



Strength Reduction Factors for ACI 318 Strut-and-Tie Method for Deep Beams

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IN STRUCTURAL RESEARCH

□ Introduction

Reliability

Strut-and-tie (STM)

STM provisions in ACI 318

□ Reliability Analysis of Deep Beams

□ Conclusions

Strength Reduction

Factors Recommendation

ACI STRUCTURAL JOURNAL

TECHNICAL PAPER

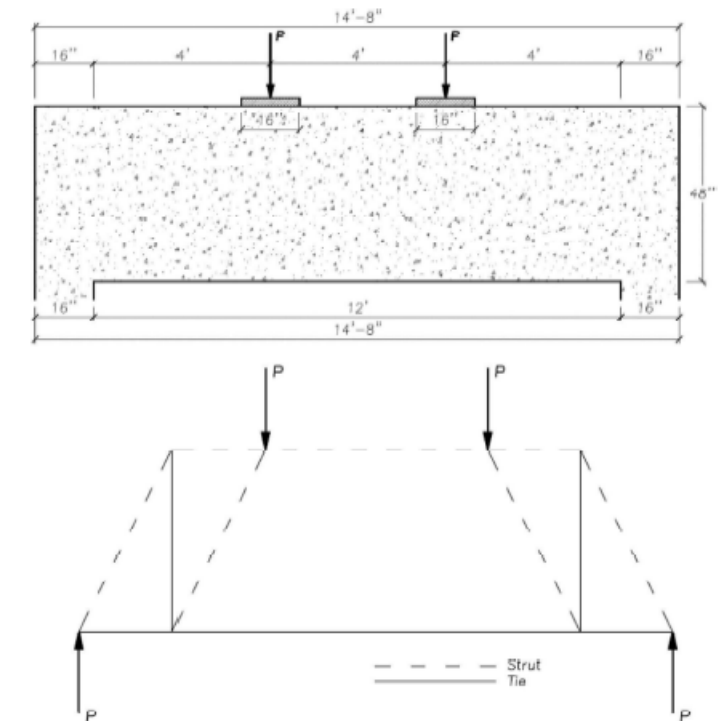
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Strength Reduction Factors for ACI 318 Strut-and-Tie Method for Deep Beams

by Victor Aguilar, Robert W. Barnes, and Andrzej Nowak

The strut-and-tie approach has gained importance in reinforced concrete design practice in the United States in the last two decades. This method has proven suitable for designing shear-critical structural members where beam theory is not applicable. However, the strength reduction factors specified for the ACI 318 strut-and-tie method have not been calibrated based on the structural reliability approach. Therefore, the reliability of members designed according to these provisions is unknown. In this study, the reliability of deep beams designed using the strut-and-tie method according to ACI 318 building code requirements for structural concrete was determined. Statistical parameters employed for loads, material uncertainty, and fabrication uncertainty were based on published literature. The uncertainty in the analytical model was characterized based on available test results. The findings indicate that current design practice using the strut-and-tie method promotes the likelihood of a nonductile failure mode relative to a ductile failure mode. Inconsistencies in reliability with respect to concrete strength are highlighted. The following reliability-based strength reduction factors are suggested: $\phi = 0.65$ for struts and nodal zones and $\phi = 0.90$ for ties.

Keywords: D-region; design strength; ductility; nodal zone; reinforced concrete; reliability; safety; shear; strut-and-tie model.

