

A SPREADSHEET-BASED INVERSE ANALYSIS PROCEDURE FOR FLEXURAL SPECIMENS

-STRAIN SOFTENING OR HARDENING SAMPLES-

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Introduction

The uniqueness of the spreadsheet format is that adjustments to the curve fitting parameters are updated in real time, reducing analysis time and allowing a more accurate simulation. Using a model the parameters can be manipulated in a spreadsheet environment to model and best fit an experimental load deflection response. Units are not necessary as long as they are consistent. For English units ensure that all inputs are in inches, pounds force, or psi. For SI units ensure all inputs are in millimeters, Newtons or MPa. This rule of consistent units applies to every entry (including the sample dimensions) in the spreadsheet. The sample values that are given in the examples are only related to this specific sample set, values for input parameters and material properties will vary with respect to the shape of the experimental load deflection curve. This user guide uses the ASTM C-1609 (or 4-point bend test) as an example, the 3-point bend test can also be simulated using this spreadsheet by changing the test method option from 4 to 3 point bending (see figure 3).

There are 2 variations of this spreadsheet that have been developed the first includes the simultaneous modeling of flexural and tensile experimental data. The second is the prediction of associative JCI-SF4, RILEM TC 162-TDF and ASTM C-1609 standard residual strength parameters. The comparison of residual strength parameters from different standards provides a far more accurate determination of the residual strength of the FRC.

This user guide uses the following versions of the back-calculation spreadsheet. The sample data has been pre-loaded into the spreadsheet in order familiarize the user with the effect of changing the model parameters on the simulation curves. It can also be noted that parameters are interchangeable. If for example; if a fit is done using the standard Load-Deflection spreadsheet (v1.7) and you need the residual strength parameters for JCI, RILEM and ASTM, just input the parameters from v1.7 into v2.5 to generate the residual strengths. Because the spreadsheet uses the simulated response to generate the data, the experimental data is needed in this case, making the transition quick and straight forward.

Below are the associative spreadsheets included with this package;

Load-Deflection - FRC_Back_Calculation_Spreadsheet_LD_v1.7_(Dist).xlsx

Load-Deflection and Tension - FRC_Back_Calculation_Spreadsheet_TENSION_v1.5_(Dist).xlsx

Load-Deflection Standards - FRC_Back_Calculation_Spreadsheet_Std_v2.5_(Dist).xlsx

CHAPTER 1

Model

1.1 Derivation of the Moment-Curvature Relationship

Figure 1 presents a constitutive model for homogenized strain softening and hardening fiber reinforced concrete. As shown in Figure 1(a), the linear portion of an elastic-perfectly-plastic compressive stress-strain response terminates at yield point $(\varepsilon_{cy}, \sigma_{cy})$ and remains constant at compressive yield stress σ_{cy} until the ultimate compressive strain ε_{cr} . The tension model in figure 1(b) is described by a trilinear response with an elastic range defined by E , and then post cracking modulus E_{cr} . By setting E_{cr} to either a negative or a positive value, the same model can be used to simulate strain softening or strain hardening materials. The third region in the tensile response is a constant stress range defined with stress σ_{cst} in the post crack region. The constant stress level μ can be set to any value at the transition strain, resulting in a continuous or discontinuous stress response. Two strain measures are used to define the first cracking and transition strains $(\varepsilon_{cr}, \varepsilon_{tm})$. The tensile response terminates at the ultimate tensile strain level of ε_{tu} . The stress-strain relationship for compression and tension can be expressed as:

$$\sigma_c(\varepsilon_c) = \begin{cases} E_c \varepsilon_c & 0 \leq \varepsilon_c \leq \varepsilon_{cy} \\ E_c \varepsilon_{cy} & \varepsilon_{cy} \leq \varepsilon_c \leq \varepsilon_{cu} \\ 0 & \varepsilon_c > \varepsilon_{cu} \end{cases} \quad (1)$$

$$\sigma_t(\varepsilon_t) = \begin{cases} E\varepsilon_t & 0 \leq \varepsilon_t \leq \varepsilon_{cr} \\ E\varepsilon_{cr} + E_{cr}(\varepsilon_t - \varepsilon_{cr}) & \varepsilon_{cr} \leq \varepsilon_t \leq \varepsilon_{trn} \\ \mu E\varepsilon_{cr} & \varepsilon_{trn} \leq \varepsilon_t \leq \varepsilon_{tu} \\ 0 & \varepsilon_t > \varepsilon_{tu} \end{cases} \quad (2)$$

where, σ_c , σ_t , ε_c and ε_t are compressive and tensile stresses and strains respectively. In order to derive the closed form solutions for moment-curvature response in non-dimensional forms, the material parameters shown in Figure 1 (a)&(b) are defined as a combination of two intrinsic material parameters: the first cracking tensile strain ε_{cr} and tensile modulus E in addition to seven normalized parameters with respect to E and ε_{cr} as shown in Eqs (3)-(8).

$$\omega = \frac{\varepsilon_{cy}}{\varepsilon_{cr}}; \quad \alpha = \frac{\varepsilon_{trn}}{\varepsilon_{cr}}; \quad \beta_{tu} = \frac{\varepsilon_{tu}}{\varepsilon_{cr}}; \quad \lambda_{cu} = \frac{\varepsilon_{cu}}{\varepsilon_{cr}} \quad (3)$$

$$\gamma = \frac{E_c}{E}; \quad \eta = \frac{E_{cr}}{E} = -\frac{(1-\mu)}{(\alpha-1)} \quad (4)$$

$$\mu = \frac{\sigma_{cst}}{E\varepsilon_{cr}} \quad (5)$$

$$\mu_{crit} = \frac{\omega}{3\omega-1} \quad (6)$$

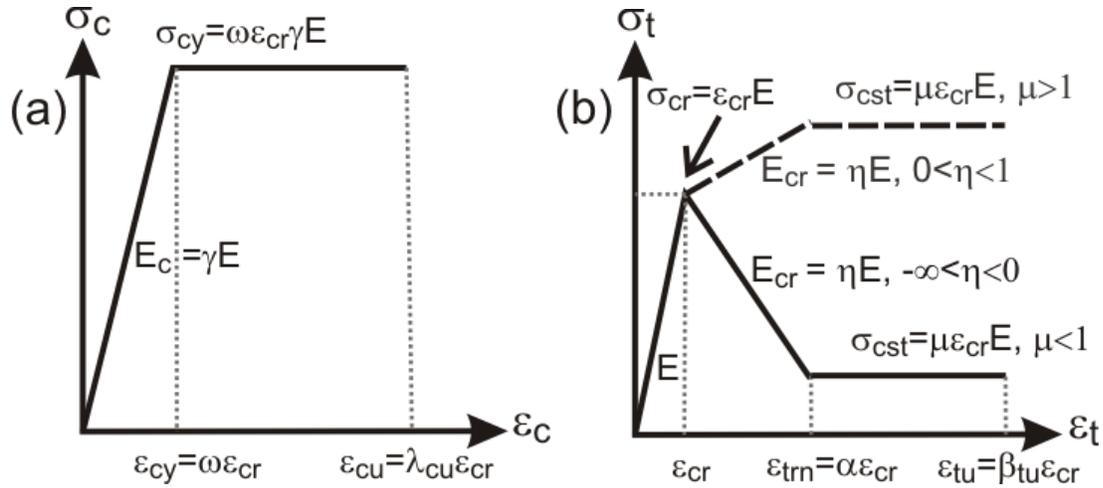


Figure 1 - Material model for homogenized fiber reinforced concrete (a) compression model and (b) tension model.

