CHAPTER 13 — BEAMS

In 2014 Chapter 13 Code, Approved Version, Revised 2013-07-25

Included Changes:

CD025
CG035
CE200
Chapter Y

13.1 — Scope

13.1.1 — Provisions of this chapter shall apply to the design of nonprestressed and prestressed beams. The provisions shall also apply to the design of:

(a) composite beams of concrete elements constructed in separate placements but connected so that all elements resist loads as a unit;
(b) one-way joist systems with additional requirements in accordance with 13.8.

13.1.2 — Deep beams shall be designed in accordance with 18.2. <11.7.1>

Ref to 18.2 based on LB12-1

13.2 — General

13.2.1 — Materials

13.2.1.1 — Design properties for concrete shall conform to Chapter 5. <~>

13.2.1.2 — Design properties for steel reinforcement shall conform to Chapter 6. <~>

13.2.2 — Connection to other members

13.2.2.1 — For cast-in-place beams construction, beam-column joints shall be in accordance with satisfy the requirements of 17.2. <10.12>

Ref to 17.2 based on LB11-3

13.2.2.2 — For precast beams construction, connections shall be in accordance with satisfy the force transfer requirements

Ref to 17.3 based on LB11-3

13.2.3 — Stability

13.2.3.1 — If a beam is not continuously laterally braced, (a) and (b) shall be satisfied:

(a) spacing of lateral bracing shall not exceed 50 times the least width of compression flange or face; <10.4.1>
(b) spacing of lateral bracing shall take into account effects of eccentric loads.  <10.4.2>

13.2.3.2 — In prestressed beams, buckling of thin webs and flanges shall be considered. If there is intermittent contact between prestressed reinforcement and an oversize duct, member buckling between contact points shall be considered.  <18.2.5>

13.2.4 — T-Beam construction

13.2.4.1 — In T-beam construction, concrete in the flange and web shall be placed monolithically or made composite in accordance with 17.7.  <8.12.1>  

13.2.4.2 — Effective flange width shall be in accordance with 8.3.3.  <8.12.2>  <8.12.3>  <8.12.4>

13.2.4.3 — For torsional design according to 9.7, the overhanging flange width used to calculate \(A_{cp}, A_g,\) and \(P_{cp}\) shall be in accordance with (a) and (b):

(a) The overhanging flange width shall include that portion of slab on each side of the beam extending a distance equal to the projection of the beam above or below the slab, whichever is greater, but not greater than four times the slab thickness.  <11.5.1.1>

(b) The overhanging flanges shall be neglected in cases where the parameter \(A_{cp}^2/P_{cp}\) for solid sections or \(A_g^2/P_{cp}\) for hollow sections calculated for a beam with flanges is less than that calculated for the same beam ignoring the flanges.  <11.5.1.1>

13.3 — Design limits

13.3.1 — Minimum beam depth

13.3.1.1 — For nonprestressed beams not supporting or attached to partitions or other construction likely to be damaged by large deflections, overall beam depth \(h\) shall not be less than the limits in Table 13.3.1.1, unless the calculated deflection limits of 13.3.3 are satisfied.  <9.5.2.1>

<table>
<thead>
<tr>
<th>Support condition</th>
<th>Minimum (h), in. ([1])</th>
</tr>
</thead>
<tbody>
<tr>
<td>Simply supported</td>
<td>(\ell ;/16)</td>
</tr>
<tr>
<td>One end continuous</td>
<td>(\ell ;/18.5)</td>
</tr>
<tr>
<td>Both ends continuous</td>
<td>(\ell ;/21)</td>
</tr>
<tr>
<td>Cantilever</td>
<td>(\ell ;/8)</td>
</tr>
</tbody>
</table>
Expressions applicable for normalweight concrete and \( f_y = 60,000 \) psi. Minimum \( h \) shall be modified by 13.3.1.1 through 13.3.1.3, as appropriate.

13.3.1.1.1 — For \( f_y \) other than 60,000 psi, the expressions in Table 13.3.1.1 shall be multiplied by \((0.4 + f_y/100,000)\). <Table 9.5a>

13.3.1.1.2 — For nonprestressed beams made of lightweight concrete having \( w_c \) in the range of 90 to 115 lb/ft\(^3\), the expressions in Table 13.3.1.1 shall be multiplied by the greater of (a) and (b): <Table 9.5a>

(a) \( 1.65 - 0.005w_c \)

(b) \( 1.09 \)

13.3.1.1.3 — For nonprestressed composite beams made of a combination of lightweight and normalweight concrete, shored during construction, and where the lightweight concrete is in compression, the modifier of 13.3.1.1.2 shall apply. <9.5.5.1>

13.3.2 — Calculated deflection limits

13.3.2.1 — For nonprestressed beams not satisfying 13.3.1 and for prestressed beams, immediate and time-dependent deflections shall be calculated in accordance with 10.2 and shall not exceed the limits in 10.2.2. <9.5.2.1> <9.5.2.6> <9.5.4.4> <9.5.5.3> <17.2.7>

13.3.2.2 — For nonprestressed composite concrete beams satisfying 13.3.1, deflections occurring after the member becomes composite need not be calculated. Deflections occurring before the member becomes composite shall be investigated unless the precomposite depth also satisfies 13.3.1. <9.5.5.2>

13.3.3 — Reinforcement strain limit in nonprestressed beams

13.3.3.1 — For nonprestressed beams with \( P_u < 0.10 f_c' A_g \), \( \varepsilon_t \) shall be at least 0.004. <10.3.5>

13.3.4 — Stress limits in prestressed beams

13.3.4.1 — Prestressed beams shall be classified as Class U, T, or C in accordance with 10.5.2. <18.3.3>

13.3.4.2 — Stresses in prestressed beams immediately after transfer and at service loads shall not exceed the permissible stresses in 10.5. <18.4>
13.4 — Required strength

13.4.1 — General

13.4.1.1 — Required strength shall be calculated in accordance with the factored load combinations defined in Chapter 7 and analysis procedures defined in Chapter 8. <~>

13.4.1.2 — For prestressed beams, effects of reactions induced by prestressing shall be considered in accordance with 7.3.12. <18.10.3> <CG034>

13.4.2 — Factored moment

13.4.2.1 — For beams built integrally with supports, $M_u$ at the support shall be permitted to be calculated at face of support. <8.9.3>

13.4.3 — Factored shear

13.4.3.1 — For beams built integrally with supports, $V_u$ at the support shall be permitted to be calculated at face of support. <R11.1.3.1>

13.4.3.2 — Sections between the face of support and a critical section located $d$ from the face of support for nonprestressed beams and $h/2$ from the face of support for prestressed beams shall be permitted to be designed for $V_u$ at that critical section if (a) through (c) are satisfied: <11.1.3> <11.1.3.1> <11.1.3.2>

(a) Support reaction, in direction of applied shear, introduces compression into the end region of the beam;

(b) Loads are applied at or near the top surface of the beam;

(c) No concentrated load occurs between the face of support and critical section.

13.4.4 — Factored torsion

13.4.4.1 — Unless determined by a more detailed analysis, it shall be permitted to take the torsional loading from a slab as uniformly distributed along the beam. <11.5.2.3>

13.4.4.2 — For beams built integrally with supports, $T_u$ at the support shall be permitted to be calculated at face of support. <11.5.2.4><11.5.2.5>

13.4.4.3 — Sections between the face of support and a critical section located $d$ from the face of support for nonprestressed beams and $h/2$ from the face of support for prestressed beams shall be permitted to be designed for $T_u$ at that critical section unless a concentrated torque occurs within this distance. In that case, the critical section shall be taken at the face of the support. <11.5.2.4> <11.5.2.5>
13.4.4.4 — It shall be permitted to reduce \( T_u \) in accordance with 9.7.4.3, <11.5.2.2>

13.5 — Design strength

13.5.1 — General

13.5.1.1 — Design strength at all sections along the beam shall satisfy \( \phi S_n \geq U \), including be in accordance with (a) through (d) for each applicable factored load combination. Interaction between load effects shall be considered. <9.1.1> <11.1.1> <11.5.3.5>

(a) \( \phi M_n \geq M_u \)

(b) \( \phi V_n \geq V_u \)

(c) \( \phi T_n \geq T_u \)

(d) \( \phi P_n \geq P_u \)

13.5.1.2 — \( \phi \) shall be determined in accordance with Y2 <~> <ChY>

13.5.2 — Flexure Moment

13.5.2.1 — If \( P_u < 0.10 f_c A_g \), \( \phi M_n \) shall be calculated in accordance with 9.3. If \( P_u \geq 0.10 f_c A_g \), \( \phi M_n \) shall be calculated in accordance with 9.4. <~> <ChY>