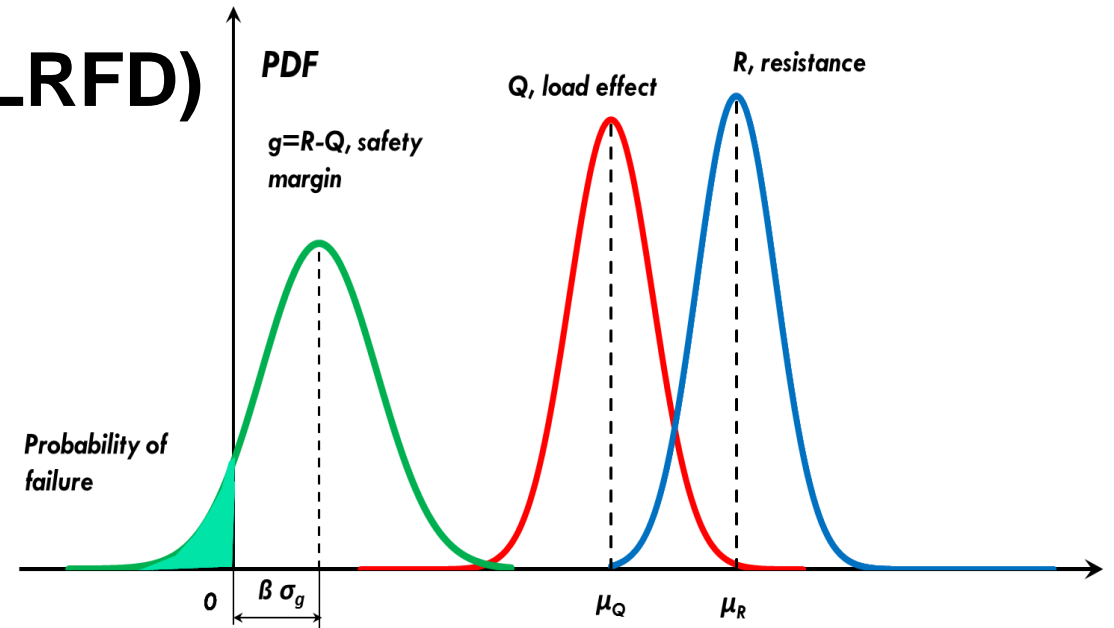


How do we deal with risk?

Load and Resistance Factor Design (LRFD)

$$\phi R_n \geq \gamma_1 Q_1 + \gamma_2 Q_2 + \dots + \gamma_k Q_k$$

$$0.75V_n \geq 1.2D_n + 1.6L_n$$



γ : Load factors are [based on the uncertainty of loads components](#)

ϕ : Resistance factors (**strength reduction factors**) are [based on the uncertainty in resistance](#)

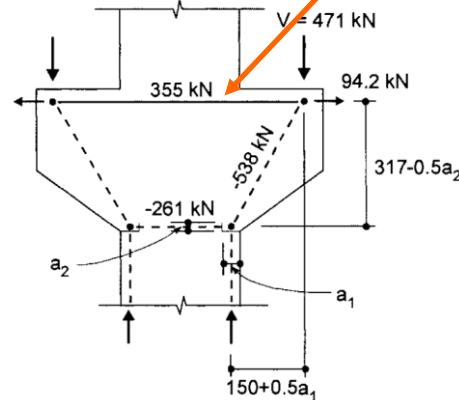
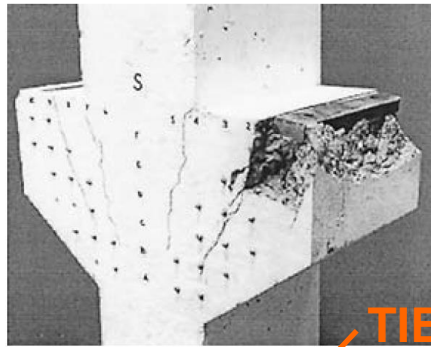
β_T : The combination of load and resistance factors should assure to reach a certain predefined probability of failure or reliability index (**target reliability index**)

