CHAPTER 9 — SECTIONAL STRENGTH

9.1 — Scope

9.1.1 — Provisions of this chapter shall govern the calculation of nominal strength at a section of a member, unless the member or region of the member is designed in accordance with Chapter 17. <8.1.1> <8.1.2> <9.1.3>

9.1.2 — Design strength at a section shall be taken as the nominal strength multiplied by the applicable strength reduction factor \( \phi \). <8.1.1> <9.1.1> <9.3.1>

9.1.3 — Nominal strength at a section of a member shall be calculated in accordance with: <~>

(a) 9.3 for flexure
(b) 9.4 for combined flexure and axial force
(c) 9.5 for one-way shear
(d) 9.6 for two-way shear
(e) 9.7 for torsion

9.2 — Design assumptions for flexural and axial strength

9.2.1 — Strain compatibility and equilibrium

9.2.1.1 — Strain compatibility and equilibrium shall be satisfied at each section. <10.2.1> <10.3.1> <18.3.1>

9.2.1.2 — Strain in concrete and nonprestressed reinforcement shall be assumed proportional to the distance from neutral axis. <10.2.2>

9.2.1.3 — Strain in bonded and unbonded prestressed reinforcement and concrete shall include the strain due to effective prestress. <~>

9.2.1.4 — Changes in strain for bonded prestressed reinforcement shall be assumed proportional to the distance from neutral axis. <18.3.2.1>

9.2.1.5 — Strain in unbonded prestressed reinforcement shall include cumulative effect of strain along the unbonded length. <~>
9.2.2 — Design assumptions for concrete

9.2.2.1 — Maximum strain at the extreme concrete compression fiber shall be assumed equal to 0.003. <10.2.3>

9.2.2.2 — Shape of the concrete compressive stress distribution with respect to concrete strain shall be assumed to be rectangular, trapezoidal, parabolic, or any other shape that results in prediction of strength in substantial agreement with results of comprehensive tests. <10.2.6>

9.2.2.3 — Tensile strength of concrete shall be neglected in flexural and axial strength calculations. <10.2.5> <18.3.2.2>

9.2.2.4 — An equivalent rectangular concrete stress distribution, defined in 9.2.2.5 through 9.2.2.7, satisfies 9.2.2.2. <10.2.7>

9.2.2.5 — Concrete stress of \( 0.85f'_c \) shall be assumed uniformly distributed over an equivalent compression zone bounded by edges of the cross section and a line parallel to the neutral axis located a distance \( a = \beta c \) from the fiber of maximum compressive strain. <10.2.7.1>

9.2.2.6 — Distance from the fiber of maximum compressive strain to the neutral axis, \( c \), shall be measured perpendicular to the neutral axis. <10.2.7.2>

9.2.2.7 — Values of \( \beta \) shall be in accordance with Table 9.2.2.7. <10.2.7.3>

Table 9.2.2.7 — Values of \( \beta \) for equivalent rectangular concrete stress distribution

<table>
<thead>
<tr>
<th>( f'_c ), psi</th>
<th>( \beta )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 2500 \leq f'_c &lt; 4000 )</td>
<td>0.85</td>
</tr>
<tr>
<td>( 4000 \leq f'_c &lt; 8000 )</td>
<td>( 0.85 - 0.05(f'_c - 4000)/1000 )</td>
</tr>
<tr>
<td>( f'_c \geq 8000 )</td>
<td>0.65</td>
</tr>
</tbody>
</table>

9.2.3 — Design assumptions for nonprestressed reinforcement

References to 6.3.1, 6.3.2, and 6.3.4 per LB11-1.

9.2.3.1 — Deformed reinforcement conforming to Table 6.3.4(a) shall be permitted to resist tensile or compressive forces in members.

9.2.3.2 — Stress-strain relationship and modulus of elasticity for deformed reinforcement shall be permitted to be idealized in accordance with 6.3.1 and 6.3.2. <10.6.4>

9.2.3.3 — Deformed reinforcement conforming to Table 6.3.4(a) used with prestressed reinforcement shall be permitted to be considered to contribute to the tensile force and be included in flexural strength calculations at a stress equal to \( f_y \). <18.7.3>
9.2.4 — Design assumptions for prestressing reinforcement

References to 6.4.1, 6.5.3, and 6.5.4 per LB11-1.

9.2.4.1 — For members with bonded prestressing reinforcement conforming to 6.4.1, stress at nominal flexural strength, \( f_{ps} \), shall be calculated in accordance with 6.5.4. <18.7.2>

9.2.4.2 — For members with unbonded prestressing reinforcement conforming to 6.4.1, \( f_{ps} \) shall be calculated in accordance with 6.5.3. <18.7.2>

9.2.5 — Design assumptions for nonprestressed prestressing reinforcement

9.2.5.1 — Nonprestressed prestressing reinforcement shall be permitted to be included in flexural strength calculations if a strain compatibility analysis is performed to determine stresses in such reinforcement. <18.7.3>

Note: 18.7.3 in ACI 318-08 permits the use of “other nonprestressed reinforcement” that does not conform to 3.5.3. This section now refers explicitly to nonprestressed prestressing reinforcement. Sub E believes that a limit to the value of strand stress that may be used in design and the minimum embedded length for nonprestressed strand should be should be added to 9.2.5 as NEW BUSINESS.

