ACI Sub-Committee 544-E
Mechanical Properties of FRCs

Agenda

Monday, March 31, 2008
2:00 PM – 3:30 PM
Hyatt Regency Century Plaza
Los Angeles, CA
Room: Constellation 2

1. Call to Order and Approval of the Agenda

2. Introductions with Review of Membership and Task Assignments

3. **TASK 1:** Report on the “Indirect Method for Obtaining a Model Stress-Strain Curve of Strain Softening FRCs”
   - Balloting result
   - Discussion and resolution of ballot results

4. **Presentation:** “Approaches to Modeling FRC Under Blast Loads” by Andrew M. Coughlin, Masters Candidate at the Pennsylvania State University

   Analysis of FRC members is characterized by complexities not present in traditional concrete. Dynamic loading such as blast or impact introduces additional complexities with strain rate effects becoming significant. Two modeling techniques will be presented: (1) a finite element analysis (FEA) using the code LS-DYNA with concrete material models and empirically generated blast loads, and (2) an equivalent single degree of freedom (SDOF) generated by a cross section analysis, beam theory, and equivalent mass-spring loading. Use of both methods for a series of upcoming experimental blast tests will be discussed.

5. Performance Based FRC Classification and Related Nomenclature

6. Other topics of interest

7. New Business
   - Technical Sessions
   - Reports and Publications

8. Adjourn
# Report on “INDIRECT METHOD FOR OBTAINING A MODEL STRESS-STRAIN CURVE OF STRAIN SOFTENING FRCs”

## Approved Sections

<table>
<thead>
<tr>
<th>REPORT</th>
<th>Vote</th>
<th>Comments</th>
<th>Author</th>
<th>RESPONSE</th>
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</thead>
<tbody>
<tr>
<td>General</td>
<td>Ballot 103 - 11/2008</td>
<td>Looks too brief! All sections should be explained a little better, so that it looks like a code rather than a technical paper. Maybe ok as a draft.</td>
<td>Islam, Akm Anwarul</td>
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<tr>
<td>General</td>
<td>Ballot 103 - 11/2008</td>
<td>1. The document represents an early step toward establishing design procedures for FRC as a structural material, a noble cause indeed. In the current form, however, the methods seem very complex, and much of the document is incomprehensible to all but experts in the research community. Somehow, I think, an ACI document should strike a different tone. An ACI document must bridge the span, and communicate with informed engineers such as make up the body of Committee 544. I think the current draft falls short of that goal of speaking to a reasonably broad audience.</td>
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<tr>
<td>General</td>
<td>Ballot 103 - 11/2008</td>
<td>2. Since the audience seems to be limited to researchers, what is the benefit of this document beyond the currently available journal articles from which they are derived? At the beginning, the document includes the disclaimer “Finally, it should be noted that this document does not attempt to recommend or favour any of the presented methods.” But it seems like the document IS intended to recommend these methods.</td>
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<td>General</td>
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<td>3. Figure 3 uses the slope of the drop from peak load as an important parameter that defines region DbBZ. This rapid drop during a test is NOT a reliable material property. It is an unstable (or nearly unstable) moment of the test where the machine is quickly correcting for the initial crack propagation. Most literature assumes this slope is a vertical drop, and I would think that is an appropriate assumption.</td>
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<td>General</td>
<td>Ballot 103 - 11/2008</td>
<td>4. The appendices in this document are hard to navigate and understand. Section 3 refers several times to implementation in Excel, and Appendix A is titled “Spreadsheet-Based Inverse Analysis Procedure”, but neither the body of the document nor the Appendix tells the reader how to do this in a spreadsheet. [Should the spreadsheet itself be made available?]</td>
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<td>Page 2, lines 25 &amp; 26</td>
<td>Ballot 103 - 11/2008</td>
<td>NEGATIVE: This definition is not specimen size-independent. A paragraph needs to be added to define the specimen size used for obtaining the load deflection response. I suggest to use beams with 6” cross section since RILEM, EN and ASTM 1609 uses this specimen size. Smaller specimen sizes could be permissible if the specimens are cut to size from a beam with a cross section of at least 6”.</td>
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<td>Page 4, line 16</td>
<td>Ballot 103 - 11/2008</td>
<td>According to EN 14651 which is based on the RILEM procedure, the deflection should be 3.44 mm and not 3.0 mm for a CMOD of 4 mm</td>
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<td>Page 4, Figure 3</td>
<td>Ballot 103 - 11/2008</td>
<td>Negative: This figure indicates that the residual strength is based on the average energy. However, the adapted RILEM method in the EN standard defines the residual flexural strength as the strength obtained at a specific beam deflection or CMOD. Deflection is also 3.45 mm not 3.0 mm.</td>
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<td>Page 7, line 12</td>
<td>Ballot 103 - 11/2008</td>
<td>Note a thorough technical editing of this document is needed before it can be published. The following will be noted as examples:</td>
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## Negative Vote

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<td>Page 3, line 18</td>
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<td>page 3, line 43</td>
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<td>Page 4, Figure 2.</td>
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### REPORT

