Shear Strength Modeling for Squat Walls with Boundary Elements

Resilience, Failure Mitigation, and Preservation of Concrete Bridges and Structures, Part 1 of 2

Yosemite B (1:30 PM – 3:30 PM)

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- Shear Failure Modes of Flanged Squat Walls
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Research Background

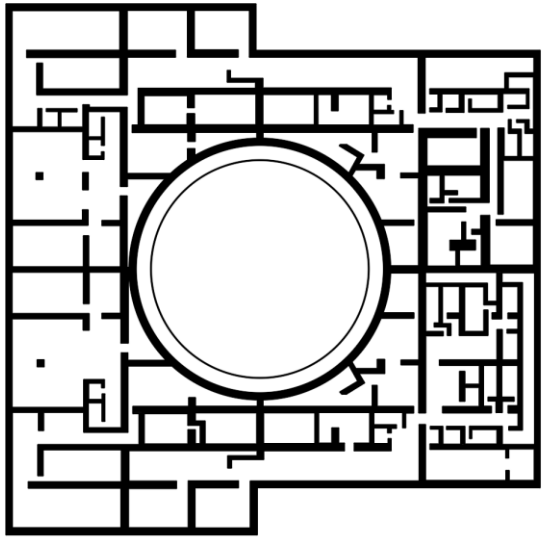
Problem statement

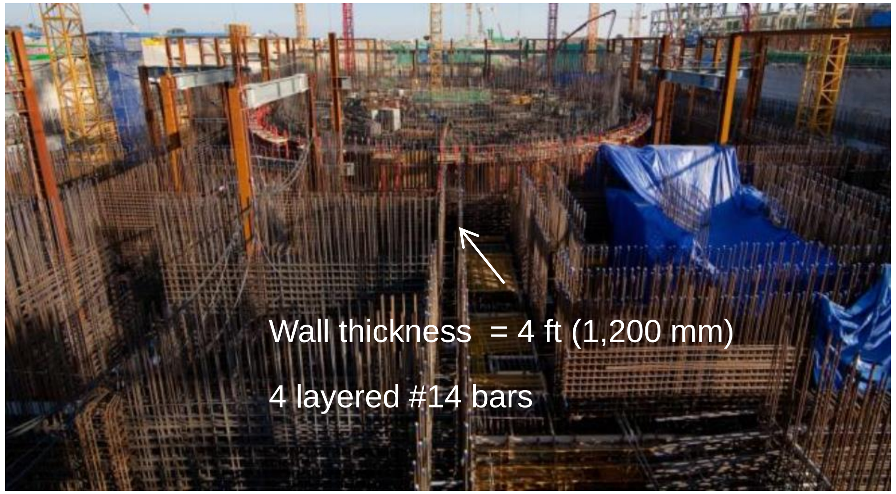
- Seismic Design: Overly reinforced structures in nuclear power plants (elastic behavior under design basis EQ.)

  + complex geometry → construction quality degradation → 264 voids were detected in two containment buildings (Source: Nuclear Safety and Security Commission)

- Seismic performance evaluation: Uncertainties in the evaluation of the actual seismic capacity of RC walls (how safe ?)
- Limitations of empirical models for squat walls including shear strength equation of ACI 318-19 (or ACI349-13)

  (minimize safety margins & uncertainties)





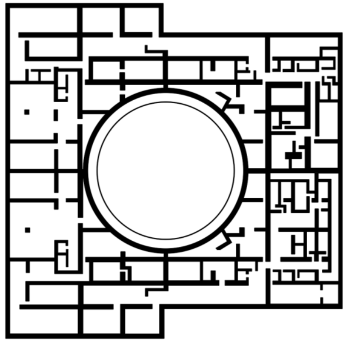
Wall thickness = 4 ft (1,200 mm)

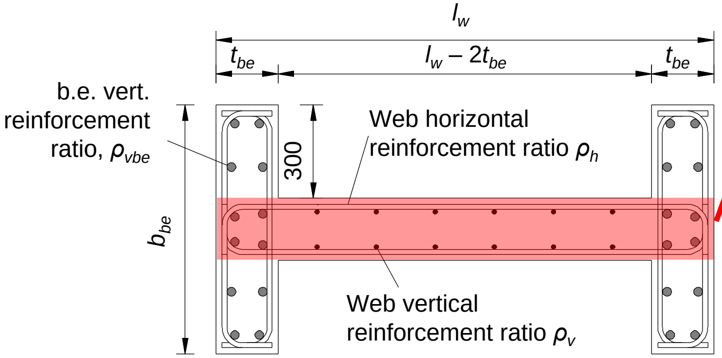
4 layered #14 bars

Research Background

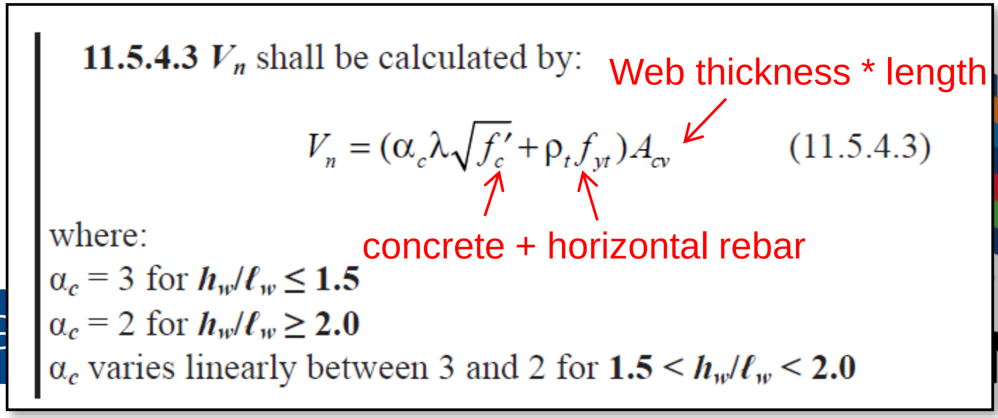
Discrepancies in shear strength models of squat walls + boundary elements

Shear strength model	Wall geometry as a modeling parameter					Reinforcement ratio as a modeling parameter				Material strength		Axial load
						Web		Boundary element				
	Wall height h_w	Wall length l_w	Web thickness t_w	Width of B.E. b_{be}	Thickness of B.E. t_{be}	Horizontal ρ_h	Vertical ρ_v	Horizontal ρ_{hbe}	Vertical ρ_{vbe}	f_c'	f_y	
ACI 318-19 (ACI 2019)	O	O	O	X	X	O	X	X	X	O	O	X
ASCE/SEI 43-05 (ASCE 2005)	O	O	O	X	X	O	O	X	X	O	O	O
Barda et al. (1977)	O	O	O	X	X	X	O	X	X	O	O	O
Wood (1990)	X	O	O	X	X	X	O	X	X	O	O	X
Gulec and Whittaker (2011)	O	O	O	O	O	X	O	X	O	O	O	O
Kassem (2015)	O	O	O	X	X	O	O	X	X	O	O	X
Moehle (2015)	X	O	O	X	X	X	O	X	X	O	O	O
Luna and Whittaker (2019)	O	O	O	X	X	O	O	X	X	O	O	O





ACI 318-19 11.5.4 In-plane shear



11.5.4.3 V_n shall be calculated by:

Web thickness * length

$$V_n = (\alpha_c \lambda \sqrt{f_c'} + \rho_t f_{yt}) A_{cv} \qquad (11.5.4.3)$$

concrete + horizontal rebar

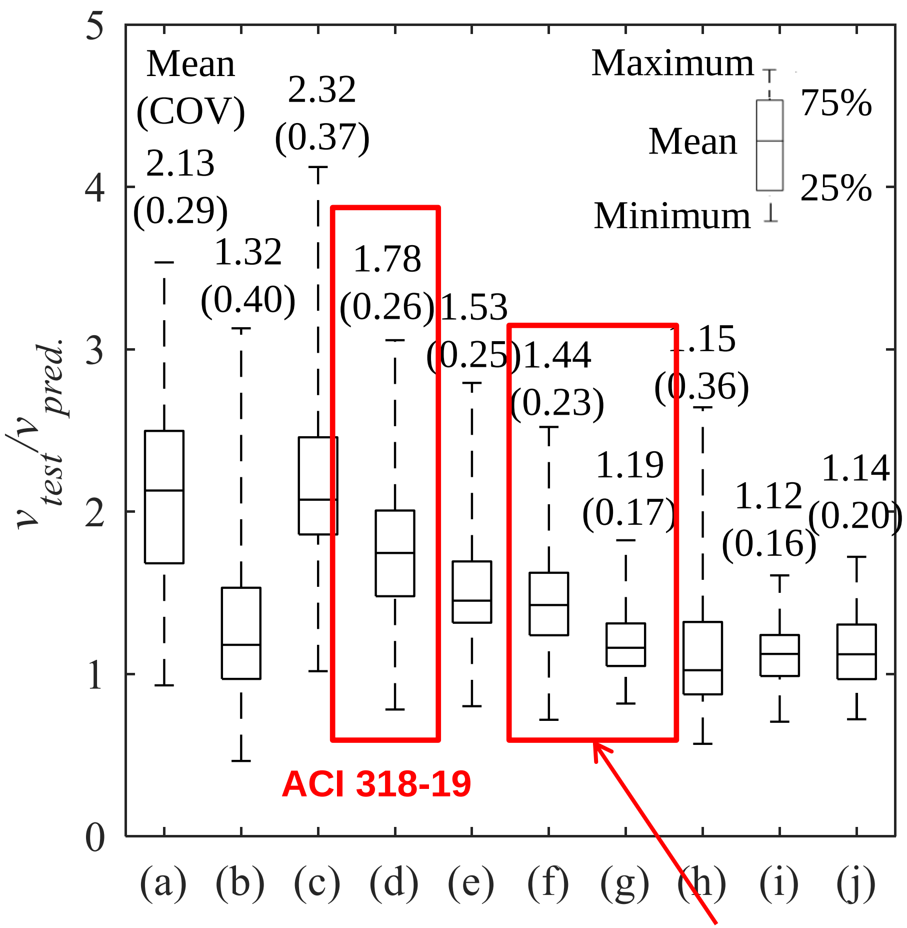
where:

