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Guide for the Use of ASTM A1035/A1035M Type CS Grade 100 (690) Steel Bars for Structural Concrete

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This guide provides recommendations on design provisions for the use of ASTM A1035/ASTM A1035M Type CS Grade 100 (690) deformed steel bars for reinforced concrete members. The recommendations address only those requirements of ACI 318-14 that limit efficient use of such steel bars. Other code requirements are not affected. Any other ACI 318 versions will be explicitly specified. Although there are limiting ACI 318 requirements, ACI 318-14 Section 1.10 would allow the use of high-strength reinforcement. "Sponsors...shall have the right to present the data on which their design is based to the building official or to a board of examiners appointed by the building official."

The International Building Code (IBC 2012) would allow the same under Section 104.11, "Alternative materials, design and methods of construction and equipment". To approve an alternative material under this section, a building department would typically require an ICC Evaluation Service (ICC-ES) Evaluation Report, which would be based on an ICC-ES Acceptance Criteria (AC) document. An AC document (ICC-ES AC429) and an Evaluation Report (ICC-ES ESR-2107) exist, permitting the use of ASTM A1035/A1035M Grade 100 reinforcement.

This guide includes a discussion of the material characteristics of Grade 100 (690) ASTM A1035/A1035M (CS) deformed steel bars and recommends design criteria for beams, columns, slab, systems, walls, and footings for Seismic Design Category (SDC) A, B, or C, and for structural components not designated as part of the seismic-force-resisting system for SDC D, E, or F.

A structure assigned to SDC A, B, or C is required to be designed for all applicable gravity and environmental loads. In the case of SDC A structures, seismic forces are notional structural integrity forces. This guide addresses all design required for SDC A, B, and C structures.

Because the modulus of elasticity for ASTM A1035/A1035M (CS) is similar to that of carbon steel (ASTM A615/A615M) using higher specified minimum yield strength f_y may result in higher steel stress at service load condition and potentially cause wider cracks and larger deflections, which may be objectionable if aesthetics and water-tightness are critical design requirements. Higher deflection can also contribute to serviceability issues. Also, with higher f_y, the required development length will be longer.

Keywords: bar; design; guide; high-strength steel; structural.

ACI 439.6R-19 supersedes ACI ITG-6R-10 and was adopted and published February 2019.

CONTENTS

CHAPTER 1—INTRODUCTION

1.1—Objective

This guide is based on ACI ITG-6R-10, "Design Guide for the Use of ASTM A1035/A1035M Grade 100 (690) Steel Bars for Structural Concrete," reported by ACI Innovation Task Group 6. The ACI ITG-6R guide provides design provisions for the use of ASTM A1035/A1035M Type CS Grade 100 (690) deformed steel bars in reinforced structural members. This guide, which is based on ACI ITG-6R-10, is a stand-alone document, references and addresses only those requirements in ACI 318-14 that limit the use of such steel bars, and should not affect the application of other code requirements. Any other ACI 318 version will be explicitly specified. This guide includes a discussion of the material characteristics of Grade 100 (690) ASTM A1035/A1035M Type CS (Chromium content 8.0 to 10.9 percent) deformed steel bars, and the design provisions are based on the specific material properties and stress-strain behavior of these bars.

Although there are limiting ACI 318 requirements, ACI 318-14 Section 1.10 would allow the use of high-strength reinforcement. "Sponsors...shall have the right to present the data on which their design is based to the building official or to a board of examiners appointed by the building official."

The IBC International Building Code (IBC 2012) would allow the same under Section 104.11, "Alternative materials, design and methods of construction and equipment". To approve an alternative material under this section, a building department would typically require an ICC Evaluation Service (ICC-ES) Evaluation Report that would be based on an ICC-ES Acceptance Criteria (AC) document. An AC document (ICC-ES AC429) and an Evaluation Report (ICC-ES ESR-2107) exist, permitting the use of ASTM A1035/A1035M Grade 100 reinforcement.

Since publication of ACI ITG-6R, additional Grade 100 (690) ASTM A1035/A1035M Types CM (Chromium content 4.0 percent to 7.9 percent) and CL (chromium content 2.0 percent to 3.9) were introduced to the market. The mechanical properties and stress-strain characteristics of these steels may affect the design criteria herein and limit the applicability of the guide. Additionally, ASTM A615/A615M Grade 100 (690) was introduced to the market. These bars will be addressed in future editions when research on concrete members reinforced with these bar types become available and the impact of the mechanical properties have been documented. Grade 120 (830) ASTM A1035/A1035M Type CS, CM and CL are available, but are not addressed under this guide.

1.2—Scope

This guide presents the material characteristics of ASTM A1035/A1035M (CS) steel bars and recommends design criteria for beams, columns, slab systems, walls, and footings for Seismic Design Category (SDC) A, B, or C. A structure assigned to SDC A, B or C is required to be designed for all applicable gravity and environmental loads. In the case of SDC A structures, seismic forces are notional structural integrity forces. This guide addresses all design required for SDC A, B, and C structures. Due to lack of adequate data, the application of this guide for SDC D, E, or F is limited to slab systems, foundations, and structural components not designated as part of the seismic-force-resisting system, but explicitly checked for the induced effects of the design displacements. The only exception is the use of transverse reinforcement for concrete confinement with a specified minimum yield strength f_y up to 100,000 psi (690 MPa) in special moment frames, special structural walls, and all components of special structural walls including coupling beams and wall piers as permitted by ACI 318. Refer to 10.1 of this guide for more information on seismic design considerations. Shells, folded plate members, and prestressed concrete are beyond the scope of this guide. However, ASTM A1035/A1035M Type CS can be used as reinforcement in prestressed concrete but not as the prestressing strands. Design examples are included to illustrate design procedures and proper application of the design criteria. Modifications to these design criteria may be justified where the design adequacy within the scope of this guide is demonstrated by successful use, analysis, or test.

1.3—Historical perspective and background

For several decades prior to ACI 318-71, the design of structural concrete was restricted to using specified minimum yield strength f_y of 60,000 psi (420 MPa) or less for reinforcing bars. Section A603(e) of ACI 318-56 specified that "Stress in tensile and compressive reinforcement at ultimate load shall not be assumed greater than the yield point or 60,000 psi, whichever is smaller."

Section 1505 of ACI 318-63, specified two requirements:

"(a) When reinforcement is used that has a yield strength, f_y, in excess of 60,000 psi (420 MPa), the yield strength to be used in design shall be reduced to $0.85f_y$ or 60,000 psi (420 MPa), whichever is greater, unless it is shown by tension tests that at a proof stress equal to the specified minimum yield strength, f_y, the strain does not exceed 0.003;

(b) Designs shall not be based on a yield strength, f_y, in excess of 75,000 psi (520 MPa). Design of tension reinforcement shall not be based on a yield strength, f_y, in excess of 60,000 psi (420 MPa) unless tests are made in compliance with Section 1508(b)."

The Commentary on Section 1505 of ACI 318-63 states that:

> This section provides limitations on the use of high strength steels to assure safety and satisfactory performance. High strength steels frequently have a strain at yield strength or yield point in excess of the 0.003 assumed for the concrete at ultimate. The requirements of Section 1505(a) are to adjust to this condition.
>
> The maximum stress in tension of 60,000 psi (420 MPa) without test is to control cracking. The absolute maximum is specified as 75,000 psi (520 MPa) to agree with present ASTM specifications and as a safeguard until there is adequate experience with the high stresses.

Then, the Commentary on Section 1508 of ACI 318-63 states that:

> When the design yield point of tension reinforcement exceeds 60,000 psi (420 MPa), detailing for crack control becomes even more important. Entirely acceptable structures have been built, particularly in Sweden, with a design yield strength approaching 100,000 psi (690 MPa) but more design criteria for crack control and considerable American practical experience with 60,000 psi (420 MPa) yield strength tension reinforcement are needed before higher yield strengths are approved for general use. The Code, therefore limits *tension* reinforcement to 60,000 psi (420 MPa) yield strength, unless special full-scale tests are made. It was thought that 75,000 psi (520 MPa) yield strength tension reinforcement should be permitted where full-scale testing is economically feasible, such as in precast members. The crack width criteria are not too difficult to meet by proper attention to reinforcing details.

When the use of Grade 40 (280) reinforcing bars in the 1930s and 1940s was replaced by the use of Grade 60 (420) bars in the 1950s and 1960s, there were concerns about fatigue resistance of the higher-strength steel bars. Similar concerns were expressed about the use of ASTM A1035/A1035M (CS) steel bars when they were introduced. El-Hacha and Rizkalla (2002) and DeJong et al. (2006) conducted studies on the fatigue behavior of ASTM A1035/A1035M (CS) steel bars. Their results indicated that a fatigue life of 1×10^6 cycles was observed at a stress range of approximately 44,000 psi (310 MPa) for ASTM A1035/A1035M (CS) steel bars. The ASTM A1035/A1035M (CS) steel bars showed comparable fatigue resistance to Grade 60 (420) reinforcing bars even though their stress at service would be higher than that of Grade 60 (420) bars.