9.3 — Flexural strength

9.3.1 — General

9.3.1.1 — Calculation of nominal flexural strength \( Mn \) shall be in accordance with the assumptions of 9.2. <~>

9.3.2 — Strength reduction factor

9.3.2.1 — Strength reduction factor for flexural strength, \( \phi_f \), in nonprestressed members and at sections in pretensioned members where strand embedment equals or exceeds the development length shall be calculated in accordance with Table 9.3.2.1, where \( \varepsilon_{tb} \) is the net tensile strain in extreme tension steel corresponding to balanced strain conditions. <9.3.1> <9.3.2.1> <10.3.2> <10.3.3> <10.3.4> <18.8.1>
### Table 9.3.2.1 — Strength reduction factor

<table>
<thead>
<tr>
<th>Net tensile stain, $\varepsilon_t$</th>
<th>Classification</th>
<th>$\phi_f$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Transverse reinforcement</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Spirals conforming to 13.6.3</td>
</tr>
<tr>
<td>$\varepsilon_t \leq \varepsilon_{tb}$</td>
<td>Compression controlled</td>
<td>0.75</td>
</tr>
<tr>
<td>$\varepsilon_{tb} &lt; \varepsilon_t &lt; 0.005$</td>
<td>Transition</td>
<td>$0.75 + 0.15 \left(\frac{\varepsilon_t - \varepsilon_{tb}}{0.005 - \varepsilon_{tb}}\right)$</td>
</tr>
<tr>
<td>$\varepsilon_t \geq 0.005$</td>
<td>Tension controlled</td>
<td>0.90</td>
</tr>
</tbody>
</table>

Add to Chapter 2:

$\varepsilon_{tb} = $ net tensile strain in extreme tension steel corresponding to balanced strain conditions.

$\phi_f = $ strength reduction factor for flexural and axial strength

$\phi_v = $ strength reduction factor for shear and torsional strength

*balanced strain conditions* — strain distribution in cross section at which strain in the extreme concrete compression fiber is 0.003 and strain at centroid of tension reinforcement is equal to yield strain.

### 9.3.2.2 — It shall be permitted to take $\varepsilon_{tb}$ equal to 0.002 for members with Grade 60 deformed reinforcement or prestressing steel.

### 9.3.2.3 — For sections in pretensioned, flexural members where strand embedment from that section is less than $\ell_d$ of the strand, $\phi_f$ shall be calculated in accordance with Table 9.3.2.3. Where bonding of strand does not extend to the end of the member, strand embedment shall be assumed to begin at end of the debonded length. <9.3.2.7>

### Table 9.3.2.3 — Strength reduction factor for sections within strand development length

<table>
<thead>
<tr>
<th>Net tensile strain, $\varepsilon_t$</th>
<th>$\phi_f$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Location of section</td>
<td></td>
</tr>
<tr>
<td>All cases</td>
<td>0.75</td>
</tr>
<tr>
<td>Within $\ell_{ur}$</td>
<td></td>
</tr>
<tr>
<td>Between $\ell_{ur}$ and $\ell_d$</td>
<td>Linear interpolation from 0.75 to 0.90 is permitted</td>
</tr>
</tbody>
</table>

### 9.3.3 — Concrete composite flexural members

### 9.3.3.1 — Provisions of 9.3.3 apply to precast concrete elements, cast-in-place concrete elements, or both, constructed in separate placements but so interconnected that all elements respond to loads as a unit. <17.1.1>

### 9.3.3.2 — For calculation of $M_n$ for composite concrete slabs and beams, use of the entire composite section shall be permitted. <17.2.1>
9.3.3.3 — For calculation of $M_n$ for composite concrete slabs and beams, no distinction shall be made between shored and unshored members. <17.2.4>

9.3.3.4 — For calculation of $M_n$ for composite concrete slabs and beams where the specified concrete compressive strength, unit weight, or other properties of different elements vary, properties of the individual elements or the critical values shall be used in design. <17.2.3>

9.4 — Combined flexural and axial strength

9.4.1 — General

9.4.1.1 — Nominal strength calculations for combined flexure and axial force shall be in accordance with the assumptions of 9.2. <18.11.1>

9.4.2 — Strength reduction factors

9.4.2.1 — Strength reduction factor for combined flexural and axial strength, $\phi_f$, shall be in accordance with Table 9.3.2.1. <9.3.1> <10.3.2> <10.3.3> <10.3.4> <18.8.1>

9.4.3 — Maximum axial strength

9.4.3.1 — Nominal axial strength of column, $P_n$, shall not be taken greater than $P_{n,max}$, as defined in Table 9.4.3.1. <10.3.6> <10.3.6.1> <10.3.6.2> <10.3.6.3>

<table>
<thead>
<tr>
<th>Table 9.4.3.1 — Maximum axial strength</th>
</tr>
</thead>
<tbody>
<tr>
<td>Column</td>
</tr>
<tr>
<td>--------</td>
</tr>
<tr>
<td>Non prestressed</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>Prestressed</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>Composite conforming to 10.13</td>
</tr>
</tbody>
</table>

Add to Chapter 2:

$A_{pc} = \text{total area of prestressed longitudinal reinforcement, in}^2$
9.5 — One-way shear strength

9.5.1 — General

9.5.1.1 — Nominal one-way shear strength at a section, $V_n$ shall be calculated as:

\[ V_n = V_c + V_s \]  \hspace{1cm} (9.5.1.1)

9.5.1.2 — Cross-sectional dimensions shall be selected such that Eq. (9.5.1.2) is satisfied.

\[ V_u / \phi_v - V_c \leq 8\sqrt{f'_c b_w d} \]  \hspace{1cm} (9.5.1.2)

9.5.1.3 — For nonprestressed members, $V_c$ shall be calculated in accordance with 9.5.6, 9.5.7, or 9.5.8.

9.5.1.4 — For prestressed members, $V_c$, $V_{ci}$, and $V_{cw}$ shall be calculated in accordance with 9.5.9 or 9.5.10.

9.5.1.5 — $V_s$ shall be calculated in accordance with 9.5.11.

9.5.1.6 — Effect of any openings in members shall be considered in calculating $V_c$ and $V_s$.