The normalized tensile strain at the bottom fiber β and compressive strain at the top fiber λ are defined as:

$$\beta = \frac{\varepsilon_{ibot}}{\varepsilon_{cr}}; \quad \lambda = \frac{\varepsilon_{ctop}}{\varepsilon_{cr}} \quad (7)$$

They are linearly related through the normalized neutral axis parameter, k .

$$\frac{\lambda \varepsilon_{cr}}{kd} = \frac{\beta \varepsilon_{cr}}{d - kd} \quad \text{or} \quad \lambda = \frac{k}{1 - k} \beta \quad (8)$$

Substitution of all normalized parameters defined in Eqs. (3) - (5) into Eq. (1)&(2) results in the following normalized stress strain models:

$$\frac{\sigma_c(\lambda)}{E\varepsilon_{cr}} = \begin{cases} \gamma\lambda & 0 \leq \lambda \leq \omega \\ \gamma\omega & \omega < \lambda \leq \lambda_{cu} \\ 0 & \lambda_{cu} < \lambda \end{cases} \quad \frac{\sigma_t(\beta)}{E\varepsilon_{cr}} = \begin{cases} \beta & 0 \leq \beta \leq 1 \\ 1 + \eta(\beta - 1) & 1 < \beta \leq \alpha \\ \mu & \alpha < \beta \leq \beta_{tu} \\ 0 & \beta_{tu} \leq \beta \end{cases} \quad (9)$$

In the derivation of moment curvature diagram for a rectangular cross section with a width b and depth d , the Kirchhoff hypothesis of plane section remaining plane for flexural loading is applied. By assuming linear strain distribution across the depth and ignoring shear deformation, the stress- strain relationships in Figure 1(a)&(b) are used to obtain the stress distribution across the cross section at three stages of imposed tensile strain: $0 < \beta \leq 1$, $1 < \beta \leq \alpha$ and $\alpha < \beta \leq \beta_{tu}$. For stage 2 and 3 there are two possible scenarios: the compressive strain at top fiber is either elastic ($0 < \lambda \leq \omega$) or plastic ($\omega < \lambda \leq \lambda_{cu}$). The neutral axis depth ratio k is then found by solving equilibrium of forces. Finally the moment capacity is calculated by taking tension and compression forces around the neutral axis and the corresponding curvature is obtained by dividing the top compressive strain with the neutral axis depth. Moment M and curvature ϕ are then normalized with their cracking moment M_{cr} and cracking curvature ϕ_{cr} to obtain a normalized moment M' and curvature ϕ' , respectively. Note that, from now on, the primed terms (M' or ϕ') refer to the normalized quantities with respect to cracking moment M_{cr} or cracking curvature ϕ_{cr} . For example $M' = M/M_{cr}$ and $\phi' = \phi/\phi_{cr}$. The expressions for calculating the moment curvature and neutral axis for all stages of applied strain are given in Eq. (10)&(11) and Table 1.

$$M(\lambda, k, \omega, \mu) = M_{cr} M'(\lambda, k, \omega, \mu) \quad M_{cr} = \frac{1}{6} b d^2 E \varepsilon_{cr} \quad (10)$$

$$\phi(\lambda, k, \omega, \mu) = \phi_{cr} \phi'(\lambda, k, \omega, \mu) \quad \phi_{cr} = \frac{2\varepsilon_{cr}}{d} \quad (11)$$

Stage	Parameters	k	M'	ϕ'
1	$0 < \beta \leq 1$	$k_1 = \begin{cases} \frac{1}{2} & \text{for } \gamma=1 \\ \frac{-1+\sqrt{\gamma}}{-1+\gamma} & \text{for } \gamma \neq 1 \end{cases}$	$m_1 = \frac{2\beta[(\gamma-1)k_1^3 + 3k_1^2 - 3k_1 + 1]}{1-k_1}$	$\phi'_1 = \frac{\beta}{2(1-k_1)}$
2.1	$1 < \beta \leq \alpha$ $0 < \lambda \leq \omega$	$k_{21} = \frac{D_{21} - \sqrt{D_{21}\gamma\beta^2}}{D_{21} - \gamma\beta^2}$ $D_{21} = \eta(\beta^2 - 2\beta + 1) + 2\beta - 1$	$M'_{21} = \frac{(2\gamma\beta^3 - C_{21})k_{21}^3 + 3C_{21}k_{21}^2 - 3C_{21}k_{21} + C_{21}}{1-k_{21}}$ $C_{21} = \frac{(2\beta^3 - 3\beta^2 + 1)\eta + 3\beta^2 - 1}{\beta^2}$	$\phi'_{21} = \frac{\beta}{2(1-k_{21})}$
2.2	$1 < \beta \leq \alpha$ $\omega < \lambda \leq \lambda_{cu}$	$k_{22} = \frac{D_{22}}{D_{22} + 2\omega\gamma\beta}$, $D_{22} = D_{21} + \gamma\omega^2$	$M'_{22} = (3\gamma\omega\beta^2 + C_{22})k_{22}^2 - 2C_{22}k_{22} + C_{22}$, $C_{22} = C_{21} - \frac{\gamma\omega^3}{\beta^2}$	$\phi'_{22} = \frac{\beta}{2(1-k_{22})}$
3.1	$\alpha < \beta \leq \beta_{tu}$ $0 < \lambda \leq \omega$	$k_{31} = \frac{D_{31} - \sqrt{D_{31}\gamma\beta^2}}{D_{31} - \gamma\beta^2}$ $D_{31} = \eta(\alpha^2 - 2\alpha + 1) + 2\mu(\beta - \alpha) + 2\alpha - 1$	$M'_{31} = \frac{(2\gamma\beta^3 - C_{31})k_{31}^3 + 3C_{31}k_{31}^2 - 3C_{31}k_{31} + C_{31}}{1-k_{31}}$ $C_{31} = \frac{(2\alpha^3 - 3\alpha^2 + 1)\eta - 3\mu(\alpha^2 - \beta^2) + 3\alpha^2 - 1}{\beta^2}$	$\phi'_{31} = \frac{\beta}{2(1-k_{31})}$
3.2	$\alpha < \beta \leq \beta_{tu}$ $\omega < \lambda \leq \lambda_{cu}$	$k_{32} = \frac{D_{32}}{D_{32} + 2\omega\gamma\beta}$, $D_{32} = D_{31} + \gamma\omega^2$	$M'_{32} = (3\gamma\omega\beta^2 + C_{32})k_{32}^2 - 2C_{32}k_{32} + C_{32}$, $C_{32} = C_{31} - \frac{\gamma\omega^3}{\beta^2}$	$\phi'_{32} = \frac{\beta}{2(1-k_{32})}$

Table 1 - Neutral axis parameter k , normalized moment M' and normalized curvature ϕ' for each stage of normalized tensile strain at bottom fiber (β)

For a given set of material parameters and dimension of the beam section, the moment curvature diagram can be generated by substituting an incremental normalized top compressive strain λ from zero up to failure. Two possible moment curvature responses, deflection hardening ($\mu > \mu_{crit}$) and deflection softening ($\mu < \mu_{crit}$), with the critical value for normalized post peak tensile strength is given in Eq. (6).

1.2 Algorithm to Predict Load Deflection Response

The load–deflection response of a beam can be obtained by using the moment–curvature response, crack localization rules, and moment-area method as follows.

1. For a given cross section and material properties, the normalized tensile strain at the bottom fiber β is incrementally imposed to generate the moment–curvature response using Eqs. (10), (11), and the expressions given in Table 1. For each value of β in stage 2 and 3, the condition for compressive stress

$\lambda < \omega$ or $\lambda > \omega$ is verified in advance of moment–curvature calculation.

2. Since a moment curvature diagram determines the maximum load allowed on a beam section, the discrete moments along the diagram are used to calculate the applied load vector $P = 2M/S$. Where S is a spacing between the support and loading point, $S=L/2$ for three point bending and $S=L/3$ for four point bending.

3. The beam is segmented into finite sections. For a given load step, static equilibrium is used to calculate moment distribution along the beam and moment–curvature relationship along with crack localization rules to identify the curvature.

4. The deflection at mid-span is calculated by numerical moment-area method of discrete curvature between the support and mid-span. This procedure is applied at each load step to until a complete load deflection response is obtained. A simplified procedure for direct calculation of the deflection is presented in the next section.