$\alpha_c = 3$ for $h_w/\ell_w \le 1.5$

$\alpha_c = 2$ for $h_w/\ell_w \ge 2.0$

α_c varies linearly between 3 and 2 for $1.5 < h_w/\ell_w < 2.0$

Research Background



ACI 318-19

ASCE 43-05 & Gulec and Whittaker (2011)

(currently used in seismic fragility analysis of wall structures in nuclear power plants)

Current practices rely on approximate (empirical) methods to determine the capacity of squat walls with boundary elements, leading to discrepancies in shear strength prediction.

Research Background

How ?

- Existing studies demonstrated that shear strength was increased by virtue of boundary elements.
- How to quantify the shear strength contribution of the boundary elements ?

  → Mechanics based methodologies to predict the shear strength of squat walls with boundary elements without empirical coefficients

What ?

- Proposed analytical model-based shear strength equation for flanged squat walls (incorporating the shear strength contribution of boundary elements)

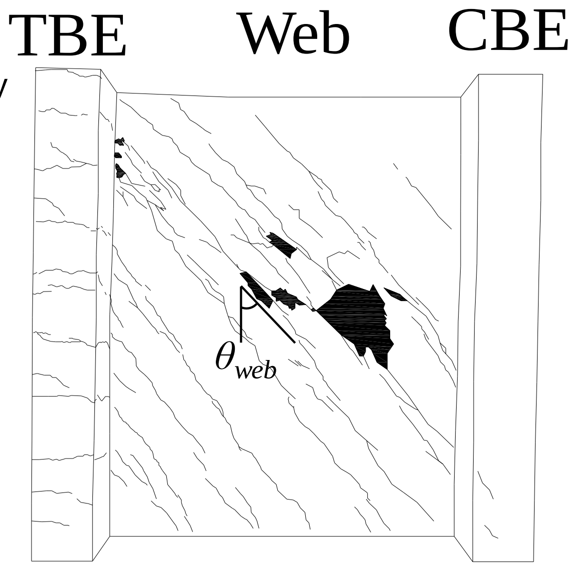
Shear Failure Modes of Flanged Squat Walls

Experimental programs

Kim and Park. 2020.11. Shear and shear-friction strengths of squat walls with flanges. ACI Structural Journal, 117(6), 269-280.

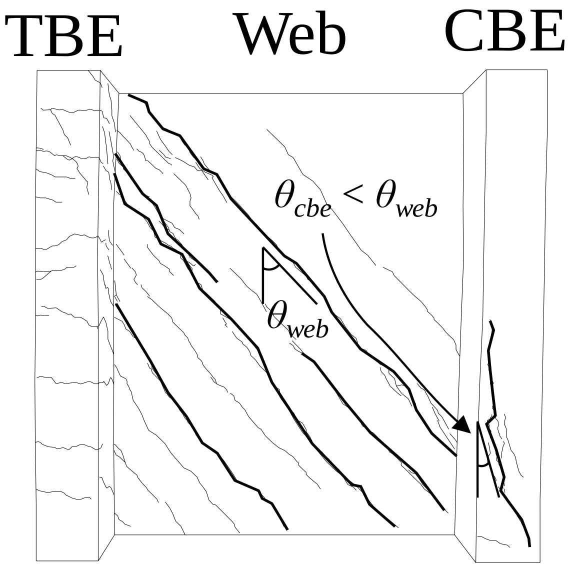
Kim and Park. 2022.03. Shear strength of flanged squat walls with 690 MPa reinforcing bars. ACI Structural Journal, 119(2), 209-220.

