FLAT PLATE AND FLAT SLAB CONSTRUCTION

Neil M. Hawkins, Professor Emeritus, University of Illinois

A Tribute to the Lasting Contributions and Legacy of Our Friend And Colleague Dr. W Gene Corley

ACI Convention, Phoenix, AZ , Sunday October 20, 2013
DISCUSSION TOPICS
Gene’s Early Professional Years

• Equivalent Frame Analysis
  SRS 218 Univ. of Illinois – Ph.D. Thesis – June 1961
  Concrete International – Dec. 1983- w. Dan Vanderbilt

• Testing and Analysis of Flat Plate and Flat Slab System Shear Strengths
  ACI Journal – Oct. 1968- Shearhead Reinforcement – w. NMH
  ACI SP-30 – 1971– Moment and Shear Transfer to Columns–w. NMH
  ACI SP-42- 1974- Moment Transfer with Shearheads – w. NMH
  WCEE 1973–Ductile Flat-Plate Structures to Resist EQ–w.JEC & PHK
  ACI SP-59- 1979– Shear in Two-Way Slabs – ACI Approach
EARLY PROFESSIONAL YEARS

National Science Foundation Fellow 1958-1961

Ph.D Structural Engineering, University of Illinois, 1961

US Army Corps of Engineers, 1961-1964

Structural Research Manager, PCA R & D Division 1964 - 1972
EQUIVALENT FRAME ANALYSIS FOR FLAT PLATES AND FLAT SLABS

• First introduced in ACI 318-71 and based on U of I Ph. D theses by Corley (1961) and Jirsa (1963).

• Early ACI Codes permitted an “empirical method” of design only; Slab properties were restricted to those load tested in the early 1900s.

To overcome that restriction the 1941 ACI code introduced an “elastic design method” giving similar results to the “empirical method” for the loaded tested floors but useable for slabs with dissimilar properties.

The 71 Code frame similar to the 41 Code frame except for stiffness definitions for frame members.
1971 AND 1941 DEFORMATION ASSUMPTIONS

(a) DEFLECTED SHAPE OF SLAB

(b) DEFLECTED SHAPE OF SLAB BY ACI CODE ASSUMPTIONS

FIG. 49 DEFLECTED SHAPE OF A SLAB PANEL UNDER UNIFORM LOAD

(a) DEFORMATION OF EDGE BEAM

(b) "DEFORMATION" OF EDGE BEAM ACCORDING TO ASSUMPTIONS OF ACI CODE FRAME ANALYSIS

FIG. 50 ILLUSTRATION OF BEAM DEFORMATIONS CAUSED BY TWISTING MOMENT
Fig. 13-7 – Simplified physical models illustrating the intent of Section 13.7.4
Where \( C = \text{Torsional Constant} \)

\[
K_t = \sum \frac{9EcsC}{\ell_2 \left(1 - \frac{c_2}{\ell_2}\right)^3}
\]

\[
C = \sum \left(1 - 0.63 \frac{x}{y}\right) \frac{x^3 y}{3}
\]

In Corley’s thesis the unit twisting moment, Fig 3(B), was uniform over the length \( L_2 \). Jirsa modified Corley’s distribution to that shown based on pattern loading considerations.
EQUIVALENT COLUMN STIFFNESS

For moment distribution procedures the equivalent column stiffness $K_{ec}$ was defined by:

$$\frac{1}{K_{ec}} = \frac{1}{K_c} + \frac{1}{K_t}$$

$K_c$ = column flexural stiffness

$K_t$ = torsional stiffness of members framing into column

Fig. R13.7.4—Equivalent column (column plus torsional members).
LAYOUT OF 9 PANEL U of I \( \frac{1}{4} \) SCALE MODEL

FIG. 58 LAYOUT OF NINE-PANEL REINFORCED CONCRETE FLAT PLATE
### Table 14: Comparison of Measured Moments with Moments Computed for 9-Panel Reinforced Concrete Flat Plate Model