1.3 Determination of Deflection Response

With the moment-area method, the curvature diagram of a half-model according to the internal moment in Figure 2 (c-e) is divided into several areas. By taking the moment of the areas around the left support, the mid-span deflection is obtained. A set of equations for calculating the mid-span deflection δ of the three-point bending at the first cracking (δ_{cr}), at ultimate (δ_u) when material has $\mu > \mu_{crit}$, and at ultimate when material has $\mu < \mu_{crit}$ are presented in Eqs.12 (a-c).

$$\delta_{cr} = \frac{1}{12} L^2 \phi_{cr} \quad (12.a)$$

$$\delta_u = \frac{L^2}{24M_u^2} \left[(2M_u^2 - M_u M_{cr} - M_{cr}^2) \phi_u + (M_u^2 + M_u M_{cr}) \phi_{cr} \right] \quad \mu > \mu_{crit} \quad (12.b)$$

$$\delta_u = \frac{\phi_u L_p}{8} (2L - L_p) + \frac{M_u \phi_{cr} L}{12M_{cr}} (L - 2L_p) \quad \mu < \mu_{crit} \quad (12.c)$$

Similarly, a set of equations for four point bending can be written as:

$$\delta_{cr} = \frac{23}{216} L^2 \phi_{cr} \quad (13.a)$$

$$\delta_u = \frac{L^2}{216M_u^2} \left[(23M_u^2 - 4M_u M_{cr} - 4M_{cr}^2) \phi_u + (4M_u^2 + 4M_u M_{cr}) \phi_{cr} \right] \quad \mu > \mu_{crit} \quad (13.b)$$

$$\delta_u = \frac{5L^2 \phi_u}{72} + \frac{M_u L^2 \phi_{cr}}{27M_{cr}} \quad \mu < \mu_{crit} \quad (13.c)$$

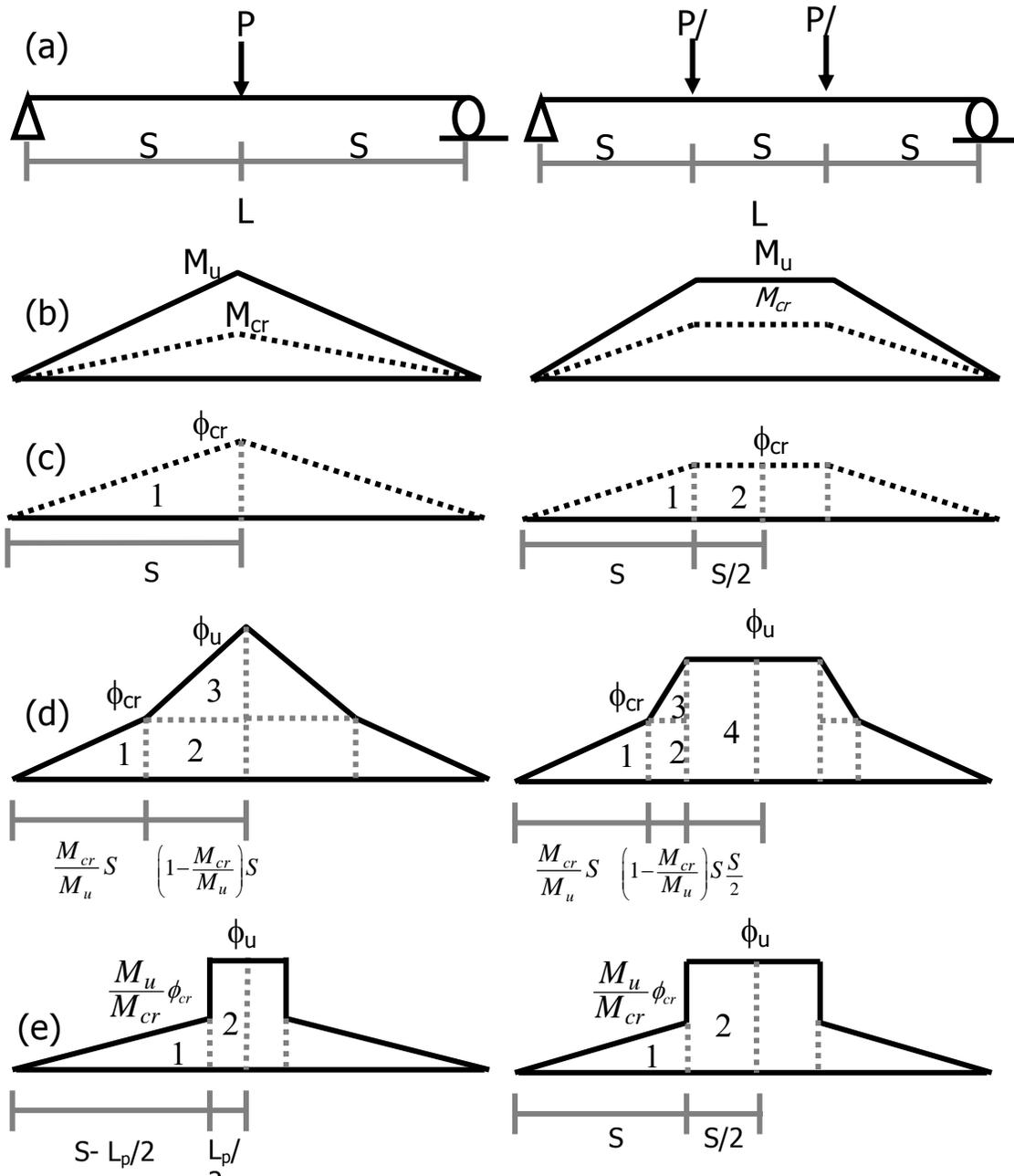


Figure 2: Three and four point bending test: (a) Experimental setup; (b) Moment distribution; (c) Curvature distribution at first bilinear cracking; (d) Curvature distribution at ultimate moment for high normalized post peak tensile strength ($\mu > \mu_{crit}$); (e) Curvature distribution at ultimate moment for low normalized post peak tensile strength ($\mu < \mu_{crit}$).

CHAPTER 2

Flexural Modeling

2.1 Four Point Bending – Strain Hardening

The four point bending test ASTM-1609 is carried out on standard beam dimensions with load applied at third points along the span (see Figure 2a). The load deflection curve will show an increase in load capacity after the linear elastic portion has ended. The hardening continues to increase as shown in Figure 3 or it can plateau as the sample continues to carry load.

Step 1:

Paste reduced load deflection data (up to 1000 points) into the columns labeled "Load" and "Deflection". This will populate the experimental load deflection curve we are trying to model as shown in Figure 3.

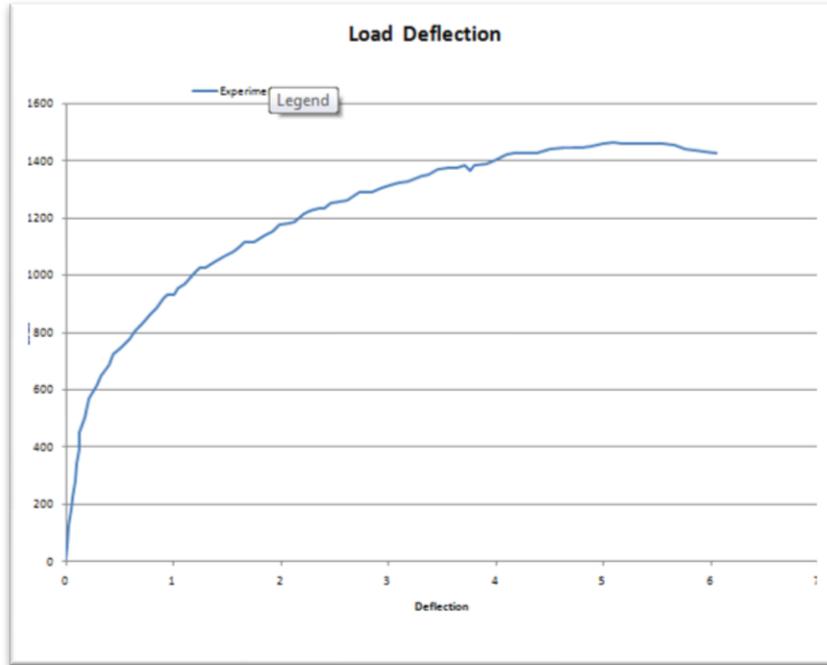


Figure 3 – Experimental load deflection response, showing deflection hardening characteristics.

Step 2:

Input the sample dimensions and test method. Table 2 shows an example.

Input	Definition	Value
Test Method	Type of test	4
b	Width of sample	50
d	Depth of sample	25
L	Length of span	300

Table 2 – Dimensions and test method definitions.

Model Parameters		
Beam Size		
Test Method	4	Point Bending
b =	50	
d =	25	
L =	300	

Figure 3 – Beam size and test method inputs.

Step 3:

Determine the best fit for the Young’s modulus for the linear elastic phase. This done by increasing or decreasing the value of the Elastic modulus (Young’s modulus) E . Fiber reinforced cement (FRC) has a Young’s modulus in the range of 3000000-5000000 psi (20000-35000 MPa).

