Footings Example 1—Design of a square spread footing of a seven-story building

Design and detail a typical square spread footing of a six bay by five bay seven-story building, founded on stiff soil, supporting a 24 in. square column. The building has a 10 ft high basement. The bottom of the footing is 13 ft below finished grade. The building is assigned to Seismic Design Category (SDC) B.

**Given:**

**Column load**
- Service dead load $D = 541$ kip
- Service live load $L = 194$ kip
- Seismic load $E = \pm 18$ kip
  (Column force due to the building frame resisting the seismic load)

**Material properties**
- Concrete compressive strength $f'_{c} = 4$ ksi
- Steel yield strength $f_{y} = 60$ ksi
- Normalweight concrete $\lambda = 1$
- Density of concrete = 150 lb/ft$^3$

**Allowable soil-bearing pressures**
- $D$ only: $q_{all,D} = 4000$ psf
- $D + L$: $q_{all,D+L} = 5600$ psf
- $D + L + E$: $q_{all,L+E} = 6000$ psf

<table>
<thead>
<tr>
<th>ACI 318-14</th>
<th>Procedure</th>
<th>Computation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Step 1: Foundation type</td>
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<tr>
<td>13.1.1</td>
<td>This bottom footing is 3 ft below basement slab. Therefore, it is considered a shallow foundation.</td>
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</tbody>
</table>

| Step 2: Material requirements | | |
| 13.2.1.1 | The mixture proportion must satisfy the durability requirements of Chapter 19 (318) and structural strength requirements. The designer determines the durability classes. Please see Chapter 2 of SP-17 for an in-depth discussion of the categories and classes. ACI 301 is a reference specification that is in sync with ACI 318. ACI encourages | By specifying that the concrete mixture shall be in accordance with ACI 301 and providing the exposure classes, Chapter 19 requirements are satisfied. Based on durability and strength requirements, and experience with local mixtures, the compressive strength of concrete is specified at 28 days to be at least 4000 psi. |
Referencing 301 into job specifications.
There are several mixture options within ACI 301, such as admixtures and pozzolans, which the designer can require, permit, or review if suggested by the contractor.
Example 1 provides a more detailed breakdown on determining the concrete compressive strength and exposure categories and classes.

Step 3: Determine footing dimensions

13.3.1.1 To calculate the footing base area, divide the service load by the allowable soil pressure.

Area of footing = \( \frac{\text{total service load} \times \sum P}{\text{allowable soil pressure, } q_a} \)

Assuming a square footing.

The footing thickness is calculated in Step 5, footing design.

The unit weights of concrete and soil are 150 pcf and 120 pcf; close. Therefore, footing self-weight will be ignored:

\[
\begin{align*}
D &= \frac{541 \text{ k}}{4 \text{ ksf}} = 135 \text{ ft}^2 \quad \text{(controls)} \\
(D + L) &= \frac{541 \text{ k} + 194 \text{ k}}{5.6 \text{ ksf}} = 131 \text{ ft}^2 \\
(D + L + E) &= \frac{541 \text{ k} + 194 \text{ k} + 18 \text{ k}}{6 \text{ ksf}} = 126 \text{ ft}^2 \\
l &= \sqrt{135 \text{ ft}^2} = 11.6 \text{ ft}
\end{align*}
\]

Therefore, provide 12 x 12 ft square footing.

Step 4: Soil pressure

Footing stability
Because there is no overturning moment, overall footing stability is assumed.

Calculate factored soil pressure

\[
q_u = \sum \frac{P_u}{\text{Area}}
\]
5.3.1(a) Calculate the soil pressures resulting from the applied factored loads.

Load Case I: \( U = 1.4D \)

\[
\begin{align*}
q_u &= \frac{757 \text{ kip}}{144 \text{ ft}^2} = 5.3 \text{ ksf}
\end{align*}
\]

Load Case II: \( U = 1.2D + 1.6L \)

\[
\begin{align*}
q_u &= \frac{960 \text{ kip}}{144 \text{ ft}^2} = 6.7 \text{ ksf} \quad \text{(controls)}
\end{align*}
\]

Load Case IV: \( U = 1.2D + 1.0E + 1.0L \)

\[
\begin{align*}
q_u &= \frac{861 \text{ kip}}{144 \text{ ft}^2} = 6.0 \text{ ksf}
\end{align*}
\]

Load Case IV: \( U = 0.9D + 1.0E \)

\[
\begin{align*}
q_u &= \frac{505 \text{ kip}}{144 \text{ ft}^2} = 3.5 \text{ ksf}
\end{align*}
\]

The load combinations includes the seismic uplift force. In this example, uplift does not occur.