<table>
<thead>
<tr>
<th>Section</th>
<th>R</th>
<th>S</th>
<th>T</th>
<th>Sum</th>
<th>U</th>
<th>V</th>
<th>U'</th>
<th>Sum</th>
<th>T'</th>
<th>S'</th>
<th>R'</th>
<th>Sum</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Moments Measured</strong> from Strains⁺</td>
<td>0.029</td>
<td>0.052</td>
<td>0.069</td>
<td>0.101</td>
<td>0.063</td>
<td>0.038</td>
<td>0.062</td>
<td>0.101</td>
<td>0.064</td>
<td>0.048</td>
<td>0.035</td>
<td>0.098</td>
</tr>
<tr>
<td><strong>Moments Measured</strong> from Reactions⁺</td>
<td>0.030</td>
<td>0.053</td>
<td>0.078</td>
<td>0.107</td>
<td>0.071</td>
<td>0.037</td>
<td>0.070</td>
<td>0.108</td>
<td>0.078</td>
<td>0.052</td>
<td>0.041</td>
<td>0.112</td>
</tr>
<tr>
<td><strong>Difference</strong> Solutions (UI94)*</td>
<td>0.045</td>
<td>0.043</td>
<td>0.062</td>
<td>0.096</td>
<td>0.061</td>
<td>0.039</td>
<td>0.061</td>
<td>0.100</td>
<td>0.062</td>
<td>0.043</td>
<td>0.046</td>
<td>0.097</td>
</tr>
<tr>
<td><strong>Proposed Frame Analysis</strong></td>
<td>0.024</td>
<td>0.051</td>
<td>0.090</td>
<td>0.108</td>
<td>0.068</td>
<td>0.038</td>
<td>0.068</td>
<td>0.0106</td>
<td>0.092</td>
<td>0.052</td>
<td>0.031</td>
<td>0.114</td>
</tr>
<tr>
<td><strong>ACI Code Frame Analysis</strong>**</td>
<td>0.058</td>
<td>0.036</td>
<td>0.066</td>
<td>0.098</td>
<td>0.061</td>
<td>0.034</td>
<td>0.061</td>
<td>0.095</td>
<td>0.066</td>
<td>0.036</td>
<td>0.058</td>
<td>0.098</td>
</tr>
<tr>
<td><strong>ACI Code Empirical Moments</strong></td>
<td>0.049</td>
<td>0.031</td>
<td>0.071</td>
<td>0.091</td>
<td>0.063</td>
<td>0.041</td>
<td>0.063</td>
<td>0.104</td>
<td>0.071</td>
<td>0.031</td>
<td>0.052</td>
<td>0.093</td>
</tr>
</tbody>
</table>
## Table 1 — Comparison of Measured with Computed Moments (Flat Plate Structures)

