig. 1) and modified by the coefficient obtained from Table 3.10 (Fig. 2). (The compression steel modifier is normally inappropriate). Only the conditions at the centre of the span in the width of slab under consideration should be considered to influence deflection.

Rules for flat slabs are generally similar to those for one-way and two-way slabs provided that the drops (if present) have a gross width, in both directions, at least equal to 1/3 of the respective spans. Otherwise, a multiplier of 0.9 must be applied to the Span/Effective Depth ratios. The check should be performed for the more critical direction.
CALCULATION OF DEFLECTIONS

Part 2 of BS8110 gives guidance for calculating the deflection of a reinforced concrete member which may be followed as an alternative to using Span/Effective Depth ratios. The procedure, in its most general form, involves numerical integration of the following equation relating deflected shape and curvature:

\[
\frac{1}{r_x} = \frac{d^2a}{dx^2} \tag{1}
\]

where \(1/r_x\) is the curvature at \(x\)
\(a\) is the deflection at \(x\)

As an alternative, a simplified approach may be used as given below:

\[
a = K\left[\frac{1}{r_b}\right]^2 \tag{2}
\]

where \(1/r_b\) is the curvature at mid-span (or support for cantilevers)
\(K\) is a constant that depends on the shape of the bending moment diagram

Values of \(K\) for common shapes of bending moment diagram are given in Table 3.1 of BS8110 Part 2.

Long-term deflections are dealt with by using an effective modulus of elasticity which has a value of \(1/(1+\phi)\) times the short-term modulus, where \(\phi\) is a creep coefficient. Section 7.3 of BS8110 Part 2 gives a procedure for determining \(\phi\).
PRESTRESSED CONCRETE

The use of Span/Effective Depth ratios is inappropriate due to the major influence of the level of prestress. As a consequence BS8110 requires that deflections be calculated and gives guidance concerning this. Deflection limits are similar to those for reinforced concrete beams but, in addition, limits to upward deflections are prescribed (BS8110, Part 2, Clause 3.2.1.2).

DEVELOPMENTS

High Strength Concrete

Tables 3.9 and 3.10 of BS8110 (Figs. 1 and 2) are effectively based on a characteristic compressive cube strength of 35MPa. However, in recent years developments in concrete technology have rendered strengths considerably in excess of this to be easily obtainable. Important advantages of high strength concrete when considering deflection control are its higher Young’s modulus and, more importantly, its higher tensile strength which gives a higher cracking moment. Consequently, a revised Table 3.10 has been published by the Concrete Society in the UK in their Technical Report No. 49 (6). Modification factors for tension reinforcement are presented for a range of characteristic compressive cube strengths up to, and including, 100MPa. The report shows that modification factors can be more than doubled with a consequent doubling of the Span/Effective Depth ratio. This obviously has significant implications for deflection control and it is to be hoped that the results of this work can be incorporated into Codes and Standards in due course.

Eurocode 2 (EC2)

The drive for harmonisation of technical standards across the European Union is leading to the development of Eurocodes which will eventually replace national codes and standards. The first of these, Eurocode 2: Design of Concrete Structures Part 1: General Rules for Buildings (7), was published in 1992. At present, this version of EC2 is classed as a draft for development but a revised document is under development which, when published in 2001/2, will run in parallel with national standards, such as BS88110, before superseding them around 2006/7. Consequently, a brief description of the provisions for deflection control in EC2 are appropriate here since, in due course, they will become UK practice.
EC2, like BS8110, permits deflection control to be achieved either through the use of Span/Effective Depth ratios or by calculation. Deflections for quasi-permanent loads are limited to Span/250 and deflections of elements, such as finishes, which could suffer damage are limited to Span/500.

Basic Span/Effective depth ratios are tabulated in EC2 for beams and slabs for two levels of concrete stress under serviceability conditions (but textbooks usually tabulate values for nominally reinforced sections as well). These ratios are then modified for spans longer than 7m (8.5m for flat slabs), for characteristic steel strengths other than 400MPa and for situations where the area of tension reinforcement provided exceeds the area actually required. Guidance is given for checking deflections by calculation.

Procedures in the current draft of the 2001 revision to EC2 are very similar. One detail difference is the recasting of the Span/Effective Depth ratios for a characteristic steel strength of 500MPa.

At present, there is no provision in EC2 for modifying Span/Effective Depth ratios to take account of the beneficial effects of compression reinforcement. Overall, current rules for deflection control in EC2 are less sophisticated than those in BS8110.

CONCLUSIONS

1. UK Code requirements for deflection control date from 1948.

2. Current requirements in BS8110 permit deflection control of reinforced concrete beams and slabs to be achieved either by using Span/Effective Depth ratios (BS8110, Part 1) or by calculation (BS8110, Part 2). It is anticipated that Span/Effective Depth ratios will be the method normally used.

3. Deflection control of prestressed concrete beams in BS8110 is achieved by calculation.

4. Recent developments indicate that the use of high strength concrete can significantly improve Span/Effective Depth ratios compared with those achieved with normal strength concrete.

5. The provisions for deflection control in Eurocode 2 follow a broadly similar approach to those in BS8110.
ACKNOWLEDGEMENTS

Extracts from the British Standard BS8110, Part 1, 1997, are produced with the permission of BSI under licence number 2000SK/0223. British Standards can be obtained by post from BSI Customer Services, 389 Chiswick High Road, London, W4 4AL, UK (Tel. +44 20 8996 9001). The author also acknowledges the assistance of Arup Research and Development, London.

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2. CP115, "The Structural Use of Prestressed Concrete in Buildings", British Standards Institution, 1959
3. CP116, "The Structural Use of Precast Concrete in Buildings", British Standards Institution
4. CP110, "The Structural Use of Concrete", Parts 1-3, British Standards Institution, 1972
5. BS8110, "Structural Use of Concrete", Parts 1-3, British Standards Institution, 1985 (Part 1 republished 1997)
Table 3.9 Basic span/effective depth ratio for rectangular or flanged beams

<table>
<thead>
<tr>
<th>Support conditions</th>
<th>Rectangular sections</th>
<th>Flanged beams with $\frac{b_w}{b} \leq 0.3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cantilever</td>
<td>7</td>
<td>5.6</td>
</tr>
<tr>
<td>Simply supported</td>
<td>20</td>
<td>16.0</td>
</tr>
<tr>
<td>Continuous</td>
<td>26</td>
<td>20.8</td>
</tr>
</tbody>
</table>

Figure 1. Basic span/effective depth ratios
(BS8110, Part 1, Table 3.9)

Table 3.10 Modification factor for tension reinforcement

<table>
<thead>
<tr>
<th>Service stress</th>
<th>$M/bd^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.50</td>
</tr>
<tr>
<td>100</td>
<td>2.00</td>
</tr>
<tr>
<td>150</td>
<td>2.00</td>
</tr>
<tr>
<td>($f_y = 250$ 157)</td>
<td>2.00</td>
</tr>
<tr>
<td>200</td>
<td>2.00</td>
</tr>
<tr>
<td>250</td>
<td>1.90</td>
</tr>
<tr>
<td>300</td>
<td>1.60</td>
</tr>
<tr>
<td>($f_y = 460$ 307)</td>
<td>1.56</td>
</tr>
</tbody>
</table>

NOTE 1. The values in the table derive from the equation:

\[
\text{Modification factor} = 0.55 + \frac{(377 - f_y)}{120 (3.9 + \frac{M}{bd^2})} \leq 2.0 \quad \text{equation 7}
\]

where

$M$ is the design ultimate moment at the centre of the span or, for a cantilever, at the support.

NOTE 2. The design service stress in the tension reinforcement in a member may be estimated from the equation:

\[
f_s = \frac{2Mf_y}{A} \times \frac{1}{A_b} \quad \text{equation 8}
\]

NOTE 3. For a continuous beam, if the percentage of redistribution is not known but the design ultimate moment at mid-span is obviously the same or greater than the elastic ultimate moment, the stress $f_s$ in this table may be taken as $2f_y$.

Figure 2. Modification factors for tension reinforcement
(BS8110, Part 1, Table 3.10)
<table>
<thead>
<tr>
<th>$\frac{100A'_{\text{sum}}}{bd}$</th>
<th>Factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00</td>
<td>1.00</td>
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<tr>
<td>0.15</td>
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<tr>
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<td>1.08</td>
</tr>
<tr>
<td>0.35</td>
<td>1.10</td>
</tr>
<tr>
<td>0.50</td>
<td>1.14</td>
</tr>
<tr>
<td>0.75</td>
<td>1.20</td>
</tr>
<tr>
<td>1.0</td>
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<td>1.30</td>
</tr>
<tr>
<td>2.0</td>
<td>1.40</td>
</tr>
<tr>
<td>2.5</td>
<td>1.45</td>
</tr>
<tr>
<td>$\geq 3.0$</td>
<td>1.50</td>
</tr>
</tbody>
</table>

NOTE 1. The values in this table are derived from the following equation:

\[
\text{Modification factor for compression reinforcement} = \\
1 - \frac{100A'_{\text{sum}}}{bd} / \left(2 + \frac{100A'_{\text{sum}}}{bd}\right) = 1.5
\]
equation 9

NOTE 2. The area of compression reinforcement $A$ used in this table may include all bars in the compression zone, even those not effectively tied with links.

Figure 3. Modification factors for compression reinforcement
(BS8110, Part 1, Table 3.11)
<table>
<thead>
<tr>
<th>PROCEDURE</th>
<th>COMMENTS</th>
</tr>
</thead>
<tbody>
<tr>
<td>&quot;Final&quot; SPAN/EFFECTIVE DEPTH RATIO</td>
<td>BS8110 Table 3.9 (Fig. 1)</td>
</tr>
<tr>
<td>= BASIC RATIO X 250/NEW LIMIT or NEW DEFLECTION/20 X 10/SPAN X MODIFICATION FACTOR FOR TENSION REINFORCEMENT X MODIFICATION FACTOR FOR COMPRESSION REINFORCEMENT</td>
<td>BS8110 Table 3.10 (Fig. 2) BS8110 Table 3.11 (Fig. 3)</td>
</tr>
<tr>
<td></td>
<td>Modifiers for non-standard deflection limits Modifier for spans greater than 10m</td>
</tr>
</tbody>
</table>

Figure 4: Summary of procedure for determining span/effective depth ratios in beams
Deflection Calculation and Control – Australian Code Amendments and Improvements

by R. I. Gilbert

Synopsis: This paper describes the behavior of reinforced and prestressed concrete flexural members under sustained service loads and outlines recent developments in the design of concrete structures for the serviceability limit states, particularly with regard to deflection and crack control. The effects of concrete cracking, creep and shrinkage on cross-sectional stresses and deformation are demonstrated and discussed for a wide range of actions and reinforcement layouts. Recent amendments to the serviceability provisions of the Australian Standard for Concrete Structures AS3600 are presented and the background to, and reasons for, the proposed changes are explained. The paper also highlights the inadequacies of the existing deflection calculation procedure in ACI 318M-99 and suggests ways to improve it. A method is proposed for calculating the time-dependent deflection of reinforced and prestressed concrete members taking into account the time-dependent effects of creep and shrinkage, including the loss of stiffness caused by shrinkage induced cracking and the breakdown of tension stiffening with time. The method is illustrated by several examples.

Keywords: cracking; creep; deflection; design; prestressed concrete; reinforced concrete; serviceability; service loads; shrinkage; structural behavior; time-dependent behavior
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INTRODUCTION

For a concrete structure to be serviceable, cracking must be controlled and deflections must not be excessive. It must also not vibrate excessively. Service load behavior depends primarily on the properties of the concrete and these are often not known reliably at the design stage. Moreover, concrete behaves in a non-linear and inelastic manner at service loads. The non-linear behavior that complicates serviceability calculations is due to cracking, tension stiffening, creep, and shrinkage. Of these, shrinkage is the most problematic. Restraint to shrinkage causes time-dependent cracking and gradually reduces the beneficial effects of tension stiffening. It results in a gradual widening of existing cracks and, in flexural members, a significant increase in deflections with time.

The control of cracking in a reinforced or prestressed concrete structure is usually achieved by limiting the stress increment in the bonded reinforcement to some appropriately low value and ensuring that the bonded reinforcement is suitably distributed. Many codes of practice specify maximum steel stress increments after cracking and maximum spacing requirements for the bonded reinforcement. Some codes include procedures for calculating design crack widths. However, few existing code procedures, if any, account adequately for the gradual increase in crack widths with time, due primarily to shrinkage.

For deflection control, the structural designer should select maximum deflection limits that are appropriate to the structure and its intended use. The calculated deflection (or camber) must not exceed these limits. Codes of practice give general guidance for both the selection of the maximum deflection limits and the calculation of deflection. However, the simplified procedures for calculating deflection in most codes were developed from tests on simply supported reinforced concrete beams and often produce grossly inaccurate predictions when applied to more complex structures. Again, the existing code procedures do not provide real guidance on how to adequately model the time-
dependent effects of creep and shrinkage in deflection calculations. This is particularly true for prestressed and partially-prestressed concrete.

Serviceability failures of concrete structures involving excessive cracking and/or excessive deflection are relatively common. Numerous cases have been reported, in Australia and elsewhere, of structures that complied with code requirements but still deflected or cracked excessively. In a large majority of these failures, shrinkage of concrete is primarily responsible or, probably more precisely, failure to adequately account for shrinkage (and creep) in the design. Clearly, the serviceability provisions embodied in our codes do not adequately model the in-service behavior of structures and, in particular, fail to account adequately for shrinkage.

The quest for serviceable concrete structures must involve the development and widespread use of more reliable design procedures. It must also involve designers giving more attention to the specification of an appropriate concrete mix, particularly with regard to the creep and shrinkage characteristics of the mix, and sound engineering input is required in the construction procedures. High performance concrete structures require the specification of high performance concrete (not necessarily high strength concrete, but concrete with relatively low shrinkage, not prone to plastic shrinkage cracking) and a high standard of construction, involving suitably long stripping times, adequate propping, effective curing procedures and rigorous on-site supervision.

This paper addresses some of these problems, particularly those related to designing for the time-dependent effects of creep and shrinkage. It outlines how the time-dependent deformation of concrete affects the in-service behavior of structures and what to do about it in design. An improved method for calculating time-dependent deflection and an overview of the recent amendments to the serviceability provisions of the Australian Standard for Concrete Structures AS 3600 are presented.

THE IN-SERVICE BEHAVIOR OF FLEXURAL MEMBERS

The short-term or instantaneous behavior of a reinforced or prestressed concrete cross-section subjected to combined bending and axial force can be readily determined using simple Modular Ratio Theory. The long-term or time-dependent behavior can also be determined using a variety of analytical procedures described in numerous texts, including (1), (2), (3) and (4). Arguably, the most attractive of these methods is the Age-Adjusted Effective Modulus Method (AEMM), which is based on the work of Trost (5) and further developed by Dilger and Neville (6) and Bazant (7). Using the AEMM, the strain and curvature on individual cross-sections at any time can be calculated, as can the stress in the concrete and bonded reinforcement or tendons.
The routine use of the AEMM in the design of concrete structures for the serviceability limit states is strongly encouraged. It will greatly reduce the incidence of serviceability failures in concrete beams and slabs, but perhaps more importantly, it provides an insight into how concrete structures behave under sustained service loads and imparts a much clearer understanding of the time-dependent interaction between the concrete and the steel. It also allows the effects of creep and shrinkage of the concrete to be determined in a rational and realistic way.

Some typical results obtained for a variety of uncracked and cracked rectangular cross-sections under sustained actions are provided here to illustrate the effects of creep and shrinkage on structural behavior. In each of the cases presented and discussed below, the rectangular cross-section (with overall depth $D = 800$ mm and width $b = 400$ mm) shown in Fig. 1 is analysed. The areas of bonded non-prestressed reinforcement ($A_{11}$ and $A_{12}$), the area of the prestressing tendons ($A_p$) and the sustained actions ($M_s$ and $N_s$) are varied from case to case. The material properties assumed in each analysis are also given in Fig. 1.

For the prestressed cross-sections, it is assumed that the prestressing steel is bonded to the surrounding concrete, i.e. the section is pretensioned. Similar results would be obtained for the time-dependent behaviour of a post-tensioned section with grouted ducts. It is also assumed that the axial force $N_s$ acts at the centroid of the uncracked transformed cross-section.

In all cases, tensile forces, stresses and deformations are positive. A positive bending moment produces tensile stresses in the bottom fibres of the cross-section and the corresponding curvature is also positive.

**Case 1 - Unreinforced, Uncracked Cross-section**

\[(A_{11} = A_{12} = A_p = 0, M_s = 200 \text{ kNm and } N_s = -2000 \text{ kN})\]

The instantaneous and time-dependent strains and stresses on the unreinforced section subjected to a sustained bending moment of 200 kNm and a sustained axial force of -2000 kN (compression) are shown in Fig. 2.1. The case of zero shrinkage is also included to illustrate the effect of creep.

Under constant sustained actions, in the absence of any bonded reinforcement, the concrete stress at any point remains constant with time. The instantaneous curvature (the slope of the instantaneous strain diagram) is $\kappa = +0.410 \times 10^{-6} \text{ mm}^{-1} (M_s/E)$. Under a constant sustained stress, the creep strain at any point is the product of the instantaneous
(elastic) strain and the creep coefficient (in this example, $\phi = 2.5$). Therefore, with time, the creep induced curvature is $\phi \kappa$, and the final curvature (instantaneous plus creep) is $\kappa = \kappa_i (1 + \phi) = 1.436 \times 10^{-6} \text{ mm}^{-1}$. In the absence of any restraint, shrinkage causes an increase of compressive strain at every point ($-600 \times 10^{-6}$ in this case), but does not cause any change in the final curvature.

The time-dependent analysis of an unreinforced, uncracked section is therefore a simple, almost trivial exercise. For an unreinforced, uncracked member, creep will cause an increase in deflection with time, with the final deflection equal to $(1 + \phi)$ times the instantaneous deflection. Shrinkage will cause a shortening of the member, but no increase in deflection (provided shrinkage is uniform over the depth of each section). In the absence of any restraint, the concrete stresses remain constant with time.

Case 2 - Uncracked, Symmetrically Reinforced Concrete Cross-section

\[
\begin{align*}
(A_{s1} = A_{s2} = 3200 \text{ mm}^2, \ A_p = 0, \ M_i = 200 \text{ kNm} \text{ and } N_i = -2000 \text{ kN})
\end{align*}
\]

The instantaneous and time-dependent strains and stresses on the doubly reinforced concrete section ($A_{s1} = A_{s2}$) subjected to a sustained bending moment of 200 kNm and a sustained axial force of -2000 kN (compression) are shown in Fig. 2.2. The case of zero shrinkage is also included to illustrate the effects of creep. A comparison of Figs. 2.1 and 2.2, illustrates the effects of equal quantities of bonded non-prestressed reinforcement ($A_{s1}/bD = A_{s2}/bD = 0.01$) on time-dependent behavior (where $b$ and $D$ are the width and overall depth of the cross-section, respectively).

The inclusion of the bonded reinforcement increases the instantaneous flexural stiffness by 27.5%, so that the instantaneous curvature reduces to $\kappa_i = +0.322 \times 10^{-6} \text{ mm}^{-1}$. With time, the bonded reinforcement restrains the free development of creep and shrinkage. As the concrete creeps and shrinks, it compresses the reinforcement with steel stresses increasing dramatically with time. The steel in turn imposes gradually increasing tensile restraining forces on the concrete at the level of the top and bottom steel (equal and opposite to the gradually increasing compressive force in the steel) so that the concrete compressive stresses gradually reduce with time. The final creep strain is significantly less than that observed in the unreinforced section of Case 1. With equal quantities of steel top and bottom, the restraint to shrinkage at the top and at the bottom of the section is the same and no shrinkage induced curvature results.
The change of curvature with time is entirely due to creep, with a final value of \( \kappa = 0.721 \times 10^6 \text{ mm}^{-1} \) \( (= \kappa_i (1 + \phi(\alpha)) \). The quantity \( \alpha \) accounts for the braking action of the bonded reinforcement on creep. In the unreinforced section of Case 1, \( \alpha = 1.0 \). In this case, \( \alpha = 2.02 \). The significance of these values for \( \alpha \) will be discussed subsequently.

Initially, when the 2000 kN compressive axial force is first imposed on the cross-section, the concrete carries 1750 kN (87.5%) and the steel reinforcement carries 250 kN (197 kN in the top steel and 53 kN in the bottom steel). Due to creep (when \( \varepsilon_{sh} = 0 \)), the force in the concrete reduces to 1313 kN (65.6%), while the forces in the top and bottom steel increases to 505 kN and 182 kN, respectively. When shrinkage is included, the final forces are 775 kN (concrete), 774 kN (top steel) and 451 kN (bottom steel). The concrete sheds 55.7% of its initial compressive force into the bonded reinforcement with time.

This is typical of the time-dependent behavior of concrete structures, with a similar redistribution of stress between the concrete and the bonded reinforcement taking place in every reinforced concrete column, in every prestressed concrete girder and in the compression zone of every reinforced concrete beam. In fact, in every location where the time-dependent deformation of concrete is restrained by bonded reinforcement, there is a gradually shedding of compression from the concrete into the bonded reinforcement or tendons.

**Case 3 - Uncracked, Non-symmetrically Reinforced Concrete Cross-section**

\[
A_{x1} = 1600 \text{ mm}^2, \quad A_{x2} = 3200 \text{ mm}^2, \quad A_p = 0, \quad M_s = 200 \text{ kNm} \quad \text{and} \quad N_s = -2000 \text{ kN}
\]

The instantaneous and time-dependent strains and stresses on the doubly reinforced concrete section (with \( A_{x1} = 0.5 \ A_{x2} \)) subjected to a sustained bending moment of 200 kNm and a sustained axial force of -2000 kN (compression) are shown in Fig. 2.3. The case of zero shrinkage is also included to illustrate the effects of creep. A comparison of Figs. 2.2 and 2.3, illustrates the effects of reducing the quantity of top steel \( (A_{x1}) \) on time-dependent behavior.

With less restraint to creep and shrinkage near the top of the beam, the top fibre strain and the curvature increase with time by a greater amount than for the section in Case 2, but the reduction in top fibre concrete stress is less. Due to creep, the final curvature is \( \kappa = 0.994 \times 10^6 \text{ mm}^{-1} \) \( (= \kappa_i (1 + \phi(\alpha)) \), a 38% increase from Case 2. The quantity \( \alpha \), used to account for the braking action of the bonded reinforcement on creep, is 1.30 (cf. 2.01 in Case 2). The restraint to shrinkage in this case is greater in the bottom of the section than in the
top and, consequently, a larger tensile restraining force develops at the bottom steel level than at the top steel level. Consequently, a positive curvature is induced by shrinkage and is equal to (1.185 - 0.994) \times 10^{-6} = 0.191 \times 10^{-6} \text{ mm}^{-1}.

Case 4 - Cracked, Singly Reinforced Concrete Cross-section in bending

\( (A_{s1} = 0, A_{s2} = 3200 \text{ mm}^2, A_p = 0, M_s = 400 \text{ kNm and } N_s = 0 \text{ kN}) \)

The instantaneous and time-dependent strains and stresses on the fully cracked reinforced concrete section (with \( A_{s1} = 0 \)) subjected to a sustained bending moment of 400 kNm (with zero axial force) are shown in Fig. 2.4. The case of zero shrinkage is also included to illustrate the effects of creep.

For this particular cross-section, the applied moment exceeds the instantaneous cracking moment (172 kNm) and, in the analysis, it is assumed that the cracked tensile concrete carries zero stress and the depth of the uncracked compressive concrete remains constant with time. In this case, the depth of the uncracked compressive concrete is calculated to be \( d_u = 239.2 \text{ mm} \). In reality, the depth to the neutral axis of strain gradually increases with time (as shown in Fig. 2.4) and the depth of the concrete compression zone gradually increases. However, by ignoring the change in the depth of the compressive zone with time, the analysis is greatly simplified and the error introduced is very small.

For this fully-cracked section, tension stiffening has been ignored. Between the cracks in the tension zone, the concrete does carry some tension and tensile creep will cause a further increase in deformation. However, on the fully-cracked section of Fig. 2.4, creep causes an increase of curvature of 53% (\( \alpha = 4.71 \)) and a redistribution of the compressive stress in the concrete as shown. Restraint to shrinkage provided by the bottom reinforcement causes a further increase in curvature of 0.925 \times 10^{-6} \text{ mm}^{-1}, so that the final curvature is 3.721 \times 10^{-6} \text{ mm}^{-1} (2.04 times greater than the instantaneous curvature).

The tensile strain in the bottom fibre of the section increases only slightly with time, with the strain (and hence stress) in the tensile steel changing by less than 4%.

Case 5 - Cracked, Doubly Reinforced Concrete Cross-Section in Bending

\( (A_{s1} = 1600 \text{ mm}^2, A_{s2} = 3200 \text{ mm}^2, A_p = 0, M_s = 400 \text{ kNm and } N_s = 0 \text{ kN}) \)

The instantaneous and time-dependent strains and stresses on a cracked, doubly reinforced concrete section (with \( A_{s1} = 0.5 A_{s2} \)) subjected to a
sustained bending moment of 400 kNm are shown in Fig. 2.5. A comparison of Figs. 2.4 and 2.5, illustrates the effects of including compressive steel on time-dependent behavior.

As can be seen, the inclusion of compression steel results in a significant reduction in time-dependent deformation, but the depth of the concrete compressive zone decreases slightly to 224.6 mm. With the restraint provided by the top steel, creep causes an increase in curvature of only 33% (cf. 53% in Case 4 where $A_{s1} = 0$) and shrinkage induced curvature is $0.600 \times 10^6$ mm$^{-1}$ (65% of that in Case 4).

As the concrete in the compressive zone creeps and shrinks, the top steel is compressed and the tensile restraining force imposed on the concrete at the level of the top steel causes a significant reduction in the compressive stresses in the concrete. In this case, the compressive force carried by the concrete is reduced from 491.4 kN (initially) to 215.8 kN after creep and shrinkage, while the force carried by the top steel increases from 98.0 kN to 364.8 kN. As in the previous cases, there is a dramatic redistribution of stress in the compressive concrete when bonded reinforcement is included.

As in Case 4, creep causes a slight increase in strain in the tensile steel and shrinkage causes a slight decrease, but in this case these changes are less than 2%.

**Case 6 - Uncracked Prestressed Concrete Cross-section in bending**

$(A_{s1} = 0, A_{s2} = 3200 \text{ mm}^2, A_p = 1500 \text{ mm}^2 (P = 2025\text{kN}), M_i = 700 \text{kNm} \& N_i = 0)$

The instantaneous and time-dependent strains and stresses on a prestressed concrete section (containing a single layer of prestressing tendons and a single layer of non-prestressed reinforcement in the bottom of the section) subjected to a sustained bending moment of 700 kNm are shown in Fig. 2.6. The case of zero shrinkage is also included to illustrate the effects of creep.

In this case, creep causes an increase in deformation and, because the bonded steel (both prestressed and non-prestressed) is near the bottom of the section where the concrete stress is small, relatively little restraint to creep is provided. The redistribution of concrete stresses due to creep is therefore relatively small. However, the tensile restraining force imposed by the bottom steel causes a slight increase in top fibre concrete compressive stress and the ratio of the creep induced curvature to the initial curvature actually exceeds the creep coefficient, with $\alpha = 0.84$.

With bonded steel only in the bottom of the section, the shrinkage induced tensile restraining forces exerted by $A_{s2}$ and $A_p$ cause
a significant increase in tensile stress in the bottom fibres and a further increase in curvature ($\kappa_{sh} = (2.333 - 1.741) \times 10^4 = 0.592 \times 10^4$ mm$^{-1}$). Due to shrinkage, the bottom fibre stress becomes tensile and, in this case, almost reaches the tensile strength of the concrete.

The inclusion of non-prestressed bottom reinforcement, often provided to satisfy the strength requirements of a member, significantly increases sagging curvature, and hence the deflection of the beam, and may cause cracking of the section at an applied moment significantly less than the short-term cracking moment.

Case 7 - Uncracked Prestressed Concrete Cross-section in bending

($A_{si} = 1600$ mm$^2$, $A_{s2} = 3200$ mm$^2$, $A_p = 1500$ mm$^2$ ($P = 2025$ kN), $M_s = 700$ kNm)

The instantaneous and time-dependent strains and stresses on the doubly reinforced prestressed concrete section (with $A_{s1} = 0.5 A_{s2}$) subjected to a sustained bending moment of 700 kNm are shown in Fig. 2.7. The case of zero shrinkage is also included to illustrate the effects of creep. A comparison of Figs. 2.6 and 2.7, illustrates the effects on time-dependent behavior of introducing non-prestressed reinforcement in the highly stressed region at the top of the section ($A_{s1}$).