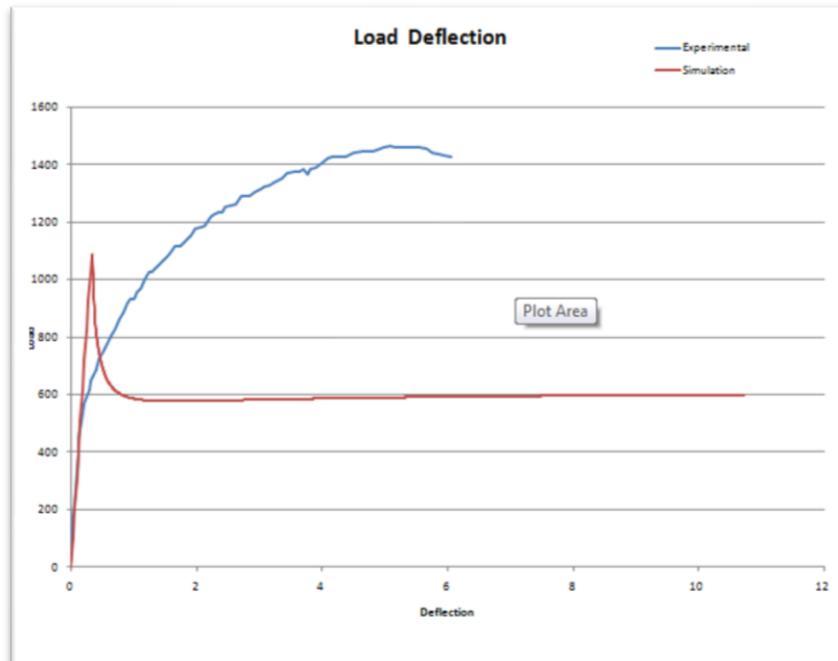


Figure 4 – Linear section of load deflection clearly modeled by $E=25000$ MPa.

Step 4:

Now ϵ_{cr} will need to be fitted to the approximate point where the linear elastic behavior ceases and non-linear behavior begins. This parameter will depend on the type of FRC being tested; cement performance, fiber type and fiber dosage all contribute to the point where cracking is observed. In this case a value of 0.00013 was used. Figure 5 shows how the simulated curve has changed in response to the change in ϵ_{cr} .



Figure 5– Fitting of ϵ_{cr} to load deflection curve

Step 5:

The post cracking slope for strain hardening is represented by E_{cr} is related through the parameter η . This parameter is dependent on the values of α and μ through equation (4). By manipulating α and μ the post cracking slope can be fitted to the load deflection curve. The parameter α adjusts the horizontal location of the transition point, while μ adjusts the vertical position of the transition point and the vertical position of the tail of the curve. Increasing μ will raise the post crack residual portion of the simulated curve. In this case the values of α and μ are given in Figure 6. Figure 7 shows the change in the simulated curve with respect to the parameter changes. The parameter η is automatically calculated per equation (4) and should reflect the post crack slope E_{cr} as negative in the softening case and positive in the hardening case (see Figure 6).

The relationship between the parameters α and μ has other effects on the shape of the simulated curve. It should be noted that changes made in ε_{cr} will also require changes to the parameters α and μ to realign the simulation curve. The value of L_p automatically populates based on units, crack localization rules and the type of test method chosen.

Model Parameters		
Beam Size		
Test Method	4	Point Bending
b =	50	
d =	25	
L =	300	
Don't need Lp	1.00	
Material Model		
E =	25000	
ϵ_{Cr} =	0.00013	
α	40	
Tension		
γ =	0.95	
η =	0.0187179	
μ =	1.73	
β_{tu} =	30	
Compression		
ω =	10.8	
λ_{cu} =	40	

Figure 6 – Parameters α and μ are highlighted



Figure 7 – Change in simulated curve with respect to α and μ .

Step 6:

Now that most of the curve has been fitted we need to determine the ending point to the simulated curve, β_{tu} is used as the normalized ending point parameter.

Model Parameters		
Beam Size		
Test Method	4	Point Bending
b =	50	
d =	25	
L =	300	
Don't need Lp	1.00	
Material Model		
E =	25000	
ϵ_{Cr} =	0.00013	
α	40	
Tension		
γ =	0.95	
η =	0.0187179	
μ =	1.73	
β_{tu} =	105	
Compression		
ω =	10.8	
λ_{cu} =	40	

Figure 8 – Parameter inputs with β_{tu} updated to reflect end point.



Figure 9 – Load deflection curve response to change in β_{tu} .

The parameter β_{tu} is dependent on the user's preference on when the ending point of the simulation curve needs to be. In some cases the entire deflection curve is not necessary, through this parameter adjustments can be made.

Step 7:

The determination of softening or hardening deflection behavior is determined by the parameter μ_{crit} which is the normalized post peak tensile strength; see equation (6). The parameter ω determines the cross over point from deflection softening to hardening. In this case;

$$\mu_{crit} = \frac{(10.8)}{(3(10.8) - 1)} = 0.344$$

Since $\mu=1.73 > \mu_{crit}$ we see deflection hardening behavior in the simulated curve. Again, we see the interaction of the normalized parameters. By adjusting the value of ω (normalized compressive yield strain), we adjust the value of μ_{crit} which effects which deflection equation is used; see equation (13b-c).

2.2 Other Generated Data

There is a lot of data being modeled in this spreadsheet, with the simulated load deflection curve being the product of other useful processes. The spreadsheet generates a simulated tensile and compressive stress strain response under the *stress strain* worksheet; see Figure (10) and (11). The spreadsheet also generates the simulated moment curvature response and the stand alone simulated load deflection response under the *moment curvature* worksheet; see Figure (12) and (13).

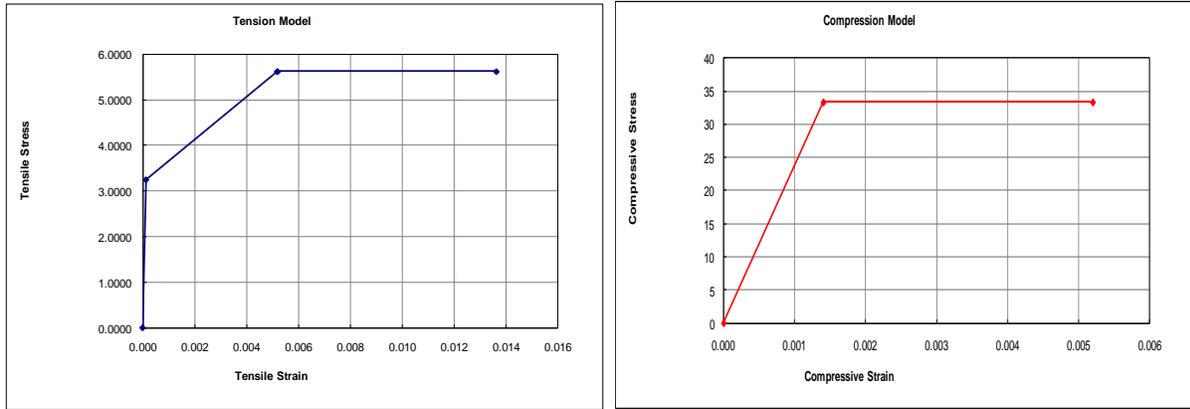


Figure 10 – Simulated tensile and compressive stress strain response for deflection hardening case.

Beam Size			
Test Method	4	Point Bending	b = 50.000
Don't need Lp	1		d = 25.000
			L = 300
			S = 100.0000
Material Model			
E =	25000.00		
ϵ_{cr} =	0.0001300		
α	40.00		
Tension			
γ	0.950	Strain	0.000000
η	0.019	ϵ_{cr}	0.0001300
μ	1.730	ϵ_{trn}	0.0052000
β_{tu}	105.00	ϵ_{trn}	0.0052000
		ϵ_{tu}	0.0136500
		Stress	0.0000
		σ_{cr}	3.25
		σ_{trn}	5.62
		σ_{cst}	5.62
		σ_{tu}	5.62
Compression			
ω	10.80	Strain	0.000000
λ_{cu}	40.0000	ϵ_{cy}	0.0014040
		ϵ_{cu}	0.0052000
		Stress	0.00000
		σ_{cy}	33.35
		σ_{cy}	33.35
Critical Parameters			
μ_{crit}	0.343949045		

Figure 11 – Stress strain screenshot of computed parameter values for hardening case.