13.5.2.2 — For prestressed beams, external tendons shall be considered as unbonded tendons in calculating flexural strength unless the external tendons are effectively bonded to the concrete section along its entire length. <18.22.2>

13.5.3 — Shear

13.5.3.1 — \( \phi V_n \) shall be calculated in accordance with 9.5. <11.1.1> <ChY>

13.5.3.2 — For composite concrete beams, horizontal shear strength \( \phi V_{nh} \) shall be calculated in accordance with 17.7. <17.5> <ChY>

13.5.4 — Torsion

13.5.4.1 — If \( T_u < \phi T_{th} \) where \( T_{th} \) is defined in 9.7 in accordance with 9.7, it shall be permitted to neglect torsional effects. The minimum reinforcement requirements of 13.6.4 and the detailing requirements of 13.7.5 and 13.7.7.3 need not be satisfied. <11.5.1> <ChY>

13.5.4.2 — \( \phi T_n \) shall be calculated in accordance with 9.7. <~> <ChY>
13.5.4.3 — Longitudinal and transverse reinforcement required for torsion shall be added to that required for the shear, moment, and axial force that act in combination with the torsion. <11.5.3.8>

13.5.4.4 — For prestressed beams, the total area of longitudinal reinforcement, $A_s$ and $A_{ps}$, at each section shall be designed to resist $M_u$ at that section plus an additional concentric longitudinal tensile force equal to $A_{ps}f_y$, based on $T_u$ at that section. <11.5.3.10>

13.5.4.5 — It shall be permitted to reduce the area of longitudinal torsional reinforcement in the flexural compression zone by an amount equal to $M_u/(0.9d_f)$, where $M_u$ occurs simultaneously with $T_u$ at that section, except that the longitudinal reinforcement provided shall not be less than the minimum required in 13.6.4. <11.5.3.9> <11.5.3.11>

13.5.4.6 — For solid sections with an aspect ratio, $h/b_t \geq 3$, of at least 3, it shall be permitted to use an alternative design procedure, provided the adequacy of which the procedure has been shown verified by analysis and substantial agreement with results of comprehensive tests. The minimum reinforcement requirements of 13.6.4 need not be satisfied, but the detailing requirements of 13.7.5 and 13.7.7.3 shall remain applicable. <11.5.7> <CE200>

13.5.4.7 — For solid precast sections with an aspect ratio $h/b_t \geq 4.5$, it shall be permitted to use an alternative design procedure and open web reinforcement, provided the adequacy of the design procedure and reinforcement details have been verified by analysis and substantial agreement with results of comprehensive tests. The minimum reinforcement requirements of 13.6.4 and detailing requirements of 13.7.5 and 13.7.7.3 need not be satisfied. <~><CE200>

13.6 — Reinforcement limits

13.6.1 — Minimum flexural reinforcement in nonprestressed beams

13.6.1.1 — A minimum area of flexural reinforcement $A_{s,min}$ shall be provided at every section where tension reinforcement is required by analysis. <10.5.1>

13.6.1.2 - $A_{s,min}$ shall be the greater of (a) and (b), except as provided in 13.6.1.3. For a statically determinate beam with a flange in tension, the value of $b_w$ shall be taken as the lesser of $b_f$ and $2b_w$. <10.5.1 > <10.5.2>

(a) $\frac{3f'_c}{f_y} b_w d$

(b) $\frac{200}{f_y} b_w d$

13.6.1.3 — If $A_t$ provided at every section is at least one-third greater than $A_s$ required by analysis, 13.6.1.1 and 13.6.1.2 need not be satisfied. <10.5.3>
13.6.2 — Minimum flexural reinforcement in prestressed beams

13.6.2.1 — For beams with bonded prestressed reinforcement, total quantity of $A_s$ and $A_{ps}$ shall be adequate to develop a factored load at least 1.2 times the cracking load calculated on the basis of $f'_c$ defined in 5.2.3. <18.8.2>

13.6.2.2 — For beams with both flexural and shear design strength at least twice the required strength, 13.6.2.1 need not be satisfied. <18.8.2>

13.6.2.3 — For beams with unbonded prestressed reinforcement tendons, a minimum area of bonded deformed longitudinal reinforcement $A_{s,min}$ shall be calculated provided according to Eq. 13.6.2.3.

$$A_{s,min} = 0.004 A_{ci} \quad (13.6.2.3)$$

where $A_{ci}$ is the area of that part of the cross section between the flexural tension face and the centroid of the gross section.<18.9.1> <18.9.2> <18.9.2.2>

13.6.3 — Minimum shear reinforcement

13.6.3.1 — A minimum area of shear reinforcement $A_{v,min}$ shall be provided in all regions where $V_u > 0.5 \phi V_c$ except for the cases listed in Table 13.6.3.1, where at least $A_{v,min}$ shall be provided where $V_u > \phi V_c < 11.4.6.1$, <CD025>

<table>
<thead>
<tr>
<th>Beam Type</th>
<th>Conditions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shallow depth</td>
<td>$h \leq 10$ in.</td>
</tr>
<tr>
<td>Integral with slab</td>
<td>$h \leq$ greater of $2.5 t_f$ or $0.5 b_w$ and $h \leq 24$ in.</td>
</tr>
<tr>
<td>Constructed with steel fiber-reinforced normalweight concrete conforming to 22.5 and with $f'_c \leq 6000$ psi</td>
<td>$h \leq 24$ in and $V_u \leq \phi 2 \sqrt{f'_c b_w d}$</td>
</tr>
<tr>
<td>One-way joist system</td>
<td>defined by 13.8</td>
</tr>
</tbody>
</table>

Table 13.6.3.1: Cases where $V_u > 0.5 \phi V_c$ requirement $A_{v,min}$ not required if $0.5 \phi V_c < V_u \leq \phi V_c$

Add new definition to Chapter 2:

$t_f =$ thickness of flange, in.
13.6.3.2 — If shown by testing that the required $M_n$ and $V_n$ can be developed, 13.6.3.1 need not be satisfied. Such tests shall simulate effects of differential settlement, creep, shrinkage, and temperature change, based on a realistic assessment of these effects occurring in service. 

13.6.3.3 — If shear reinforcement is required and torsional effects can be neglected according to 13.5.4.1, $A_{v,min}/s$ shall be in accordance with Table 13.6.3.3 <11.4.6.3> <11.4.6.4>

<table>
<thead>
<tr>
<th>Beam type</th>
<th>$A_{v,min}/s$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nonprestressed and Prestressed with effective prestress force &lt; 40 percent of the tensile strength of the flexural reinforcement</td>
<td>Greater of: 0.75$\sqrt{f_c \frac{b_w}{f_{yt}}}$ (a), 50$\frac{b_w}{f_{yt}}$ (b)</td>
</tr>
<tr>
<td>Prestressed with effective prestress force ≥ 40 percent of the tensile strength of the flexural reinforcement</td>
<td>Lesser of: Greater of: 0.75$\sqrt{f_c \frac{b_w}{f_{yt}}}$ (c), 50$\frac{b_w}{f_{yt}}$ (d)</td>
</tr>
<tr>
<td></td>
<td>$\frac{A_{pu} f_{pu}}{80 f_{yt} d \sqrt{b_w}}$ (e)</td>
</tr>
</tbody>
</table>

13.6.4 — Minimum torsional reinforcement

13.6.4.1 — A minimum area of torsional reinforcement shall be provided in all regions where $T_u \geq \phi T_{th}$ in accordance with 9.7 <11.5.5.1>

13.6.4.2 — If torsional reinforcement is required, minimum transverse reinforcement $(A_v + 2A_t)_{min}/s$ shall be the greater of (a) and (b). <11.5.5.2>

\[(a) \ 0.75\sqrt{f_c \frac{b_w}{f_{yt}}} \]
\[(b) \ 50\frac{b_w}{f_{yt}} \]

13.6.4.3 — If torsional reinforcement is required, minimum longitudinal reinforcement $A_{e,min}$ shall be the lesser of (a) and (b). <11.5.5.3>
13.7 — Reinforcement detailing

13.7.1 — General

13.7.1.1 — Concrete cover for reinforcement shall be in accordance with 6.118.1. <~>

13.7.1.2 — Development lengths of deformed and prestressed reinforcement shall be calculated in accordance with 21.4. <~>

13.7.1.3 — Splice lengths of deformed reinforcement shall be calculated in accordance with 21.5. <~>