Target Reliability Index

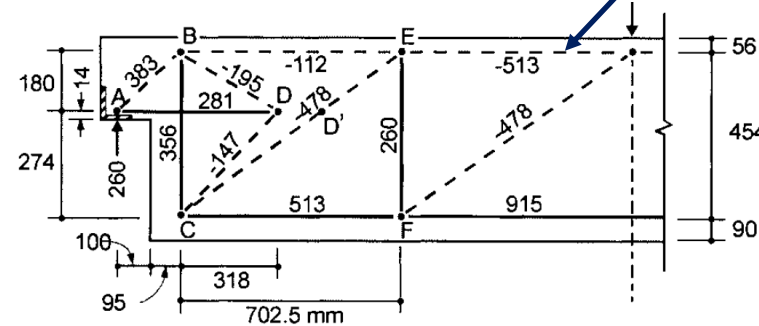
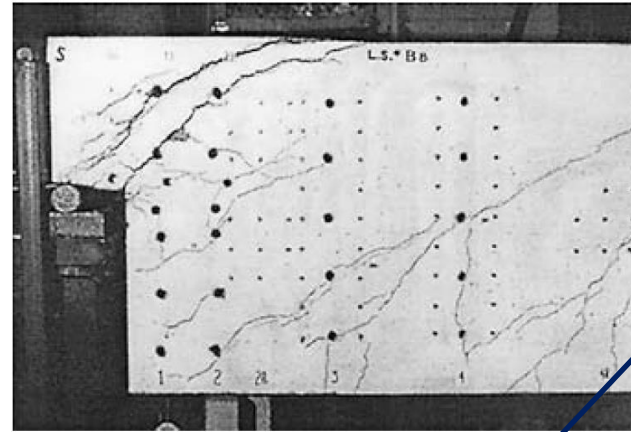
- ❑ Acceptable minimum safety margin is selected in terms of target reliability index, β_T .
- ❑ $\beta_T = 3.5$ for previous code calibration for components (beams or girders) and strength limit states.
- ❑ $\beta_T = 3.5$ corresponds to a probability of failure of 1/4300.
 $\beta_T = 3.0 \rightarrow 1/740$; $\beta_T = 4.0 \rightarrow 1/32,000$.
- ❑ Greater values of β_T can be selected to reduce probability of failure for less desirable failure modes, as is the case with shear in structural concrete.

Strut-and-tie

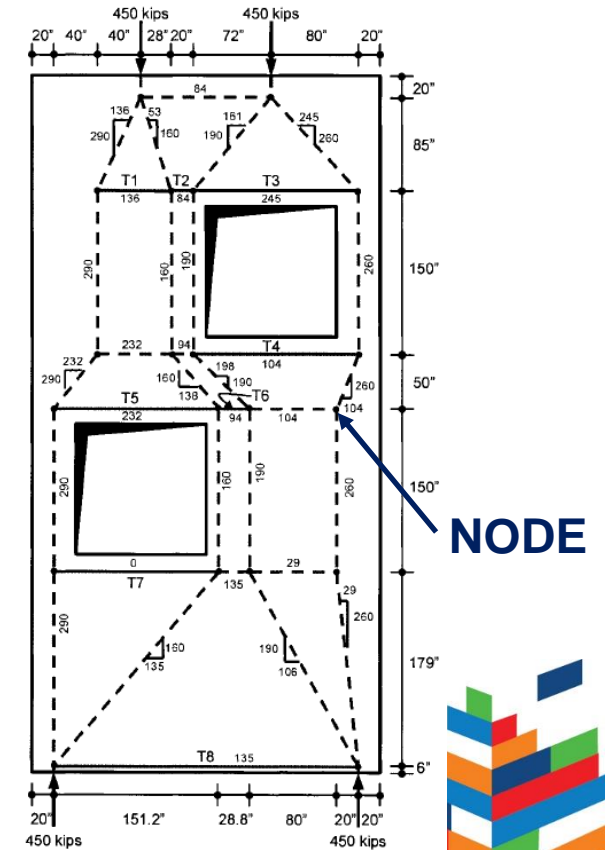
Complex stress field of a concrete structure is represented by an equivalent truss, with elements in compression (struts) and elements in tension (ties), and intersections are called nodes.



