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Strut-and-Tie Method Guidelines for ACI 318-19— Guide

Reported by Joint ACI-ASCE Committee 445

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Strut-and-Tie Method Guidelines for ACI 318-19—Guide

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American Concrete Institute
38800 Country Club Drive
Farmington Hills, MI 48331
Phone: +1.248.848.3700
Fax: +1.248.848.3701

www.concrete.org

Strut-and-Tie Method Guidelines for ACI 318-19—Guide

Reported by Joint ACI-ASCE Committee 445

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Biroł Fitik
Josef Hegger
Patrick Huber
Karin Reissen
Udo Wiens
Konrad Zilch

*Subcommittee 445-A Members who Developed this Guide

†Chair of Subcommittee 445-A

Additional Subcommittee 445-A Members not noted above:

Sergio Brena
I-Kuang Fang
Katrin Habel

Matthew Huizinga
Carin Roberts-Wollmann
Rafael Alves de Souza

Bozidar Stojadinovic
Nancy Larson Verney
Fernando Yanez

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Strut-and-tie models (STMs) were first used at the end of the nineteenth century as a concrete design method. The method was added to ACI 318 in 2002 as Appendix A. In 2014, STM provisions were moved into the main body of the code as Chapter 23, Strut-and-Tie Method. This document focuses on the ACI 318-19 implementation of strut-and-tie modeling. The main objectives of this document are to: 1) explain the intent and application of ACI 318 STM provisions; 2) provide additional design guidance for the STM based on other design codes, specifications, and committee documents; and 3) present design recommendations from recent research publications. This document provides practical guidance to the structural design community.

Keywords: D-regions; design; disturbed regions; model; node; strut; tie.

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CONTENTS**CHAPTER 1—INTRODUCTION, p. 2**

- 1.1—Background, p. 2
- 1.2—Objectives, p. 2

CHAPTER 2—NOTATION AND DEFINITIONS, p. 2

- 2.1—Notation, p. 2
- 2.2—Definitions, p. 4

CHAPTER 3—OVERVIEW OF THE STRUT-AND-TIE MODEL DESIGN PROCEDURE, p. 5

- 3.1—Introduction, p. 5
- 3.2—Overview of the strut-and-tie method, p. 5
- 3.3—Conceptual example, p. 6
- 3.4—Historical development of the strut-and-tie method, p. 9

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CHAPTER 4—DESIGN STRENGTH OF STRUTS, TIES, AND NODES, p. 10

- 4.1—Overview, p. 10
- 4.2—Strut and node types and factors influencing strength, p. 11
- 4.3—STM strength requirements, p. 14
- 4.4—Strength and requirements for struts, p. 14
- 4.5—Strength and requirements for ties, p. 17
- 4.6—Strength and requirements for nodes, p. 19

CHAPTER 5—MODELING AND DESIGN CONSIDERATIONS, p. 23

- 5.1—Introduction, p. 23
- 5.2—Influence of model topology on performance of complex D-regions, p. 23
- 5.3—Guidance to selection of STM shapes, p. 25
- 5.4—Calculating member forces and stresses in complex D-regions, p. 28
- 5.5—Comparison of sectional and STM design of transverse reinforcement in a B-region, p. 30
- 5.6—Refined models for cracking of struts and the size effect, p. 31
- 5.7—States of stress in nodes, p. 32
- 5.8—Maximum shear stress limits in beams, p. 34
- 5.9—Member dimensions and constitutive relationships, p. 36
- 5.10—Reinforcement arrangement in relation to tie orientation, p. 37
- 5.11—Consideration of prestressing, p. 37
- 5.12—Capacity evaluation of existing structures, p. 39
- 5.13—Computer-based design aids, p. 40

CHAPTER 6—SERVICEABILITY CONSIDERATIONS, p. 41

- 6.1—Introduction, p. 41
- 6.2—Cracking at service limit states, p. 41
- 6.3—Determination of required distributed reinforcement, p. 42
- 6.4—Distributed reinforcement placement and detailing, p. 42
- 6.5—Deflections, p. 43