13.7.1.4 — Bundled bars shall be detailed in accordance with 21.6. <~>

13.7.1.5 — The most restrictive requirements for reinforcement spacing and placement shall apply. <11.5.3.8>

13.7.2 — Reinforcement spacing

13.7.2.1 — Minimum spacing $s$ shall be in accordance with 21.2. <~>

13.7.2.2 — For nonprestressed and Class C prestressed beams, spacing of bonded longitudinal reinforcement closest to the tension face shall not exceed $s$ required in 10.3. <10.6.1> <10.6.4>

13.7.2.3 — For nonprestressed and Class C prestressed beams with $h$ exceeding 36 in., longitudinal skin reinforcement shall be provided and uniformly distributed on both side faces of the beam for a distance $h/2$ from the tension face. Spacing of skin reinforcement shall not exceed $s$ required in 10.3.2, where $c_c$ is the clear cover from the skin reinforcement to the side face. It shall be permitted to include skin reinforcement in strength calculations if a strain compatibility analysis is made. <10.6.7>

13.7.3 — Flexural reinforcement in nonprestressed beams

13.7.3.1 — Calculated tension or compression force in reinforcement at each section of the beam shall be developed on each side of that section. <12.1.1>
13.7.3.2 — Critical locations for development of reinforcement are points of maximum stress and points along the span where bent or terminated tension reinforcement is no longer required to resist flexure. <12.10.2>

13.7.3.3 — Reinforcement shall extend beyond the point at which it is no longer required to resist flexure for a distance equal to the greater of \( d \) and \( 12d_b \), except at supports of simply-supported spans and at free ends of cantilevers. <12.10.3>

13.7.3.4 — Continuing flexural tension reinforcement shall have an embedment length not less than \( \ell_d \) beyond the point where bent or terminated tension reinforcement is no longer required to resist flexure. <12.10.4>

13.7.3.5 — Flexural tension reinforcement shall not be terminated in a tensile zone unless (a), (b), or (c) is satisfied. <12.10.5>

(a) \( V_u \leq (2/3)\phi V_n \) at the cutoff point; <12.10.5.1>

(b) For No. 11 bars and smaller, continuing reinforcement provides double the area required for flexure at the cutoff point and \( V_u \leq (3/4)\phi V_n \); <12.10.5.3>

(c) Stirrup or hoop area in excess of that required for shear and torsion is provided along each terminated bar or wire over a distance \( \frac{3}{4} d \) from the termination point. Excess stirrup or hoop area shall not be less than \( 60b.ws_{fy} \). Spacing \( s \) shall not exceed \( d/(8\beta_b) \). <12.10.5.2>

13.7.3.6 — Adequate anchorage shall be provided for tension reinforcement where reinforcement stress is not directly proportional to moment, such as in sloped, stepped, or tapered beams; or where tension reinforcement is not parallel to the compression face. <12.10.6>

13.7.3.7 — Development of tension reinforcement by bending across the web to be anchored or made continuous with reinforcement on the opposite face of beam shall be permitted. <12.10.1>

13.7.3.8 — Termination of reinforcement

13.7.3.8.1 — At simple supports, at least one-third the maximum positive moment reinforcement shall extend along the beam bottom into the support a minimum of 6 in. except for precast beams, where such reinforcement shall extend at least to the center of the bearing length. <12.11.1> <16.6.2.3>

13.7.3.8.2 — At other supports, at least one-fourth the maximum positive moment reinforcement shall extend along the beam bottom into the support a minimum of 6 in. <12.11.1>

13.7.3.8.3 — At simple supports and points of inflection, \( d_b \) for positive moment tension reinforcement shall be limited such that \( \ell_d \) for that reinforcement satisfies (a) or (b). If reinforcement terminates beyond the centerline of supports by a standard hook or a mechanical anchorage at least equivalent to a standard hook, (a) or (b) need not be satisfied. <12.11.3>
(a) \( \ell_d \leq (1.3M_n/V_u + \ell_a) \) if end of reinforcement is confined by a compressive reaction.

(b) \( \ell_d \leq (M_n/V_u + \ell_a) \) if end of reinforcement is not confined by a compressive reaction.

where:

- \( M_n \) is calculated assuming all reinforcement at the section is stressed to \( f_y \).
- \( V_u \) is calculated at the section.
- At a support, \( \ell_a \) is the embedment length beyond the center of the support.
- At a point of inflection, \( \ell_a \) is the embedment length beyond the point of inflection limited to the greater of \( d \) and \( 12d_b \).

**13.7.3.8.4** — At least one-third the negative moment reinforcement at a support shall have an embedment length beyond the point of inflection at least the greatest of \( d \), \( 12d_b \), and \( \ell_n/16 \).<12.12.3>

**13.7.4** — Flexural reinforcement in prestressed beams

**13.7.4.1** — External tendons shall be attached to the member in a manner that maintains the specified eccentricity between the tendons and the concrete centroid through the full range of anticipated member deflections. <18.22.3>

**13.7.4.2** — If nonprestressed reinforcement is provided to satisfy flexural strength requirements, the detailing requirements of 13.7.3 shall be satisfied. <18.9.4.3>

**13.7.4.3** — Termination of prestressed reinforcement

**13.7.4.3.1** Post-tensioned anchorage zones shall be designed and detailed in accordance with 18.4. Ref to 18.4 based on LB12-1

**13.7.4.3.2** — Post-tensioning anchorages and couplers shall be designed and detailed in accordance with 21.7. Ref to 21.7 based on LB12-2

**13.7.4.4** — Termination of deformed reinforcement in beams with unbonded prestressed tendons

**13.7.4.4.1** — Length of deformed reinforcement required by 13.6.2.3 shall be in accordance with (a) and (b). <18.9.4>

(a) In positive moment areas, length of reinforcement shall be at least \( \ell_n/3 \) and be centered in that area. <18.9.4.1>

(b) In negative moment areas, reinforcement shall extend at least \( \ell_n/6 \) on each side of the face of support. <18.9.4.2>

**13.7.5** — Longitudinal torsional reinforcement
13.7.5.1 — If torsional reinforcement is required, longitudinal torsional reinforcement shall be distributed around the perimeter of closed stirrups meeting 21.8.1.6 or hoops with a spacing not greater than 12 in. The longitudinal reinforcement shall be inside the stirrup or hoop and at least one longitudinal bar or tendon shall be placed in each corner. <11.5.6.2>

13.7.5.2 — Longitudinal torsional reinforcement shall have a diameter at least 0.042 times the transverse reinforcement spacing, but not less than 3/8 in. <11.5.6.2>

13.7.5.3 — Longitudinal torsional reinforcement shall be provided for a distance of at least \((b_t + d)\) beyond the point required by analysis. <11.5.6.3>

13.7.5.4 — Longitudinal torsional reinforcement shall be developed at the face of the support at both ends of the beam. <11.5.4.3>

Note: Section 13.7.6 and 13.7.7 swapped as approved in San Antonio at the Summer 2013 meeting. For this reason, the provisions below are renumbered.

13.7.6 — Transverse reinforcement

13.7.6.1 — General

13.7.6.1.1 — Transverse reinforcement shall be provided in accordance with 13.7.7. The most restrictive requirements shall apply. <11.5.3.8>

13.7.6.1.2 — Details of geometry and anchorage of transverse reinforcement shall be in accordance with 21.8. <7.11.2> Ref to 21.8 based on LB12-2

13.7.6.2 — Shear

13.7.6.2.1 — If required, shear reinforcement shall be provided using stirrups, hoops, or longitudinal bent bars. <7.11.3> <7.11.2>

13.7.6.2.2 — Maximum spacing of shear reinforcement shall be in accordance with Table 13.7.6.2. <11.4.5> <11.4.5.1> <11.4.5.3>

Table 13.7.6.2.2 — Maximum spacing of shear reinforcement

<table>
<thead>
<tr>
<th>(V_s)</th>
<th>Maximum (s), in.</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\leq 4\sqrt{f_c b_w d})</td>
<td>Lesser of: (\frac{d}{2})</td>
</tr>
<tr>
<td>(\geq 4\sqrt{f_c b_w d})</td>
<td>Lesser of: (\frac{d}{4})</td>
</tr>
</tbody>
</table>
13.7.23 — Inclined stirrups and longitudinal bars bent to act as shear reinforcement shall be spaced so that every 45-degree line, extending $d/2$ toward the reaction from mid-depth of member to longitudinal tension reinforcement, shall be crossed by at least one line of shear reinforcement. <11.4.5.2>

