FLAT PLATE AND FLAT SLAB CONSTRUCTION

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A Tribute to the Lasting Contributions and Legacy of Our Friend And Colleague Dr. W Gene Corley

ACI Convention, Phoenix, AZ , Sunday October 20, 2013

DISCUSSION TOPICS
Gene's Early Professional Years

- **Equivalent Frame Analysis**
  - SRS 218 Univ. of Illinois – Ph.D. Thesis –June 1961
  - ACI Journal – Nov. 1970 – w. James Jirsa
  - Concrete International – Dec. 1983- w. Dan Vanderbilt
- **Testing and Analysis of Flat Plate and Flat Slab System Shear Strengths**
  - ACI Journal Sept.1971- NY World's Fair Waffle Slab Tests- with DM
  - ACI Journal – Oct. 1968- Shearhead Reinforcement – w. NMH
  - ACI SP-30 –1971–Moment and Shear Transfer to Columns–w. NMH
  - ACI SP-42- 1974- Moment Transfer with Shearheads – w. NMH
  - WCEE 1973–Ductile Flat-Plate Structures to Resist EQ–w.JEC & PHK
  - ACI SP-59- 1979– Shear in Two-Way Slabs – ACI Approach

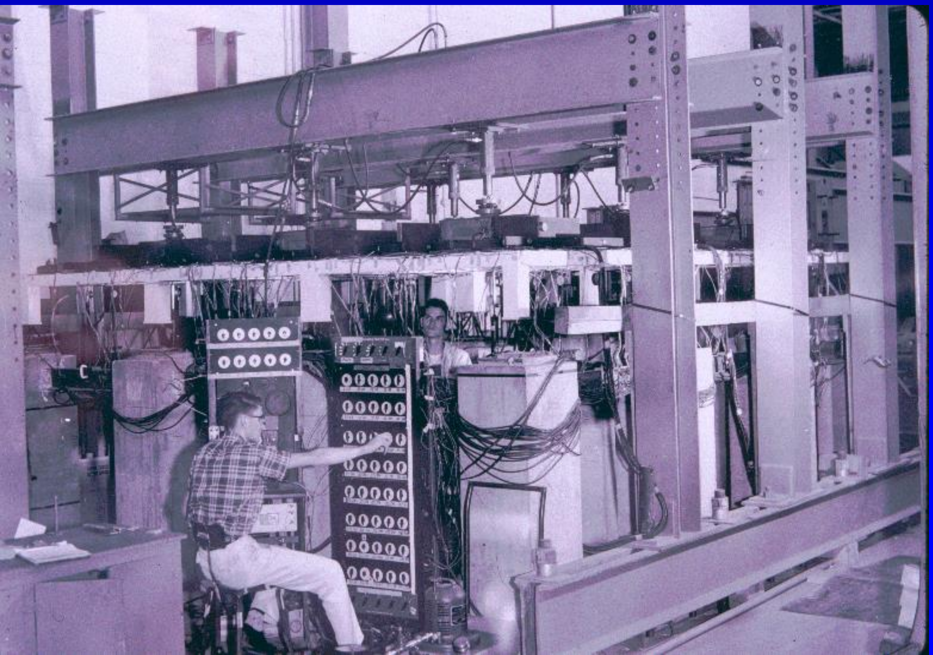
EARLY PROFESSIONAL YEARS

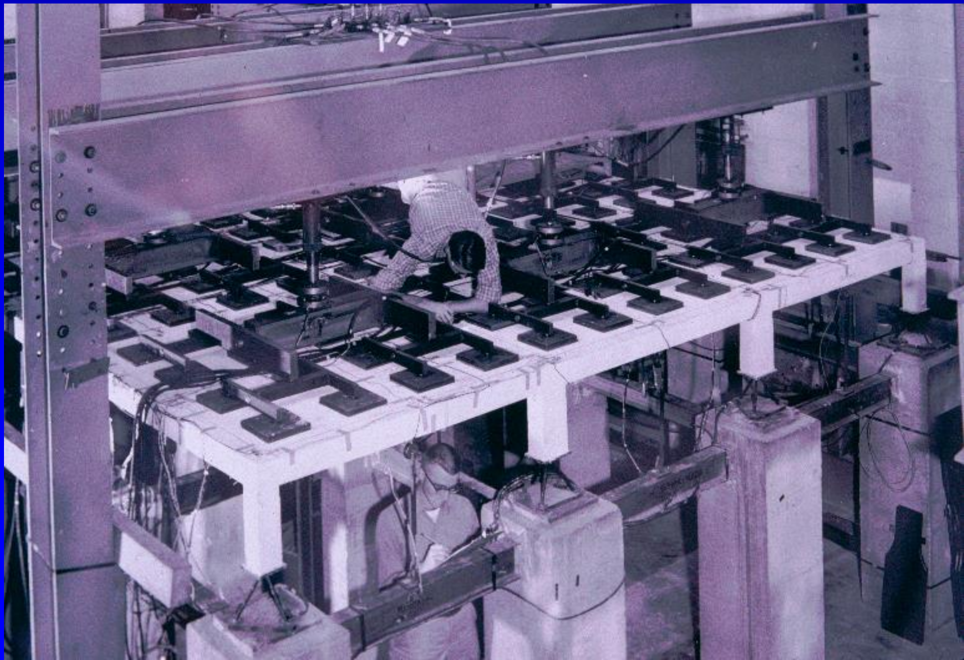
National Science Foundation Fellow 1958-1961

Ph.D Structural Engineering, University of Illinois, 1961

US Army Corps of Engineers, 1961-1964

Structural Research Manager, PCA R & D Division 1964 - 1972





EQUIVALENT FRAME ANALYSIS FOR FLAT PLATES AND FLAT SLABS

- First introduced in ACI 318-71 and based on U of I Ph. D theses by Corley (1961) and Jirsa (1963).
- Early ACI Codes permitted an “empirical method” of design only; Slab properties were restricted to those load tested in the early 1900s.

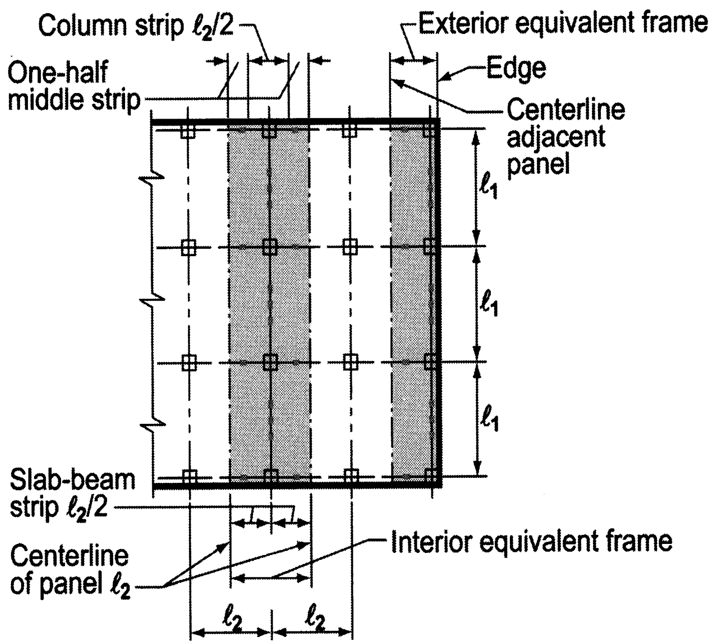
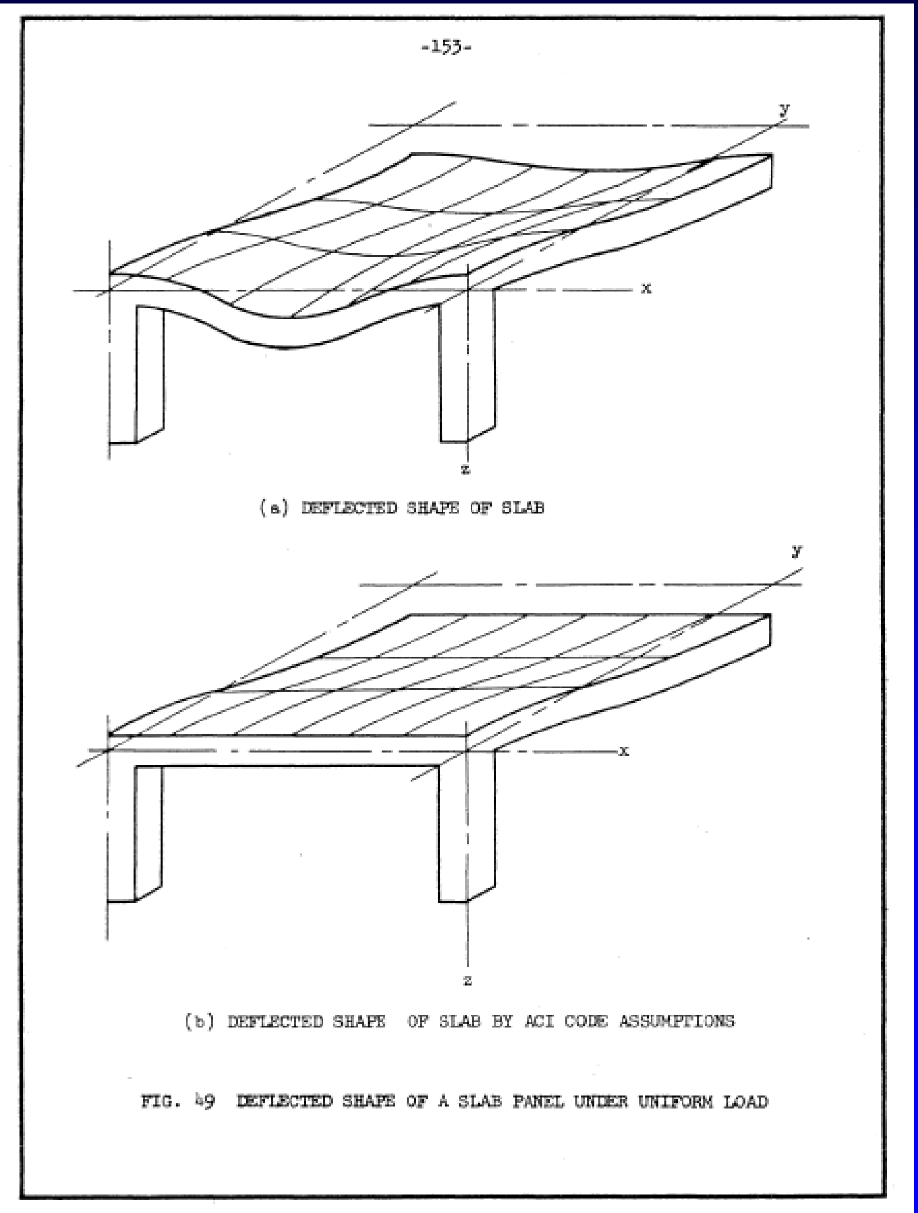