CHAPTER 7—IMPLEMENTATION OF STM DESIGN AND CONSTRUCTION DETAILS, p. 44

- 7.1—Introduction, p. 44
- 7.2—Deep beams, p. 44
- 7.3—Frame corners and beam-column joints, p. 46
- 7.4—Corbels, p. 48
- 7.5 Dapped-end beams, p. 50
- 7.6—Beams supporting beams (indirect supports), p. 53
- 7.7—Inverted T-beams, p. 56
- 7.8—Walls and beams with openings, p. 58
- 7.9—Coupling beams, p. 60
- 7.10—Footings, p. 62
- 7.11—Pile caps, p. 62
- 7.12—Post-tensioned anchorage zones, p. 64

CHAPTER 8—OTHER STM DESIGN SPECIFICATIONS, CODES, AND GUIDE DOCUMENTS, p. 66

- 8.1—Introduction, p. 66
- 8.2—Strut-and-tie design requirements in CSA A23.3-19, p. 66
- 8.3 American Association of State Highway and Transportation Officials (AASHTO), p. 69
- 8.4—FIP Recommendations (FIP 1999), p. 71
- 8.5—Eurocode 2, p. 74
- 8.6—*fib* Model Code for Concrete Structures 2010 (*fib* 2013), p. 75

CHAPTER 9—REFERENCES, p. 75

- 9.1—Referenced standards and reports, p. 76
- 9.2—Authored references, p. 76

CHAPTER 1—INTRODUCTION

1.1—Background

While the origins of strut-and-tie models date back to the end of the nineteenth century (Ritter 1899; Considère 1900; Mörsch 1909), the strut-and-tie design method was not formally introduced into ACI 318 until 2002. Up until the 2014 version of ACI 318, strut-and-tie model (STM) provisions were included in Appendix A of ACI 318. With the reorganization of ACI 318 in 2014, STM provisions were moved into the main body of the code as Chapter 23, in part to recognize the importance of this method in structural design practice. Subsequently, significant changes to the STM provisions occurred in ACI 318-19. Chapter 23 was renamed to “Strut-and-Tie Method.” The abbreviation of STM only stands for strut-and-tie model.

In an effort to provide information on the use of strut-and-tie models in structural design, explanatory notes are included with the ACI 318 provisions. Additionally, two ACI special publications—ACI SP-208 and ACI SP-273—were developed by Subcommittee A, Strut & Tie, of Joint ACI-ASCE Committee 445, Shear and Torsion, in which the application of STM was illustrated through various design examples. Nevertheless, there is a need to provide additional guidance on the use of STM techniques.

1.2—Objectives

This document focuses on the ACI 318-19 implementation of strut-and-tie modeling. The main objectives of this document are to: 1) explain the intent and application of ACI 318 STM provisions; 2) provide additional design guidance for the STM based on other design codes, specifications, and committee documents; and 3) present design recommendations from recent research publications. This document is intended to provide practical guidance to the structural design community.

CHAPTER 2—NOTATION AND DEFINITIONS

2.1—Notation

This section defines notations used in this guide.