13.7.24 — Longitudinal bars bent to act as shear reinforcement, if extended into a region of tension, shall be continuous with longitudinal reinforcement and, if extended into a region of compression, shall be anchored $d/2$ beyond mid-depth of member. <12.13.4>

13.7.3 — Torsion

13.7.3.1 — If required, transverse torsional reinforcement shall be provided using closed stirrups meeting 21.8.1.6 or hoops. <11.5.4>

13.7.3.2 — Transverse torsional reinforcement shall be provided for a distance of at least $(b_t + d)$ beyond the point required by analysis. <11.5.6.3> <11.5.4.1> <11.5.4.2>

13.7.3.3 — Spacing of transverse torsional reinforcement shall not exceed the lesser of $p_b/8$ and 12 in. <11.5.6.1>

13.7.3.4 — For hollow sections, the distance from the centerline of the transverse torsional reinforcement to the inside face of the wall of the hollow section shall be at least $0.5A_{oh}/p_b$. <11.5.4.4>

13.7.4 — Lateral support of compression reinforcement

13.7.4.1 — Transverse reinforcement shall be provided throughout the distance where longitudinal compression reinforcement is required. Lateral support of longitudinal compression reinforcement shall be provided by closed stirrups or hoops in accordance with 13.7.4.4 through 13.7.4.45. <7.11.1>

13.7.4.2 — Size of transverse reinforcement shall be at least (a) or (b). Deformed wire or welded wire reinforcement of equivalent area shall be permitted. <7.10.5.1> <7.11.1>

(a) No. 3 for longitudinal bars No. 10 and smaller;

(b) No. 4 for longitudinal bars No. 11 and larger and for longitudinal bundled bars.

13.7.4.3 — Spacing of transverse reinforcement shall not exceed the least of (a) through (c): <7.10.5.2> <7.11.1>

(a) $16d_b$ of longitudinal reinforcement;
(b) \(48d_b\) of transverse reinforcement;

(c) least dimension of beam.

13.7.6.4.4 — Longitudinal compression reinforcement shall be arranged such that every corner and alternate compression bar shall be enclosed by the corner of the transverse reinforcement with an included angle of not more than 135 degrees and no bar shall be farther than 6 in. clear on each side along the transverse reinforcement from such an enclosed bar. <7.10.5.3>

13.7.6.7 — Structural integrity reinforcement in cast-in-place beams <7.13.1> <7.13.2> <12.1.3>

13.67.1 — For beams along the perimeter of the structure, structural integrity reinforcement shall be in accordance with (a), (b), and (c):

(a) at least one-quarter the maximum positive moment reinforcement, but not less than two bars or strands, shall be continuous; <7.13.2.2>

(b) at least one-sixth the negative moment reinforcement at the support, but not less than two bars or strands, shall be continuous; <7.13.2.2>

(c) longitudinal structural integrity reinforcement shall be enclosed by closed stirrups meeting 21.8.1.6 or hoops along the clear span of the beam. <7.13.2.3>

13.67.2 — For other than perimeter beams, structural integrity reinforcement shall be in accordance with (a) or (b):

(a) at least one-quarter the maximum positive moment reinforcement, but not less than two bars or strands, shall be continuous; <7.13.2.5>

(b) longitudinal reinforcement shall be enclosed by closed stirrups meeting 21.8.1.6 or hoops along the clear span of the beam. <7.13.2.5>

13.67.3 — Longitudinal structural integrity reinforcement shall pass through the region bounded by the longitudinal reinforcement of the column. <7.13.2.2> <7.13.2.5>

13.67.4 — Longitudinal structural integrity reinforcement at noncontinuous supports shall be anchored to develop \(f_y\) at the face of the support. <7.13.2.2> <7.13.2.5>

13.67.5 — If splices are necessary in continuous structural integrity reinforcement, the reinforcement shall be spliced according to (a) and (b): <7.13.2.4> <7.13.2.5>

(a) positive moment reinforcement shall be spliced at or near the support;

(b) negative moment reinforcement shall be spliced at or near midspan.
13.7.6 Splices shall be full mechanical, full welded, or Class B tension lap splices.  

13.8 — One-way joist systems

13.8.1 — General

13.8.1.1 — One-way joist construction consists of a monolithic combination of regularly spaced ribs and a top slab designed to span in one direction.  

13.8.1.2 — Width of ribs shall not be less than 4 in. at any location along the depth.  

13.8.1.3 — Overall depth of ribs shall not exceed 3.5 times the minimum width.  

13.8.1.4 — Clear spacing between ribs shall not exceed 30 in.  

13.8.1.5 — $V_c$ shall be permitted to be taken as 1.1 times the value calculated in 9.5.  

13.8.1.6 — For structural integrity, at least one bottom bar in each joist shall be continuous and shall be anchored to develop $f_y$ at the face of supports.  

13.8.1.7 — Reinforcement perpendicular to the ribs shall be provided in the slab as required for flexure, considering load concentrations, but shall not be less than that required for shrinkage and temperature in accordance with 10.4.  

13.8.1.8 — One-way joist construction not meeting the limitations of 13.8.1.1 through 13.8.1.4 shall be designed as slabs and beams.  

13.8.2 — Joist systems with structural fillers

13.8.2.1 — If permanent burned clay or concrete tile fillers of material having a unit compressive strength at least equal to $f_c$ in the joists are used, 13.8.2.1.1 and 13.8.2.1.2 shall apply.  

13.8.2.1.1 — Slab thickness over fillers shall be at least the greater of 1/12 the clear distance between ribs and 1.5 in.  

13.8.2.1.2 — For calculation of shear and negative moment strength, it shall be permitted to include the vertical shells of fillers in contact with the ribs. Other portions of fillers shall not be included in strength calculations.  

13.8.3 — Joist systems with other fillers

13.8.3.1 — If fillers not complying with 13.8.2.1 or removable forms are used, slab thickness shall be at least the greater of 1/12 the clear distance between ribs and 2 in.
CHAPTER 13—Beams

COMMENTARY

Notes:
1. Written for 2013-03-28 revision of Beams chapter available on the reorganization website as the Latest Version.
2. Purple highlights indicate reference numbers that will need to be updated.
3. Approved at the San Antonio meeting on 7/24/2013, includes CE200.

**Note - All references to Chapter 13 will need to be changed to chapter 9 in accordance with new chapter numbering.**

R13.1 — Scope

R13.1.1 — Composite structural steel-concrete beams are not covered in this chapter. Design provisions for such composite beams are covered in Reference 17.1. <R17.1>

**Note – this will need to be modified or removed after the move of 18.2 to this chapter.**

R13.1.2 — Deep beams are defined in 18.2.1.1. Chapter 13 does not apply to deep beams.

R13.2 — General

R13.2.3.1 — Tests have shown that laterally unbraced reinforced concrete beams, even when very deep and narrow, will not fail prematurely by lateral buckling provided the beams are loaded without lateral eccentricity that causes torsion. <R10.4>

Laterally unbraced beams are frequently loaded off center eccentrically or with slight inclination. Stresses and deformations by such loading become detrimental for narrow, deep beams, with long unsupported lengths. Lateral supports spaced closer than 50b may be required by for such loading conditions. <R10.4>

R13.2.3.2 — In post-tensioned members where the tendon—prestressed reinforcement has intermittent contact with an oversize duct, the member may—can buckle due to the axial Prestressing force, as the member can deflect laterally while the tendon—prestressed reinforcement does not. If the tendon—prestressed reinforcement is in continuous contact with the member being prestressed, or is part of an unbonded tendon with the sheathing not excessively larger than the prestressed reinforcement, the prestressing force cannot buckle the member. <R18.2.5>

R13.2.4 — T-Beam construction

R13.2.4.1 — For monolithic or fully composite construction, the beam includes a portion of the slab as flanges.
R13.2.3.4.3 — For monolithic or fully composite construction, the beam includes a portion of the slab as flanges. Two examples of the section to be considered in torsional design are provided in Fig. R13.2.3.4.3. <R13.2.4>

![Diagram of beam and slab](image)

Fig. R13.2.3.4.3—Examples of the portion of slab to be included with the beam for torsional design.

R13.3 — Design limits

R13.3.1 — Minimum beam depth

R13.3.1.1 — For application of this provision to composite concrete beams, see R13.3.2.2.