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<th>Comments</th>
<th>Author</th>
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<tbody>
<tr>
<td>5, Fig. 4.</td>
<td>ballot 103 - 11/2008</td>
<td>Regarding Fig. 4. Question to Barzin and co-authors. Did you use a “size factor” adjustment when you correlated beam test data with tensile data in your validation procedure? If so, you should mention it one way or the other. That is: “The validation given in Refs. Xx, used (or did not use) a size factor to accommodate the difference in cross section between tensile and bending specimens.” Figure 4 is important not only to warn the reader but as a given order of magnitude.</td>
<td>Naaman, Antoine</td>
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<td>5 – line 6</td>
<td>ballot 103 - 11/2008</td>
<td>Must number equations: “(6 - 1)”, “(6 - 2)”, etc. throughout per Style Manual.</td>
<td>Tamail, Peter</td>
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<td>5, line 13, formula</td>
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<td>“moment-curvature response. A spreadsheet based iterative inverse analysis approach presented herein”</td>
<td>Naaman, Antoine</td>
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<td>5, Figure 5a.</td>
<td>ballot 103 - 11/2008</td>
<td>Here again, it is important to point out “Equivalent strain”.</td>
<td>Naaman, Antoine</td>
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<td>6, line 24 &amp; 25</td>
<td>ballot 103 - 11/2008</td>
<td>“Equivalent flexural strength” is the term used in the JSCE, SF-4 test method. Seems confusing to use it here - could use: “[i.e., post-crack]”.</td>
<td>Tamail, Peter</td>
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<td>6, line 25</td>
<td>ballot 103 - 11/2008</td>
<td>“ASTM C78 and ASTM C293 are based on using un-notched third-point and center-point loading.”</td>
<td>Naaman, Antoine</td>
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<td>10, Line 18</td>
<td>ballot 103 - 11/2008</td>
<td>Page 10, Line 16. There is a reference to Chapter 6. There is no Chapter 6 in the report.</td>
<td>Naaman, Antoine</td>
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<td>10 – line 19</td>
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<td>How will the Committee distribute the spread sheet?</td>
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<td>Page 11, line 7</td>
<td>Ballot 103 – 11/2008</td>
<td>“… point loaded test beam, and (b) specific relationship developed by Rilem TC162-TDF committee that can...”</td>
<td>Gupta, Rishi</td>
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<td>Page 11, line 8</td>
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<td>“… be used for specific beam types presented in sections 6.2.”</td>
<td>Gupta, Rishi</td>
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<td>Page 11, Conclusions Section. Line 11</td>
<td>Ballot 103 – 11/2008</td>
<td>“Finally it should be noted that the purpose of this document is to provide the reader with complete guidelines necessary...”</td>
<td>Naaman, Antoine</td>
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<td>Page 11 – line 16</td>
<td>Ballot 103 – 11/2008</td>
<td>“the presented methods. Performance and comparison of the presented methods will be discussed in...”</td>
<td>Tatnall, Peter</td>
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<td>page 11 – line 20+</td>
<td>Ballot 103 – 11/2008</td>
<td>“References are listed in Cited References in 5.2.”</td>
<td>Tatnall, Peter</td>
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<td>Page 13, Appendix A</td>
<td>Ballot 103 – 11/2008</td>
<td>Appendix A. Although the solutions given in Appendix A are what is needed, they may be too overwhelming. Should we simply refer to a web site where a program can be downloaded to give the results?. The original reference remains of course available in the references.</td>
<td>Naaman, Antoine</td>
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<td>Figure 9</td>
<td>Ballot 103 – 11/2008</td>
<td>Figure should be removed or comments added since it is not realistic</td>
<td>Massicotte, Bruno</td>
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<td>Validation</td>
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<td>3. Validation against experimental data. There is a great need for validation in all these indirect methods. Validation implies that similar tests were carried out in tension and bending with sizes of cross-section of similar order. Very few such tests exists to my knowledge. Moreover, if many such tests existed, then it means that they are easy to carry out (the tensile part) and therefore there is little need to recourse to a bending test. I would like to see a special section titled “validation” prior to conclusions, addressing the issue. In it we should indicate that “Limited validation of these inverse method has been carried out by those who have developed them. However, validation by the technical profession at large has not been undertaken and will be one of the focus of future work of the committee. It is reasonable to assume that for a validation to be acceptable, series of tests of similar specimens should be carried out in both tension and bending with specimens of about equal cross sections...”</td>
<td>Naaman, Antoine</td>
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<td>New section on other methods</td>
<td>Ballot 103 – 11/2008</td>
<td>New section on other methods. To my knowledge Van Mier has also described an indirect procedure for interpreting bending tests into tensile response. Dr. Barros also mentioned the name of another researcher. The subcommittee should therefore prepare a special section about these other methods to explain them (since they are numerical no details need to be given) and refer the reader to the proper references.</td>
<td>Naaman, Antoine</td>
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Indirect Method for Obtaining a Model Stress-Strain Curve of Strain Softening FRCs

Reported by ACI Committee 544

This document presents existing methods for estimating uniaxial stress-strain response of strain softening FRCs using beam-test data. In this approach uniaxial compressive response is determined directly from standard compression cylinder tests. The uniaxial tensile response is calculated from test-beam data using an approach originally developed by Rilem TC162-TDF committee. The document both (a) provide specific formulas reported in the literature for various test beam types, (b) and summarizes an inverse-analysis method for obtaining the stress-strain values when test beams differ from those for which the formulas have been develop.

CONTENTS

Chapter 1 – General
1.1 – Scope
1.2 – General

Chapter 1 – Method
2.1 – General
2.2 – Stress-Strain Model
2.3 - Calculating Flexural Tensile Strength, $f_t$, and Residual Flexural Strengths, $f_{t-1}$ and $f_{t-4}$
2.3.1 - Flexural Tensile Strength, $f_t$
Chapter 1 – General

1.1 – Scope
The document covers existing methods for estimating uniaxial stress-strain response of strain softening FRCs using beam-test data. The methods are valid only for strain-softening FRCs, as defined in Figure 1.

![Figure 1: Classification of FRCs based on their tensile response [Naaman and Reinhardt 2005].](image)

1.2 – General
The document presents existing methods for obtaining uniaxial stress-strain curves from beam-test data. In this approach uniaxial compressive response is determined directly from standard compression cylinder tests. The uniaxial tensile response is calculated from test-beam data using an approach originally developed by Rilem TC162-TDF committee.

It should be noted that the Rilem TC162-TDF method specifically uses notched 3-point beam tests. The approach outlined herein is presented assuming that any general 3 or 4 point beam-test is used. Values of the necessary coefficients must then be determined from the experimental data using an inverse analysis method. The document both (a) summarizes specific coefficient values reported in the literature for various test beam types, (b) and provides an inverse-analysis method for obtaining the values when the specific coefficient values are not known.