By comparison with Case 6 (where $A_{s1} = 0$), the introduction of $A_{s1} = 1600$ mm$^2$ results in a significant reduction in both instantaneous and time-dependent deformation, with a reduction of instantaneous curvature of 14% and a reduction of final curvature of 38% (comparing Figs. 2.6 and 2.7). The final top fibre strain (after creep and shrinkage) is reduced from $-2142 \times 10^{-6}$ (see Fig. 2.6) to $-1599 \times 10^{-6}$. The creep induced curvature is reduced from $1.305 \times 10^{-6}$ mm$^{-1}$ (in Case 6) to 0.790 $\times 10^{-6}$ mm$^{-1}$ (with $\alpha = 1.19$). The shrinkage induced curvature is reduced from $0.592 \times 10^{-6}$ mm$^{-1}$ to 0.283 $\times 10^{-6}$ mm$^{-1}$.

The stress in the top reinforcement increases dramatically with time. Located in a region of high compressive stress, and hence high creep strains, the final stress in the top steel is -305.3 MPa (and this is not an unusual situation). The build up of compression in the bonded steel is matched by an equal and opposite tensile restraining force imposed on the concrete at the steel level, thereby reducing the compressive stress in the concrete as shown.

Immediately after first loading, the stress in the prestressing steel is 1328 MPa and the corresponding prestressing force $P_s$ is 1992 kN ($\sigma_p A_p = 1328 \times 1500 \times 10^3$). The compressive forces in the top and bottom non-prestressed steel are 107 kN and 47 kN, respectively, and in the concrete, the compressive force is 1838 kN. After creep and shrinkage in the concrete (and relaxation in the prestressing steel), the
prestressing force reduces to 1778 kN (ie. a 10.7% loss of prestress). The compressive forces in the non-prestressed top and bottom steel increase to 488 kN and 328 kN, respectively, and the compressive force in the concrete decreases to 962 kN. Although, the prestressing force reduced with time by only 10.7% (in this case), the compressive force in the concrete reduced by 45.9%, with much of the prestressing force transferred from the concrete to the bonded non-prestressed reinforcement with time.

Case 8 - Uncracked Prestressed Concrete Cross-section in bending

\( A_{s1} = 3200 \text{ mm}^2, A_{s2} = 3200 \text{ mm}^2, A_p = 1500 \text{ mm}^2 \) \((P = 2025 \text{kN}), M_c = 700 \text{kNm})

The instantaneous and time-dependent strains and stresses on the doubly reinforced prestressed concrete section (with \( A_{s1} = A_{s2} \)) subjected to a sustained bending moment of 700 kNm are shown in Fig. 2.8. A comparison of Figs. 2.7 and 2.8, illustrates the effects on time-dependent behavior of increasing the quantity of non-prestressed reinforcement in the highly stressed region at the top of the section \((A_{s1})\).

From a comparison with Case 7 (where \( A_{s1} = 1600 \text{ mm}^2 \)), increasing the quantity of top steel to 3200 \text{ mm}^2 results in a further significant reduction in both instantaneous and time-dependent deformation. The instantaneous curvature is 13.7% less (25.8% less than when \( A_{s1} = 0 \)) and the final curvature after creep and shrinkage is 37.5% less (61.2% less than when \( A_{s1} = 0 \)) (comparing Figs. 2.6 and 2.7 with 2.8). The creep induced curvature is reduced from \( 1.305 \times 10^{-6} \text{ mm·}^{-1} \) (in Case 6) and \( 0.790 \times 10^{-6} \text{ mm·}^{-1} \) (in Case 7) to \( 0.489 \times 10^{-6} \text{ mm·}^{-1} \) here (with \( \alpha = 1.65 \)). The shrinkage induced curvature is reduced from \( 0.592 \times 10^{-6} \text{ mm·}^{-1} \) (in Case 6) and \( 0.283 \times 10^{-6} \text{ mm·}^{-1} \) (in Case 7) to \( 0.092 \times 10^{-6} \text{ mm·}^{-1} \) here. Although the quantities of top and bottom non-prestressed steel are the same, the restraint provided by the eccentric bonded tendons results in the small increase in positive curvature with time due to shrinkage.

The increased restraint to creep and shrinkage provided by the increased quantity of top steel, results in a further reduction of concrete stresses with time, as shown in Fig. 2.8. The magnitude of the top fibre stress reduces from -9.26 MPa to -4.79 MPa with time and the bottom fibre stress from -1.87 MPa to +0.87 MPa.

Immediately after first loading, the stress in the prestressing steel is 1327 MPa and the corresponding prestressing force \( P \) is 1991 kN. The compressive forces in the top and bottom non-prestressed steel are 197 kN and 52 kN, respectively, and in the concrete, the compressive force is 1742 kN. After creep and shrinkage, the prestressing force reduces to 1772 kN (ie. 11.0% loss of prestress). The compressive forces in the non-prestressed top and bottom steel increase to 781 kN and 376 kN,
respectively, and the compressive force in the concrete decreases to 615 kN. The compressive force in the concrete is reduced by 65.3%.

For partially-prestressed concrete members containing significant quantities of bonded reinforcement, the actual cracking moment at some time after first loading is often much less than that usually calculated in design. It is common practice in design to assume that the loss of prestress in the concrete is the same as the loss of prestressing force in the tendon. As can be seen, this is not so.

**Case 9 - Cracked Prestressed Concrete Cross-section in bending**

\( A_{s1} = 0, A_{s2} = 3200 \text{ mm}^2, A_p = 1500 \text{ mm}^2 (P = 2025\text{kN}), M_0 = 1000 \text{ kNm} \)

The instantaneous and time-dependent strains and stresses on the cracked singly reinforced prestressed concrete section (with \( A_{s1} = 0 \)) subjected to a sustained bending moment of 1000 kNm are shown in Fig. 2.9. The case of zero shrinkage is also included.

When the applied moment induces a stress in the bottom fibre equal to the tensile strength of the concrete, the section cracks. Unlike reinforced concrete beams in bending, the depth of the intact concrete in compression in a prestressed section depends on the magnitude of the applied moment and gradually reduces under increasing moment. At a moment of 1000 kNm, the depth to the neutral axis for the section considered here is 582.5 mm and the instantaneous curvature is \( 1.118 \times 10^{-6} \text{ mm}^{-1} \).

With creep occurring only in the unreinforced compressive zone, the change in curvature due to creep (without shrinkage) is \( 2.25 \times 10^{-6} \text{ mm}^{-1} \) (with \( \alpha = 1.31 \)). The shrinkage curvature is \( 0.913 \times 10^{-6} \text{ mm}^{-1} \). With no restraint (bonded reinforcement) in the compression zone, the redistribution of compressive stress in the concrete is relatively small.

The losses of prestress in the cracked section due to both creep and shrinkage are very small. With a tendon stress of 33.3 MPa lost due to relaxation, the creep loss is negligible (4 MPa or 0.3% of initial prestress), while the shrinkage loss is 42 MPa (or 3.1% of the initial prestress).

**Case 10 - Cracked Prestressed Concrete Cross-section in bending**

\( A_{s1} = 3200 \text{ mm}^2, A_{s2} = 3200 \text{ mm}^2, A_p = 1500 \text{ mm}^2 (P = 2025\text{kN}), M_0 = 1000 \text{ kNm} \)

The instantaneous and time-dependent strains and stresses on the cracked doubly reinforced prestressed concrete section (with \( A_{s1} = A_{s2} = 3200 \text{ mm}^2 \)) subjected to a sustained bending moment of 1000 kNm are shown in Fig. 2.10. A comparison of Figs. 2.9 and 2.10, illustrates the
effects on time-dependent behavior of the inclusion of non-prestressed reinforcement in the highly stressed region at the top of the section ($A_n$).

The inclusion of compressive steel increases the stiffness of the cracked section. The depth to the neutral axis increases from 582.5 mm (Case 9) to 604.3 mm. The instantaneous curvature reduces from $1.118 \times 10^{-6}$ mm$^{-1}$ (Case 9) to $0.887 \times 10^{-6}$ mm$^{-1}$ and the change in curvature due to creep (without shrinkage) reduces from $2.250 \times 10^{-6}$ mm$^{-1}$ (Case 9) to $0.977 \times 10^{-6}$ mm$^{-1}$ (with $\alpha = 2.27$). The shrinkage induced curvature reduces from $0.913 \times 10^{-6}$ mm$^{-1}$ (Case 9) to $0.306 \times 10^{-6}$ mm$^{-1}$. Evidently, the inclusion of compressive steel greatly reduces both the creep and shrinkage induced curvature and, hence, the time-dependent deflection.

The inclusion of restraint (bonded reinforcement) in the compression zone causes a significant redistribution of compressive stress from the concrete to the top steel (with a final top steel stress of -325 MPa). Creep alone causes a reduction in top fibre stress in the concrete from -15.31 MPa (initially) to -9.53 MPa, while creep plus shrinkage result in a top fibre stress of -8.25 MPa and tensile stress is induced at the bottom of the intact concrete. Initially, the compressive forces carried by the concrete and the top steel are 1805 kN and 315 kN, respectively. After creep and shrinkage, these forces become 770 kN and 1040 kN, respectively.

As in Case 9, the losses of prestress due to creep and shrinkage are very small. The creep loss is only (20 MPa or 1.4% of the initial prestress) and the shrinkage loss is 53 MPa (or 3.9%).

**CALCULATION OF DEFLECTION**

*AS 3600 - 1994 and Amendment 2 - 2000*

The control of deflections may be achieved by limiting the calculated deflection to an acceptably small value. Two alternative general approaches for deflection calculation are specified in AS3600 (8), namely 'deflection by refined calculation' and 'deflection by simplified calculation'. The former is not specified in detail but allowance should be made for cracking and tension stiffening, the shrinkage and creep properties of the concrete, the expected load history and, for slabs, the two-way action of the slab. The use of the AEMM to determine the instantaneous and time-dependent deformation of the critical cross-sections in a beam or slab (similar to the results presented in Section 2), and then integrating the curvatures to obtain deflection, is such a refined calculation method and is recommended. However, the latter approach (deflection by simplified calculation) is generally used in design. The method and its limitations are discussed in detail below. Prior to Amendment 2 of AS3600, introduced in 2000, the simplified deflection
calculation approach was similar to that specified for reinforced concrete beams in ACI318M99 (Clause 9.5.2) (9).

Deflection by Simplified Calculation - AS3600 (Amendment 2)

The instantaneous or short-term deflection of a beam may be calculated using the mean value of the elastic modulus of concrete at the time of first loading, \( E_c \), together with the effective second moment of area of the member, \( I_e \). The effective second moment of area involves an empirical adjustment of the second moment of area of a cracked member to account for tension stiffening (the stiffening effect of the intact tensile concrete between the cracks). For a given cross-section, \( I_e \) is calculated using the formula developed by Branson (10),

\[
I_e = I_c + (I - I_c)(M_c/M) \leq I_{c,max}
\]

(1)

where \( I_c \) is the second moment of area of the fully-cracked section (calculated using modular ratio theory); \( I \) is the second moment of area of the gross concrete section about its centroidal axis; \( M \) is the maximum bending moment at the section, based on the short-term serviceability design load or the construction load; and \( M_c \) is the cracking moment given by

\[
M_c = Z(f'_{cf} - f_{cs} + P/A_y) + Pe \geq 0.0
\]

(2)

Z is the section modulus of the uncracked section \((I/y)\), where \( y \) is the distance from the centroidal axis of the uncracked section to the extreme tensile fibre; \( f'_{cf} \) is the characteristic flexural tensile strength of concrete (specified as \( 0.6f'_{cu} \)); \( f_{cs} \) is the maximum shrinkage induced tensile stress on the uncracked section at the extreme fibre at which cracking occurs and may be taken as

\[
f_{cs} = \left( \frac{1.5p}{1 + 50p} \right) E_c \epsilon_{sh}
\]

(3)

where \( p \) is the tensile reinforcement ratio \((A_{sf}/bd)\) and \( \epsilon_{sh} \) is the design final shrinkage strain.

The maximum value of \( I_e \) at any cross-section, \( I_{c,max} \) in Equation 1, is \( I \) when \( p - A_{sf}/bd \geq 0.005 \) and \( 0.6I \) when \( p < 0.005 \).

Alternatively, as a further simplification but only for reinforced concrete members, \( I_e \) at each nominated cross-section for rectangular sections may be taken as equal to

\((0.02 + 2.5p)bd^3\) when \( p \geq 0.005 \) and \((0.1 - 13.5p)bd^3\)
when \( p < 0.005 \).

The value of \( I_{cf} \) for the member is determined from the value of \( I_{cf} \) at midspan for a simple-supported beam. For interior spans of continuous beams, \( I_{cf} \) is half the midspan value plus one quarter of the value at each support, and for end spans of continuous strips, \( I_{cf} \) is half the midspan value plus half the value at the continuous support. For a cantilever, \( I_{cf} \) is the value at the support.

By comparison, for reinforced concrete members, ACI 318M99 (9) also specifies the use of Equation 1, except that \( M_c \) is replaced by \( M_a \) (the maximum moment in the member at the stage deflection is computed) and no allowance is made for shrinkage induced tension, with \( M_a = Zf_t \) (where \( f_t \) is specified as \( 0.7f'_c \)).

The term \( f_c \) in Equation 2 was introduced in Amendment 2 to AS3600 to allow for the tension that inevitably develops due to the restraint to shrinkage provided by the bonded tensile reinforcement. Equation 3 is based on the expression proposed by Gilbert (8), by assuming conservative values for the elastic modulus and the creep coefficient of concrete and assuming about 40% of the final shrinkage has occurred at the time of cracking. In the calculation of \( f_c \), using Equation 3, the final or long-term value of \( e_{cr} \) in the concrete should be used.

This allowance for shrinkage induced tension is particularly important in the case of lightly reinforced members (including slabs) where the tension induced by the full service moment alone might not be enough to cause cracking. In such cases, failure to account for shrinkage may lead to deflection calculations based on the uncracked section properties. This usually grossly underestimates the actual deflection. For heavily reinforced sections, the problem is not so significant, as the service loads are usually well in excess of the cracking load and the ratio of cracked to uncracked stiffness is larger.

For the calculation of long-term deflection, one of two approaches may be used. For reinforced or prestressed beams, the creep and shrinkage deflections can be calculated separately (using the material data specified in the Standard and the principles of mechanics). Alternatively, for reinforced concrete beam, long-term deflection can be crudely approximated by multiplying the immediate deflection caused by the sustained load by a multiplier \( k_s \) given by

\[
k_s = [2 - 1.2(A_{cf}/A_{ct})] \geq 0.8
\]

\( (4) \)

where the ratio of the compressive to tensile reinforcement areas \( A_{cf}/A_{ct} \) is taken at midspan for a simple or continuous span and at the support for a cantilever.
ACI 318M99 (9) also allows the use of a deflection multiplier $\lambda$ to calculate long-term deflection, with $\lambda = \xi/(1+50(A_{cd}/bd))$ and $\xi$ depends on the time under load and equals 2.0 for 5 years or more, 1.4 for 12 months, 1.2 for 6 months and 1.0 for 3 months.

What is Wrong with the AS3600 Simplified Procedure and How to Improve it:

The current simplified approach for the calculation of final deflection fails to adequately predict the long-term or time-dependent deflection (by far the largest portion of the total deflection in most reinforced and prestressed concrete members). Shrinkage induced curvature and the resulting deflection is not adequately accounted for when using $k_{\text{c}}$ (or $\lambda$) and no account is taken of the actual creep and shrinkage properties of the concrete. The introduction of $f_{\text{cr}}$ in the estimation of the cracking moment is a positive step in improving the procedure, by recognising that early shrinkage can induce tension that significantly reduces the cracking moment and significantly reduces the instantaneous stiffness with time. However, the gradual reduction in $k_{\text{f}}$ with time due to shrinkage and cyclic loading is still not fully accounted for. To better model the breakdown of tension stiffening with time, Equation 3 should be replaced by Equation 5, which was originally proposed by Gilbert (11) but was modified by Standards Australia (for political, rather than technical, reasons).

$$f_{\text{cr}} = \left( \frac{2.5p}{1+50p} E_e e_{ab} \right)$$

Equation 5

A further criticism of the simplified approach is the use of the second moment of area of the gross concrete section $I$ in Equation 1. It is unnecessarily conservative to ignore the stiffening effect of the bonded reinforcement in the calculation of the properties of the uncracked cross-section.

The use of the deflection multiplier $k_{\text{c}}$ (or $\lambda$) to calculate time-dependent deflections is simple and convenient and, provided the section is initially cracked under short term loads, it sometimes provides a ‘ball-park’ estimate of final deflection. However, to calculate the shrinkage induced deflection by multiplying the load induced short-term deflection by a long-term deflection multiplier is fundamentally wrong. Shrinkage can cause significant deflection even in unloaded members. The approach ignores the creep and shrinkage characteristics of the concrete, the environment, the age at first loading etc. At best, it must be seen as providing a very approximate estimate. At worst, it is not worth the time involved in making the calculation.
It is, however, not too much more complicated to calculate long-term creep and shrinkage deflection separately. As mentioned previously, well-established and reliable methods are available for calculating the time-dependent behavior of reinforced and prestressed concrete cross-sections. Favre et al. (12) and Ghali and Favre (2) have proposed equations developed from the AEMM for this purpose.

A simple method suitable for routine use in design has been proposed by the author for incorporation into AS3600 and is outlined below.

Proposed Method for Deflection Calculation

The load induced curvature, \( \kappa(t) \), (instantaneous plus creep) at any time \( t \) due to a sustained service actions may be expressed as

\[
\kappa(t) = \kappa_i(t)(1 + \phi/\alpha)
\]

(6)

where \( \kappa_i(t) \) is the instantaneous curvature due to the sustained service moment \( M_s \) (\( \kappa_i(t) = M_s/EI \)); for an uncracked cross-section \( I_s \) should be taken as the second moment of area of the uncracked transformed section, while for a cracked section, \( I_s \) should be calculated from Equation 1 (with \( f_{c,t} \) calculated using Equation 5 when estimating the final long-term curvature); \( \phi \) is the creep coefficient at time \( t \); and \( \alpha \) is a term that accounts for the effects of cracking and the ‘braking’ action of the reinforcement on creep and may be estimated from Equations 7a, 7b or 7c.

For a cracked reinforced concrete section in pure bending, \( \alpha = \alpha_1 \), where

\[
\alpha_1 = [0.48 p^{-0.5}] [1 + (125 p + 0.1)(A_{t,cr}/A_{t})^{1.2}]
\]

(7a)

For an uncracked reinforced or prestressed concrete section, \( \alpha = \alpha_2 \), where

\[
\alpha_2 = [1.0 - 1.5 p] [1 + (140 p - 0.1)(A_{t,cr}/A_{t})^{1.2}]
\]

(7b)

where \( p = A_{t,cr} b d_t / d_r \) and \( A_{t,cr} \) is the equivalent area of bonded reinforcement in the tensile zone at depth \( d_t \) (the depth from the extreme compressive fibre to the centroid of the outermost layer of tensile reinforcement). The area of any bonded reinforcement in the tensile zone (including bonded tendons) not contained in the outermost layer of tensile reinforcement (ie. located at a depth \( d_r \) less than \( d_t \)) should be included in the calculation of \( A_{t,cr} \) by multiplying that area by \( d_r/d_t \). For the purpose of
the calculation of \( A_{\text{cr}} \), the tensile zone is that zone that would be in tension due to the applied moment acting in isolation. \( A_{\text{cr}} \) is the area of the bonded reinforcement in the compressive zone.

For a cracked, partially prestressed section or for a cracked reinforced concrete section subjected to bending and axial compression, \( \alpha \) may be taken as

\[
\alpha = \alpha_2 + (\alpha_1 - \alpha_2)(d_n/d_c)^{2.4}
\]

(7c)

where \( d_n \) is the depth of the intact compressive concrete on the cracked section and \( d_c \) is the depth of the intact compressive concrete on the cracked section ignoring the axial compression and/or the prestressing force (i.e. the value of \( d_c \) for an equivalent cracked reinforced concrete section containing the same quantity of bonded reinforcement).

The shrinkage induced curvature on a reinforced or prestressed concrete section can be approximated by

\[
\kappa_{sh} = \frac{k_{sh} c_{sh}}{D}
\]

(8)

where \( D \) is the overall depth of the section, \( A_n \) and \( A_{nc} \) are as defined under Equation 7b above, and the factor \( k_s \) depends on the quantity and location of the bonded reinforcement and may be estimated from Equations 9a, 9b, 9c or 9d.

For an uncracked cross-section, \( k_s = k_{s1} \), where

\[
k_{s1} = (100p - 2500p^2) \left( \frac{d_n}{0.5D} - 1 \right) \left( 1 - \frac{A_{nc}}{A_n} \right)^{13} \quad \text{when } p = A_n/b \ d_n \leq 0.01
\]

(9a)

\[
k_{s1} = (40p + 0.35) \left( \frac{d_n}{0.5D} - 1 \right) \left( 1 - \frac{A_{nc}}{A_n} \right)^{13} \quad \text{when } p = A_n/b \ d_n > 0.01
\]

(9b)

For a cracked reinforced concrete section in pure bending, \( k_s = k_{s2} \), where

\[
k_{s2} = 1.2 \left( 1 - 0.5 \frac{A_{nc}}{A_n} \right) \left( \frac{D}{d_n} \right)
\]

(9c)

For a cracked, partially prestressed section or for a cracked reinforced concrete section subjected to bending and axial compression, \( k_s \) may be taken as
\[ k_s = k_{i1} + (k_{i2} - k_s)(d_t/d_n) \]
\[ (9d) \]

where \( d_n \) is the depth of the intact compressive concrete on the cracked section and \( d_{nt} \) is the depth of the intact compressive concrete on the cracked section ignoring the axial compression and/or the prestressing force (i.e., the value of \( d_n \) after cracking for an equivalent cracked reinforced concrete section containing the same quantity of bonded reinforcement).

Equations 7, 8 and 9 have been developed from parametric studies of a wide range of cross-sections analysed using the Age-Adjusted Effective Modulus Method of analysis (with typical results of such analyses discussed in Section 2 and illustrated in Figs. 2.1 to 2.10).

When the load induced and shrinkage induced curvatures are calculated at selected sections along a beam or slab, the deflection may be obtained by double integration. For a reinforced or prestressed concrete continuous span with the degree of cracking varying along the member, the curvature at the left and right supports, \( \kappa_L \) and \( \kappa_R \) and the curvature at midspan \( \kappa_M \) may be calculated at any time after loading and the deflection at midspan may be approximated by assuming a parabolic curvature diagram along the span, \( \ell \):

\[ \Delta = \frac{\ell^2}{96} (\kappa_L + 10\kappa_M + \kappa_R) \]
\[ (10) \]

The above equation will give a reasonable estimate of deflection even when the curvature diagram is not parabolic and is a useful expression for use in deflection calculations.

**WORKED EXAMPLES**

**Example 1:**

A reinforced concrete beam of rectangular section (800 mm deep and 400 mm wide) is simply-supported over a 12 m span and is subjected to a uniformly distributed sustained service load of 22.22 kN/m. The longitudinal reinforcement is uniform over the entire span and consists of four 32 mm diameter bars located in the bottom at an effective depth of 750 mm \( (A_{i2} = A_{it} = 3200 \text{ mm}^2) \) and two 32 mm diameter bars in the top at a depth of 50 mm below the top surface \( (A_{i1} = A_{it} = 1600 \text{ mm}^2) \). For each cross-section, therefore, \( p = A_i/bd = 0.0107 \). Calculate the instantaneous and long-term deflection at midspan, assuming the following material properties:
$f'_c = 32 \text{ MPa}; f'_{cy} = 3.39 \text{ MPa}; E_c = 28,570 \text{ MPa}; E_s = 2 \times 10^5 \text{ MPa}; \phi = 2.5; \text{ and } \varepsilon_n = 0.0006.$

The section at midspan:

The sustained bending moment is $M_0 = 400 \text{ kNm}$. This cross-section was analysed in Section 2.5 using the age-adjusted effective modulus method and the instantaneous and final strain and stress distributions are illustrated in Fig. 2.5.

The second moments of area of the uncracked transformed cross-section, $I$, and the full-cracked transformed section, $I_c$, are $I = 20,560 \times 10^6 \text{ mm}^4$ and $I_c = 7,990 \times 10^6 \text{ mm}^4$. The bottom fibre section modulus of the uncracked section is $Z = I/y_b = 52.7 \times 10^6 \text{ mm}^3$. From Equation 5,

$$f_{is} = \left( \frac{2.5 \times 0.0107}{1 + 50 \times 0.0107} \times 2 \times 10^5 \times 0.0006 \right) = 2.09 \text{ MPa}$$

and the time-dependent cracking moment is obtained from Equation 2:

$$M_{cr} = 52.7 \times 10^6 (3.39 - 2.09) = 68.5 \text{ kNm}.$$  

From Equation 1, the effective second moment of area is

$$I_{ef} = [7990 + (20560 - 7990)(68.5/400)] \times 10^6 = 8050 \times 10^6 \text{ mm}^4.$$  

The instantaneous curvature due to the sustained service moment is

$$\kappa(t) = \frac{M_0}{E_s I_{ef}} = \frac{400 \times 10^6}{28570 \times 8050 \times 10^6} = 1.74 \times 10^{-6} \text{ mm}^{-1}.$$  

From Equation 7a:

$$\alpha_1 = [0.48 \times 0.0107^{0.5}][1 + (125 \times 0.0107 + 0.1)(1600/3200)^{1.2}] = 7.55$$

and the load induced curvature (instantaneous plus creep) is obtained from Equation 6:

$$\kappa(t) = 1.74 \times 10^{-6} (1 + 2.5/7.55) = 2.32 \times 10^{-6} \text{ mm}^{-1}.$$  

(This agrees very well with the result obtained from the AEMM analysis illustrated in Fig. 2.5 where $\kappa(t) = 2.325 \times 10^{-6} \text{ mm}^{-1}$).
From Equation 9c:  \( k = k_2 = 1.2 \times (1 - 0.5 \times \frac{1600}{3200} \times \frac{800}{750}) = 0.96 \)

and the shrinkage induced curvature is obtained from Equation 8:

\[
\kappa_{sh} = \left[ \frac{0.96 \times 600 \times 10^{-6}}{800} \right] = 0.72 \times 10^{-6} \text{mm}^{-1}
\]

(c.f. 0.60 x 10^{-6} mm^{-1} shown in Fig. 2.5).

The instantaneous and final time-dependent curvatures at midspan are therefore

\( \kappa_i = 1.74 \times 10^{-6} \text{ mm}^{-1} \) and \( \kappa = \kappa(t) + \kappa_{sh} = 3.04 \times 10^{-6} \text{ mm}^{-1} \).

The section at each support:

The sustained bending moment is zero and the section remains uncracked. The load-dependent curvature is therefore zero. However, shrinkage curvature develops with time. From Equation 9b:

\[
k_r = k_r = k_2(40 \times 0.0107 + 0.35)(\frac{750}{0.5 \times 800} - 1)(1 - \frac{1600}{3200})^{1/3} = 0.276
\]

and the shrinkage induced curvature is estimated from Equation 8:

\[
\kappa_{sh} = \left[ \frac{0.276 \times 600 \times 10^{-6}}{800} \right] = 0.21 \times 10^{-6} \text{mm}^{-1}
\]

(c.f. 0.191 x 10^{-6} mm^{-1} obtained from the AEMM of the uncracked section, as shown in Fig. 2.3).

Deflections:

The instantaneous and final long-term deflections at midspan, \( \Delta_i \) and \( \Delta_{LT} \), respectively, are obtained from Equation 10:

\[
\Delta_i = \frac{12000^2}{96} (0 + 10 \times 1.74 + 0) \times 10^{-6} = 26.1 \text{ mm}
\]

\[
\Delta_{LT} = \frac{12000^2}{96} (0.21 + 10 \times 3.04 + 0.21) \times 10^{-6} = 46.0 \text{ mm} \quad (= \text{span/260})
\]
It is of interest to note that using the current approach in AS3600, with $k_c = 1.4$ (from Equation 4), the calculated final deflection is 60.9 mm.