Cook & Mitchell (1988)



Cook & Mitchell (1988)



ACI 318-19 STM Provisions

□ Strut

$$F_{us} \leq \phi F_{ns}$$

where,

F_{us} : factored force in the strut

F_{ns} : nominal strength of the strut

□ Node

$$F_{un} \leq \phi F_{nn}$$

where,

F_{un} : factored force in the nodal zone

F_{nn} : nominal strength of the nodal zone

□ Tie

$$F_{ut} \leq \phi F_{nt}$$

where,

F_{ut} : factored force in the tie

F_{nt} : nominal strength of the tie

ACI 318-19 in all cases $\phi = 0.75$

[AASHTO LRFD $\phi_{\text{compression}} = 0.70$ and $\phi_{\text{tension}} = 0.90$ or 1.0]

□ Strut strength

Unreinforced: $F_{ns} = f_{ce} A_{cs}$

Reinforced: $F_{ns} = f_{ce} A_{cs} + A'_s f'_s$

where effective compressive strength:

$$f_{ce} = 0.85 \beta_c \beta_s f'_c$$

β_s to account for the effect of tensile stresses and cracking reinforcement

□ Nodal strength

$$F_{nn} = f_{ce} A_{nz}$$

where effective compressive strength:

$$f_{ce} = 0.85 \beta_c \beta_n f'_c$$

β_n to account for the effect of anchoring ties in the node

□ Tie strength

$$F_{nt} = A_{ts} f_y$$

f'_c uncertainty is about 11–17%

f_y uncertainty is about 2–4%

Objectives

- ❑ Investigate the strut-and-tie provisions in ACI 318-19:

Reliability

Likelihood of ductile failure

- ❑ Current safety margin?

Reliability of individual components (struts, ties, nodes)

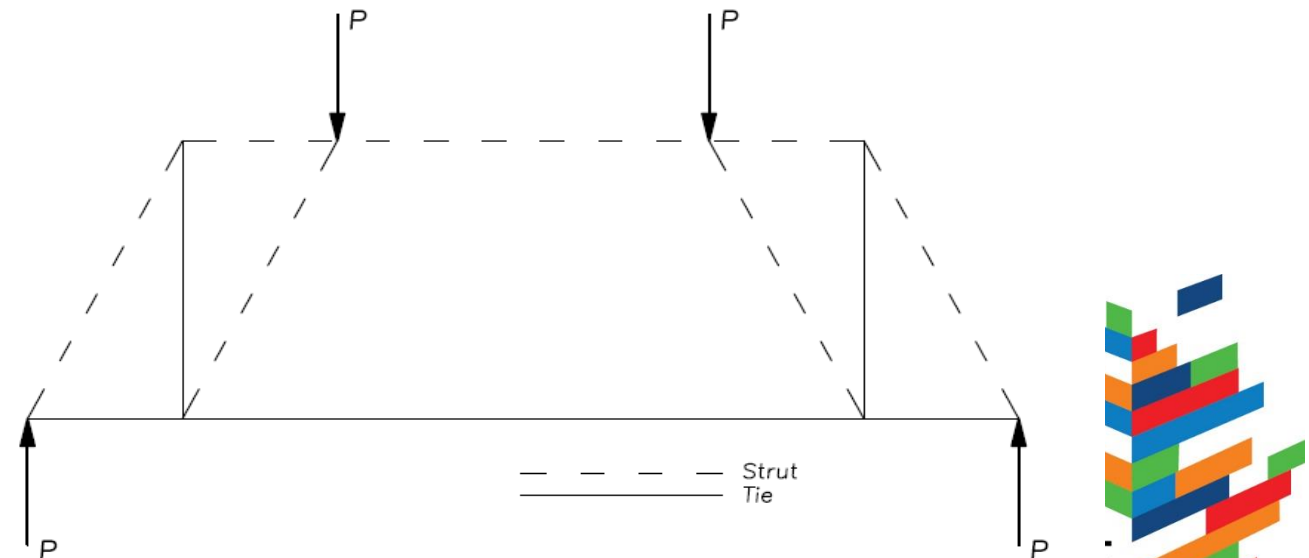
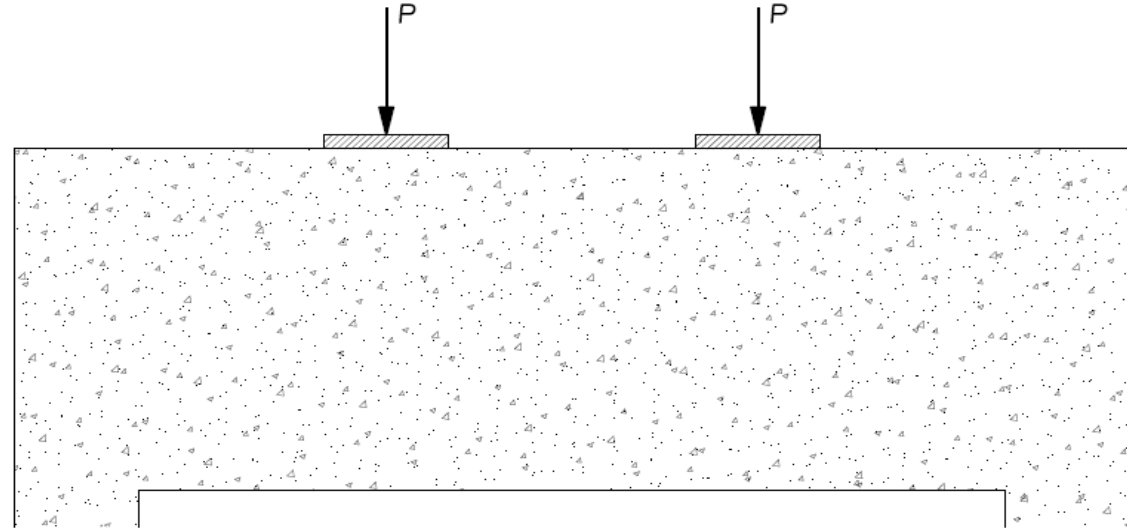
Reliability of the member (deep beam system)