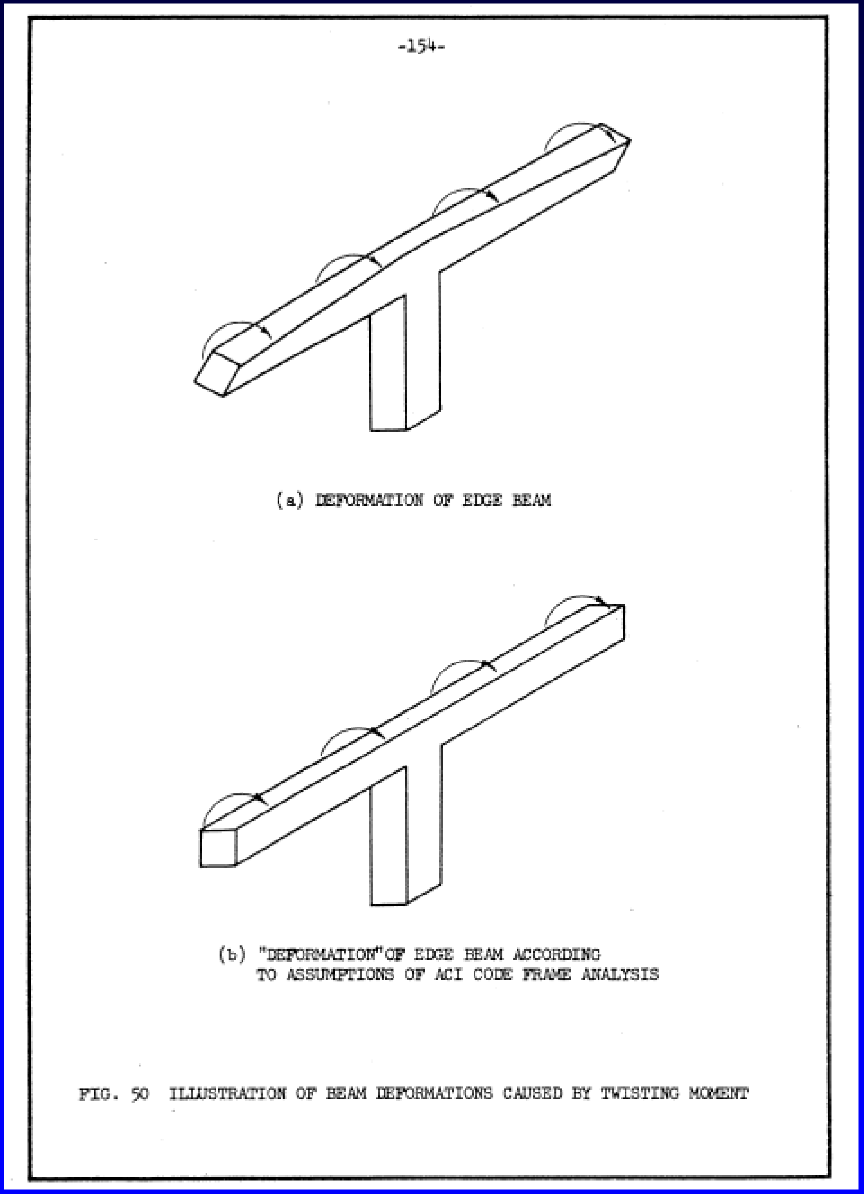
Fig. R13.7.2—Definitions of equivalent frame.

To overcome that restriction the 1941 ACI code introduced an “elastic design method” giving similar results to the “empirical method” for the loaded tested floors but useable for slabs with dissimilar properties

The 71 Code frame similar to the 41 Code frame except for stiffness definitions for frame members

1971 AND 1941 DEFORMATION ASSUMPTIONS





1971 SLAB STIFFNESS ASSUMPTIONS

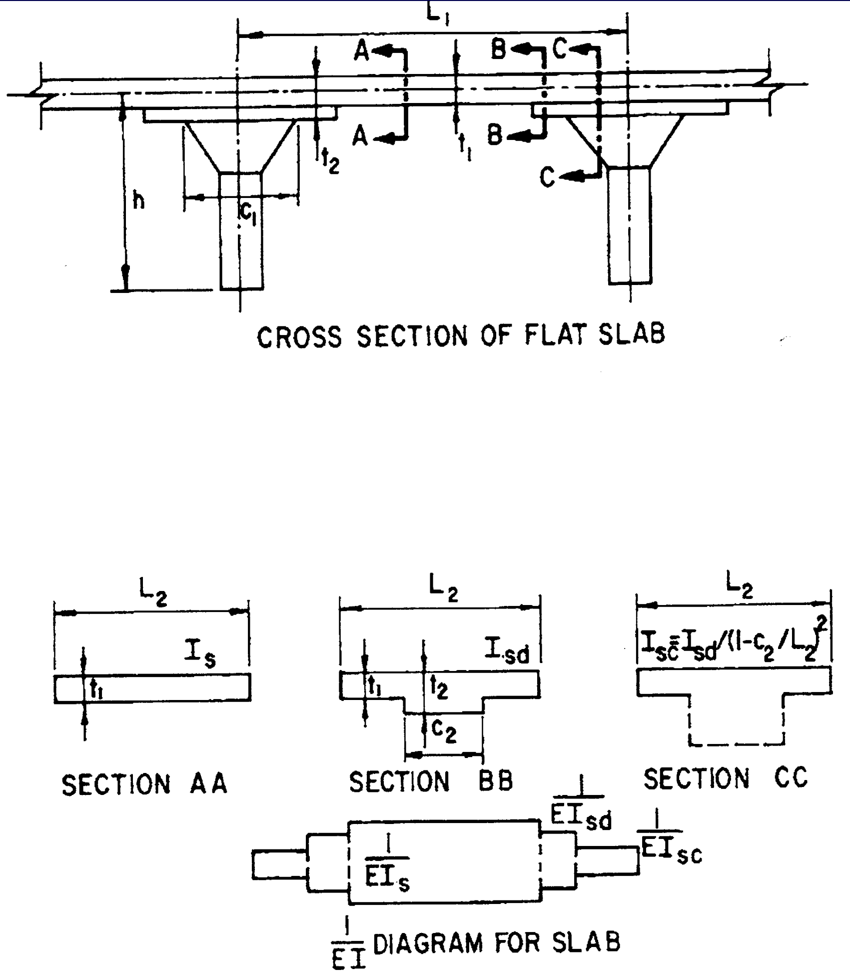


Fig 1 — Cross sections for calculating stiffnesses of equivalent frame

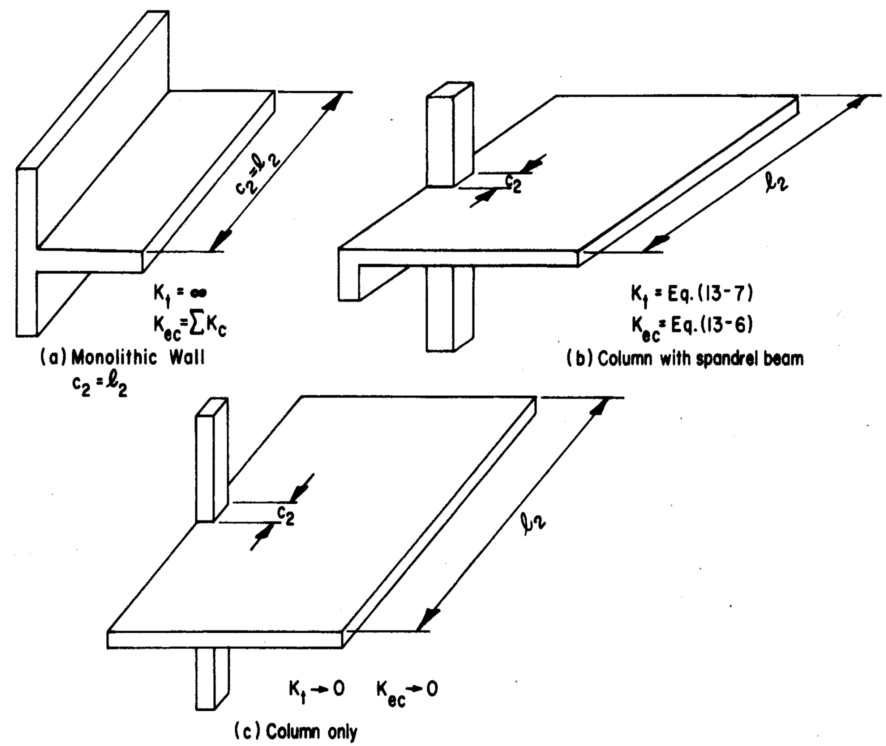


Fig. 13-7 – Simplified physical models illustrating the intent of Section 13.7.4

TORSIONAL MEMBER STIFFNESS ASSUMPTIONS

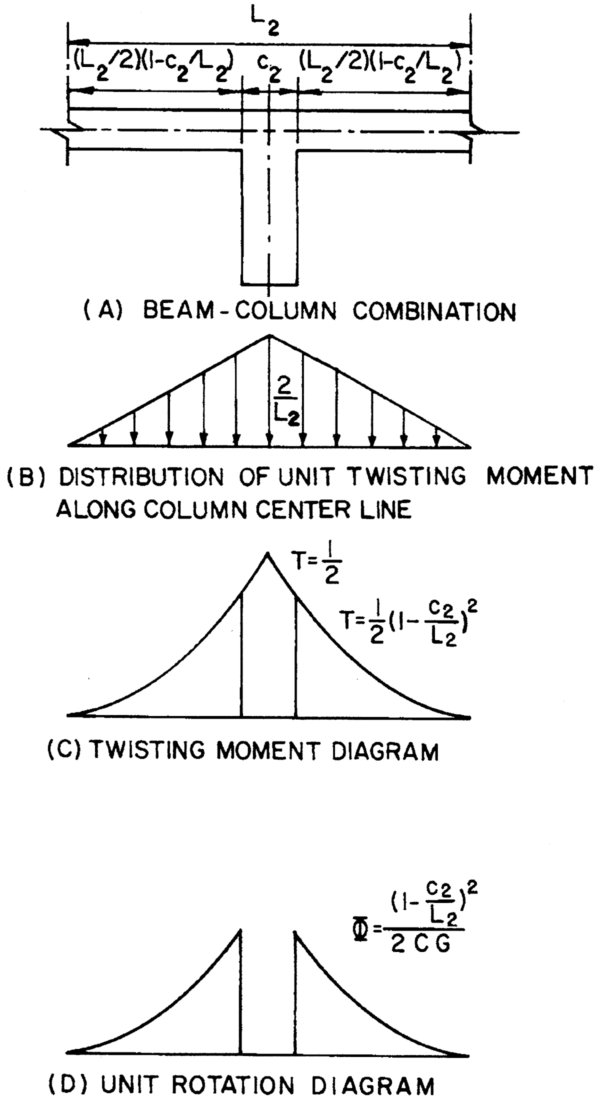


Fig. 3 — Rotation of beam under applied unit twisting moment

$$K_t = \sum \frac{9E_{cs}C}{\ell_2\left(1-\frac{c_2}{\ell_2}\right)^3}$$

Where C = Torsional Constant

$$C = \sum\left(1-0.63\frac{x}{y}\right)\frac{x^3y}{3}$$

In Corley's thesis the unit twisting moment, Fig 3(B), was uniform over the length L_2. Jirsa modified Corley's distribution to that shown based on pattern loading considerations