R13.3.1.1.1 — The modification for $f_y$ is approximate but should provide conservative results for typical reinforcement ratios and for values of $f_y$ between 40,000 and 80,000 psi. <R9.5.2.1>

R13.3.1.1.2 — The modification for lightweight concrete is based on the results and discussions in Reference 9.15. No correction is given for concretes with $w_c$ greater than 115 lb/ft$^3$ because the correction term would be close to unity in this range. <R9.5.2.1>

R13.3.1.2 — A concrete floor finish may be considered for strength purposes if it is cast monolithically with the slab. Permission is given to include a separate concrete finish in the structural thickness if composite action is provided in accordance with 17.7. <R8.14>

R13.3.2 — Calculated deflection limits

R13.3.2.2 — The limits in Table 13.3.1.1 apply to the entire depth of nonprestressed composite beams shored during construction so that, after removal of temporary supports, dead load is resisted by the full composite section. In unshored construction, the beam depth of concern depends on whether the deflection being considered occurs before or after the attainment of effective composite action. See also R13.3.2.2. <R9.5.5>

Additional deflections due to excessive creep and shrinkage caused by premature loading of precast beams should be considered. This is especially important at early ages when the moisture content is high and the strength is low. <R17.2.7>
The transfer of horizontal shear by direct bond is important if excessive deflection from slippage is to be prevented. Shear keys provide a secondary means of transferring shear but will not be engaged until slippage occurs. <R17.2.7>

**R13.3.3 — Reinforcement strain limit in nonprestressed beams**

**R13.3.3.1** — The effect of this limitation is to restrict the reinforcement ratio in nonprestressed beams to mitigate brittle flexural behavior in case of an overload. This limitation does not apply to prestressed beams. <R10.3.5> <Commentary consistent with approved Chapter 12>

*Note: Delete definition of $\rho_b$ has been deleted from Chapter 2.*

**R13.4 — Required strength**

**R13.4.3 — Factored shear**

**R13.4.3.2** — The closest inclined crack to the support of the beam in Fig. R13.4.3.2(a) will extend upward from the face of the support reaching the compression zone about $d$ from the face of the support. If loads are applied to the top of the beam, the stirrups across this crack need only resist the shear force due to loads acting beyond $d$ (lower right free body in Fig. R13.4.3.2(a)).

The loads applied to the beam between the face of the support and the point $d$ away from the face are transferred directly to the support by compression in the web above the crack. Accordingly, the Code permits design for a maximum factored shear $V_u$ at a distance $d$ from the support for nonprestressed beams and at a distance $h/2$ for prestressed beams. <R11.1.3.1> <R11.1.3.2>

In Fig. R13.4.3.2(b), loads are shown acting near the bottom of a beam. In this case, the critical section is taken at the face of the support. Loads acting near the support should be transferred across the inclined crack extending upward from the support face. The shear force acting on the critical section should include all loads applied below the potential inclined crack.

*Fig. R13.4.3.2(a) — Free body diagrams of the end of a beam.*
Fig. R13.4.3.2(b)—Location of critical section for shear in a beam loaded near bottom.

Fig. R13.4.3.2(c), (d), (e), (f)—Typical support conditions for locating factored shear force \( V_u \).

Typical support conditions where the shear force at a distance \( d \) from the support may be used include:

1. beams supported by bearing at the bottom of the beam, such as shown in Fig. R13.4.3.2(c);
2. beams framing monolithically into a column as illustrated in Fig. R13.4.3.2(d).

Typical support conditions where the critical section is taken at the face of support include:

< R11.1.3.1 >
139 (1) beams framing into a supporting member in tension, such as shown in Fig. R13.4.3.2(e). Shear within the connection should also be investigated and special corner reinforcement should be provided. <R11.1.3.1>

142 (2) beams for which loads are not applied at or near the top as previously discussed and shown in Fig. R13.4.3.2(b). <R11.1.3.1>

145 (3) beams loaded such that the shear at sections between the support and a distance \( d \) from the support differs radically from the shear at distance \( d \). This commonly occurs in brackets and in beams where a concentrated load is located close to the support, as shown in Fig. R13.4.3.2(f). <R11.1.3.1>

R13.4.4 — Factored torsion

R13.4.4.3 — It is not uncommon for a beam to frame into one side of a girder near the support of the girder. In such a case, a concentrated shear and torque are applied to the girder. <R11.5.2.4 and R11.5.2.5>

R13.5 — Design strength

R13.5.2 — Moment

R13.5.2.1 — Beams resisting significant axial forces require consideration of the combined effects of axial forces and moments. These beams are not required to satisfy the provisions of Chapter 14, but are required to satisfy the additional requirements for ties or spirals defined in Table 9.4.3.1. For slender beams with significant axial loads, consideration should be given to slenderness effects as required for columns in 8.2.4. <~>

R13.5.4 — Torsion

R13.5.4.3 — The requirements for torsion reinforcement and shear reinforcement are added and stirrups are provided to supply at least the total amount required. Because the reinforcement area \( A_v \) for shear is defined in terms of all the legs of a given stirrup while the reinforcement area \( A_t \) for torsion is defined in terms of one leg only, the addition of transverse reinforcement area is calculated as follows: <R11.5.3.8>

\[
\begin{align*}
\text{Total} \left( \frac{A_{v-t}}{s} \right) &= \frac{A_v}{s} + 2 \frac{A_t}{s} \\
\end{align*}
\] (R13.5.4.3)

If a stirrup group has more than two legs for shear, only the legs adjacent to the sides of the beam should be included in this summation because the inner legs would be ineffective for resisting torsion. <R11.5.3.8>

The longitudinal reinforcement required for torsion is added at each section to the longitudinal reinforcement required for bending moment that acts concurrently with the torsion. The longitudinal reinforcement is then chosen for this sum, but should not be less than the amount
required for the maximum bending moment at that section if this exceeds the moment acting concurrently with the torsion. If the maximum bending moment occurs at one section, such as midspan, while the maximum torsional moment occurs at another, such as the face of the support, the total longitudinal reinforcement required may be less than that obtained by adding the maximum flexural reinforcement plus the maximum torsional reinforcement. In such a case, the required longitudinal reinforcement is evaluated at several locations. <R11.5.3.8>

R13.5.4.4 — Torsion causes an axial tension force in the longitudinal reinforcement balanced by the force in the diagonal concrete compression struts. In a nonprestressed beam, the tension force must be resisted by longitudinal reinforcement having an axial tensile strength of $A_\ell f_y$. This reinforcement is in addition to the required flexural reinforcement and is distributed uniformly inside and around the perimeter of the closed transverse reinforcement so that the resultant of $A_\ell f_y$ acts along the axis of the member. <R11.5.3.10>

In a prestressed beam, the same approach (providing additional reinforcing bars with strength $A_\ell f_y$) may be followed, or overstrength of the prestressed reinforcement can be used to resist some of the axial force $A_\ell f_y$ as discussed below. <R11.5.3.10>

In a prestressed beam, the stress in the prestressed reinforcement at nominal strength will be between $f_{se}$ and $f_{ps}$. A portion of the $A_\ell f_y$ force can be resisted by a force of $A_{ps}\Delta f_{pt}$ in the prestressed reinforcement, where $\Delta f_{pt}$ is the difference between the stress that can be developed in the strand at the section under consideration and the stress required to resist the bending moment at this section, $M_u$. The stress required to resist the bending moment can be calculated as $[M_u/(0.9d_pA_{ps})]$. For pretensioned strands, the stress that can be developed near the free end of the strand can be calculated using the procedure illustrated in Fig. R21.4.8.3. <R11.5.3.10>

NOTE - Figure R21.4.8.3 is reproduced here for reviewing convenience.

R13.5.4.5 — The longitudinal tension due to torsion is offset in part by the compression in the flexural compression zone, allowing a reduction in the longitudinal torsion reinforcement required in the compression zone. <R11.5.3.9>
R13.5.4.6 — Examples of such procedures can be found in References 11.39 to 11.41, which have been extensively and successfully used for design of precast, prestressed beams with ledges. The seventh edition of the PCI Design Handbook describes the procedure of References 11.39 and 11.40. This procedure was experimentally verified by the tests described in Reference 11.42. <R11.5.7>

Note: Section below replaces above based on CE200.

An example of such an alternative design that satisfies 13.5.4.6 procedures are to be found in References 11.39 and 11.40, which have been extensively and successfully used for design of precast, prestressed concrete spandrel beams with \( \frac{h}{b_t} \geq 3 \) and closed stirrups. The procedure described in References 11.39 and 11.40 is an extension to prestressed concrete sections of the torsion procedures of pre-1995 editions of the Code. The sixth seventh edition of the PCI Design Handbook describes the procedure of References 11.39 and 11.40. This procedure was experimentally verified by the tests described in Reference 11.42. <11.5.7> 

R13.5.4.7 – The experimental program described in Reference 13.XX demonstrates that properly designed open web reinforcement is a safe and effective alternative to traditional closed stirrups for precast spandrels with \( \frac{h}{b_t} > 4.5 \). Reference 13.YY presents a design procedure that satisfies 13.5.4.7 for slender spandrels and describes the limited conditions to which the procedure applies. <CE200>

Note: Add and update references <CE200>

13.XX. Lucier, Gregory; Walter, Catrinia; Rizkalla, Sami; Zia, Paul; and Klein, Gary, "Development of a Rational Design Methodology for Precast Slender Spandrel Beams, Part 1: Experimental Results,” PCI Journal, Precast/Prestressed Concrete Institute, Chicago, IL, V. 56, No. 2, Spring 2011, pp. 88-112.