Example 2:

A post-tensioned concrete beam of rectangular section (800 mm deep and 400 mm wide) is simply-supported over a 12 m span and is subjected to a uniformly distributed sustained service load of 38.89 kN/m. The beam is prestressed with a single parabolic cable consisting of 15/12.7 mm diameter strands ($A_p = 1500 \text{ mm}^2$) with $d_p = 650$ mm at midspan and $d_p = 400$ mm at each support. The 92 mm diameter duct containing the tendons is filled with grouted soon after transfer, thereby bonding the prestressing tendons to the surrounding concrete. The longitudinal non-prestressed reinforcement is uniform over the entire span and consists of 4 - 32 mm diameter bars located in the bottom at an effective depth of 750 mm ($A_{l_2} = 3200 \text{ mm}^2$) and 2 - 32 mm diameter bars in the top at a depth of 50 mm below the top surface ($A_{l_1} = A_{tc} = 1600 \text{ mm}^2$). For the purpose of this exercise, assume the initial prestressing force in the tendon is 2025 kN throughout the member and the relaxation loss is 50 kN. Calculate the instantaneous and long-term deflection at midspan, assuming the following material properties:

\[ f'_c = 32 \text{ MPa}; \quad f'_t = 3.39 \text{ MPa}; \quad E_c = 28,570 \text{ MPa}; \quad E_s = 2 \times 10^4 \text{ MPa}; \quad \phi = 2.5; \quad \text{and} \quad c_{nh} = 0.0006. \]

The section at midspan:

The sustained bending moment is $M_s = 700 \text{ kNm}$. This cross-section was analysed in Section 2.7 using the age-adjusted effective modulus method and the instantaneous and final strain and stress distributions are illustrated in Fig. 2.7. The section remains uncracked throughout.

The centroidal axis of the uncracked transformed cross-section is located at a depth of 415.7 mm below the top fibre and the second moment of area is $I = 21,070 \times 10^6 \text{ mm}^4$.

The instantaneous curvature due to the sustained service moment is

\[ \kappa_i(t) = \frac{M_s - P e}{E_c I} = \frac{700 \times 10^6 - 2025 \times 10^3 \times 234.3}{28570 \times 21070 \times 10^6} = 0.375 \times 10^{-6} \text{ mm}^{-1}. \]

From Equation 7a, with $A_{tc} = 1600 \text{ mm}^2$, $A_{s} = A_{s_1} + A_p \frac{d_p}{d_n} = 3200 + 1500 \times 650/750 = 4500 \text{ mm}^2$ and, therefore $p = A_s/b d_n = 4500/(400 \times 750) = 0.015$: 
\[ \alpha_2 = [1.0 - 15.0 \times 0.015] [1 + (140 \times 0.015 - 0.1)(1600/4500)^{1/2}] = 1.22 \]

and the load induced curvature (instantaneous plus creep) is obtained from Equation 6:

\[ \kappa(t) = 0.375 \times 10^{-6} (1 + 2.5/1.22) = 1.14 \times 10^{-6} \text{ mm}^{-1}. \]

(This agrees very well with the result obtained from the AEMM analysis illustrated in Fig. 2.7 where \( \kappa(t) = 1.165 \times 10^{-6} \text{ mm}^{-1} \)).

From Equation 9b:

\[ k_r = k_{r1} = (40 \times 0.015 + 0.35)(\frac{750}{0.5 \times 800} - 1)(1 - \frac{1600}{4500}) = 0.536 \]

and the shrinkage induced curvature is obtained from Equation 8:

\[ \kappa_{sh} = \left[ \frac{0.54 \times 600 \times 10^{-6}}{800} \right] = 0.40 \times 10^{-6} \text{ mm}^{-1} \]

(c.f. \( 0.283 \times 10^{-6} \text{ mm}^{-1} \) shown in Fig. 2.7).

The instantaneous and final time-dependent curvatures at midspan are therefore

\[ \kappa_i = 0.375 \times 10^{-6} \text{ mm}^{-1} \text{ and } \kappa = \kappa(t) + \kappa_{sh} = 1.54 \times 10^{-6} \text{ mm}^{-1}. \]

The section at each support:

The sustained bending moment is zero and the section remains uncracked. The centroidal axis of the uncracked transformed cross-section (with \( A_p \) located at a depth of 400 mm) is located at a depth of 409.4 mm below the top fibre and the second moment of area is \( I = 20,560 \times 10^6 \text{ mm}^4 \). The prestressing steel is located 9.4 mm above the centroidal axis of the transformed section, so that the prestressing force induces a small instantaneous positive curvature. Shrinkage (and creep) curvature develops with time.

The instantaneous curvature is

\[ \kappa_i(t) = \frac{M_i - Pe}{E_c I} = \frac{0 - 2025 \times 10^3 \times -9.4}{28570 \times 20560 \times 10^6} = 0.032 \times 10^{-6} \text{ mm}^{-1}. \]

From Equation 7a, with \( A_w = 1600 \text{ mm}^2 \), \( A_{cr} = A_{r1} + A_p \frac{d_i}{d_o} = 3200 + 1500 \times 400/750 = 4000 \text{ mm}^2 \) and, therefore \( p = A_i/b \frac{d_o}{d_i} = 4000/(400 \times 750) = 0.0133 \):
and the load induced curvature (instantaneous plus creep) is obtained from Equation 6:

$$\kappa(t) = 0.032 \times 10^{-6} (1 + 2.5/1.27) = 0.09 \times 10^{-6} \text{ mm}^{-1}.$$  

From Equation 9b:

$$k_i = k_{ij} = (40 \times 0.0133 + 0.35)(-750/0.5 \times 800 - 1)(1 - 1600/4000)^{1/3} = 0.398$$

and the shrinkage induced curvature is estimated from Equation 8:

$$\kappa_{sh} = \left[ \frac{0.398 \times 600 \times 10^{-6}}{800} \right] = 0.30 \times 10^{-6} \text{ mm}^{-1}$$

The instantaneous and final time-dependent curvatures at the supports are therefore

$$\kappa_i = 0.032 \times 10^{-6} \text{ mm}^{-1} \quad \text{and} \quad \kappa = \kappa(t) + \kappa_{sh} = 0.39 \times 10^{-6} \text{ mm}^{-1}.$$  

**Deflections:**

The instantaneous and final long-term deflections at midspan, $\Delta_i$ and $\Delta_{LT}$, respectively, are obtained from Equation 10:

$$\Delta_i = \frac{12000^2}{96} (0.032 + 10 \times 0.375 + 0.032) \times 10^{-6} = 5.7 \text{ mm}$$

$$\Delta_{LT} = \frac{12000^2}{96} (0.39 + 10 \times 1.54 + 0.39) \times 10^{-6} = 24.3 \text{ mm}$$

In this example, the ratio of final to instantaneous deflection is 4.3.

**Example 3:**

To test the proposed calculation procedure against experimental observations, the long-term deflection is calculated for a simply-supported one-way slabs tested under a constant, sustained, uniformly-distributed load for a period of 260 days. The slab was previously described and analysed by Gilbert (11). The slab specimen was prismatic with a cross-section 1000 mm wide and 180 mm deep and contained tensile reinforcement consisting of 6 – 12 mm diameter bars ($A_{sr} = 660$
mm²) at an effective depth \( d = 150 \) mm. The reinforcement ratio \( p = A_{st} / bd = 0.0044 \). The specimen contained no compressive reinforcement. The slab span was 5 m and the sustained load was 2 kPa plus the self-weight of the concrete, ie. \( w_{su} = 6.4 \) kPa. The slab was moist cured for two weeks after casting and, at age 14 days, the sustained load was first applied and testing commenced. The load remained constant throughout the test.

The characteristic concrete compressive strength at 28 days was determined from cylinder tests to be 31 MPa. The shrinkage strain in the concrete, \( \varepsilon_{sh} \), was measured on companion specimens of the same cross-sectional dimensions as the slab and with the same curing conditions. The creep coefficient, \( \phi \), was measured on cylinders under a constant sustained compressive stress of 5.0 MPa, first applied at age 14 days. After 260 days, the measured values were \( \varepsilon_{sh} = 690 \) \( \mu \)s and \( \phi = 2.6 \).

Immediately after the sustained load was applied to the specimen, the deflection at midspan was 3.9 mm and no evidence of cracking was observed. The first cracks in the slab were noticed on the slab soffit after 6 days under load and over the next 45 days additional cracks developed. No new cracks were recorded after 51 days under load, but the width of the cracks increased throughout the period of testing. The deflection at midspan after 260 days under load was 28.9 mm, with only 2.1 mm occurring in the last 100 days of the test. With a ratio of span to time-dependent deflection of 200, the slab is outside the code's minimum allowable span to time-dependent deflection ratio and would be unserviceable for most applications.

With \( f' = 31 \) MPa, the calculated values of elastic modulus and flexural tensile strength of concrete (in accordance with AS3600) are \( E_c = 28120 \) MPa and \( f_{ct} = 3.34 \) MPa, respectively.

The section at midspan:

For this specimen, the moment at midspan under the full service load is \( M_s = 20.0 \) kNm. The second moments of area of the uncracked transformed cross-section, \( I \), and the full-cracked transformed section, \( I_{cr} \), are \( I = 500.2 \times 10^6 \) mm⁴ and \( I_{cr} = 76.2 \times 10^6 \) mm⁴. The bottom fibre section modulus of the uncracked section is \( Z = I / y_b = 5.64 \times 10^6 \) mm³. From Equation 5,

\[
f_{ct} = \left( \frac{2.5 \times 0.0044}{1 + 50 \times 0.0044} \times 2 \times 10^4 \times 0.00069 \right) = 1.24 \text{ MPa}
\]

and the time-dependent cracking moment is obtained from Equation 2:

\[
M_{cr} = 5.64 \times 10^6 (3.34 - 1.24) = 11.84 \text{ kNm}.
\]
From Equation 1, the effective second moment of area is

\[ I_{ef} = [76.2 + (500.2 - 76.2)(11.84/20.0)^2] \times 10^6 = 164.2 \times 10^6 \text{ mm}^4 \]

The instantaneous curvature due to the sustained service moment is therefore

\[ \kappa_i(t) = \frac{M_s}{E_{ef}I_{ef}} = \frac{20.0 \times 10^6}{28120 \times 164.2 \times 10^6} = 4.33 \times 10^{-6} \text{ mm}^{-1}. \]

From Equation 7a (with \( A_{sc} = 0 \)):

\[ \alpha_i = [0.48 \times 0.0107 \times 0.5] = 7.24 \]

and the load induced curvature (instantaneous plus creep) is obtained from Equation 6:

\[ \kappa(t) = 4.33 \times 10^{-6} (1 + \frac{2.6}{7.24}) = 5.89 \times 10^{-6} \text{ mm}^{-1}. \]

From Equation 9c:

\[ k_s = k_{s2} = 1.2 \times (1 - 0.5 \times \frac{0.0044}{660} \times \frac{180}{150}) = 1.44 \]

and the shrinkage induced curvature is obtained from Equation 8:

\[ \kappa_{sh} = \left[ \frac{1.44 \times 690 \times 10^{-6}}{180} \right] = 5.52 \times 10^{-6} \text{ mm}^{-1} \]

The final time-dependent curvature at midspan is therefore

\[ \kappa = \kappa(t) + \kappa_{sh} = 11.41 \times 10^{-6} \text{ mm}^{-1}. \]

The section at each support:

The sustained bending moment is zero and the section remains uncracked. The load-dependent curvature is therefore zero. However, shrinkage curvature develops with time. From Equation 9b:

\[ k_s = k_{si} = \left( 100 \times 0.0044 - 2500 \times 0.0044^2 \times \frac{150}{0.5 \times 180} - 1 \right) = 0.261 \]

and the shrinkage induced curvature is estimated from Equation 8:

\[ \kappa_{sh} = \left[ \frac{0.261 \times 690 \times 10^{-6}}{180} \right] = 1.00 \times 10^{-6} \text{ mm}^{-1}. \]
Deflections:

The final long-term deflection at midspan, $\Delta_{LT}$, is obtained from Equation 10:

$$\Delta_{LT} = \frac{5000^2}{96}(1.00 + 10 \times 11.41 + 1.00) \times 10^{-6} = 30.2 \text{ mm}.$$

This compares very well with the measured final deflection of 28.9 mm.

CONCLUSIONS

The behavior of reinforced and prestressed concrete flexural members under sustained service loads has been discussed. In particular, the time-dependent effects of creep and shrinkage have been outlined and suggestions have been made on how to consider these effects in design. The deficiencies in the existing procedures for deflection calculations in the Australian Standard AS3600 and ACI318M99 have been discussed and an alternative, simplified procedure for calculating both the short-term and long-term deflections has been proposed and illustrated by examples. The proposed procedure has also been tested against experimental results.

ACKNOWLEDGEMENTS

The study is part of an on-going research program at The University of New South Wales funded by the Australian Research Council on the serviceability of concrete structures. This support is gratefully acknowledged.

REFERENCES


(9) ACI 318M-99. *Building Code Requirements for Structural Concrete*. American Concrete Institute. Michigan. USA.


Characteristic compressive strength of the concrete, $f'_c = 32$ MPa;
Flexural tensile strength of the concrete (modulus of rupture), $f_t = 3.39$ MPa;
Elastic modulus of the concrete, $E_c = 28,570$ MPa;
Elastic modulus of non-prestressed and prestressed steel, $E_s = E_p = 2 \times 10^5$ MPa;
Creep coefficient of concrete, $\phi = 2.5$;
Ageing coefficient of concrete, $\chi = 0.8$;
Shrinkage strain of the concrete, $\varepsilon_{st} = -0.0006$ or zero;
Relaxation loss in the prestressing steel is about 2.5% of the initial prestressing force (50kN in the examples considered here).

Figure 1 Cross-section analysed in Section 2.
Code Provisions for Deflection Control in Concrete Structures

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**Figure 2.1** Unreinforced, uncracked section ($A_{s1} = A_{s2} = A_p = 0$; $M_s = 200$ kNm, $N_s = -2000$ kN)

- $\kappa = +0.410 \times 10^{-6}$ mm$^{-1}$
- $\kappa = +1.436 \times 10^{-6}$ mm$^{-1}$

---

**Figure 2.2** Uncracked reinforced section ($A_{s1} = A_{s2} = 3200$ mm$^2$, $A_p = 0$; $M_s = 200$ kNm, $N_s = -2000$ kN)

- $\kappa = 0.322 \times 10^{-6}$ mm$^{-1}$
- $\kappa = 0.721 \times 10^{-6}$ mm$^{-1}$
Figure 2.3 Uncracked reinforced section ($A_{s1} = 1600 \text{ mm}^2$, $A_{s2} = 3200 \text{ mm}^2$, $A_p = 0$, $M_s = 200 \text{ kNm}$, $N_s = -2000 \text{ kN}$)

Figure 2.4 Cracked reinforced section ($A_{s1} = 0$, $A_{s2} = 3200 \text{ mm}^2$, $A_p = 0$, $M_s = 400 \text{ kNm}$, $N_s = 0 \text{ kN}$)
Figure 2.5  Cracked reinforced section ($A_{s1} = 1600 \text{ mm}^2$, $A_{s2} = 3200 \text{ mm}^2$, $A_p = 0$; $M_s = 400 \text{ kNm}$, $N_s = 0$)

Figure 2.6  Uncracked partially prestressed section in bending ($A_{s1} = 0$, $A_{s2} = 3200 \text{ mm}^2$, $A_p = 1500 \text{ mm}^2$; $P = 2025 \text{kN}$; $M_s = 700 \text{ kNm}$)
Figure 2.7 Uncracked partially-prestressed section in bending ($A_{s1} = 1600 \text{ mm}^2$, $A_{s2} = 3200 \text{ mm}^2$, $A_p = 1500 \text{ mm}^2$; $P = 2025 \text{kN}$; $M_s = 700 \text{kNm}$)

Figure 2.8 Uncracked partially-prestressed section in bending ($A_{s1} = 3200 \text{ mm}^2$, $A_{s2} = 3200 \text{ mm}^2$, $A_p = 1500 \text{ mm}^2$; $P = 2025 \text{kN}$; $M_s = 700 \text{kNm}$)
Figure 2.9  Cracked partially-prestressed section in bending (A_{s1} = 0, A_{s2} = 3200 \text{ mm}^2, A_p = 1500 \text{ mm}^2; P = 2025\text{kN}; M_s = 1000 \text{kNm})

Figure 2.10  Cracked partially-prestressed section in bending (A_{s1} = 3200 \text{ mm}^2, A_{s2} = 3200 \text{ mm}^2, A_p = 1500 \text{ mm}^2; P = 2025\text{kN}; M_s = 1000 \text{kNm})
Deflection Provisions in the Draft Brazilian Code

by M. C. C. Guarda, J. S. Lima, and L. M. Pinheiro

Synopsis: This paper presents an overview of the requirements for deflection of beams and slabs in the draft Brazilian Code for concrete structures, NBR 6118:2000. Firstly, the Brazilian Code recommendations for deflection computations and allowable limits are described, and then these provisions are compared with those of section 9.5 of ACI 318-99. The paper also includes a numerical example to illustrate how the new proposals are applied to structural design. It is observed that the general criteria of the two codes are similar, but some differences are pointed out regarding minimum thickness requirements, load conditions, effective moment of inertia for continuous beams and the values of modular ratio, modulus of rupture and allowable deflections.

Keywords: allowable deflections; building codes; code provisions; limit state design; reinforced concrete; serviceability
INTRODUCTION

It is well-known that serviceability limit states do not involve the collapse of a structure but the disruption of its functional use. One of the major serviceability limit states is excessive deflection, which may cause loss of esthetic appearance and damage to structural and nonstructural elements. As present-day structures are designed for high concrete and steel stresses, design by efficient ultimate-load procedures implies more slender members, which need better control for serviceability deflection performance.

The draft Brazilian Code NBR 6118:2000 (1) gives special attention to deflection control, specifying in-service load conditions and allowable deflections for beams and slabs according to the use of the structure. This draft is currently under evaluation and the final draft is expected to be published in 2001.

The aim of this paper is to present the deflection provisions in the draft Brazilian Code (1) and compare them with section 9.5 of ACI 318:99 (2).

PROVISIONS FOR CALCULATION OF DEFLECTIONS IN THE DRAFT BRAZILIAN CODE

The previous version of the Brazilian Code included some minimum thickness requirements based on the yield strength of reinforcement, support conditions and span lengths. However, this approach was abandoned in the new Code, which determines that deflections be calculated regardless of the member thickness.
Load Conditions

The draft Brazilian Code (1) is in a limit state format. In checking the serviceability limit states, unfactored loads are always used, but live load reductions may be considered. This means that instead of using the maximum live load in the lifetime of the structure, some fraction of the mean maximum lifetime load may be taken.

This reduction depends on the serviceability limit state that is to be checked, on the use of the structure and on the characteristics of the nonstructural elements attached to the structural members.

In checking excessive deflections, it may be desirable to use a quasi-permanent live load, which is between 20 and 60% of the specified live load. Whenever these deflections could cause any damage to nonstructural elements, a frequent live load may be used (30 to 70% of the specified live load). Typical fractions of live loads ($\psi_1$ and $\psi_2$) are shown in Table 1.

In calculating cracking moment, however, no reduction of the live load is permitted.

Cracking Moment

The cracking moment is calculated from the following equation:

$$M_c = \frac{I_f}{y_t}$$

where $I_f$ is the moment of inertia of the uncracked concrete section; $f_r$ is the modulus of rupture of concrete, $f_r = 0.315 (f'_c)^{1/2}$ for rectangular beams or $f_r = 0.252 (f'_c)^{1/2}$ for T beams, in MPa; $f'_c$ is the specified compressive strength of concrete, in MPa; and $y_t$ is the distance from centroidal axis of cross section to extreme tension fiber.

Moments of Inertia

For the precracking stage or Stage I, the draft Code allows the use of gross concrete section, neglecting the contribution of the steel reinforcement. If more accuracy is desired, the transformed moment of inertia can be obtained from a transformed section by assuming a modular ratio of elasticity $n$ that reflects long-term behavior. The recommended values of $n$ of each load condition are:

<table>
<thead>
<tr>
<th>Load condition</th>
<th>n</th>
</tr>
</thead>
<tbody>
<tr>
<td>Full live load</td>
<td>10</td>
</tr>
<tr>
<td>Frequent or quasi-permanent live load</td>
<td>15</td>
</tr>
</tbody>
</table>
For the postcracking stage or Stage II, the transformed moment of inertia is used, neglecting the contribution of concrete in tension.

The effective moment of inertia is based on Branson's equation:

\[ l_e = \left( \frac{M_{cr}}{M_a} \right) l_b + \left[ 1 - \left( \frac{M_{cr}}{M_a} \right) \right] l_{cr} \leq l_b \]

where \( M_{cr} \) is the cracking moment; 
\( M_a \) is the maximum moment in member under the appropriate load condition (quasi-permanent or frequent); 
\( l_b \) is the moment of inertia of the uncracked concrete section; and 
\( l_{cr} \) is the moment of inertia of the cracked section transformed to concrete.

For continuous beams, the draft Code suggests the use of the weighted-average value of \( l_e \) based on the bending moment diagram (Fig. 1):

\[ l_e = \frac{a_1}{\ell} l_{e,1} + \frac{a_m}{\ell} l_{e,m} + \frac{a_2}{\ell} l_{e,2} \]

where \( a_m \) is the length of the positive moment region; 
\( a_1, a_2 \) are the lengths of the left and right negative moment regions, respectively; 
\( \ell \) is the span length considered as the clear span plus depth of member as long as it does not exceed the distance between centers of supports; 
\( l_{e,m} \) is the effective moment of inertia for the positive moment region; and 
\( l_{e,1}, l_{e,2} \) are the effective moments of inertia for the left and right negative moment regions, respectively.

The values of \( a_1/\ell \) and \( a_2/\ell \) can be taken as 0.15 for beams continuous at both ends.

**Modulus of Elasticity**

For normal-weight concrete, the value of \( E_c \) can be estimated from the empirical expression:

\[ E_c = 4734 \sqrt{f_c'} \quad \text{(MPa)} \]

where \( f_c' \) is the specified compressive strength of concrete, in MPa.
Calculation of Deflections

Two kinds of deflection are considered: immediate deflections and additional long-term deflections due to creep and shrinkage.

Immediate deflections - Immediate deflections may be calculated using equations from elastic methods. However, more accurate procedures may be used if they result in reasonable predictions of deflections.

Additional long-term deflections - Additional long-term deflections due to creep and shrinkage may be evaluated using an empirical approach for both one-way and two-way systems. They are given by the immediate deflection multiplied by the factor:

$$\lambda = \frac{\Delta \xi}{1 + 50p'}$$

where

$$\Delta \xi = \xi(t) - \xi(t_o)$$

$$\xi$$ is the time-dependent factor between 0 and 2 (Table 2), depending on the period over which sustained loads are of interest:

$$\xi(t) = 0.68 \times 0.996^t \times t^{0.32} \quad \text{if} \quad t < 70 \text{ months}$$

$$\xi(t) = 2 \quad \text{if} \quad t \geq 70 \text{ months}$$

$$t$$ is the time in months when long-term deflection is required;

$$t_o$$ is the time in months when sustained loads are applied to the structure. If sustained loads are applied at different times, $$t_o$$ should be taken as the average factor:

$$t_o = \frac{\sum P_i t_{i,t}}{\sum P_i}$$

$$P_i$$ is a portion of sustained load;

$$t_{i,t}$$ is the time in months when portion $$i$$ of sustained load is applied;

$$p'$$ is the ratio of compression reinforcement calculated at midspan, $$p' = \frac{A'_c}{bd}$$.

For a continuous beam, the weighted-average value of $$p'$$ according to the bending moment diagram (Fig. 1) may be considered;

$$A'_c$$ is the area of compression reinforcement;

$$b$$ is the width of cross section; and

$$d$$ is the distance from extreme compression fiber to centroid of tensile reinforcement.

If more accuracy is desired, deflections due to creep and shrinkage may be estimated separately, taking other parameters into account as described in the draft Code.

Total deflection - The total deflection of the member is the sum of the immediate deflection and the additional long-term deflection, and it is given by:

$$\Delta_t = (1 + \lambda)\Delta_u$$
ALLOWABLE DEFLECTIONS

As excessive deflections can be undesirable for various reasons, the draft Code separates the suggested values of maximum permissible computed deflections into categories. Depending on the nature and function of the structure, the deflection limit that applies is selected (Table 3).

Sensory Acceptability

Although sensory acceptability tends to be a matter of personal opinion, it is important to observe that deflections greater than \( l/250 \) are generally visible. The critical deflection is the total immediate plus the additional long-term deflection.

Serviceability of Structure

Excessive deflections may interfere with the use of the structure, for example by causing problems in the drainage of floors and roofs and the operation of sensitive equipment.

Effects on Nonstructural Elements

Malfunctioning of partitions, doors and windows can be avoided by limiting the deflection that occurs after installation of the nonstructural elements. The critical deflection is the immediate deflection due to live load plus the additional long-term deflection.

Effects on Structural Elements

The structural behavior may be different from that assumed in the design if excessive deflections occur. In this case, they must be considered in design for strength.

COMPARISON WITH ACI 318:99

The flow-chart in Fig. 2 summarizes the procedure for deflection control in the draft Brazilian Code (1), which is essentially similar to section 9.5 of ACI 318:99 (2). Some differences are pointed out in the following.

Minimum Thickness Requirements

The Brazilian Code requires that deflections be calculated and compared with allowable limits for each member, irrespective of its thickness. According to the ACI Code, on the other hand, deflection calculation may be avoided in some cases. When minimum thickness requirements for both beams and slabs are satisfied, no checking of the deflection serviceability performance is necessary.
Sustained Portion of Live Load

The Brazilian Code suggests live load reductions in the load conditions used for the calculation of deflections, as shown in Table 1. Therefore, the sustained portions of live load are already defined in the code. In the ACI Code, from a different standpoint, the limit state of excessive deflection is based on service loads and no reduction of live load is recommended. Thus, it is necessary to make an assumption regarding the portion of live load to be considered as sustained.

Modular Ratio

The modular ratio in the Brazilian Code depends only on the load conditions. When reduction of live load is permitted, as for example when the uncracked transformed section is used in calculating the cracking moment, \( n \) should be taken as 10. When the reduction of live load is permitted (quasi-permanent or frequent values), as for example when calculating deflections, \( n \) is always 15. In the ACI Code, as the modular ratio is given directly by \( E_i/E_c \), lower values of \( n \) are used. Consequently, the value of the effective moment of inertia would be higher according to the Brazilian Code than according to the ACI Code.

Modulus of Rupture

The values of the modulus of rupture calculated from the Brazilian Code equations are lower than those given by the ACI Code equation, especially for low-strength concrete. As a result, values of the cracking moment according to the Brazilian Code are lower than those according to the ACI Code.

Effective Moment of Inertia for Continuous Beams

For continuous beams, the draft Brazilian Code (1) suggests a weighted-averaging of effective moment of inertia values for the negative and positive moment sections, based on the bending moment diagram. The ACI 318:99 (2), on the other hand, suggests the use of a simple averaging of \( I_a \) values for the negative and positive moment sections.

The Brazilian Code recommendations are similar to those in ACI 435.2 (3).

Allowable Deflections

The Brazilian Code deflection limitations given in Table 2 cover a variety of specific situations. It seems that these limits are more stringent than those in Table 9.5b in ACI 318:99 (2). Besides, it should be noticed that the Brazilian Code places a limit on total deflections, which is not directly recommended in the ACI Code.

The categories of allowable deflections in the Brazilian Code are similar to those in ACI 435.3 (4).
Values of $\lambda$

The empirical multiplier for additional long-term deflection $\lambda$ is the same for dead and live loads in the Brazilian Code, even if these loads are applied at different times. In the latter, instead of using more than one value of $\lambda$, the multiplier is calculated taking an average of the times when loads are applied. In the ACI Code, according to references (5) and (6), different values of $\lambda$ should be taken for dead and live loads. Therefore, while in the Brazilian Code the additional long-term deflection is given by $\lambda(\Delta_{D} + \Delta_{L})$, in the ACI Code it is given by $\lambda_{1}\Delta_{D} + \lambda_{2}\Delta_{L}$.

Some of these differences can be observed in the following numerical example.