EQUIVALENT COLUMN STIFFNESS

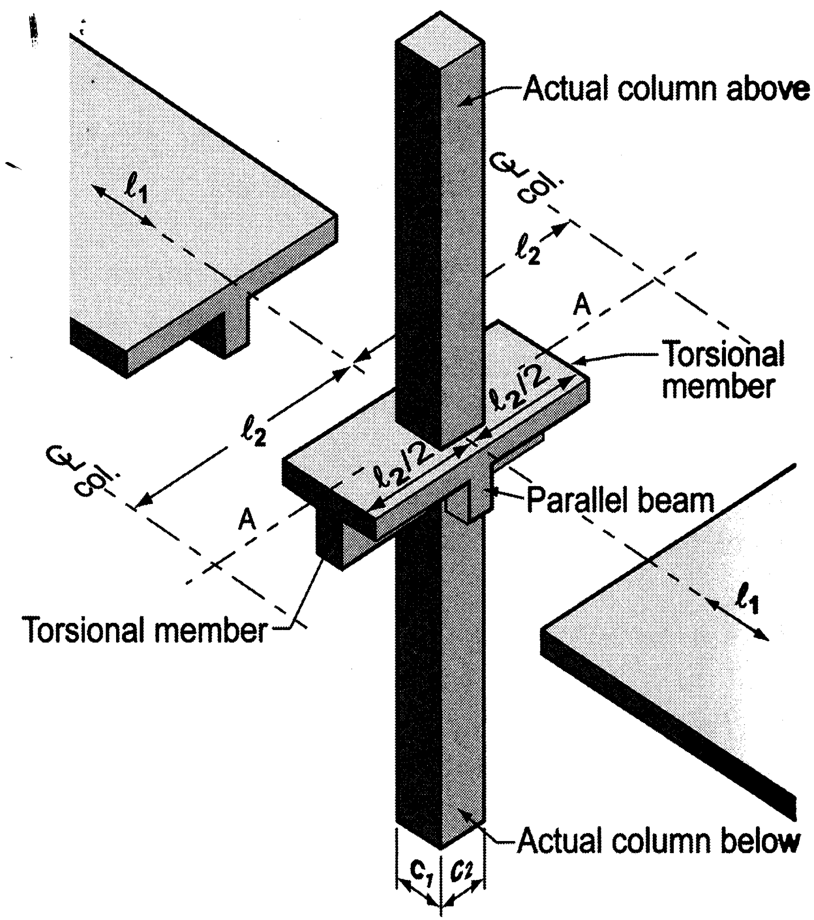


Fig. R13.7.4—Equivalent column (column plus torsional members).

For moment distribution procedures the equivalent column stiffness K_{ec} was defined by:

$$1/K_{ec} = 1/K_c + 1/K_t$$

K_c = column flexural stiffness

K_t = torsional stiffness of members framing into column

LAYOUT OF 9 PANEL U of I ¼ SCALE MODEL

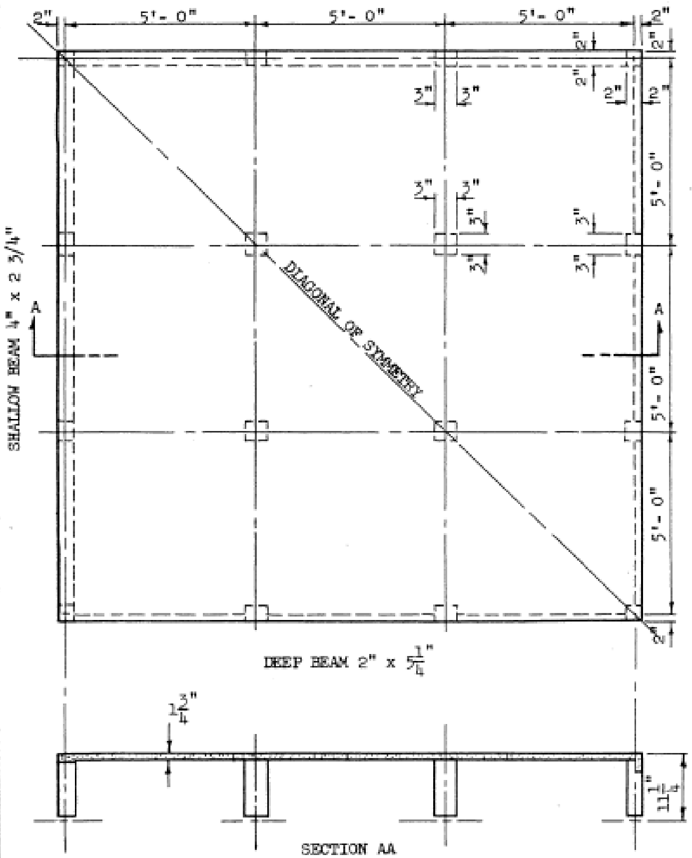
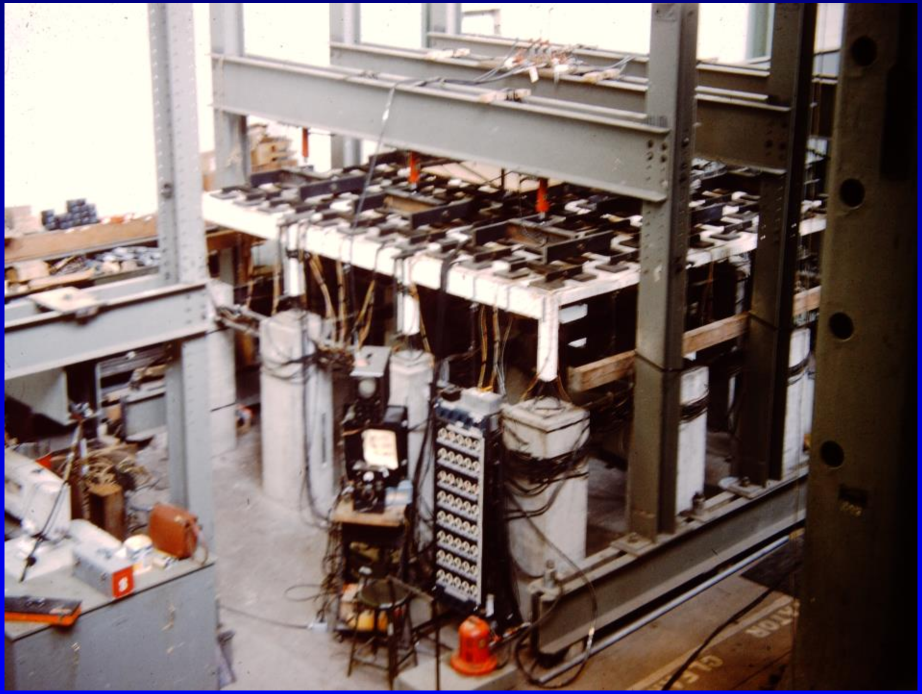


FIG. 58 LAYOUT OF NINE-PANEL REINFORCED CONCRETE FLAT PLATE

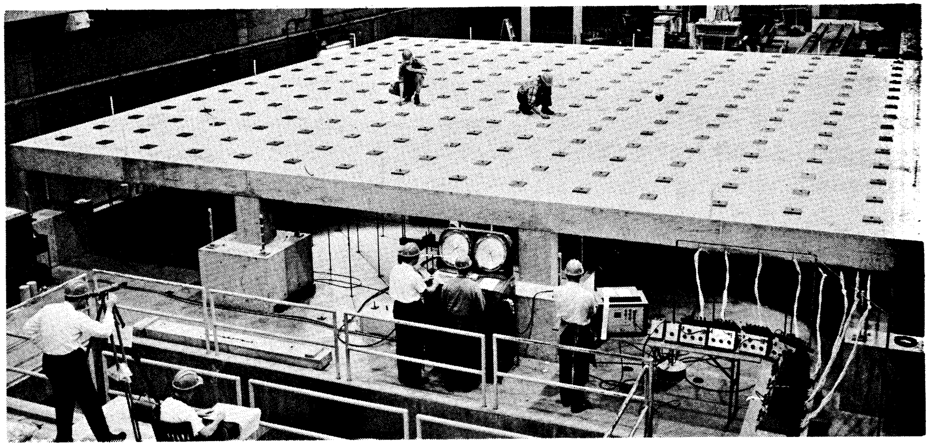


COMPARISON OF MEASURED AND COMPUTED SERVICE LOAD MOMENTS

TABLE 14. COMPARISON OF MEASURED MOMENTS WITH MOMENTS COMPUTED FOR 9-PANEL REINFORCED CONCRETE FLAT PLATE MODEL

Moment Coefficients of WL

Section	R	S	T	Sum	U	V	U'	Sum	T'	S'	R'	Sum
					Entire Structure							
Moments Measured from Strains+	0.029	0.052	0.069	0.101	0.063	0.038	0.062	0.101	0.064	0.048	0.035	0.098
Moments Measured from Reactions+	0.030	0.053	0.078	0.107	0.071	0.037	0.070	0.108	0.078	0.052	0.041	0.112
Difference Solutions (UI94)*	0.045	0.043	0.062	0.096	0.061	0.039	0.061	0.100	0.062	0.043	0.046	0.097
Proposed Frame Analysis	0.024	0.051	0.090	0.108	0.068	0.038	0.068	0.0106	0.092	0.052	0.031	0.114
ACI Code Frame Analysis**	0.058	0.036	0.066	0.098	0.061	0.034	0.061	0.095	0.066	0.036	0.058	0.098
ACI Code Empirical Moments	0.049	0.031	0.071	0.091	0.063	0.041	0.063	0.104	0.071	0.031	0.052	0.093



COMPARISON WITH PCA ¾ SCALE FLAT PLATE RESULTS

TABLE I — COMPARISON OF MEASURED WITH COMPUTED MOMENTS (FLAT PLATE STRUCTURES)

Section	M^- Shallow beam edge	M^+	M^-	M^+	M^-	M^+	M^- Deep beam edge		
University of Illinois structure, F1 (1/4 scale), $w_m/w_p = 2.5$	Moment coefficients, 1000 M/WL_1								
Calculated uniform load design moment	47	44	72	66	34	67	73	44	46
Calculated maximum design moment	54	50	75	73	45	73	76	50	52
Ratio maximum to uniform load moment	1.15	1.14	1.04	1.11	1.32	1.09	1.04	1.13	1.13
Measured uniform load moment	27	49	65	64	40	58	58	47	34
Measured maximum moment	21	52	68	67	44	63	63	48	26
Ratio maximum to uniform load moment	—	1.06	1.04	1.05	1.10	1.09	1.09	1.02	—
Ratio design to measured uniform load moment	1.74	0.90	1.11	1.03	0.85	1.16	1.26	0.94	1.35
PCA structure (3/4 scale)									
Calculated uniform load design moment	44	48	67	62	38	62	68	49	43
Measured uniform load moment	37	47	68	68	31	73	73	42	31
Ratio design to measured uniform load moment	1.19	1.02	0.99	0.91	1.22	0.85	0.85	1.16	1.39