13.YY. Lucier, Gregory; Walter, Catrinia; Rizkalla, Sami; Zia, Paul; and Klein, Gary, "Development of a Rational Design Methodology for Precast Slender Spandrel Beams, Part 2: Analysis and Design Guidelines,” PCI Journal, Precast/Prestressed Concrete Institute, Chicago, IL, V. 56, No. 4, Fall 2011, pp. 106-133.

Update Reference 11.16 to Seventh Edition of PCI Design Handbook
R13.6 — Reinforcement limits

R13.6.1— Minimum flexural reinforcement in nonprestressed beams

R13.6.1.1 — This provision is a precaution against abrupt flexural failure developing immediately after cracking. A flexural member designed according to Code provisions requires considerable additional load beyond cracking to reach its flexural strength. Thus, considerable deflection would warn that the member strength is approaching. If the flexural strength were reached shortly after cracking, the warning deflection would not occur.

This provision is intended to result in flexural strength exceeding the cracking strength by a margin. The objective is to produce a beam that will be able to sustain loading after the onset of flexural cracking, with visible cracking and deflection, thereby warning of possible overload. Beams with less reinforcement may sustain sudden failure with the onset of flexural cracking.

In practice, this provision only controls reinforcement design for beams which for architectural or other reasons, are larger in cross section than required for strength. With a very small amount of tension reinforcement required for strength, the calculated moment strength of a reinforced concrete section using cracked section analysis becomes less than that of the corresponding unreinforced concrete section calculated from its modulus of rupture. Failure in such a case could occur at first cracking and without warning. To prevent such a failure, a minimum amount of tension reinforcement is required in both positive and negative moment regions. <R10.5>

R13.6.1.2 — If the flange of a section is in tension, the amount of tension reinforcement needed to make the strength of the reinforced section equal that of the unreinforced section is about twice that for a rectangular section or that of a flanged section with the flange in compression.

A greater amount of minimum tension reinforcement is particularly necessary in cantilevers and other statically determinate beams where there is no possibility for redistribution of moments. <R10.5>

R13.6.2 — Minimum flexural reinforcement in prestressed beams

R13.6.2.1 — Minimum flexural reinforcement is required for reasons similar to nonprestressed beams as discussed in R13.6.1.1. <~>

Abrupt flexural failure immediately after cracking does not occur when the prestressing reinforcement is unbonded; therefore, this requirement does not apply to members with unbonded tendons. <R18.8.2>

R13.6.2.3 — Some bonded reinforcement is required by the Code in beams prestressed with unbonded tendons to ensure flexural behavior at ultimate beam strength, rather than tied arch behavior, and to limit crack width and spacing at service load when concrete tensile stresses exceed the modulus of rupture. Providing minimum bonded reinforcement helps to ensure acceptable behavior at all loading stages. The minimum amount of bonded reinforcement is based on research comparing the behavior of bonded and unbonded post-tensioned beams. The minimum bonded reinforcement areas required by Eq. 13.6.2.3 are absolute minimum areas independent of reinforcement grade of steel or design yield strength. <R18.9.1> <R18.9.2>
R13.6.3 — Minimum shear reinforcement

R13.6.3.1 — Shear reinforcement restrains the growth of inclined cracking so that ductility of the beam is improved and a warning of failure is provided. In an unreinforced web, the formation of inclined cracking might lead directly to failure without warning. Such reinforcement is of great value if a beam is subjected to an unexpected tensile force or an overload. Accordingly, a minimum area of shear reinforcement not less than that given by Table 13.6.3.1 is required wherever $V_u$ is greater than $0.5\phi V_c$, or greater than $1.0\phi V_c$ for the cases indicated in Table 13.6.3.1. <R11.4.6.1>

Research has shown that unbonded post-tensioned members do not inherently provide large capacity for energy dissipation under severe earthquake loadings because the member response is primarily elastic. For this reason, unbonded post-tensioned structural elements reinforced in accordance with the provisions of this section should be assumed to carry only vertical loads and to act as horizontal diaphragms between energy dissipating elements under earthquake loadings of the magnitude defined in 21.1.1. <R18.9.1>

The commentary was approved for removal at the main committee meeting in San Antonio.

The exception for beams constructed using steel fiber-reinforced concrete is intended to provide a design alternative to the use of shear reinforcement, as defined in 9.5.11.5, for beams with longitudinal flexural reinforcement in which $V_u$ does not exceed $\phi 2\sqrt{f_c b_w d}$. Fiber-reinforced concrete beams with hooked or crimped steel fibers in dosages as required by 22.5.6.3 have been shown, through laboratory tests, to exhibit shear strengths greater than $3.5\sqrt{f_c b_w d}$. There are no data for the use of steel fibers as shear reinforcement in concrete beams exposed to chlorides from deicing chemicals, salt, salt water, brackish water, seawater, or spray from these sources. Where steel fibers are used as shear reinforcement in corrosive environments, corrosion protection should be considered. <R11.4.6.1(f)>
Joists are excluded from the minimum shear reinforcement requirement for 0.5 $\phi V_c < V_u \leq \phi V_c$ because there is a possibility of load sharing between weak and strong areas. <R11.4.6.1>

Even when $V_u$ is less than 0.5 $\phi V_c$, the use of some web reinforcement is recommended in all thin-web, post-tensioned members (joists, waffle slabs, beams, and T-beams) to reinforce against tensile forces in webs resulting from local deviations from the design tendon profile, and to provide a means of supporting the tendons in the design profile during construction. If sufficient support is not provided, lateral wobble and local deviations from the smooth parabolic tendon profile assumed in design may result during placement of the concrete. In such cases, the deviations in the tendons tend to straighten out when the tendons are stressed. This process may impose large tensile stresses in webs, and severe cracking may develop if no web reinforcement is provided. Unintended curvature of the tendons, and the resulting tensile stresses in webs, may be minimized by securely tying tendons to stirrups that are rigidly held in place by other elements of the reinforcement cage. The recommended maximum spacing of stirrups used for this purpose is the smaller of 1.5$h$ or 4 ft. If applicable, the shear reinforcement provisions of 13.6.3 and 13.7.7.2.2 will require closer stirrup spacings. <R11.4.6.1>

For repeated loading of flexural members, the possibility of inclined diagonal tension cracks forming at stresses appreciably smaller than under static loading should be taken into account in design. In these instances, it would be prudent to use at least the minimum shear reinforcement expressed by 13.6.3.3, even though tests or calculations based on static loads show that shear reinforcement is not required. <R11.4.6.1>

**R13.6.3.2** — When a beam is tested to demonstrate that its shear and flexural strengths are adequate, the actual beam dimensions, and material strengths, and behavior are known. Therefore, the test strengths are considered the nominal strengths $V_u$ and $M_u$. Considering these strengths as nominal values ensures that if the actual material strengths in the field were less than specified, or the member dimensions were in error such as to result in a reduced member strength, a satisfactory margin of safety will be retained due to the strength reduction factor, $\phi$.