9.5.1.7 — Effects of axial tension due to creep and shrinkage in restrained members shall be considered in calculating $V_c$.  \hspace{1cm} (11.1.1.2)

9.5.1.8 — Effect of inclined flexural compression in variable depth members shall be considered in calculating $V_c$.  \hspace{1cm} (11.1.1.2)

9.5.2 — Strength reduction factor

9.5.2.1 — Strength reduction factor for one-way shear, $\phi_v$, shall be 0.75.  \hspace{1cm} (9.3.2.3)

9.5.3 — Effective depth for one-way shear

9.5.3.1 — For calculation of $V_c$ and $V_s$ in prestressed members, $d$ shall be taken as the distance from extreme compression fiber to centroid of prestressed and nonprestressed longitudinal reinforcement, if any, but need not be taken less than $0.8h$.  \hspace{1cm} (11.3.1)  \hspace{1cm} (11.4.3)

9.5.3.2 — For calculation of $V_c$ and $V_s$ in solid, circular sections, $d$ shall be permitted to be taken as 0.8 times the diameter and $b_w$ shall be permitted to be taken as the diameter.  \hspace{1cm} (11.2.3)  \hspace{1cm} (11.4.7.3)
9.5.4 — Limiting material strengths for one-way shear

9.5.4.1 — For one-way shear, the value of $\sqrt{f'_c}$ used to calculate $V_c$, $V_{ci}$, and $V_{cw}$ shall not exceed 100 psi. \(<11.1.2> <11.1.2.1>\)

9.5.4.2 — Values of $f_y$ and $f_{ym}$ used to calculate $V_s$ shall be in accordance with Table 6.3.4(a). \(<11.4.2>\)

Reference to 6.4.3 per LB11-1.

9.5.5 — Composite concrete members

9.5.5.1 — For calculation of $V_n$ for composite concrete members, no distinction shall be made between shored and unshored members. \(<17.2.4>\)

9.5.5.2 — For calculation of $V_c$ for composite concrete members where the specified concrete compressive strength, unit weight, or other properties of different elements vary, properties of the individual elements or the critical values shall be used in design. \(<17.2.3>\)

9.5.5.3 — If an entire composite concrete member is assumed to resist vertical shear, $V_c$ shall be permitted to be calculated assuming a monolithically cast member of the same cross-sectional shape. \(<17.2.1> <17.4.1>\)

9.5.5.4 — If an entire composite concrete member is assumed to resist vertical shear, $V_s$ shall be permitted to be calculated assuming a monolithically cast member of the same cross-sectional shape if shear reinforcement is fully anchored into the interconnected elements in accordance with 20.8. \(<17.4.1> <17.4.2>\)

Reference to 20.8 per LB10-3.

9.5.6 — $V_c$ for nonprestressed members without axial force

9.5.6.1 — $V_c$ shall be permitted to be calculated using the simplified or detailed options in Table 9.5.6.1. \(<11.2.1.1> <11.2.2> <11.2.2.1>\)

Table 9.5.6.1 — $V_c$ for nonprestressed members without axial force

<table>
<thead>
<tr>
<th>Calculation options</th>
<th>$V_c$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Simplified</td>
<td>$2\lambda \sqrt{f'_c b_w d}$</td>
</tr>
<tr>
<td>Detailed</td>
<td>Least of: $\left(1.9\lambda \sqrt{f'<em>c + 2500 \rho_w \frac{V</em>{id}}{M_u}}\right) b_w d$</td>
</tr>
<tr>
<td></td>
<td>$\left(1.9\lambda \sqrt{f'_c + 2500 \rho_w}b_w d\right.$</td>
</tr>
<tr>
<td></td>
<td>$3.5\lambda \sqrt{f'_c} b_w d$</td>
</tr>
</tbody>
</table>

* $M_u$ and $V_u$ occur simultaneously at section considered.
9.5.7 — \( V_c \) for nonprestressed members with axial compression

9.5.7.1 — \( V_c \) shall be permitted to be calculated using the simplified or detailed options in Table 9.5.7.1, where \( N_u \) is positive for compression. <11.2.1.1> <11.2.2> <11.2.2.1>

Table 9.5.7.1 — \( V_c \) for nonprestressed members with axial compression

<table>
<thead>
<tr>
<th>Calculation options</th>
<th>( V_c )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Simplified</td>
<td>( 2 \left( 1 + \frac{N_u}{2000A_g} \right) \lambda \sqrt{f'_c b_w d} ) (a)</td>
</tr>
<tr>
<td>Detailed</td>
<td>Lesser of:</td>
</tr>
<tr>
<td></td>
<td>( 1.9 \lambda \sqrt{f'_c} + 2500 \rho_w \frac{V_u d}{M_u - N_u \left( \frac{4h - d}{8} \right)} ) b_w d (b)*</td>
</tr>
<tr>
<td></td>
<td>( 3.5 \lambda \sqrt{f'_c} b_w d \left( 1 + \frac{N_u}{500A_g} \right) ) (c)</td>
</tr>
</tbody>
</table>

* Not applicable if \( M_u - N_u \left( \frac{4h - d}{8} \right) \leq 0 \). \( M_u \), \( N_u \), and \( V_u \) occur simultaneously at section considered.

9.5.8 — \( V_c \) for nonprestressed members with axial tension

9.5.8.1 — \( V_c \) shall be permitted to be calculated using the simplified or detailed options in Table 9.5.8.1, where \( N_u \) is negative for tension. <11.2.1.1> <11.2.2> <11.2.2.1>

Table 9.5.8.1 — \( V_c \) for nonprestressed members with axial tension

<table>
<thead>
<tr>
<th>Calculation options</th>
<th>( V_c )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Simplified</td>
<td>0.0 (a)</td>
</tr>
<tr>
<td>Detailed</td>
<td>Greater of:</td>
</tr>
<tr>
<td></td>
<td>( 2 \left( 1 + \frac{N_u}{500A_g} \right) \lambda \sqrt{f'_c b_w d} ) (b)</td>
</tr>
<tr>
<td></td>
<td>0.0 (c)</td>
</tr>
</tbody>
</table>

9.5.9 — \( V_c \) for pretensioned members beyond transfer length and post-tensioned members

9.5.9.1 — For prestressed flexural members with effective prestress force not less than 40 percent of the tensile strength of the flexural reinforcement, \( V_c \) shall be permitted to be calculated in accordance with Table 9.5.9.1. <11.3.2>
### Table 9.5.9.1 — $V_c$ for prestressed members

<table>
<thead>
<tr>
<th>$V_c$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Greater of:</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>Least of:</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>(a) $2\lambda\sqrt{f'_c b_w d}$</td>
</tr>
<tr>
<td>(b) $0.6\lambda\sqrt{f'_c + 700} \frac{V_u d_p}{M_u} b_w d$</td>
</tr>
<tr>
<td>(c) $0.6\lambda\sqrt{f'_c + 700} b_w d$</td>
</tr>
<tr>
<td>(d) $5\lambda\sqrt{f'_c b_w d}$</td>
</tr>
</tbody>
</table>

* Actual value of $d_p$ shall be used in row (b). $M_u$ and $V_u$ occur simultaneously at the section considered.