EQUIVALENT FRAME PROCEDURE LIMITATIONS

Discussed in " Frame Analysis of Concrete Buildings" Vanderbilt and Corley, *Concrete International,* Dec. 1983

- **Method assumes analysis by moment distribution methods.**
- **Method calibrated for gravity loadings only by comparison to U of I ¼ scale and PCA ¾ scale tests**
- **Method based on stiffness of uncracked sections**
- **Method not calibrated for lateral loadings but theoretical studies suggest using a cracked section stiffness equal to 1/3rd uncracked section stiffness. See ACI 318R13.5.1.2**
- **The method is extensively used and remains essentially unchanged since 1971.**

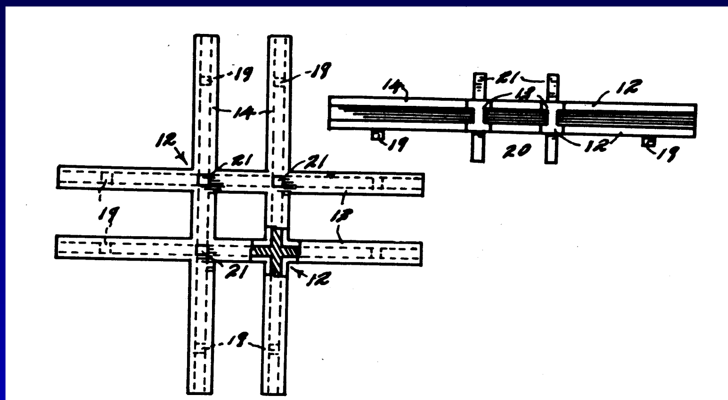
PUNCHING SHEAR

- Flat plate for PCA and U of I tests designed for 70 psf LL and 86 psf DL. Grade 40 steel: 3000 psi concrete.
- Both slabs failed by punching at an interior column. Strains in the top steel at the column face ≥ 7 times the yield strain at punching. Failure load of 369 psf and was only 85% of the ACI $4\sqrt{f'_c}$ value.
- Computed yield line strength was 350psf. Based on shape of the load-slab midspan deflection curves and the limited spread of reinforcement yielding across the width of the slab a capacity greater than the 369psf was likely if not for the punching failure.
- Punching was classified as a "secondary" failure due to the extensive yielding of the top reinforcement around the column prior to failure.

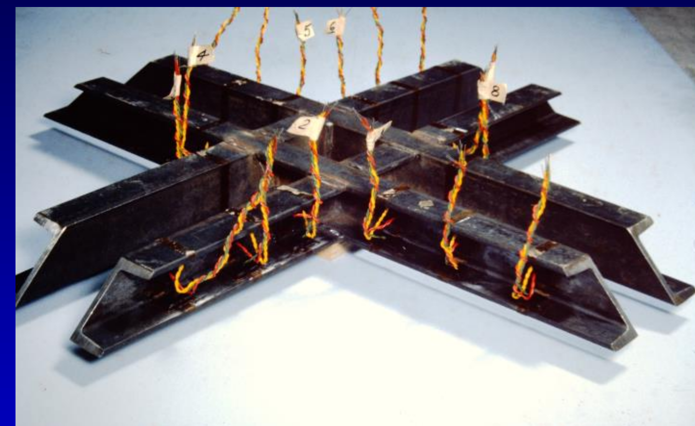
PUNCHING SHEAR ISSUES

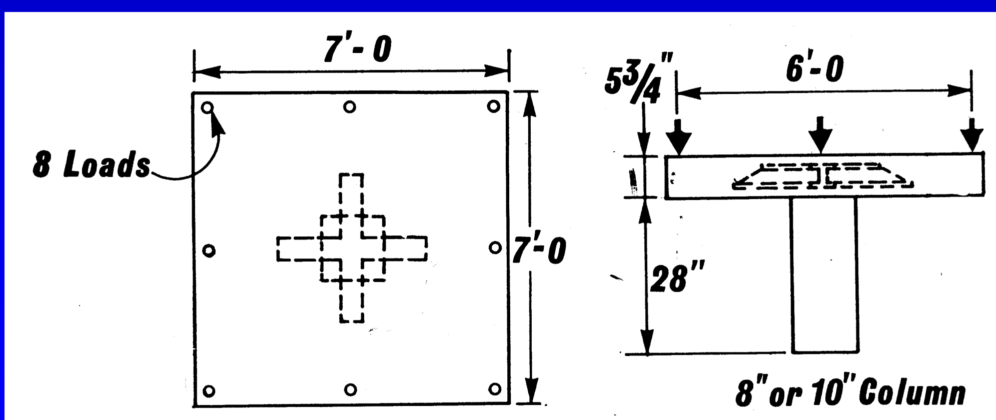
- **How to prevent the “secondary” punching failure and enable large slab deflections before failure? Answer: Shear reinforcement but what type?**
- **How to evaluate punching strength when there is also moment being transferred from slab to column?**
- **Under Gene’s leadership PCA set out to make significant contributions to addressing both those issues.**

SHEAR REINFORCEMENT STUDIES
Shearheads

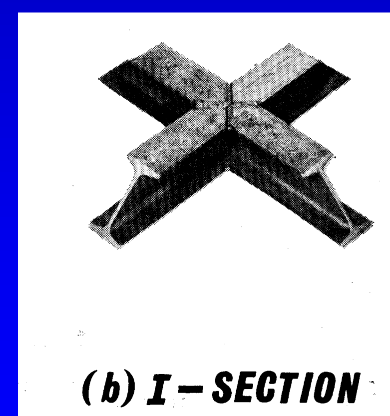


1930 Wheeler Patent Shearhead



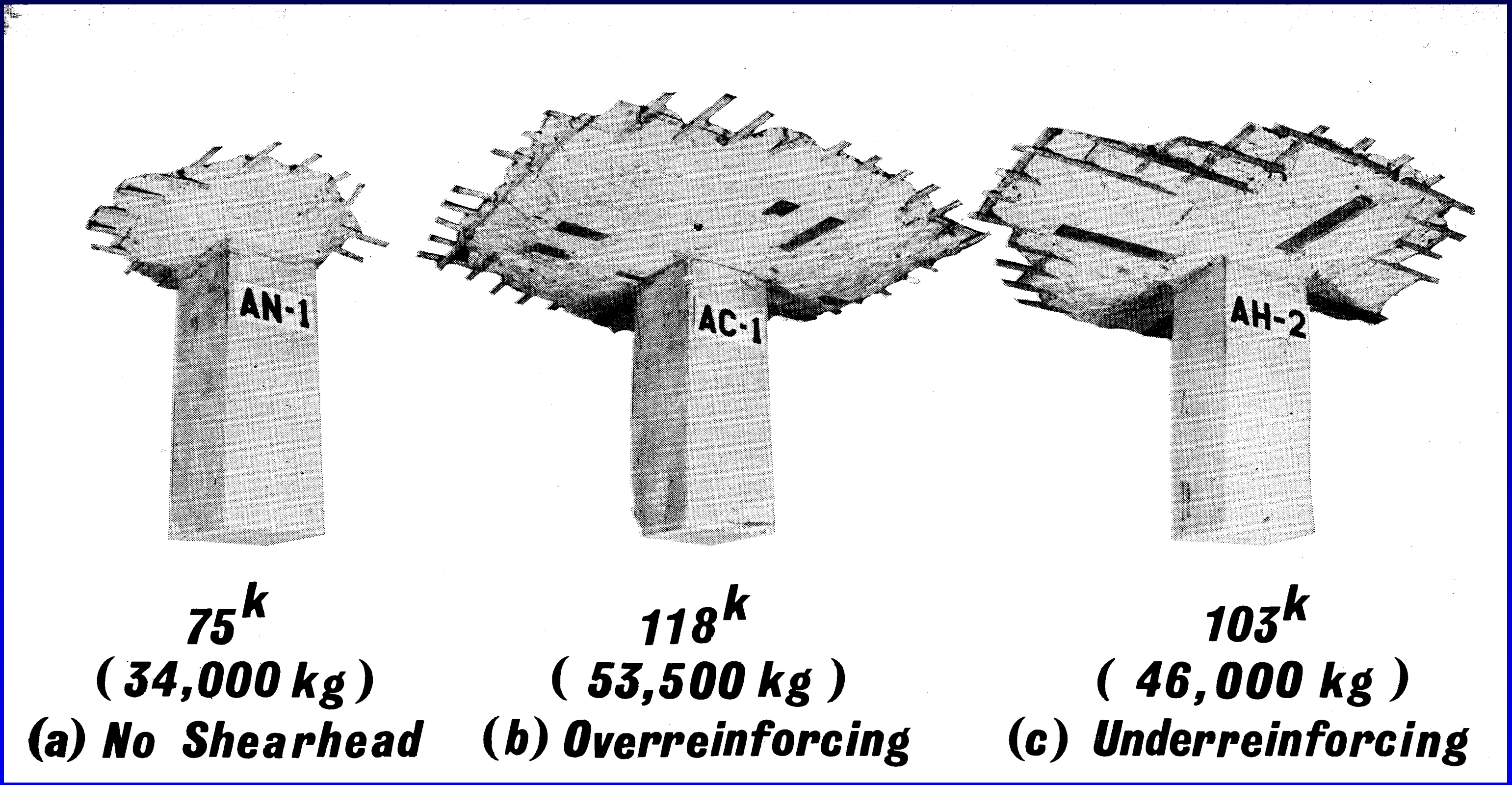


PCA TEST SPECIMENS



1966 PCA TEST SHEARHEADS

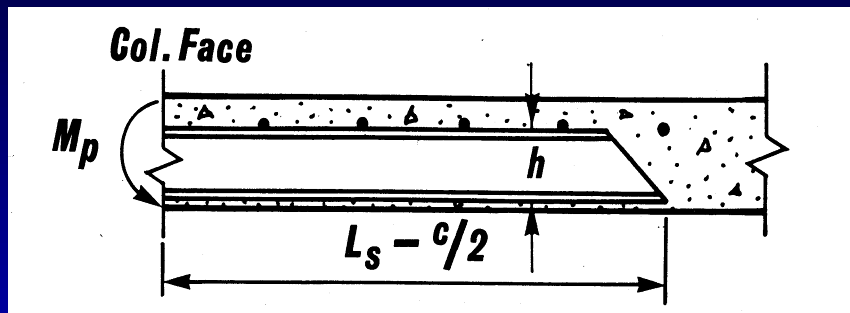
SHEAR REINFORCEMENT STUDIES
10 Specimens with Shearheads Tested