1.4—Reinforcing steel grades availability

The most widely used deformed reinforcing bars conform to ASTM A615/A615M, which include Grade 40 (280), Grade 60 (420), Grade 75 (520), and Grade 80 (550). The Grade 60 (420) reinforcement exhibits minimum yield strength of 60,000 psi (420 MPa) with a distinct yield plateau. ACI 318 permits use of reinforcing bars with a specified minimum yield strength f_y exceeding 60,000 psi (420 MPa), but f_y is limited to the lesser of 80,000 psi (550 MPa) or the stress corresponding to a strain of 0.0035, except as follows. ACI 318 limits the specified minimum yield strength for deformed bars used as shear reinforcement to 60,000 psi (420 MPa). For deformed bars used as confinement reinforcement (ties or spirals) in compression members, ACI 318 permits the use of specified minimum yield strength of up to 100,000 psi (690 MPa).

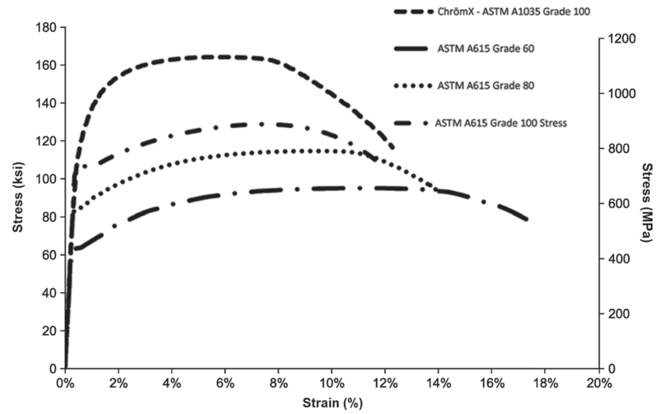

Fig. 1.5—Comparison of typical stress-strain curves for ASTM A615/A615M and ASTM A1035/A1035M (CS) reinforcing bars.

1.5—Introduction of ASTM A1035/A1035M Type CS Grade 100

The introduction of higher-strength steel reinforcing bars with a specified minimum yield strength, $f_y = 100$, the designers might be able to reduce the total cross-sectional area of required reinforcement. The reduced area of reinforcement could result in fewer bars and reduce reinforcement congestion often encountered in mat foundations, shear walls, beam-column joints, and many precast concrete elements. The reduction in reinforcement congestion facilitates concrete placement and consolidation.

The ASTM A1035/A1035M (CS) bar Grade 100 (690) (Fig. 1.5) exhibits a linear stress-strain relationship up to a proportional limit ranging from 60,000 to 80,000 psi (420 to 550 MPa), without a well-defined yield plateau. Refer to Appendix C of this guide for a discussion on how the lack of a well-defined yield plateau affects the flexural behavior of beams. Actual yield strength, determined by the 0.2 percent offset method, typically exceeds 115,000 psi (790 MPa) for Grade 100 (690) bars. The tensile strength typically exceeds 155,000 psi (1070 MPa) for Grade 100 (690) bars. The corresponding strain at the peak of the stress-strain curve ranges from 0.04 to 0.06. Refer to Chapter 3 of this guide for more details on the material characteristics of ASTM A1035/A1035M (CS) reinforcing bars.

CHAPTER 2—NOTATION AND DEFINITIONS

2.1—Notation

A_s = area of nonprestressed longitudinal tension reinforcement, in.^{2} (mm^2)

A_s' = area of nonprestressed longitudinal compression reinforcement, in.^{2} (mm^2)

A_{tr} = total cross-sectional area of all transverse reinforcement that is within the spacing s and crosses the potential plane of splitting through the reinforcement being developed or lap spliced, in.^{2} (mm^2); refer to Eq. (10.2b)

a = depth of equivalent rectangular stress block as defined in ACI 318, in. (mm)

b = width of compression face of member, in. (mm)
b_w = web width, in. (mm)
C = force in the compression zone of a beam, lbf (N)
c = distance from extreme compression fiber to neutral axis, in. (mm)
c_b = $c_{min} + 0.5d_b$, in. (mm); refer to Eq. (10.2a)
c_{bb} = clear cover of reinforcement being developed or lap spliced, measured to tension face of member, in. (mm)
c_c = clear cover to tension steel, in. (mm)
c_{max} = maximum value of c_s or c_{bb}, in. (mm)
c_{min} = minimum value of c_s or c_{bb}, in. (mm)
c_s = minimum value of c_{si} + 0.25 in. (6.35 mm) or c_{so}, in. (mm); c_{si} may be used instead of c_{si} + 0.25 in. (6.35 mm)
c_{si} = one-half of average clear spacing between bars or lap splices in a single layer, in. (mm)
c_{so} = clear cover of reinforcement being developed or lap spliced, measured to side face of member, in. (mm)
d = distance from extreme compression fiber to centroid of longitudinal tension reinforcement, in. (mm)
d' = distance from extreme compression fiber to centroid of longitudinal compression reinforcement, in. (mm)
d_b = bar diameter, in. (mm)
d_t = distance from extreme compression fiber to centroid of extreme layer of longitudinal tension reinforcement, in. (mm)
E_c = modulus of elasticity of concrete, psi (MPa)
E_s = modulus of elasticity of reinforcement, psi (MPa)
f_c' = specified compressive strength of concrete, psi (MPa)
f_s = calculated tensile stress in reinforcement, psi (MPa)
f_u = specified ultimate strength of reinforcement, psi (MPa)
f_y = specified minimum yield strength of reinforcement, psi (MPa)
f_{yt} = specified minimum yield strength of transverse reinforcement, psi (MPa)
h = overall height or thickness of member, in. (mm)
h_f = flange thickness of T-beam, in. (mm)
I_{cr} = moment of inertia of cracked section transformed to concrete, in.^{4} (mm^4)
I_e = effective moment of inertia for computation of deflection, in.^{4} (mm^4)
I_g = moment of inertia of gross concrete section about centroidal axis, neglecting reinforcement, in.^{4} (mm^4)
K_{tr} = transverse reinforcement index; refer to Eq. (10.2b)
ℓ_d = development length (also splice length), in. (mm)
M = applied moment at critical section, in.-lbf (N·mm)
M_a = maximum moment in member due to service loads at stage deflection is computed, in.-lbf (N·mm)
M_{cr} = cracking moment, in.-lbf (N·mm)
M_n = nominal flexural strength at section, in.-lbf (N·mm)
n = number of bars being developed or lap spliced along plane of splitting; refer to Eq. (10.2b)
P_n = nominal axial strength of cross section, lbf (N)
R_r = relative rib area of the reinforcement = ratio of projected rib area normal to bar axis to the product of the nominal bar diameter and the center-to-center rib spacing, may be taken conservatively as 0.07 for design
s = spacing of transverse (shear) reinforcement, in. (mm)
T = force in tension reinforcement of a beam, lbf (N)
T_s = additional bond strength provided by the transverse steel, lbf (N)
t_d = bar diameter factor = $0.78d_b + 0.22$ in. (mm); refer to Eq. (10.2d)
t_r = term representing the effect of relative rib area on T_s = $9.6R_r + 0.28 \leq 1.72$; refer to Eq. (10.2c)
V = applied shear at critical section, lbf (N)
V_c = nominal shear strength provided by concrete, lbf (N)
w_u = factored load per unit length of beam or one-way slab, lb/ft^3 (kg/m^3)
α = reinforcement location factor
β = ratio of the distance from the neutral axis to the extreme tension face to the distance from the neutral axis to the center of the tension reinforcement factor relating depth of equivalent rectangular compressive stress block to neutral axis depth, as defined in ACI 318
β_c = coating factor; refer to Eq. (10.2a)
Δ = deflection, in. (mm)
ε_s = strain in reinforcement
ε_t = net tensile strain in extreme layer of longitudinal tension steel at nominal strength, excluding strains due to creep, shrinkage, and temperature
ϕ = strength reduction factor
λ = lightweight aggregate concrete factor
ρ_b = bias factor = ratio of the average actual value to the specified minimum value of a property being analyzed
ρ = reinforcement ratio = A_s/bd
ρ_b = reinforcement ratio producing balanced strain conditions as defined in ACI 318
ρ_s = volumetric spiral reinforcement ratio
ψ = curvature
ω = factor reflecting benefit of large cover/spacing perpendicular to controlling cover/spacing = $0.1(c_{max}/c_{min}) + 0.9 \leq 1.25$; refer to Eq. (10.2a)

2.2—Definitions

ACI provides a comprehensive list of definitions through an online resource, ACI Concrete Terminology. Definitions provided herein complement that resource.