The document outlines only the calculation method for determining a model stress-strain response. Statistical aspects of processing actual test data are not addressed here and will be presented elsewhere. It should also be noted that presented methods have already been validated against experimental data and, therefore, this validation is not presented herein. Additional information on this issue can be obtained from the referenced publications. Finally, it should be noted that this document does not attempt to recommend or favour any of the presented methods. Performance and comparison of the presented methods will be discussed in future publications.

The document first presents (a) a model stress-strain diagram, (b) calculation procedure for obtaining flexural tensile and residual flexural strengths from beam test data, and (c) a relationship between the stress-strain parameters and the experimentally determined flexural strengths. The relationship is presented in terms of stress coefficients $C_1$, $C_2$, $C_3$. The coefficients are to be determined using an inverse analysis procedure summarized in chapter 6. Specific coefficient values are provided next for the case of the notched RILEM TC 162-TDF and Belgian NBN B15-238 third-point beams. The coefficient values have not thus far been reported in the literature for other beam types, such as ASTM C293 and C78 test beams. Therefore, a complete, step-by-step inverse analysis procedure to be used for other beam types is provided last in Appendix A.

CHAPTER 2 - METHOD

2.1 - General
Various test methods exist for evaluating directly or indirectly the uniaxial-tensile properties of FRCs. These include uniaxial tensile tests, wedge splitting tests and beam tests. Because of the ease of use in conventional field laboratories, presented approach is based on the use of beam tests.

Common flexural-beam tests are performed under either (a) 3-point bending, such as prescribed by Rilem TC162-TDF and ASTM C293, or (b) 4-point bending conditions, such as prescribed in ASTM C78 (i.e., ASTM C1399 and C 1609 for FRC), DBV 1992, JCI-SF4, NBN B15-238 1992, etc. Beam tests can further be subdivided into notched beams, as is the case with the 3-point bending Rilem TC162-TDF test, or un-notched, as is the case with ASTM beams. Use of notched beams decreases significantly data scatter.

Further detailed description of each beam type and test procedure has been presented elsewhere and is, thus, beyond the scope of this document.

2.2 - Stress-Strain Model
The model stress-strain diagram used in this approach is the same as proposed by RILEM TC 162-TDF [Vandewalle 2003] and shown in Figure 2.
Figure 2: Model stress – strain diagram for FRCs in uniaxial tension and compression, according to RILEM TC 162-TDF [Vandewalle 2003].

The key points of the compression side of the diagram are obtained directly from the standard compressive cylinder test. The key points of the tension side of the diagram, i.e.:

- $\sigma_1$ and $\varepsilon_1$ – tensile strength and corresponding strain
- $\sigma_2$ and $\varepsilon_2$ – stress and strain at the onset of the stable strain softening branch
- $\sigma_3$ and $\varepsilon_3$ – stress and strain at the end of the softening branch

can be determined from the following stress values obtained from the load-displacement response of a test beam [Vandewalle 2000, 2002, Barros 2004]:

- (average) flexural tensile strength, $f_t$, reached at the beam at the peak beam strength, i.e., the limit of proportionality of the beam response,
- residual flexural strength at deflection of 0.46 mm, $f_r-1$, and
- residual flexural strength at deflection of 3.0 mm, $f_r-4$.

Expressions for first obtaining the flexural tensile strength, $f_t$, and residual flexural strengths, $f_r-1$ and $f_r-4$, and then obtaining tensile stress and strain values defined in Figure 2, are presented next.

2.3 - Calculating Flexural Tensile Strength, $f_t$, and Residual Flexural Strengths, $f_r-1$ and $f_r-4$

The flexural tensile strength, $f_t$, and residual flexural strengths, $f_r-1$ and $f_r-4$, values are calculated directly from the load displacement response of the test beam, shown in Figure 3.

Figure 3: Load – displacement response of a test beam, used in evaluating flexural tensile strength parameters [Vandewalle 2000, 2002, Barros 2004].
2.3.1 - Flexural Tensile Strength, $f_t$

The flexural tensile strength, $f_t$, is defined as the maximum tensile stress within the critical section at a load level equal to the limit of proportionality, $F_L$. The limit of proportionality is the load at the end of the linear-elastic response, as shown in Figure 3. However, if there is no clear end point to the linear-elastic portion of the load-displacement curve, the limit of proportionality is then defined as the highest load up to a deflection of 0.05 mm [Barros 2004].

Similar to the modulus of rupture, $f_r$, of the conventional concrete materials, the flexural tensile strength, $f_t$, is calculated assuming linear stress distribution within the critical section. When a three-point bending test beam is used, where variables $L$, and $b$ define span and width of the test beam, and $h_{unc}$ defines the uncracked section height, the flexural tensile strength, $f_t$, can be calculated as:

$$f_t := \frac{3F_L \cdot L}{2b \cdot h_{unc}^2}$$

In the case when a notched test beam is used, the uncracked section height, $h_{unc}$, equals the distance between the notch tip and the cross-section top.

2.3.2 - Residual Flexural Strengths, $f_{R-1}$ and $f_{R-2}$

Residual flexural strengths, $f_{R-1}$ and $f_{R-2}$, are obtained from beam loads $F_{R-1}$ and $F_{R-2}$ that correspond to beam deflections of 0.46 and 3.0 mm, respectively [Barros 2004], as shown in Figure 3. It should be noted that even though at this deflection levels both (a) the stress distribution is no longer linear within the critical section, and (b) the uncracked beam height is smaller than its value at the limit of proportionality, $h_{unc}$, the same relationship to that used for calculating the flexural tensile strength, $f_t$, is still used:

$$f_{R-1} := \frac{3F_{R-1} \cdot L}{2b \cdot h_{unc}^2}$$

$$f_{R-2} := \frac{3F_{R-2} \cdot L}{2b \cdot h_{unc}^2}$$

2.4 - Relationship Between Uniaxial-Tensile Stresses and Flexural Tensile Strengths

The flexural tensile strength, $f_t$, and residual flexural strengths, $f_{R-1}$ and $f_{R-2}$, can be directly related to the key tensile stresses in the model stress-strain diagram shown in Figure 2. The relationship is a function of the specimen type and size. The effect of these factors is introduced through:

a) stress coefficients $C_1$, $C_2$, $C_3$, calculated from test data using inverse analysis, and
b) a size-dependent safety factor $k_h$ defined in Figure 4.