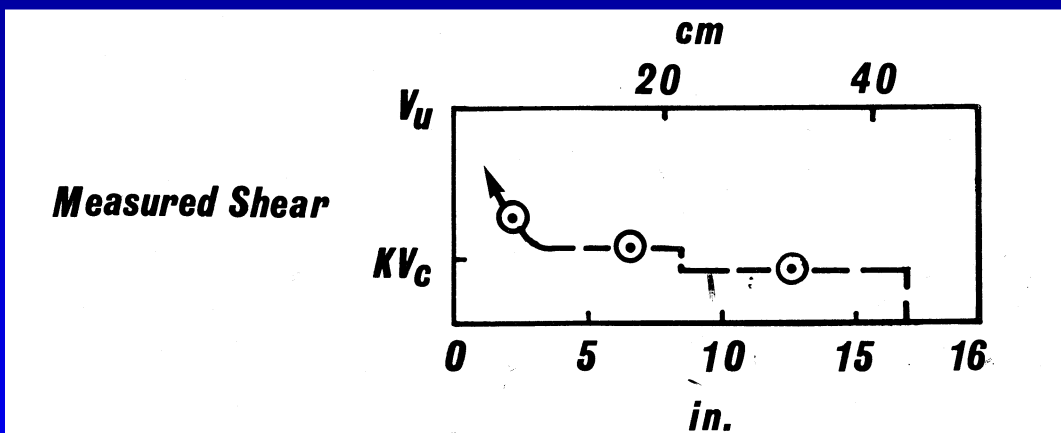
Ls = 0 Ls = 18 in Ls = 20 in

Shearhead increases shear capacity in the same way as a larger column.
For warning of failure shearhead should yield before punching.
Then critical section for shear does not extend to end of shearhead

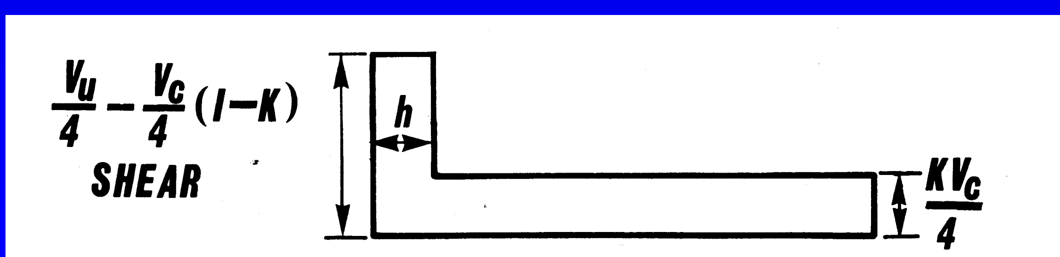
SHEAR REINFORCEMENT STUDIES
Shearhead – Determination of Required Capacity



$$M_p = \frac{V_u}{8\phi}\left[h + K\left(L_s - \frac{c}{2}\right)\right]$$



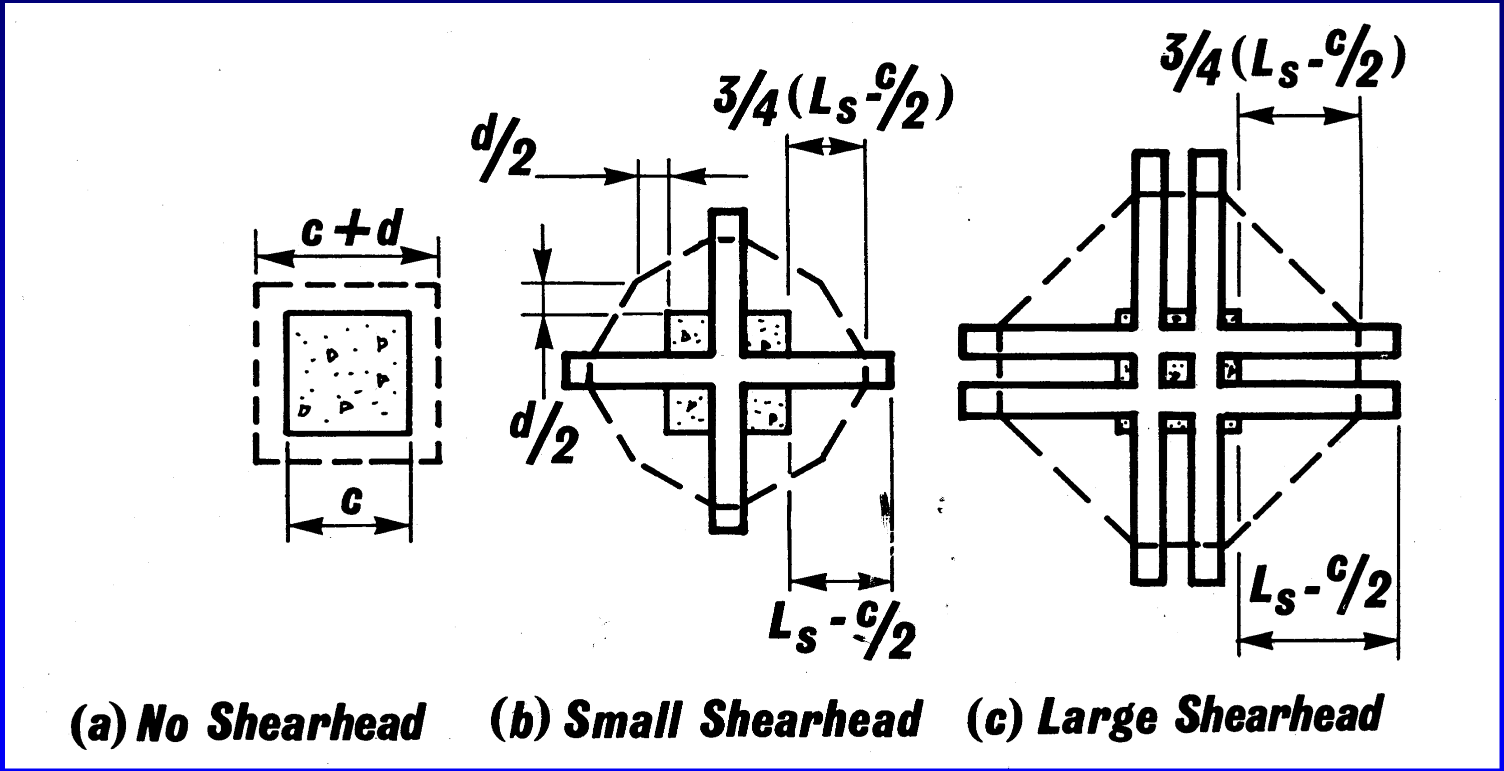
SHEAR DETERMINED FROM STRAIN GAGE READINGS



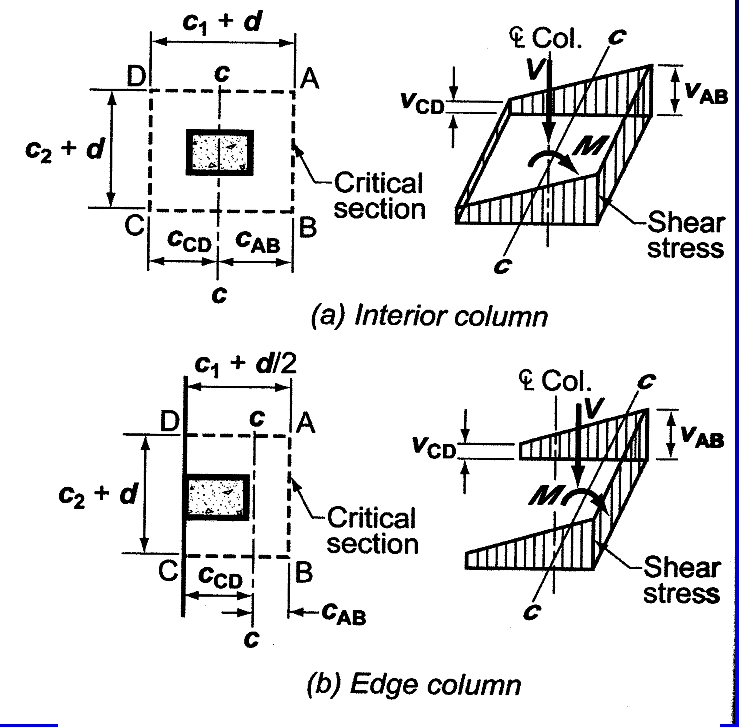
IDEALIZED SHEAR

K= <u>EI OF SHEARHEAD</u>
EI COMPOSITE SECTION
WIDTH (c + d)   K ≥ 0.15

SHEAR REINFORCEMENT STUDIES
Shearhead – Location of Critical Section for Shear



SHEAR REINFORCEMENT STUDIES
Shear and Moment Transfer – Existing ACI Code



$$v_{u(AB)} = \frac{V_u}{A_c} + \frac{\gamma_v M_u c_{AB}}{J_c}$$

$$\gamma_v = (1 - \gamma_f)$$

Additional "v" Caused by M

Fraction $\gamma_f M_u$ to be transferred by flexure within lines 1.5h either side of column

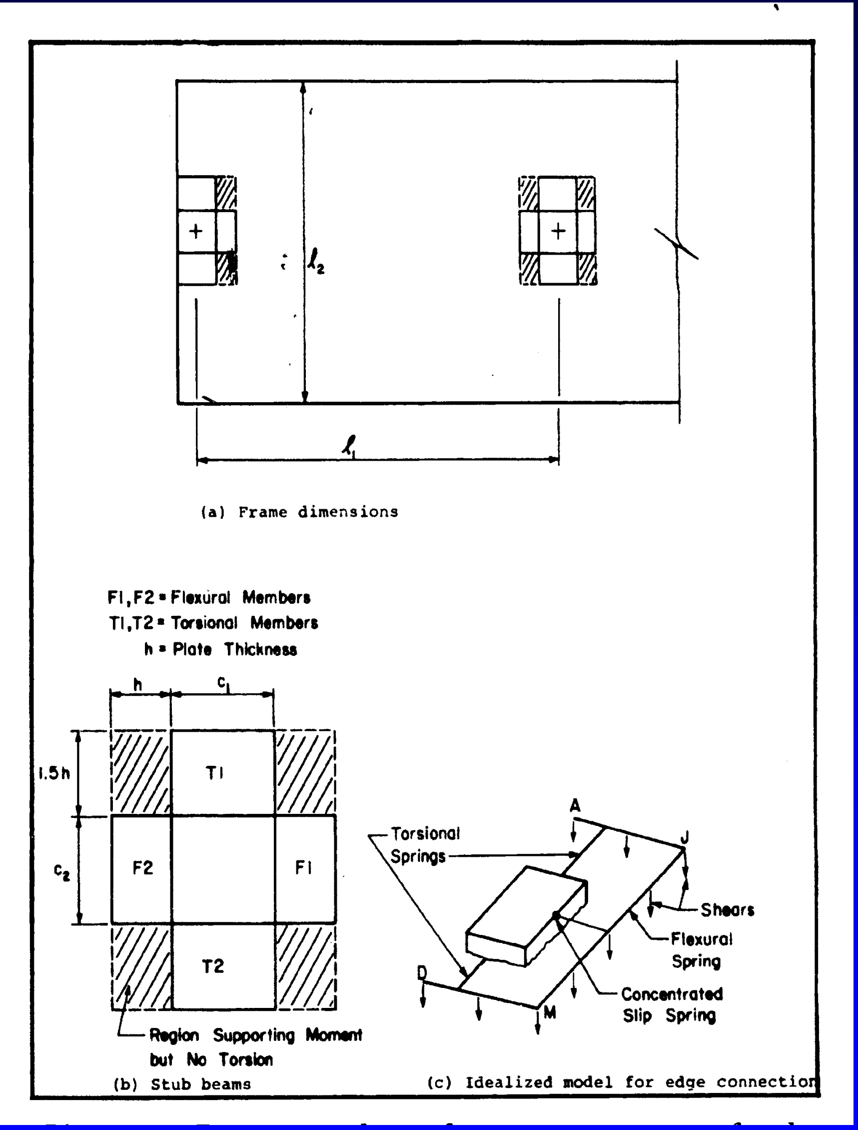
where $$\gamma_f = \frac{1}{1 + (2/3)\sqrt{b_1/b_2}}$$