0.2% offset method—method for determining a yield strength value for a material that does not exhibit a distinct yield plateau. The yield strength is the stress on the engineering stress-strain curve at its intersection with a line having a slope equal to the initial modulus of elasticity and offset from the linear elastic portion of the engineering stress-strain curve by a strain of 0.2%.

bias factor—ratio of the average actual value to the specified minimum value of a property being considered in risk analysis.

coefficient of variation (COV)—the standard deviation divided by the mean value of a variable.

cumulative distribution function (CDF)—a function or graph describing the probability distribution of a real-valued random variable taking on a value less than or equal to a particular value.

Seismic Design Category (SDC)—classification assigned to a structure based on its occupancy category, and the severity of the design earthquake ground motion. The category assignment can range from A to F.

CHAPTER 3—MATERIAL PROPERTIES

3.1—Introduction

This guide addresses high-strength deformed reinforcing bars of Type CS as defined by ASTM A1035/A1035M, "Standard Specification for Deformed and Plain, Low-Carbon, Chromium, Steel Bars for Concrete Reinforcement." As with other specifications for steel reinforcing bars, this standard includes requirements for nominal weights and dimensions, tensile properties, chemical composition, and deformations. This chapter reviews properties of primary interest to the structural designer who may specify the use of ASTM A1035/A1035M bars.

Since the publication of ACI ITG-6R, additional Grade 100 (690) reinforcing steels were introduced to the market, including ASTM A1035/A1035M Types CM and CL and ASTM A615/A615M Grade 100 (690). The mechanical properties and stress-strain characteristics of these steels may impact the applicability of this design guide. These bars will be addressed in future editions when research on concrete members reinforced with these bar types become available and the impact of the mechanical properties have been documented.

3.2—Weights, dimensions, and deformations

ASTM A1035/A1035M (CS) specifies nominal weights (mass), areas, and dimensions. Deformation requirements are the same as those specified by ASTM A615/A615M for carbon-steel bars and ASTM A706/A706M for low-alloy steel bars. Consequently, this section does not discuss bar deformations and related aspects.

3.3—Specified tensile properties

Tensile properties are paramount for structural design. Tables 3.3a and 3.3b contain a summary of the specified tensile properties for ASTM A1035/A1035M (CS) and other reinforcement (ACI 439.4R). Table 3.3a shows that the tensile strength properties for ASTM A1035/A1035M (CS) bars are significantly greater than those for ASTM A615/A615M Grade 60 (420), Grade 80 (550), and Grade 100 (690) bars. The requirements for elongation in 8 in. (200 mm) across the fracture, as shown in Table 3.3b, are comparable to those for ASTM A615/A615M Grade 80 (550) and Grade 100 (690) reinforcement, and in some cases lower than those for ASTM A615/A615M Grade 60 (420) reinforcement. As described in 1.3 of this guide, ASTM A1035/A1035M (CS) reinforcing steel does not exhibit a distinct yield plateau (Fig. 1.5). Consequently, ASTM A1035/A1035M (CS) specifies minimum yield strength according to the 0.2 percent offset method.

Table 3.3a—Specified tensile and yield strength

Bar type	Tensile strength, minimum, psi (MPa)	Yield strength*	
		Minimum, psi (MPa)	Maximum, psi (MPa)
ASTM A615/A615M Grade 60 (420)	90,000 (620)	60,000 (420)	—
ASTM A615/A615M Grade 80 (550)	105,000 (725)	80,000 (550)	—
ASTM A615/A615M Grade 100 (690)	115,000 (790)†	100,000 (690)	—
ASTM A706/A706M Grade 60 (520)	80,000‡ (550)‡	60,000 (420)	78,000 (540)
ASTM A706/A706M Grade 80 (520)	100,000‡ (690)‡	80,000 (550)	98,000 (675)
ASTM A1035/A1035M (CS, CM, and CL) Grade 100 (690)	150,000 (1030)	100,000 (690)	—

*Observed yield point for ASTM A615/A615M and ASTM A706/A706M bars, and yield strength according to 0.2 percent offset method for ASTM A1035/A1035M bars, which is applicable to ASTM A615/A615M and ASTM A706/A706M bars only when steel bar tested does not exhibit a well-defined yield point.

†Grade 100 (690) reinforcing bars have a ratio of specified tensile strength to specified yield strength of 1.15. Designers should be aware that there will, therefore, be a lower margin of safety and reduced warning of failure following yielding when Grade 100 (690) bars are used in structural members.

‡Tensile strength for ASTM A706/A706M bars should also be not less than 1.25 times measured yield strength.

Table 3.3b—Specified elongation in 8 in. (200 mm) across fracture

Bar type	Bar size no.			
	3, 4, 5, 6 (10, 13, 16, 19)	7, 8 (22, 25)	9, 10, 11 (29, 32, 36)	14, 18 (43, 57)
	Elongation in 8 in. (200 mm) across fracture, minimum, percent			
ASTM A615/A615M Grade 60 (420)	9	8	7	7
ASTM A615/A615M Grade 80 (550), 100 (690)	7	7	6	6
ASTM A706/A706M Grade 60 (420)	14	12	12	10
ASTM A706/A706M Grade 80 (550)	12	12	12	10
ASTM A1035/A1035M (CS, CM, and CL) Grade 100 (690)	7	7	7	6

3.4—Measured tensile properties

Understanding the differences in tensile behavior of higher strength steels is essential for the safe and serviceable design

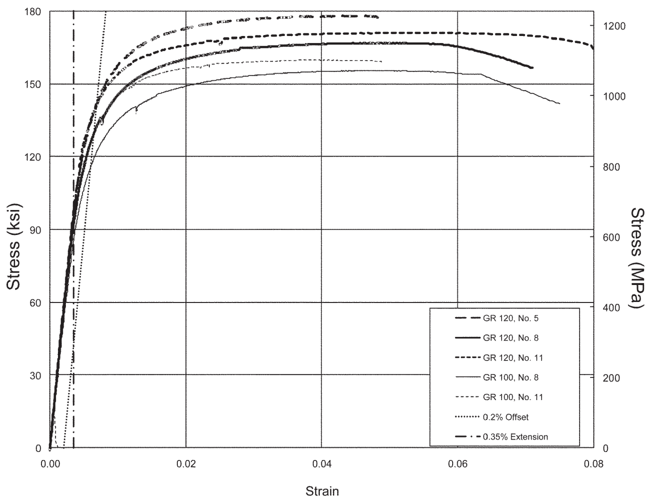

Fig. 3.4a—Actual stress-strain curves for ASTM A1035/A1035M Type CS reinforcing bars of different grades and sizes (WJE 2008).

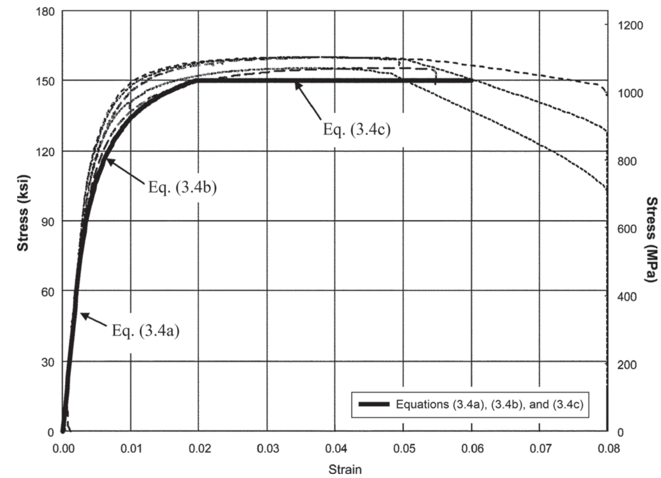

Fig. 3.4b—Equations (3.4a), (3.4b), and (3.4c) compared with actual stress-strain curves from samples of No. 8 (25) and No. 11 (36) bars of ASTM A1035/A1035M Grade 100 (690) Type CS.

of concrete members. Tensile properties are necessary in the probabilistic studies that establish the strength reduction factors used in reinforced concrete design.

Figure 1.5 shows comparisons of stress-strain curves recorded for samples of ASTM A1035/A1035M (CS) Grade 100 (690) bars to similar curves for samples of ASTM A615/A615M bars in Grades 60 (420) and 80 (550). ASTM A1035/A1035M (CS) bars have a greater tensile strength and lack a well-defined yield point and yield plateau. ASTM A1035/A1035M (CS) bars reach a proportional limit at a stress from 60,000 to 80,000 psi (420 to 550 MPa), which is similar to the yield stress of ASTM A615/A615M Grade 60 (420) and ASTM A706/A706M Grade 60 (420) bars (WJE 2008) and the strain at the peak tensile stress in the bar ranges from 0.04 to 0.06. By comparison, strains at peak tensile stress for ASTM A615/A615M Grade 60 (420) bars range from 0.07 to 0.10, and those of ASTM A706/A706M Grade 60 (420) bars range from 0.10 to 0.14.