NUMERICAL EXAMPLE

The rectangular beam shown in Fig. 3 supports dead and live loads of 5.80 kN/m (0.40 kip/ft) and 7.35 kN/m (0.50 kip/ft), respectively. Determine if the deflection criteria are satisfied, given:

- $f'_{c} = 34.5$ MPa (5000 psi)
- $f_{y} = 414$ MPa (60,000 psi)
- $A_{s} = 1135$ mm$^2$ (1.76 in.$^2$)
- $E_{s} = 200 \times 10^{3}$ MPa (29 x 10$^8$ psi)

Assume that:

- this is a residential building;
- the partitions are installed 3 months after the shoring is removed;
- the beam supports partitions likely to be damaged by excessive deflections.

<table>
<thead>
<tr>
<th>Load Conditions</th>
<th>Load (kN/m)</th>
<th>$M_{x}$ (kN-m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 D</td>
<td>5.80</td>
<td>26.98</td>
</tr>
<tr>
<td>2 D + 0.3L</td>
<td>8.00*</td>
<td>37.24</td>
</tr>
<tr>
<td>3 D + L</td>
<td>13.15</td>
<td>61.17</td>
</tr>
</tbody>
</table>

* when calculating the additional long-term deflection according to the ACI, it will be assumed that 30% of the live load is sustained.

<table>
<thead>
<tr>
<th>Cracking Moment</th>
<th>NBR 6118:2000</th>
<th>ACI 318:99</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_{r}$</td>
<td>3.34</td>
<td>3.66</td>
</tr>
<tr>
<td>$I_{o}$</td>
<td>$14.17 \times 10^{8}$</td>
<td>$14.17 \times 10^{8}$</td>
</tr>
<tr>
<td>$y_{t}$</td>
<td>203</td>
<td>203</td>
</tr>
<tr>
<td>$M_{cr}$</td>
<td>23.31</td>
<td>25.54</td>
</tr>
<tr>
<td>$M_{a}$</td>
<td>61.17</td>
<td>61.17</td>
</tr>
</tbody>
</table>

Therefore, the midspan section will be cracked at service loads.
### Transformed Section

<table>
<thead>
<tr>
<th>n</th>
<th>Modular ratio</th>
<th>NBR 6118:2000</th>
<th>ACI 318:99</th>
</tr>
</thead>
<tbody>
<tr>
<td>n</td>
<td>Cracked transformed moment of inertia (mm⁴)</td>
<td>9.17 x 10⁸</td>
<td>5.52 x 10⁸</td>
</tr>
</tbody>
</table>

### Effective Stiffness

<table>
<thead>
<tr>
<th>lₐf</th>
<th>Effective moment of inertia (mm⁴) for load condition 1</th>
<th>NBR 6118:2000</th>
<th>ACI 318:99</th>
</tr>
</thead>
<tbody>
<tr>
<td>lₐf</td>
<td>Effective moment of inertia (mm⁴) For load condition 2</td>
<td>10.39 x 10⁸</td>
<td>8.31 x 10⁸</td>
</tr>
<tr>
<td>lₐf</td>
<td>Effective moment of inertia (mm⁴) For load condition 3</td>
<td>-</td>
<td>6.15 x 10⁸</td>
</tr>
<tr>
<td>Ec</td>
<td>Modulus of elasticity of concrete (MPa)</td>
<td>27,959</td>
<td>27,804</td>
</tr>
</tbody>
</table>

### Multiplier for Long-term Deflections

<table>
<thead>
<tr>
<th>λ</th>
<th>Multiplier factor</th>
<th>NBR 6118:2000</th>
<th>ACI 318:99</th>
</tr>
</thead>
<tbody>
<tr>
<td>λ</td>
<td></td>
<td>1.17**</td>
<td>1.00 (for D)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>2.00 (for L)</td>
</tr>
</tbody>
</table>

** assuming t₡ = 0.5 month for dead loads and t₇ = 3 months for live loads

### Deflections (mm)

<table>
<thead>
<tr>
<th>Δₐ₀D</th>
<th>Immediate dead-load deflection</th>
<th>NBR 6118:2000</th>
<th>ACI 318:99</th>
</tr>
</thead>
<tbody>
<tr>
<td>Δₐ₁</td>
<td>Total immediate deflection</td>
<td>4.97</td>
<td>13.86</td>
</tr>
<tr>
<td>Δₐ₃L</td>
<td>Immediate live-load deflection</td>
<td>1.95</td>
<td>10.93</td>
</tr>
<tr>
<td>Δₐ₅₃L</td>
<td>Immediate live-load deflection Considering sustained portion of live load only</td>
<td>-</td>
<td>3.32</td>
</tr>
<tr>
<td>Δₐ₂D</td>
<td>Additional long-term deflection due to dead loads</td>
<td>3.53</td>
<td>2.93</td>
</tr>
<tr>
<td>Δₐ₃L</td>
<td>Additional long-term deflection due to sustained live loads</td>
<td>2.28</td>
<td>6.64</td>
</tr>
<tr>
<td>Δₐ₄</td>
<td>Total deflection</td>
<td>10.78</td>
<td>-</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Δₐ₀D + Δₐ₃L + Δₐ₄</th>
<th>Deflection after the installation of partitions</th>
<th>NBR 6118:2000</th>
<th>ACI 318:99</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>7.76</td>
<td>20.50</td>
</tr>
</tbody>
</table>

### Verification of allowable deflections (mm)

**NBR 6118:2000**

<table>
<thead>
<tr>
<th>Reasons</th>
<th>allowable</th>
<th>calculated</th>
<th>Ok</th>
</tr>
</thead>
<tbody>
<tr>
<td>Visual</td>
<td>24.40</td>
<td>10.78</td>
<td>Ok</td>
</tr>
<tr>
<td>Tactile</td>
<td>17.43</td>
<td>1.85</td>
<td>Ok</td>
</tr>
<tr>
<td>Effects on masonry or plastered walls</td>
<td>10.00</td>
<td>7.76</td>
<td>Ok</td>
</tr>
</tbody>
</table>
ACI 318:99

<table>
<thead>
<tr>
<th>Type of member</th>
<th>allowable</th>
<th>Calculated</th>
</tr>
</thead>
<tbody>
<tr>
<td>Roof or floor construction supporting or attached to nonstructural elements likely to be damaged by large deflections</td>
<td>12.71</td>
<td>20.50</td>
</tr>
</tbody>
</table>

Thus, the deflection criteria are satisfied according to the draft Brazilian Code (1). According to ACI 318:99 (2), however, the floor is not satisfactory.

The reasons for the discrepancy have already been pointed out before, when the Brazilian Code was compared with the ACI Code. In this example, the main differences are due to the use of a frequent live load in the Brazilian Code, which results in lower total loads than when the ACI Code is applied, and due to the use of a higher modular ratio, which leads to a higher cracked transformed moment of inertia.

Therefore, the deflection criteria of the Brazilian Code tend to be less conservative than those of the ACI Code.

CONCLUDING REMARKS

In this paper, the provisions for deflection control in the draft Brazilian Code (1) have been described. The procedures recommended in this draft Code are similar to those in section 9.5 of ACI 318:99 (2), but some differences were indicated regarding minimum thickness requirements, load conditions, effective moment of inertia for continuous beams and the values of modular ratio, modulus of rupture, $\lambda$ and allowable deflections.

In addition, it may be noticed that those provisions are only general rules for deflection control. If conditions of loading during construction can drastically alter the subsequent deflections or if deflection requirements are more stringent than for normal buildings, deflections should always be estimated by a more comprehensive analysis and lower values of allowable deflections should be used.

References


2. ACI Committee 318, "Building Code Requirements for Reinforced Concrete (ACI 318-99) and Commentary (ACI 318R-99)", American Concrete Institute, Detroit, 1999, 391 p.

3. ACI Committee 435, "Deflections of Reinforced Concrete Flexural Members (ACI 435.2R-66) (Reapproved 1989)", ACI Manual of Concrete Practice, Part 4, American Concrete Institute, Detroit, Mich.

4. ACI Committee 435, "Allowable Deflections (ACI 435.3R-68) (Reapproved 1989)", ACI Manual of Concrete Practice, Part 3, American Concrete Institute, Detroit, Mich.


NOTATION

\( a_1, a_2 \) = lengths of the negative moment regions (continuous beam)

\( a_m \) = length of the positive moment region (continuous beam)

\( A_s \) = area of tension reinforcement

\( A_{s'} \) = area of compression reinforcement

\( b \) = width of cross section

\( d \) = distance from extreme compression fiber to centroid of tension reinforcement

\( D \) = dead loads

\( E_c \) = modulus of elasticity of concrete

\( E_s \) = modulus of elasticity of steel

\( f'c \) = specified compressive strength of concrete, MPa.

\( f_c \) = modulus of rupture of concrete

\( f_y \) = yield strength of reinforcement

\( h \) = height of cross section

\( I_{cr} \) = moment of inertia of the cracked section transformed to concrete

\( I_e \) = effective moment of inertia

\( I_{e,1}, I_{e,2} \) = effective moments of inertia for the negative moment regions (continuous beam)

\( I_{e,m} \) = effective moment of inertia for the positive moment region (continuous beam)

\( I_o \) = moment of inertia of the uncracked concrete section

\( \ell \) = span length of beam or one-way slab or span in short direction of two-way slab, considered as the clear span plus depth of member as long as it does not exceed distance between centers of supports

\( L \) = live loads

\( M_a \) = maximum moment in member under the appropriate load condition

\( M_{cr} \) = cracking moment

\( n \) = modular ratio of elasticity

\( P_i \) = portion of sustained load

\( t \) = time in months when long-term deflection is required

\( t_o \) = time in months when sustained loads are applied to the structure

\( t_{oi} \) = time in months when portion i of the sustained loads is applied

\( y_l \) = distance from centroidal axis of cross section to extreme tension fiber

\( \Delta_{D} \) = immediate dead-load deflection

\( \Delta_{L} \) = total immediate deflection

\( \Delta_{L,s} \) = immediate live-load deflection

\( \Delta_{L,s} \) = immediate live-load deflection considering sustained portion of live load only
\[ \Delta_{d,0} = \text{additional long-term deflection due to dead loads} \]
\[ \Delta_{d,L} = \text{additional long-term deflection due to sustained live loads} \]
\[ \Delta_t = \text{total deflection} \]
\[ \lambda = \text{empirical multiplier for additional long-term deflection} \]
\[ \xi = \text{time-dependent factor for sustained loads} \]
\[ \rho' = \text{ratio of compression reinforcement calculated at midspan} \]
\[ \psi_1, \psi_2 = \text{fractions of live-load used in checking excessive deflections} \]

**CONVERSION FACTORS**

- 1 in. = 25.4 mm
- 1 in.\(^2\) = 645 mm\(^4\)
- 1 in.\(^4\) = 4.16 x 10\(^5\) mm\(^4\)
- 1 ft = 0.3048 m
- 1 kip = 4.448 kN
- 1 kip/ft = 14.6 kN/m
- 1 psi = 6895 Pa
- 1 ft-kip = 1.356 kN-m

**TABLE 1 – FRACTIONS OF LIVE LOAD IN CHECKING EXCESSIVE DEFLECTIONS**

<table>
<thead>
<tr>
<th>Use of Structure</th>
<th>Frequent Live load ((\psi_1))</th>
<th>Quasi-permanent live load ((\psi_2))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Places with no heavy fixed equipment or not crowded for long periods</td>
<td>30%</td>
<td>20%</td>
</tr>
<tr>
<td>Places with heavy fixed equipment or crowded for long periods</td>
<td>60%</td>
<td>40%</td>
</tr>
<tr>
<td>Libraries, archives, garages</td>
<td>70%</td>
<td>60%</td>
</tr>
</tbody>
</table>

**TABLE 2 – VALUES OF \(\xi(t)\), t IN MONTHS**

<table>
<thead>
<tr>
<th>t</th>
<th>0</th>
<th>0.5</th>
<th>1</th>
<th>2</th>
<th>5</th>
<th>10</th>
<th>20</th>
<th>40</th>
<th>70</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\xi(t))</td>
<td>0</td>
<td>0.54</td>
<td>0.68</td>
<td>0.84</td>
<td>1.12</td>
<td>1.36</td>
<td>1.64</td>
<td>1.89</td>
<td>2.00</td>
</tr>
</tbody>
</table>
**TABLE 3 – DEFLECTION LIMITATIONS**

<table>
<thead>
<tr>
<th>Reasons for limiting deflections</th>
<th>Examples</th>
<th>Deflection limitation</th>
<th>Portion of the total deflection to be considered</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Sensory acceptability</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Visual</td>
<td>Sagging floors that can be seen</td>
<td>$\ell/250$</td>
<td>Total deflection</td>
</tr>
<tr>
<td>Tactile</td>
<td>Vibrations of floors that can be felt</td>
<td>$\ell/350$</td>
<td>Full live load deflection</td>
</tr>
<tr>
<td><strong>Serviceability of structure</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Surfaces which should drain water</td>
<td>Roofs and outdoor decks</td>
<td>$\ell/250^{(1)}$</td>
<td>Total deflection</td>
</tr>
<tr>
<td>Floors which should remain plane</td>
<td>Gymnasia and bowling alleys</td>
<td>$\ell/350 + \text{camber}^{(2)}$</td>
<td>Total deflection</td>
</tr>
<tr>
<td><strong>Effects on nonstructural elements</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Walls</td>
<td>Masonry and plaster</td>
<td>$\ell/500^{(3)}$ or 10 mm (0.4 in.) or $\phi = 0.0017 \text{ rad}^{(4)}$</td>
<td>Incremental deflections after walls are constructed</td>
</tr>
<tr>
<td></td>
<td>Metal movable partitions</td>
<td>$\ell/250^{(3)}$ or 25 mm (1 in.)</td>
<td>Incremental deflections after partitions are installed</td>
</tr>
<tr>
<td></td>
<td>Lateral building movement</td>
<td>$H/500$ or $H/1250^{(5)}$ offset per story$^{(6)}$</td>
<td>Wind load deflection on frequent combination</td>
</tr>
<tr>
<td></td>
<td>Horizontal thermal movements</td>
<td>$H/500$ offset per story</td>
<td>Temperature differential deflection</td>
</tr>
<tr>
<td></td>
<td>Vertical thermal movements</td>
<td>$\ell/400^{(7)}$ or 15 mm (0.6 in.)</td>
<td>Temperature differential deflection</td>
</tr>
<tr>
<td>Ceilings</td>
<td>Plaster</td>
<td>$\ell/350$</td>
<td>Incremental deflections after ceiling is built</td>
</tr>
<tr>
<td></td>
<td>Tiles</td>
<td>$\ell/175$</td>
<td>Incremental deflections after ceiling is built</td>
</tr>
<tr>
<td>Crane</td>
<td>Rail misalignment</td>
<td>$\ell/400$</td>
<td>Braking action deflection</td>
</tr>
</tbody>
</table>
Notes:

- All deflection limitations are given for members supported at both ends and it is assumed that the supports do not move. For cantilevers, the span \( t \) may be considered twice as long as the real one.
- For two-way slabs, the span \( t \) should be considered in the short direction.
- The deflection of floors may be partially compensated by camber.

(1) Surfaces should be sloped or total deflection corrected by camber to prevent ponding of water.
(2) The camber itself cannot produce a floor which deviates from plane by more than \( t/350 \).
(3) The span \( t \) should be considered parallel to the direction of partition.
(4) \( \phi \) is the rotation of member supporting a wall.
(5) \( H \) is the building height and \( H_1 \) is the distance between adjacent floors.
(6) This limitation does not include the deflection caused by axial strain of columns. It also applies to the vertical offset deflection between two shear walls in the same plane.
(7) The span \( t \) is the distance between the exterior column and the first interior column.

FIGURE 1 - CONTINUOUS BEAMS
Calculate load conditions:
- D only
- D + \psi_2L or D + \psi_1L
- D + L

Calculate $M_a$ (for each load condition)

Calculate $E_c = 4760f_c'$

Calculate $y_1, l_0, f_c = 0.315 (f_c')^{2/3}$ (for rectangular beams)
or $f_c = 0.252 (f_c')^{2/3}$ (for T beams) and $M_{cr} = \frac{l_0 f_c}{y_1}$

If $M_a \leq M_{cr}$ (for each load condition)

Calculate $l_a$

Calculate $p' = \frac{A_{st}}{bd}$, $\xi = \xi(l) - \xi(l_0)$ and $\lambda = \frac{\Delta \xi}{1 + 50p'}$

Calculate deflections:
- $\Delta_t = (1 + \lambda)\Delta_{UL}$
- $\Delta_t = (1 + \lambda)\Delta_{LL}$
- $\Delta_t = \lambda \Delta_{LD} + (1 + \lambda)\Delta_{UL}$

End

FIGURE 2 - FLOW CHART FOR DEFLECTION EVALUATION
FIGURE 3 - EXAMPLE BEAM
Synopsis:

Predicting the deflection serviceability of reinforced concrete members in service is fraught with uncertainties which include imperfect knowledge of the limiting serviceability criteria, the material properties and the load history including construction loads and the service load. The serviceability criteria can be immediate deflection/curvature or incremental deflection/curvature. Most codes offer two methods for control of deflections. The designer may choose to calculate the deflections and check that these computed deflections are less than arbitrary, specified allowable limits. Calculating the immediate deflections of reinforced concrete members is difficult due to the concrete cracking in the tension zones of such members. Calculating the additional deflections due to the shrinkage and creep of the concrete and the consequent redistribution of stress is extremely difficult. Alternatively the codes give specified maximum span/depth ratios for which serviceability can be assumed to be satisfied and deflections do not need to be calculated. This paper compares the deemed-to-comply span/thickness limits of ACI 318-99, CSA A23.3-94, BS 8110-85, AS 3600-94, Eurocode 2 (1992 draft), ACI 435-78 and the proposals of Gardner and Zhang (1995), Thompson and Scanlon (1988), Asamoah and Gardner (1997) and Scanlon and Choi (1999).

Keywords: code provisions; deflections; long term deflections; serviceability
INTRODUCTION

The object of structural design is to achieve acceptable probabilities that structures will perform satisfactorily during their intended service life. For safety, the structure must have adequate strength with a low probability of collapse. The required probability against collapse is achieved by increasing the specified loads by appropriate load factors and reducing the member strengths by strength reduction factors. Although safety is the most important limit state it is not sufficient without satisfying the requirements of serviceability. Service load deflections/curvatures may be excessive, or long-term deflections/curvatures due to sustained loads may cause damage to partitions, visual discomfort and/or perception etc.. With the increasing use of higher strength concretes and reinforcing steels, as well as more efficient design procedures, there is a tendency towards designing shallower section members in reinforced concrete structures with attendant reductions in stiffness and hence larger deflections.

Most codes offer two methods for control of deflections. The designer may choose to calculate the deflections and check that these computed deflections are less than specified allowable limits. Calculating the immediate deflections of reinforced concrete members is difficult due to the concrete cracking in the tension zones of such members. Calculating the additional deflections due to shrinkage, creep and the consequent redistribution of stress is extremely difficult. Alternatively the codes give specified maximum span/depth ratios for which serviceability can be assumed to be satisfied and deflections do not need to be calculated. This alternative is usually more attractive to designers because of its simplicity.

CODE REQUIREMENTS FOR DEFLECTION CONTROL

ACI 318-99 - Building Code Requirements for Reinforced Concrete,
CSA A23.3-M94 - Design of Concrete Structure for Buildings

The American Code ACI 318-99 (1) and the Canadian Code CSA A23.3-94 (2) are the commonly used design codes for reinforced concrete structures in North America. For beams their provisions are effectively identical to those in ACI 318-71. The deflection limits and minimum thicknesses required by the two codes are reproduced in Tables 1 and 2.
Table 3 is an extended version of Table 2 recommended by ACI 435-1978 (3) which distinguishes between members that support, or are attached to, non-structural elements likely to be damaged by large deflections and those that do not. Grossman (4) noted that the minimum member thicknesses provided in Table 2 (ACI 318-99 Table 9-5(a)), to eliminate the need to calculate deflections, do not correlate with the requirements of Table 1 (ACI 318-99 Table 9-5(b)) of the same code. It can be noted that Table 2 does not take account of several parameters which may play important roles in the long-term behaviour of reinforced concrete members. For example, the guideline seems to be too conservative for beams with compression reinforcement, since the effect of compression steel is not considered. Consideration should also be taken of the effect of concrete compressive strength and the relative load levels in the members.

For flat slabs the provisions of the two codes are slightly different. If the slab thickness of interior panels is larger than that calculated from equation (1), ACI 318-99 (1) does not require that deflections be calculated and deflection serviceability can be assumed to be satisfied.

\[ h_{\text{min}} = \frac{l_n (800 + f_y / 1.38)}{36,000} \]

where \( l_n = \) longer clear span (m) 
\( f_y = \) yield strength of tensile flexural reinforcement (MPa)

The minimum thickness has to be increased by 10% for edge panels unless edge beams of a specified stiffness are provided. For flat slabs with drop panels meeting code specified minimum thickness and dimensions, the slab thickness beyond the drop panel may be reduced by 10%.

For slabs with beams spanning between the supports on all sides the minimum thickness is

\[ h_{\text{min}} = \frac{l_n (800 + f_y / 1.38)}{36,000 + 5000\beta (\alpha_n - 0.2)} \]

for \( 0.2 < \alpha < 2.0 \)

\[ h_{\text{min}} = \frac{l_n (800 + f_y / 1.38)}{36,000 + 9000\beta} \]

for \( \alpha > 2.0 \)

\( \alpha = \) ratio of flexural stiffness of beam to flexural stiffness of slab
\[ \alpha_m = \text{average value of } \alpha \]
\[ \beta = \text{ratio of long side to short side} \]

CSA A23.3-94 (2) has adopted the more conservative provisions proposed by Thompson and Scanlon (5).

\[ h_{\text{min}} = \frac{l_n (0.6 + f_y /1000)}{30 + 4 \beta \alpha_m} \]

[4]

For flat slabs without edge beams use \( \alpha_m = 0 \). At discontinuous edges if an edge beam with a stiffness \( \alpha \) greater than 0.8 is not provided the thickness calculated by equation [4] should be increased by 10%.

For slabs with drop panels the minimum thickness is given by equation [5] where \( h_s \) is the slab thickness, \( h_d \) is the total depth of the drop panel and \( x_d \) is the distance from the face of the column to the edge of the drop panel.

\[ h_{\text{min}} = \frac{l_n (0.6 + f_y /1000)}{30 \left[ 1 + 2 \frac{x_d}{l_n} \left( \frac{h_d - h_s}{h_s} \right) \right]} \]

[5]

For two way slabs the minimum thickness is given by equation 4, or slab perimeter/140 for slabs with discontinuous edges and perimeter/160 for slabs continuous on all four edges.

The span/thickness provisions of ACI 318-99 and CSA A23.3-94 do not address the sensitivity of slab deflections to early-age construction loads, rate of construction or concrete strength.

**BS 8110-1985 - Code of Practice for Design and Construction of Concrete Structures**

The British Standard BS 8110-85 (6) span/effective depth requirements for rectangular or flanged beams are based on limiting the total deflection to span/250. These span depth ratios should normally ensure that the part of the deflection occurring after construction of finishes and partitions will be limited to span/350 or 20mm, whichever is less, for spans up to 10m. The basic ratios are given in Table 4.

The basic ratios should be modified according to the ratios of tension and compression reinforcement provided and the service load steel stress at the centre of the span (or at the support in the case of a cantilever). These factors are listed in Tables 5 and 6. The span/effective depth ratios take
account of normal shrinkage (<750x10^6) and normal creep (creep coefficient <3).

Tables 4 and 5 can also be used for slabs using the reinforcement ratio at mid span. The ratio for a two way slab supported by walls or stiff beams should be based upon the shorter span and the reinforcement ratio in that direction and on the longer span for flat slabs.

**AS 3600-1994 - Australian Standard - Concrete Structures Code**

The serviceability requirements of the Australian Standard AS 3600-1994 (7) limits the total deflection to span/250 and the incremental deflection to span/500 where provision is made to minimise the effect of movement, otherwise span/1000.

Limiting deemed-to-comply span-to-depth ratios for beams can be calculated from the following equation;

\[
\frac{L_{\text{eff}}}{d} = \left[ \frac{k_1 (\Delta/\ell_{\text{eff}}) b_{\text{eff}} E_c}{k_2 F_{d,\text{eff}}} \right]^{1/3}
\]

\( \Delta/\ell_{\text{eff}} = \) total or incremental deflection limit
\( D = \) dead load, (kN/m)
\( F_{d,\text{eff}} = \) effective design load/unit length
\( (1.0+k_{cs})D + (\psi_s+k_{cs}\psi_l)L \) for total deflection
\( k_{cs}D + (\psi_s+k_{cs}\psi_l)L \) for incremental deflection
\( k_1 = 0.045 \) for rectangular sections
\( 0.045(0.7 + 0.3b/b)^3 \) for T and L sections
\( k_2 = \) deflection constant 5/384, 1/384 and 2/384 for simply supported, both ends continuous and one end continuous beams respectively.
\( k_{cs} = [2-1.2A'/A_s] > 0.8 \)
\( L_{\text{eff}} = \) effective span
\( L = \) live load, (kN/m)
\( \psi_s = 0.25 \) for offices and domestic occupancy (0.5 to 0.8 for storage)
\( \psi_s = 0.5 \) for offices (1.0 for storage)

A similar equation is given for deem-to-comply span depth ratios for one-way, flat slabs and slabs supported on four sides by walls or stiff beams.

\[
\frac{L_{\text{eff}}}{d} = k_1 k_s \left[ \frac{(\Delta/\ell_{\text{eff}}) E_c}{F_{d,\text{eff}}} \right]^{1/3}
\]

[6] [7]
D = dead load, (kPa)
k_3 = 1.0 for a one-way slab
  = 0.95 for a two-way flat slab without drop panels
  = 1.05 for a two-way flat slab with drop panels
k_4 = deflection constant 1.6 for simply supported slabs, 2.0 in an end span or 2.4 in an interior span.
L = live load, (kPa)

For two-way slabs supported by walls or stiff beams k_3 = 1.0 and k_4 is given in a table as a function of boundary condition and panel aspect ratio.

**Eurocode 2**

The 1992 draft Eurocode 2 (8) gives, Table 7, limiting span/effective depth ratios for beams spanning up to 7 metres and flat slabs spanning up to 8.5 metres (presumably for a deflection limit of span/250). The values were derived on the assumption that the steel stress at mid-span is 250 N/mm². For flanged sections where the ratio of flange breadth to web breadth exceeds 3, the values should be multiplied by 0.8. For two-way slabs the calculation should be based upon the shorter span. For flat slabs the calculation should be based upon the longer span. The limits for flat slabs correspond to a less severe limitation than a mid-span deflection of span/250 relative to the columns. Deflection limits are span/250 for quasi-permanent loads and an incremental deflection of span/500 for fixtures and finishes which could suffer damage.

**Gardner and Zhang - Beams**

Using a layered, nonlinear finite element program Gardner and Zhang (9) developed the span thickness requirements to satisfy a specified deflection limit in terms of specified, or characteristic, concrete strength, tension and compression steel ratios, and the ratio of the sustained moment to the moment capacity of the beam. The span/thickness ratio requirements for simply supported beams, satisfying the span/500 deflection criterion under a service load/ultimate load ratio of 50%, are given in Table 8. For deflection limits other than span/500 the limiting span thickness ratio is simply multiplied by 500/required span deflection ratio. Span/thickness ratios for a span/250 deflection criterion can be obtained by doubling the values for the span/500 deflection criterion. The immediate deflection limit of span/375 was not found to be critical.

Span/thickness ratios for continuous beams may be obtained by multiplying the values for simple supported beams, using the positive moment steel ratio, by the following factors:
Support Condition | Factor
---|---
Simply supported: | 1.0
One end continuous:  
   discontinuous end unrestrained: | 1.2
   discontinuous end integral with support: | 1.3
Both ends continuous: | 1.4
Cantilever: | 0.35

Increasing the service moment, as a fraction of the beam design ultimate moment, reduces the limiting span/thickness ratio. As a first approximation the limiting span/thickness ratio is inversely proportional to the cube root of the ratio of the moment levels. Similarly, using a higher yield strength steel, which will increase the concrete stress for a given service moment/design ultimate moment ratio, will also result in smaller permissible span/thickness ratios.