13.3.2.1 Because the footing has equal dimension in plan, it will be designed in one direction and symmetry is assumed.

Step 5: One-way shear design

Fig. 1.2—One-way shear in longitudinal direction.
| 21.2.1(b) | Shear reduction factor: | $\phi V_n \geq V_u$ |
| 7.5.1.1 | | $V_n = V_c + V_s$ |
| 7.5.3.1 | | $\phi_{\text{shear}} = 0.75$ |
| 22.5.1.1 | | Assume $V_s = 0$ (no shear reinforcement) $V_n = V_c$ |
| 22.5.5.1 | Therefore: | $V_c = 2\sqrt{f'_c b_w d}$ |
| 7.4.3.2 | And satisfying: | $V_u \leq \phi V_c$ |
| 22.5.1.1 | The critical section for one-way shear is at a distance $d$ from the face of the column (Fig. 1.2). |
| 7.4.3.2 | The engineer could either assume a value for $d$ that satisfies the strength Eq. (22.5.5.1) by iteration or solve Eq. (7.5.1.1). |
| 20.6.1.3.1 | In this example, the first approach is followed: |
| 20.6.1.3.1 | Assume that the footing is 30 in. thick. |
| 20.6.1.3.1 | The cover requirement is 3 in. to bottom of reinforcement. Assume that No. 8 bars are used in the both directions and design for the more critical case (upper layer). Therefore, the effective depth $d$: |
| 20.6.1.3.1 | $d = 30 \text{ in.} - 3 \text{ in.} - 1 \text{ in.} - 1 \text{ in.}/2$ |
| 20.6.1.3.1 | $= 25.5 \text{ in.}$ |
| 20.6.1.3.1 | $\phi V_n \geq V_u = \left(\frac{l - c}{2} - d\right) b q u$ |
| | $V_u = \left(\frac{(12 \text{ ft})}{2} - \frac{24 \text{ in.}}{2(12 \text{ in./ft})} - \frac{25.5 \text{ in.}}{12 \text{ in./ft}}\right)$ |
| | $(12 \text{ ft})(6.7 \text{ ksf}) = 231 \text{ kip}$ |
| | $\phi V_c = 0.75(2) \sqrt{4000 \text{ psi}} (12 \text{ ft})(25.5 \text{ in.})$ |
| | $(12 \text{ in./ft}) = 348 \text{ kip}$ |
| | $\phi V_c = 348 \text{ kip} > V_u = 231 \text{ kip}$ OK |
| | Therefore, assumed depth is adequate: |
| | $h = 30 \text{ in.}$ |
Step 6: Two-way shear design

The foundation will not be reinforced with shear reinforcement for two-way action. Therefore, the nominal shear strength for two-way foundation without shear reinforcement is equal to the concrete shear strength:

\[ v_n = v_c \]

22.6.1.2

Under punching shear theory, inclined cracks are assumed to originate and propagate at 45 degrees away and down from the column corners. The punch area is calculated at an average distance of \( d/2 \) from column face on all sides (Fig. 1.3).

\[ b_o = 4(c + d) \]

22.6.1.4

ACI 318 permits the engineer to take the average of the effective depth in the two orthogonal directions when designing the footing, but in this example the smaller effective depth will be used.

22.6.4.1

22.6.5.1

The two-way shear strength equations for nonprestressed members must be satisfied and the least calculated value of (a), (b), and (c) controls:

\[ v_c = 4\lambda \sqrt{f_c} \]  
\[ v_c = (2 + \frac{4}{\beta})\lambda \sqrt{f_c} \]  
\[ v_c = (\alpha_s d/b_o + 2)\lambda \sqrt{f_c} \]

where \( \beta \) is ratio of the long side to short side of column; \( \beta = 1 \)

\[ v_c = 4(1.0)(\sqrt{4000 \text{ psi}}) = 253 \text{ psi} \]

\[ v_c = (2 + \frac{4}{1})(1.0)(\sqrt{4000 \text{ psi}}) = 379.5 \text{ psi} \]

\[ v_c = (\frac{40(25.5 \text{ in.})}{198 \text{ in.}} + 2)(\sqrt{4000 \text{ psi}}) = 452 \text{ psi} \]

Equation (a) controls; \( v_c = 253 \text{ psi} \)
### Step 7: Flexure design

**α_s = 40, considered interior column**

\[ V_c = 4\lambda\sqrt{f_c'\cdot b_o\cdot d} \]

**Use a reduction factor of 0.75:**

\[ \phi V_c = (0.75)4\lambda\sqrt{f_c'\cdot b_o\cdot d} \]

**Check if design strength exceeds required strength:**

\[ \phi V_c \geq V_u? \]

**\[ (0.75)4\lambda\sqrt{f_c'\cdot b_o\cdot d} \]**

**\[ V_c = \frac{(253 \text{ psi})(198 \text{ in.})(25.5 \text{ in.})}{1000 \text{ lb/kip}} = 1277 \text{ kip} \]

\[ \phi = 0.75 \]

\[ \phi V_c = 0.75(1277 \text{ kip}) = 958 \text{ kip} \]

\[ V_u = [(12 \text{ ft})(12 \text{ ft}) - \left(\frac{24 \text{ in.} + 25.5 \text{ in.}}{12 \text{ in./ft}}\right)^2] (6.7 \text{ ksf}) = 851 \text{ kip} \]

\[ \phi V_c = 958 \text{ kip} > V_u = 851 \text{ kip} \quad \text{OK} \]

Two-way shear strength is adequate.