For ASTM A1035/A1035M (CS) bars, the elongation in 8 in. (200 mm) across the fracture ranges from 0.08 to 0.13, whereas the elongation in 8 in. (200 mm) across the fracture for ASTM A615/A615M Grade 60 (420) and ASTM A706/A706M Grade 60 (420) bars range from 0.09 to 0.12 and 0.14 to 0.20, respectively. The modulus of elasticity value of 29,000,000 psi (200,000 MPa) as defined in ACI 318 is applicable to define the elastic modulus for ASTM A1035/A1035M (CS) Grade 100 (690) bars.

Figure 3.4a shows actual stress-strain curves recorded for samples of ASTM A1035/A1035M reinforcing bars (WJE 2008). Yield strength of ASTM A1035/A1035M (CS) bars, determined by the 0.2 percent offset method, exceeds 115,000 psi (790 MPa). The tensile strength for ASTM A1035/A1035M (CS) Grade 100 (690) bar exceeds 155,000 psi (1070 MPa).

An approximate lower bound for the stress-strain curves of Grade 100 (690) bars can be represented by the following three equations, as shown in Fig. 3.4b.

Equations (3.4a) through (3.4c) represent a lower bound to the stress-strain behavior of ASTM A1035/A1035M (CS) Grade 100 (690) bar. The equations are based on an assumed proportional limit of 70,000 psi (480 MPa) and an assumed tensile strength of 150,000 psi (1030 MPa), which is reached at the strain of 0.02, the upper limit for Eq. (3.4b).

$$f_s = 29{,}000\varepsilon_s \text{ (ksi) for } \varepsilon_s \leq 0.0024 \text{ (in.-lb)}$$
$$f_s = 200{,}000\varepsilon_s \text{ (MPa) for } \varepsilon_s \leq 0.0024 \text{ (SI)} \tag{3.4a}$$

$$f_s = 170 - \frac{0.4317}{\varepsilon_s + 0.0019} \text{ (ksi) for } 0.0024 < \varepsilon_s \leq 0.02 \text{ (in.-lb)}$$

$$f_s = 1170 - \frac{2.9670}{\varepsilon_s + 0.0019} \text{ (MPa) for } 0.0024 < \varepsilon_s \leq 0.02 \text{ (SI)} \tag{3.4b}$$

$$f_s = 150 \text{ (ksi) for } 0.02 < \varepsilon_s \leq 0.06 \text{ (in.-lb)}$$
$$f_s = 1040 \text{ (MPa) for } 0.02 < \varepsilon_s \leq 0.06 \text{ (SI)} \tag{3.4c}$$

Accordingly, an analysis was performed on the results of 137 mill tests on ASTM A1035/A1035M (CS) Grade 100 (690) bars produced after 2004 that were provided by a manufacturer. Based on the analysis of the mill test data, it was shown that Eq. (3.4b) and (3.4c) would provide a lower tolerance limit on actual stress corresponding to a strain of 0.0035, actual yield strength, and actual tensile strength for ASTM A1035/A1035M (CS) Grade 100 (690) bars such that at least 95 percent of the data are greater than the corresponding values calculated by these equations with a confidence level of 90 percent.

Cumulative distribution functions (CDFs) for the yield strength by the 0.2 percent offset method and the tensile strength were developed from the ASTM A1035/A1035M

Table 3.4—Statistical analysis of mill test data for ASTM A1035/A1035M (CS) Grade 100 (690) steel reinforcing bars*

Bar designation	No. of samples	0.2% offset yield strength			Tensile strength	
		Mean, ksi (MPa)	Bias factor λ_b	COV	Mean, ksi (MPa)	COV
No. 3 (10)	10	139.8 (964)		0.044	174.3 (1202)	0.046
No. 4 (13)	20	129.9 (895)		0.026	162.1 (1118)	0.027
No. 5 (16)	28	130.9 (903)		0.036	164.7 (1135)	0.035
No. 6 (19)	16	129.9 (895)		0.067	164.1 (1132)	0.045
No. 7 (22)	15	124.2 (857)		0.061	162.5 (1120)	0.028
No. 8 (25)	9	128.4 (886)		0.032	161.6 (1114)	0.019
No. 9 (29)	8	127.1 (877)		0.029	161.7 (1115)	0.039
No. 10 (32)	3	133.7 (922)		0.050	169.5 (1169)	0.051
No. 11 (36)	28	132.8 (916)		0.040	168.5 (1162)	0.031
All sizes	137	130.6 (901)		0.051	165.2 (1139)	0.040
Lower tail of data for all sizes	—	—	1.159	0.043	—	—
Lower tail of data for ASTM A615/A615M Grade 60 (420)*	—	—	1.145	0.055	—	—

*Statistical determination used for establishing strength reduction factors ϕ as reported by Nowak and Szerszen (2003).

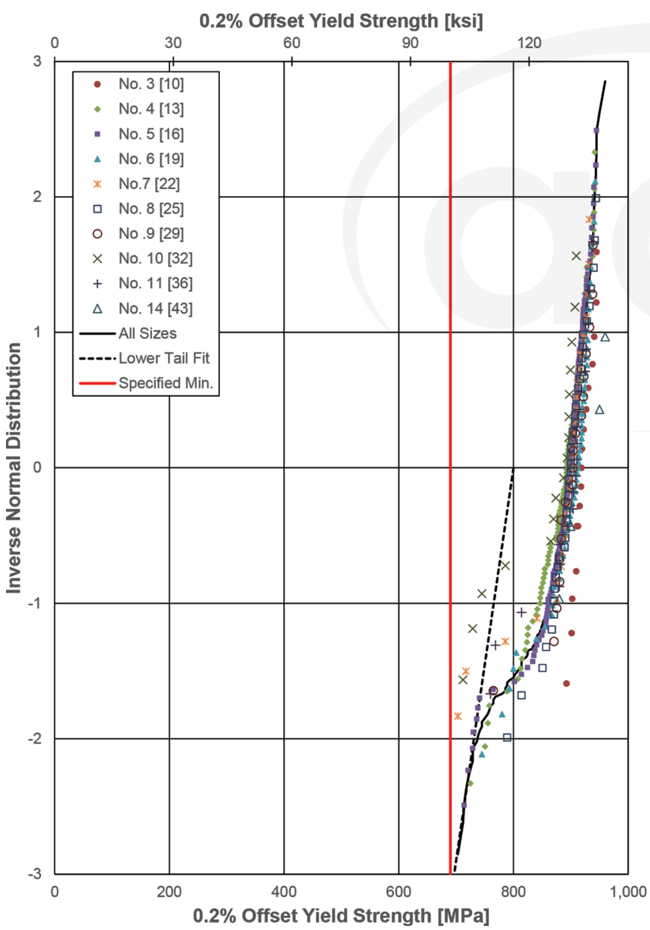

Fig. 3.4c—Cumulative distribution function for yield strength of ASTM A1035/A1035M (CS) Grade 100 (690) bars by 0.2 percent offset method. (Note: 1 MPa = 145 psi.)

(CS) Grade 100 (690) bar mill test data. The results are summarized in Table 3.4, Fig. 3.4c, and Fig. 3.4d. The summarized statistical data include the bias factor λ_b and the coefficient of variation (COV).

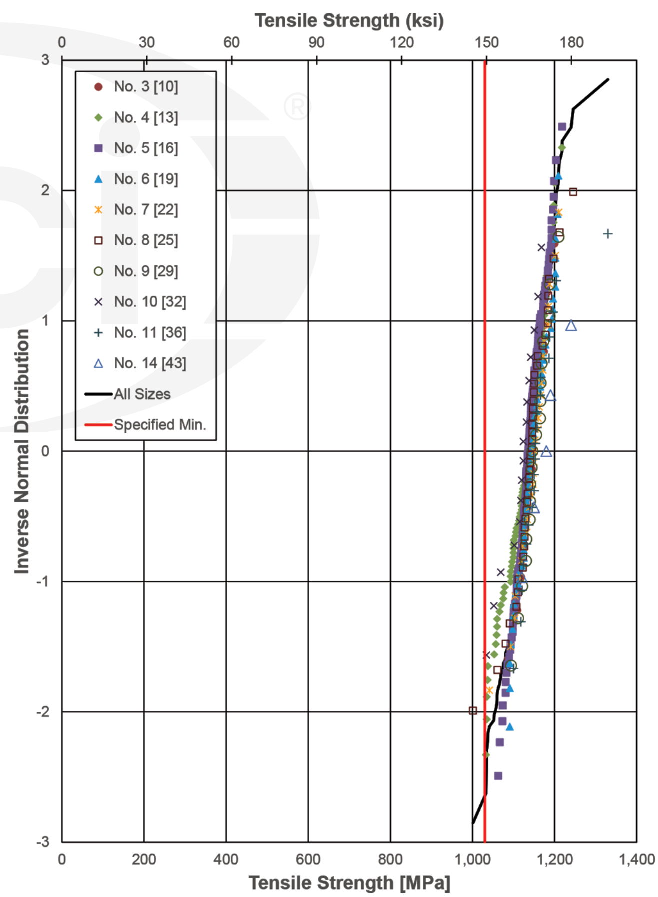

Fig. 3.4d—Cumulative distribution function for tensile strength of ASTM A1035/A1035M (CS) Grade 100 (690) bars. (Note: 1 MPa = 145 psi.)