Tension
Boundary
Element



Compression
Boundary
Element

Web-crushing

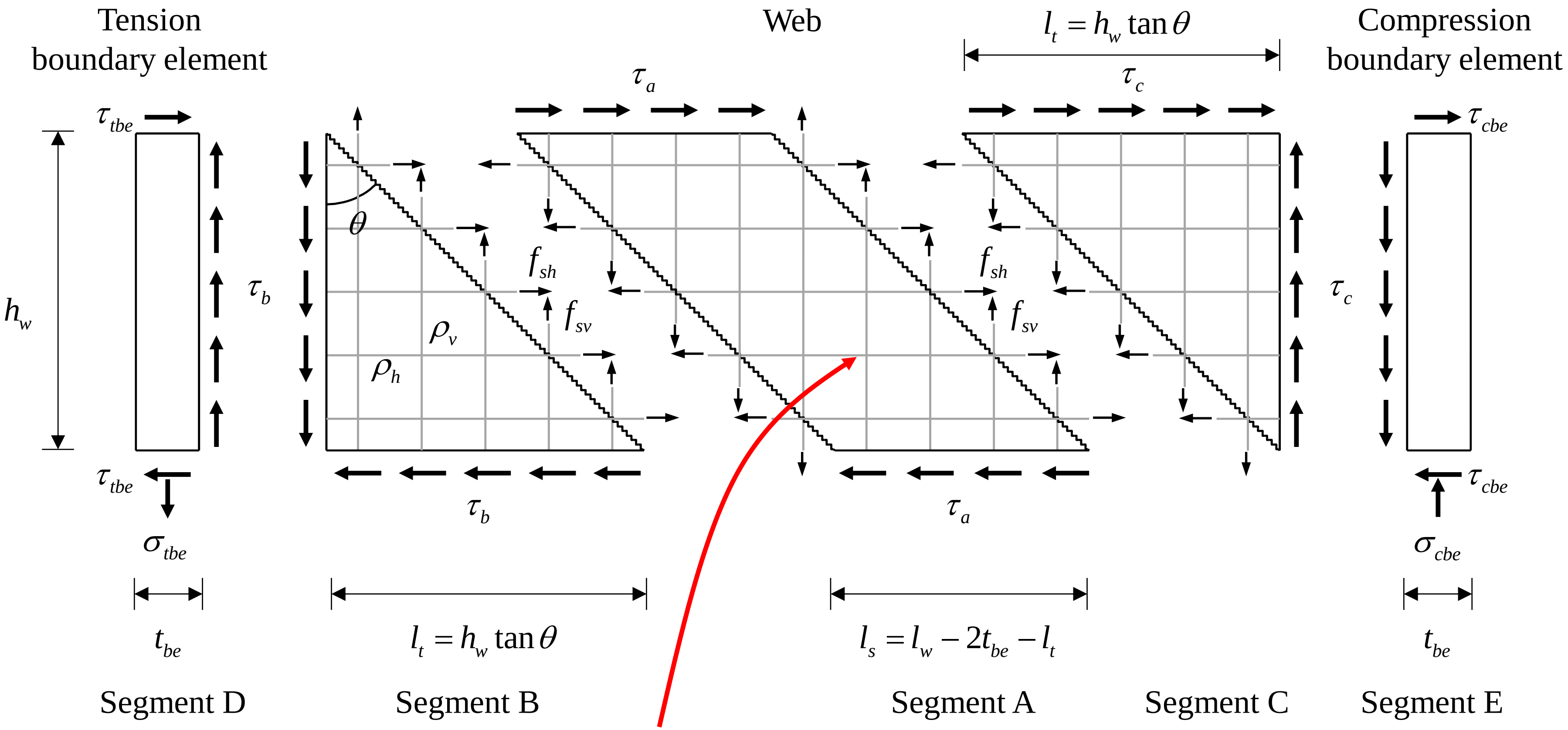


Web diagonal tension
+ shear-compression at CBE

Typical cracking patterns of squat walls with boundary elements failed in shear

Analytical Model

Force equilibrium based on a simplified crack pattern



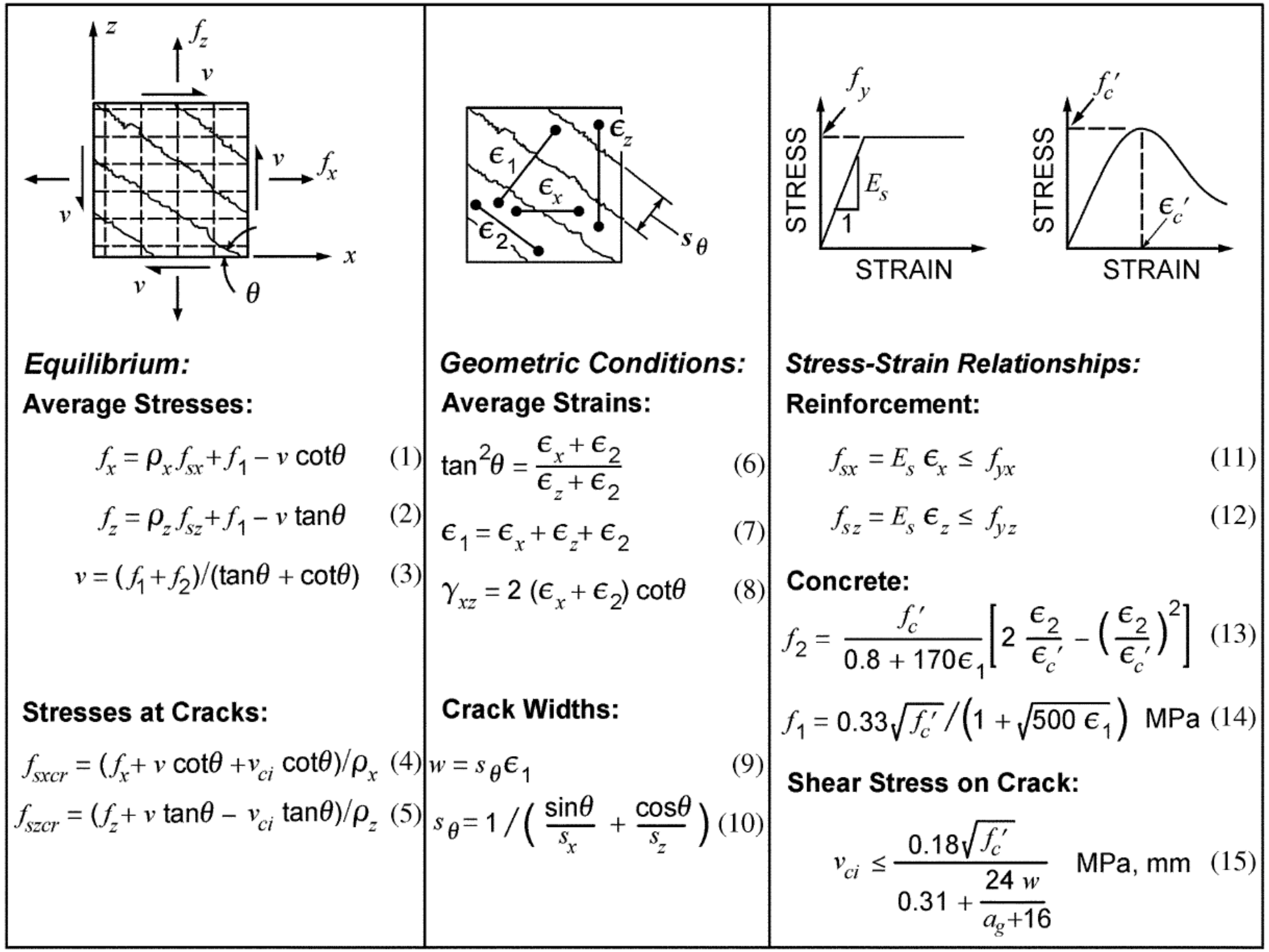
≈ Response of shear panel subjected to pure shear:

Modified Compression Field Theory (Vecchio and Collins 1986)

Analytical Model

Summary of MCFT

Bentz. et al. (2006)



Equilibrium:

Average Stresses:

$$f_x = \rho_x f_{sx} + f_1 - v\cot\theta \quad (1)$$

$$f_z = \rho_z f_{sz} + f_1 - v\tan\theta \quad (2)$$

$$v = (f_1 + f_2)/(\tan\theta + \cot\theta) \quad (3)$$

Stresses at Cracks:

$$f_{sxcr} = (f_x + v\cot\theta + v_{ci}\cot\theta)/\rho_x \quad (4)$$

$$f_{szcr} = (f_z + v\tan\theta - v_{ci}\tan\theta)/\rho_z \quad (5)$$

Geometric Conditions:

Average Strains:

$$\tan^2\theta = \frac{\epsilon_x + \epsilon_2}{\epsilon_z + \epsilon_2} \quad (6)$$

$$\epsilon_1 = \epsilon_x + \epsilon_z + \epsilon_2 \quad (7)$$

$$\gamma_{xz} = 2(\epsilon_x + \epsilon_2)\cot\theta \quad (8)$$

Crack Widths:

$$w = s_\theta \epsilon_1 \quad (9)$$

$$s_\theta = 1 \Big/ \left(\frac{\sin\theta}{s_x} + \frac{\cos\theta}{s_z}\right) \quad (10)$$

Stress-Strain Relationships:

Reinforcement:

$$f_{sx} = E_s\,\epsilon_x \le f_{yx} \quad (11)$$

$$f_{sz} = E_s\,\epsilon_z \le f_{yz} \quad (12)$$

Concrete:

$$f_2 = \frac{f_c'}{0.8 + 170\epsilon_1}\left[2\frac{\epsilon_2}{\epsilon_c'} - \left(\frac{\epsilon_2}{\epsilon_c'}\right)^2\right] \quad (13)$$

$$f_1 = 0.33\sqrt{f_c'} \Big/ \left(1 + \sqrt{500\,\epsilon_1}\right) \text{ MPa} \quad (14)$$

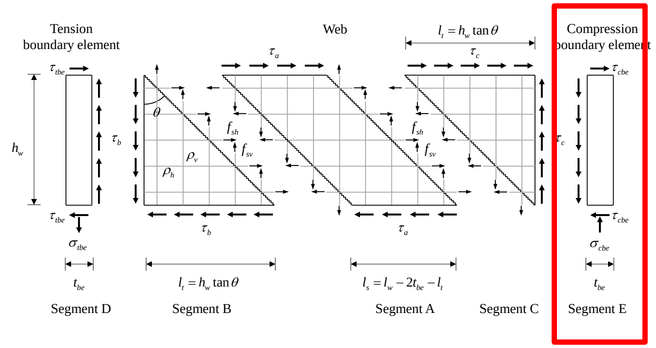
Shear Stress on Crack:

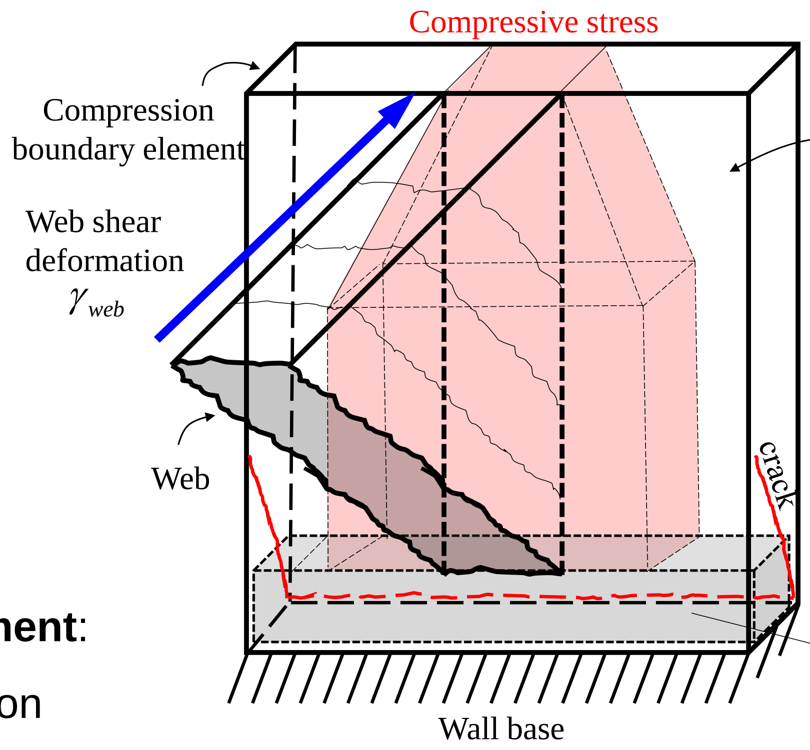
$$v_{ci} \le \frac{0.18\sqrt{f_c'}}{0.31 + \dfrac{24\,w}{a_g + 16}} \text{ MPa, mm} \quad (15)$$

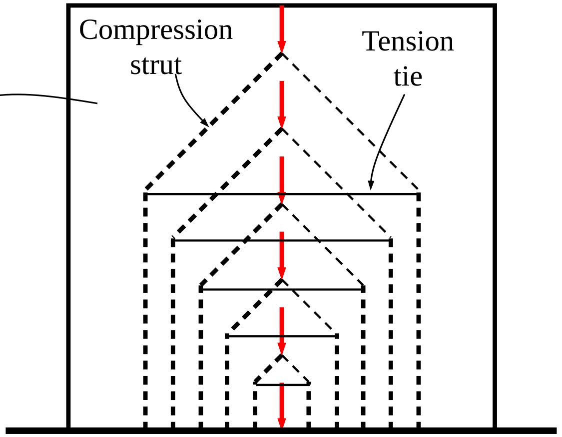
Fig. 2—Equations of modified compression field theory.