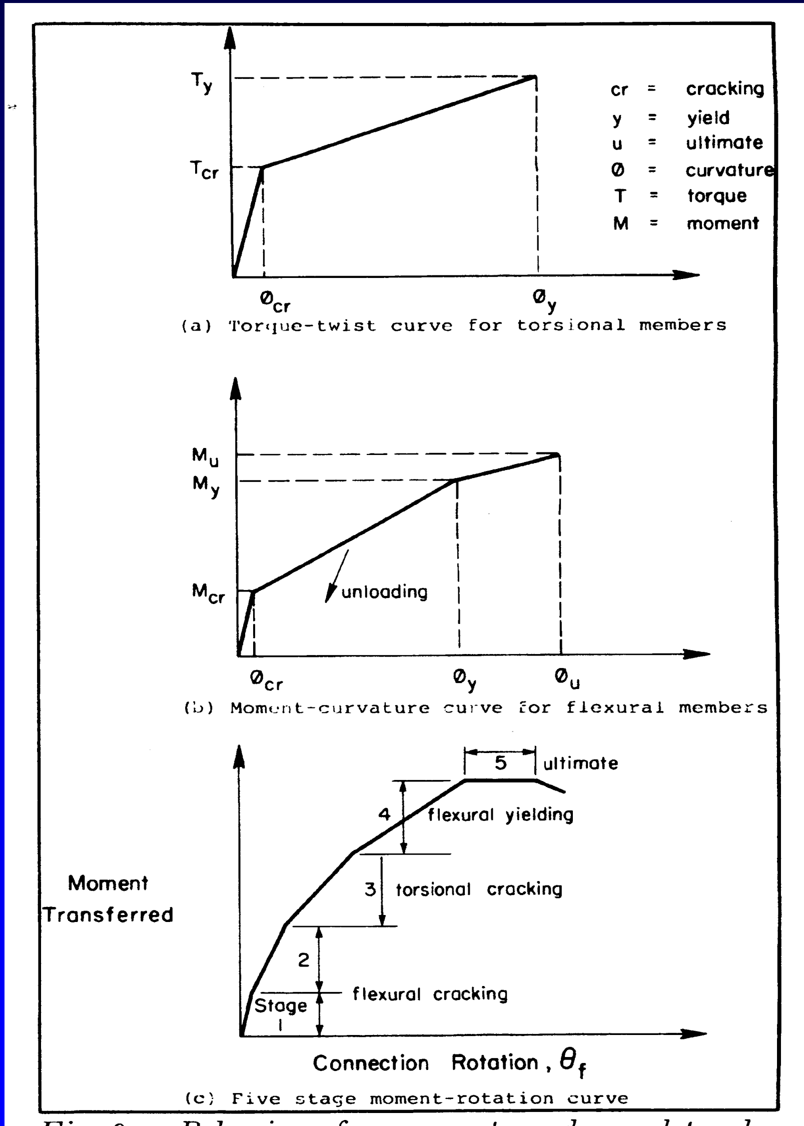
and $b_1 = c_1 + d$

For RC slabs and exterior columns γ_f can be increased to 1.0 provided V_u does not exceed $0.75\phi Vc$ for edge columns and 0.50 ϕVc for corner columns. At interior columns γ_f can be increased by 25% but to not greater than 1.0 provided $V_u \leq 0.40\ \phi Vc$ and $\varepsilon_t \geq 0.010$.

Determining Fraction of M Transferred by Reinforcement

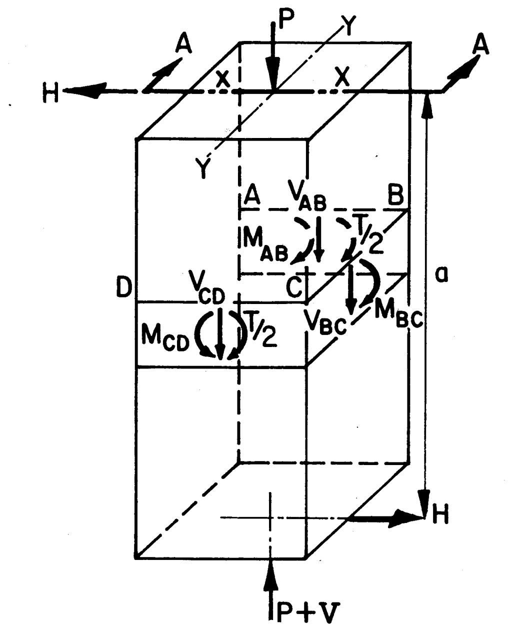
UNDERSTANDING SHEAR AND MOMENT TRANSFER
BEAM ANALOGY



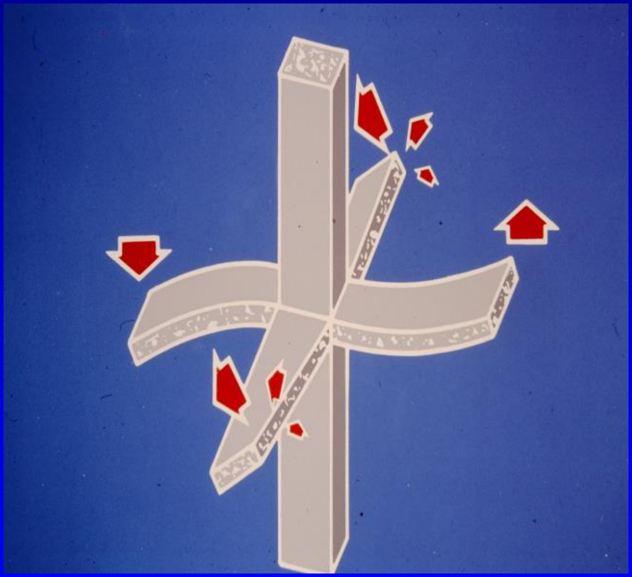


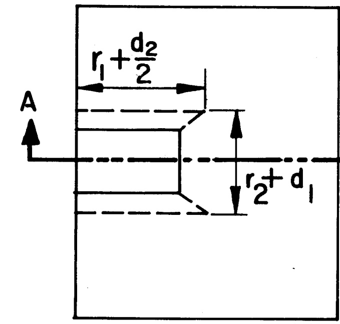
Model **Torsional, Flexural and Overall Response**

UNDERSTANDING SHEAR AND MOMENT TRANSFER BEAM ANALOGY - EXTERIOR COLUMN STRENGTH

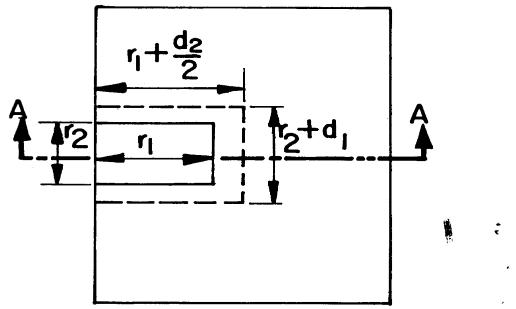


(a) MOMENTS AND FORCES



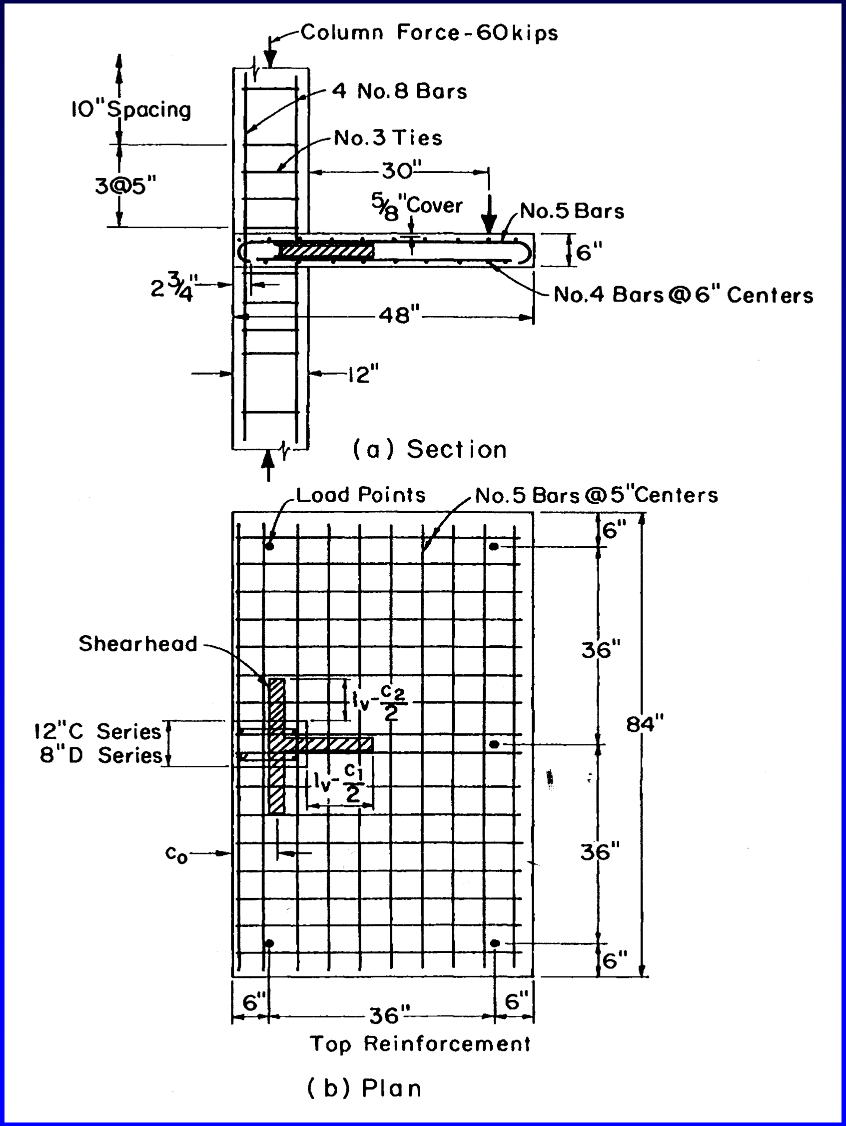


(c) CRITICAL SECTION FOR MOMENT–TORSION



(d) CRITICAL SECTION FOR SHEAR–TORSION

SHEAR REINFORCEMENT STUDIES
Exterior Column Connections -Dimensions



VARIABLES:

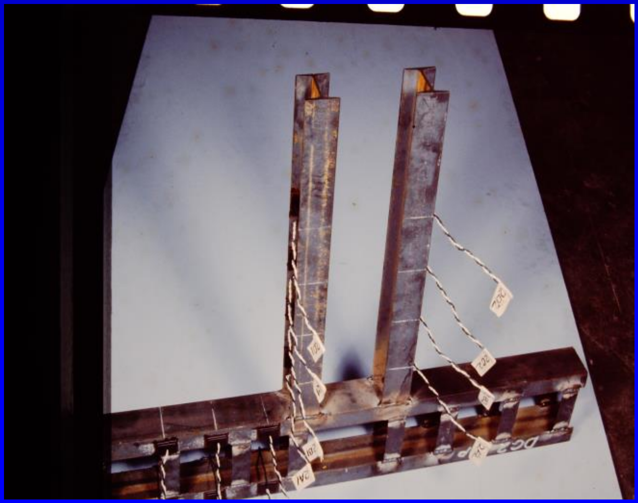
Sheahead - Shape, Length, Area

Column Size -3 with 12 x 8 in

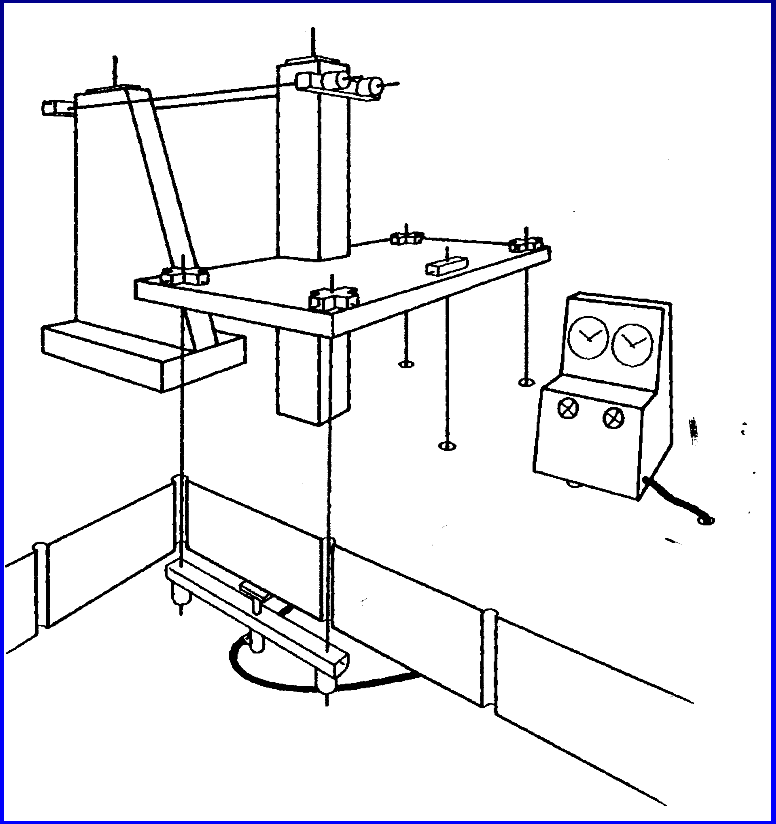
-11 with 12 x 12 in

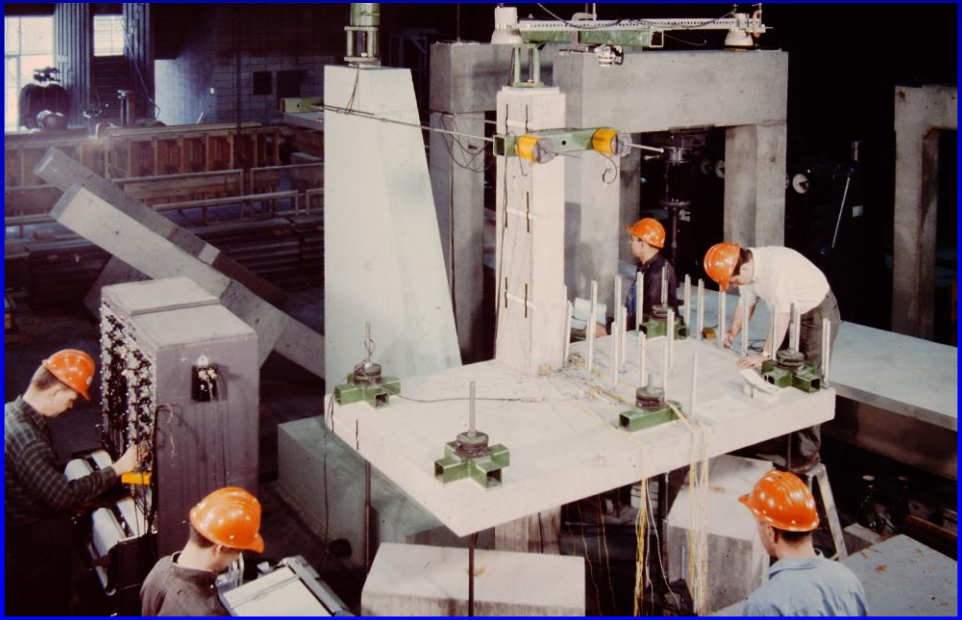
Grade 60 Steel

Sanded Lightweight Concrete 3,000 psi

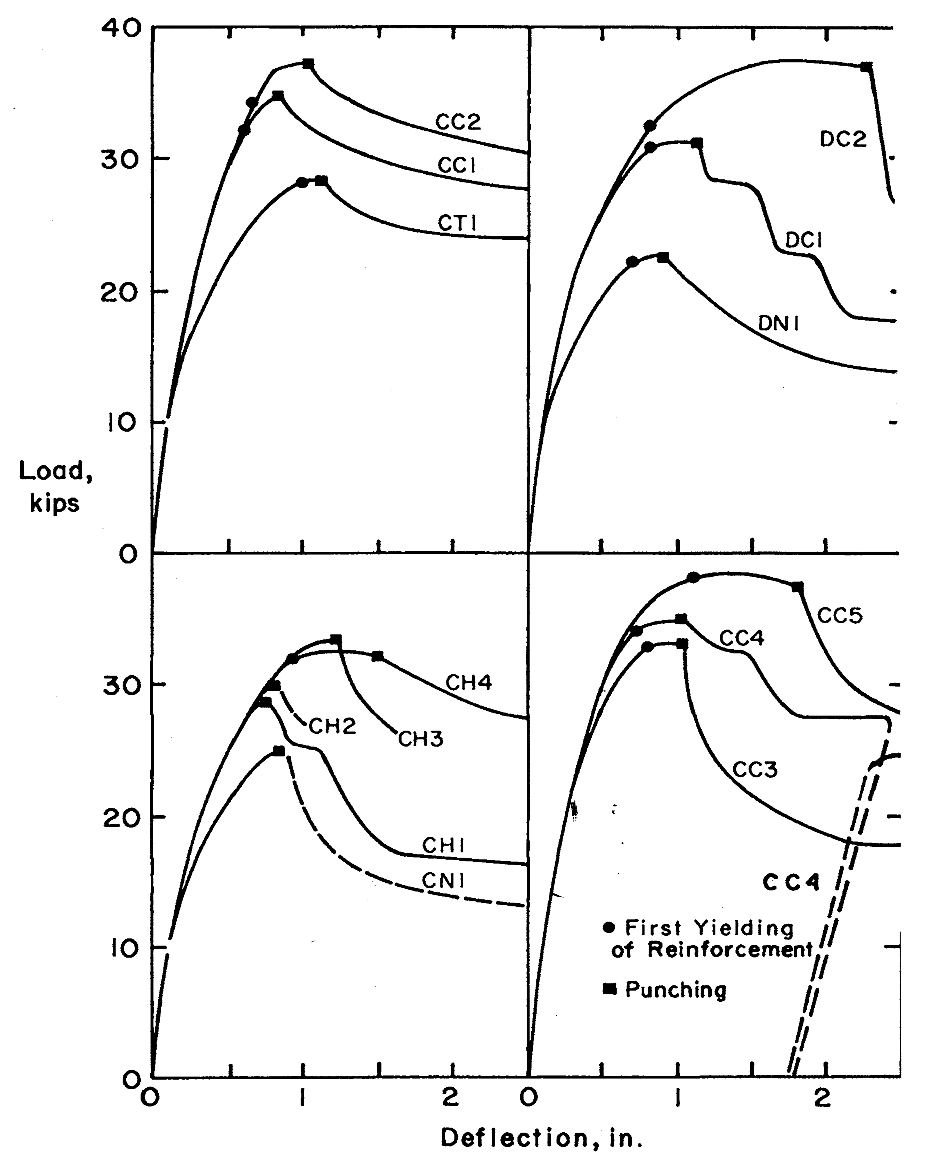


SHEAR REINFORCEMENT STUDIES
Exterior Column Connections – Test Setup





SHEAR REINFORCEMENT STUDIES
Exterior Column Connections – Loading Response



D = 12 x 8; C = 12 x12 column

N = No Shearhead

C = Channel Sections

H = I Sections

<u>Under-reinforced</u> CH4; CC5; DC2

Projections: 17.5; 21; 21 in

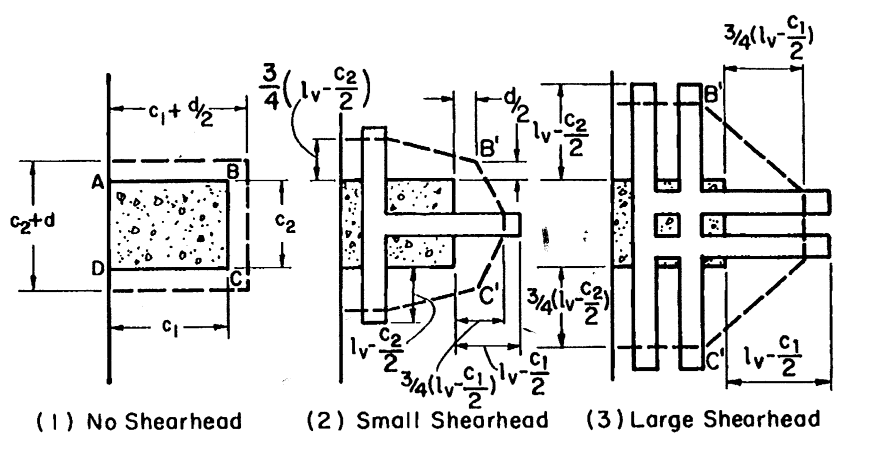
<u>Over-reinforced</u> CH1,2,3

Projections: 8.5, 11.5, 14.5 in

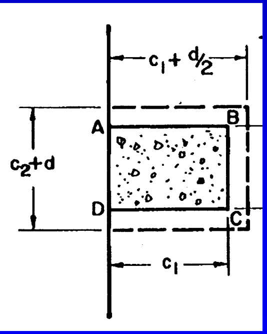
<u>Over-reinforced</u> CT1, CC1, CC2

Projections: 14.5, 21, 21 in

SHEAR REINFORCEMENT STUDIES
Exterior Column Connections – Critical Sections



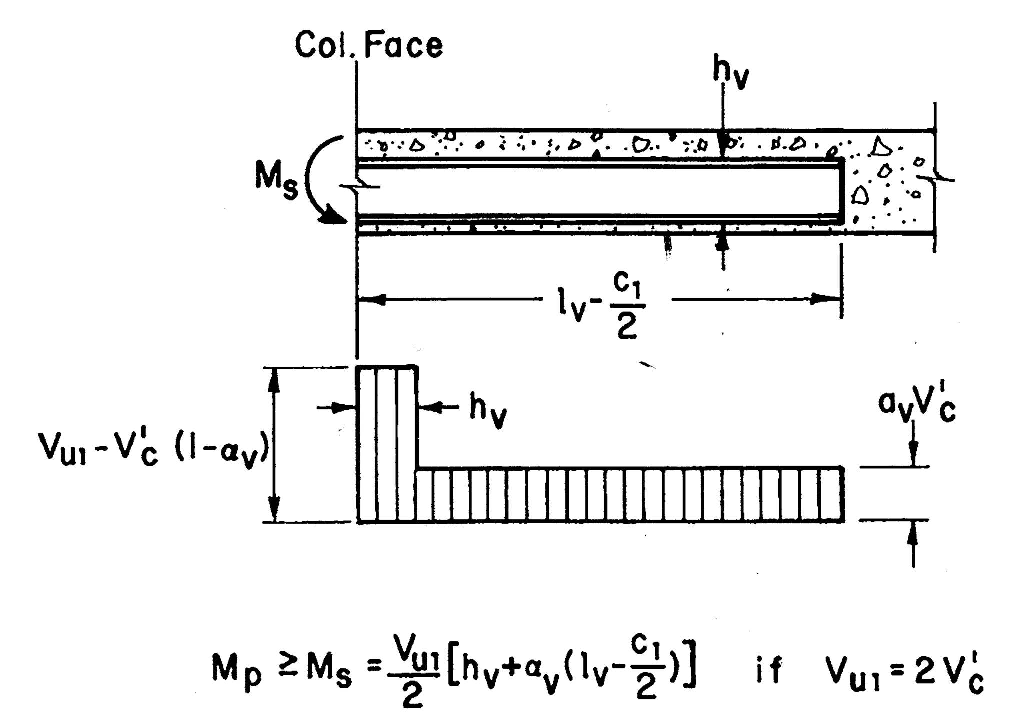
For shear stress v_1 due to Shear



For shear stress v_2 due to Moment Transfer

For Design $v_1 + v_2 = v_u \leq \phi\, v_n$

SHEAR REINFORCEMENT STUDIES