### 9.5.9.2 — For prestressed members, $V_c$ shall also be permitted to be taken as the lesser of $V_{ci}$ and $V_{cw}$ calculated in accordance with Table 9.5.9.2 and $M_{cre}$ shall be calculated as:

$$M_{cre} = \left(\frac{I}{y_t}\right) \left(6\lambda\sqrt{f'_c + f_{pe} - f_d}\right) \quad (9.5.9.2)$$

### Table 9.5.9.2 — Alternate method for calculating $V_c$ for prestressed members

<table>
<thead>
<tr>
<th>Types of shear cracking investigated</th>
<th>$V_c$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_{ci}$</td>
<td>Greater of:</td>
</tr>
<tr>
<td>(a)*,† $0.6\lambda\sqrt{f'<em>c b_w d} + V_d + \frac{V_i M</em>{cre}}{M_{max}}$</td>
<td></td>
</tr>
<tr>
<td>(b) $1.7\lambda\sqrt{f'_c b_w d}$</td>
<td></td>
</tr>
<tr>
<td>$V_{cw}$</td>
<td>(c)† $3.5\lambda\sqrt{f'<em>c + 0.3 f</em>{pe}} b_w d_p + V_p$</td>
</tr>
</tbody>
</table>

* $M_{max}$ and $V_i$ correspond to the load combination causing maximum factored moment at the section.
† $d_p$ need not be taken less than $0.8h$ in Rows (a) and (c).

### 9.5.9.3 — Alternatively, $V_{cw}$ shall be permitted to be calculated as the shear force corresponding to dead load, $D$, plus live load, $L$, that results in a principal tensile stress of $4\lambda\sqrt{f'_c}$ at the centroidal axis of member, or at the intersection of flange and web when the centroidal axis is in the flange. <11.3.3.2>

### 9.5.9.4 — In composite members, the composite cross section shall be used in calculating the principal tensile stress defined in 9.5.9.3 due to post-composite loads. <11.3.3.2>
9.5.10 — \( V_c \) for pretensioned members in regions of reduced prestress

9.5.10.1 — Between the member end and transfer length, \( l_w \), (a), (b), and (c) shall be satisfied. <11.3.4>

(a) A reduced effective prestress shall be considered when calculating \( V_c \) in accordance with 9.5.9.2.

(b) Value of \( V_c \) calculated in accordance with Table 9.5.9.1 shall not exceed \( V_{cw} \) calculated with the reduced effective prestress.

(c) Effective prestress force shall be assumed to vary linearly from zero at the end of the prestressing steel, to a maximum at a distance from end of the prestressing steel equal to the transfer length, assumed to be 50 diameters for strand and 100 diameters for single wire.

9.5.10.2 — Where bonding of some strands does not extend to the end of member, (a), (b), and (c) shall be satisfied. <11.3.5>

(a) A reduced effective prestress shall be considered when calculating \( V_c \) in accordance with 9.5.9.1 or 9.5.9.2.

(b) Value of \( V_c \) calculated in accordance with Table 9.5.9.1 shall not exceed \( V_{cw} \) calculated with the reduced effective prestress.

(c) Effective prestress force in strands for which bonding does not extend to the end of member shall be assumed to vary linearly from zero at the point at which bonding commences to a maximum at a distance from this point equal to the transfer length, assumed to be 50 diameters for strand and 100 diameters for single wire.

9.5.11 — One-way shear reinforcement

9.5.11.1 — At each section where \( V_u > \phi V_c \), transverse reinforcement shall be provided such that Eq. (9.5.11.1) is satisfied. <11.4.7.1>

\[
V_s \geq \frac{V_u}{\phi} - V_c \quad (9.5.11.1)
\]

9.5.11.2 — For one-way members reinforced with rectangular ties, circular ties, stirrups, hoops, crossties, or spirals, \( V_s \) shall be calculated in accordance with 9.5.11.5. <~>

9.5.11.3 — For one-way members reinforced with bent-up longitudinal bars, \( V_s \) shall be calculated in accordance with 9.5.11.6. <~>

9.5.11.4 — If more than one type of shear reinforcement is provided to reinforce the same portion of a member, \( V_s \) shall be calculated as the sum of the \( V_s \) values calculated for the various types of shear reinforcement. <11.4.7.8>
9.5.11.5 — One-way shear strength provided by ties, stirrups, hoops, crossties, and spirals

9.5.11.5.1 — Shear reinforcement conforming to (a), (b), or (c) shall be permitted in prestressed and nonprestressed members: <11.4.1.1>

(a) Stirrups, ties, or hoops perpendicular to axis of member
(b) Welded wire reinforcement with wires located perpendicular to axis of member
(c) Spiral reinforcement

9.5.11.5.2 — Shear reinforcement conforming to (a), (b), or (c) shall also be permitted in nonprestressed members: <11.4.1.2>

(a) Inclined stirrups making an angle of at least 45 degrees with longitudinal tension reinforcement
(b) Bent portion of longitudinal reinforcement making an angle of at least 30 degrees with the longitudinal tension reinforcement
(c) Combinations of stirrups and bent longitudinal reinforcement

9.5.11.5.3 — If spiral reinforcement or shear reinforcement perpendicular to the axis of the member is provided, $V_s$ shall be calculated as: <11.4.7.2>

$$V_s = \frac{A_y}{s} f_{yr} d$$  \hspace{1cm} (9.5.11.5.3)

where $s$ is the spiral pitch or the longitudinal spacing of the shear reinforcement and $A_y$ is defined in 9.5.11.5.5 or 9.5.11.5.6.