**Figure 4:** Size-dependent safety factor, $k_h$, as defined by RILEM TC 162 TDF [Vandewalle 2003, Barros 2004]. Variable h is the full height of the test beam.
2.4.1 – Tensile Stress Values $\sigma_1$, $\sigma_2$, and $\sigma_3$

Tensile stresses values $\sigma_1$, $\sigma_2$, and $\sigma_3$, as defined in Figure 2, can now be calculated as:

\[
\sigma_1 := C_1 \left( 1.6 - \frac{d}{1.0m} \right) f_t
\]

\[
\sigma_2 := C_2 f_{R_1} \kappa_h
\]

\[
\sigma_3 := C_3 f_{R_4} \kappa_h
\]

Where all the variables are as defined above.

2.4.2 – Tensile Strain Values $\varepsilon_1$, $\varepsilon_2$, and $\varepsilon_3$

Tensile strain value $\varepsilon_1$, is obtained following the Hook’s law and using Young modulus of FRC, $E_c$, which is assumed to be the same in compression and tension, as shown in Figure 1, i.e.:

\[
\varepsilon_1 := \frac{\sigma_1}{E_c}
\]

All the other strain values, $\varepsilon_2$, and $\varepsilon_3$, must be determined from test data using an inverse analysis procedure.

CHAPTER 3 – EVALUATING STRESS COEFFICIENTS AND STRAIN PARAMETERS

3.1 – Inverse Analysis Procedure for a General Beam Type

The goal of inverse analysis is to determine material properties using experimentally determined moment-curvature response. A spreadsheet based iterative inverse analysis approach presented herein was developed by Soranakom and Mobasher (2006).

It should be noted that Soranakom and Mobasher (2006) used somewhat different nomenclature from that used in Figure 2. The relationship between the two is defined as:

\[
\sigma_1 = \sigma_{cr}
\]

\[
\sigma_2 = \sigma_3 = \sigma_p
\]

\[
\varepsilon_1 = \varepsilon_2 = \varepsilon_{CR}
\]

\[
\varepsilon_3 = \varepsilon_m
\]

The approach is based on a closed form solution connecting measured flexural response of FRC test beams to assumed material stress-strain response. Tensile stress-strain properties are obtained through an iterative process, as explained next. The procedure has been implemented in an Excel spreadsheet for ease of calculation.

Compressive stress-strain properties are determined from a separate compressive cylinder test. Tensile material parameters are assumed. The moment curvature diagram is generated next by incrementally increasing the top compressive strain, finding neutral axis and finally calculating moment and curvature. The procedure is repeated until predicted moment-curvature response matches measured values. The readers are advised to consult reference [Soranakom and Mobasher 2006] for in-depth information.
3.1.1 – Material Model
FRC material model used in Soranakom and Barzin’s [2006] is shown in Figure 5.

![Figure 5: FRC model used by Soranakom and Barzin’s [2006].](image)

Following nomenclature is used:
- cracking strength in tension \( \sigma_{cr} \)
- post cracking strength in tension \( \sigma_p \)
- ultimate tensile strain \( \varepsilon_{tu} \)
- yield strength in compression \( \sigma_{cy} \)
- corresponding yield strain in compression \( \varepsilon_{cy} \)
- ultimate compressive strain \( \varepsilon_{cu} \)

Ultimate tensile and compressive strains are used to limit the strength of the models. In this approach all strains and stresses are normalized in terms of the cracking strain \( \varepsilon_{cr} \) and cracking strength \( \sigma_{cr} = \varepsilon_{cr} E \), respectively. Corresponding normalized strain parameters \( \beta_{tu} \), \( \omega \), \( \lambda \), and \( \lambda_{cu} \) as well as stress parameters \( \mu \) and \( \omega \), are defined in Figure 5.

3.1.2 – Moment - Curvature Relationship
Normalized compressive strain at the top fiber \( \varepsilon_c \) is selected first. Values are assumed for three different phases: \( 0 < \lambda < 1 \), \( 1 < \lambda < \omega \), and \( \omega < \lambda < \lambda_{cu} \). The neutral axis depth ratio \( k \) is determined next using assumed stress strain diagram and imposing conditions of equilibrium. Corresponding moment of the tension - compression force couple taken around the neutral axis is determined next. Corresponding curvature is determined by dividing the top compressive strain with the neutral axis depth.

Moment \( M \) and curvature \( \phi \) are normalized next by dividing them with the cracking moment \( M_{cr} \) and cracking curvature \( \phi_{cr} \) respectively. Obtained values are termed normalized moment \( M' \) and curvature \( \phi' \), respectively.

Two simplified moment-curvature diagrams, as shown in Figure 6, are used to optimize the inverse analysis. The intersection points \( (\phi', M') \) of the linear elastic response and the linear post crack response is determined using the following regression equation:
\[ M' = \frac{(M/M_{cr})}{M'_{it}} \]

\[ M'_{it} = \frac{M'_{cr}}{M'_{bcr}} \]

\[ M'_{ucr} = \frac{M'_{cr}}{M'_{bcr}} \]

\[ \phi_{ucr}^{'} (= \phi_{bcr}^{'}) \]

\[ \phi_{it}^{'} (= \phi_{bcr}^{'}) \]

\[ (a) \]

\[ \phi = \begin{bmatrix} 0 \\ \phi_{bcr}^{'} \\ \phi_{ucr}^{'} \\ \phi_{it}^{'} \end{bmatrix} \]

\[ M = \begin{bmatrix} 0 \\ M'_{bcr} \\ M'_{cr} \\ M_{cr} \end{bmatrix} \]

\[ \text{where normalized bilinear cracking moment curvature is equal to the intersection point, } \phi_{bcr} = \phi_{it}^{'} \text{ and } M'_{bcr} = M'_{it}. \]