Exterior Column Connections – Shearhead Strength Requirements

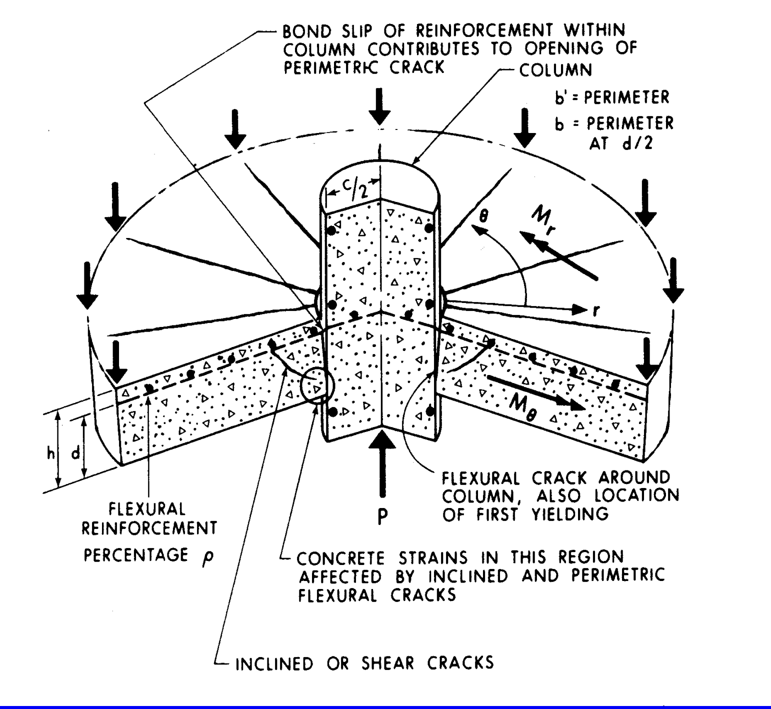


Current Code Requirement For Plastic Moment Strength

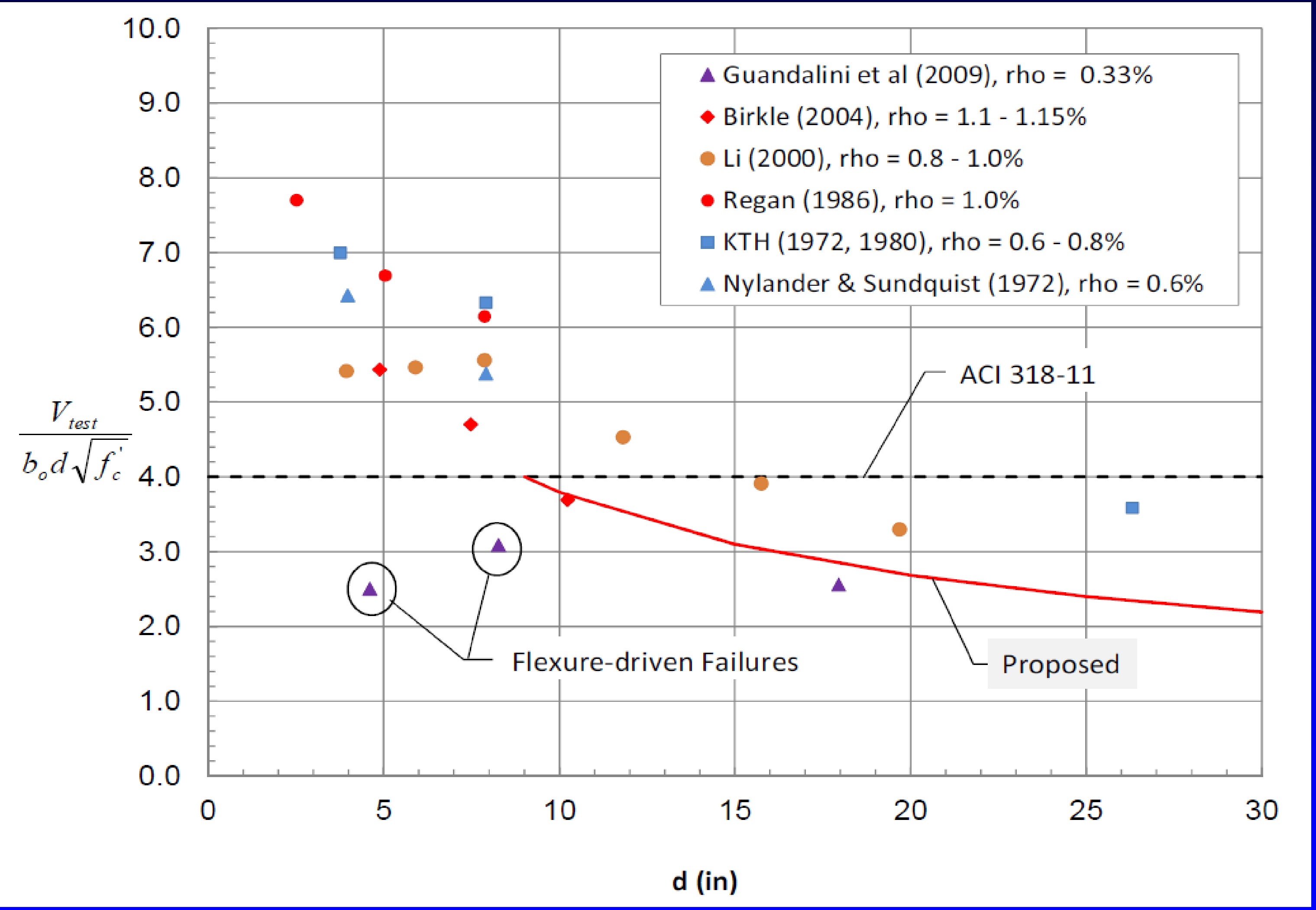
WHAT STILL NEEDS TO BE ADDRESSED?

Slabs Without Shear Reinforcement – Flexural Strength Limit

- **Recognize Relevance of Muttoni's Critical Shear Crack (CSC) Theory**
- **Aggregate Interlock Along CSC Is Lost When There Is General Yielding of Reinforcement in the Vicinity of Column**
- **Per Ghali, Strength for General Yielding is 8m where m is flexural strength per unit width**
- **Require $\phi_v V_{shear} \leq \phi_f V_{flex} = \phi_f\, 8m$ – Needed for low ρ**



WHAT STILL NEEDS TO BE ADDRESSED?
Slabs Without Shear Reinforcement – Depth Effect



$k_v = 3/\sqrt{d}$

WHAT STILL NEEDS TO BE ADDRESSED?

Slabs With Shear Reinforcement

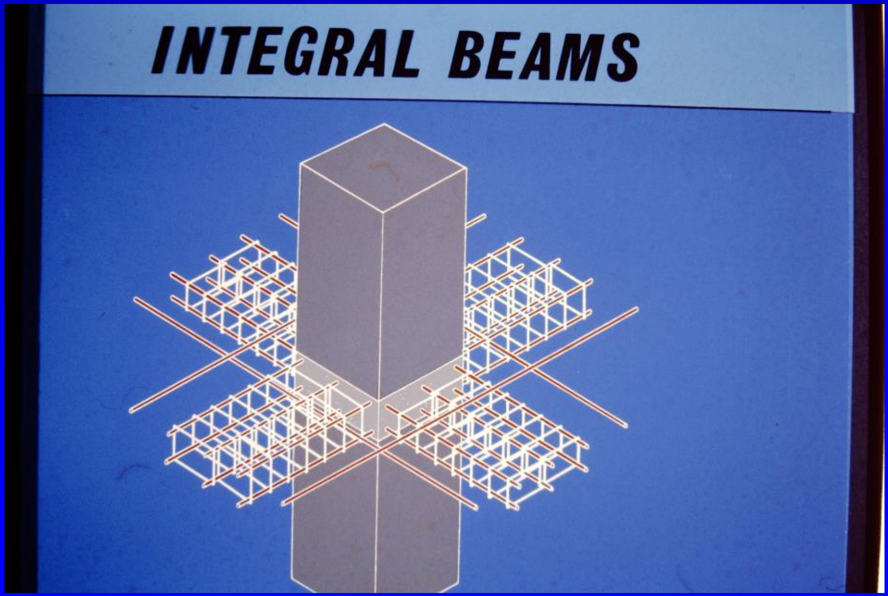
- **Develop Conceptually Consistent Punching Shear, Moment Transfer, and Ductility Provisions For Connections With Shear Reinforcement**

Cover Stirrup Reinforcement,

Stud Rail Reinforcement,

Fortress Reinforcement,

Shearhead Reinforcement.



Thank You