<table>
<thead>
<tr>
<th>Section</th>
<th>M&lt;sup&gt;-&lt;/sup&gt;</th>
<th>M&lt;sup&gt;+&lt;/sup&gt;</th>
<th>M&lt;sup&gt;-&lt;/sup&gt;</th>
<th>M&lt;sup&gt;+&lt;/sup&gt;</th>
<th>M&lt;sup&gt;-&lt;/sup&gt;</th>
<th>M&lt;sup&gt;+&lt;/sup&gt;</th>
<th>M&lt;sup&gt;-&lt;/sup&gt;</th>
<th>M&lt;sup&gt;+&lt;/sup&gt;</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shallow beam edge</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>University of Illinois structure, F1 (1/4 scale), ( w_m/w_p = 2.5 )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Calculated uniform load design moment</td>
<td>47</td>
<td>44</td>
<td>72</td>
<td>66</td>
<td>34</td>
<td>67</td>
<td>73</td>
<td>44</td>
</tr>
<tr>
<td>Calculated maximum design moment</td>
<td>54</td>
<td>50</td>
<td>75</td>
<td>73</td>
<td>45</td>
<td>73</td>
<td>75</td>
<td>50</td>
</tr>
<tr>
<td>Ratio maximum to uniform load moment</td>
<td>1.15</td>
<td>1.14</td>
<td>1.04</td>
<td>1.11</td>
<td>1.32</td>
<td>1.09</td>
<td>1.04</td>
<td>1.13</td>
</tr>
<tr>
<td>Measured uniform load moment</td>
<td>27</td>
<td>49</td>
<td>65</td>
<td>64</td>
<td>40</td>
<td>58</td>
<td>58</td>
<td>47</td>
</tr>
<tr>
<td>Measured maximum moment</td>
<td>21</td>
<td>52</td>
<td>68</td>
<td>67</td>
<td>44</td>
<td>63</td>
<td>63</td>
<td>48</td>
</tr>
<tr>
<td>Ratio maximum to uniform load moment</td>
<td>—</td>
<td>1.06</td>
<td>1.04</td>
<td>1.05</td>
<td>1.10</td>
<td>1.09</td>
<td>1.09</td>
<td>1.02</td>
</tr>
<tr>
<td>Ratio design to measured uniform load moment</td>
<td>1.74</td>
<td>0.90</td>
<td>1.11</td>
<td>1.03</td>
<td>0.85</td>
<td>1.16</td>
<td>1.26</td>
<td>0.94</td>
</tr>
<tr>
<td>Deep beam edge</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>PCA structure (3/4 scale)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Calculated uniform load design moment</td>
<td>44</td>
<td>48</td>
<td>67</td>
<td>62</td>
<td>38</td>
<td>62</td>
<td>68</td>
<td>49</td>
</tr>
<tr>
<td>Measured uniform load moment</td>
<td>37</td>
<td>47</td>
<td>68</td>
<td>68</td>
<td>31</td>
<td>73</td>
<td>73</td>
<td>42</td>
</tr>
<tr>
<td>Ratio design to measured uniform load moment</td>
<td>1.19</td>
<td>1.02</td>
<td>0.99</td>
<td>0.91</td>
<td>1.22</td>
<td>0.85</td>
<td>0.85</td>
<td>1.16</td>
</tr>
</tbody>
</table>
EQUIVALENT FRAME PROCEDURE LIMITATIONS

Discussed in “Frame Analysis of Concrete Buildings”
Vanderbilt and Corley, Concrete International, Dec. 1983

- Method assumes analysis by moment distribution methods.
- Method calibrated for gravity loadings only by comparison to U of I ¼ scale and PCA ¾ scale tests.
- Method based on stiffness of uncracked sections.
- Method not calibrated for lateral loadings but theoretical studies suggest using a cracked section stiffness equal to 1/3\textsuperscript{rd} uncracked section stiffness. See ACI 318R13.5.1.2
- The method is extensively used and remains essentially unchanged since 1971.
• Flat plate for PCA and U of I tests designed for 70 psf LL and 86 psf DL. Grade 40 steel: 3000 psi concrete.

• Both slabs failed by punching at an interior column. Strains in the top steel at the column face $\geq 7$ times the yield strain at punching. Failure load of 369 psf and was only 85% of the ACI $4\sqrt{f'c}$ value.

• Computed yield line strength was 350psf. Based on shape of the load-slab midspan deflection curves and the limited spread of reinforcement yielding across the width of the slab a capacity greater than the 369psf was likely if not for the punching failure.

• Punching was classified as a “secondary” failure due to the extensive yielding of the top reinforcement around the column prior to failure.
PUNCHING SHEAR ISSUES

• How to prevent the “secondary” punching failure and enable large slab deflections before failure? Answer: Shear reinforcement but what type?

• How to evaluate punching strength when there is also moment being transferred from slab to column?

• Under Gene’s leadership PCA set out to make significant contributions to addressing both those issues.
SHEAR REINFORCEMENT STUDIES
Shearheads

1930 Wheeler Patent Shearhead

PCA TEST SPECIMENS

8 Loads

7'-0

53/4"  6'-0

28"

8" or 10" Column

(b) I - SECTION

1966 PCA TEST SHEARHEADS
SHEAR REINFORCEMENT STUDIES
10 Specimens with Shearheads Tested

Shearhead increases shear capacity in the same way as a larger column. For warning of failure shearhead should yield before punching. Then critical section for shear does not extend to end of shearhead.
SHEAR REINFORCEMENT STUDIES
Shearhead – Determination of Required Capacity

Shear determined from strain gage readings

\[ M_p = \frac{V_u}{8\phi} \left[ h + K \left( L_s - \frac{c}{2} \right) \right] \]

Idealized shear

\[ \frac{V_u}{4} - \frac{V_c}{4} (1-K) \]

Shear

\[ KV_c \]