Analytical Model

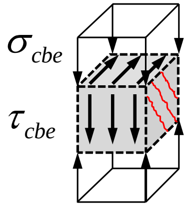
Force equilibrium based on a simplified crack pattern

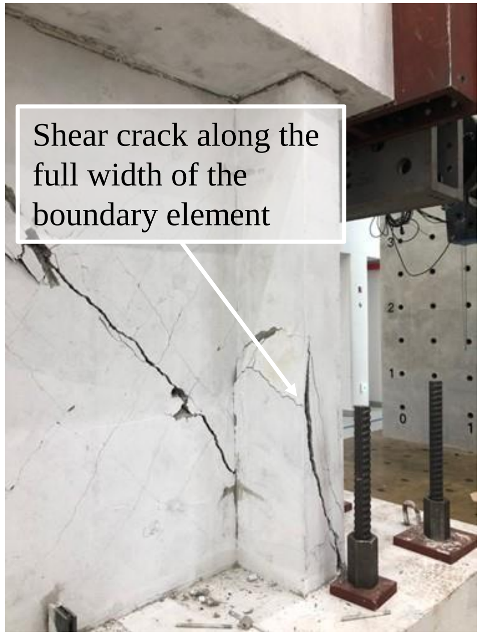


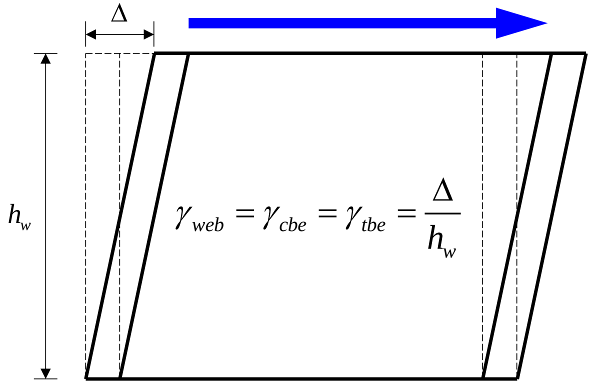




Combined shear-compression stresses at compression boundary element







$$\gamma_{web} = \gamma_{cbe} = \gamma_{tbe} = \frac{\Delta}{h_w}$$

Compression boundary element:

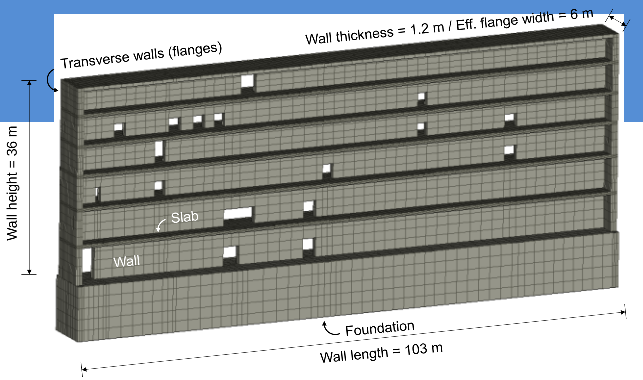
combined shear – compression

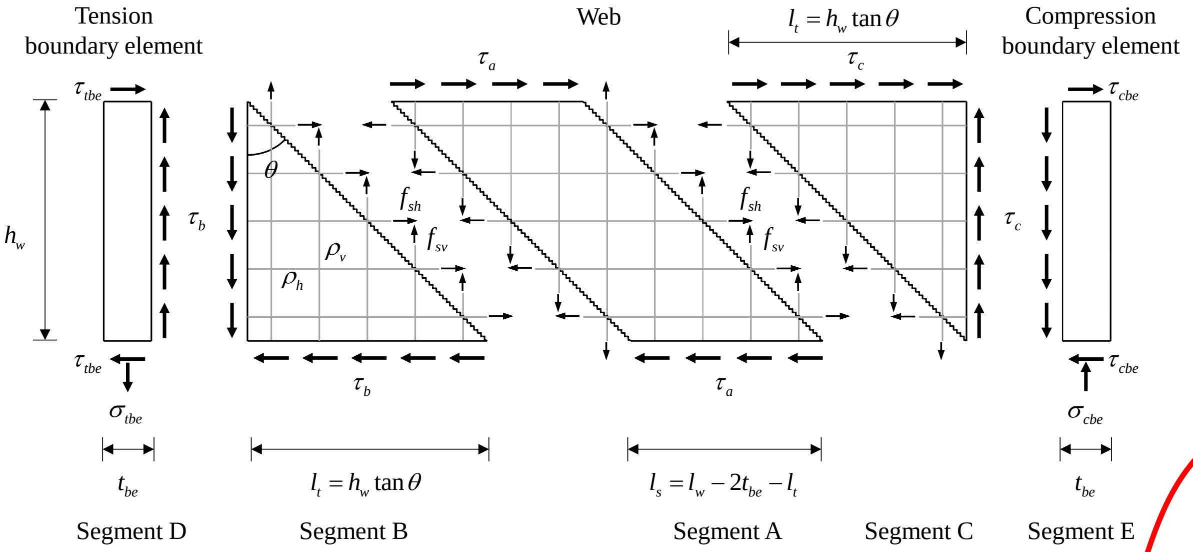
Tension boundary element:

combined shear – tension

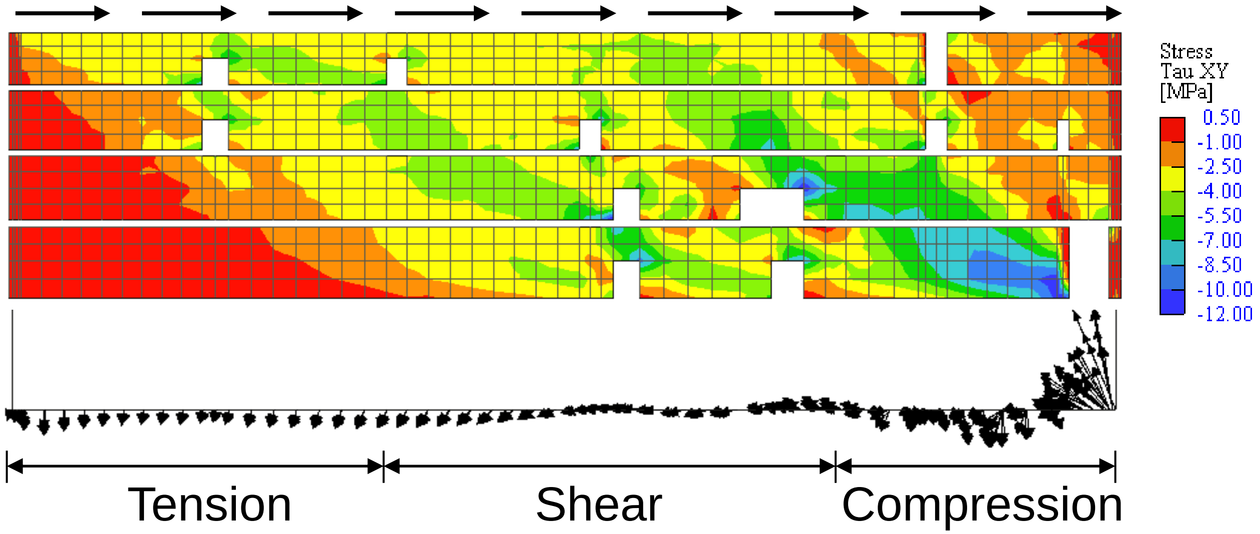
Analytical Model

Force equilibrium based on a simplified crack pattern





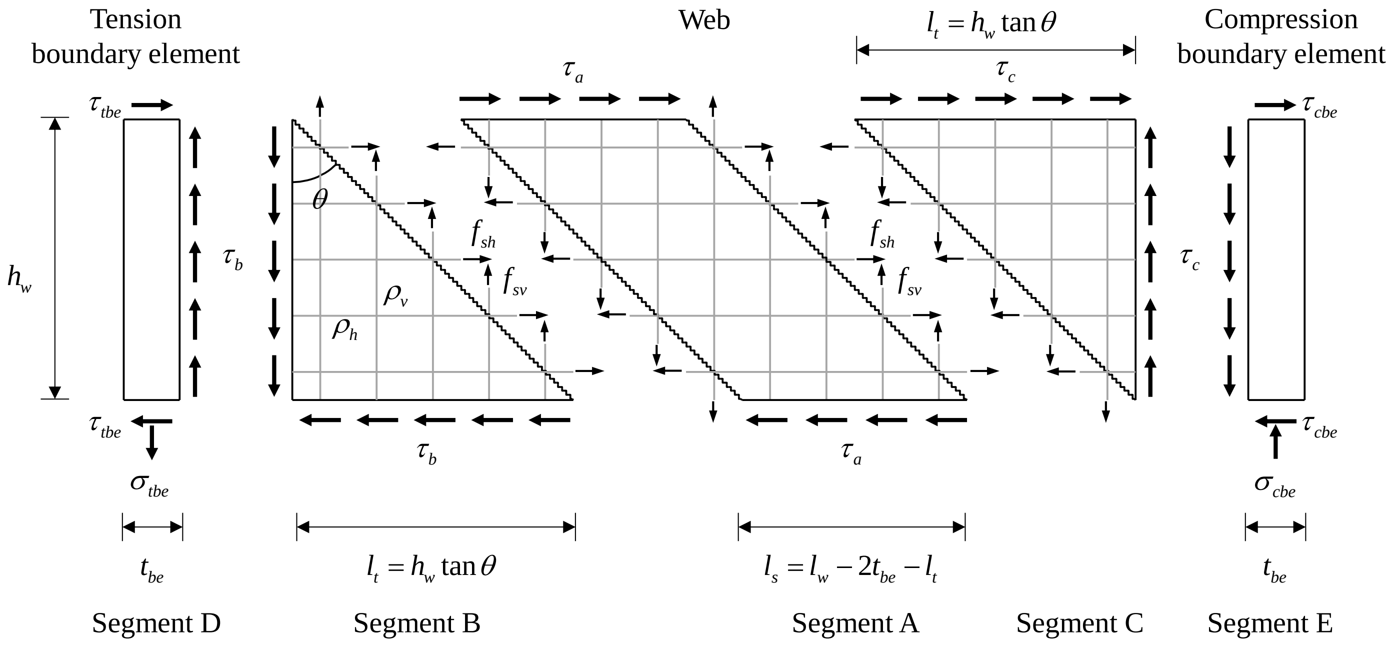
Lateral loading



Reaction force vectors of a wall structure in a nuclear power plant

Analytical Model

Force equilibrium based on a simplified crack pattern



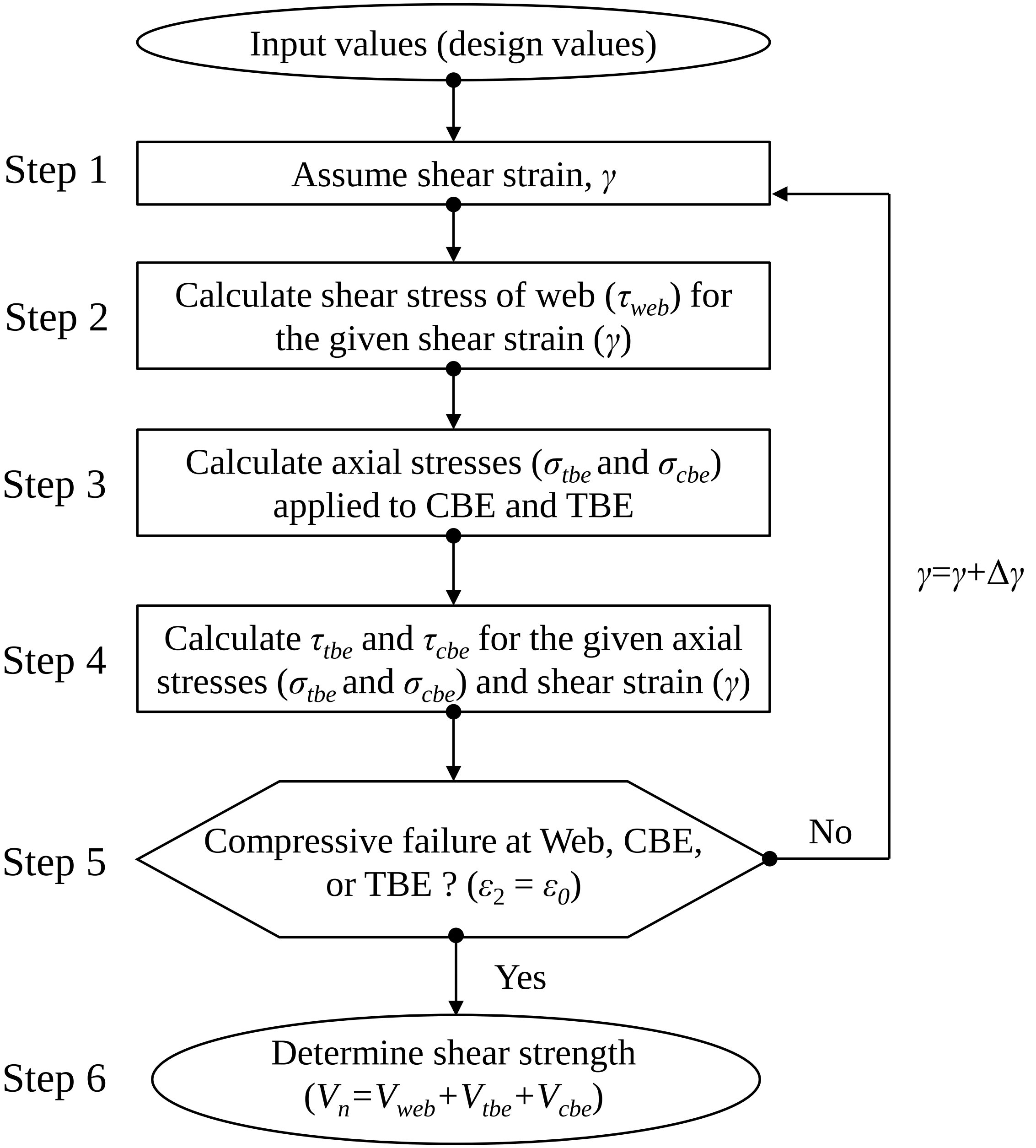
$$V_{model} = V_A + V_B + V_C + V_D + V_E = V_{web} + V_{cbe} + V_{tbe}$$

$$= \tau_{web} A_{web} + \tau_{cbe} A_{cbe} + \tau_{tbe} A_{tbe}$$

Requires iteration to determine the ultimate strength (until failure of one segment)

Analytical Model

Solution algorithm



Input values (design values)

Step 1 Assume shear strain, γ

Step 2 Calculate shear stress of web (τ_{web}) for the given shear strain (γ)

Step 3 Calculate axial stresses (σ_{tbe} and σ_{cbe}) applied to CBE and TBE

Step 4 Calculate τ_{tbe} and τ_{cbe} for the given axial stresses (σ_{tbe} and σ_{cbe}) and shear strain (γ)

Step 5 Compressive failure at Web, CBE, or TBE ? ($\varepsilon_2 = \varepsilon_0$)

No: $\gamma=\gamma+\Delta\gamma$

Yes

Step 6 Determine shear strength ($V_n=V_{web}+V_{tbe}+V_{cbe}$)

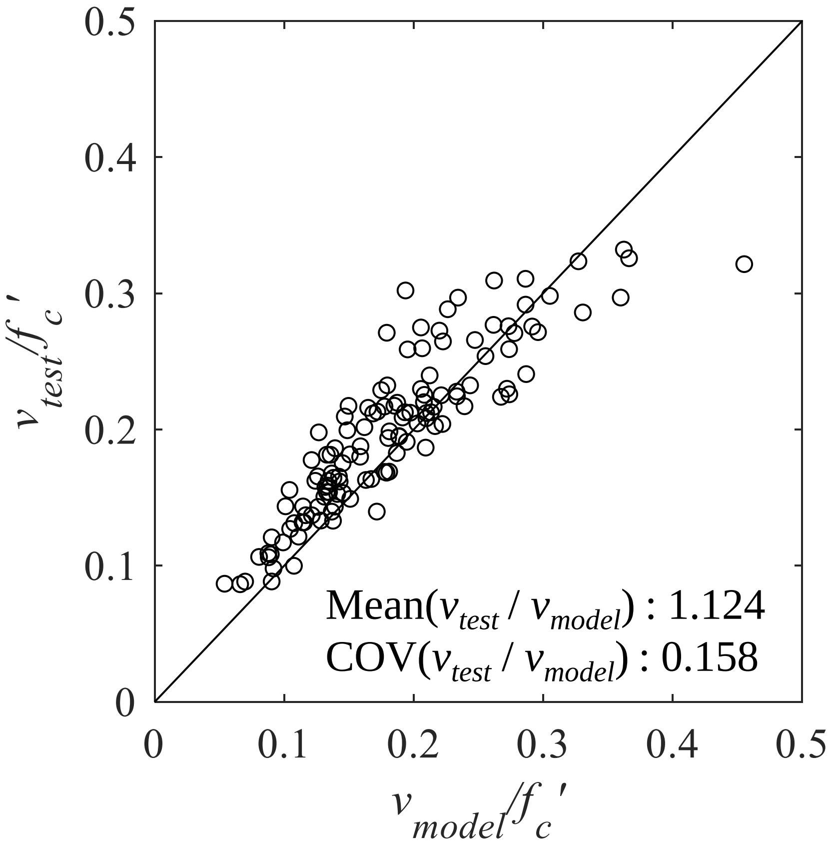
Analytical Model

Validation

Database (123 squat wall test specimens)