**Thompson and Scanlon 1988 - Flat Slabs**

Thompson and Scanlon (5) reported the results of a parametric study of the effects of restraint cracking, concrete strength, design live load, construction load and panel aspect ratio on the deflections of flat slabs. Deflections were calculated using a plate bending, finite element program with an effective second moment of area to account for the reduced stiffness due to cracking. It was observed that the calculated deflections were sensitive to the assumed value of the modulus of rupture. Thompson and Scanlon used serviceability criteria of incremental deflection less than span/480 and total deflection less than span/240. From their parametric study Thompson and Scanlon proposed a more conservative minimum thickness requirement, for both interior and edge panels, for the control of deflections in two way slabs. The live load deflection limit of span/360 was not found to be critical.

$$h_{min} = \frac{l_n}{30} k(\beta)$$  \[8\]

where $k(\beta) = (1.20 - 0.20 \beta) > 0.9$

$\beta$ = ratio of longer clear span to shorter clear span

Thompson and Scanlon also recommended that the minimum slab thickness could be reduced by 10% for flat slabs with drop panels whose thickness is greater than or equal to 1.25 times the slab thickness, or by 20% if the drop panel thickness is greater than 1.50 times the slab thickness. Thompson and Scanlon did not investigate the effect of age at which the construction load was applied on the calculated deflections.
The shore-reshore procedure used to construct reinforced concrete flat slab structures leads to the imposition of large early-age construction loads (10), typically of the same order of magnitude as the service loads, on the partially cured supporting slabs. Consequently, it is necessary that both the design and construction load be taken into account during the design phase of reinforced concrete floor slab construction.

A layered finite element program was used to study the effects of age of loading, span, panel aspect ratio, live load to dead load ratio and concrete strength on the deflection serviceability of flat slab systems (11). It was observed that the age of loading (construction cycle) and span have significant effects on the slab thickness required to satisfy serviceability.

The following equation summarizes the slab thicknesses required to satisfy an exterior panel, interior column line incremental deflection of clear span/240.

$\frac{h}{k_1k_2} = \frac{38}{53.4} \left[ \frac{38}{1.7D} \right]^{0.6} \left[ \frac{1.4 + 1.7L/D}{2.25} \right]^{0.25} (1.15 - 0.15\beta)$

where $\beta$ = ratio of long clear span to short clear span

$h$ = slab thickness, (m)

$k_1 = 0.9$ for an interior panel

$k_2 = 0.9$ for slabs with drop panels

$l_n$ = longer clear span, (m)

$t_0$ = age at which the construction load is applied to the slab

$L$ = live load, (kPa)

$D$ = dead load, (kPa)

$f_{ck28}$ = 28 day characteristic concrete strength, (MPa)

$f_{cm28} = f_{ck28} + 8$ = 28 day characteristic concrete strength, (MPa)

The use of the code recommended minimum drop panel thickness of 1.25 times the slab thickness reduces the slab thickness required to satisfy the serviceability criterion by approximately 18%; hence the ACI 318-99 recommendation of a 10% reduction in slab thickness is conservative. For interior panels the thickness given by Equation [9] can be reduced by 10% as recommended by ACI 318-99.

Camber, equal to some multiple of the immediate deflection, can be used to reduce the total deflection of the slab system which may be visually disturbing or lead to ponding. Equation (10) was developed to estimate the mid panel 28 day deflections, after removal of the construction load, of corner and interior panel slab systems whose thickness satisfies equation [9].
Code Provisions for Deflection Control in Concrete Structures

\[
\delta = \frac{0.00425 k}{h^2} \left[ \frac{l_n^t}{t_0} \right] \left[ \frac{27.1}{E_{cm28}} \right] \left[ \frac{1}{\beta} \right]
\]  

where \( k = 1.0 \) for a corner panel and 0.4 for an interior panel
\( E_{cm28} \) = mean modulus of elasticity at 28 days

The column line camber should be the mid-panel camber multiplied by the cosine of the angle between the diagonal and the column line.

Scanlon and Choi 1999 - One-way Slabs

Scanlon and Choi (12) proposed the following equation based upon an incremental deflection limit.

\[
\frac{l_n}{h} = \left[ \frac{\Delta_{inc}}{l_n} \right]^{1/3}
\]  

\( W_t \) = variable portion of live load
\( W_s \) = sustained load
\( \alpha \) = ratio of \( l_{effective} \) to \( l_{gross} \)
\( \lambda \) = ACI 318 long term deflection multiplier
\( \kappa \) = deflection coefficient 5, 2, 1.4 for simply supported, one end continuous and both ends continuous respectively.

DISCUSSION

Obviously the deflections calculated for members designed using thicknesses given by deemed-to-comply provisions should satisfy the code specified deflection limitations. The characteristic live load, live load not exceeded 95% of the time, should be used for ultimate limit state calculations. However the experimental work of Choi(13) indicated that an average live load of 50% of the extreme, assumed specified/characteristic, live load would be reasonable for serviceability limit state calculations. BS 8110-85 states that when calculating deflections the portion of the load to be considered permanent should be 25 to 30% for office use but at least 75% for storage. AS 3600-1994 suggests that for deflection calculations the live load can be multiplied by 0.6 for offices (1.0 for storage) for immediate deflections and 0.25 for long term deflections (0.5 to 0.8 for storage). Further the expected, average, value of the concrete strength, not the lower bound characteristic concrete strength, should be used in deflection calculations.

There appears to be agreement that the incremental deflection after construction of partitions and finishes is more critical than immediate
deflection, AS 3600-1994, Scanlon and Thompson and Gardner and Zhang. There is also general agreement that the limiting incremental deflections are span/500 for brittle partitions otherwise span/250.

Table 9 compares the deem-to-comply span thickness ratios for simply supported, rectangular section beams for a total deflection criterion of span/250. It is reassuring that all the ratios are circa span/20. It must be noted that ACI, CSA and Gardner and Zhang use span/thickness but BS 8110, AS 3600 and Eurocode 2 use span to effective depth. Only the proposals of AS 3600-94 and Gardner and Zhang formally accommodate incremental deflection limits other than span/250. The provisions of ACI, CSA and Eurocode 2 do not include the effect of compression reinforcement. The modifying factors for boundary conditions other than simply supported, given in Table 10, are similar for all proposals. The modifying factors were calculated assuming curvature is proportional to the moment coefficients given in ACI 318-99 clause 8.3.3 and CSA A23.3-94 clause 9.3.3.

Table 11 compares the limiting span thickness ratios for the interior panels of flat slabs. The limiting span thickness ratios are similar except AS 3600 is more liberal; which may be due to the use of effective depth rather than slab thickness. Only the provisions of Gardner and Asamoah take account of the construction cycle - age of loading. In their calculations Gardner and Asamoah assumed a form plus 3 reshores construction schedule.

**RECOMMENDATIONS**

The live loads for which deflections should be calculated should be clearly specified in the codes taking note that of the difference between expected live load and extreme or characteristic live load. For purposes of calculating the incremental deflection, it is suggested that the service load be calculated from the equation below which is a compromise between the provisions of BS 8110-85 and AS 3600-94.

\[
Service \ Load = D + \alpha L
\]

\[
\alpha = 0.4 \ for \ offices, \ apartments \ etc. \ and \ 0.8 \ for \ storage.
\]

The deemed-to-comply span/thickness provisions of ACI 318 and CSA A23.3 should be updated to reflect recent research work and the concepts available in more recent codes.

Incremental deflections limits are more conservative than immediate (elastic) deflection limits.

For beams, the deemed-to comply minimum thicknesses given in Table 8 should be adopted. For incremental deflection limits other than span/500 the limiting span thickness ratio is simply multiplied by
500/required span deflection ratio. As a first approximation the limiting span/thickness ratio is inversely proportional to the cube root of the ratio of the moment levels. Similarly it can be deduced that using a higher yield strength steel, which will increase the concrete stress for a given service moment/design ultimate moment ratio, will also result in smaller permissible span/thickness ratios. For other than simple spans the modification factors suggested by Gardner and Zhang should be used.

For flanged sections where the ratio of flange breadth to web breadth exceeds 3 the values should be multiplied by 0.8.

For flat slabs the minimum thickness for deflection serviceability should be calculated using equation 9.

ACKNOWLEDGEMENT

The author gratefully acknowledges the financial support of the Natural Sciences and Engineering Research Council of Canada under OPG0005645.

CONVERSION FACTORS

1 mm = 0.4 ins
1 m = 39.4 ins
1 MPa = 145 psi
1 kPa = 20.8 psf

NOTATION

$A_s$ area of tension steel
$A'_{s}$ area of compression steel
$b$ width of beam
d effective depth of beam, taken as equal to 0.85 $h$
d' depth to compression steel
$D$ dead load, (kPa)
$E_{cm28}$ mean modulus of elasticity at 28 days
$f'_{ck28}$ 28 day characteristic concrete strength, (MPa)
$f_{cm28}$ $f'_{ck28} + 8 = 28$ day mean concrete strength, (MPa)
$f_y$ yield strength of tensile flexural reinforcement (MPa)
$h$ slab thickness, (m)
l_n clear span for beams, longer clear span for flat slabs(m)
$L$ live load, (kPa)
$M_s$ sustained moment
$M_u$ Design ultimate moment calculated using ACI 318-99
$t_o$ age at which the construction load is applied to the slab
$\alpha$ various definitions - identified in text
$\beta$ ratio of longer clear span to shorter clear span
δ deflection
Δ specified deflection limit
ρ tension steel ratio
ρ' compression steel ratio

REFERENCES

1 ACI Committee 318, Building Code Requirements for Reinforced Concrete (ACI 318-99). American Concrete Institute, Detroit, 1999.


11 Ofosu-Asamoah K. and N. J. Gardner, Flat slab thickness required to satisfy serviceability including early age construction loads. ACI Structural
Code Provisions for Deflection Control in Concrete Structures


Table 1 Maximum Permissible Computed Deflection
(ACI 318-99 and CSA A23.3-94)

<table>
<thead>
<tr>
<th>Type of member</th>
<th>Deflection to be considered</th>
<th>Deflection limitation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flat roofs not supporting or attached to nonstructural elements likely to be damaged by large deflections</td>
<td>Immediate deflection due to live load L</td>
<td>$l_n/180$</td>
</tr>
<tr>
<td>Floors not supporting or attached to nonstructural elements likely to be damaged by large deflections</td>
<td>Immediate deflection due to live load L</td>
<td>$l_n/360$</td>
</tr>
<tr>
<td>Roof or floor construction supporting or attached to nonstructural elements likely to be damaged by large deflections</td>
<td>That part of the total deflection occurring after attachment of nonstructural elements (sum of the long-time deflection due to all sustained loads and the immediate deflection due to any additional live load).</td>
<td>$l_n/480$</td>
</tr>
<tr>
<td>Roof or floor construction supporting or attached to nonstructural elements not likely to be damaged by large deflections</td>
<td></td>
<td>$l_n/240$</td>
</tr>
</tbody>
</table>

ACI 318-99 $l_n = \text{clear span} + \text{depth of member} < \text{center-to-center of supports}$

CSA A23.3-94 $l_n = \text{clear span}$
Table 2 Minimum Thickness of Non-Prestressed Beams and One-Way Slabs Unless Deflections Are Computed (ACI 318-99 and CSA A23.3-94)

<table>
<thead>
<tr>
<th>Member</th>
<th>Simply supported</th>
<th>One end continuous</th>
<th>Both ends continuous</th>
<th>Cantilever</th>
</tr>
</thead>
<tbody>
<tr>
<td>Solid one-way slabs</td>
<td>$l_n/20$</td>
<td>$l_n/24$</td>
<td>$l_n/28$</td>
<td>$l_n/10$</td>
</tr>
<tr>
<td>Beams or ribbed one-way slabs</td>
<td>$l_n/16$</td>
<td>$l_n/18.5$</td>
<td>$l_n/21$</td>
<td>$l_n/8$</td>
</tr>
</tbody>
</table>

ACI 318-99 $l_n =$ clear span + depth of member < center-to-center of supports
CSA A23.3-94 $l_n =$ clear span
Table 3 Minimum Thickness of Beams and One-Way Slabs used in Roof and Floor Construction  
(ACI 435, 1978)

<table>
<thead>
<tr>
<th>Member</th>
<th>Members not supporting, or not attached to, non-structural elements likely to be damaged by large deflections</th>
<th>Members supporting, or attached to, non-structural elements likely to be damaged by large deflections</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Simply supported</td>
<td>One end continuous</td>
</tr>
<tr>
<td>Roof slab</td>
<td>$L/22$</td>
<td>$L/28$</td>
</tr>
<tr>
<td>Floor slab and roof beam or ribbed roof slab</td>
<td>$L/18$</td>
<td>$L/23$</td>
</tr>
<tr>
<td>Floor beam or ribbed floor slab</td>
<td>$L/14$</td>
<td>$L/18$</td>
</tr>
</tbody>
</table>
### Table 4 Basic Span/Effective Depth Ratios For Beams (Table 3.10, BS 8110-85)

<table>
<thead>
<tr>
<th>Support Conditions</th>
<th>Rectangular Sections</th>
<th>Flanged beams $b_v/b &lt; 0.3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cantilever</td>
<td>7</td>
<td>5.6</td>
</tr>
<tr>
<td>Simply Supported</td>
<td>20</td>
<td>16.0</td>
</tr>
<tr>
<td>Continuous</td>
<td>26</td>
<td>20.8</td>
</tr>
</tbody>
</table>

### Table 5 Modification Factor for Tension Reinforcement (Table 3.11, BS 8110-1985)

<table>
<thead>
<tr>
<th>Steel service stress (MPa)</th>
<th>Non-dimensional moment $M_{n}/bd^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.50</td>
</tr>
<tr>
<td>100</td>
<td>2.00</td>
</tr>
<tr>
<td>150</td>
<td>2.00</td>
</tr>
<tr>
<td>200</td>
<td>2.00</td>
</tr>
<tr>
<td>250</td>
<td>1.90</td>
</tr>
<tr>
<td>288</td>
<td>1.68</td>
</tr>
<tr>
<td>300</td>
<td>1.60</td>
</tr>
</tbody>
</table>
Table 6 Modification Factor for Compression Reinforcement (Table 3.12, BS 8110-1985)

<table>
<thead>
<tr>
<th>Reinforcement ratio of compression reinforcement $100 A_s$/bd</th>
<th>0.15</th>
<th>0.25</th>
<th>0.50</th>
<th>0.75</th>
<th>1.0</th>
<th>1.5</th>
<th>2.0</th>
<th>3.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>Factor to be applied</td>
<td>1.05</td>
<td>1.08</td>
<td>1.14</td>
<td>1.20</td>
<td>1.25</td>
<td>1.33</td>
<td>1.40</td>
<td>1.50</td>
</tr>
</tbody>
</table>

Table 7 Basic ratios of span/effective depth for reinforced concrete members (Table 4.14, Eurocode 2-1992)

<table>
<thead>
<tr>
<th>Structural System</th>
<th>steel ratio = 1.5%</th>
<th>steel ratio = 0.5%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Simply supported beam or two-way simply supported slab</td>
<td>18</td>
<td>25</td>
</tr>
<tr>
<td>End span of continuous beam or one-way continuous slab or two-way slab continuous over one long side</td>
<td>23</td>
<td>32</td>
</tr>
<tr>
<td>Interior span of continuous beam or two-way slab</td>
<td>25</td>
<td>35</td>
</tr>
<tr>
<td>Flat slab based upon longer span</td>
<td>21</td>
<td>30</td>
</tr>
<tr>
<td>Cantilever</td>
<td>7</td>
<td>10</td>
</tr>
</tbody>
</table>
Table 8 Proposed Span/Depth Requirements to Ensure the Span/500
Deflection Limit
[for a deflection limit of span/250 multiply the above values by 2]

<table>
<thead>
<tr>
<th>ρ (%)</th>
<th>ρ' (%)</th>
<th>( f_y = 400 \text{ Mpa} )</th>
<th>M = 30% ( M_u )</th>
<th>M = 50% ( M_u )</th>
<th>M = 70% ( M_u )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>30 MPa</td>
<td>20 MPa</td>
<td>30 MPa</td>
<td>40 Mpa</td>
</tr>
<tr>
<td>&lt;0.5</td>
<td>0</td>
<td>12.7</td>
<td>8.2</td>
<td>9.8</td>
<td>10.9</td>
</tr>
<tr>
<td>1.0</td>
<td>0</td>
<td>11.1</td>
<td>8.4</td>
<td>9.9</td>
<td>10.8</td>
</tr>
<tr>
<td>1.5</td>
<td>0</td>
<td>10.8</td>
<td>7.7</td>
<td>8.8</td>
<td>10.1</td>
</tr>
<tr>
<td>2.0</td>
<td>0</td>
<td>9.7</td>
<td>7.1</td>
<td>8.3</td>
<td>9.3</td>
</tr>
<tr>
<td>1.5</td>
<td>0.5</td>
<td>13.9</td>
<td>11.0</td>
<td>11.6</td>
<td>12.7</td>
</tr>
<tr>
<td>2.0</td>
<td>0.5</td>
<td>12.6</td>
<td>9.9</td>
<td>10.8</td>
<td>11.6</td>
</tr>
<tr>
<td>2.5</td>
<td>0.5</td>
<td>11.8</td>
<td>9.2</td>
<td>10.2</td>
<td>10.8</td>
</tr>
<tr>
<td>2.0</td>
<td>1.0</td>
<td>15.0</td>
<td>12.6</td>
<td>13.3</td>
<td>14.2</td>
</tr>
<tr>
<td>2.5</td>
<td>1.0</td>
<td>14.2</td>
<td>11.5</td>
<td>12.4</td>
<td>12.6</td>
</tr>
<tr>
<td>3.0</td>
<td>1.0</td>
<td>13.7</td>
<td>11.0</td>
<td>11.4</td>
<td>12.3</td>
</tr>
<tr>
<td>2.5</td>
<td>1.5</td>
<td>16.6</td>
<td>14.0</td>
<td>15.0</td>
<td>14.6</td>
</tr>
<tr>
<td>3.0</td>
<td>1.5</td>
<td>14.7</td>
<td>13.3</td>
<td>13.7</td>
<td>14.0</td>
</tr>
</tbody>
</table>
Table 9 Comparison of Simple Span Beam Span/thickness Ratios

<table>
<thead>
<tr>
<th>ρ</th>
<th>ρ'</th>
<th>ACI 318</th>
<th>CSA A23.3</th>
<th>BS 8110</th>
<th>Euro code</th>
<th>Gardner &amp; Zhang</th>
</tr>
</thead>
<tbody>
<tr>
<td>&lt; 0.5</td>
<td>0</td>
<td>20</td>
<td>20</td>
<td>25.2</td>
<td>25</td>
<td>19.4</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>20</td>
<td>20</td>
<td>19.6</td>
<td>18</td>
<td>17.6</td>
</tr>
<tr>
<td>1.5</td>
<td>0</td>
<td>20</td>
<td>20</td>
<td>18.1</td>
<td>18</td>
<td>17.6</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>20</td>
<td>20</td>
<td>16.7</td>
<td></td>
<td>16.6</td>
</tr>
<tr>
<td>1.5</td>
<td>0.5</td>
<td>20</td>
<td>20</td>
<td>20.6</td>
<td></td>
<td>23.2</td>
</tr>
<tr>
<td>2.5</td>
<td>0.5</td>
<td>20</td>
<td>20</td>
<td>18.7</td>
<td></td>
<td>20.4</td>
</tr>
<tr>
<td>2.5</td>
<td>1</td>
<td>20</td>
<td>20</td>
<td>20.5</td>
<td></td>
<td>24.8</td>
</tr>
</tbody>
</table>

* Ratio of span/effective depth
1. Calculated assuming steel stress 250 MPa
2. Incremental deflection of span/240

Table 10 Span/thickness Factors for Other than Simple Beams
use midspan +ve moment steel ratio in Table 9.

<table>
<thead>
<tr>
<th>Moment</th>
<th>ACI-CSA</th>
<th>BS 8110</th>
<th>AS 3600</th>
<th>Euro code</th>
<th>Gardner and Zhang</th>
</tr>
</thead>
<tbody>
<tr>
<td>One end Continuous - discontinuous end unrestrained ( \frac{wL^2}{11} )</td>
<td>1.2</td>
<td>1.3</td>
<td>1.3</td>
<td>1.3</td>
<td>1.2</td>
</tr>
<tr>
<td>One end Continuous - other end integral with support ( \frac{wL^2}{14} )</td>
<td>1.2</td>
<td>1.3</td>
<td>1.3</td>
<td>1.3</td>
<td>1.3</td>
</tr>
<tr>
<td>Both ends continuous ( \frac{wL^2}{16} )</td>
<td>1.4</td>
<td>1.3</td>
<td>1.4</td>
<td>1.4</td>
<td>1.4</td>
</tr>
<tr>
<td>Cantilever ( \frac{wL^2}{16} )</td>
<td>0.5</td>
<td>0.35</td>
<td>0.4</td>
<td>0.4</td>
<td>0.35</td>
</tr>
</tbody>
</table>
Table 11 Comparison Flat Slab Interior Panel Span/thickness Ratios

<table>
<thead>
<tr>
<th>Span (m)</th>
<th>ACI 318-99</th>
<th>CSA A23.3 1994</th>
<th>BS 8110²</th>
<th>AS 3600-94²</th>
<th>Euro code²</th>
<th>OA&amp;G¹ 7 day</th>
<th>OA&amp;G¹ 3 day</th>
</tr>
</thead>
<tbody>
<tr>
<td>Live Load 2.4 kPa</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6.00</td>
<td>33</td>
<td>30</td>
<td>28</td>
<td>39</td>
<td>26</td>
<td>40.5</td>
<td>30</td>
</tr>
<tr>
<td>7.00</td>
<td>33</td>
<td>30</td>
<td>29</td>
<td>38</td>
<td>27</td>
<td>33.8</td>
<td>28.3</td>
</tr>
<tr>
<td>8.50</td>
<td>33</td>
<td>30</td>
<td>29</td>
<td>36</td>
<td>27</td>
<td>31</td>
<td>26.2</td>
</tr>
<tr>
<td>Live Load 4.8 kPa</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6.00</td>
<td>33</td>
<td>30</td>
<td>28</td>
<td>35</td>
<td>26</td>
<td>33</td>
<td>28.4</td>
</tr>
<tr>
<td>7.00</td>
<td>33</td>
<td>30</td>
<td>29</td>
<td>34</td>
<td>27</td>
<td>31</td>
<td>27.3</td>
</tr>
<tr>
<td>8.50</td>
<td>33</td>
<td>30</td>
<td>29</td>
<td>33</td>
<td>27</td>
<td>29</td>
<td>25.2</td>
</tr>
</tbody>
</table>

Edge panel thickness/interior panel thickness

|          | +10% | +10% | +30% | +20% | -   | +10% | +10% |

Drop panel – Reduce slab thickness by 10%

¹ Ofosu-Asamoah and Gardner 1997, *f<sub>ck</sub> = 30 MPa*

² Corrected to total thickness by adding 30 mm to effective depth
Effect of Flange Width on Beam Deflections

by H. H. Nassif, M. Sanders, and W. Cao

Synopsis: Current concrete design codes use an effective stiffness model to calculate the deflections of T-Beams. Based on the effective flange width, an effective moment of inertia, as well as section properties is computed. This effective flange width is limited by a criterion that is not consistent between various codes and which directly affects the computed deflections, moments, shears, and torques for the beam. In particular, the effect of flange width on serviceability limit states such as deflection, fatigue, cracking, and vibration is evident. There is a need to use more rational as well as realistic effective flange width criteria that would lead to more accurate predictions of beam deflections.

An analytical and experimental study has been initiated to assess the contribution of the flange width to the calculation of deflection in concrete T-beams. This paper presents part of the results of an analytical study to model the exact behavior of T-beams with various effective flange widths. The Finite Element Method (FEM) is utilized to model the overall pre-cracking, cracking, and post-cracking, non-linear behavior. Actual data for material properties are used to model concrete and reinforcing steel. The FE Model is validated using available test results from literature and then used to analyze T-beams with various parameters. The beams are incrementally loaded to failure under a two point loading system. The load-deflection relationships are determined. A parametric study is undertaken to determine the effect of overall flange width, and other parameters such as reinforcement ratio, and concrete compressive strength, on deflections. Results show that deflections for beams subjected to service loads (service load ranging between cracking and 30-40% of ultimate loads) are mainly affected by using the effective flange width rather than full flange width. It is observed that using the actual flange width would provide a better estimate of deflection at service loads.

Keywords: deflection; effective flange width; finite element analysis; reinforced concrete; stiffness
ACI member Hani H. Nassif is an assistant Professor of Civil and Environmental Engineering at Rutgers, The State University of New Jersey. He received his BS (1981), and ME (1983) from the University of Detroit and his Ph.D. from the University of Michigan at Ann Arbor, Michigan. He is the Chairman of ACI 348—Structural Safety, and is active on several ACI committees including Deflection, Concrete bridges, and prestressed Concrete. His current research interests include prestressed concrete, external presetressing, high performance concrete, and concrete bridges.

Mark Sanders, is a former Graduate Research Assistant. He received his BS and MS (1998) from Villanova University in Pennsylvania.

William Cao, is currently a graduate student research assistant at the Department of Civil and Environmental Engineering of Rutgers, The State University of New Jersey.

INTRODUCTION

The American Concrete Institute (ACI 318-95), (1), uses an effective stiffness model to determine the deflection of concrete beams after cracking. This model has been shown to work well for rectangular beams (Al Shaikh and Al-Zaid, 1993), (2). However, in the case of T-beams, an effective flange width must be selected in order to calculate the moment of inertia of the section before and after cracking. Section 8.10.2 in ACI specifies that the effective flange width to be used in design should be the smallest of: 1) One-quarter the span length, (L/4), 2) web width + the clear spacing between webs, (bw +L/2), and 3) web width plus sixteen times the flange thickness, (bw +16t). There are further requirements for isolated T-beams and T-beams with a flange on one side only. Once the effective width is calculated, Branson’s equation (Branson, 1963), (3), is used to calculate the effective moment of inertia, Ie, and elastic deflection equations are used to calculate the immediate deflection. Although other equations have been suggested by Grossman (1981), (4), and Rangan (1982), (5), ACI currently recommends Branson’s equation only. Questions have been raised in ACI Committee 435, Deflections, as to the validity of using the effective flange width at various load levels. There is a need to determine the effect of using effective flange width, be, rather than the actual flange width, b, on the beam deflection computations at various load levels. The procedure should be applicable to pre-cracking, cracking and post cracking stages of loading. The finite element method is used to model beams with various parameters. Results from beams tested in the laboratory are used to validate the accuracy of the model. Also, results from using both finite element analysis and the current ACI beam deflection method used to determine deflection of concrete T-beam are presented.
LITERATURE REVIEW

The use of finite element has been established for the last three decades. Many authors have used the finite element to model concrete beams and frames including the non-linear cracking behavior. Ngo and Scordelis (1967), (6), presented the first published application of the finite element (FE) method to reinforced concrete structures. Simply supported beams were analyzed by representing the steel and concrete with two-dimensional, triangular elements. Bond-link elements were created to model the steel-concrete interface. Beams with pre-defined crack patterns were then analyzed using a linear-elastic procedure. Moreover, Nilson (1968), (7), first applied non-linear material properties to the FE analysis of concrete along with non-linear bond-slip at the concrete-steel interface. Quadrilateral plane stress elements were used. Cracking was accounted for by stopping the analysis when the stress in an element exceeded the tensile strength and redefining a new, cracked structure.

Franklin (1970), (8), developed a non-linear analysis procedure that automatically accounted for cracking within the structure and redistributed stresses accordingly. Using an incremental loading, the response of the system could be traced from beginning to end in one continuous computer analysis. The elements used included frame elements, plane stress elements, axial bar elements, and bond-link elements. Bresler and Bertero (1968) (9) studied the effects of repeated loading on concrete structures. Zienkiewicz et al (1972) (10) has included tensile cracking and elasto-plastic compressive behavior in analyses using an initial-stress approach. Cervenka and Gerstle (1971), (11) used composite concrete-steel material properties in a FE analysis and compared their results to experimental tests on reinforced concrete wall panels and spandrel beams.

For slabs and plates, Jofriet and McNiece (1973), (12), studied plate-bending systems using triangular and quadrilateral elements and incorporating progressive cracking. Comparisons were made with experimental results. Scanlon and Murray (1974), (13), developed a method of incorporating cracking and time-dependent effects in the analysis of slabs. Layered, rectangular elements were used, allowing the slab to crack progressively layer by layer.