The code specified $\phi$ values ($\phi < 1.0$) ensure that if the actual material strengths in the field were less than specified, the beam dimensions were in error, or the Code strength equations overestimated beam strength such as to result in a reduced beam strength, a satisfactory margin of safety will be retained. Tested beam strengths are not affected by these errors, and hence the strength reduction factor is not applied. <R11.4.6.2>

**R13.6.3.3** — Tests have indicated the need to increase the minimum area of shear reinforcement as the concrete strength increases to prevent sudden shear failures when inclined cracking occurs. Therefore, Equations in table rows 13.6.3.3(a) and (c) provide for a gradual increase in the minimum area of transverse reinforcement with increasing concrete strength. Equations in table rows 13.6.3.3(b) and (d) provide for an absolute minimum area of transverse reinforcement independent of concrete strength and control for concrete strengths less than 4400 psi. <R11.4.6.3>
Tests\textsuperscript{11.33} of prestressed beams with minimum web reinforcement based on 13.6.3.3 indicated that the smaller $A_v$ from equations (c) and (e) is sufficient to develop ductile behavior. Equation (e) is discussed in Reference 11.28. <R11.4.6.4>

**R13.6.4 — Minimum torsional reinforcement**

**R13.6.4.2** — The differences in the definition of $A_v$ and the symbol $A_t$ should be noted; $A_v$ is the area of two legs of a closed stirrup whereas $A_t$ is the area of only one leg of a closed stirrup. If a stirrup group has more than two legs, only the legs adjacent to the sides of the beam are considered as discussed in R13.5.4.3. <R11.5.5.1 & R11.5.5.2>

Tests\textsuperscript{11.31} of high-strength reinforced concrete beams have indicated the need to increase the minimum area of shear reinforcement to prevent shear failures when inclined cracking occurs. Although there are a limited number of tests of high-strength concrete beams in torsion, the equation for the minimum area of transverse closed stirrups has been made consistent with calculations required for minimum shear reinforcement. <R11.5.5.1 & R11.5.5.2>

**R13.6.4.3** — Minimum longitudinal reinforcement is provided to prevent abrupt torsional failure immediately after torsional cracking. The torsional cracking moment decreases with applied shear; therefore, shear reduces the torsional reinforcement required to prevent brittle failure immediately after cracking. Equation 13.6.4.3(a) is based on test results of beams subject to pure torsion modified by mechanics of materials to consider shear and an assumption that the ratio of torsion stresses to shear stresses is 2:1. \textsuperscript{11.33} Equation 13.6.4.3(a) has been found to be appropriate for a practical range of reinforced beams satisfying 13.5.1.1(c) \textsuperscript{11.31}. <~> Under combined torsion and shear, the torsional cracking moment decreases with applied shear, which leads to a reduction in torsional reinforcement required to prevent brittle failure immediately after cracking. When subjected to pure torsion, reinforced concrete beam specimens with less than 1 percent torsional reinforcement by volume have failed at first torsional cracking.\textsuperscript{11.31} Equation 13.6.4.3(a) is based on a 2:1 ratio of torsion stress to shear stress and results in a torsion reinforcement volumetric ratio of about 0.5 percent. \textsuperscript{11.33} <R11.5.5.3> <~>

Tests of prestressed concrete beams have shown that a similar amount of longitudinal reinforcement is required. \textsuperscript{11.31} <~>

**R13.7 — Reinforcement detailing**

**R13.7.1 — General**

**R13.7.2 — Reinforcement spacing**

**R13.7.2.3** — For relatively deep flexural beams, some reinforcement should be placed near the vertical faces of the tension zone to control cracking in the web \textsuperscript{10.20,10.21} as shown in Fig.
R13.7.2.3. Without such auxiliary reinforcement, the width of the cracks in the web may exceed the crack widths at the level of the flexural tension reinforcement. <R10.6.7>

The size of the skin reinforcement is not specified; research has indicated that the spacing rather than bar size is of primary importance. Bar sizes No. 3 to No. 5, or welded wire reinforcement with a minimum area of 0.1 in.² per foot of depth, are typically provided. <R10.6.7>

Fig. R13.7.2.3—Skin reinforcement for beams and joists with h > 36 in.

R13.7.3 — Flexural reinforcement in nonprestressed beams

R13.7.3.2 — In codes prior to 2014, the one of the critical sections was defined as the location “where adjacent reinforcement terminates, or is bent”. This definition was inconsistent with other provisions in the code as well as the commentary figure illustrating the critical section. In the 2014 code, the this critical section is redefined as the location “where bent or terminated tension reinforcement is no longer required to resist flexure.” This wording is now consistent throughout the Code. <>

Critical sections for a typical continuous beam are indicated with a “c” for points of maximum stress or an “x” for points where bent or terminated tension reinforcement is no longer required to resist flexure in Fig. R13.7.3.2. For uniform loading, the positive reinforcement extending into the support is more apt to be governed by the requirements of 13.7.8.1 or 13.7.3.8.3 rather than by development length measured from a point of maximum moment or bar cutoff. <R12.10.2>
Fig. R13.7.3.2—Development of flexural reinforcement in a typical continuous beam.

R13.7.3.3 — The moment diagrams customarily used in design are approximate; some shifting of the location of maximum moments may occur due to changes in loading, settlement of supports, lateral loads, or other causes. A diagonal tension crack in a flexural member without stirrups may shift the location of the calculated tensile stress approximately a distance $d$ toward a point of zero moment. If stirrups are provided, this effect is less severe, although still present to some extent. <R12.10.3>

To provide for shifts in the location of maximum moments, the Code requires the extension of reinforcement a distance $d$ or $12d_b$ beyond the point at which it is calculated to be no longer required to resist flexure, except as noted. Cutoff points of bars to meet this requirement are illustrated in Fig. R13.7.3.2. When bars of different sizes are used, the extension should be in accordance with the diameter of bar being terminated. <R12.10.3>

R13.7.3.4 — Local peak stresses exist in the remaining bars wherever adjacent bars are cut off in tension regions. In Fig. R13.7.3.2, an “x” is used to indicate the point where terminated tension reinforcement is no longer required to resist flexure. If bars were cut off at this location (the required cut-off location is beyond this point in accordance by 13.7.3.3), peak stresses in the
continuing bars would reach $f_y$ at "x". Therefore, the continuing reinforcement is required to have a full $L_d$ extension as indicated. The peak stress points remaining in continuing bars after part of the bars have been cut off. If bars are cut off as short as the moment diagrams allow, these peak stresses reach $f_y$, which requires a full $L_d$ extension as indicated. This extension may exceed the length required to resist flexural tension forces. <R12.10.4>

**R13.7.3.5** — Reduced shear strength and loss of ductility when bars are cut off in a tension zone, as in Fig. R13.7.3.2, have been reported. The Code does not permit flexural reinforcement to be terminated in a tension zone unless additional conditions are satisfied. Flexural cracks tend to open at low load levels wherever any reinforcement is terminated in a tension zone. If the stress in the continuing reinforcement and the shear strength are each near their limiting values, diagonal tension cracking tends to develop prematurely from these flexural cracks. Diagonal cracks are less likely to form where shear stress is low (13.7.3.5(a)) or flexural reinforcement stress is low (13.7.3.5(b)). Diagonal cracks can be restrained by closely spaced stirrups (13.7.3.5(c)). These requirements are not intended to apply to tension splices that are covered by 21.5 and 21.8.1.7. <R12.10.5>

<Note: 21.8.1.7 is deleted as this provision is not applicable. 21.8.1.7 is for splices of lapped U-shaped stirrups and not relevant for this discussion.>

**R13.7.3.7** — A bar bent to the far face of a beam and continued there may be considered effective in satisfying 13.7.3.3 to the point where the bar crosses the mid-depth of the member. <R12.10.3>

**R13.7.3.8** — Termination of reinforcement

**R13.7.3.8.1** — Positive moment reinforcement is extended into the support to provide for some shifting of the moments due to changes in loading, settlement of supports, and lateral loads. It also enhances structural integrity. <R12.11.1>

For precast beams, tolerances and reinforcement cover distances need to be considered to avoid bearing on plain concrete where reinforcement has been discontinued. <R16.6.2.3>

**R13.7.3.8.3** — The diameter of the positive moment tension reinforcement is limited to assure/ensure that the bars are developed in a length short enough such that the moment capacity is greater than the applied moment over the entire length of the beam. As illustrated in the moment diagram of Fig. R13.7.3.8.3(a), the slope of the moment diagram is $V_u$ while the slope of moment development is $M_n / \ell_d$ where $M_n$ is the nominal flexural strength of the cross section without the $\phi$ factor. By sizing the reinforcement such that the capacity slope, $M_n / \ell_d$, equals or exceeds the demand slope, $V_u$, proper development is provided. Therefore, $M_n / V_u$ represents the required/available development length. Under favorable support conditions, a 30 percent increase for $M_n / V_u$ is permitted when the ends of the reinforcement are confined by a compressive reaction. <~>

The application of this provision is illustrated in Figure R13.7.3.8.3(ab) for simple supports and in Fig. R13.7.3.8.3(bc) for points of inflection. For example, a bar size is provided at a simple
support such that $\ell_d$ is calculated in accordance with 21.4.2. The bar size provided is satisfactory only if calculated $\ell_d$ does not exceed $1.3M_n/V_u + l_a$. <R12.11.3>

The $\ell_a$ to be used at points of inflection is limited to the effective depth of the member $d$ or 12 bar diameters ($12d_b$), whichever is greater. The $\ell_a$ limitation is provided because test data are not available to show that a long end anchorage length will be fully effective in developing a bar that has only a short length between a point of inflection and a point of maximum stress. <R12.11.3>