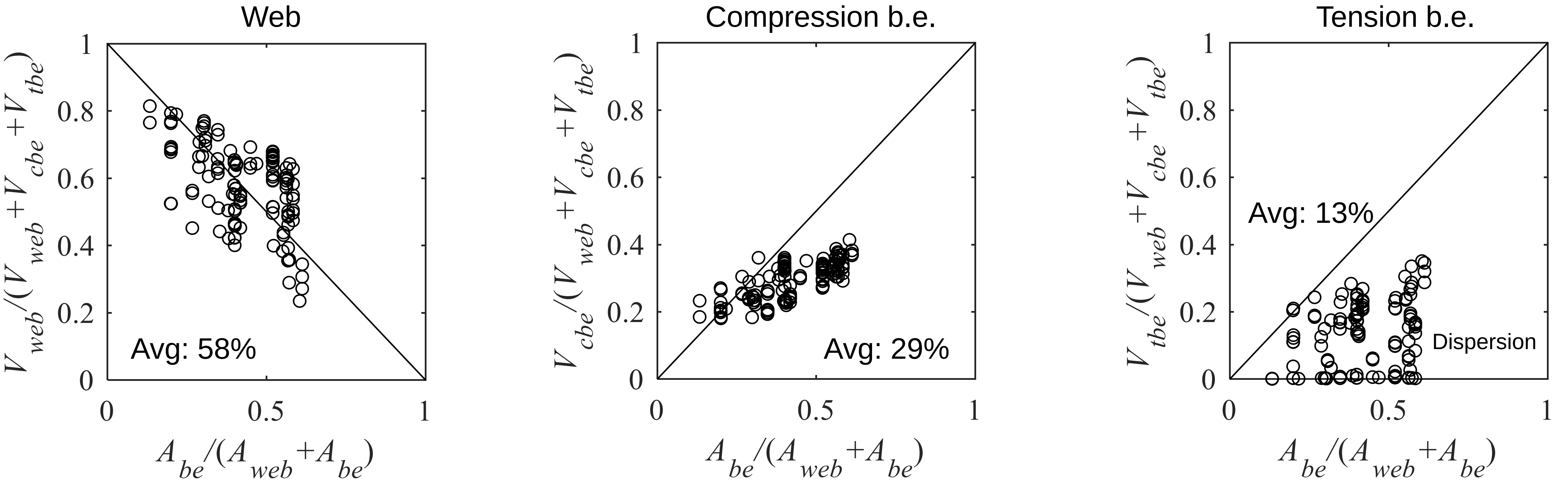
Design variables	Minimum	Maximum
Wall thickness, t_w (mm)	70	203
Wall length, l_w (mm)	800	3960
Wall height, h_w (mm)	401	2619
Aspect ratio, h_w/l_w	0.21	1.38
Thickness of boundary element, t_{be} (mm)	75	360
Width of boundary element, h_{be} (mm)	79	1500
Concrete compressive strength, f_c' (MPa)	13.7	110.7
Rebar yield strength, f_y (MPa)	272.3	754.2
Horizontal reinforcement ratio of web, ρ_h (%)	0.25	2.80
Vertical reinforcement ratio of web, ρ_v (%)	0.26	2.80
Boundary element horizontal reinforcement ratio, ρ_{hbe} (%)	0.05	4.93
Boundary element vertical reinforcement ratio, ρ_{vbe} (%)	0.48	14.35
Axial load ratio, $N/(A_g f_c')$ (%)	0	27

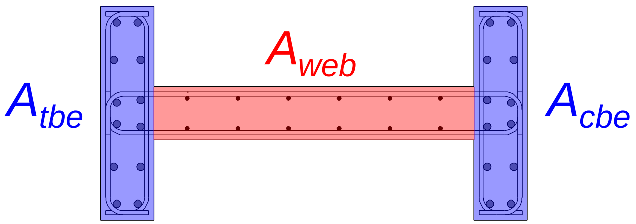
Predicted (v_{model}) vs. Tested (v_{test}) shear strengths



Analytical Model

Fraction of shear resistance





$A_{cbe} + A_{tbe} = A_{be}$

Simplified Design Equation

Derivation of design shear equation from the analytical model

$$V_{proposed} = V_{web} + V_{cbe} + \cancel{V_{tbe}} = V_{web} + V_{cbe}$$

From force equilibrium (MCFT),

$$v_{web} = \left(f_1 + \rho_h f_{sh} \right) \cot\theta$$

$$v_{cbe} = \left(f_1 + f_2 \right) / \left(\tan\theta + \cot\theta \right)$$

f_1 = tensile stress of concrete

f_2 = compressive stress of concrete

ρ_h = horizontal reinforcement ratio in web

f_{sh} = tensile stress of horizontal rebars

θ = crack angle

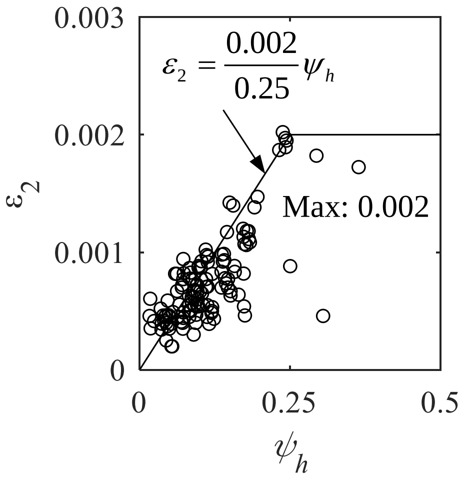
$$V_{proposed} = v_{web} A_{web} + V_{cbe} A_{cbe}$$

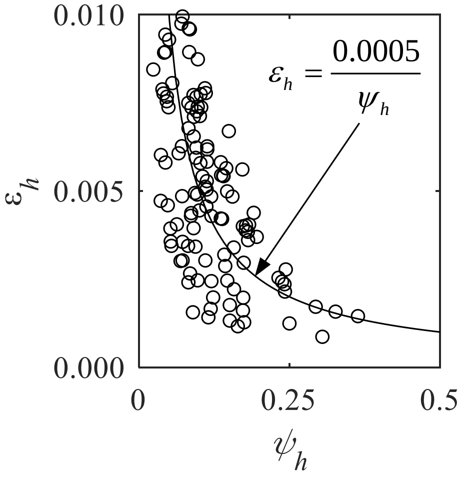
Simplified Design Equation

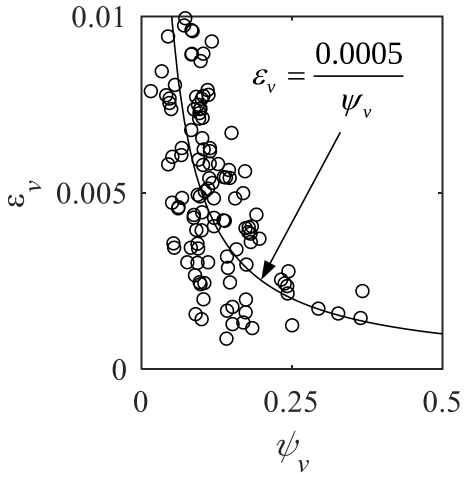
Derivation of design shear equation from the analytical model

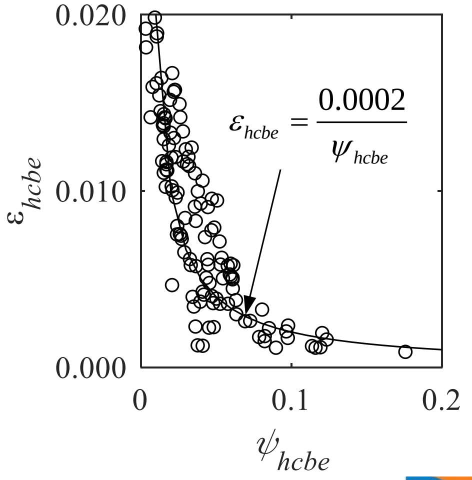
$$v_{web} = \left(f_1 + \rho_h f_{sh} \right) \cot\theta \qquad v_{cbe} = \left(f_1 + f_2 \right) / \left(\tan\theta + \cot\theta \right)$$

Strains at the maximum strength of walls (from analytical model)









compression b.e.

web

Simplified Design Equation

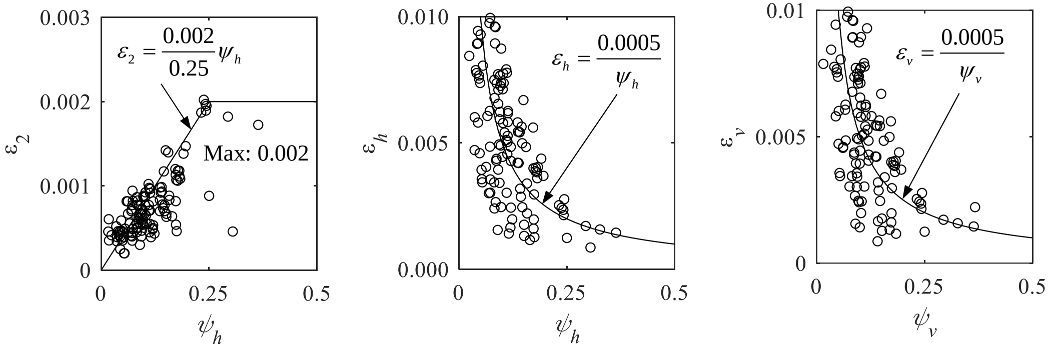
v_{web} (shear strength of web at failure) $= v_c + v_s$

$$v_{web} = v_c + v_s = \left(f_1 + \rho_h f_{sh} \right) \cot\theta = \left(\frac{0.33\sqrt{f_c^{'}}}{1+\sqrt{500\varepsilon_1}} + \rho_h E_s \varepsilon_h \right) \cot\theta$$

$$\cot\theta = \sqrt{\frac{\varepsilon_h + \varepsilon_2}{\varepsilon_v + \varepsilon_2}} \approx \sqrt{\frac{\psi_v}{\psi_h}} = R_{vh}$$

$$v_c = \frac{0.33\sqrt{f_c^{'}}}{1+0.5\sqrt{\frac{1}{\psi_v}+\frac{1}{\psi_h}}} R_{vh} \quad (\psi_h > 0 \text{ and } \psi_h > 0)$$

$$v_s = \rho_h E_s \frac{0.0005}{\psi_h} R_{vh} \le \rho_h \mathrm{f}_{yh} \mathrm{R}_{vh} \quad (\psi_h > 0 \text{ and } \psi_h > 0)$$



f_c' = compressive strength of concrete

ψ_v = normalized vertical reinforcement ratio ($\rho_v f_{yv}/f_c'$)