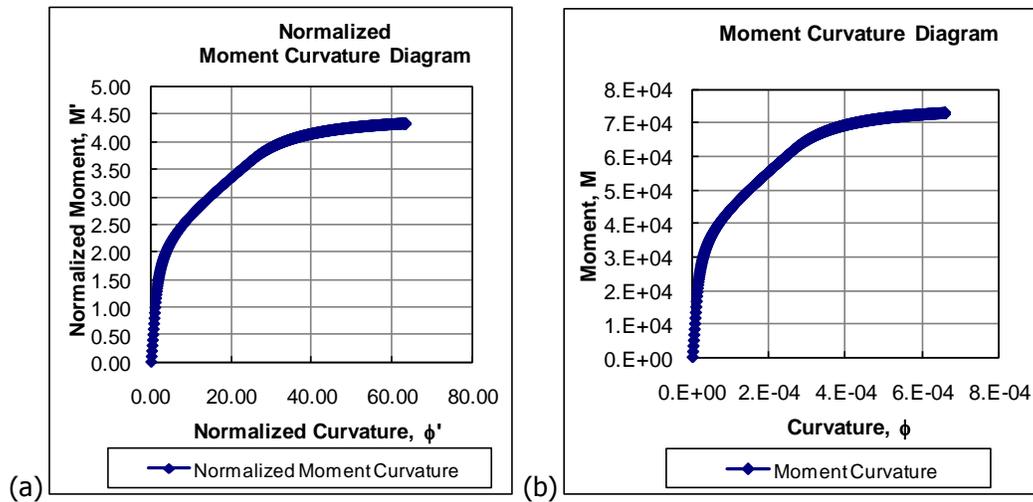


Figure 12 – Strain hardening (a) normalized moment curvature and (b) moment curvature.

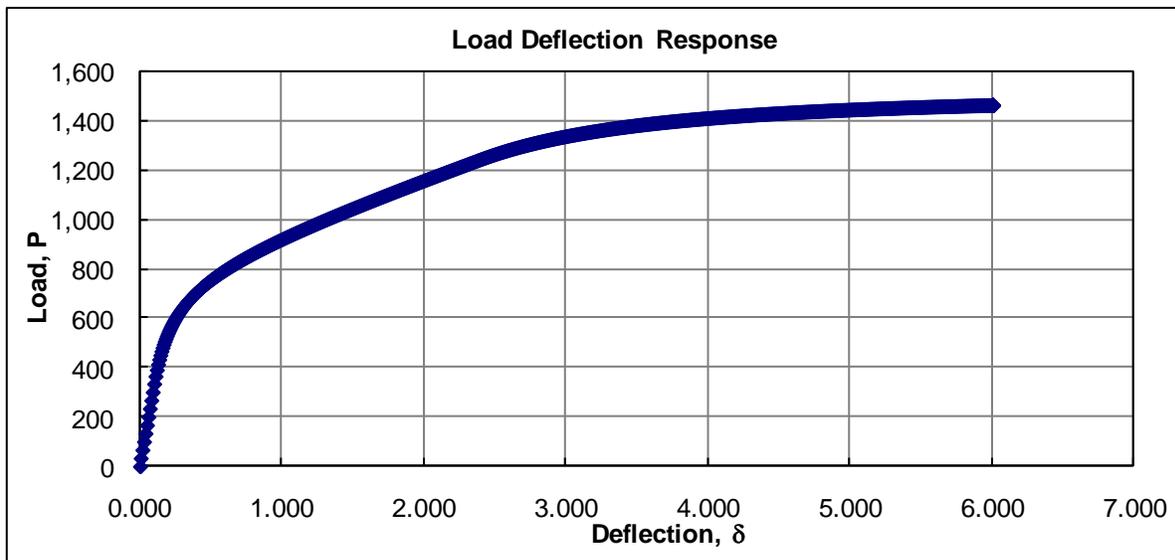


Figure 13 – Simulated load deflection response

2.3 Four Point Bending – Strain Softening

The four point bending test ASTM-1609 is carried out on standard beam dimensions with load applied at third points along the span (see Figure 2a). The load deflection curve will show a decrease in load

capacity after the linear elastic portion has ended. The softening continues as shown in Figure 14 or it can plateau as the sample continues to sustain load as deflection continues.

STEP 1:

Paste reduced load deflection data (up to 1000 points) into the columns labeled "Load" and "Deflection". This will populate the experimental load deflection curve we are trying to model as shown in Figure 14.

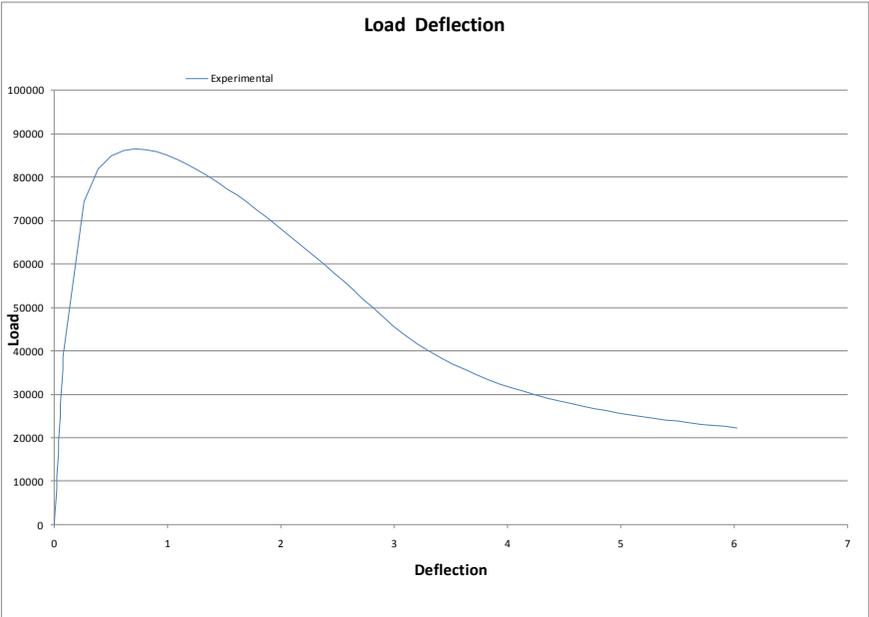


Figure 14 - Experimental load deflection response, showing deflection softening characteristics.

Step 2:

Input the sample dimensions and test method. Table 3 shows an example.

Input	Definition	Value
Test Method	Type of Test	4
b	Width of sample	150
d	Depth of sample	150
L	Length of span	450

Table 3 – Dimensions and test method definitions.

Beam Size		
Test Method	4	Point Bending
b =	150	
d =	150	
L =	450	

Figure 15 – Beam size and test method inputs.

Step 3:

Determine the best fit for the Young's modulus for the linear elastic phase. This done by increasing or decreasing the value of the Elastic modulus (Young's modulus) E . Fiber reinforced cement (FRC) has a Young's modulus in the range of 3000000-5000000 psi (20000-35000 MPa).



Figure 16 – Linear section of load deflection clearly modeled by $E=20000$ MPa.

Step 4:

Now ϵ_{cr} will need to be fitted to the approximate point where the linear elastic behavior ceases and non-linear behavior begins. This parameter will depend on the type of FRC being tested; cement performance, fiber type and fiber dosage all contribute to the point where cracking is observed. In this case a value of 0.00013 was used. Figure 17 shows how the simulated curve has changed in response to the change in

ϵ_{cr} .

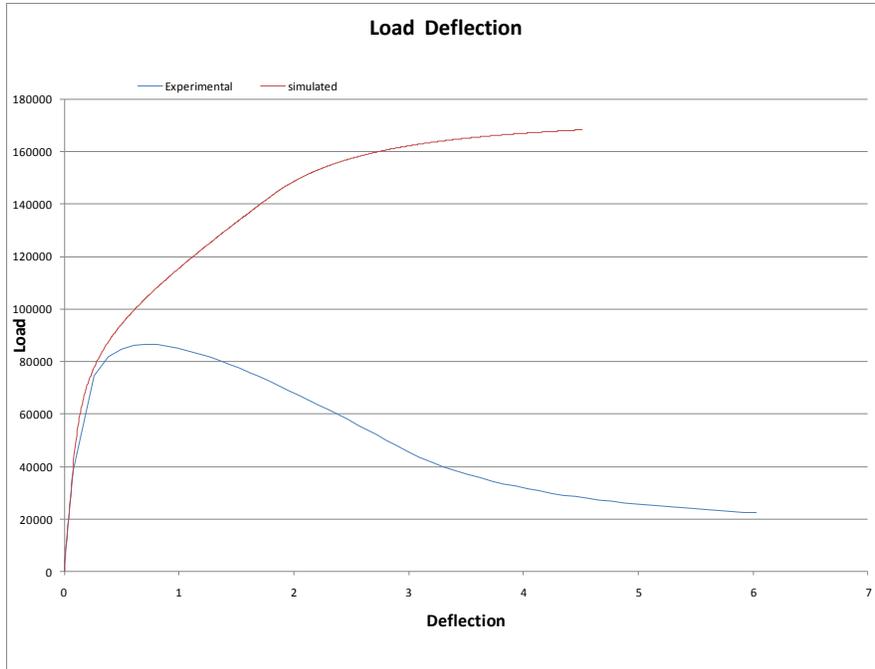


Figure 17– Fitting of ε_{cr} to load deflection curve

Step 5:

The post cracking slope for strain hardening is represented by E_{cr} is related through the parameter η . This parameter is dependent on the values of α and μ through equation (4). By manipulating α and μ the post cracking slope can be fitted to the load deflection curve. The parameter α adjusts the horizontal location of the transition point, while μ adjusts the vertical position of the transition point and the vertical position of the tail of the curve. Increasing μ will raise the post crack residual portion of the simulated curve. In this case the values of α and μ are given in Figure 18. Figure 19 shows the change in the simulated curve with respect to the parameter changes. The parameter η is automatically calculated per equation (4) and should reflect the post crack slope E_{cr} of a negative value in the softening case (see Figure 18).