\[ \phi = \begin{bmatrix} 0 \\ \phi_{bcr}^{'} \\ \phi_{it}^{'} \\ \phi_{ucr}^{'} \end{bmatrix} \]

\[ M = \begin{bmatrix} 0 \\ M'_{bcr} \\ M'_{cr} \\ M'_{ucr} \end{bmatrix} \]

\[ \text{where normalized bilinear cracking moment curvature is equal to the original normalized cracking moment curvature, } \phi_{bcr}^{'} = \phi_{cr}^{'} = 1 \text{ and } M'_{bcr} = M'_{cr} = 1. \]

\[ M'_{cr2} = (1 - \xi) M'_{ucr} + \xi M'_{it} \text{ where } \xi = \frac{\phi_{cr}^{'} - \phi_{it}^{'}}{\phi_{ucr}^{'} - \phi_{it}^{'}} \]

**Figure 6:** Normalized moment curvature diagrams and their approximate bilinear models: (a) deflection hardening \((\mu > \mu_{crit})\); (b) deflection softening \((\mu < \mu_{crit})\).

\[ M'_{it} = 0.7425 M'_{ucr} + 0.1739 \] and \( \phi_{it}^{'} = M'_{it} \)

For deflection hardening, three controlling points are needed.

\[ 3.2 - \text{Specific Values when the Notched RILEM Beam is Used} \]

If notched RILEM TC162-TDF 3-point bending is used, the specimen can be approximated as shown in Figure 9.
In this case, the following stress coefficients and strain values can be adopted [Barros 2004].

$$C_1 := 0.52$$

$$C_2 := 0.36$$

$$C_3 := 0.27$$

$$\varepsilon_2 := 0.12\%$$

$$\varepsilon_3 := 10.4\%$$

### 3.3 - Specific Values when the Unnotched Belgian Standard Beam is Used

Nemegeer-Harelbeke [1998] used deformation-controlled bending test defined by Belgian Standard NBN B15-238 (Tests on Fiber-Reinforced Concrete – Bending Test on Prismatic Samples) to evaluate FRC material properties. The test is a 4-point bending test, with the span 0.45 m, as well as beam width and height of 0.15 m.

Contrary to the approach presented above in which case loads are recorded at deflection of 0.46 mm and 3.0 mm [Barros 2004], Belgian Standard NBN B15-238 records loads at deflection of 1.5 mm and 3 mm, i.e., $F_{R_{15}}$ and $F_{R_{30}}$, respectively. Corresponding residual (i.e., "equivalent") flexural strengths, $f_{R_{15}}$ and $f_{R_{30}}$, are calculated as:

$$f_{R_{15}} := \frac{3F_{R_{15}}L}{2b \cdot h_{unc}}$$

$$f_{R_{30}} := \frac{3F_{R_{30}}L}{2b \cdot h_{unc}}$$

In this approach it is assumed that once a beam has cracked, depth of the compressive zone equals 10% of the beam height. Hence, the equivalent flexural tensile stress in the post-cracking range can now be estimated as 37% of the corresponding linear-elastic stress. Resulting tensile stresses values $s_{1}$, $s_{2}$, and $s_{3}$, as defined in Figure 2, thus equal:

$$\sigma_1 := 0.2 \text{ MPa}$$

$$\left( \frac{f_{c}}{1 \text{ MPa}} \right)^2$$

$\sqrt{3}$
3.4 - Use of ASTM C-78 and C-293 Beams

ASTM C78 and ASTM C293 are based on using un-notched third-point and center-point loading, respectively, as shown in Figures 10 and 11. While these beams could also be used to obtain the uniaxial stress-strain response of FRC as described in Chapters 3 to 5, no values for the stress coefficients C₁, C₂ and C₃ have been reported in the literature. In this case the user must employ one of the standard inverse-analysis procedures described in chapter 6. For the reader’s benefit, one such spreadsheet-based inverse analysis procedures is described in detail in Appendix A. While the step-by-step process is provided in the appendix, the actual spreadsheet file can be obtained directly from the authors or the ACI 544 committee.

Figure 10: Third-point loading test layout used in the ASTM C78 method [ASTM C78].

Figure 14: Center-point loading test layout used in the ASTM C293 method [ASTM C293].

\[
\begin{align*}
\sigma_2 &= 0.37 f_{R_{\text{15}}} \\
\sigma_3 &= 0.37 f_{R_{\text{30}}} \\
\varepsilon_1 &= \frac{\sigma_1}{E_C} \\
\varepsilon_2 &= 0.1\% \\
\varepsilon_3 &= 1\%
\end{align*}
\]
CHAPTER 4 – CONCLUSIONS

This document presents existing methods for estimating uniaxial stress-strain response of strain softening FRCs using beam-test data. The methods are based on an inverse analysis of beam data. Validity of the presented approaches has been experimentally validated in the cited references. The document presents both (a) a complete spreadsheet-based inverse analysis that can be used for any center-point or third-point loaded test beam, and (b) specific relationship developed by Rilem TC162-TDF committee that can be used for specific beam types presented in sections 6.2. The relationship can also be used for other beam types, but the user must determine stress coefficients C_1, C_2 and C_3 using one of the described inverse analysis methods.

Finally it should be noted that the purpose of this document is to provide the reader with complete guidelines necessary for obtaining uniaxial stress-strain curves of strain softening FRCs from beam test data. To evaluate how well these methods perform as compared to experimental data, the reader is directed to the referenced publications. This document does not attempt to recommend or favour any of the presented methods. Performance and comparison of the presented methods will be discussed in future publications.

CHAPTER 5 – REFERENCES

ASTM C78  ASTM Committee C09 on Concrete and Concrete Aggregates, Standard Test Method for Flexural Strength of Concrete (Using Simple Beam with Third-Point Loading), ASTM International, 2007.

ASTM C293  ASTM Committee C09 on Concrete and Concrete Aggregates, Standard Test Method for Flexural Strength of Concrete (Using Simple Beam with Center-Point Loading), ASTM International, 2007.