ACI 318-95 MODEL

The ACI method involves incrementing the live load and calculating the deflections using elastic deflection equations and the effective moment of inertia, Ie. The ACI method of calculating deflections is derived from elastic methods that are applied to both the determination of effective flange width and the calculation of the curvature of the loaded beam. Elementary bending theory assumes that the compressive stresses in the top of a beam under a positive moment do not vary horizontally through the cross-section. However, it has been shown that if a beam has a wide flange, the amount of bending moment taken by the outer portions of the flange is reduced. Thus, an effective width (be) is used. The moment of inertia of the gross section is replaced by Branson's effective moment of inertia equation to account for reduction in the section due to cracking. The effective moment of inertia is calculated as follows:
Where

\[ I_e = I_{cr} + (I_g - I_{cr}) \left( \frac{M_{ac}}{M_a} \right)^3 \]

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FINITE ELEMENT ANALYSIS

The Finite Element Method (FEM), in comparison with the most common analytical methods of concrete behavior, is a powerful tool to study the behavior of the reinforced concrete T-beams. A general-purpose finite element code, ABAQUS, (14), was utilized in this study. ABAQUS includes a variety of routines that allows for the implementation of specific material models (concrete and steel), boundary conditions, and bond behavior. The interaction between the reinforcing steel and concrete is also considered. The concrete material model consists of an isotropically hardening yield surface, active when the stress is dominantly compressive, and an independent “crack detection surface” which determines if a point fails by cracking. After such a cracking failure, oriented damaged elasticity concepts are used to describe the reversible part of the material’s response. The model is a smeared crack model, in the sense that it does not track individual “macro” cracks. Instead, constitutive calculations are performed independently at each integration point in the model.

The analysis is performed by applying an incremental load, with iterations in each increment. The modified RIKS algorithm with an assumed proportional loading history is used. This approach determines the static equilibrium solution for unstable response in concrete, due to cracking in tension, yielding of reinforcement, or concrete softening in compression. It neglects any permanent strains associated with cracking. This implies that the cracks can close completely if they are exposed to compressive stress. The plain concrete model is treated independently of the reinforcing steel that can be added to the concrete. The one-dimensional elements (rods) can be incorporated to model rebars in oriented surfaces. The concrete/steel interface is influenced by effects such as bond slip and dowel action. These effects can be approximated by introducing a tension-stiffening model.

Tension stiffening will simulate load transfer through the rebar across cracks. The tension stiffening model depends on many factors: the density of the reinforcement, quality of bond between the concrete and the rebars, the relative size of concrete aggregate
compared to the rebar diameter, and mesh size. It is assumed that the stresses are zero at ten times the failure strain, as shown in Figure 2. However, this value should be calibrated for a particular case, otherwise, low tension stiffening may cause temporarily unstable behavior in the overall response of the model. Also, the reduction in shear stress modulus can be introduced as a function of the opening strain across the crack by incorporating shear retention into the model. The tensile strength was taken equal to the modulus of rupture, which is defined by the American Concrete Institute (ACI) as \( f_t = 7.5 \sqrt{f_c} \). The steel stress-strain relationship shown in Fig. 3 is implemented in the analysis, including the elastic, yield, plastic and strain hardening.

The analysis is carried out using two different types of elements: beam and a combination of beam and shell elements. Both models yielded the same cracking and ultimate load capacity as shown in Fig. 6. Therefore, a beam element with nine Gaussian integration points is used throughout the parametric study. The results are compared in terms of the cracking moment, ultimate moment capacity and load-deflection relationships. The analyses of T-beam sections, based on the model proposed do not show a distinct cracking point. As the moment applied to the section approaches the theoretical cracking moment in an incremental analysis and the section begins to crack, the stiffness of the section changes slowly. This slow change is due to the smeared crack approach to modeling the cracking behavior of concrete used by ABAQUS as well as many other FE packages.

The finite element model is composed of two parts. The flanges of the T-beams were represented by 8-node, two-dimensional shell elements if the flange width is large compared to its thickness. However, for beams with flange widths not much greater than the thickness, the use of two-dimensional shell elements would not yield accurate results. When the flange width is much greater than the thickness, bending of the flange as a thin plate can be neglected, and the forces transmitted to the flange by bending can be represented by forces along the middle plane of the flange. However, when the flange width is not much greater than the thickness, the shear varies significantly through the section and it is not adequate to represent the flange with a two-dimensional model. The webs of the T-beams were represented by 3-noded beam elements.

These elements appear to be an adequate representation of the web, as no deep beam behavior was encountered in the analysis. The web was attached to the flange via beam-type multi-point constraints. Due to the rigidity of the concrete in the system, this type of connection seems the most accurate. Figure 4 shows an example of the FE model used in the analysis.

The shell-beam model is the model used in the T-beam analysis. It is compared with experimental data and with a simpler model composed solely of beam sections. The shell model performs well because the depth of the compression zone is small compared to the beam widths. In the T-beam analysis, the model did not perform very well when the depth of compression is of the same order of magnitude as the width of the flange. In these cases, the model could not converge upon a solution early in the load-deflection
analysis. However, as long as the flange width was at least ten times the depth of compression, the model performed fairly well.

VALIDATION OF FINITE ELEMENT MODEL

Few experimental studies are available regarding the deflection of T-beams that are constrained as part of a floor system. Therefore, a comparison is made between the FE model described earlier and a set of rectangular beams. The beams used for the validation of the model are tested by Al-Shaikh and Al-Zaid (1993), (2), as part of an investigation regarding the calculation of the effective moment of inertia of concrete beams. Three beams are considered: BN12, a normally reinforced beam, BH13 with high reinforcement ratio, and BL11, with a low reinforcement ratio. Each beam was modeled using beam as well as shell elements. The experimental load-deflection curves from experimental data and FE model are shown in figures 6-8. It is observed that the FE results and experimental data are close especially for BL11 and BH13. For simplicity and efficiency of the model, beam elements will be used in the parametric study. However, in the major study, various types of elements have been used including three-dimensional brick elements.

PARAMETRIC STUDY

The effects of various parameters on the behavior of T-beams are studied. Parameters such as: the amount of steel reinforcement, $A_s$, concrete type, $f'_c$, reinforcing steel grade, $f_y$, are investigated. However, for brevity, only the effect of flange width as a parameter will be presented. Results obtained from FEM are compared to those from the ACI analysis. The T-beam used in this study is 8 feet long and subjected to two-point loading as shown in Fig. 9. Due to symmetry, only half of the beam was modeled as shown previously in Figs. 4-5.

Effect of Using a Variable Flange Width

The beam shown in figure 9 has been designed according to ACI code using a $b_e = 24"$. The flange widths considered were 14", 24", 34", and 44". Each beam is modeled using the FE model described in section 4 while keeping all other parameters constants. The response of each beam is illustrated in figs. 10-13 for various flange widths. Figure 14 shows the variation of the ratio of deflections between ACI and FEM with the Flange width. It is observed that up to $b_f = b_e$ the ratios are constant. However, for beams having a flange width larger than the code specified effective flange width, $b_e$, the variation is almost linear. It is also important to note that this variation is dependent on the load level. The major difference is observed for loads at cracking, $P_{cr}$, and at about 30% of the ultimate load. For loads at 45% of ultimate loads the change is small while it is negligible at 60% of the ultimate load.

CONCLUSIONS AND RECOMMENDATIONS

Based on the above results, the following conclusions are made:
1. The FEM predicted the ultimate moment as well the cracking moment capacities for rectangular beams very well.

2. For beams designed according to ACI318-95, and for calculating deflections at service loads (in this study service loads range between Pcr to 30% of ultimate load), the full flange width, $b_f$, should be used rather than the effective width, $b_e$. Branson’s equation for $I_{eff}$ performs well prior to cracking ($I_e = I_{eff}$) as well as at ultimate. However, between cracking and ultimate load a modification of $I_e$ is needed to reflect the effect of flange width.

3. It is recommended that the full flange width, $b_f$, be used for deflection calculation instead of the effective flange width, $b_e$, for beams subjected to service loads in the range between Pcr up to 30% of ultimate load.

REFERENCES


ACI (Branson's Model) $I_{effective}$

$-(I_g) = 10(I_{cr})$

Figure 1: Variation of Effective Moment of Inertia based on FEA.
Figure 2: Tension Stiffening used in the Concrete Model.

Figure 3: Stress-Strain relationship for Grade 60 Steel (Macgregor, 1988), (ref. 15).
Shell Element for Thin flange section only

Beam Element for Web section

Figure 4: Beam and shell elements are connected by a Multi-Point Constraint.

Figure 5: FE Model of half beam with boundary restraints and applied loads.
Figure 6: Comparison of results from Finite Element Models and Experimental results for Al-Zaid's (1994) beam BL11

Figure 7: Comparison of results from Finite Element Models and Experimental results for Al-Zaid's (1994) beam BN12
Figure 8: Comparison of results from Finite Element Models and Experimental results for Al-Zaid’s (1994) beam BH13.

Figure 9: Beam Layout Used in the Parametric Analysis Using Variable Flange Width for Al-Zaid’s (1994) beam BN12.
Figure 10: Load versus deflection Relationship for a Concrete T-Beam with a Flange Width = 14".

Figure 11: Load versus deflection Relationship for a Concrete T-Beam with a Flange Width = 24."
Figure 12: Load versus deflection for a Concrete T-Beam with a Flange Width = 34".

Figure 13: Load versus deflection Relationship for a Concrete T-Beam with a Flange Width = 44".
Figure 14: Beam Layout Used in the Parametric Analysis Using Variable Flange Width.
Deflection Prediction for Reinforced Concrete Structures Under Service Load

by N. Mickleborough

Synopsis: To determine the structural response to service load of reinforced concrete structures and structural components, the consequences of cracking on the effective stiffness must be considered. Usual methods to determine structural response of tall reinforced concrete structures involve codified simplifications and global reductions in effective stiffness of the beams and columns.

Previous work (1), (2) has developed a method to determine the effective stiffness of members within a structure which can be extended to the analysis of structural frames. This method considers the consequences of cracking in tall buildings using a member element stiffness reduction model. The model is based on the ratio of the area of the member moment diagram where the applied moment exceeds the cracking moment, to the total area of the moment diagram. From this ratio the effective stiffness of the member can be estimated.

A practical cracking analysis system has been established by integrating the proposed stiffness reduction model with an iterative algorithm and the commercial package of a linear finite element analysis. With this procedure, each member can be assigned their appropriate stiffness as a function of the applied load, and the response of the structure to service loading determined.

Verification of this method of analysis, and its application to structural frames, has been achieved from an extensive experimental program considering flexural members, shear wall members and large-scale structures.

Keywords: cracking; deflection; ductility; effective stiffness; reinforced concretes; serviceability; tests
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INTRODUCTION

The design of structures requires the determination of the response to service load, and the requirement of limiting deflections and motion to remain within acceptable limits. Specifically, for tall reinforced concrete buildings the requirements to maintain satisfactory serviceability limits necessitates the determination of the lateral stiffness of the structure. If such motion and deflection requirements are not met, undesirable consequences with the structural and non-structural components can ensue due to excessive movement. Excessive movement can also increase the crack propagation within a member causing a further reduction in the stiffness and increase in deflection.

The current procedures for the determination of deformation, deflection and serviceability criteria generally, are presented in codes of practice. For tall buildings, overall lateral deflections are limited as well as restrictions being placed on inter-story drift. For structural concrete buildings the procedures for determining the effects of cracking in the concrete are based on empirical relationships. These relationships are mainly derived from tests on simple flexural members. The behaviour of tall-framed structures however, is a more complex problem, but in general, codes of practice do not recommend methods to consider these effects. The procedures most commonly used are based on linear-elastic analysis with allowances for the non-linear effects of crack formation. Alternatively the effects of cracking due to flexural tensile stresses can be represented by an arbitrarily reduced moment of inertia (3).

None of the approaches have sufficient generality to be used for specialized structures. Tension stiffening effects play a large role in the behaviour of structural concrete in the post-cracking range and needs to be considered in any serviceability behaviour analysis. For large structures the use of micro-elements to predict element behaviour is still not a viable design option, hence an effective stiffness model of a macro-element is considered appropriate since the local effects, such as tension stiffening can be incorporated.
The most common procedure for incorporating the tension stiffening, as well as loading effects, is based on the original work by Branson (4). This work forms the basis of most codes of practice for determining the effective moment of inertia used to estimate deflections. The estimate of the effective moment of inertia \( I_e \) lies between the uncracked (gross) and the fully cracked moments of inertia \( (I_{cr} \leq I_e \leq I_g) \) and is of the form presented by Eq. [1].

\[
I_e = \xi I_{cr} + [1 - \xi] I_g,
\]

(1)

where,

\[
\xi = \left[1 - \left(\frac{M_{cr}}{M_u}\right)^3\right].
\]

While the absolute maximum value of the applied moment is used in this formulation, the shape of the moment diagram, in representing the form of the applied load, affects the crack formation and hence the behaviour of the flexural member.

Current research (5) has established an effective stiffness model to determine the relationship between the member stiffness reduction and the moments due to applied service loading on the members. The model is generally applicable to structural members subject to various loading conditions and acts as a constitutive model for members in all forms of reinforced concrete structures. Results of tests to predict the stiffness of reinforced concrete shear walls under service loads (2) indicated that the proposed method provided an accurate prediction of both deflections and flexural stiffness reduction of shear walls subject to loadings greater than 50% of ultimate load.

**FLEXURAL STIFFNESS REDUCTION MODEL**

The probability of such crack formation at any section is dependent on the basic form of the shape and magnitude of the moment diagram \( (1) \). The probability that an applied moment is greater than or equal to the cracked moment \( M_{cr} \) can be represented by

\[
P\left[M(x) \geq M_{cr}\right] = \int_{x \text{ such that } M(x) \geq M_{cr}} \frac{M(x)}{S} dx = \frac{S_{cr}}{S}
\]

(3)
where, $M(x)$ is the moment function along the flexural member, $S_{cr}$ is the area of moment diagram segment over which the working moment exceeds the cracking moment $M_{cr}$, $S = \int M(x) \, dx$ = total area under the moment diagram, and $P$ is the cumulative distribution function. The probability density function represented by $P$ is, $p(x) = \frac{M(x)}{S}$, while the probability of occurrence of cracked sections associated with the outcome $I_{cr}$ is, $P_{cr} = P\{M(x) \geq M_{cr}\}$. The probability of occurrence of an uncracked section with outcome $I_{uncr}$, the gross moment of inertia, is, $P_{uncr} = P\{M(x) < M_{cr}\} = 1 - P_{cr}$.

The expected value of the, so called, effective moment of inertia for the reinforced concrete beam, subjected to a certain type of loading, is represented by

$$I_e = P_{uncr}I_{uncr} + P_{cr}I_{cr} = (1 - P_{cr})I_{uncr} + P_{cr}I_{cr}$$

(4)

The various loading patterns on the members is represented by the form of the moment diagram. A two-point loading case can be used to illustrate this formulation. The moment diagram is shown in Fig.1 and the value of $S$ and $S_{cr}$ can be calculated from simple geometry.

For the case of symmetrical loading,

$$P_{cr} = \frac{S_{cr}}{S} = \left[1 - \left(\frac{1 - a}{1 + a}\right)^2\left(\frac{M_{cr}}{M_a}\right)^2\right]$$

(5)

and loads applied at the 1/3 points, $a = L/3$ and,

$$P_{cr} = \left[1 - \frac{1}{2}\left(\frac{M_{cr}}{M_a}\right)^2\right]$$

(6)

The model has been verified by a series of experimental tests on different flexural structural members under varying loading conditions (1) as well as from the results of other recent research on prediction of deflections for reinforced concrete beams (6). The procedure can also be extended to the analysis of columns and shear walls where there is dominant flexural cracking behaviour(2).
SHEAR WALL FRAME

To assist in determining the validity and limitations of the procedures, two full-scale structural frames were tested in the side-sway loading mode. The two frames consisted of a reinforced concrete beam-column frame with two stories and a single span (5), and a shear wall framed structure of two stories and single bay. The results of the shear wall framed structure are presented. The illustrations of the frame, as tested, and structural geometry are presented in Figs. 1 and 2.

The column dimensions were 250mm wide by 375mm deep with the beams being 250mm wide by 350mm deep. The wall dimensions were 750mm by 180mm cross-section and the span between column and wall centrelines was 2100 mm. The typical story height was 1500mm. The frame was integral with a rigid base 400mm wide, 400mm thick and 3400mm attached to the laboratory strong floor. A total constant vertical load of 400 kN was applied equally to the top of both the column and the wall simulating gravity load through a spreader beam. The lateral load was applied at the level of the top beam at a constant rate of 0.05 kN/sec and all forces and deformations were recorded continuously through electronic measuring devices.

During lateral load testing the gravity load simulator applied a constant vertical load of 400 kN to the structure at the top of the column and shear wall. The aim of the test structure was simulate a single story component in a tall building and to investigate this behaviour and then to be able to extend it to all other similar components within a structure. To achieve this simulation a gravity load was applied through the gravity load simulator. This loading was to simulate the effect of upper stories acting through the members of this component. The gravity load simulator is capable of supplying a constant vertical load to the structure while moving horizontally within a range of approximately 200 mm.

ANALYTICAL INVESTIGATION

Incremental Loading

From the flexural stiffness reduction model presented, an incremental analysis can be considered to predict the behaviour of the frame under increasing load. The non-linear iteration procedure provides a history of the behaviour of the reinforced concrete structure due to the cracking of the concrete from the application of the externally applied incrementally increasing loads. In this procedure a linear finite element analysis was used to determine the distribution of forces within the structure.
At any increment of load, cracking in a member may occur and due to this crack formation the member would undergo a reduction in stiffness, measured by $I_{cr}$. Again at this increment of load, moment redistribution can occur, which in turn determines the stiffness reduction of the reinforced concrete members for the subsequent increment. The variations of the member moment and $I_{cr}$ are coupled at each load increment. These coupled variations trace the physical propagation of cracking and moment redistribution in the members during the iteration process. Fig. 4 presents the iterative procedure for the incremental load applied to the structure.

The analytical prediction using the procedure, shown in Fig. 4 using 5 kN load steps, and the load deflections at the 1st and 2nd story levels from the experimental investigation is compared in Figure 5. Very close agreement is achieved between the prediction and measured deflections at the three load levels of 50%, 60% and 70% of ultimate (175 kN, 210 kN and 245 kN respectively). The analysis is restricted to the serviceability range of loading, therefore significantly larger differences occur when the lateral load increases beyond a level greater than about 70% of ultimate load.

The effect of the incremental loading with increasing applied load, on the reduction of moment of inertia of the structural members, is illustrated in Fig. 6. The progression of cracking within the members and hence the redistribution of moments and forces within the structure is dependent on the magnitude of the vertical load and the structural boundary conditions. The first members to crack are the beams of both the 1st and 2nd stories, occurring at about 16% and 20% of the ultimate lateral load respectively. The stiffness of these beams reduce relatively rapidly and have already reduced to a stiffness of about 70% when cracking and subsequently stiffness reduction occurs in the 1st floor column, at a lateral load of about 30% of ultimate. When the lateral load has reached approximately 60% of ultimate the initial cracks start to occur in the shear walls. At 70% of ultimate lateral load the stiffness of the beams at the 1st and 2nd story have reduced to 45% and 48%, the columns to 58% and 79%, and the shear walls to 89% and 85% of their uncracked stiffness, respectively.

Direct Iteration

Further analysis has been considered to calculate the lateral deflection of the wall frame structure at a typical load level of 50% of ultimate. The analysis was conducted by direct iteration procedure and applying the lateral load of 175 kN. The complete load was applied to the structure at one time, rather than as in the previous case where it was applied incrementally. The analysis then proceeded by considering the forces in the members and adjusting their effective moments of inertia in line with the flexural stiffness reduction model.
Once the $I_e$ had been determined for each member, a complete analysis was conducted on the structure and the forces and deflections determined. With these values the new effective moments of inertia were determined for the structure and the analysis repeated. Consideration is then given to the rate of convergence of the lateral deflections and the convergence of the ratio of effective to uncracked moments of inertia.

Figs. 7 and 8 plot the process of convergence by considering the variations of deflection and the ratio of stiffness as a function of the number of iterations at the load stage of 50% of ultimate load. Some fluctuations of deflection occur even though the value of deflection is approached after a relatively few cycles of iteration.

The fluctuations shown in Fig. 8 appear to be due to the exchange of stiffness between the column and the shear wall at the 1st story level, as evidenced by the small oscillations of the two values in line with each other. The fluctuations however, are relatively small and good prediction of the overall behaviour of the structure due to the direct application of the lateral force is achieved.

CONCLUSIONS

An effective stiffness model has been developed which is based on the probability of cracking in reinforced concrete members. The model was incorporated with a linear finite element package. The package incrementally and iteratively considered the cracking effects on the lateral stiffness of tall reinforced concrete buildings.

From an experimental investigation, the proposed methods give accurate predictions of lateral deflections and member stiffness throughout the range of loading to approximately 70% of ultimate load. In excess of this range of loading the material nonlinearity becomes a significant factor in the behaviour of reinforced concrete structures.

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Figure 1. Two Point Beam Loading and Moment Diagram
Figure 2. Shear Wall Frame Test
Figure 3. Shear Wall Frame Geometry

Figure 4. Incremental Load Iterative Procedure
Figure 5. Deflection Comparison for Experimental and Analytical Procedure

Figure 6. Flexural Stiffness Reduction of Structural Members
Figure 7. Convergence of Deflection by Direct Iteration Procedure

Figure 8. Convergence of Member Stiffness by Direct Iteration Procedure
Bending Stiffness of Concrete Flexural Members Reinforced with High Strength Steel

by B. Vijaya Rangan and P. Kumar Sarker

Synopsis: Reinforcing bars with a minimum yield strength of 500 MPa (72.5 ksi) are currently in use in Australia. The paper examines the effect of this high strength reinforcement on the bending stiffness of flexural members. The influence of tensile steel ratio, concrete strength and shrinkage of concrete on the bending stiffness is studied using a parametric analysis. It is found that for the same service load the bending stiffness of members with 500 MPa (72.5 ksi) reinforcement varies between 85 and 100 percent of that of members reinforced with 400 MPa (60 ksi) steel depending on tensile steel ratio. Also shrinkage of concrete should be included in the bending stiffness calculations for members with tensile steel ratio less than about 0.008.

Keywords: beams; bending; deflection; high-strength; reinforcement; shrinkage; slabs; stiffness
INTRODUCTION

Following the practice in Europe and other countries, the Australian reinforcing steel manufacturers have recently introduced high strength deformed reinforcing bars with a minimum yield strength of 500 MPa (72.5 ksi) into practice. It is claimed that the increase in the minimum yield strength from 400 MPa (60 ksi) to 500 MPa (72.5 ksi) enables more efficient and cost-effective reinforced concrete designs, with direct material cost savings of around 10 percent obtainable on most projects. The cost saving is primarily due to less area of reinforcing steel needed to carry given forces. However, a decrease in the area of tensile steel in beams and slabs may lead to a decrease in bending stiffness and hence an increase in deflections of such members.

This paper studies the effect of 500 MPa (72.5 ksi) reinforcing steel on the bending stiffness of concrete flexural members in comparison with the stiffness of members with 400 MPa (60 ksi) steel. The parameters involved in the study are tensile steel ratio, compressive strength, and shrinkage of concrete.
CALCULATION OF BENDING STIFFNESS

The bending stiffness of reinforced concrete flexural members is closely related to the moments of inertia of cracked and gross concrete sections, \( I_{cr} \) and \( I_g \), respectively, and several other factors such as the extent of stiffening offered by the concrete in tension zone of the member, load stage, load intensity, etc. There are a number of methods available in the literature to calculate bending stiffness. The accuracy of the methods varies.

In codes of practice, the bending stiffness is usually calculated using simple expressions. Both the American Concrete Institute Building Code, ACI 318-99 (1) and the Australian Standard for Concrete Structures, AS 3600-1998 (2) use the expression proposed by Branson (3,4). Accordingly, the effective moment of inertia \( I_e \) is given by

\[
I_e = \left( \frac{M_{cr}}{M} \right)^3 (I_g - I_{cr}) + I_{cr}
\]

(1)

but not greater than \( I_g \), where \( M_{cr} \) is the flexural cracking moment and \( M \) is the maximum (unfactored) bending moment at the load stage and the section for which the bending stiffness is being calculated. In Eq. 1, the first term empirically accounts for the stiffening effect of the concrete in the tension zone of the member (often referred to as tension stiffening effect), and \( M_{cr} \) is calculated by

\[
M_{cr} = \frac{f'_{ct} I_g}{y_t}
\]

(2)

where \( y_t \) is the distance of the extreme fibre in tension from the centroidal axis of the uncracked concrete section and \( f'_{ct} \) is the flexural strength of concrete taken as \( 0.6 \sqrt{f'_c} \) where \( f'_c \) is expressed in terms of MPa (1,2).

Branson's expression (Eq. 1) gives reasonably good estimates of bending stiffness provided that the tensile reinforcement ratio \( p \) is not less than about 0.005. For low values of \( p \), the maximum in-service bending moment (i.e., \( M \) in Eq. 1) may be little more than the cracking moment \( M_{cr} \) as given by Eq. 2. In such cases, the first term of Eq. 1 grossly overestimates the tension stiffening effect, which in turn can lead to
significant over-estimation of $I_e$ and therefore under-estimation of deflections. To counteract this inadequacy of Eq. 1, an upper limit of $0.6I_g$ on the value of $I_e$ was proposed earlier by the first author (5) for members with $p \leq 0.005$. This upper limit has been verified by an independent analytical study by Gilbert (6). It approximately accounts for the loss of bending stiffness in members with low reinforcement ratios, due to the tensile stresses produced by shrinkage of concrete.

To account rationally for the breakdown of tension stiffening due to shrinkage induced tension, Gilbert (7) proposed a simple method to modify Eq. 1. Based on this modification, Eq. 1 in the Australian Standard AS 3600 has been amended as follows:

$$I_e = \left(\frac{M_{cr}}{M}\right)^3 \left(I_g - I_{cr}\right) + I_{cr} \leq I_{e,\text{max}}$$

(3)

where

$$I_{e,\text{max}} = I_g \quad \text{when } p \geq 0.005$$

(4)

$$= 0.6I_g \quad \text{when } p < 0.005$$

(5)

$$M_{cr} = \frac{(f'_{cf} - f_{cs})I_g}{\gamma_t} \geq 0$$

(6)

$$f'_{cf} = 0.6\sqrt{f_c} \quad \text{when } f_c \text{ is in MPa}$$

(7)

$$f_{cs} = \frac{1.5p}{1 + 50p} E_s \varepsilon_{cs}$$

(8)

$\varepsilon_{cs}$ is the design shrinkage strain and $E_s$ is the modulus of elasticity of reinforcing steel taken as equal to $200 \times 10^3 \text{ MPa (29000 ksi)}$. The Australian Standard AS 3600 (2) gives data to estimate $\varepsilon_{cs}$, with typical values in the range of 500 to 1000 microstrains.

It is prudent that the calculation of $I_e$ is based on full service load conditions so as to make some allowance for the loss of bending stiffness due to cracking produced by the construction loads. Information available in
the literature (8) indicates that the construction loads can be as large as full service loads. Therefore, in Eq. 3 the value of $M$ is equal to the bending moment at the section when full service loads are acting. If $\sigma_u$ is the stress in the tensile steel reinforcement at this stage and at the section where $M$ is acting, then $M$ can be expressed as

$$M = A_s \sigma_u (0.87d)$$

(9)

where $A_s$ is the area of tensile steel and $0.87d$ is the approximate lever arm. Because $A_s$ will be determined based on ultimate flexural strength considerations, a reasonable value for $\sigma_u$ is $0.6f_y$, where $f_y$ is the yield strength of steel reinforcement. Therefore, for rectangular sections $M$ is given by

$$M = pb d (0.6f_y)(0.87d)$$

(10)

where $p = \frac{A_s}{bd}$, $b$ is the width of a rectangular beam and $d$ is the effective depth.

In this paper, Eqs. 3 to 10 are used to calculate the bending stiffness $E_s I_s$.

PARAMETRIC STUDY

Using Eqs. 3 to 10, the values of $I_s$ were calculated for a number of rectangular sections. Because compression reinforcement has very little effect on $I_s$ (see examples 5.4 and 5.5 in Reference 9), $I_s$ may be taken corresponding to a singly reinforced section as

$$I_s = \frac{bd^3}{12} \left[ 4k^3 + 12np(1-k)^3 \right]$$

(11)

where

$$k = \sqrt{(np)^2 + 2np} - np$$

(12)
\[ n = \frac{E_s}{E_c} \]  
(13)

and \( E_c \) is the modulus of elasticity of concrete taken approximately for normal weight concrete as

\[ E_c = 5050 \sqrt{f'_c} \]  
(14)

Also, \( I_g = \frac{bD^3}{12} \) and take \( d = 0.9D \), where \( D \) is the overall depth of a rectangular section.