**13.2.71**

The critical section is permitted to be at the face of the column (Fig. 1.4).

**\[ M_u = q_u\left(\frac{l-c}{2}\right)^2(b)/2 \]**

**\[ M_u = q_u\left(\frac{l-c}{2}\right)^2(b)/2 \]**

**Fig. 1.4—Flexure in the longitudinal direction.**

**\[ M_u = (6.7 \text{ ksf})(\frac{12 \text{ ft}}{2}) \left(\frac{24 \text{ in.}}{12 \text{ in./ft}}\right)^2(12 \text{ ft}) / 2 \]**
22.2.1.1 Set compression force equal to tension force at the column face: $C = T$

$C = 0.85 f'_c ba$ and $T = A_s f_y$

$a = \frac{A_s f_y}{0.85 f'_c b}$ and

22.3.1.1 $\phi M_n = \phi A_s f_y (d - \frac{a}{2})$

22.2.2.2 Substitute for $a$ in the equation above.

21.2.1(a) Use reduction factor from Table 21.2.1.

8.5.1.1(a) Setting $\phi M_n \geq M_u = 1005$ ft-k and solving for $A_s$:

$\phi M_n = (0.9) A_s (60 \text{ ksi})(25.5 \text{ in.} - \frac{(0.1) A_s}{2})$

$A_s = 8.91 \text{ in.}^2$

8.6.1.1 Check the minimum reinforcement ratio: $\rho_l = 0.0018$

13.3.3.3(a) Use 13 No. 8 bars distributed uniformly across the entire 12 ft width of footing.

21.2.1(a) Check if the tension controlled assumption and the use of $\phi = 0.9$ is correct.

To answer the question, the calculated tensile strain in reinforcement is compared to the values in Table 21.2.2. The strain in reinforcement is calculated from similar triangles (refer to Fig. 1.5):

21.2.2 $\epsilon_t = \frac{\epsilon_c}{c} (d - c)$

where: $c = \frac{a}{\beta_1}$ and $a = 0.28 A_s$

22.2.2.4.1

22.2.2.4.3

$M_u = 1005$ ft-kip

$a = \frac{A_s (60 \text{ ksi})}{0.85(5 \text{ ksi})(12 \text{ ft})} = 0.1 A_s$

$\phi = 0.9$

$\phi M_n \leq (0.9) A_s (60 \text{ ksi})(25.5 \text{ in.} - \frac{(0.1) A_s}{2})$

$A_s = 8.91 \text{ in.}^2$

$A_{s, min} = 0.0018(12 \text{ ft})(12 \text{ in./ft})(30 \text{ in.}) = 7.8 \text{ in.}^2 < A_{s, req'd} = 8.91 \text{ in.}^2 \text{ OK}$

Note: Although not required by code, some practitioner distribute half the required bars in the mid third of the footing and distribute the remaining bars in the equally on both sides.

$c = \frac{0.1(13)(0.79 \text{ in.}^2)}{0.85} = 1.21 \text{ in.}$

$\epsilon_t = \frac{0.003}{1.21 \text{ in.}} (25.5 \text{ in.} - 1.21 \text{ in.}) = 0.06$

$\epsilon_t = 0.06 > 0.005$
Section is tension controlled and $\phi = 0.9$ 

![Diagram of strain distribution across footing.](image)

**Fig. 1.5—Strain distribution across footing.**

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**Step 8: Transfer of column forces to the base**

16.3.1.1 Factored forces are transferred to the foundation at the base of the column by bearing on concrete and the reinforcement dowels.

22.8.3.2 The foundation is wider on all sides than the loaded area. Therefore, the nominal bearing strength, $B_n$, is the smaller of the two equations.