A_1	= loaded area for consideration of bearing strength, in. ² (mm ²)	F_{nn}	= nominal strength at face of a nodal zone, lb (N)
A_2	= area of the lower base of the largest frustum of a pyramid, cone, or tapered wedge contained wholly within the support and having its upper base equal to the loaded area. The sides of the pyramid, cone, or tapered wedge should be sloped one vertical to two horizontal, in. ² (mm ²)	F_{ns}	= nominal strength of a strut, lb (N)
A_b	= area of an individual bar or wire, in. ² (mm ²)	F_{nt}	= nominal strength of a tie, lb (N)
A_{core}	= cross-sectional area of a member measured to the outside edges of transverse reinforcement, in. ² (mm ²)	F_u	= factored force acting in a strut, tie, or nodal zone in a strut-and-tie model, lb (N)
A_{cs}	= cross-sectional area at one end of a strut in a strut-and-tie model, taken perpendicular to the axis of the strut, in. ² (mm ²)	f_c	= compressive stress in concrete, psi (MPa)
A_{cv}	= gross area of concrete section bounded by web thickness and length of section in the direction of shear force considered in the case of walls, and gross area of concrete section in the case of diaphragms. Gross area is total area of the defined section minus area of any openings, in. ² (mm ²)	f_c'	= specified compressive strength of concrete, psi (MPa)
A_g	= gross area of concrete section, in. ² (mm ²)	f_{ce}	= effective compressive strength of concrete in a strut or a nodal zone, psi (MPa)
A_{nz}	= area of a face of a nodal zone or a section through a nodal zone, in. ² (mm ²)	f_{pu}	= specified tensile strength of prestressing reinforcement, psi (MPa)
A_{ps}	= area of prestressing reinforcement in flexural tension zone, in. ² (mm ²)	f_{py}	= specified yield strength of prestressing reinforcement, psi (MPa)
A_s	= area of nonprestressed longitudinal tension reinforcement, in. ² (mm ²)	f_r	= modulus of rupture of concrete, psi (MPa)
A_s'	= area of compression reinforcement, in. ² (mm ²)	f_s	= calculated tensile stress in reinforcement at service loads, psi (MPa)
A_{tp}	= area of prestressing reinforcement in a tie, in. ² (mm ²)	f_s'	= stress in compression reinforcement under factored loads, psi (MPa)
A_{ts}	= area of nonprestressed reinforcement in a tie, in. ² (mm ²)	f_{se}	= effective stress in prestressing reinforcement (after allowance for all prestress losses), psi (MPa)
A_v	= area of shear reinforcement within spacing s , in. ² (mm ²)	f_{si}	= stress in the i -th layer of surface reinforcement, psi (MPa)
A_{vd}	= total area of reinforcement in each group of diagonal bars in a diagonally reinforced coupling beam, in. ² (mm ²)	f_y	= specified yield strength of reinforcement, psi (MPa)
$A_{v,min}$	= minimum area of shear reinforcement within spacing s , in. ² (mm ²)	f_{yt}	= specified yield strength of transverse reinforcement, psi (MPa)
a	= shear span to near support, in. (mm)	h	= overall thickness or height of member, in. (mm)
b	= shear span to far support, in. (mm)	I	= moment of inertia of section about centroidal axis, in. ⁴ (mm ⁴)
b_s	= effective width of strut, in. (mm)	I_{cr}	= moment of inertia of cracked section transformed to concrete, in. ⁴ (mm ⁴)
b_w	= web width or wall thickness, in. (mm)	I_e	= effective moment of inertia for computation of deflection, in. ⁴ (mm ⁴)
C	= compression force acting on a nodal zone, lb (N)	I_g	= moment of inertia of gross concrete section about centroidal axis, neglecting reinforcement, in. ⁴ (mm ⁴)
D	= effect of service dead load	L	= effect of service live load
d	= distance from extreme compression fiber to centroid of longitudinal tension reinforcement, in. (mm)	ℓ	= span length of beam or one-way slab; clear projection of cantilever, in. (mm)
d'	= distance from extreme compression fiber to centroid of longitudinal compression reinforcement, in. (mm)	ℓ_{anc}	= length along which anchorage of a tie should occur, in. (mm)
E	= effect of horizontal and vertical earthquake-induced forces	ℓ_b	= width of bearing, in. (mm)
E_c	= modulus of elasticity of concrete, psi (MPa)	ℓ_c	= segment length of curved reinforcement, in. (mm)
E_s	= modulus of elasticity of reinforcement and structural steel, psi (MPa)	ℓ_d	= development length in tension of deformed bar, deformed wire, plain and deformed welded wire reinforcement, or pretensioned strand, in. (mm)
F_n	= nominal strength of a strut, tie, or nodal zone, lb (N)	ℓ_{dc}	= development length in compression of deformed bar and deformed wire, in. (mm)
		ℓ_{dh}	= development length in tension of deformed bar or deformed wire with a standard hook, measured from critical section to outside end of hook (straight embedment length between critical section and start of hook [point of tangency] plus inside radius of bend and one bar diameter), in. (mm)
		ℓ_{dt}	= development length in tension of headed deformed bar, measured from the critical section to the bearing face of the head, in. (mm)