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Vandewalle 2000a  

Vandewalle 2000b  

Vandewalle 2002a  

Vandewalle 2002b  

Vandewalle 2003  

Vandewalle 2002  

Vandewalle 2003  
APPENDIX A: SPREADSHEET-BASED INVERSE ANALYSIS PROCEDURE

A.1 - Derivation of the Moment-Curvature Diagram

Moment curvature diagram is derived by first drawing stress strain diagram according to the applied normalized compressive strain at the top fiber $\lambda$ in 3 stages: $(0<\lambda\leq 1)$, $(1<\lambda<\omega)$ and $(\omega<\lambda\leq \lambda_{cu})$. The neutral axis depth ratio $k$ is then found by solving equilibrium of forces. Finally the moment capacity is obtained by dividing the top compressive strain with the neutral axis depth. Moment $M$ and curvature $\phi$ are then normalized with their cracking moment $M_{cr}$ and cracking curvature $\phi_{cr}$ to obtained a normalized moment $M'$ and curvature $\phi'$, respectively. Note that, from now on, the primed terms ($M'$ or $\phi'$) refer to the normalized quantities with respect to cracking moment $M_{cr}$ or cracking curvature $\phi_{cr}$. For example $M'_{cr} = M_{cr}/M_{cr} = 1$ and $\phi'_{cr} = \phi_{cr}/\phi_{cr} = 1$. The expressions for calculating moment curvature and neutral axis for three stages of applied top compressive strain $\lambda$ are given in Eq. (1) - (5).

$$M(\lambda, k, \omega, \mu) = M_{cr}, \quad M'(\lambda, k, \omega, \mu) = \frac{1}{6}bd^2E\varepsilon_{cr}$$

$$\phi(\lambda, k, \omega, \mu) = \phi_{cr}, \quad \phi'(\lambda, k, \omega, \mu) = \frac{2E_{cr}}{d}$$

$$M'(\lambda, k, \omega, \mu) = \begin{cases} \frac{\lambda}{2k} & \text{for } 0 \leq \lambda \leq 1 \\ \frac{(2\lambda^3 + 3\mu\lambda^2 - 3\mu + 2)k^2}{\lambda^2} - 3\mu(2k - 1) & \text{for } 1 < \lambda \leq \omega \\ \frac{(3\mu\lambda^2 + 3\omega\lambda^2 - 3\mu - \omega^3 + 2)k^2}{\lambda^2} - 3\mu(2k - 1) & \text{for } \omega < \lambda \leq \lambda_{cu} \end{cases}$$

$$\phi'(\lambda, k, \omega, \mu) = \frac{\lambda}{2k}$$

$$k = \begin{cases} \frac{1}{2} & \text{for } 0 \leq \lambda \leq 1 \\ \frac{2\mu\lambda}{\lambda^2 + 2\mu(\lambda + 1) - 1} & \text{for } 1 < \lambda \leq \omega \\ \frac{2\mu\lambda}{-\omega^2 + 2\lambda(\omega + \mu) + 2\mu - 1} & \text{for } \omega < \lambda \leq \lambda_{cu} \end{cases}$$

For a given set of material parameters and dimension of the beam section, the moment curvature diagram can be generated by substituting an incremental normalized top compressive strain $\lambda$ from zero up to failure in Eq. (1) - (5). Two possible moment curvature responses, deflection hardening ($\mu > \mu_{crit}$) and deflection softening ($\mu < \mu_{crit}$), are shown by solid curves in Figure 6. The critical value for normalized post peak tensile strength is given in Eq. (6).

$$\mu_{crit} = \frac{\omega}{3\omega - 1}$$

In calculation of moment curvature diagram, the ultimate normalized top compressive strain $\lambda_u$ needs to be identified by comparing the normalized top compressive strain at compressive failure $\lambda_{cu}$ and the normalized top compressive strain at tensile failure $\lambda_{tu}$. The smaller of these two limits the flexural capacity.

$$\lambda_u = \min(\lambda_{cu}, \lambda_{tu})$$

where
The normalized ultimate moment $M'_u$ is then be found by using Eq (3), (5), (7) and (8). Table A1 presents a summary of the depth of Neutral axis, the normalized moment, and the curvature for general loading condition as a function of various tensile post peak strength levels, $\mu$.

### A.1.1 – Simplified Moment Curvature Diagram

Figure 6 show the simplification of normalized moment curvature diagrams to normalized bilinear moment curvature diagrams for deflection hardening and deflection softening. In the simplified models, the intersection points $(\phi'_b, M'_b)$ of the linear elastic response and the linear post crack response is found by using a regression equation (9). It should be noted that the regression equation is dimensionless, independent of the unit used.

$$M'_{it} = 0.7425 M'_u + 0.1739$$

For deflection hardening, three controlling points are needed.

$$\phi = \begin{cases} 0 \\ \phi'_b \\ \phi'_u \end{cases}, \quad M = \begin{cases} 0 \\ M'_b \\ M'_{u} \end{cases}$$

where normalized bilinear cracking moment curvature is equal to the intersection point, $\phi'_b = \phi'_t$ and $M'_b = M'_t$. For deflection softening, four controlling points are needed.

$$\phi = \begin{cases} 0 \\ \phi'_b \\ \phi'_b \\ \phi'_u \end{cases}, \quad M = \begin{cases} 0 \\ M'_b \\ M'_{cr} \\ M'_{cr2} \end{cases}$$

where normalized bilinear cracking moment curvature is equal to the original normalized cracking moment curvature, $\phi'_b = \phi'_c = 1$ and $M'_b = M'_c = 1$. The additional normalized reduced cracking moment $M'_{cr2}$ at normalized cracking curvature $\phi'_c$ is obtained by linear interpolate between $\phi'_t$ and $\phi'_u$.