- ❑ Select β_T and suggest ϕ

Deep Beam

Generic Deep Beam

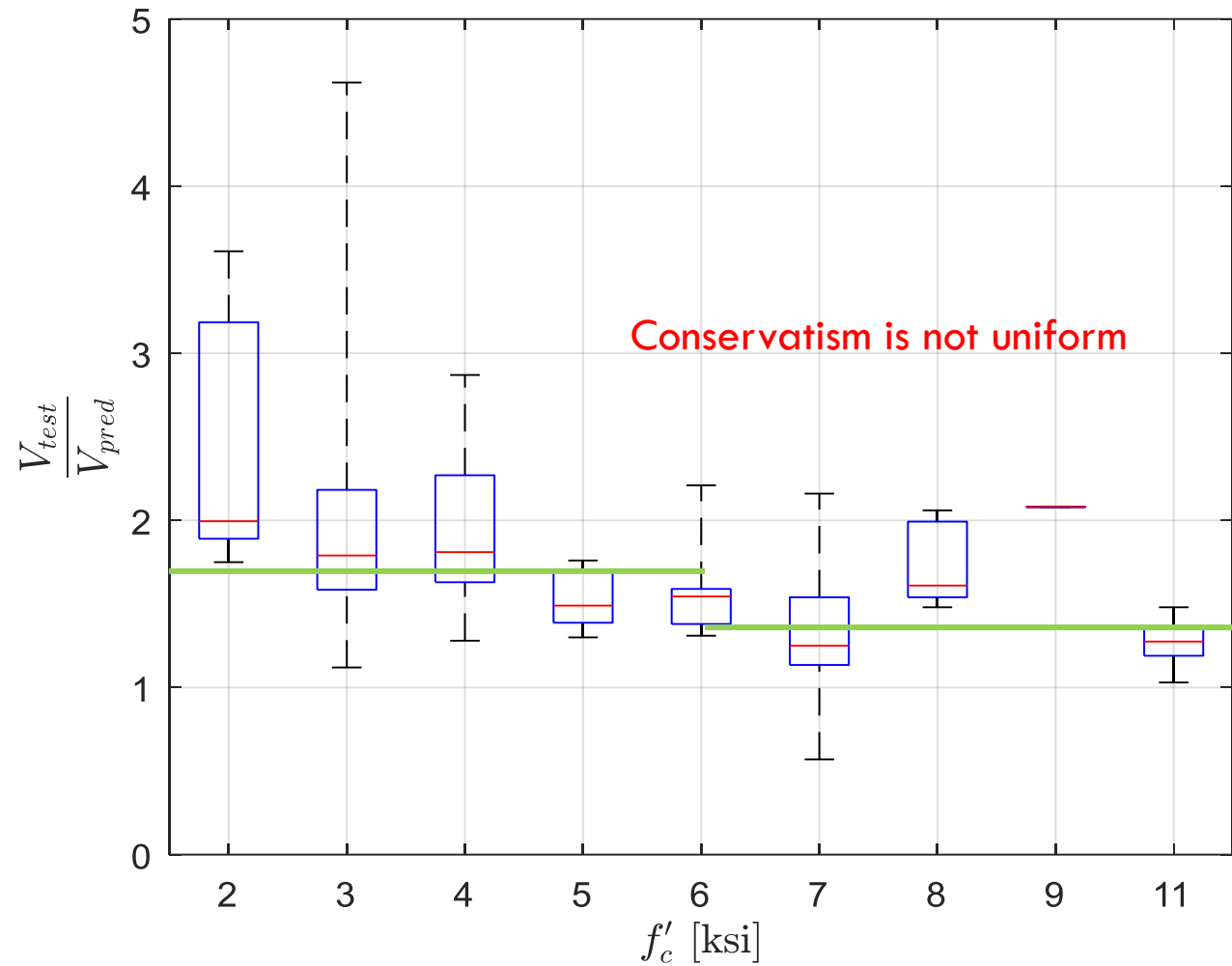
- ❑ *Statically determinate model*
- ❑ *Series system*
- ❑ *Component forces for design*
 $COV = 20\%$



Analysis Uncertainty

Park and Kuchma (2007)

- ❑ #214 tests on deep beams from 8 sources
- ❑ Predictions based on STM



V_{test}/V_{pred} as a function of f'_c

Probabilistic Models for Reliability Analysis

❑ Load models are same as in former ACI calibration (Nowak and Szerszen, 2003)

❑ Dead $\lambda = 1.05$ COV = 0.10 (normal)

❑ Live $\lambda = 1.00$ COV = 0.18 (extreme I)

❑ $R = R_n MFP$

❑ M and F based on available data materials properties and fabrication tolerances documented by Nowak et al. (2012)

$$\lambda = \frac{\bar{x}}{x_n} \quad COV = \frac{\sigma_x}{\bar{x}}$$

Item	λ	COV	Distribution	Source
Compressive concrete strength				
$f_c' = 3,000$ psi	1.31	0.170		
$f_c' = 4,000$ psi	1.24	0.150		
$f_c' = 5,000$ psi	1.19	0.135		
$f_c' = 6,000$ psi	1.15	0.125		
$f_c' = 7,000$ psi	1.13	0.115	Normal	Nowak et al. ⁴²
$f_c' = 8,000$ psi	1.11	0.110		

Analysis uncertainty

Normal strength concrete	1.76	0.142		
High strength concrete	1.31	0.145	Normal	This study
Member forces	1.00	0.200		

# 7	1.14	0.030		
# 8	1.13	0.025	Lognormal	Nowak et al. ⁴²
# 9	1.14	0.020		
# 10	1.13	0.020		
# 11	1.13	0.020		
# 14	1.14	0.020		