For reinforced concrete flexural members used in practice, \( p \) may vary from 0.002 to 0.035 and \( f'_c \) from 20 MPa (2900 psi) to 50 MPa (7250 psi). Therefore, the following sets of values were selected for the parametric study:

\[ f'_c : 20, 30, 40 \text{ and } 50 \text{ MPa} \]
\[ p_{500} : 0.002, 0.004, 0.008, 0.012, 0.016, 0.020, 0.024, 0.028, 0.032 \text{ and } 0.035 \]
\[ \varepsilon_{cs} : 0, 0.00050, 0.00075 \text{ and } 0.0010 \]

In these, \( p_{500} \) is the tensile steel ratio using reinforcing steel with a yield strength of 500 MPa (72.5 ksi). The service moment \( M \) (Eq. 10) is therefore,

\[ M = 261 p_{500} b d^2 \]  
(15)

In order to study the effect of 500 MPa (72.5 ksi) steel on \( I_c \) relative to 400 MPa (60 ksi) steel, another sets of calculations were also performed. To make the comparison valid, the service load bending moment \( M \) was kept a constant. Obviously, for the same bending moment, use of 400 MPa (60 ksi) steel will require approximately 1.25 (= 500/400) times the tensile steel needed when using 500 MPa (72.5 ksi) reinforcement. Therefore, \( I_c \) calculations for 400 MPa (60 ksi) steel were made using tensile steel ratios equal to 1.25 \( p_{500} \) corresponding to each \( p_{500} \) value selected above for the parametric study. From these, the ratios of \( I_{c_{500}} / I_{c_{400}} \) were calculated, where \( I_{c_{500}} \) and \( I_{c_{400}} \) correspond to the values of \( I_c \) for the respective reinforcing steel.
RESULTS AND DISCUSSION

The results obtained from the parametric study are given in Figs. 1 to 8. Figs. 1 to 4 show the variation of $I_{e,500}/bd^3$ versus $p_{500}$ for different values of $f'_c$ and shrinkage strain $\varepsilon_{cs}$. The ratio of bending stiffnesses of members with 500 MPa (72.5 ksi) and 400 MPa (60 ksi) reinforcement, $I_{e,500}/I_{e,400}$ is plotted against $p_{500}$ in Figs 5 to 8. These results are discussed below.

Effect of Shrinkage of Concrete

Figs. 1 to 4 show that when $p_{500} > 0.008$, the effect of shrinkage of concrete on the bending stiffness $I_{e,500}/bd^3$ is negligible and for all practical purposes, $I_e$ can be taken approximately equal to the cracked moment of inertia, $I_{cr}$. For $p_{500} \leq 0.008$, the shrinkage of concrete has noticeable influence on the bending stiffness, especially for higher grades of concrete (i.e., $f'_c = 40$ and 50 MPa). Because floor slabs contain low tensile steel ratios in the range of $p < 0.008$, the importance of introducing the effect of shrinkage in the calculation of $I_e$ (i.e., Eq. 6) is once again confirmed by the present study. For very low values of $p_{500}$, the upper limit of 0.6 $I_g$ on $I_{e,500}$ controls in all cases.

The ratio of stiffness, $I_{e,500}/I_{e,400}$ plotted in Figs. 5 to 8 indicate negligible influence of shrinkage of concrete on this value in all cases. This is to be expected because the effect of shrinkage of concrete is included in the calculation of both $I_{e,500}$ and $I_{e,400}$.

Effect of Tensile Steel Ratio

The influence of tensile steel ratio $p_{500}$ on the bending stiffness $I_{e,500}/bd^3$ is illustrated in Figs. 1 to 4. It can be seen that $I_{e,500}/bd^3$ increases with the tensile steel ratio for medium and high values of $p_{500}$. This is due to the increase in the cracked moment of inertia as the tensile steel ratio increases. The curves reach a dip close to $p_{500} = 0.005$. For low values of tensile steel
ratio, \( I_{e500} / bd^3 \) increases as \( p_{500} \) decreases due to increase in the contribution of tension stiffening in such cases. The maximum value in this part of the curves is dictated by the upper limit of 0.6 \( I_g \) on \( I_e \) (see Eq. 5).

**Effect of Yield Strength of Reinforcement**

Figs. 5 to 8 show the effect of yield strength of reinforcement on bending stiffness. In these Figures, \( I_{e500} \) and \( I_{e400} \) are, respectively, the effective moments of inertia of sections reinforced with either 500 MPa (72.5 ksi) steel or 400 MPa (60 ksi) steel for the same service load moment.

Note that the tensile steel ratio \( p_{500} \) for sections reinforced with 500 MPa (72.5 ksi) steel is only 0.8 (=400/500) times the tensile steel ratio \( p_{400} \) for sections with 400 MPa (60 ksi) steel in order to maintain the same service load moment in both cases. However, the ratio \( I_{e500} / I_{e400} \) is not a constant and varies with \( f'_c \) and \( p_{500} \) (Figs. 5 to 8).

When \( f'_c = 40 \) and 50 MPa (5800 and 7250 psi), \( I_{e500} / I_{e400} \) is approximately equal to 0.85 for values of \( p_{500} > 0.008 \). For \( p_{500} \leq 0.008 \), in all cases, the tension stiffening effect influences the trend of curves shown in Figs. 5 to 8. When \( f'_c = 20 \) MPa (2900 psi) and \( p_{500} > 0.008 \), the disproportionate changes in the value of cracked moment of inertia \( I_{cr} \) with tensile steel ratio \( p \) is the reason for the trend of the curve shown in Fig. 5. When \( f'_c = 30 \) MPa (4350 psi), the curve in Fig. 6 shows a transition in the trend observed in Fig. 5 and Figs. 7 and 8. In all cases, the effect of shrinkage of concrete on the trends is insignificant.

**CONCLUSIONS**

A parametric study was carried out to study the effects of concrete strength, tensile steel ratio, yield strength of reinforcement, and shrinkage of concrete on the bending stiffness of reinforced concrete flexural members. Based on this study the following conclusions are drawn:

1. For members with tensile steel ratio greater than 0.008, the shrinkage of concrete has insignificant effect on the bending stiffness (Figs. 1 to 4). For all practical purposes, the effective moment of inertia of such members may be taken as the cracked moment of inertia.
2. For members with tensile steel ratio less than or equal to 0.008, the effect of shrinkage of concrete should be included in the calculation of effective moment of inertia. Most concrete floor slabs belong to this category. Eqs. 3 to 8 can be used to account for the effect of shrinkage of concrete in such cases.

Conclusions 1 and 2 apply to members reinforced with 500 MPa (72.5 ksi) as well as 400 MPa (60 ksi) steel.

3. The bending stiffness of members varies with the tensile steel ratio as shown in Figs. 1 to 4. The effective moment of inertia increases with the tensile steel ratio for medium and high values, due to increase in the cracked moment of inertia, and reaches the upper limit equal to the moment of inertia of uncracked section for very high values of tensile steel ratio. For low values of tensile steel ratio, the tension stiffening influences the trend of curves shown in Figs. 1 to 4.

4. The effect of high strength reinforcement on bending stiffness, as illustrated in Figs. 5 to 8, varies with the concrete strength and the tensile steel ratio. For low values of tensile steel ratio, the difference in bending stiffnesses diminishes rapidly due to the presence of tension stiffening. A similar trend is observed for concrete strengths of 20 MPa (2900 psi) and 30 MPa (4350 psi) when tensile steel ratio is high. This is due to the disproportionate increase in the cracked moment of inertia with tensile steel ratio in such cases.

REFERENCES

1. ACI Committee 318, "Building Code Requirements for Structural Concrete", ACI Standards 318-99 and Commentary, American Concrete Institute, Farmington Hills, MI 2000, 392 pp.


NOTATION

\( A_s \) = area of tensile reinforcement
\( b \) = width of rectangular section
\( D \) = overall depth of rectangular section
\( d \) = effective depth
\( E_c \) = modulus of elasticity of concrete
\( E_s \) = modulus of elasticity of steel
\( f_c \) = cylinder compressive strength of concrete
\( f_y \) = yield strength of steel
\( I_{cr} \) = moment of inertia of cracked concrete
\( I_e \) = effective moment of inertia
\( I_g \) = moment of inertia of gross concrete section
\( k \) = neutral axis depth parameter
\( M \) = bending moment at service loads
\( M_{cr} \) = cracking moment
\( n \) = modular ratio (= \( E/E_c \))
\( p \) = tensile steel ratio (= \( A_s/bd \))
\( y_t \) = depth of extreme fiber in tension from the neutral axis
\( \varepsilon_{cs} \) = final shrinkage strain of concrete
\( \sigma_{st} \) = stress in the tensile steel at service loads
Fig. 1 - Variation of bending stiffness with tensile steel ratio

Fig. 2 - Variation of bending stiffness with tensile steel ratio
Fig. 3 - Variation of bending stiffness with tensile steel ratio

Fig. 4 - Variation of bending stiffness with tensile steel ratio
Fig. 5 - Effect of yield strength of reinforcement on bending stiffness

Fig. 6 - Effect of yield strength of reinforcement on bending stiffness
Fig. 7 - Effect of yield strength of reinforcement on bending stiffness

Fig. 8 - Effect of yield strength of reinforcement on bending stiffness
Time-Dependent Deflection and Cracking of Reinforced Concrete Flat Slabs

by R. I. Gilbert

Synopsis: An experimental program of long-term testing of large-scale reinforced concrete flat slab structures is described and the results from the first series of tests on five continuous flat slab specimens are presented. Each specimen was subjected to sustained service loads for periods up to 500 days and the deflection, extent of cracking and column loads were monitored throughout. The measured long-term deflection is many times the initial short-term deflection, due primarily to the loss of stiffness associated with time-dependent cracking under the combined influences of transverse load and drying shrinkage. This effect is not accounted for in the current code approaches for deflection calculation and control. Recently proposed procedures to improve deflection calculation (1) are evaluated against the test results and good agreement between the measured and calculated deflections is obtained.

Keywords: cracking; creep; deflection; flat slabs; laboratory tests; reinforced concrete; restraint; serviceability; service loads; shrinkage; structural behavior; time-dependent behavior
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INTRODUCTION

The design of a reinforced concrete structure is complicated by the difficulties involved in estimating the service load behavior. The deflection and extent of cracking in reinforced concrete flexural members depend primarily on the non-linear and inelastic properties of concrete and, as such, are difficult to predict with confidence. The problem is particularly difficult in the case of slabs, which are typically thin in relation to the their spans and are therefore deflection sensitive. It is stiffness rather than strength that usually governs the design of slabs, particularly in the cases of flat slabs and flat plates.

In most concrete codes (including ACI 318-95 (2) and AS3600-1994 (3)), the procedures specified for calculating the final deflection of a slab are necessarily design-oriented and simple to use, involving crude approximations of the complex effects of cracking, tension stiffening, concrete creep and shrinkage and the load history. Unfortunately, in most code procedures, the effects of cracking in a slab are not adequately taken into account, particularly those resulting from the time-dependent cracking caused by restraint to shrinkage and temperature deformations.

The final deflection of a slab depends on the extent of initial cracking which, in turn, depends on the construction procedure (shoring and re-shoring), the amount of early shrinkage, the temperature gradients in the first few weeks after casting, the degree of curing and so on. It also depends on the degree of restraint and the quality of the concrete, in particular the magnitude and rate of development of shrinkage.

Many of these parameters are, to a large extent, out of the control of the designer. In field measurements of the deflection of many identical flat slab panels (4), (5), large variability was reported. Final deflections of identical panels differed by over 100% in some cases.

Over the past several years, numerous cases have been reported, in Australia and elsewhere, of flat slabs for which the calculated deflection is
far less than the actual deflection. Many of these slabs complied with the code’s serviceability requirements, but still deflected excessively. Evidently, the deflection calculation procedures embodied in the code do not adequately model the in-service behavior of slabs.

Surprisingly, no laboratory controlled long-term measurements of deflection and crack propagation in large-scale reinforced concrete slabs have been reported in the literature to date. This lack of reliable experimental data has hindered both analytical research and the development of reliable design procedures. In this paper, an extensive experimental program of long-term testing of large-scale flat slab structures currently underway at the University of New South Wales is described and the results are presented for the first series of tests. These bench-mark results include the measured time-varying material properties, slab deflection, extent of cracking and column loads and will be used to develop and evaluate alternative design procedures.

With the trend towards higher strength reinforcing steels, the design for serviceability will increasingly assume a more prominent role in the design of slabs. Designers will need to pay more attention to the specification of both the concrete mix, particularly the creep and shrinkage characteristics, and a suitable construction procedure, involving acceptably long stripping times, adequate propping, effective curing and rigorous on-site supervision.

EXPERIMENTAL PROGRAM

A three-year experimental program to measure the time-dependent in-service behavior of reinforced concrete flat slabs commenced at the University of New South Wales in 1998. The work is funded by the Australian Research Council and will eventually involve testing eight large-scale flat plate structures. At the time of writing this paper, five slab specimens (designated S1 to S5) have been tested under sustained service loads for periods up to 500 days. S3 to S5 remain under load. The tests are described and some significant initial results are presented here.

Slab specimens and test set-up

Each slab is continuous over two spans in two orthogonal directions and is supported on nine 200 mm x 200 mm x 1250 mm long columns below the slab. The plan dimensions of each slab and the top and bottom reinforcement layouts are identical and are shown in Fig. 1. Also shown are the 16 points (#1 - #16) on the slab soffits at which deflections were measured. Slabs S1 and S2 are 100 mm thick, while S3, S4 and S5 are 90 mm thick. The slab reinforcement consists of 10 mm diameter deformed bars (Y10) and the clear concrete cover to the outer layer of bars is 15 mm.
As shown in Fig. 1, the spacing of the bottom bars in each direction is 220 mm throughout. The spacing of the top reinforcement in the column strips is 140 mm, except for the top bars perpendicular to the discontinuous east and west slab edges where the spacing is 250 mm. No top reinforcement is included in the middle strip regions of each slab. The concrete for each specimen has significantly different properties (except for S4 and S5 where the same batch of concrete was used). Fig. 2 shows slabs S4 and S5 under construction.

The base of each column for specimen S1 is pinned with all exterior columns mounted on roller supports to minimize restraint to drying shrinkage in each direction (see Fig. 3). The base of each exterior column in specimens S2 to S5 is poured monolithically with a 700 x 700 x 300 mm pad footing fixed to the laboratory floor, as shown in Figs. 2 and 4. These supports prevent translation at the column bases and thereby provide restraint to shrinkage in the slab. This is more typical of the support conditions in practical slabs.

All slabs were cast and initially moist cured for nine days (14 days for slab S3), at which time the formwork was removed and the slab backpropped to the laboratory floor. At age 14 days (age 28 days for slab S3), each slab was subjected to a uniformly distributed load applied via concrete blocks carefully constructed and arranged to ensure uniform loading and uninhibited air flow over both the top and bottom surfaces of the slab. The props were removed and testing commenced. The loading arrangements for several of the slabs are shown in Figs. 4 and 5.

**Load histories of slab specimens**

The load history of each slab is given in Fig. 6. For example, in Fig. 6a, S1 was initially loaded at age 14 days with a uniformly distributed load consisting of self-weight (2.40 kPa) and a single layer of concrete blocks (3.15 kPa). The load was sustained until age 169 days, when a second layer of blocks was added (3.10 kPa). The load was then held constant until age 301 days when the second layer of blocks was removed. At age 433 days, the remaining layer of blocks was removed and, until the test ended at age 512 days, the slab carried just its self-weight.

Between ages 100 and 225 days, the top surface of S2 was exposed to wetting and drying. Initially the exposure to water was accidental, but it unexpectedly and significantly influenced slab behavior (see Fig. 7b). To explore this effect further, the top surface of S1 was intentionally wetted at age 279 days.
Test measurements

For the duration of the tests, deflections were measured at the center of each bay and at the mid-span on each column line. Strains on slab edges at various locations were recorded, as were the loads in columns C1 to C4 (Fig. 1). The time-dependent development of cracking on both the top and bottom slab surfaces was also monitored.

Concurrent with the slab tests, material properties were measured on companion specimens. The compressive strength and elastic modulus of concrete were measured at various ages on cylinders, and the flexural tensile strength was measured on prisms. Creep was measured on cylinders loaded at 14 days in standard creep rigs. Shrinkage strains were recorded from 600 x 600 x 100 mm thick slab specimens (with edges sealed to ensure that drying only took place at the top and bottom surfaces).

Test results

The measured compressive strength, \( f_c \), elastic modulus, \( E_c \), and flexural tensile strength, \( f_t \), for the concrete at first loading (averaged over four test specimens for each batch) are given in Table 1. The average yield stress for the reinforcement is \( f_y = 650 \text{ MPa} \) and its elastic modulus is \( E_s = 219,000 \text{ MPa} \). The measured creep coefficient, \( \phi \), and shrinkage strain, \( \varepsilon_{sh} \times 10^6 \), are given in Table 2.

The mid-panel deflection versus time curves at points #4, #6, #11 and #13 on each slab are plotted in Fig. 5 and the average deflections at these four points, \( \Delta_i \), for each slab are given in Tables 3 and 4. Also given in Table 3 for S1 are the average deflections at the symmetric points #8 and #9 (designated \( \Delta_2 \)), #1, #2, #15 and #16 (\( \Delta_3 \)), #5 and #12 (\( \Delta_4 \)) and #3, #7, #10 and #14 (\( \Delta_5 \)).

The vertical reactions at the bases of the columns C1 to C4 in S1 were measured throughout the test using load cells and are presented in Table 5. Also presented are the measured loads in column C4 in specimens S2, S3, S4 and S5.

At first loading, no cracking was observed in S1, although very fine flexural cracks occurred within the first two weeks under load on the top surface of the slab, radiating from the interior column C4. These cracks increased in width with time and extended, but remained quite serviceable with a maximum recorded crack width of less than 0.15mm at age 169 days, prior to increasing the load. When the superimposed load was doubled at age 169 days, the existing cracks extended and widened but no new cracks developed. The maximum crack width at age 279 days, prior to wetting the top surface of the slab was 0.375 mm. No cracking occurred on the soffit of
the slab up to this time. The crack patterns on the top surface of S1 are shown in Figs. 6a, 6b, and 6c.

At age 279 days, the top surface of slab S1 was wetted for 48 hours. Over the next week, fine cracks occurred in the midspan regions on the soffit of the slab as shown in Fig. 6d, with a maximum crack width of 0.125mm. The extent and width of the top cracks during this period did not change appreciably.

In specimen S2, flexural cracks formed at first loading on the top surface of the slab over columns C2, C3 and C4 and additional cracking occurred on the top surface at all columns with time and the crack widths gradually increased. The soffit of S2 remained uncracked throughout the test. The maximum crack width on the top surface was 0.35mm. Crack patterns on the top surface of S2 at various ages are shown in Figs. 6e and 6f.

The top surfaces of both slabs S4 and S5, initially loaded with two layers of concrete blocks (8.39 kPa), cracked at first loading. The initial cracks, radiating from the interior columns on the top surface, had a maximum width in the range 0.15 - 0.18mm.

Under the sustained load of 8.39 kPa, the maximum width of the cracks in the top surface of S4 increased with time to 0.25 - 0.375mm at age 25 days, 0.35 - 0.625mm at 40 days and 0.65 - 1.13mm at 150 days. Clearly, the cracks on the top surface of S4 became unserviceable with time. By age 25 days, fine cracks (of maximum width 0.1mm) had developed on the soffit of S4 in the positive moment regions, increasing in width to 0.175 - 0.225 mm at age 40 days and 0.2 - 0.275mm at age 150 days.

For S5, initially heavily loaded and thereafter subjected only to self-weight, no cracks appeared at any stage on the slab soffit. The cracks on the top surface almost closed on removal of the superimposed load. However, the width of the top cracks gradually increased with time, reaching 0.25 - 0.375 mm at 150 days.

Discussion of results

For the slab reinforcement shown in Fig. 1, taking \( f_y = 550 \text{ MPa} \) and \( f'_c = 20 \text{ MPa} \), the calculated collapse load in flexure for a 90mm thick slab is 14.34 kPa. The test loads applied to all slab specimens, therefore, were typical service loads, with the maximum applied load being 58.5% of the collapse load in S4 and S5, and significantly less in S1, S2 and S3.

Of the slabs tested, S1 and S5 behaved in a serviceable manner, with maximum final deflections less than Span/250 and reasonably fine crack widths. The concrete in the 100 mm thick specimen S1 had relatively high
tensile strength and elastic modulus at first loading and relatively low shrinkage. S5 also had relatively low shrinkage and zero superimposed sustained load. S2 (100 mm thick) and S3 (90 mm thick), despite being relatively lightly loaded, suffered final deflections in excess of Span/250 and significant time-dependent cracking. The concrete in S2 had high shrinkage. S4 was subjected to a sustained load of 58.5% of the collapse load and suffered excessive deflection and excessively wide cracks (increasing in width by a factor of about 5 by age 150 days).

The results confirm that time-dependent cracking greatly affects the serviceability of flat slabs. In all specimens, new cracking occurred with time and existing cracks extended and widened (usually on the top surface). The loss of stiffness resulting from time-dependent cracking, caused the final deflection of each specimen to be significantly larger than that predicted by the code (ACI 318-95). For the slabs under constant sustained load (S2, S3, S4 and S5), the ratio of the measured incremental or time-dependent deflection, \( \Delta_{t,\text{time}} \), to the initial deflection due to the sustained load, \( \Delta_{t,\text{init}} \), is shown in Table 6, together with that specified in ACI 318. Clearly, the code greatly underestimates the time-dependent deflection in all cases (even for the unloaded S5).

The significantly larger long-term deflections and the more extensive distribution of cracking in S2 (compared to S1) are due to many factors including lower concrete tensile strength, higher shrinkage, higher creep and increased restraint to shrinkage provided by the footing pads under each exterior column. Research is underway to ascertain the relative significance of each of these factors.

In all specimens the column reactions in C1 and C2 decreased with time and the reactions at C3 and C4 increased. Therefore, the peak negative moment over the interior column (in the E-W direction) increased with time and the positive span moments decreased. This moment redistribution is primarily due to shrinkage and depends to a large extent on the reinforcement layout (6). The increase in negative moments with time is consistent with the gradual increase in flexural cracking observed on the top surface of all slab specimens and the relatively small amount of cracking in the positive moment regions. It is also typical of observations on flat slabs in service, particularly those with serviceability problems (1), where the slab soffit is often free from cracking while the top surface is extensively and excessively cracked.

The average mid-panel deflection of S2 increased from 2.84 mm at first loading to 10.66 mm at age 100 days (Table 4). The relatively high shrinkage in the first 100 days (in excess of 750 \( \mu e \), see Table 2), was responsible for significant time-dependent cracking and the resulting rapid increase in deflection. At age 100 days, a severe hailstorm damaged the roof of the laboratory over S2 and the top surface of the slab was thoroughly
wetted. Unexpectedly, within 24 hours, the average mid-panel deflection decreased by 1.51 mm (more than the instantaneous deflection due to the superimposed load). For the next 120 days, the slab was subjected to repeated periods of wetting and drying and deflection did not increase substantially (see Fig. 5). No further wetting occurred after age 220 days and the average mid-panel deflection increased from 9.49 mm at age 225 days to 13.67 mm at 400 days. The immediate recovery of deflection on first wetting the slab was due to the rapid absorption of water on the top surface, thus reducing the positive curvature in the mid-span regions. Apparently, the degree of exposure is of paramount importance to the serviceability of slabs.

To explore this effect further, the top surface of S1 was wetted intentionally at age 279 days. During the previous 110 days under a heavy superimposed load, the average mid-panel deflection had increased by just 0.78 mm (see Table 3) and the slab appeared to be approaching its final deflection. Within 24 hours of wetting, the average mid-panel deflection decreased by 1.10 mm (which is greater than the instantaneous increase in deflection when the superimposed load was doubled at age 169 days). Within two days of wetting flexural cracking occurred in the soffit of the slab (see Fig. 6d) and deflection increased significantly (from 8.35 mm at 280 days to 10.52 mm at 301 days, see Table 3). The rapid absorption of water and the resulting reduction of positive curvature on wetting caused a redistribution of slab moments, with the positive span moments increasing and the negative support moments decreasing. The increase in positive bending in the heavily loaded slab caused cracking on the slab soffit and a sudden loss of stiffness. These significant effects on deflection and cracking need further research, but clearly should be considered in design.

COMMENTS ON DEFLECTION CALCULATION PROCEDURES

Current design approaches for the calculation of the deflection of flat slabs have recently been shown to be inadequate, (1). They fail to adequately account for the loss of stiffness due to cracking, in particular time-dependent cracking resulting from shrinkage. The tests described here confirm the significance of time-dependent cracking and its influence on long-term deflection.

In general, slabs are lightly reinforced and lose a large percentage of their stiffness when cracking occurs. Shrinkage (and, in practical slabs, temperature changes as well) usually results in a continuing, gradual expansion of the cracked region in flat slabs, with a resultant gradual reduction in slab stiffness with time. Consequently, the ratio of final deflection to short-term deflection in flat slabs is often greater than 4.
Recently several alternative methods were proposed to include the influence of shrinkage induced cracking in deflection circulations, (1). The experimental results for slabs S4 and S5 are here used to check the preferred calculation procedures, designated Alternatives 2 and 3 by Gilbert (1). Deflections are calculated using two different models, an elastic layered finite element model (ELFE) and the 'wide beam' method for flat slabs (WBM) originally proposed by Nilson and Walters (7) and subsequently described by Gilbert and Mickleborough (8). In both analysis procedures, the columns are assumed fully fixed to the laboratory floor and the measured material properties for slabs S4 and S5 (as given in Tables 1 and 2) are used.

The calculated instantaneous elastic deflections obtained using gross section properties under the initial load of 8.39 kN/m² (see Figures 6d and 6e) are given in Rows 6 and 8 of Table 7. The measured instantaneous deflections immediately after the application of the superimposed load are given in rows 1 and 2 of Table 7 and the remaining deflections of slab S5 after the superimposed load was removed within 24 hours of first loading are given in Row 3. As defined earlier, 1, is the average midpanel deflection at points #4, #6, #11 and #13 on each slab (see Fig. 1), while 2, 3, 4 and 5 are, respectively, the average deflections at points #8 and #9, points #1, #2, #15 and #16, points #5 and #12, and points #3, #7, #10 and #14.

With the tensile strength of concrete at the age of first loading measured at $f_t = 2.76$ MPa, the instantaneous cracking moment of the 90mm thick slab is $M_{cr} = f_t Z_g = 3.73$ kNm/m. The calculated instantaneous deflection of the cracked slabs immediately after first loading are shown in Rows 6 and 9 of Table 7. A standard layering technique to reduce the stiffness of the cracked tensile concrete and to allow for tension stiffening was used in the finite element model (9). In the WBM, the stiffness of the slab was reduced to account for cracking by the factor $I_e/I_g$, where $I_g$ is the gross moment of inertia of the wide beam in the midspan region carrying the maximum positive frame moment and $I_e$ is the effective moment of inertia for the wide beam taken as the sum of the values of $I_e$ for the column and middle strips forming the wide beam). For any slab strip, $I_e$ is calculated in accordance with ACI 318-95 (2) and is given by

$$I_e = \left(\frac{M_{cr}}{M_u}\right)^3 I_g + \left[1 - \left(\frac{M_{cr}}{M_u}\right)^3\right] I_{cr} \leq I_g$$

(1)

The calculated instantaneous deflections of the cracked slabs using and the wide beam method are in reasonable agreement with the measured instantaneous deflections, overestimating the maximum midpanel deflection by about 20%. The calculated instantaneous deflections of the cracked slabs
obtained using the non-linear layered finite element model are in excellent agreement with the experimental results.

The measured final deflections of each slab after 236 days under load are given in Rows 3 and 4 of Table 7. For slab S4, subjected to a sustained load of 8.39 kN/m², the maximum incremental or time-dependent deflection, occurring in the midpanel regions (points #4, #6, #11 and #13 in Figure 1), is $20.6 - 4.5 = 16.1$ mm and very significant additional cracking occurred with time. According to ACI 318-95 (2), the ratio of time-dependent deflection to instantaneous deflection may be taken as $\lambda = 1.3$ for this load duration. With a maximum instantaneous deflection of 4.50 mm in S4, the anticipated incremental deflection is therefore $1.3 \times 4.5 = 5.9$ mm (only 36% of the measured value). Clearly the code's long-term deflection multiplier, does not adequately model the time-dependent loss of stiffness associated with shrinkage cracking and the gradual breakdown of tension stiffening with time.