ψ_h = normalized horizontal reinforcement ratio ($\rho_h f_{yh}/f_c'$)

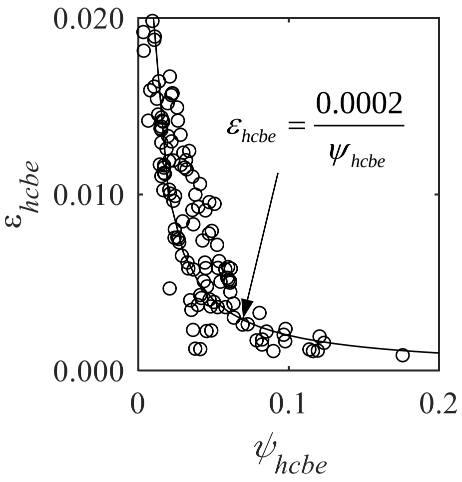
R_{vh} = simplified crack angle

Simplified Design Equation

v_{cbe} (shear strength of compression b.e.)

$$v_{cbe} = \left(\cancel{f_1} + f_2\right) / \left(\tan\theta + \cot\theta\right) = f_2 / \left(\tan\theta + \cot\theta\right) = \frac{f_c^{'}}{0.8 + 170\varepsilon_1} / \left(\tan\theta + \cot\theta\right)$$

$$\cot\theta = \sqrt{\frac{\varepsilon_h + \varepsilon_2}{\varepsilon_v + \varepsilon_2}} = \sqrt{\frac{0.0002/\psi_{hcbe} + 0.002}{0.002}} = \sqrt{0.1/\psi_{hcbe} + 1}$$



$$v_{cbe} = \frac{f_c^{'}}{0.8 + 170\varepsilon_1} / \left(\tan\theta + \cot\theta\right) = \frac{f_c^{'}}{1.14 + 0.034/\psi_{hcbe}} \frac{\sqrt{1 + 0.1/\psi_{hcbe}}}{2 + 0.1/\psi_{hcbe}}$$

Simplified Design Equation

Summary of $V_{proposed} = v_{web}A_{web} + v_{cbe}A_{cbe}$

$$v_{web} = v_c + v_s$$

$$v_c = \frac{0.33\sqrt{f_c^{'}}}{1+0.5\sqrt{\frac{1}{\psi_v}+\frac{1}{\psi_h}}} R_{vh} \quad (\psi_h > 0 \text{ and } \psi_h > 0)$$

$$v_s = \rho_h E_s \frac{0.0005}{\psi_h} R_{vh} \le \rho_h \mathrm{f}_{yh} \mathrm{R}_{vh} \quad (\psi_h > 0 \text{ and } \psi_h > 0)$$

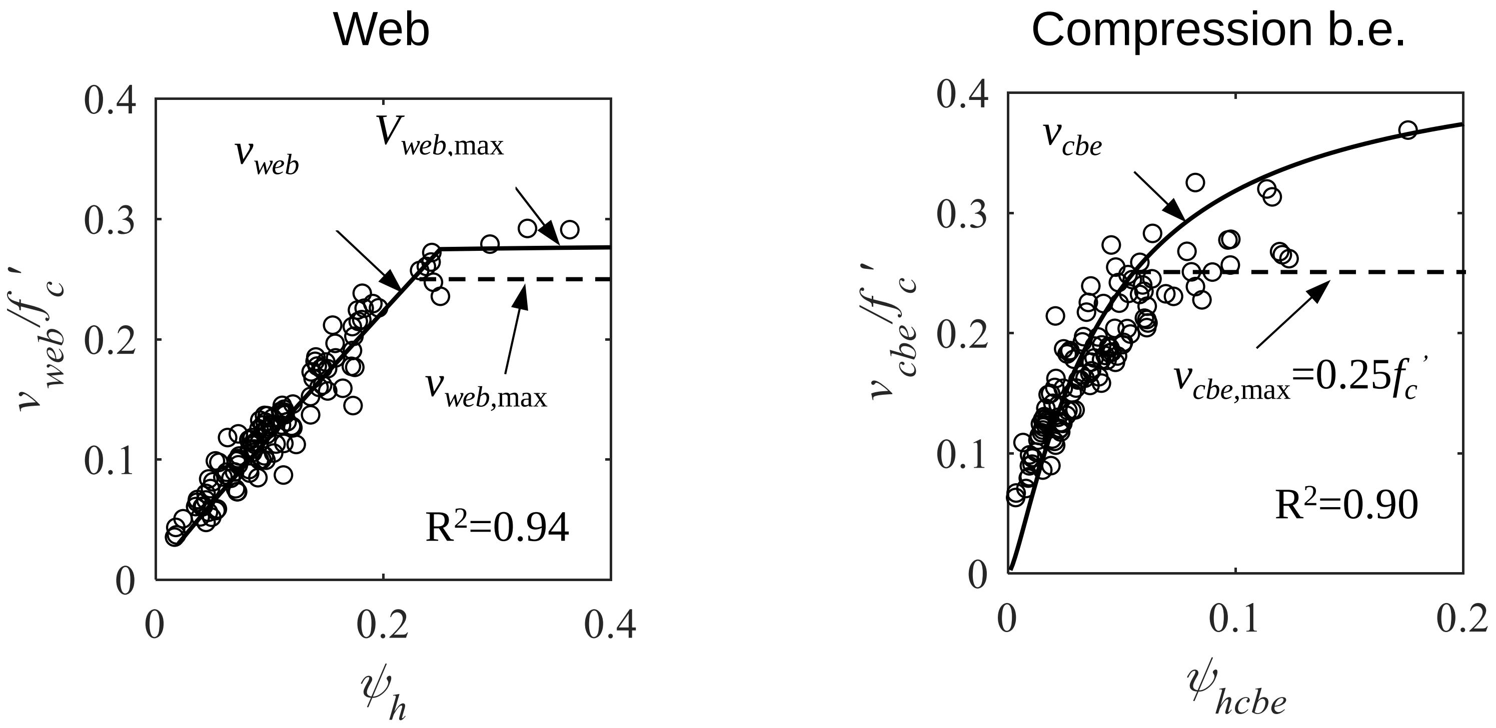
$$v_{cbe} = \frac{f_c^{'}}{1.14+0.034/\psi_{hcbe}} \frac{\sqrt{1+0.1/\psi_{hcbe}}}{2+0.1/\psi_{hcbe}}$$

Inputs:

- sectional area (web and b.e.)
- material strength (f_c', f_y)
- reinforcement ratio (ρ)

Simplified Design Equation

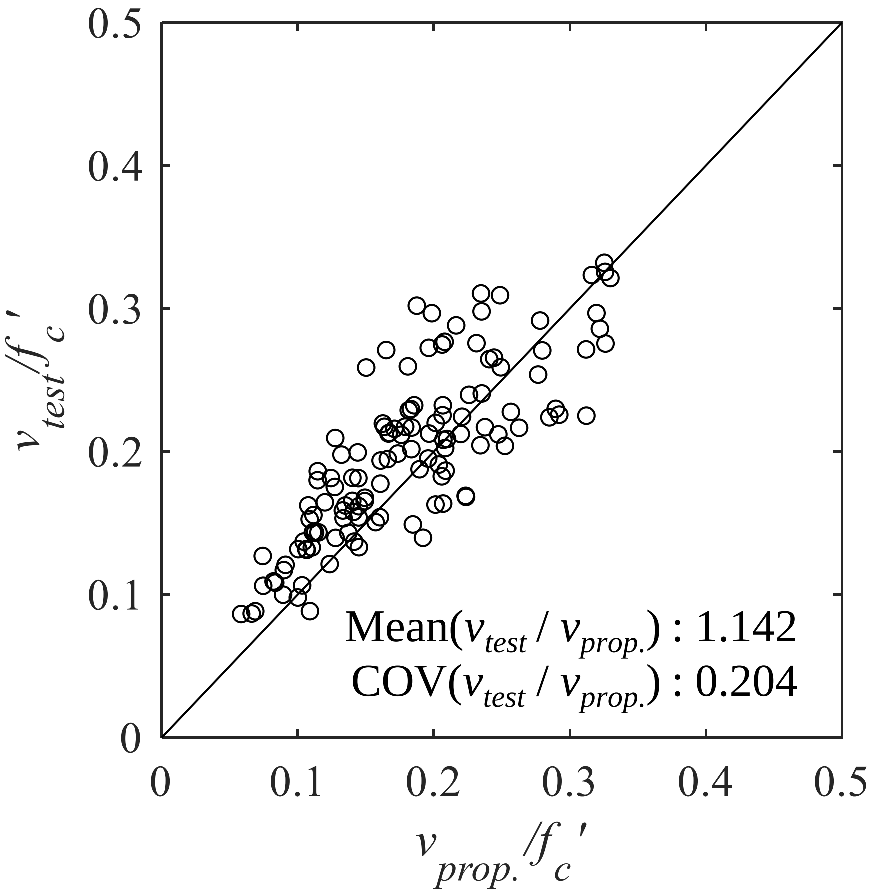
Implementation



Comparison of shear strength between analytical model and simplified design equation