The relationship between the parameters α and μ has other effects on the shape of the simulated curve. It should be noted that changes made in ϵ_{cr} will also require changes to the parameters α and μ to realign the simulation curve. The value of L_p automatically populates based on units, crack localization rules and the type of test method chosen.

Beam Size		
Test Method	4	Point Bending
b =	150	
d =	150	
L =	450	
Don't need Lp	1.00	
Material Model		
E =	20000	
ϵ_{cr} =	0.00026	
α	105	
Tension		
γ =	0.95	
η =	-0.008365	
μ =	0.13	
β_{tu} =	105	
Compression		
ω =	10.8	
λ_{cu} =	40	

Figure 18 – Parameters α and μ are highlighted

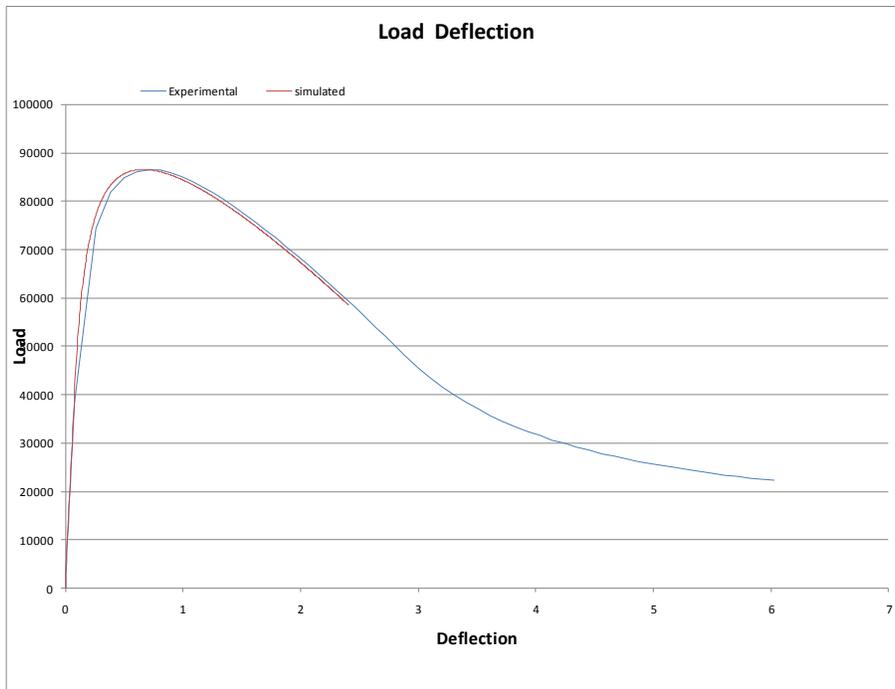


Figure 19 – Change in simulated curve with respect to α and μ .

Step 6:

Now that most of the curve has been fitted we need to determine the ending point to the simulated curve, β_{tu} is used as the normalized ending point parameter.

Model Parameters		
Beam Size		
Test Method	4	Point Bending
b =	150	
d =	150	
L =	450	
Don't need Lp	1.00	
Material Model		
E =	20000	
ϵ_{cr} =	0.00026	
α	105	
Tension		
γ =	0.95	
η =	-0.008365	
μ =	0.13	
β_{tu} =	235	
Compression		
ω =	10.8	
λ_{cu} =	40	

Figure 20 – Parameter inputs with β_{tu} updated to reflect end point.

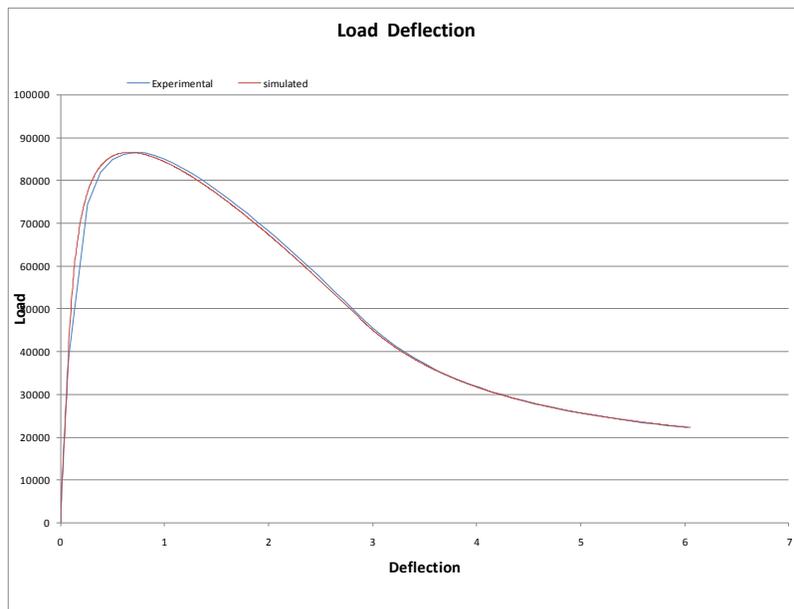


Figure 21 – Load deflection curve response to change in β_{tu} .

The parameter β_{tu} is dependent on the user's preference on when the ending point of the simulation curve needs to be. In some cases the entire deflection curve is not necessary, through this parameter adjustments can be made.

Step 7:

The determination of softening or hardening deflection behavior is determined by the parameter μ_{crit} which is the normalized post peak tensile strength; see equation (6). The parameter ω determines the cross over point from deflection softening to hardening. In this case;

$$\mu_{crit} = \frac{(10.8)}{(3(10.8) - 1)} = 0.344$$

Since $\mu=0.13 < \mu_{crit}$ we see deflection softening behavior in the simulated curve. Again, we see the interaction of the normalized parameters. By adjusting the value of ω (normalized compressive yield strain), we adjust the value of μ_{crit} which effects which deflection equation is used; see equation (13b-c).

2.4 Other Generated Data

There is a lot of data being modeled in this spreadsheet, with the simulated load deflection curve being the product of other useful processes. The spreadsheet generates a simulated tensile and compressive stress strain response under the *stress strain* worksheet; see Figure (22) and (23). The spreadsheet also generates the simulated moment curvature response and the stand alone simulated load deflection response under the *moment curvature* worksheet; see Figure (24) and (25).

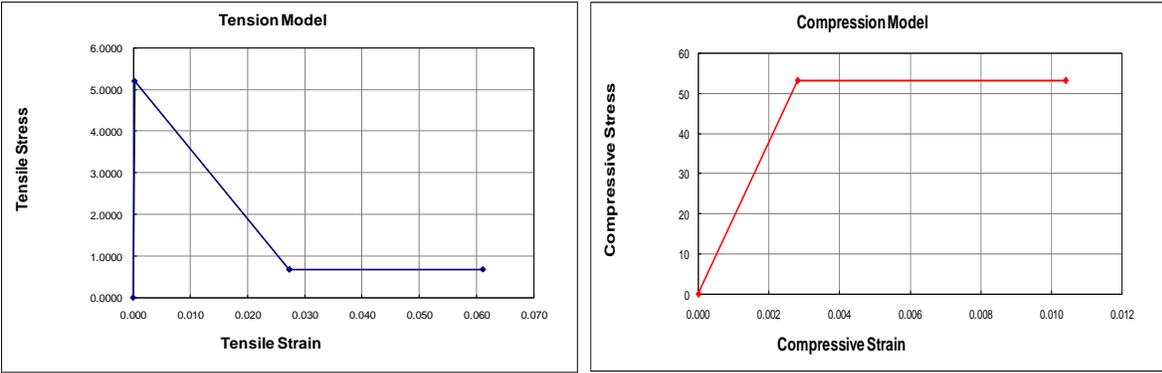


Figure 22 – Simulated tensile and compressive stress strain response for deflection softening case.