$$M'_{cr2} = (1 - \xi) M'_{tu} + \xi M'_{tu}$$

where

$$\xi = \frac{\phi'_c - \phi'_u}{\phi'_u - \phi'_t}$$

### A.1.2 – Load – Deflection Response

Figure A1 shows the set up for three and four point bending tests. Figures A1(b) and (c) show their moment distributions at cracking and ultimate levels. With the area moment method, the corresponding curvature diagrams shown in Figures A1 (c) - (e) are divided into several areas and taken around the left support to obtain the mid-span deflection $\delta$. A set of equations for calculating mid span deflection of three point bending at first bilinear cracking, at ultimate when material has $\mu > \mu_{crit}$ and at ultimate when material has $\mu < \mu_{crit}$ are presented in Eqs. 13(a) – 13(c).
\[ \omega = \begin{cases} 1 \\ \frac{3}{2} \end{cases} \ \\ \mu = \begin{cases} 0.75 \\ 3.5 \lambda - 0.5 \\ \frac{\lambda}{3 \lambda - 1} \\ \frac{\lambda}{5 \lambda - 3} \end{cases} \ \\ \mu = 0.5 \ \\ \mu = 0.25 \ \\ \mu = 0.1 \]

<table>
<thead>
<tr>
<th>\kappa</th>
<th>\omega</th>
<th>\mu</th>
<th>0.75</th>
<th>0.5</th>
<th>0.25</th>
<th>0.1</th>
</tr>
</thead>
</table>
| 1 | \frac{1}{2} | \frac{1.5 \lambda}{3.5 \lambda - 0.5} | \frac{\lambda}{3 \lambda - 1} | \frac{\lambda}{5 \lambda - 3} | \frac{0.2 \lambda}{2.2 \lambda - 1.8} | \ 
| 5 | \frac{\lambda}{6 \lambda - 12} | \frac{1.5 \lambda}{11.5 \lambda - 24.5} | \frac{\lambda}{11 \lambda - 25} | \frac{\lambda}{21 \lambda - 51} | \frac{0.2 \lambda}{10.2 \lambda - 25.8} | \ 

\[ \frac{M}{\lambda^2} \left( 3(\omega + \mu) \lambda^2 - \omega^3 - 3 \mu + 2 \right) \kappa^2 - 6 \mu \kappa + 3 \mu \]

<table>
<thead>
<tr>
<th>\varphi</th>
<th>\omega</th>
<th>\mu</th>
<th>0.75</th>
<th>0.5</th>
<th>0.25</th>
<th>0.1</th>
</tr>
</thead>
</table>
| 1 | \frac{\lambda}{2 \kappa} | \frac{3.5 \lambda - 0.5}{3} | \frac{3 \lambda - 1}{2} | \frac{5 \lambda - 3}{2} | \frac{22 \lambda - 18}{4} | \ 
| 5 | \frac{3 \lambda - 6}{2 \kappa} | \frac{11.5 \lambda - 24.5}{3} | \frac{11 \lambda - 25}{2} | \frac{21 \lambda - 51}{2} | \frac{10.2 \lambda - 25.8}{0.4} | \ 

**Table A1:** Representations for the depth of Neutral axis, Moment and curvature as a function of applies strain beyond the first cracking point (\( \epsilon = \frac{\epsilon}{\epsilon_o} \)) bilinear compression, bilinear tension
\[ \delta_{bcr} = \frac{1}{12} L^2 \phi_{bcr} \]  
(13.a)

\[ \delta_u = \frac{L^2}{24 M_u^2} \left[ \left( 2 M_u^2 - M_u M_{bcr} - M_{bcr}^2 \right) \phi_u + \left( M_u^2 + M_u M_{bcr} \right) \phi_{bcr} \right] \quad \mu > \mu_{crit} \]  
(13.b)

\[ \delta_u = \frac{\phi_u L_p}{8} \left( 2L - L_p \right) + \frac{M_u \phi_{bcr} L}{12 M_{bcr}} \left( L - 2L_p \right) \quad \mu < \mu_{crit} \]  
(13.c)

Similarly, a set of equations for four point bending can be written as:

\[ \frac{23}{216} L^2 \phi_{bcr} \]  
(14.a)

\[ \delta_u = \frac{L^2}{216 M_u^2} \left[ \left( 23 M_u^2 - 4 M_u M_{bcr} - 4 M_{bcr}^2 \right) \phi_u + \left( 4 M_u^2 + 4 M_u M_{bcr} \right) \phi_{bcr} \right] \quad \mu > \mu_{crit} \]  
(1.b)

\[ \delta_u = \frac{5 L^2 \phi_u}{72} + \frac{M_u L^2 \phi_{bcr}}{27 M_{bcr}} \quad \mu < \mu_{crit} \]  
(2.c)

Load step \( P_i \) can be back calculated from a discrete point \( i \) along the moment curvature diagram as follows.

\[ P_i = \frac{2 M_i}{S} \quad \text{for} \quad \phi_i = 0...\phi_u \]  
(20)

where \( S \) is a spacing between the support and loading point, \( S = L/2 \) for three point bending and \( S = L/3 \) for four point bending.

### A.1.3 - Example: Three Point Bending Test

Determine the moment curvature diagram and load deflection response of a beam size 100x100 mm tested under three point bending at clear span \( L = 300 \) mm. Assume that the plastic length for crack localized zone under the point load \( L_p = 100 \) mm. The ultimate uniaxial compressive strength \( f'_c = 30 \) MPa, uniaxial tensile strength \( \sigma_{cr} = 2.50 \) MPa and post peak tensile strength \( \sigma_p = 0.75 \) MPa. The ultimate compressive strain \( \varepsilon_{cu} = 0.003 \) and the ultimate tensile strain \( \varepsilon_{tu} = 0.02 \). The Young modulus \( E = 25,000 \) MPa.

In the Excel worksheet, users need to provide the input parameters in the green highlighted cells. Other cells will automatically calculate according to the user input. The unit used is a consistent unit.