9.5.11.5.4 — If inclined stirrups are used as shear reinforcement, $V_s$ shall be calculated as: <11.4.7.4>

$$V_s = \frac{A_y}{s} f_{yr} (\sin \alpha + \cos \alpha) d$$  \hspace{1cm} (9.5.11.5.4)

where $\alpha$ is the angle between inclined stirrups and longitudinal axis of the member, $s$ is measured parallel to the longitudinal reinforcement, $A_y$ is defined in 9.5.11.5.5.

9.5.11.5.5 — For each rectangular tie, stirrup, hoop, or crosstie, $A_y$ shall be taken as the area of each bar or wire within spacing $s$ times the number of legs. <~>

9.5.11.5.6 — For each circular tie or spiral, $A_y$ shall be taken as two times the area of the bar or wire within spacing $s$. <11.4.7.3>

9.5.11.6 — One-way shear strength provided by bent-up longitudinal bars

9.5.11.6.1 — If shear reinforcement consists of a single bar or a single group of parallel bars having an area $A_y$, all bent the same distance from the support, $V_s$ shall be taken as the lesser of (a) and (b): <11.4.7.5>

(a) $A_y f_y \sin \alpha$
(b) $3 \sqrt{f'_{c}} b_w d$
where $\alpha$ is the angle between bent-up reinforcement and longitudinal axis of the member.

9.5.11.6.2 — Only the center three-fourths of the inclined portion of any longitudinal bent bar shall be considered effective for shear reinforcement. <11.4.7.7>

9.5.11.6.3 — If shear reinforcement consists of a series of parallel bent-up bars or groups of parallel bent-up bars at different distances from the support, $V_s$ shall be calculated in accordance with Eq. (9.5.11.5.4). <11.4.7.6>

9.6 — Two-way shear strength

9.6.1 — General

9.6.1.1 — Two-way members reinforced with structural steel I- or channel-shaped sections used as shearheads shall be designed in accordance with 9.6.11. <--> <11.11.4>

9.6.1.2 — Nominal shear strength for two-way members not reinforced for shear shall be calculated in accordance with Eq. 9.6.1.2. <--> <11.11.7.2>

$$v_n = \frac{V_c}{b_o d}$$  \hspace{1cm} (9.6.1.2)

9.6.1.3 — Nominal shear strength for two-way members with shear reinforcement other than shearheads shall be calculated in accordance with Eq. 9.6.1.3. <11.1.1> <11.11.7.2>

$$v_n = \frac{(V_c + V_s)}{b_o d}$$  \hspace{1cm} (9.6.1.3)

9.6.1.4 — Two-way shear shall be resisted by a section with a depth $d$ and an assumed critical perimeter $b_o$ that wraps completely or partially around the column, concentrated load, or reaction area. <--> 

9.6.1.5 — The shear stress resulting from moment transfer by eccentricity of shear shall be assumed to vary linearly about the centroid of $b_o$. <11.11.7.2>

9.6.1.6 — $V_c$ shall be calculated in accordance with 9.6.8. <--> 

9.6.1.7 — For calculation of $V_c$, effects of axial tension due to creep and shrinkage in restrained members shall be considered if applicable. <11.1.1.2>

9.6.1.8 — For two-way members reinforced with single- or multi-leg stirrups, $V_s$ shall be calculated in accordance with 9.6.9. <--> 

9.6.1.9 — For two-way members reinforced with headed shear stud reinforcement, $V_s$ shall be calculated in accordance with 9.6.10. <-->
9.6.2 — Strength reduction factor

9.6.2.1 — Strength reduction factor for two-way shear, $\phi_v$, shall be 0.75. <9.3.2.3>

9.6.3 — Effective depth for two-way shear

9.6.3.1 — For calculation of $V_c$ and $V_s$ in two-way, prestressed members, $d$ shall be taken as the distance from extreme compression fiber to centroid of prestressed and nonprestressed longitudinal reinforcement, if any, but need not be taken less than $0.8h$. <11.3.1> <11.4.3>

9.6.4 — Limiting material strengths for two-way shear

9.6.4.1 — For two-way shear, the value of $\sqrt{f'_c}$ used to calculate $V_c$ shall not exceed 100 psi. <11.1.2> <11.1.2.1>

9.6.4.2 — Value of $f_{ym}$ used to calculate $V_s$ shall be in accordance with Table 6.3.4(a). <11.4.2>

Reference to 6.3.4 per LB11-1.

9.6.5 — Inner critical sections for two-way members

9.6.5.1 — For two-way shear, each of the critical sections to be considered shall be located so that its perimeter $b_o$ is a minimum but need not be closer than $d/2$ to: <11.11.1.2>

(a) Edges or corners of columns, concentrated loads, or reaction areas

(b) Changes in slab or footing thickness, such as edges of shear capitals and drop panels

9.6.5.2 — For square or rectangular columns, concentrated loads, or reaction areas, critical section for two-way shear shall be permitted to be calculated assuming straight sides. <11.11.1.3>

9.6.5.3 — For a circular or regular polygon-shaped column, critical section for two-way shear shall be permitted to be calculated assuming a square column of equivalent area. <15.3>

9.6.5.4 — If an opening is located within a column strip or closer than $10h$ from a concentrated load or reaction area, a portion of $b_o$ enclosed by straight lines projecting from the centroid of the column, concentrated load or reaction area and tangent to the boundaries of the opening shall be considered ineffective. <11.11.6>
9.6.6 — Outer critical section for two-way members with shear reinforcement

9.6.6.1 — For two-way members reinforced with single- or multi-leg stirrups, a critical section located \( d/2 \) outside the outermost line of stirrup legs that surround the column shall be considered. <11.11.7.2>