Fabrication uncertainty

Width cast-in- place beam	1.01	0.040	Normal	Ellingwood et al. ⁴⁹
Area of reinforcement	1.00	0.015		Nowak et al. ⁴²

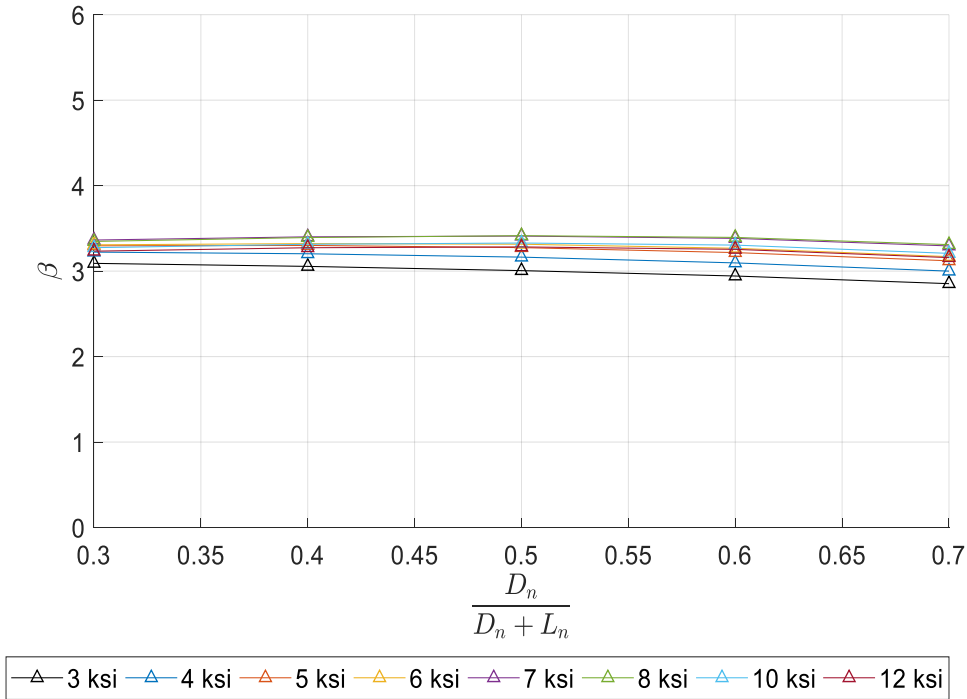
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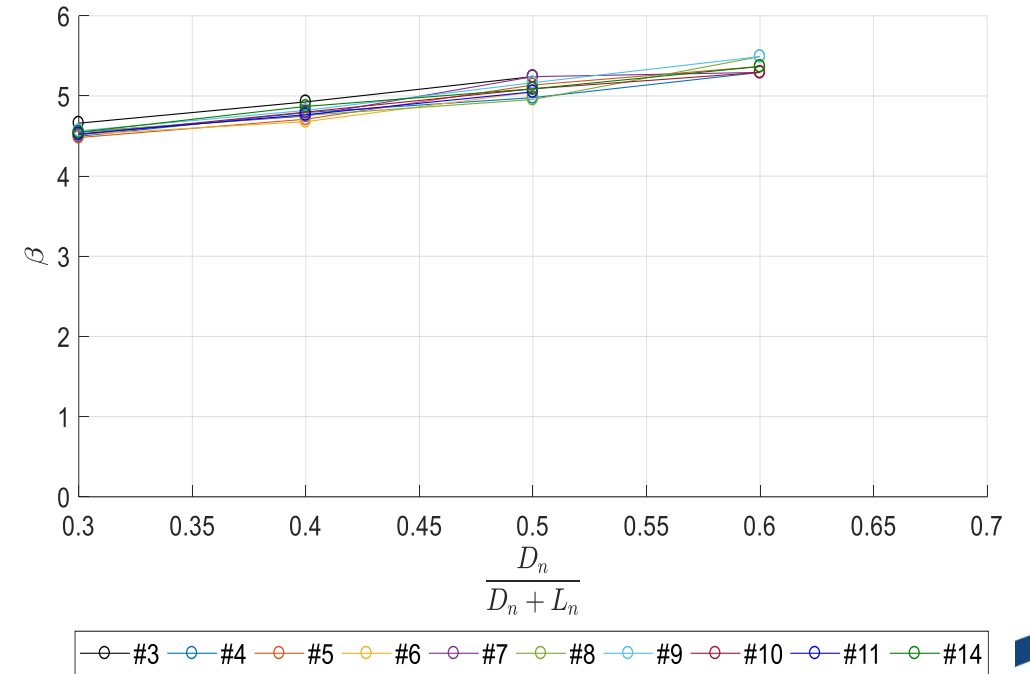