Nowak and Szerszen (2003) reported the comparable statistical analysis for yield strength of ASTM A615/A615M Grade 60 (420) bars as used for code-calibration reliability analysis. They recommended using a bias factor λ_b of 1.145

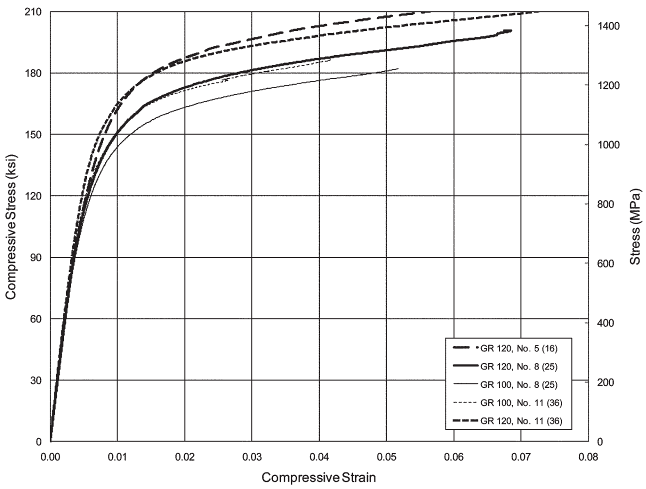

Fig. 3.5—Actual stress-strain curves in compression for ASTM A1035/A1035M (CS) reinforcing bars (WJE 2008).

and a COV of 0.05, based on the lower tail of distribution of the data used for their analysis, in the development of strength reduction factors for ACI 318. The summarized statistical data in Table 3.4 include the bias factor λ_b of 1.159 and COV of 0.043 for ASTM A1035/A1035M (CS) Grade 100 (690) reinforcement, based on a curve fit to the lower tail of the pooled data. The lower tail fit curve, shown in Fig. 3.4c, was determined in the same manner as used by Nowak and Szerszen (2003) for ASTM. The values of the bias factors for ASTM A1035/A1035M (CS) Grade 100 (690) bar is larger than 1.145 established for ASTM A615/A615M Grade 60 (420) bars, whereas the value for COV is less than the recommended value of 0.05 for ASTM A615/A615M Grade 60 (420) bars. Use of the same strength reduction factors with ASTM A1035/A1035M (CS) Grade 100 (690) bars as used for ASTM A615/A615M Grade 60 (420) bars is therefore recommended.

3.5—Actual compressive properties

WJE (2008) studied the stress-strain behavior in compression of representative samples of ASTM A1035/A1035M (CS) bars in size No. 5 (16), 8 (25), and 11 (36) in both Grades 100 (690) and 120 (830). The typical length of a compression test specimen was twice its nominal diameter. Figure 3.5 shows recorded compressive stress-strain curves. The proportional limit ranges from 60,000 to 80,000 psi (420 to 550 MPa) for both grades. Yield strength in compression by the 0.2 percent offset method is approximately 130,000 psi (900 MPa) for Grade 100 (690) bars and ranges from 135,000 to 140,000 psi (930 to 970 MPa) for Grade 120 (830) bars. The initial slopes of the stress-strain curves in compression are consistent with 29,000,000 psi (200,000 MPa) defined by ACI 318 for the modulus of elasticity.

3.6—Chemical composition

Table 3.6 shows the comparison of the specified chemical composition for ASTM A1035/A1035M (CS) low-carbon, chromium-steel bars with that of ASTM A615/A615M carbon-steel bars and ASTM A706/A706M low-alloy steel bars. The specified chemical content of ASTM A1035/A1035M (CS) bars differs from the other two primarily in that there is a required amount of chromium (8.0 to 10.9 percent). In addition, carbon content is limited to 0.15 percent and a maximum limit is specified for nitrogen (0.05 percent). Other than the chromium requirement, the chemical composition specified by ASTM A1035/A1035M (CS) is similar to ASTM A706/A706M. ASTM A1035/A1035M (CS) bars are not readily weldable because of metallurgical reasons, as discussed in 10.4 of this guide.

Table 3.6—Specified chemical composition by heat analysis

	Bar type		
	ASTM A1035/ A1035M (CS)	ASTM A615/ A615M	ASTM A706/ A706M
Element	Maximum content, percent		
Carbon	0.15	†	0.30
Chromium	8.0 to 10.9*	—	—‡
Manganese	1.50	†	1.50‡
Nitrogen	0.05	—	—
Phosphorus	0.035	0.06	0.035
Sulfur	0.045	†	0.045
Silicon	0.50	—	0.50

*Chromium content should also be a minimum of 8.0 percent and maximum of 10.9 percent.

†Content should be reported but no limit is established.

‡Regulated by maximum carbon equivalent of 0.55 percent according to the following formula: CE = %C + %Mn/6 + %Cu/40 + %Ni/20 + %Cr/10 – %Mo/50 – %V/10.

CHAPTER 4—BEAMS

4.1—Introduction

Although the previous chapters address Grade 100 (690) and Grade 120 (830) of ASTM A1035/A1035M (CS) bars, the procedures in 4.2 through 4.11 of this guide apply only to the use of ASTM A1035/A1035M (CS) Grade 100 (690) bars. Research data and experience are insufficient to allow proper evaluations of the procedures for Grade 120 (830) bars.

With the simplified strength design procedures in 4.2 and 4.3, flexural strength design using ASTM A1035/A1035M (CS) Grade 100 (690) bars is similar to that using ASTM A615/A615M Grade 60 (420) bars, whether by manual calculations or by using design software. Only certain inputs and limitations are modified.

4.2—Flexural strength

As discussed in 3.4 of this guide, Eq. (3.4a) through (3.4c) represent a lower bound to the stress-strain behavior of ASTM A1035/A1035M (CS) Grade 100 (690) bar.

For a given reinforced concrete section, the designer may perform a nonlinear flexural analysis by using the stress-strain relationship defined by Eq. (3.4a) and (3.4b) and satisfying the requirements of force equilibrium and strain compatibility. However, the analysis procedure requires an iterative process of trial and error. If such a nonlinear analysis is used for flexural design, the tensile stress in the reinforcement under service load should be determined to evaluate satisfaction of serviceability criteria such as crack control or control of member deflections. Furthermore, evaluation procedures for other strength criteria at ultimate limit state, such as shear strength, should consider nonlinear behavior of the reinforcement. Refer to Appendix B of this guide for detailed discussions of a nonlinear flexural analysis approach and for design examples comparing the nonlinear flexural analysis approach with elastic-plastic approach.

A simplified design method using ASTM A1035/A1035M (CS) bars based on the study by Mast et al. (2008) uses an idealized elastic-plastic stress-strain curve, similar to that used for ASTM A615/A615M Grades 60 (420) and 75 (520) bars (Fig. 4.2). Limitations are set on the range of applicability for this method to protect against harmful effects resulting from the actual steel stress being higher than assumed, and effects from larger steel strains at service load. Refer to 4.3 of this guide for a discussion of these limitations.

The idealized elastic-plastic stress-strain curve shown in Fig. 4.2 has an elastic portion with a modulus E_s of 29,000,000 psi (200,000 MPa) (the same as for ASTM A615/A615M bar), and a perfectly plastic behavior after reaching f_y equal to 100,000 psi (690 MPa). These parameters allow computation of flexural strength with f_y set at 100,000 psi (690 MPa) for a beam that is tension-controlled. Appendix A illustrates a series of design examples using the idealized elastic-plastic stress-strain curve of ASTM A1035/A1035M (CS) Grade 100 (690) reinforcement.

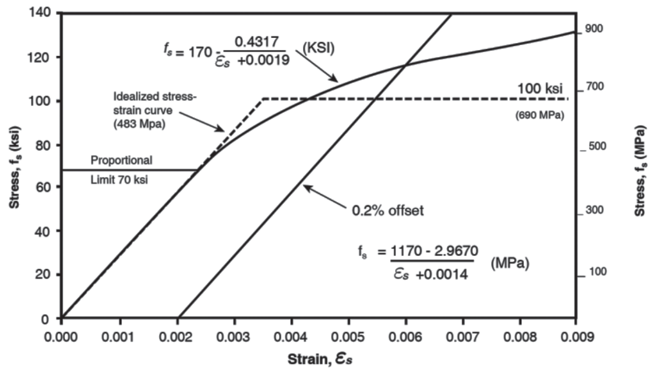

Fig. 4.2—Approximated nonlinear stress-strain relationship of ASTM A1035/A1035M (CS) Grade 100 (690) steel and idealized bilinear elastic-plastic stress-strain relationship for simplified design. (Note: 1 ksi = 6.9 MPa.)