9.6.6.2 — For two-way members reinforced with headed shear stud reinforcement, a critical section located \( d/2 \) outside the outermost peripheral line of shear reinforcement shall be considered. <11.11.5.4>

9.6.7 — Maximum shear for two-way members with shear reinforcement

9.6.7.1 — For two-way members with shear reinforcement, value of \( V_c \) calculated at inner critical sections defined in 9.6.5 shall not exceed the values given in Table 9.6.7.1. <11.11.7.2>

Table 9.6.7.1 — Maximum \( V_c \) at inner critical sections for two-way members with shear reinforcement

<table>
<thead>
<tr>
<th>Type of shear reinforcement</th>
<th>Maximum ( V_c )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stirrups</td>
<td>( 2\mu \sqrt{f'_c b_o d} )</td>
</tr>
<tr>
<td>Headed shear stud reinforcement</td>
<td>( 3\mu \sqrt{f'_c b_o d} )</td>
</tr>
</tbody>
</table>

9.6.7.2 — For two-way members with shear reinforcement, values of \( v_n \) at inner critical sections defined in 9.6.5 and outer critical sections defined in 9.6.6 shall not exceed the values given in Table 9.6.7.2. <11.11.5.4> <11.11.7.2> <11.11.7.3>

Table 9.6.7.2 — Maximum \( v_n \) for two-way members with shear reinforcement

<table>
<thead>
<tr>
<th>Type of shear reinforcement</th>
<th>Maximum ( v_n ) at inner critical sections</th>
<th>Maximum ( v_n ) at outer critical section</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stirrups</td>
<td>( 6\sqrt{f'_c} )</td>
<td>( 2\mu \sqrt{f'_c} )</td>
</tr>
<tr>
<td>Headed shear stud reinforcement</td>
<td>( 8\sqrt{f'_c} )</td>
<td>( 2\mu \sqrt{f'_c} )</td>
</tr>
</tbody>
</table>
9.6.8 — Two-way shear strength provided by concrete

9.6.8.1 — For two-way shear, \( V_c \) shall be the least of (a), (b), and (c):

(a) \( 4 \lambda \sqrt{f'_c b_o d} \)

(b) \( \left( 2 + \frac{4}{\beta} \right) \lambda \sqrt{f'_c b_o d} \)

(c) \( 2 + \frac{\alpha_s d}{b_o} \lambda \sqrt{f'_c b_o d} \)

where \( \beta \) is the ratio of long side to short side of the column, concentrated load, or reaction area and \( \alpha_s \) is defined in 9.6.8.4. <11.11.2.1>

9.6.8.2 — For prestressed, two-way members satisfying (a), (b), and (c), it shall be permitted to calculate \( V_c \) using 9.6.8.3. <11.11.2.2>

(a) Bonded reinforcement is provided in accordance with 12.6.3.4 and 12.7.2.4

(b) No portion of the column cross section is closer to a discontinuous edge than four times the slab thickness \( h \)

(c) Effective prestress, \( f_{pc} \), in each direction is not less than 125 psi

Reference to 12.6.3.4 and 12.7.2.4 per LB11-1.

9.6.8.3 — For prestressed, two-way members conforming to 9.6.8.2, \( V_c \) shall be permitted to be calculated as the lesser of (a) and (b): <11.11.2.2>

(a) \( \left( 3.5 \lambda \sqrt{f'_c} + 0.3 f_{pc} \right) b_o d + V_p \)

(b) \( \left[ \left( 1.5 + \frac{\alpha_s d}{b_o} \right) \lambda \sqrt{f'_c} + 0.3 f_{pc} \right] b_o d + V_p \)

where \( \alpha_s \) is defined in 9.6.8.4, the value of \( f_{pc} \) is the average of \( f_{pc} \) in the two directions and shall not be taken greater than 500 psi, \( V_p \) is the vertical component of all effective prestress forces crossing the critical section, and the value of \( \sqrt{f'_c} \) shall not exceed 70 psi.

9.6.8.4 — For calculating \( V_c \) using 9.6.8.1(c) or 9.6.8.3(b), \( \alpha_s \) shall be in accordance with Table 9.6.8.4. <11.11.2.1>
Table 9.6.8.4—Value of $\alpha_s$

<table>
<thead>
<tr>
<th>Location of column</th>
<th>$\alpha_s$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Interior</td>
<td>40</td>
</tr>
<tr>
<td>Edge</td>
<td>30</td>
</tr>
<tr>
<td>Corner</td>
<td>20</td>
</tr>
</tbody>
</table>

9.6.9 — Two-way shear strength provided by single- or multiple-leg stirrups

9.6.9.1 — Single- or multiple-leg stirrups fabricated from bars or wires shall be permitted as shear reinforcement in slabs and footings conforming to (a) and (b): <11.11.3>

(a) $d$ is at least 6 in.
(b) $d$ is at least 16 times the diameter of the shear reinforcement

9.6.9.2 — For two-way members with stirrups, $V_s$ shall be calculated as: <11.11.3.1> <11.4.7.2>

$$V_s = \frac{A_v}{s} f_y d$$

where $A_v$ is the sum of the area of all stirrup legs on a peripheral line that is approximately parallel to the perimeter of the column section, and $s$ is the spacing of the shear reinforcement in the direction perpendicular to the column face.

9.6.10 — Two-way shear strength provided by headed shear stud reinforcement

9.6.10.1 — Headed shear stud reinforcement shall be permitted to be used as shear reinforcement in slabs and footings if the studs are placed perpendicular to the plane of the two-way member and if the geometry of the headed shear stud reinforcement satisfies 12.7.7 for slabs or 16.7.7 for footings. <11.11>

Reference to 12.7.7 is per LB11-1. Reference to 16.7.7 is a guess.