![Diagram](image)

\[ \text{Capacity slope} \left( \frac{M_u}{\ell_d} \right) \leq \text{Demand slope} \left( V_u \right) \]

\[ \ell_d \leq \frac{M_n}{V_u} \]

\[(a) \ Positive \ M_u \ Diagram\]

![Diagram](image)

Note: The 1.3 factor is usable only if the reaction confines the ends of the reinforcement.

\[(b) \ Maximum \ \ell_d \ at \ simple \ support\]

| Note: Change “usable” to “applicable” in Note in (b) |
R13.7.4 — Flexural reinforcement in prestressed beams

R13.7.4.1 — External tendons are often attached to the concrete beam at various locations between anchorages (such as midspan, quarter points, or third points) for desired load balancing effects, for tendon alignment, or to address tendon vibration concerns. Consideration should be given to the effects caused by the tendon profile shifting in relationship to the concrete centroid as the member deforms under effects of post-tensioning and applied load. <R18.22.3>

R13.7.4.2 — Nonprestressed reinforcement should be adequately developed to achieve factored load forces. The requirements of 13.7.3 will ensure that bonded reinforcement required for flexural strength under factored loads will be adequately developed to achieve tension or compression forces. <R18.9.4>

R13.7.4.4.1 — The minimum lengths apply for bonded reinforcement required by 13.6.2.3 only. Research on continuous spans shows that these minimum lengths provide satisfactory behavior under service load and factored load conditions. <R18.9.4>

R13.7.5 — Longitudinal torsional reinforcement

R13.7.5.1 — Longitudinal reinforcement is needed to resist the sum of the longitudinal tensile forces due to torsion. Because the force acts along the centroidal axis of the section, the centroid of the additional longitudinal reinforcement for torsion should approximately coincide with the centroid of the section. The Code accomplishes this by requiring the longitudinal torsional reinforcement to be distributed around the perimeter of the closed stirrups. Longitudinal bars or tendons are required in each corner of the stirrups to provide anchorage for the legs of the stirrups. Corner bars have also been found to be very effective in developing torsional strength and in controlling cracks. <R11.5.6.2>
R13.7.5.3 — The distance \((b_t + d)\) beyond the point at which it—longitudinal torsional reinforcement is calculated to be no longer required for torsional reinforcement is greater than that used for shear and flexural reinforcement because torsional diagonal tension cracks develop in a helical form. The same distance is required by 13.7.7.3.2 for Note that 13.7.7.3.2 requires the same distance for transverse torsional reinforcement. <R11.5.6.3>

R13.7.5.4 — Longitudinal torsion reinforcement required at a support should be adequately anchored into the support. Sufficient embedment length should be provided outside the inner face of the support to develop the needed tension force in the bars or tendons. In the case of bars, this may require hooks or horizontal U-shaped bars lapped with the longitudinal torsion reinforcement. <R11.5.4.3>

Note: Structural integrity moved to 13.7.7 and transverse reinforcement to 13.7.6 in the code. The following section numbers have been adjusted.

R13.7.6 — Transverse reinforcement

R13.7.6.3 — Torsion

R13.7.6.3.1 — The stirrups must be closed because inclined cracking due to torsion may occur on all faces of a member. <R11.5.4.1>

In the case of sections subjected primarily to torsion, the concrete side cover over the stirrups spalls off at high torques. This renders lapped-spliced stirrups ineffective, leading to a premature torsional failure. Therefore, closed stirrups should not be made up of pairs of U-stirrups lapping one another. <R11.5.4.1>

R13.7.6.3.2 — The distance \((b_t + d)\) beyond the point at which it—transverse torsional reinforcement is calculated to be no longer required for torsional reinforcement is greater than that used for shear and flexural reinforcement because torsional diagonal tension cracks develop in a helical form. The same distance is required by 13.7.5.3 for longitudinal torsional reinforcement. <R11.5.6.3>

R13.7.6.3.3 — The spacing of the closed stirrups or hoops transverse torsional reinforcement is limited to ensure the development of the nominal torsional strength of the beam, to prevent excessive loss of torsional stiffness after cracking, and to control crack widths. For a square cross section, the \(p_h/8\) limitation requires stirrups at approximately \(d/2\), which corresponds to 13.7.6.2. <R11.5.6.1>

R13.7.6.3.4 — The closed stirrups or hoops provided for torsion transverse torsional reinforcement in a hollow section should be located in the outer half of the wall thickness effective for torsion where the wall thickness can be taken as \(A_{oh}/p_h\). <R11.5.4.4>

R13.7.6.4 — Lateral support of compression reinforcement
R13.7.6.1 — Compression reinforcement in beams should be enclosed by transverse reinforcement to prevent buckling. <R7.11.1>

R13.7.6.7 — Structural integrity reinforcement in cast-in-place beams

Experience has shown that the overall integrity of a structure can be substantially enhanced by minor changes in detailing of reinforcement and connections. It is the intent of this section of the Code to improve the redundancy and ductility in structures so that in the event of damage to a major supporting element or an abnormal loading event, the resulting damage may be confined to a relatively small area and the structure will have a better chance, higher probability of maintaining overall stability. <R7.13>

With damage to a support, top reinforcement that is continuous over the support, but not confined by stirrups, will tend to tear out of the concrete and will not provide the catenary action required to bridge the damaged support. By making a portion of the bottom reinforcement continuous, catenary action can be provided. <R7.13.2>

If the depth of a continuous beam changes at a support, the bottom reinforcement in the deeper member should be terminated with a standard hook and bottom reinforcement in the shallower member should be extended into and fully developed in the deeper member. <R7.13.2>

If the depth of a continuous beam changes at a support, the bottom reinforcement in the deeper member should be terminated into the support with a standard hook or headed bar and the bottom reinforcement in the shallower member should be extended into and fully developed in the deeper member. <R7.13.2>

Requiring continuous top and bottom reinforcement in perimeter or spandrel beams provides a continuous tie around the structure. It is not the intent to require a tension tie of continuous reinforcement of constant size around the entire perimeter of a structure, but rather to require that one-half of the top flexural reinforcement, required to extend past the point of inflection by 13.7.3.8.4, be further extended and spliced at or near midspan in 13.7.6.5. Similarly, the bottom reinforcement required to extend into the support in 13.7.3.8.2 should be made continuous or spliced with bottom reinforcement from the adjacent span. <R7.13.2>

R13.7.67.1 — Requiring continuous top and bottom reinforcement in perimeter or spandrel beams provides a continuous tie around the structure. It is not the intent to require a tension tie of continuous reinforcement of constant size around the entire perimeter of a structure, but rather to require that one-half of the top flexural reinforcement, required to extend past the point of inflection by 13.7.3.8.4, be further extended and spliced at or near midspan in 13.7.7.5. Similarly, the bottom reinforcement required to extend into the support in 13.7.3.8.2 should be made continuous or spliced with bottom reinforcement from the adjacent span. <R7.13.2>

Figure R13.7.67.1 shows an example of a two-piece stirrup that satisfies the requirement of Sections 13.7.67.1(c) and 13.7.67.2(b). The 90-degree hook of the cap tie is located on the slab side so that it can be better confined. Pairs of U-stirrups lapping one another as defined in 21.8.1.7 are not permitted in perimeter or spandrel beams. In the event of damage to the side
concrete cover, the top longitudinal reinforcement may tend to tear out of the concrete and will not be adequately restrained by the exposed lap splice of the stirrup. Thus, the top longitudinal reinforcement will not provide the catenary action needed to bridge over a damaged region. Further, lapped U-stirrups will not be effective at high torque as discussed in R13.7.6.3.1.<R7.13.2>

Fig. R13.7.6.1—Example of a two-piece stirrup that complies with the requirements of 13.7.6.1(c) and 13.7.6.2(b)

R13.8 — One-way joist systems

R13.8.1 — General

The size and spacing limitations for concrete joist construction are based on successful past performance of joist systems meeting the limitations of 13.8.1. <R8.13>

R13.8.1.4 — A limit on the maximum spacing of ribs is required because of the provisions permitting higher shear strengths and less concrete protection for the reinforcement for these relatively small, repetitive members. <R8.13.3>

R13.8.1.5 — This increase in shear strength is justified on the basis of: (1) satisfactory performance of joist construction designed with higher calculated shear strengths specified in previous Codes, which allowed comparable shear stresses, and (2) potential for redistribution of local overloads to adjacent joists. <R8.13.8>