4.3—Tension- and compression-controlled limits

4.3.1 *Historical*—It has long been accepted that reinforced concrete flexural members should behave elastically at service load but have the capability to deform inelastically before reaching maximum capacity. This is accomplished by limiting the reinforcement ratio so that the reinforcement yields before the concrete crushes. Beginning with ACI 318-63, flexural member reinforcement ratio was limited to 75 percent of the balanced reinforcement ratio ρ_b. By 1995, this criterion had been used for over 30 years, and flexural member behavior was judged to be satisfactory.

4.3.2 *Tension- and compression-controlled strain limits*—Beginning with ACI 318-95, the tension-controlled criterion based on tensile strain in the reinforcement was selected to provide behavior similar to that experienced under the $0.75\rho_b$ criterion. Nonlinear analyses of the behavior between service load and nominal strength were made for the old maximum reinforcement ratio and the new tension-controlled strain criteria. The ratio of deformation at nominal strength to that at service load for steel strain ε_s, curvature ψ, and deflection Δ were calculated and compared. Setting the tension-controlled strain limit at 0.005 for Grade 60 (420) steel provided more deformation than the $\rho = 0.75\rho_b$ limit. For Grade 60 (420) steel, the compression-controlled limit was set at 0.002, which is the strain (rounded) at the balanced condition (Mast 1992).

4.3.3 *Strain limits for sections with ASTM A1035/A1035M (CS) bar*—The definition of tension-controlled sections in ACI 318 is based on ASTM A615/A615M Grades 60 (420) and 75 (520) bars. These were the reinforcing bars available when ACI 318 strain limits were originally developed. For ASTM A1035/A1035M (CS) bar, the process to determine ACI 318 strain limits was repeated using equations similar to Eq. (3.4a) and (3.4b) of this guide and the idealized elastic-plastic stress-strain relationship for ASTM A1035/A1035M (CS) bar. It was found that a tension-controlled strain limit of 0.0066 (corresponding to c/d = 5/16) produced behavior similar to members designed by ACI 318 with Grades 60 (420) and 75 (520) bars and a strain limit of 0.005 (Mast et al. 2008). The strain limit for compression-controlled sections is discussed in the next section.

4.3.4 *Simplified design strain limits*—The tension- and compression-controlled strain limits need to be modified for simplified design using the idealized elastic-plastic stress-strain relationship described in 4.2. The tension-controlled strain limit should be adjusted to 0.009 (c/d = 0.25) to compensate for the fact that the actual steel stress at nominal strength is higher than the assumed stress of 100,000 psi (690 MPa). This adjustment ensures ductility and deformability are comparable to designs based on ACI 318 using ASTM A615/A615M Grade 60 (420) bars. Table 4.3.4 and Fig. 4.3.4 show that the simplified method using ε_t = 0.009 and f_y = 100,000 psi (690 MPa) results in a value of nominal flexural strength 18 percent less than the same section designed using ε_t = 0.0066 and the corresponding f_s = 119,000 psi (820 MPa) according to Eq. (3.4b). Using the simplified method avoids having to perform an analysis using the nonlinear stress-strain relationship of the steel.

ACI 318 defines the compression-controlled strain limit as the net tensile strain at balanced strain conditions. For f_y of 100,000 psi (690 MPa) and E_s = 29,000,000 psi (200,000 MPa), this is a strain of 0.00345. It is simpler and conservative to round the compression-controlled strain limit for the A1035/A1035M (CS) reinforcement to 0.004.

Using the limits defined previously for the simplified method, the tension-controlled limit occurs at a c/d = 0.25 (instead of 0.375 for Grade 60 [420] steel), and the compression-controlled limit is at a c/d = 3/7, or 0.43 (instead of 0.600 for Grade 60 [420] steel).

Table 4.3.4—Comparison of design methods using ASTM A1035/A1035M (CS) Grade 100 (690) steel

	Using Eq. (3.4a) and (3.4b)	Simplified method
Tension-controlled strain limit	0.0066	0.009
Steel tensile stress f_s, ksi (MPa)	119 (820)	100 (690)
Neutral axis depth c, in. (mm)	$0.3125d$	$0.25d$
Stress block depth $a = \beta_1 c$, in. (mm)	$0.3125\beta_1 d$	$0.25\beta_1 d$
Compression force C, kip (N)	$0.85f_c'ab$	$0.85f_c'ab$
Steel area $A_s = C/f_s$, in.^{2} (mm^2)	$0.85(f_c'/f_s)(0.3125\beta_1\delta)b$	$0.85(f_c'/f_s)(0.25\beta_1\delta)b$
Tension-controlled reinforcement ratio $\rho_t = A_s/bd$	$0.002232f_c'\beta_1$ $(0.0003239f_c'\beta_1)$	$0.002125f_c'\beta_1$ $(0.0003079f_c'\beta_1)$
$T = C = A_s f_s = \rho_t bdf_s$, kip (N)	$0.2656f_c'\beta_1 bd$	$0.2125f_c'\beta_1 bd$
Lever arm $= d - a/2$, in. (mm)	$d(1 - 0.156\beta_1)$	$d(1 - 0.125\beta_1)$
M_n for $f_c' = 5$ ksi (34.5 MPa); $\beta_1 = 0.8$, in.-kip (mm-N)	$0.186\,f_c'bd^2$	$0.153\,f_c'bd^2$

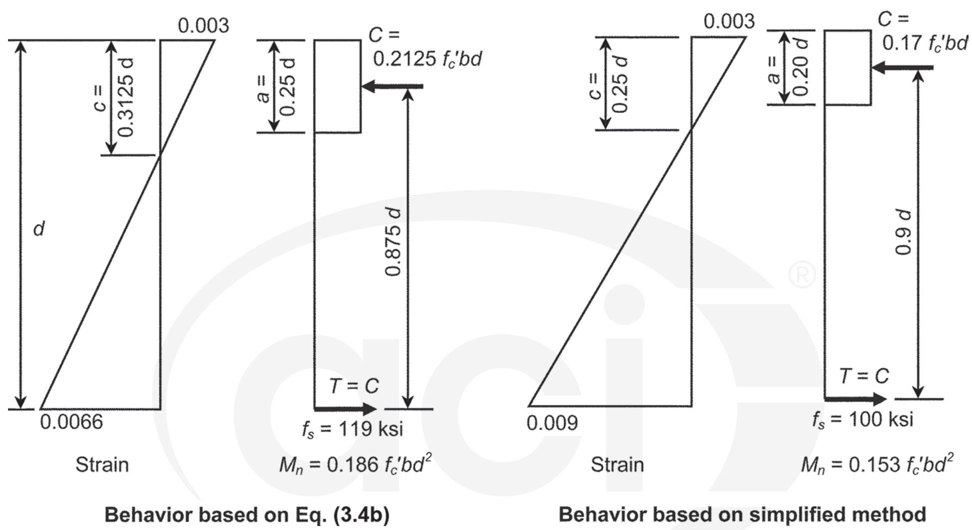

Fig. 4.3.4—Tension-controlled strain limits with $f_c' = 5$ ksi and $\beta_1 = 0.8$. (Note: 1 ksi = 6.9 MPa.)

4.4—Strength reduction factor ϕ

With the aforementioned modified tension- and compression-controlled limits for use with the simplified method, it is necessary to revise the equation for the transition in ϕ between these strain limits. The revised equation is given as follows

$$0.65 \leq (\phi = 0.45 + 50\varepsilon_t) \leq 0.9 \quad (4.4)$$

Note that for $\varepsilon_t \geq 0.009$ (tension-controlled section), $\phi = 0.9$, and for $\varepsilon_t \leq 0.004$ (compression-controlled section), $\phi = 0.65$. Figure 4.4 illustrates this relationship.

If the designer uses nonlinear analysis for flexural design, Eq. (4.4) should be modified for the value of ϕ in the transition between the tension- and compression-controlled strain limits. The modified equation for ϕ is given in Appendix B.