Reliability of Individual Components

Reliability of individual components (*does not include analysis variability*)



Struts and nodal zones



Ties

Struts and nodes are more likely to fail than ties!

Reliability of the System (Deep Beam)

❑ System probability of failure (Series)

For series systems with positive correlation, the probability of failure of the system (P_F) must satisfy

$$\max \{ P_i \} \leq P_F \leq 1 - \prod_{i=1}^n (1 - P_i)$$

Lower bound: fully correlated members, **Upper bound:** uncorrelated members

❑ Assumptions

- ❑ *Strut and nodes fully correlated*
- ❑ *Ties fully correlated*
- ❑ *Strut & nodes uncorrelated with ties*

Results Reliability Analysis

- Component reliability and system results for **normal-strength** concrete $f'_c \leq 6,000 \text{ psi}$

- 100 million simulations

Component	ϕ	β_i	$P(R_t < R_{s,nz})$	β_{sys}
Strut	0.75	3.0		
Node	0.75	3.0	0	4.0
Tie	0.75	5.0		
Strut	0.75	3.0		
Node	0.75	3.0	1%	4.0
Tie	0.90	4.0		
Strut	0.75	3.0		
Node	0.75	3.0	6%	4.0
Tie	1.00	3.4		
Strut	0.65	3.4		
Node	0.65	3.4	3%	4.2
Tie	0.90	4.0		
Strut	0.65	3.4		
Node	0.65	3.4	18%	4.2
Tie	1.00	3.4		

$= \beta_T$
Ideal for shear limit state



Results Reliability Analysis

- Component reliability and system results for **high-strength** concrete $f'_c > 6,000 \text{ psi}$

- 100 million simulations

Component	ϕ	β_i	$P(R_t < R_{s,nz})$	β_{sys}
Strut	0.75	3.3		
Node	0.75	3.3	0.3%	3.2
Tie	0.75	5.0		
Strut	0.75	3.3		
Node	0.75	3.3	70%	2.9
Tie	1.00	3.4		

$\approx 3.0 < \beta_T = 3.5$

Strut	0.65	3.9		
Node	0.65	3.9	78%	3.4
Tie	0.90	4.0		

$\approx 3.5 = \beta_T$
For flexural limit state

Strut	0.60	4.3		
Node	0.60	4.3	58%	3.8
Tie	0.80	4.7		
Strut	0.50	5.1		
Node	0.50	5.1	93%	4.1
Tie	0.75	5.0		

Conclusions

- ❑ The current resistance factors for STM in ACI 318-19 promotes the likelihood of a nonductile failure mode relative to a ductile failure mode. *ϕ factors for STM need attention.*
- ❑ The level of conservatism of strength prediction with ACI 318-19 STM varies with the concrete compressive strength, which results in inconsistencies in the reliability of STM designs
 - ❑ Normal-strength concrete $\beta \approx 4.0$
 - ❑ High-strength concrete $\beta \approx 3.0$
- ❑ **Suggested strength reduction factors:**
 - ❑ $\phi = 0.65$ for struts and nodes and $\phi = 0.90$ for ties
 - ❑ Slightly increase the system reliability across all ranges of concrete strength
 - ❑ Increase the relative probability of a ductile failure mode

Thank
you!



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