Beam Size			
Test Method	4	Point Bending	b = 150.000
Don't need Lp	1		d = 150.000
			L = 450
			S = 150.0000
Material Model			
E =	20000.00		
σcr =	0.0002600		
α	105.00		
Tension			
γ	0.950	Strain	0.000000
η	-0.008	εcr	0.0002600
μ =	0.130	εtr1	0.0273000
βtu =	235.00	εtrn	0.0273000
		εtu	0.0611000
		Stress	0.0000
		σcr	5.20
		σtrn	0.68
		σcst	0.68
		σtu	0.68
Compression			
ω =	10.80	Strain	0.000000
λcu =	40.0000	εcy	0.0028080
		εcu	0.0104000
		Stress	0.00000
		σcy	53.35
		σcy	53.35
Critical Parameters			
μcrit	0.343949045		

Figure 23 – Stress strain screenshot of computed parameter values for softening case.

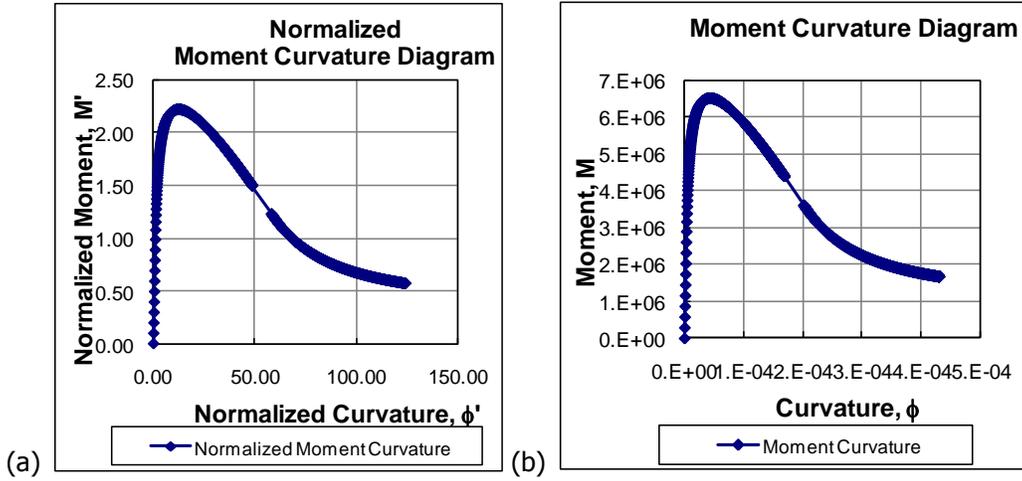


Figure 24 – Strain softening (a) normalized moment curvature and (b) moment curvature.

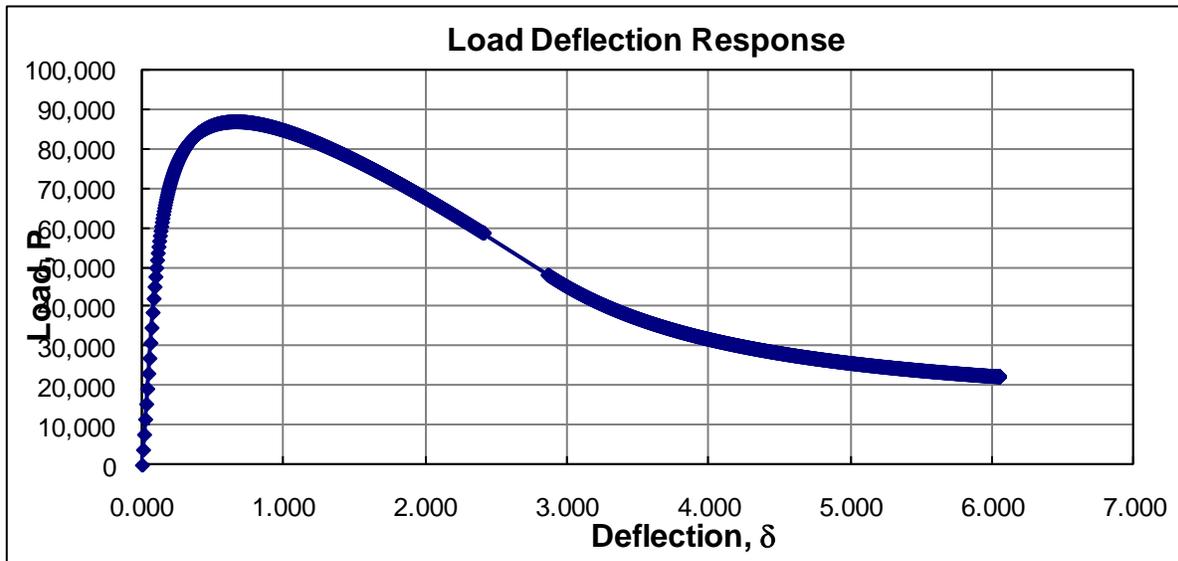


Figure 25 – Simulated load deflection response

CHAPTER 3

Flexural and Tensile Modeling

3.1 – Flexural Bending with Tensile Data

By utilizing the same modeling concepts as outlined in section 2.1 the inclusion of tensile data does not affect the functionality of the way the spreadsheet simulates the load deflection response. The following example uses the ASTM C-1609 test data and tensile data in the fitting of both curves.

STEP 1:

Upload flexural load-deflection and tensile stress strain data into the appropriate columns under the "RAW DATA" worksheet tab.

		Units are: in, lbs, psi or: mm, N, Mpa Please Use Units Consistently			
Experimental Data					
Deflection	Load	Tensile			
		Strain	Stress		
0	0		0	0.065102	
0.005929	3014		8.58E-06	0.390614	
0.01186	6029		8.58E-06	0.651024	
0.01779	9043		3.43E-05	1.04164	
0.02371	12060		4.29E-05	1.32375	
0.02964	15070		4.29E-05	1.62756	
0.03557	18090		4.29E-05	1.99647	
0.0415	21100		6.01E-05	2.31113	
0.04743	24120		8.58E-05	2.6475	
0.05336	27130		0.000103	2.87535	

Figure 3.1 – Raw data input

STEP 2:

Input flexural test method and dimensions of flexural sample. The user definition of the post cracking slope η , can be input here by placing a "Y" in the appropriate cell, a "N" will source a derived η value from α and μ (see equation 4).

Model Parameters		
User Defined η (Y or N)?		
	Y	
User Defined η =	0.015	
Beam Size		
Test Method	4	Point Bending
b =	150	
d =	150	
L =	450	

Figure 3.2 – Flexural test method, sample dimensions and user definition of parameter η .

STEP 3:

Manipulate Young's Modulus to simulate the initial slope of the load deflection curve.

Care must be taken to observe the effect on the simulated stress-strain curve also.

Figure 3.3 shows the dual effect of simultaneously modeling the two curves.

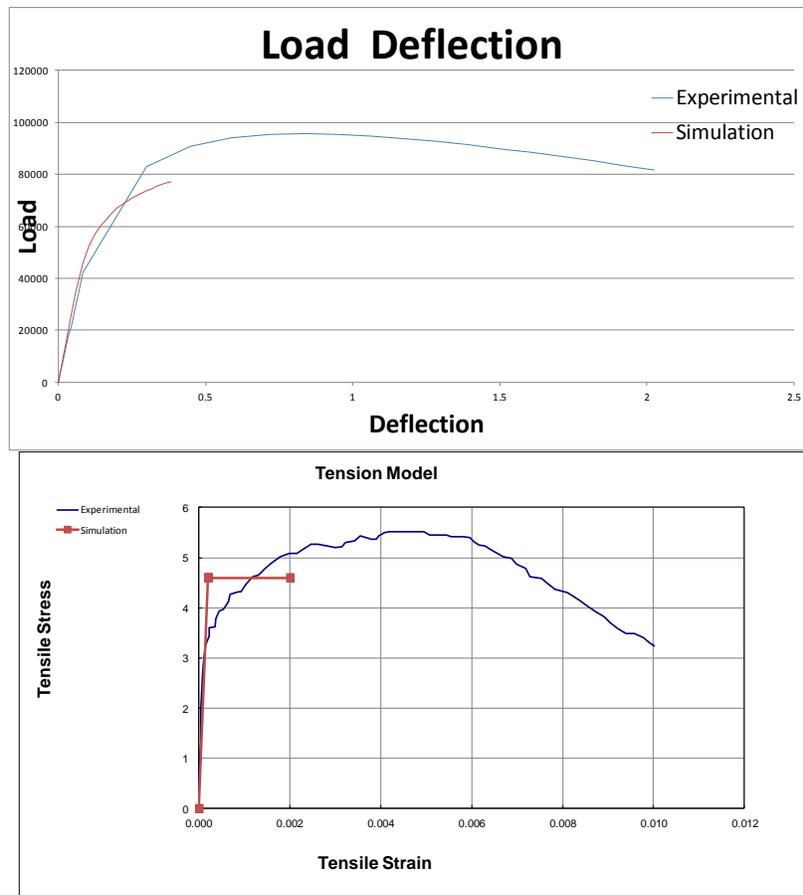


Figure 3.3 – Simultaneous load-deflection and tensile stress-strain response.