From the information given above, enter: \( test \ method = 3, \ b = 100 \ mm, \ d = 100 \ mm, \ L = 300 \ mm, \ L_p=100 \ mm, \ E = 25000 \ MPa. \)

Other parameters can be calculated as follows.
- Cracking strain \( \varepsilon_{cr} = \sigma_{cr} / E = 2.5/25000 = 0.0001 \)
- Normalized post peak tensile strength, \( \mu = \sigma_p/\sigma_{cr} = 0.75/2.5 = 0.30 \)
- Normalized ultimate tensile strain, \( \beta_{tu} = \varepsilon_{tu}/\varepsilon_{cr} = 0.02/0.0001 = 200 \)
- Normalized ultimate compressive strain, \( \lambda_{cu} = \varepsilon_{cu}/\varepsilon_{cr} = 0.003/0.0001 = 30 \)
- Assume compressive yield stress \( \sigma_{cy} = 0.8*f'_c = 0.8*30 = 24 \) MPa
- Compressive to tensile strain ratio, \( \omega = \sigma_{cy} / (E\varepsilon_{cr}) = 24/2.5 = 9.6 \)
Figure A1: Three and four point bending test: (a) Experimental setup; (b) Moment distribution; (c) Curvature distribution at first bilinear cracking; (d) Curvature distribution at ultimate moment for high normalized post peak tensile strength ($\mu > \mu_{\text{crit}}$); (e) Curvature distribution at ultimate moment for low normalized post peak tensile strength ($\mu < \mu_{\text{crit}}$).

According to the material parameters input, the tensile and compressive stress strain responses are calculated and plotted in Figure A2. The critical normalized post peak tensile strength $\mu_{\text{crit}}=0.345$ is calculated by Eq. (6). Since $\mu = 0.30 < \mu_{\text{crit}}$, the worksheet shows type 2 (deflection softening). According to the normalized ultimate tensile failure $\beta_{tu}$, the corresponding normalized top compressive strain $\lambda_{tu}$ is checked by Eq. (8), which shows that the compressive strain fails at stage 2 ($1 < \lambda \leq \omega$). $\lambda_{tu} < \lambda_{cu}$ means that the ultimate tensile strain will reach the failure before the compressive strain crushing. The
smaller of $\lambda_{tu}$ and $\lambda_{cu}$ is used as a normalized ultimate compressive strain $\lambda_u$ in the calculation of neutral axis depth ratio $k_u$ by Eq. (5).

The normalized ultimate moment $M'_{u}$ and curvature $\phi'_{u}$ are calculated by Eqs. (3) and (4), respectively. The intersection points $(\phi'_{it}, M'_{it})$ of the linear elastic and the linear post crack response for bilinear moment curvature diagram is found by regression Eq. (9). The normalized cracking moment curvature always equal to one, $(\phi'_{cr} = M'_{cr} = 1)$. Because the material is deflection softening, the normalized reduced cracking moment $M'_{cr2}$ is needed and calculated by Eq. (12). The four controlling points for deflection softening moment curvature diagram for Eq. (11) are completed.

The moment curvature response is recovered by multiplying the cracking moment curvature to their normalized moment curvature using Eqs. (1), (2) and (11). The deflections of three point bending for the deflection softening $\mu < \mu_{crit}$ are calculated by Eqs. 13(a), 13(c) and the load is calculated by Eq. (20). Note that the deflections at the cracking are the same but the load step due to $M_{bcr}$ and $M_{cr2}$ are different.

Alternatively, the accurate moment curvature diagram can be done directly by using Eqs. (1) - (8). Figure A 3 compares the accurate moment curvature diagram and their approximate bilinear models. The load deflection response according to bilinear models is shown in Figure A4.

For four point bending test, the procedure is almost the same except that $L_p$ is not needed, users can enter any value or simply zero.
Proposed Performance Based Classification and Related Nomenclature for all FRC Composites

Antoine E. Naaman

1. Introduction-Motivation

The constitutive properties of structural materials are essential for modeling structural response for analysis, evaluation, and design. This is particularly true for the tensile properties of fiber reinforced concrete should its contribution be considered in the design of structural concrete members.

2. Proposed Classification

All FRC composites can be simply classified according to two categories depending on their tensile stress-elongation response as follows: Strain Hardening or Strain Softening. Strain hardening behavior is generally accompanied by multiple cracking. This performance based specification can be achieved by any combination of parameters at the fiber, matrix and their interface bond properties.

Figure 1 summarizes the classification and Figures 2a and 2b illustrate, respectively, typical stress-elongation response curves in direct tension for a strain softening and a strain hardening FRC composite.

3. Implication for Bending Response

The bending response of FRC composites is essentially controlled by their tensile response. This is because the compressive strength of concrete is typically an order of magnitude higher than its tensile strength, and bending failure starts by tensile failure. The general relationship that ties together tensile and bending behavior of FRC composites is illustrated in Fig. 3. Note that while the tensile response represents a fundamental property of the composite, the bending response (while not fundamental in nature) is related to the most common applications of fiber reinforced cement and concrete composites. It is noted that all tension “strain-hardening” FRC composites are expected to be “deflection-hardening” while some tension “strain-softening” can lead to “deflection-hardening.” The mechanical condition for a tension strain-softening material to lead to a deflection-hardening behavior is available and has been validated by experiments.
Figure 1: Proposed general classification of FRC composites based on their tensile response.

Figure 2: Typical stress-elongation curve of FRC composites in tension. (a) Strain-softening FRC composite, and (b) Strain-hardening FRC composite.
Figure 3: Relation between tensile and bending behavior of FRC composites.

4. Proposed Nomenclature and Acronyms

- **FRC composite or FRCC**: Fiber Reinforced Cement (or Concrete) composite. This nomenclature has been already widely used in prior ACI 544 reports.
- **SH-FRC composite or SH-FRCC**: Strain Hardening FRC composite. This refers to the fundamental classification described earlier.
- **SS-FRC composite or SS-FRCC**: Strain Softening FRC composite. This refers to the fundamental classification described earlier.
- **DH-FRC composite or DH-FRCC**: Deflection Hardening FRC composite.
- **DS-FRC composite or DH-FRCC**: Deflection Hardening FRC composite.