Simplified Design Equation

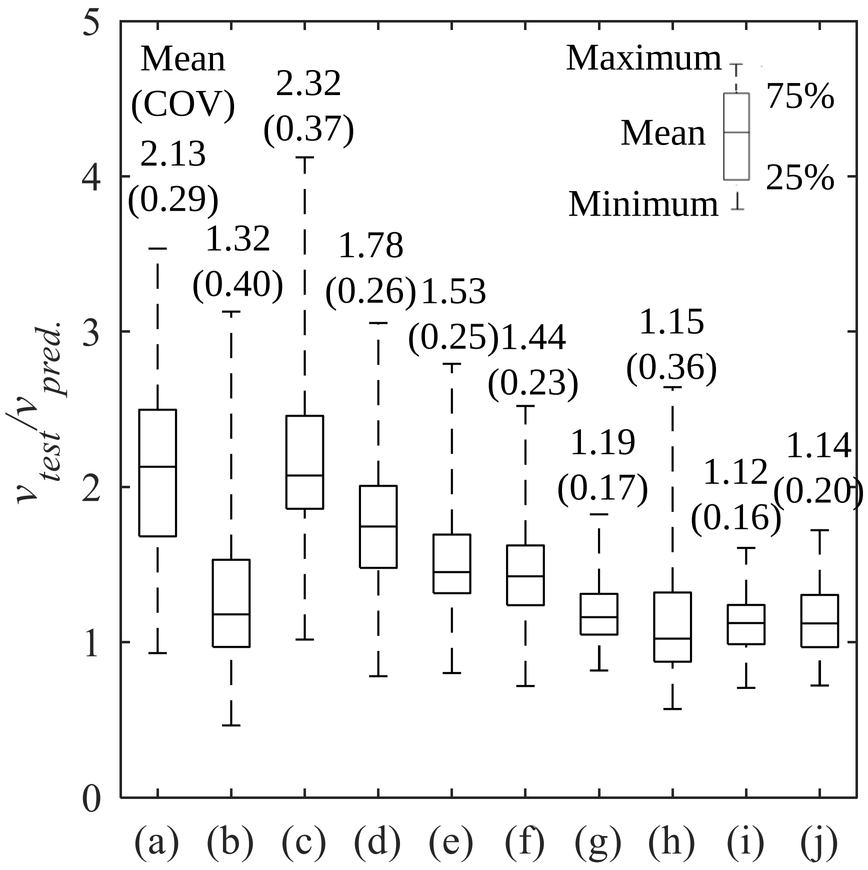
Validation



Prediction method	Analytical model (v_{test}/v_{model})	Proposed equation ($v_{test}/v_{prop.}$)
Mean	1.124	1.142
Standard deviation	0.177	0.233
COV	0.157	0.204
Minimum	0.706	0.721
Maximum	1.607	1.722

Simplified Design Equation

Evaluation of the predictability of the existing models



(a) Wood (1990)

(b) Moehle (2015)

(c) Luna and Whittaker (2019)

(d) ACI 318-19 (2019)

(e) Barda et al. (1977)

(f) ASCE/SEI 43-05 (2005)

(g) Gulec and Whittaker (2011)

(h) Kassem (2015)

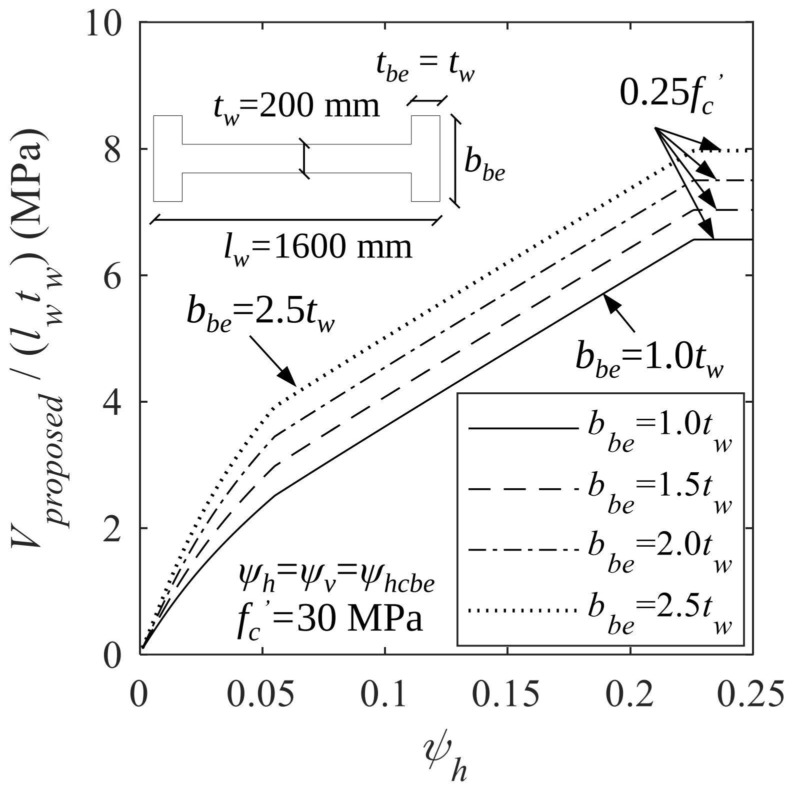
(i) Iterative analytical model (present study)

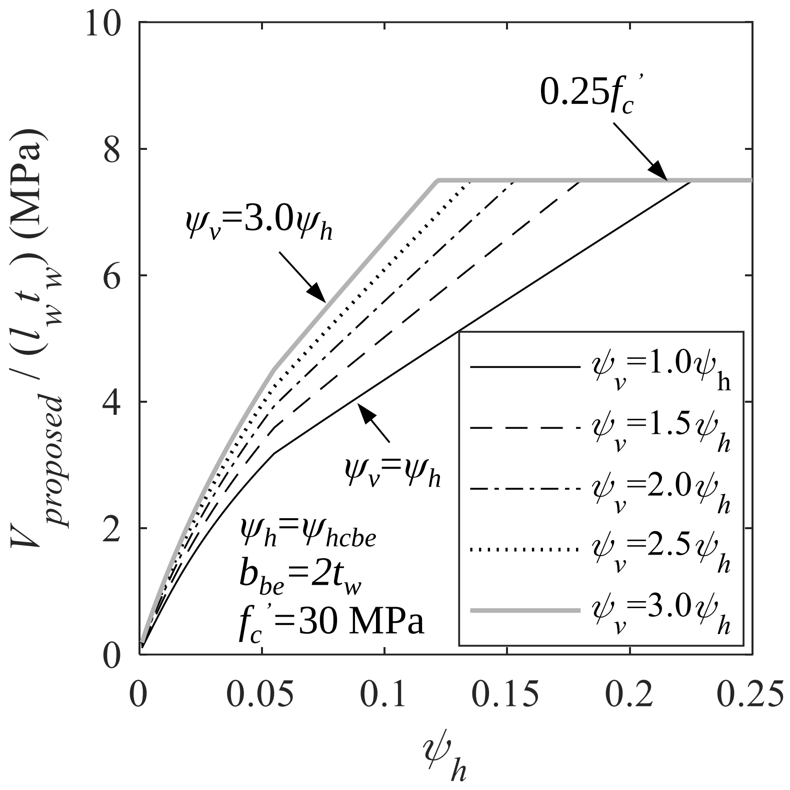
(j) Simplified design equation (present study)

Simplified Design Equation

Parametric investigation

The proposed equation Incorporates the shear strength contribution of boundary element & vertical rebars





Summary and Conclusions

- A mechanics-based model was developed to predict the shear strength of squat walls built with boundary elements.
- The model was validated with 123 experimental specimens and generated strains and stresses to understand the contribution of individual components to overall shear strength.
- Attention should be paid to boundary elements, which contribute 42% to total shear strength, but tension boundary elements can be neglected for design convenience.
- Both horizontal and vertical reinforcement ratios affect shear strength, and the provision of ACI 318-19 ignoring vertical reinforcement should be revised.
- Analytical model and simplified design equations showed comparable results with measured capacities and reduced v_{test} / v_{pred} ratio.

References

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