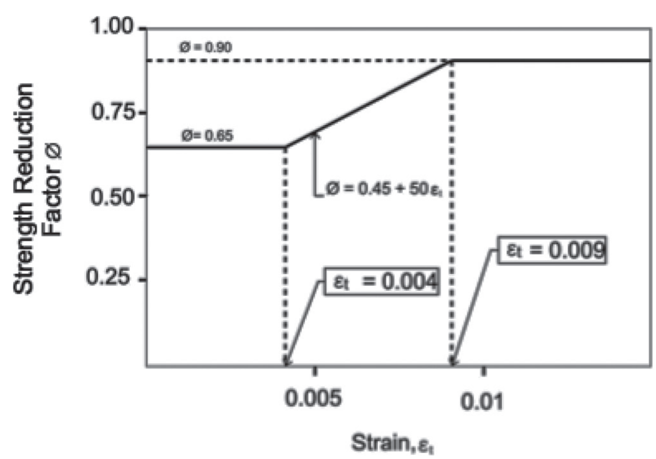

Fig. 4.4—Relationship between strength reduction factor ϕ and strain limits for use only with simplified design based on idealized elastic-plastic stress-strain relationship for A1035/A1035M (CS) Grade 100 (690) steel reinforcement.

4.5—Stress in steel due to flexure

For an efficient use of flexural reinforcement, it is advisable to proportion all flexural members as tension-controlled members. Figure 4.5 shows steel strain and stress diagrams for a member at the tension-controlled limit. The steel stress diagram shows both the idealized elastic-plastic and nonlinear stress distributions. The stress diagrams below the neutral axis are the same as in Fig. 4.2. There are two regions in the steel stress diagram where the idealized elastic-plastic design stress may exceed the nonlinear design stress given

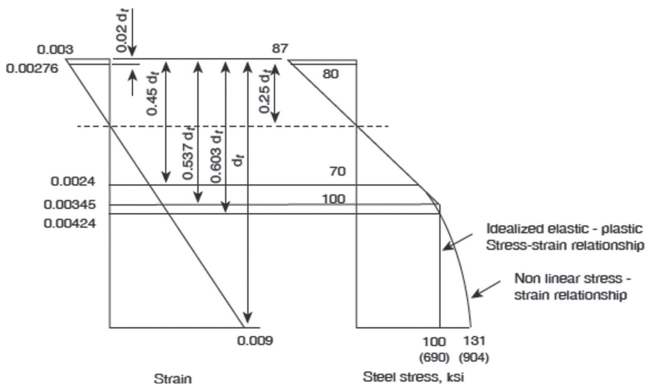

Fig. 4.5—Strain and steel stress at tension-controlled limits. (Note: 1 ksi = 6.9 MPa.)

by Eq. (3.4a) and (3.4b) of this guide. On the tension side, this occurs between $0.45d_t$ and $0.603d_t$ below the extreme compression fiber. Ordinarily, beams do not have primary tension steel at this location, but if there is, underestimating steel stress near the tension face when using the idealized elastic-plastic stress-strain model offsets strength overestimation between $0.45d_t$ and $0.603d_t$. On the compression side, in relation to a maximum usable compression steel stress of 80,000 psi (550 MPa), the steel stress is overestimated for steel with strains from 0.00276 to 0.003. For a section at the tension-controlled limit, this occurs only for steel located within $0.02d_t$ measured from the compression face. The influence of this overestimation is typically insignificant.

4.6—Compression stress limit

ACI 318 limits the maximum usable compression strain in concrete to 0.003. For this reason, ACI 318 limits the compression steel stress to 80,000 psi (550 MPa), and this limit should also apply for ASTM A1035/A1035M (CS) bars in compression.

Although most design software accepts only a single value of f_y for tension and compression, using f_y = 100,000 psi (690 MPa) as input for design software should not be a problem. For a maximum concrete compressive strain of 0.003, the steel stress in compression does not exceed 80,000 psi (550 MPa) unless the depth from the extreme compression fiber to the compression steel, d', is less than $0.08c$, where c is the depth of the compression zone. This is unlikely to happen and can be easily checked.

4.7—Moment redistribution

The moment redistribution technique in ACI 318 allows moments in continuous members to be increased or decreased by prescribed amounts relative to the magnitudes determined from the elastic theory, where sufficient rotational capacity is provided at plastic hinge locations. Corresponding adjustments are made to the moment demand envelopes in the span. As of the writing of this guide, no data are available to judge if moment redistribution is applicable to members with ASTM A1035/A1035M (CS) bars. In particular, the simplified strength design procedures in 4.2 and 4.3 of this guide may not provide adequate estimates of the actual loading conditions at which flexural hinges form. Furthermore, the inelastic rotation capacity of high-moment regions reinforced with ASTM A1035/A1035M (CS) bars has not been evaluated. Therefore, do not use moment redistribution for members containing ASTM A1035/A1035M (CS) bars until further data are available.

When determining the design forces in members with ASTM A1035/A1035M (CS) bars or other reinforcement constructed integrally with other members or elements that contain ASTM A1035/A1035M (CS) longitudinal bars, the actual forces transferred from the member containing ASTM A1035/A1035M (CS) bars may be difficult to determine at the strength limit state because ASTM A1035/A1035M (CS) bars lack a well-defined yield plateau. This situation could occur with beam-column joints of frame structures, the interface between slabs and spandrel beams, two-way slab construction, or continuous members where ASTM A1035/A1035M (CS) bars and other reinforcement types are used in different regions.

4.8—Deflection

For one-way members, ACI 318 provides two methods for controlling deflections at the service load level:

(1) Compute the expected deflections and compare against appropriate limits

(2) Implicitly control deflection through minimum thicknesses

When deflections are not computed, ACI 318 provides the minimum height or thickness h of one-way concrete members to implicitly control deflections. The minimum thickness values apply only to members not supporting or attached to construction likely to be damaged by deflections, and are given as functions of the span length, member type, and support condition. The table is based on normalweight concrete and ASTM A615/A615M Grade 60 (420) bar. For f_y other than 60,000 psi (420 MPa), footnote (b) of the table permits modification of the tabulated values by the multiplier $(0.4 + f_y/100{,}000)$. This table was first presented in ACI 318-71 where accompanying commentary stated:

> The modification for yield strength in Footnote (b) is based on judgment, experience, and studies of the results of tests and of unpublished analyses. The simple expression given is approximate but should yield conservative results for the types of members considered in the table, for typical reinforcement ratios, and for values of f_y between 40 and 80 ksi.

The range of f_y between 40,000 and 80,000 psi (280 and 550 MPa) corresponded to the range typically available in 1971.

When using ASTM A1035/A1035M (CS) longitudinal bars in designs based on the method described in 4.2 and 4.3 of this guide, the strain in the reinforcement at the service condition is higher than that in comparable members designed with Grade 60 (420) reinforcement. Under service load condition, the steel stress is usually taken as 0.67 of f_y. Using the multiplier $(0.4 + f_y/100{,}000)$ as a member thickness modification factor results in similar maximum member curvature for members with different reinforcement strengths. The guide by Mast (2006) shows that for f_y = 100,000 psi (690 MPa), the deflection at service load is 1.4 times that for members with f_y = 60,000 psi (420 MPa). This confirms the applicability of the multiplier of (0.4 + f_y/100,000) = 1.4 on the minimum height or thickness h in ACI 318 minimum thickness recommendations of Chapter 8 when using ASTM A1035/A1035M (CS) bars with f_y = 100,000 psi (690 MPa). Desalegne and Lubell (2012) recommended that direct deflection calculations should be used for the design of concrete slabs reinforced with ASTM A1035/A1035M (CS), which is discussed in Chapter 6.

For direct deflection calculation, the ACI 318 method uses an effective moment of inertia proposed by Branson (1977) to account for variable cracks at different sections along the member length. Bischoff (2005), however, showed that using Branson's equation for I_e underestimates deflections, especially when the reinforcement ratio is less than 1 percent. Bischoff (2005) proposed an alternate formulation of I_e

$$I_e = \frac{I_{cr}}{1-\left(1-\dfrac{I_{cr}}{I_g}\right)\left(\dfrac{M_{cr}}{M_a}\right)^2} \leq I_g \tag{4.8}$$

Calculated deflections using Eq. (4.8) correlate well with deflections measured in laboratory tests for members with a range of reinforcement ratios (Bischoff and Scanlon 2007). Because slabs and beams using ASTM A1035/A1035M (CS) longitudinal bars may result in lightly reinforced members, Eq. (4.8) may be used in place of ACI 318-14 Eq. (24.2.3.5a). This method is suitable when the steel stress at the load condition checked for deflection is below the proportional limit.

When ASTM A1035/A1035M (CS) steel is used as longitudinal reinforcement, the designer should check that the member deflection limits are not exceeded for lightly reinforced members.

4.9—Crack control

4.9.1 *Historical*—In the early 1960s, the developments of ultimate strength design and ASTM A615/A615M Grade 60 (420) reinforcement caused concern about crack widths. These two changes in design practice resulted in steel stress in flexural members at the service load of approximately 36,000 psi (250 MPa), almost twice the 20,000 psi (140 MPa) allowed previously for ASTM A615/A615M Grade 40 (280) steel under working stress design. Numerous research papers led to the crack control requirements of Chapter 10 in ACI 318-71. These requirements remained mostly unchanged until 1999.