9.6.10.2 — For two-way members with headed shear stud reinforcement, $V_s$ shall be calculated as: <11.4.7.2> <11.11.5.1>

$$V_s = \frac{A_v}{s} f_y d$$

where $A_v$ is the sum of the area of all shear studs on a peripheral line geometrically similar to the perimeter of the column section, and $s$ is the spacing of the peripheral lines of headed shear stud reinforcement in the direction perpendicular to the column face.
9.6.10.3 — If headed shear stud reinforcement is provided, \( \frac{A_v}{s} \) shall satisfy Eq. (9.6.10.3).

\[
\frac{A_v}{s} \geq 2 \sqrt{f'_c} \frac{b_o}{f_{yt}} \tag{9.6.10.3}
\]

9.6.11 — Design provisions for two-way members with shearheads

9.6.11.1 — Each shearhead shall consist of steel shapes fabricated with a full penetration weld into identical arms at right angles. Shearhead arms shall not be interrupted within the column section. <11.11.4.1>

9.6.11.2 — A shearhead shall not be deeper than 70 times the web thickness of the steel shape. <11.11.4.2>

9.6.11.3 — The ends of each shearhead arm shall be permitted to be cut at angles of at least 30 degrees with the horizontal if the plastic moment strength, \( M_p \), of the remaining tapered section is adequate to resist the shear force attributed to that arm of the shearhead. <11.11.4.3>

9.6.11.4 — Compression flanges of steel shapes shall be within 0.3\( d \) of compression surface of the slab. <11.11.4.4>

9.6.11.5 — The ratio \( \alpha_v \) between the flexural stiffness of each shearhead arm and that of the surrounding composite cracked slab section of width \( (c_2 + d) \) shall be at least 0.15. <11.11.4.5>

9.6.11.6 — For each arm of the shearhead, \( M_p \) shall satisfy Eq. (9.6.11.6).

\[
M_p \geq \frac{V_u}{2\phi_f n} \left[ h_v + \alpha_v \left( \ell_v - \frac{c_1}{2} \right) \right] \tag{9.6.11.6}
\]

where \( \phi_f \) corresponds to tension-controlled members in 9.3.2.1, \( n \) is the number of shearhead arms, and \( \ell_v \) is the minimum length of each shearhead arm required to comply with 9.6.11.8 and 9.6.11.10. <11.11.4.6>
9.6.11.7 — Nominal flexural strength contributed to each slab column strip by a shearhead, $M_v$, shall satisfy Eq. (9.6.11.7).

$$M_v \leq \frac{\phi_f \alpha_V V_u}{2n} \left( \ell_v - \frac{c_1}{2} \right) \quad \text{(9.6.11.7)}$$

where $\phi_f$ corresponds to tension-controlled members in 9.3.2.1, $n$ is the number of shearhead arms, and $\ell_v$ is the length of each shearhead arm actually provided. However, $M_v$ shall not be taken greater than the least of (a), (b), and (c). <11.11.4.9>

(a) 30 percent of $M_u$ in each slab column strip 

(b) Change in $M_u$ in column strip over the length $\ell_v$ 

(c) $M_p$ as defined in 9.6.11.6

9.6.11.8 — The critical section for shear shall be perpendicular to the plane of the slab and shall cross each shearhead arm at a distance $(3/4)\left[ \ell_v - (c_1/2) \right]$ from the column face. This critical section shall be located so $b_o$ is a minimum, but need not be closer to the supporting column than the perimeter closest to the column defined in 9.6.5.1(a). <11.11.4.7>

9.6.11.9 — If an opening is located within a column strip or closer than $10h$ from a column in slabs with shearheads, the ineffective portion of $b_o$ shall be one-half of that defined in 9.6.5.4. <11.11.6.2>

9.6.11.10 — $V_u/\phi_V$ shall not be taken greater than $4\sqrt{f'_c b_o d}$ on the critical section defined in 9.6.11.8 and shall not be taken greater than $7\sqrt{f'_c b_o d}$ on the critical section closest to the column defined in 9.6.5.1(a). <11.11.4.8>

9.6.11.11 — Where transfer of moment is considered, the shearhead must have adequate anchorage to transmit $M_p$ to the column. <11.11.4.10>

9.6.11.12 — Where transfer of moment is considered, the sum of factored shear stresses due to vertical load acting on the critical section defined by 9.6.11.8 and the shear stresses resulting from factored moment transferred by eccentricity of shear about the centroid of the critical section closest to the column defined in 9.6.5.1(a) shall not exceed $4\lambda \sqrt{f'_c}$. <11.11.7.3>

9.7 — Torsion

9.7.1 — General

9.7.1.1 — Nominal torsional strength shall be calculated in accordance with 9.7.7. <~>
9.7.2 — Strength reduction factor

9.7.2.1 — Strength reduction factor for torsion, \( \phi_t \), shall be 0.75. <9.3.2.3>

9.7.3 — Limiting material strengths for torsion

9.7.3.1 — For torsion, the value of \( \sqrt{f'_{c}} \) used to calculate \( T_{th} \) and \( T_{cr} \) shall not exceed 100 psi. <11.1.2>

9.7.3.2 — Value of \( f_{yt} \) used to calculate \( T_{n} \) shall be in accordance with Table 6.3.4(a). <11.4.2>

Reference to 6.3.4 per LB11-1.

9.7.4 — Threshold torsion

9.7.4.1 — Threshold torsion, \( T_{th} \), shall be calculated in accordance with Table 9.7.4.1, where \( N_{u} \) is positive for compression. <11.5.1>