ℓ_n = length of clear span measured face-to-face of supports, in. (mm)
 ℓ_s = effective length of node, in. (mm)
 N_u = factored axial force normal to cross section; to be taken as positive for compression and negative for tension, lb (N)
 R = reaction, lb (N)
 r_b = bend radius at the inside of a bar, in. (mm)
 s = center-to-center spacing of items, such as longitudinal reinforcement, transverse reinforcement, prestressing tendons, or wires, in. (mm)
 s_i = center-to-center spacing of reinforcement in the i -th layer adjacent to the surface of the member, in. (mm)
 T = tension force acting on a nodal zone, lb (N)
 W = effect of wind load
 w_a = effective height of concrete compressive stress region, used to dimension nodal zone, in.
 w_c = density (unit weight) of normalweight concrete or equilibrium density of lightweight concrete, lb/ft³ (kg/m³)
 w_s = width of a strut perpendicular to the axis of the strut and in the plane of the strut-and-tie model, in. (mm)
 w_t = effective height of concrete concentric with a tie, used to dimension nodal zone, in.
 $w_{t,max}$ = maximum effective height of concrete concentric with a tie, in. (mm)
 w_u = factored load per unit length of beam or one-way slab, lb/in. (kg/m)
 α = angle defining the orientation of reinforcement
 α_i = angle between the axis of a strut and the bars in the i -th layer of reinforcement crossing that strut
 β_1 = factor relating depth of equivalent rectangular compressive stress block to depth of neutral axis
 β_c = confinement modification factor for struts and nodes in a strut-and-tie model
 β_n = factor to account for the effect of the anchorage of ties on the effective compressive strength of a nodal zone
 β_s = factor to account for the effect of cracking and confining reinforcement on the effective compressive strength of the concrete in a strut
 Δf_p = increase in stress in prestressing reinforcement due to factored loads, psi (MPa)
 ϵ_1 = principal tensile strain in cracked concrete due to factored loads, in./in. (mm/mm)
 θ = angle between axis of strut, compression diagonal, or compression field and the tension chord of the member
 θ_c = the smaller of the two angles between the strut (or the resultant of two or more struts) and ties extending from a curved-bar node
 θ_{cr} = angle of diagonal cracking relative to the longitudinal axis of a member
 θ_v = difference between angle of diagonal cracking and compression
 λ = modification factor reflecting the reduced mechanical properties of lightweight concrete, all relative

to normalweight concrete of the same compressive strength

λ_s = factor used to modify shear strength based on the effects of member depth, commonly referred to as the size effect factor
 ν = Poisson's ratio (may be assumed to be 0.17 for concrete with f'_c up to 10,000 psi [69 MPa])
 ϕ = strength reduction factor

2.2—Definitions

Please refer to the latest version of ACI Concrete Terminology for a comprehensive list of definitions. Definitions provided herein complement that resource.

B-region—portion of a member in which it is reasonable to assume that strains due to flexure vary linearly through the section.

cover, concrete—distance between the outermost surface of embedded reinforcement and the closest outer surface of the concrete.

D-region—portion of a member within approximately a distance h of a force discontinuity or a geometric discontinuity, where nonlinear strain distributions occur.

discontinuity—abrupt change in geometry or loading.

effective depth of section—distance measured from extreme compression fiber to centroid of longitudinal tension reinforcement.

headed deformed bars—deformed bars with heads attached at one or both ends.

nodal zone—volume of concrete around a node that is assumed to transfer strut-and-tie forces through the node.

node—point in a strut-and-tie model where the axes of the struts, ties, and concentrated forces acting on the joint intersect.

node, curved bar—the bend region of a continuous reinforcing bar (or bars).

strength, design—nominal strength multiplied by a strength reduction factor ϕ .

strength, nominal—strength of a member or cross section calculated in accordance with provisions and assumptions of the strength design method before application of any strength reduction factors.

strength, required—strength of a member or cross section required to resist factored loads or related internal moments and forces in such combinations as required by codes.

strut, bottle-shaped—strut that is wider at midlength than at its ends.

strut, boundary—strut located along the boundary of a member or discontinuity region.

strut, interior—strut not located along the boundary of a member or discontinuity region.

strut-and-tie model—a framework or truss model of a structural member or of a D-region in such a member made up of struts and ties connected at nodes, capable of transferring the factored loads to the supports or to adjacent B-regions.

CHAPTER 3—OVERVIEW OF THE STRUT-AND-TIE MODEL DESIGN PROCEDURE

3.1—Introduction

The strut-and-tie method is an approach to evaluate or design structural concrete by simplifying complex states of stress within a structural member into simple load paths. These load paths are idealized as two-dimensional or three-dimensional networks of straight, pin-connected, axially loaded elements that link all applied loads with supports or with adjacent portions of the structure, such as shown in Fig. 3.1. The arrangement of struts, ties, and nodes is collectively known as a strut-and-tie model (STM) and resembles a truss or framework within the body of the member.