STEP 4:

Changing the values of α and μ will alter the post peak orientation of both curves (as described in chapter 2). However care must be taken to ensure a close enough fit for both curves.

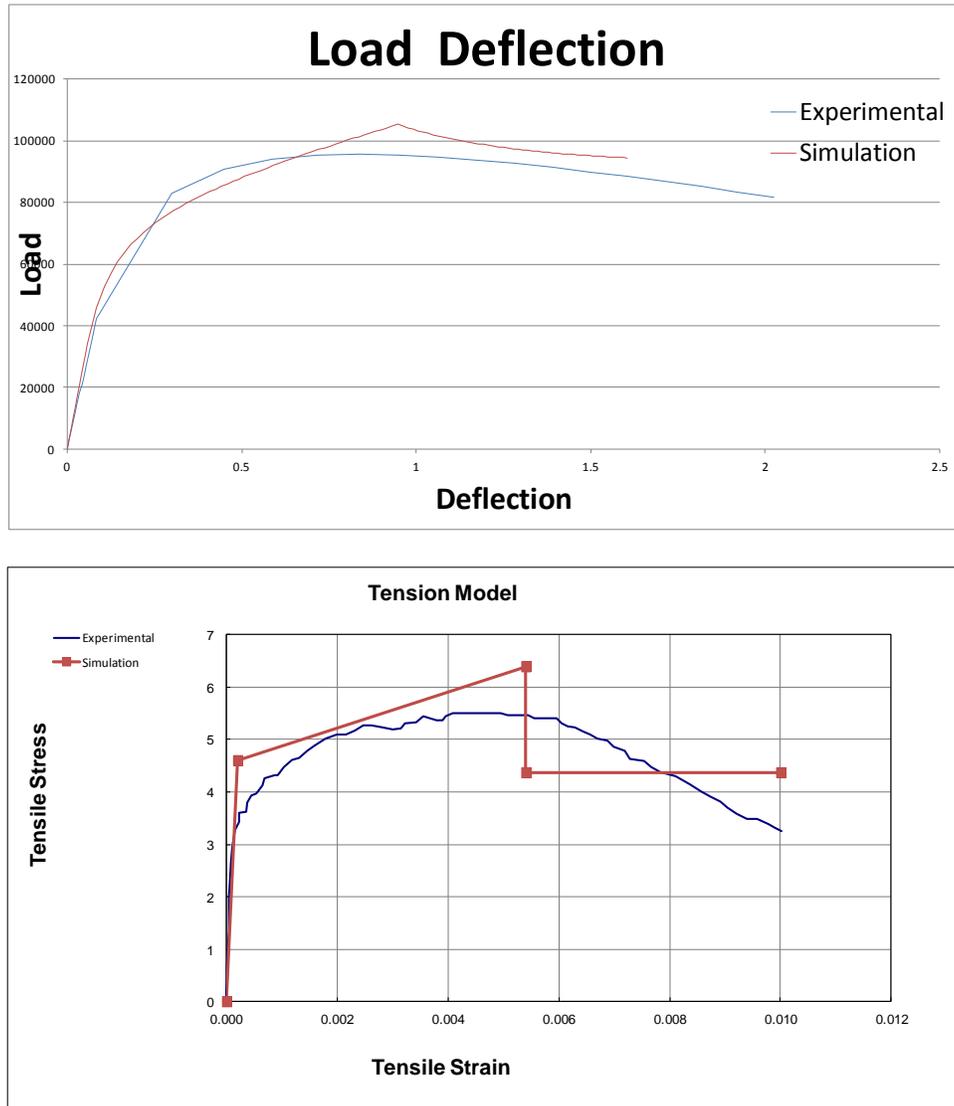


Figure 3.4 – Final curve fit for both load-deflection and tensile stress-strain response.

The simulated responses shown in Figure 3.4 are determined after considering how close to each experimental curve to go. Experimental tensile data has previously proved to underestimate the actual tensile response, consequently some judgment decisions must be made

when determining how close to fit each curve. Figure 3.5 shows the parameters (in SI) used to produce the simulated load-deflection and stress-strain responses in Figure 3.4.

Model Parameters		
User Defined η (Y or N)?		
	Y	
User Defined η =	0.015	
Beam Size		
Test Method	4	Point Bending
b =	150	
d =	150	
L =	450	
Don't need Lp	1.00	
Material Model		
E =	23000	
ϵ_r =	0.0002	
α	27	
Tension		
γ =	0.95	
η =	0.015	
μ =	0.95	
β_{tu} =	50	
Compression		
ω =	100	
λ_{cu} =	100	

Figure 3.5 – Model parameters for the dual load-deflection and tensile stress-strain response.

3.2 Other Reported Data

In addition to load-deflection, stress-strain and moment curvature plots, the spreadsheet produces experimental data analysis. Back calculation parameters are also produced and shown in Table 3.1. The spreadsheet automatically detects the units (SI or English) based on the span length of the sample, if $L > 150$ then it takes the units to be SI (flexural samples are rarely 150 inches long).

Output Data - Experimental		
Flexural Toughness =	142240	(N-mm)
Bending Strength =	14.04608	(Mpa)
Flexural Stiffness =	584711.5	(Mpa)
Load @ 1st Crack =	34057.62	(N)
Defl @ 1st Crack =	0.058247	(mm)
Max Flex Load =	105345.6	(N)
Defl. @ Max Load =	0.947497	(mm)
Deflection Capacity =	1.604958	(mm)
Output Data - Back-Calc Parameters		
Tensile Toughness =	0.043884	(Mpa)
ϵ_{cr} (μstr) =	200	
E =	23000	(Mpa)
σ_{cr} =	4.60	(Mpa)
μ =	0.95	
α =	27	
ϵ_{trn} (μstr) =	5400	
ϵ_{tu} (μstr) =	10000	
$\mu\sigma_{cr}$ =	4.37	(Mpa)

Table 3.1 – Summary of experimental analysis and back-calculation parameters.

CHAPTER 4

Flexural Modeling with Residual Strength Comparison

4.1 Back Calculation with Residual Strength Comparison

It may be necessary for the residual strength from the JCI-SF4, RILEM TC 164-TDF or ASTM C-1609 to be compared to the described model in an attempt to take into account the shifting neutral axis of a cracking sample. For a set of samples, the residual strength parameters can be plotted and the linear relationship developed as shown in [4].

STEP 1:

The process for simulating the experimental load deflection data is identical to the previous methods. Upload the experimental load-deflection data under the "RAW DATA" worksheet tab. Then follow the process of manipulating the simulated curve until a satisfactory simulation is accomplished.

STEP 2:

The difference in this version of the spreadsheet is under the "STANDARDS" worksheet tab. This is where the different residual strength parameters for each of the outlined standards is calculated and compared with the ASU model. Figure 4.1 shows the residual strength parameters in appropriate units. A summary of these results are shown in figure 4.2. This summary table makes copying large sets of analysis data into tables easier. It is interesting to note that the RILEM residual strength is calculated in SI, then converted if needed into English units.

References

- 1 Soranakom, C., and Mobasher, B., "Moment-Curvature Response of Strain Softening and Strain Hardening Cement Based Composites," *Cement and Concrete Composites*, Volume 30, Issue 6, Pages 465-477, July 2008
- 2 Soranakom C, Mobasher B. Closed-form moment-curvature expressions for homogenized fiber-reinforced concrete. *ACI Material Journal* 2007; 104(4):351-9.
- 3 Soranakom C, Mobasher B. Closed form solutions for flexural response of fiber reinforced concrete beams. *Journal of Engineering Mechanics* 2007;133(8):933-41.
- 4 Bakhshi M, Barsby C, Mobasher B, "Comparative Evaluation of Early Age Toughness Parameters in Fiber Reinforced Concrete", Submitted to *Materials and Structures*, 2011.