Table 9.7.4.1 — Threshold torsion

<table>
<thead>
<tr>
<th>Type of member</th>
<th>Cross section</th>
<th>( T_{th} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nonprestressed member</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Solid</td>
<td>( \lambda \sqrt{f'<em>{c}} \left( \frac{A</em>{cp}^2}{p_{cp}} \right) )</td>
</tr>
<tr>
<td></td>
<td>Hollow</td>
<td>( \lambda \sqrt{f'<em>{c}} \left( \frac{A</em>{g}^2}{p_{cp}} \right) )</td>
</tr>
<tr>
<td>Prestressed member</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Solid</td>
<td>( \lambda \sqrt{f'<em>{c}} \left( \frac{A</em>{cp}^2}{p_{cp}} \right) \sqrt{1 + \frac{f_{pc}}{4\lambda \sqrt{f'_{c}}}} )</td>
</tr>
<tr>
<td></td>
<td>Hollow</td>
<td>( \lambda \sqrt{f'<em>{c}} \left( \frac{A</em>{g}^2}{p_{cp}} \right) \sqrt{1 + \frac{f_{pc}}{4\lambda \sqrt{f'_{c}}}} )</td>
</tr>
<tr>
<td>Nonprestressed member subjected to an axial force</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Solid</td>
<td>( \lambda \sqrt{f'<em>{c}} \left( \frac{A</em>{cp}^2}{p_{cp}} \right) \sqrt{1 + \frac{N_{u}}{4A_{g}\lambda \sqrt{f'_{c}}}} )</td>
</tr>
<tr>
<td></td>
<td>Hollow</td>
<td>( \lambda \sqrt{f'<em>{c}} \left( \frac{A</em>{g}^2}{p_{cp}} \right) \sqrt{1 + \frac{N_{u}}{4A_{g}\lambda \sqrt{f'_{c}}}} )</td>
</tr>
</tbody>
</table>
9.7.5 — Cracking torsion

9.7.5.1 — Cracking torsion, \( T_{cr} \), shall be calculated in accordance with Table 9.7.5.1, where \( N_u \) is positive for compression. <11.5.2>

Table 9.7.5.1 — Cracking torsion

<table>
<thead>
<tr>
<th>Type of member</th>
<th>( T_{cr} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nonprestressed member</td>
<td>( 4\lambda \sqrt{f'<em>c} \left( \frac{A</em>{cp}^2}{p_{cp}} \right) )</td>
</tr>
<tr>
<td>Prestressed member</td>
<td>( 4\lambda \sqrt{f'<em>c} \left( \frac{A</em>{cp}^2}{p_{cp}} \right) \left[ 1 + \frac{f_{pc}}{4\lambda \sqrt{f'_c}} \right] )</td>
</tr>
<tr>
<td>Nonprestressed member subjected to an axial force</td>
<td>( 4\lambda \sqrt{f'<em>c} \left( \frac{A</em>{cp}^2}{p_{cp}} \right) \left[ 1 + \frac{N_u}{4A_g \lambda \sqrt{f'_c}} \right] )</td>
</tr>
</tbody>
</table>

9.7.6 — Factored design torsion

9.7.6.1 — It shall be permitted to neglect torsional effects if Eq. (9.7.6.1) is satisfied. <11.5.1>

\[
T_u < \phi_u T_{th}
\]  

(9.7.6.1)

9.7.6.2 — If \( T_u \) in a member is required to maintain equilibrium and exceeds \( T_{th} \), the member shall be designed to resist \( T_u \). <11.5.2.1>

9.7.6.3 — In a statically indeterminate structure where reduction of \( T_u \) in a member can occur due to redistribution of internal forces after torsional cracking, \( T_u \) shall be permitted to be reduced to \( \phi_u T_{cr} \). <11.5.2.2>

9.7.6.4 — If \( T_u \) is redistributed for torsional cracking in accordance with 9.7.6.3, the factored moments and shears used for design of the adjoining members shall be in equilibrium with the reduced torsion. <11.5.2.2>
9.7.7 — Torsional strength

9.7.7.1 — For prestressed and nonprestressed members, $T_n$ shall be calculated as: $\begin{equation} T_n = \frac{2A_o A_f f_{yt}}{s} \cot \theta \end{equation}$

where $A_o$ shall be determined by analysis and $\theta$ shall not be taken smaller than 30 degrees nor larger than 60 degrees.

9.7.7.2 — In Eq. (9.7.7.1), it shall be permitted to take $A_o$ equal to 0.85 $A_{oh}$. $\begin{equation} \end{equation}$

9.7.7.3 — In Eq. (9.7.7.1), it shall be permitted to take $\theta$ equal to (a) or (b): $\begin{equation} \end{equation}$

(a) 45 degrees for nonprestressed members or members with effective prestress force less than 40 percent of the tensile strength of the longitudinal reinforcement

(b) 37.5 degrees for prestressed members with an effective prestress force of at least 40 percent of the tensile strength of the longitudinal reinforcement

9.7.8 — Cross-sectional limits

9.7.8.1 — If $T_u$ exceeds $T_{th}$, cross-sectional dimensions shall be such that (a) or (b) is satisfied. $\begin{equation} \end{equation}$

(a) For solid sections

$\begin{equation} \left( \frac{V_u}{b_w d} \right)^2 + \left( \frac{T_u p h}{1.7 A_{oh}^2} \right)^2 \leq \phi_p \left( \frac{V_c}{b_w d} + 8 \sqrt{f_c'} \right) \end{equation}$

(b) For hollow sections

$\begin{equation} \left( \frac{V_u}{b_w d} \right) + \left( \frac{T_u p h}{1.7 A_{oh}^2} \right) \leq \phi_p \left( \frac{V_c}{b_w d} + 8 \sqrt{f_c'} \right) \end{equation}$

9.7.8.2 — For prestressed members, the value of $d$ used in 9.7.8.1 need not be taken less than 0.8 $h$. $\begin{equation} \end{equation}$

9.7.8.3 — For hollow sections where the wall thickness varies around the perimeter,

Eq. (9.7.8.1b) shall be evaluated at the location where the term $\left( \frac{V_u}{b_w d} \right) + \left( \frac{T_u p h}{1.7 A_{oh}^2} \right)$ is a maximum. $\begin{equation} \end{equation}$

9.7.8.4 — For hollow sections where the wall thickness is less than $\frac{A_{oh}}{P_h}$, the term $\left( \frac{T_u p h}{1.7 A_{oh}^2} \right)$ in Eq. (9.7.8.1b) shall be taken as $\left( \frac{T_u}{1.7 A_{oh} t} \right)$, where $t$ is the thickness of the wall of the hollow section at the location where the stresses are being checked. $\begin{equation} \end{equation}$