Similarly, for Slab S5, after unloading from 8.39 kN/m² to 2.16 kN/m² soon after initial loading, the instantaneous component of deflection caused by the sustained load (2.16 kN/m²) may be taken to be $2.16/8.39$ times the deflection under full load, i.e. $4.58 \times 2.16/8.39 = 1.18$ mm. The predicted maximum incremental or time-dependent deflection according to the code is $1.3 \times 1.18 = 1.53$ mm, which is only 31% of the measured maximum incremental deflection of $6.14 - 1.18 = 4.96$ mm. The difference between the measured deflections in Row 3 of Table 7 and the instantaneous deflection due to self-weight is the creep and shrinkage induced deflection that occurred during the 15 hours between first loading (at age 14 days) and unloading the following morning.

To account for time-dependent cracking in deflection calculations, Gilbert (1) proposed the inclusion of the shrinkage-induced tension at the extreme tensile concrete fibre in the estimation of the cracking moment. With the resulting reduction in $M_{cr}$ with time, the value of $I_e$ used for long-term deflection calculations will usually be significantly smaller than that used in an instantaneous deflection analysis. Using this approach (refer to Reference 1) and the measured material properties, the shrinkage induced tensile stress in the midspan region of the wide beam is $f_{sg} = 1.23$ MPa and the reduced cracking moment is $M_{cr} = 2.07$ MPa. Two alternative approaches to long-term deflection calculation incorporating the shrinkage induced tension were proposed by Gilbert (1), titled Alternatives 2 and 3. Alternative 2 uses the ACI long-term deflection multiplier and Alternative 3 calculates the creep and shrinkage induced deflection separately using the actual creep and shrinkage characteristics of the concrete.

In the WBM, the maximum positive moments in both the column and middle strips in both directions are greater than the time-dependent cracking moment and the effective moment of inertia for the wide beam in
the E-W direction, for example, reduces to 43% of its instantaneous value. Using the reduced values for $I_e$ for the wide beams in each direction and the ACI deflection multiplier of 1.3 (Alternative 2 in Reference 1), the final calculated deflections using the WBM for S4 are calculated and given in Row 10 of Table 7. Similarly, the final calculated deflections using the WBM and Alternative 2 for S5 are calculated and given in Row 11 of Table 7. The maximum slab deflection calculated using Alternative 2 is within 8% of the measured value for S4 and although overestimating deflection for the unloaded S5 the calculated results are considered acceptable.

Using Alternative 3, the approach recommended by Gilbert (1) in which the measured creep coefficient and shrinkage strain are used directly to calculate creep and shrinkage induced deflection, the calculated deflections for S4 and S5 are given in Rows 12 and 13, respectively, of Table 7. For the heavily loaded slab S4, the calculated results are in very close agreement with the experimental results, while for the unloaded slab S5, the procedure overestimates the deflection but not unreasonably so.

CONCLUSIONS

The time-dependent in-service behavior of five large-scale flat slab structures has been presented. The deflection, the extent of cracking and the column loads were monitored with time, together with the strength and deformation characteristics of the concrete. Significant time-dependent cracking was observed and the long-term to short-term deflection ratios were significantly greater than those predicted using current codes. Recently proposed procedures for calculating slab deflection (1), incorporating the loss of stiffness with time due to shrinkage induced cracking, have been shown to reasonably predict the deflection of the test specimens. The tests described here are part of an on-going experimental study at the University of New South Wales in which eight flat slabs will eventually be tested.

ACKNOWLEDGEMENTS

The study is funded by the Australian Research Council. The assistance of Dr Patrick Zou, Mr X. Guo and the staff of the Heavy Structures Laboratory at UNSW is gratefully acknowledged.

REFERENCES


(2) ACI 318-95. Building Code Requirements for Reinforced Concrete. American Concrete Institute. Michigan. USA.
Table 1. Material properties at age of first loading.

<table>
<thead>
<tr>
<th>Slab Specimen</th>
<th>Age at first loading (days)</th>
<th>£c (MPa)</th>
<th>fc (MPa)</th>
<th>fi (MPa)</th>
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<tbody>
<tr>
<td>S1</td>
<td>14</td>
<td>30,020</td>
<td>34.5</td>
<td>4.39</td>
</tr>
<tr>
<td>S2</td>
<td>14</td>
<td>29,100</td>
<td>29.0</td>
<td>2.72</td>
</tr>
<tr>
<td>S3</td>
<td>28</td>
<td>22,620</td>
<td>18.0</td>
<td>2.48</td>
</tr>
<tr>
<td>S4</td>
<td>14</td>
<td>22,010</td>
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<td>2.76</td>
</tr>
<tr>
<td>S5</td>
<td>14</td>
<td>22,010</td>
<td>19.9</td>
<td>2.76</td>
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Table 2. Creep coefficient and shrinkage strain (x 10^-6).

<table>
<thead>
<tr>
<th>Age (days)</th>
<th>S1</th>
<th>S2</th>
<th>S3</th>
<th>S4 and S5</th>
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<tr>
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<td>£c</td>
<td>£c</td>
<td>£c</td>
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<td>940</td>
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* At age 250 days.
Table 3. Average deflections for slab S1.

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<tr>
<th>Age (days)</th>
<th>$\Delta_1$ (mm)</th>
<th>$\Delta_2$ (mm)</th>
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<th>$\Delta_4$ (mm)</th>
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<td>3.85</td>
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<td>7.60</td>
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* before and ** after load applied and removed.

Table 4. Average deflection $\Delta_1$ for slab S2, S3, S4 and S5.

<table>
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<tr>
<th>S2 Age (days)</th>
<th>S2 $\Delta_1$ (mm)</th>
<th>S3 Age (days)</th>
<th>S3 $\Delta_1$ (mm)</th>
<th>S4 Age (days)</th>
<th>S4 $\Delta_1$ (mm)</th>
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<td>3.73</td>
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<tr>
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<td>10.66</td>
<td>60</td>
<td>5.57</td>
<td>150</td>
<td>18.30</td>
<td>100</td>
<td>4.94</td>
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<tr>
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<td>9.15</td>
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<td>166</td>
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<td>150</td>
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<td></td>
<td></td>
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<tr>
<td>400</td>
<td>13.67</td>
<td></td>
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</table>
* top surface first wetted

Table 5. Column loads.

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<tr>
<th>Age (days)</th>
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<th>S2 (kN)</th>
<th>S3 (kN)</th>
<th>S4 (kN)</th>
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<td>40.7</td>
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<td>41.7</td>
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<td>28.8</td>
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<td>65.0</td>
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<td>94.7</td>
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Table 6. Long-term to instantaneous deflection ratios.

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<tr>
<th>Specimen</th>
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<th>Total days under load</th>
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<td>Measured</td>
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<td>S2</td>
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<tr>
<td>S3</td>
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</tr>
<tr>
<td>S4</td>
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<td>1.3</td>
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<tr>
<td>S5</td>
<td>1.71</td>
<td>1.3</td>
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Table 7. Measured and calculated deflections for S4 and S5.

<table>
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<th>Row</th>
<th>Loading stage</th>
<th>Slab</th>
<th>Deflection (mm)</th>
<th>Δ₁</th>
<th>Δ₂</th>
<th>Δ₃</th>
<th>Δ₄</th>
<th>Δ₅</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Test Results: Instantaneous deflection at first loading (8.39 kN/m² both slabs)</td>
<td>S4</td>
<td>4.50</td>
<td>3.67</td>
<td>2.52</td>
<td>3.68</td>
<td>3.37</td>
<td></td>
</tr>
</tbody>
</table>
<pre><code> |                                                                               | S5   | 4.58            | 3.89| 2.49| 3.69| 3.09|
</code></pre>
<p>| 2   | Instantaneous deflection just after superimposed load removed (S5)             | S5   | 2.27            | 1.92| 1.15| 1.91| 1.74|
| 3   | Measured final deflection after 236 days under sustained load (8.39 kN/m² for S4 and 2.16 kN/m² for S5) | S4   | 20.6            | 14.9| 12.3| 14.7| 14.1|
|                                                                               | S5   | 6.14            | 4.88| 3.15| 5.11| 3.45|
| 4   | Layered Finite Element Analysis: Uncracked instantaneous deflection           | S4 &amp; S5 | 3.72            | 2.86| 2.03| 2.77| 2.27|
| Cracked instantaneous deflection                                               | S4 &amp; S5 | 4.43            | 3.65| 2.25| 3.60| 2.66|
| 5   | Wide Beam Method (WBM): Uncracked instantaneous deflection                    | S4 &amp; S5 | 4.07            | 3.09| 1.84| 2.62| 1.83|
| Cracked instantaneous deflection                                               | S4 &amp; S5 | 5.48            | 4.14| 2.47| 3.51| 2.45|
| 6   | Final long deflection (Alternative 2)                                         | S4   | 22.2            | 16.8| 10.0| 14.2| 9.93|
| 7   | Final long deflection (Alternative 2)                                         | S5   | 7.62            | 5.76| 3.44| 4.88| 3.41|
| 8   | Final long deflection (Alternative 3)                                         | S4   | 21.0            | 16.0| 12.5| 14.7| 12.4|
| 9   | Final long deflection (Alternative 3)                                         | S5   | 7.60            | 6.06| 5.14| 5.50| 5.13|
| 10  | Final long deflection (Alternative 3)                                         | S4   | 21.0            | 16.0| 12.5| 14.7| 12.4|
| 11  | Final long deflection (Alternative 3)                                         | S5   | 7.60            | 6.06| 5.14| 5.50| 5.13|</p>
Bottom reinforcement: 

B1 = Y10 bars @ 220 mm centers
Top reinforcement: 

T1 = Y10 bars @ 250 mm centers
T2 = Y10 bars @ 140 mm centers

E-W bars placed 1st and last. 
N-S bars placed 2nd and 3rd.

Slab thickness = 100 mm (S1, S2) and 90 mm (S3, S4, S5)

Fig. 1. Plan of slab specimens and reinforcement layout.
Fig. 2. Slabs S4 and S5 under construction.

Fig. 3. Slab S1 just prior to first loading.
Fig. 4. Slab S2 (foreground) and S1 under load.

Fig. 5. Slab S3 (foreground) and slabs S1 and S2 (background) under load.
Fig. 6. Load histories of slabs S1 to S5.
(Fig. 7. Continued on next page)
Fig. 7. Mid-panel deflection versus time.
Fig. 8. Crack patterns for S1 and S2.
Comparison of Beam Deflection Variability in Members Using High Strength Concrete and Normal Strength Concrete
by B.-S. Choi and A. Scanlon

Synopsis: Research has shown that design expressions previously developed for material properties such as modulus of elasticity, modulus of rupture and creep coefficient used in the calculation of beam deflections may not be appropriate for high-performance-high-strength concrete. Also, the uncertainties associated with regular and high-performance concrete material properties as characterized by probability distribution functions may be different. Since high strength concrete may be used to reduce the size of structural members, assessment of deflections will be an increasingly important design consideration. This paper discusses these issues and demonstrates through Monte Carlo simulation techniques some differences between the variability of deflections in beams made with regular and high-strength concretes.

Keywords: beam; concrete; deflection; slab; variability
Bong-Seob Choi is research professor of civil engineering at Seoul National University, Korea. He received his Ph.D. degree from The Pennsylvania State University. His research activities are in the fields of serviceability of concrete structures.

Andrew Scanlon, a Fellow of the American Concrete Institute is a Professor of Civil Engineering at Penn State University. He is Chairman of ACI 435 Deflections and is also a member of ACI 224 (Cracking), ACI 342 (Evaluation of Existing Bridges), ACI 348 (Safety), and ACI 437 (Strength Evaluation).

INTRODUCTION

In recent years, applications of high-strength concrete have increased, and high-strength concrete has now been used in many parts of the world. Therefore, the prediction and control of deflections may be increasingly important because the size of structural members can potentially be reduced by the use of high-strength concrete. However, the equations and procedures presented in many codes of practice or regulations for the prediction and calculation of deflection give a deterministic value, which is either smaller or larger than the deflection that would be likely to occur in a real structure. Even with the most sophisticated methods of analysis using experimentally determined material properties, the variability of short-time as well as long-time deflection is high. High-strength concretes have mechanical properties that are different from those of lower-strength concretes. Relationships developed for normal strength concrete may not be appropriate for high-strength concrete in the calculation of immediate deflections as well as long-time deflections. The variability of normal concrete deflections may also be different from that of high-strength concrete deflections.

The objective of the study described in this paper is to compare the variability of deflections in beams made with normal and high-strength concretes. A deterministic layered beam model, which includes the effects of cracking, creep and shrinkage is presented based on the finite element approach. In addition, Monte Carlo simulation is conducted to illustrate the variability of deflections for both normal strength concrete and high strength concrete. With these simulated results, comparisons are made between results for normal strength and high strength concrete.
DETERMINISTIC LAYERED BEAM MODEL

For a realistic estimate of reinforced concrete beam deflection under a given loading, nonlinear effects such as cracking, tension stiffening, creep and shrinkage must be taken into account in the analytical model. These effects result in variations in the material stiffness along the member length. The finite element method allows the variations to be taken into account by computing stiffness characteristics for localized regions of the structure and using matrix methods to assemble a global stiffness matrix for the entire structure.

For the evaluation of stiffness matrix for a beam element, the one-dimensional element used in this analysis has six degrees of freedom, consisting of three displacement components at each end of an element. Also, the beam is considered as a series of layers to allow for a variation in elastic constants with depth. The modulus of elasticity may vary from layer to layer but is considered to be constant within layers.

Tension Stiffening Effect

A reinforced concrete element subjected to uniaxial tension stress is shown in Figure 1. When the concrete stress reaches the ultimate tensile strength, primary cracks form at intervals along the length. The total load is transferred across these cracks by the reinforcement but the concrete between cracks is still capable of carrying stress because of the bond between steel and concrete. As the loads increase, more cracks form and the proportion of the load carried by the concrete gradually diminishes. A stress-strain relationship with descending branch of the concrete stress-strain curve for cracked reinforced concrete was first suggested by Scanlon and Murray [1] and was subsequently used by many researchers.

In this study, it is assumed that after tensile cracking the post-tensile response of concrete may be represented as a linear reduction in the tensile stress as shown in Figure 2. A linear stepped model for tension stiffening effect is shown in Figure 3. The ultimate tensile strain, $\nu$, is defined as $\nu = \frac{f}{E}$, where $f$ is a tension stiffening parameter. In order to apply the concept to cracking analysis, it is necessary to evaluate the reduced modulus of elasticity for concrete layers due to cracking through the following iterative procedure.

Step 1 The modulus of elasticity in the tension region, $E_t$, is assumed to be the same as one in the compression region until the tensile stress reaches the modulus of rupture, $f$, (Case 1) as shown in Figure 3.

Step 2 If the tensile stress exceeds the specified modulus of rupture, the modulus of elasticity, $E_t$, may be modified to $E_{t1}$, which is the next lower concrete modulus (Case 2) defined by projecting from the calculated stress $f_{t1}$ into the decreasing branch of the
stress-strain diagram.

Step 3 The analysis is repeated for other layers in the same manner.

Step 4 In the second iteration, if the tensile stress does not exceed the next lower rupture of modulus, \( f_{t_{11}} \), it is assumed that \( E_{11} \) does not need to be modified (Case 3).

Step 5 If exceeding \( f_{t_{1}} \), \( E_{11} \) is modified into the next lower concrete modulus, \( E_{c2} \) (Case 4).

Step 6 This iteration continues until the stress stays inside the envelope satisfying a desired convergence criterion.

Time Dependent Effects

To predict the time-dependent response of a reinforced concrete beam to given loading and environmental conditions, the material properties need to be defined and a mathematical model is required to approximate the long-term behavior. Several practical models for predicting creep and shrinkage properties for a particular concrete and environmental conditions have been developed. Equations for creep and shrinkage recommended in ACI Committee Report 209R [2] are used in this study.

For creep analysis, the age-adjusted effective modulus proposed by Trost [3] is used. This method consists of an elastic analysis with a modified elastic modulus, \( E_{ca} \), which is defined by Equation (1), and is called the age-adjusted effective modulus.

\[
E_{ca} = \frac{E_{ci}}{1 + \chi \phi_{i}}
\]

(1)

\( E_{ci} \) is the initial modulus of elasticity. The aging coefficient, \( \chi \), accounts for nonuniform sustained stress and depends on age at the loading time in days, \( t_{a} \), on the load duration \( t-t_{a} \), and on the ultimate creep coefficient, \( \phi_{i} \), defined as ratio of creep strain to initial strain. The variable \( t \) represents the time in days after the casting of concrete. Therefore, in this analysis, the initial modulus of elasticity, \( E_{i} \), is modified to the reduced modulus of elasticity, \( E_{ca} \), as shown in Figure 4. It is assumed that the modulus of elasticity in tension region, \( E_{t} \), is also changed into \( E_{ta} \). By using the same post cracking stress-strain relationship as for instantaneous loading, a reduction in tensile strength under sustained stress is accounted for.

For the case in which the reinforcement and eccentricity are constant along the span and the same in the positive and negative moment regions of continuous beam, shrinkage deflections for uniform beams are computed by Equation (2).

\[
\delta_{h} = \frac{w}{E_{ca}I_{h}}
\]
where \( \kappa \) is a deflection coefficient presented in Table 4.2.1 of ACI Committee Report 209R (1998) for different boundary conditions, and \( \kappa_{\text{sh}} \) is the curvature due to shrinkage warping. The equivalent tensile force method, as modified by Branson [4] using \( E_c / 2 \) and the gross section properties, is used for computing shrinkage curvature in this study.

\[
\kappa_{\text{sh}} = \frac{T_s e_s}{E_c I_g / 2}
\]

where \( T_s = (A_{\text{t}} + A_{\text{b}}) E_s / 2 \), \( A_{\text{t}} \) = area of top steel, \( A_{\text{b}} \) = area of bottom steel, \( e_s \) = distance between the centroid of gross concrete section and steel area and \( I_g \) = moment of inertia of the gross section.

The model was verified by comparison with available experimental results[5]. A tension stiffening parameter, \( T_s = 3.0 \) was found to give good correlation for both instantaneous and long-term deflections.

**STATISTICAL DATA FOR NORMAL AND HIGH STRENGTH CONCRETE**

In this study, Monte Carlo simulation is used to study the variability of deflections with assumed statistical data and probability distributions of variables and the deterministic layered beam model described previously. In most cases, the statistical data and probability distributions were obtained from the literature [2,6,7,8]. Fourteen design parameters are considered as random variables. These include the parameters related to the concrete and steel strength, beam dimensions and time-dependent properties such as creep and shrinkage. Information contained in ACI Committee Report 209R is used to develop statistics for the variability of creep and shrinkage.

It is assumed that modulus of rupture and modulus of elasticity are correlated with compressive strength. This is accomplished as follows. A value of compressive strength is selected randomly. This value is then used to compute a mean value for modulus of rupture and modulus of elasticity. The assumed coefficient of variation for these parameters are then used to established probability density functions. Values of modulus of rupture and modulus of elasticity are then obtained randomly from these distributions. In addition, the tension stiffening parameter, \( T_s \), is also considered as a random variable because of the variability of post-tensile cracking behavior. The statistical data of random variables for normal strength concrete are summarized in Table 1.
The statistical data of random variables for high strength concrete are assumed to be the same as those of normal strength concrete except for modulus of rupture, \( f_r \), and ultimate creep coefficient \( \varepsilon \). The mean value of modulus of rupture was reported as 0.94 \( f_c \) (MPa) in ACI Committee Report 363R [9]. Ngab et al [10] reported that creep coefficient of high strength concrete under drying condition is about 50 to 75 percent of that of normal strength. Consequently, it is assumed that the mean of ultimate creep coefficient of high strength concrete is 70 percent of that of normal strength. The assumed statistical data of random variables for high strength concrete are summarized in Table 2.

DESCRIPTION OF DESIGNED ONE-WAY SLABS AND T-BEAMS

To conduct Monte Carlo simulation, three cases are considered consisting of two simply supported one-way slabs and a simply supported T-beam designed on the basis of ultimate moment capacity. Details of nominal concrete properties and dimensions are shown in Table 3, and sketches showing for dimensions are presented in Figure 5. Table 4 shows details of the combinations of uniformly distributed dead and live loads. The live loads were selected to provide a range of live load to dead load ratio.

It is also necessary to assume a load-time history in order to evaluate long-time deflections. In this study, as shown in Figure 6, load-time history consists of the construction load \( W_{cr} \), which can be taken as equal to the service dead load plus live load, and a sustained load \( W_s \), consisting of dead load plus sustained portion of live load. The construction load is assumed to be applied instantaneously and then removed followed by application of sustained load. The load then remains constant thereafter and the variable portion of live load is applied at the end of assumed load duration. Consequently, an instantaneous deflection, \( \delta \), corresponding to initial application of sustained load, and a long-time total deflection, \( \delta_t \), including immediate deflection due to live load are considered for evaluation of variability of deflections as indicated in Figure 6(b). The loading age and load duration are taken as 7 days and 1000 days respectively.

MONTE CARLO SIMULATION RESULTS

For practical implementation of Monte Carlo Simulation, several techniques are required. Uniformly distributed random numbers may be generated using a random number generator. These are used for selecting random variates, which are sample values for random variables with given distributions. If uniform random numbers are generated between 0 and 1, a transformation technique is required for generating random variates from the
known probability distributions of random variables. A process of sampling from a known or assumed confidence interval of probability density functions is performed to select variables required for input into the mathematical model.

Based on these techniques, several examples are presented to indicate some trends and differences between the variability of deflections for regular and high-performance concrete. Figure 7 shows the probability distributions obtained for immediate deflection of slab SS324M. For normal concrete, there are two distinct parts to the distribution. The majority of the results are clustered around the mean value of 4.9 mm. There are a significant number of values spread out above the mean value as well. The majority of cases involving lower deflection values are associated with an uncracked member. If cracking is detected however, this section has a low reinforcement ratio, causing the stiffness to drop significantly after cracking resulting in higher deflections. For high performance concrete, the results are almost all clustered around the mean value of 2.18 mm associated with uncracked section. It is also shown that the probability of uncracked member for high-strength concrete is much higher than for normal strength concrete.

Figure 8 shows the distribution of total long-time deflections. In normal concrete case, the shape of the distribution is significantly different than for immediate deflection. A bi-modal distribution is indicated. The group of higher deflections is associated with a cracked section indicating that many cases that were uncracked under immediate loading have developed cracking under long time loading. On the other hand, in the case of high strength concrete, the shape of the distribution is not significantly different than for immediate deflection. The results indicate that the slab is still uncracked under long-time loading in most cases.

Figure 9 shows the probability distributions for immediate deflection for slab SS430D. This slab has the highest reinforcement ratio and highest applied moment to cracking moment ratio of the simply supported slabs. The probability distribution is approximately normal and the behavior of the slab is dominated by cracking for normal concrete under initial load. The coefficient of variation is only 13.3% compared with 84% for slab SS324M. However, for high strength concrete, two distinct parts are formed indicating that the lower deflections are associated with uncracked sections and the higher deflections are associated with cracked sections. Consequently, the C.O.V. for high strength concrete is higher than for normal strength concrete. For total deflection under long time loading, the same general shape is indicated in Figure 10(a). While the standard deviation increases from 2.11 mm to 4.93 mm, the C.O.V. decreases from 13.3 to 12.1% due to the increase in the mean deflection under long time loading. For high strength concrete, a bi-modal distribution is indicated with uncracked sections under immediate loading having developed cracking under long-time loading as shown in Figure 10(b).
As a result, C.O.V. for high strength concrete is higher than for normal concrete. For these cases, the behavior appears to be dominated by the cracked transformed section and the variability is affected primarily by the modulus of elasticity.

Results for immediate deflection are presented in Figure 11 for T-beam example TC430M. In the normal concrete case, the distribution divides into two groups. The lower deflections are associated with an uncracked beam while the higher deflections are associated with cracking. However, for high strength concrete, the majority of results are associated with uncracked members and C.O.V. is lower than one of normal concrete case.

In Figure 12(a), the probability distribution for total long time deflection is seen to be approximately normal indicating that in many cases cracking has developed under sustained loading. The trend is similar to that indicated for slab SS324M. For high strength concrete, there are two parts to the distribution as shown in Figure 12(b). While the lower deflections are associated with uncracked sections, the higher deflections are associated with cracked sections. In addition, C.O.V. of high strength concrete is higher than that of normal concrete.

CONCLUSIONS

This exploratory study has indicated some general trends and differences between normal strength and high strength concrete with respect to deflection variability. As expected, mean values of deflections are lower for members made with high strength concrete than for members made with normal strength concrete. The results of normal concrete cases indicate that for members with low reinforcement ratios and, as a result, applied moments that are close to the cracking moment, the stiffness is very sensitive to the development of cracking. While the most likely condition is for the slab to be uncracked, there is still a significant probability of cracking occurring and hence there is increased uncertainty and variability in deflection of members with low reinforcement ratios. However, it is shown that there is less likelihood of cracking occurring and less uncertainty about stiffness in the cases of high strength concrete because the applied moments are less than the cracking moments due to the increased modulus of rupture.

For members with higher reinforcement ratios there is a high likelihood of cracking occurring and as a result less uncertainty about the stiffness in cases of normal concrete. Variability of deflections is therefore lower for higher reinforcement ratios. However, the results of high strength concrete show that there is still significant probability of uncracked section occurring. As a result, the variability of deflections is higher for higher reinforcement ratios and higher applied moment to cracking moment ratio. Additional
research is needed to investigate the effects of other parameters such as variability in load and load history, size effect and modulus of rupture, creep and shrinkage models, and other beam and slab configurations.

REFERENCES


Table 1  Statistical data of random variables for normal strength concrete

<table>
<thead>
<tr>
<th>Variable</th>
<th>Mean (μ)</th>
<th>C.O.V</th>
<th>S.D. (σ)</th>
<th>Distribution</th>
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Table 2 Statistical data of random variables for high strength concrete

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Table 3  Details of one-way slabs and T-beams

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Identification Code of One-Way Slab and T-Beam : AAXBBC

AA = SS : Simply Supported One-Way Slab  
   = TC : Two Equal Continuous T-Beam  
X = $f'_c$ ksi  
BB = Span $\div$ 10 (in)  
   = M : Medium Depth (h)  
   = D : Deepest Depth (h)  
C = S : Shallow Depth (h)
Table 4  Details of loads on one-way slabs and T-beams

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<tr>
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<th>$W_{lsus}$ (kN/m)</th>
<th>$W_{lvar}$ (kN/m)</th>
<th>$W_{l}$ (kN/m)</th>
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- List of symbols on loads
  - $W_{sw}$ = Self-weight of beams
  - $W_{sd}$ = Superimposed dead load
  - $W_{d}$ = Total dead load ($=W_{sw} + W_{sd}$)
  - $W_{lsus}$ = Sustained portion of live load
  - $W_{lvar}$ = Variable portion of live load
  - $W_{l}$ = Total live load ($=W_{lsus} + W_{lvar}$)
  - $W_{co}$ = Equivalent construction load ($=W_{d} + W_{l}$) = Load level 1
  - $W_{s}$ = Instantaneous load of sustained portion ($= W_{d} + W_{lsus}$) = Load level 2
Fig. 1 Interactive effect between concrete and reinforcement

Fig. 2 Tensile stress in reinforced concrete
Fig. 3  Linear stepped model for tension stiffening
Fig. 4  Linear uniaxial stress-strain relationship for creep effect
(a) Simply supported one-way slab

(b) Two span continuous T-beam

Fig. 5 Sketch for dimensions of one-way slabs and T-beams
(a) Load-time history

(b) Deflection-time history

Fig. 6 Load-time history and corresponding deflection-time history
Fig. 7 Comparison of Probability Distributions of SS324M for Instantaneous Deflections
Fig. 8 Comparison of Probability Distributions of SS324M for Long-Time Total Deflections
Fig. 9 Comparison of Probability Distributions of SS430D for Instantaneous Deflections
Fig. 10  Comparison of Probability Distributions of SS430D for Long-Time Total Deflections
Fig. 11 Comparison of Probability Distributions of TC430M for Instantaneous Deflections
Fig. 12 Comparison of Probability Distributions of TC430M for Long-Time Total Deflections