The process of approximating the load path in a structure, or portion thereof, entails visualizing the flow of forces from the applied loads to the reactions while maintaining equilibrium of all forces. External and internal equilibrium conditions should be satisfied. The strut-and-tie method complies with the lower-bound theorem of plasticity (Baker et al. 1956; Nielsen 1971; Thürlimann et al. 1983)—the concept that a structure can resist a set of applied loads as long as there is at least one stress distribution that satisfies equilibrium without violating the yield condition. Accordingly, the STM should achieve equilibrium with the applied loads without exceeding the capacity of any model component (strut, tie, or node). Thus, for design purposes, the internal force in each element of the STM is limited to that element's design strength as prescribed by the corresponding ACI 318 strut-and-tie modeling provisions. The nominal strength of each strut, tie, or node is based on the material strength as well as the geometry and other characteristics of the STM that should fall within the bounds defined by the code provisions. Strut or overall member dimensions may be controlled by sectional shear force limits. These provisions and limits are described in Chapter 4.

The strut-and-tie method is most useful when applied to regions of a structure near localized changes in geometry or applications of external loads. In these instances, the stress and strain distributions may be complex with nonlinear variation over the cross section. Because these regions are defined by the presence of one or more *discontinuities*, they are denoted D-regions, sometimes referred to as disturbed regions. Common examples include corbels, non-slender or deep beams, anchorage zones for post-tensioned tendons, and regions near openings in structural walls. In addition to these regions of complex load paths, the strut-and-tie method can be used to model simpler stress states or load paths, including those with a linear variation of strain over the cross section. B-regions, sometimes referred to as beam regions, are those portions of the structure where the assumption that plane sections remain plane is reasonable, and Bernoulli's classical beam theory is appropriate to approximate the actual behavior. Because simplified methods for the design of B-regions are available, strut-and-tie modeling is most commonly applied to the D-regions of members.

One additional consideration of the strut-and-tie method is compatibility. In the strut-and-tie method, load paths are

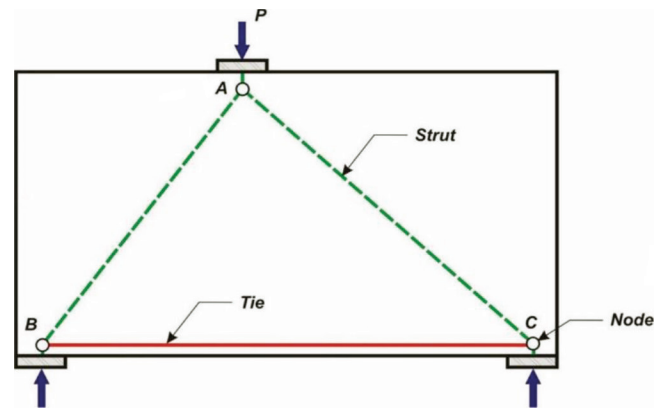


Fig. 3.1—Basic elements of a strut-and-tie model for a deep beam.

discretized into struts and ties connected at nodes, where these elements are assigned finite dimensions. The method is based on the assumption that struts carry compressive stresses, while the areas immediately adjacent to, but outside of, the strut boundaries are considered unstressed. A similar scenario exists with both ties and nodes. This simplification of the load paths leads to apparent strain discontinuities at element boundaries. These discontinuities are permissible: kinematic compatibility is not required to satisfy the lower-bound theorem of plasticity, on which the method is based. Nonetheless, compatibility considerations can be addressed by aligning the STM elements with the linear-elastic stress field to improve efficiency and service-load performance as described in Section 5.3.

3.2—Overview of the strut-and-tie method

An overview of the general strut-and-tie method for modeling and design of a structural concrete member is provided in this section. The process can be adapted to a wide variety of structure and model types. Some of the concepts described will be used later in this chapter to illustrate the application of the method through the development of an example STM. These are the basic steps of model development for each load case:

1. Determine all applied loads and reactions. This step may need to be repeated to suit the geometric layout of the STM in later steps.
2. Identify portions of a structure where the use of the strut-and-tie method may be necessary, including regions near load or geometric discontinuities. These regions are called D-regions and are discussed in more detail in Section 3.3.2.
3. Calculate all forces that should be transferred at the boundaries of the region modeled and the locations where these forces are expected to act on the STM.
4. Determine if minimum distributed reinforcement is warranted in accordance with ACI 318-19 Section 23.5. Use of distributed reinforcement may influence the allowable strut strength (Step 10).
5. Check that the member dimensions are adequate to satisfy any applicable sectional shear force limits such as those prescribed in ACI 318-19 Sections 23.4.4 or 9.9.2.