22.8.3.2(a) $B_n = \frac{A_2}{A_1} (0.85 f_c' A_1)$

and

22.8.3.2(b) $B_n = 2(0.85 f_c' A_1)$

Check if $\sqrt{\frac{A_2}{A_1}} \leq 2.0$ where $A_1$ is the bearing area of the column and $A_2$ is the area of the part of the supporting footing that is geometrically similar to and concentric with the loaded area.

\[
\sqrt{\frac{A_2}{A_1}} = \sqrt{\frac{[(12 \text{ ft})(12 \text{ in./ft})]^2}{(24 \text{ in.})^2}} = 6 > 2
\]

Therefore, Eq. (22.8.3.2(b)) controls.
<table>
<thead>
<tr>
<th>21.2.1(d)</th>
<th>The reduction factor for bearing is 0.65:</th>
<th>$\phi_{bearing} = 0.65$</th>
</tr>
</thead>
<tbody>
<tr>
<td>16.3.4.1</td>
<td>Column factored forces are transferred to the foundation by bearing and through reinforcement dowels. Provide dowel reinforcement area of at least $0.005A_g$ and at least four bars.</td>
<td>$\phi_B = (0.65)(2)(0.85)(4000 \text{ psi})(24 \text{ in.})^2$ $\phi_B = 2546 \text{ kip} &gt; 960 \text{ kip}$ (Step 4) <strong>OK</strong></td>
</tr>
<tr>
<td>16.3.5.4</td>
<td>Bars are in compression for all load combinations. Therefore, the bars must extend into the footing a compression development length, $l_{dc}$, the larger of the two:</td>
<td>$A_{s,dowel} = 0.005(24 \text{ in.})^2 = 2.88 \text{ in.}^2$ Use eight No. 6 bars</td>
</tr>
<tr>
<td>25.4.9.2</td>
<td>The footing depth must satisfy the following inequality so that the vertical reinforcement can be developed within the provided depth:</td>
<td>$h_{req,d} = 14.3 \text{ in.} + 6(0.75 \text{ in.}) + 0.75 \text{ in.}$ + 2(0.75 in.) + 3 in. = 24.05 in. $h_{req,d} = 24.1 \text{ in.} &lt; h_{prov} = 30 \text{ in.}$ <strong>OK</strong></td>
</tr>
<tr>
<td>25.3.1</td>
<td>$l_{dc} = \left{ \frac{f_y\psi_r}{50\lambda \sqrt{f_c}}db \left(0.0003f_y\psi_rdb\right) \right}$ $l_{dc} = \frac{0.02(60,000 \text{ psi})}{\sqrt{4000 \text{ psi}}} = 14.3 \text{ in.}$ $l_{dc} = 0.0003(60,000 \text{ psi})(0.75 \text{ in.}) = 13.5 \text{ in.}$ $l_{dc} = 14.3 \text{ in.}$ (controls)</td>
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Step 9: Footing details

<table>
<thead>
<tr>
<th>13.2.8.3</th>
<th>Development length</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>13.2.7.1</td>
<td>Reinforcement development is calculated at the maximum factored moment, which occurs at the column face. Bars must extend</td>
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</table>
a tension development length beyond the critical section.

\[ l_d = \left( \frac{3}{40} \frac{f_y}{\lambda \sqrt{f_c}} \frac{\psi_t \psi_e \psi_s}{c + K_{tr}} \right) d_b \]

where
\[ \psi_t = \text{bar location; not more than 12 in. of fresh concrete below horizontal reinforcement} \]
\[ \psi_e = \text{coating factor; uncoated} \]
\[ \psi_s = \text{bar size factor; No. 8 and larger} \]
\[ c_b = \text{spacing or cover dimension to center of bar, whichever is smaller} \]
\[ K_{tr} = \text{transverse reinforcement index} \]

It is permitted to use \( K_{tr} = 0 \).

But the expression: \( \frac{c_b + K_{tr}}{d_b} \) must not be taken greater than 2.5.

No. 6: \( \frac{c_b + K_{tr}}{d_b} = \frac{3.5 \text{ in.} + 0}{1.0 \text{ in.}} = 3.5 \leq 2.5 \)

No.8 bars: \( 28.5(1.0 \text{ in.}) = 28.5 \text{ in.} > 12 \text{ in.} \) OK

\[ l_d = \left( \frac{3}{40} \frac{60,000 \text{ psi}}{(1.0)(1.0)(1.0)} \right) d_b \]

\[ = 28.5d_b \]

\[ l_d \text{ in the longitudinal direction:} \]
\[ l_{d,prov.} = ((12 \text{ ft})(12 \text{ in./ft}) - 24 \text{ in.})/2 - 3 \text{ in.} \]
\[ l_{d,prov.} = 57 \text{ in.} > l_{d,req.d} = 28.5 \text{ in.} \text{ OK} \]

use straight No. 8 bars in both directions.
Step 10: Detailing

- (8) #6 DWLS
- (13) #8

Dimensions:
- 12 ft-0in. x 12 ft-0in.
- 2 ft-6 in.
- 3 in.