EI of shearhead

EI composite section width \((c + d)\)  \( K \geq 0.15 \)
SHEAR REINFORCEMENT STUDIES
Shearhead – Location of Critical Section for Shear

(a) No Shearhead  (b) Small Shearhead  (c) Large Shearhead
SHEAR REINFORCEMENT STUDIES
Shear and Moment Transfer – Existing ACI Code

Fraction $\gamma_f M_u$ to be transferred by flexure within lines 1.5h either side of column

where

$$\gamma_f = \frac{1}{1 + (2/3)\sqrt{b_1/b_2}}$$

and $b_1 = c_1 + d$

For RC slabs and exterior columns $\gamma_f$ can be increased to 1.0 provided $V_u$ does not exceed $0.75 \phi Vc$ for edge columns and $0.50 \phi Vc$ for corner columns. At interior columns $\gamma_f$ can be increased by 25% but to not greater than 1.0 provided $V_u \leq 0.40 \phi Vc$ and $\varepsilon_t \geq 0.010$.

Determining Fraction of M Transferred by Reinforcement
UNDERSTANDING SHEAR AND MOMENTTRANSFER
BEAM ANALOGY

(a) Frame dimensions

Torsional, Flexural and Overall Response

Model
UNDERSTANDING SHEAR AND MOMENT TRANSFER
BEAM ANALOGY - EXTERIOR COLUMN STRENGTH

(a) MOMENTS AND FORCES

(c) CRITICAL SECTION FOR MOMENT-TORSION

(d) CRITICAL SECTION FOR SHEAR-TORSION
SHEAR REINFORCEMENT STUDIES
Exterior Column Connections - Dimensions

VARIABLES:
Sheahead - Shape, Length, Area
Column Size -3 with 12 x 8 in
-11 with 12 x 12 in
Grade 60 Steel
Sanded Lightweight Concrete 3,000 psi
SHEAR REINFORCEMENT STUDIES
Exterior Column Connections – Test Setup
SHEAR REINFORCEMENT STUDIES
Exterior Column Connections – Loading Response

D = 12 x 8; C = 12 x 12 column
N = No Shearhead
C = Channel Sections
H = I Sections

Under-reinforced CH4; CC5; DC2
Projections: 17.5; 21; 21 in

Over-reinforced CH1,2,3
Projections: 8.5, 11.5, 14.5 in

Over-reinforced CT1, CC1, CC2
Projections: 14.5, 21, 21 in
SHEAR REINFORCEMENT STUDIES
Exterior Column Connections – Critical Sections

For shear stress $v_1$ due to Shear

For shear stress $v_2$ due to Moment Transfer

For Design $v_1 + v_2 = v_u \leq \phi v_n$
SHEAR REINFORCEMENT STUDIES
Exterior Column Connections – Shearhead Strength Requirements

Current Code Requirement For Plastic Moment Strength

\[ M_p \geq M_s = \frac{V_{u_1}}{2} \left[ h_v + a_v \left( l_v - \frac{c_1}{2} \right) \right] \text{ if } V_{u_1} = 2V'_c \]
WHAT STILL NEEDS TO BE ADDRESSED?
Slabs Without Shear Reinforcement – Flexural Strength Limit

- Recognize Relevance of Muttoni’s Critical Shear Crack (CSC) Theory
- Aggregate Interlock Along CSC Is Lost When There Is General Yielding of Reinforcement in the Vicinity of Column
- Per Ghali, Strength for General Yielding is 8m where m is flexural strength per unit width
- Require $\phi_v V_{\text{shear}} \leq \phi_f V_{\text{flex}} = \phi_f 8m$ – Needed for low $\rho$
WHAT STILL NEEDS TO BE ADDRESSED?
Slabs Without Shear Reinforcement – Depth Effect

\[ k_v = \frac{3}{\sqrt{d}} \]
WHAT STILL NEEDS TO BE ADDRESSED?
Slabs With Shear Reinforcement

• Develop Conceptually Consistent Punching Shear, Moment Transfer, and Ductility Provisions For Connections With Shear Reinforcement

Cover Stirrup Reinforcement,
Stud Rail Reinforcement,
Fortress Reinforcement,
Shearhead Reinforcement.
Thank You