Researchers agree that strain in the tension steel is one of the primary variables for controlling crack width. Because the elastic relationship between strain and stress for steel is a constant for strains smaller than the proportional limit, most crack width formulas use f_s instead of ε_s. In addition, the interaction of bond to the reinforcement and tensile stress in the concrete surrounding the reinforcement also affect crack spacing and crack width. Numerous equations have been proposed for predicting crack spacings and widths including the Gergely-Lutz equation (Gergely and Lutz 1968), which was adopted by ACI 318-71.

4.9.2 *Crack widths*—There are three reasons for limiting crack width: 1) appearance; 2) control of reinforcement corrosion; and 3) water-tightness. Aesthetic considerations by owners and designers often demand tight control of crack width. For liquid-retaining structures and building envelopes, water-tightness is essential. Control of reinforcement corrosion is paramount to achieve durability.

4.9.3 *ACI 318-99 revisions*—While the z-factor determined from Eq. (10-2) of ACI 318-71 provided adequate reinforcement distribution for crack control of normal-size beams and one-way slabs with standard concrete clear covers not exceeding 1.5 in. (38 mm), the z-factor method based on the Gergely-Lutz (Gergely and Lutz 1968) equation produced unrealistic bar spacings when larger concrete covers were used. This led designers to either ignore the provisions when larger covers were used or simply use the code-minimum cover or a standard value—typically 2.5 in. (63 mm)—rather than the actual cover in the calculations. Additionally, some building inspectors continued the old practice of comparing measured crack widths in the field against the limiting crack widths according to the z-factor as reasons for rejecting members or projects, even though research showed that there was little if any correlation between corrosion and crack width for the range of crack widths being considered. Further, the limiting crack widths implied by the z-factor—0.016 in. (0.41 mm) for interior exposure and 0.013 in. (0.33 mm) for exterior exposure (refer to Commentary on Section 10.6.4 of ACI 318-71)—represent a difference of 0.003 in. (0.08 mm), which implies an unrealistic accuracy. To find a simpler, more direct way of defining bar spacing to control cracking than Eq. (10-2), ACI Committee 318 examined the work of Kaar and Mattock (1963), Gergely and Lutz (1968), Beeby (1979), and Frosch (1999). A simple bilinear relationship between concrete clear cover and bar spacing for a maximum crack width of 0.016 in. (0.41 mm) was adopted as Eq. (10-5) in ACI 318-99. Figure 4.9.3 illustrates this bilinear relationship and three other nonlinear relationships. In Fig. 4.9.3, f_s is the bar stress at service load, assumed as 60 percent of specified minimum yield strength, and is the ratio of the distance from neutral axis to the extreme tension face to the distance from neutral axis to the center of the tension reinforcement. The bilinear relationship (ACI 318-99 Eq. (10-5)) is independent of β and bar size, whereas the other nonlinear relationships depend on these variables. A modified version of Eq. (10-5) of ACI 318-99—valid for service load steel stresses ranging from 24,000 to 48,000 psi (170 to 330 MPa)—became Eq. (10-4) of ACI 318-05.

4.9.4 *Applicability with ASTM A1035/A1035M (CS) bars*—Figure 4.9.4 is similar to the bilinear equation shown in Fig. 4.9.3, but with f_s taken as 67,000 psi (460 MPa), which is two-thirds of the specified minimum yield strength f_y. ACI

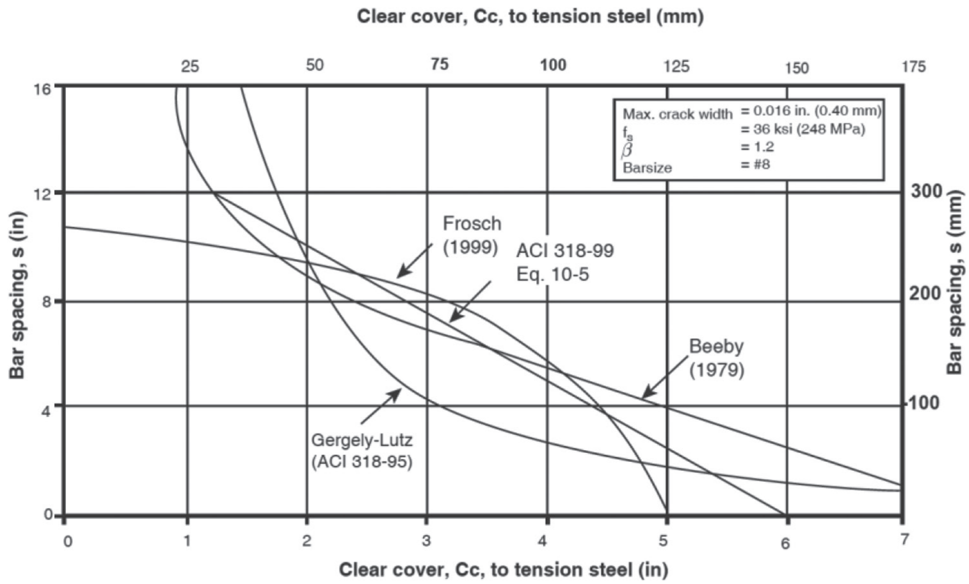

Fig. 4.9.3—Bar spacing versus clear cover for crack control. (Note: 1 in. = 25 mm; 1 ksi = 6.9 MPa.)

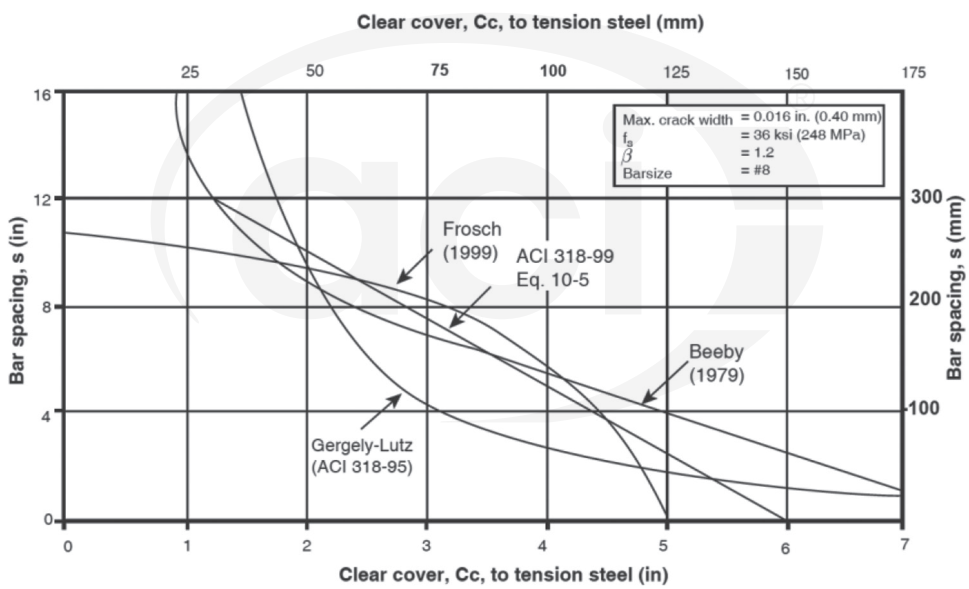

Fig 4.9.4—Bar spacing versus clear cover for crack control (f_s = 67 ksi). (Note: 1 in. = 25 mm; 1 ksi = 6.9 MPa.)

318 gives more conservative results compared with those shown in Fig. 4.9.3, but the results are restrictive in design choices. For example, the bar spacing s cannot exceed 6.5 in. (165 mm) at 1 in. (25 mm) cover c_c, and smaller bar spacing is required at greater cover to meet the 0.016 in. (0.41 mm) crack width criterion. Any combination of c_c and s above the bilinear curve leads to larger crack widths.

When crack control is critical, use of ACI 318 is appropriate. For members with minimal cover, such as slabs and walls, this is feasible when designing for f_y = 100,000 psi (690 MPa) and f_s = 67,000 psi (460 MPa). To provide crack control at a reasonable bar spacing for members with increased cover, it will be necessary to limit the steel stress at service load to less than 67,000 psi (460 MPa).

4.10—Minimum reinforcement

A minimum area of flexural reinforcement in accordance to ACI 318, As_{min}, shall be provided at every section where tension reinforcement is required by analysis with f_y = 100,000 psi (690 MPa). When the minimum reinforcing requirements for temperature and shrinkage of ACI 318 are applied, the minimum reinforcing ratio of 0.0014 will control.

4.11—Strength design for shear

Design for shear and torsion in reinforced concrete beams using ASTM A1035/A1035M (CS) bars as reinforcement should follow ACI 318, with a few exceptions noted as follows.

The shear failure of reinforced concrete beams without shear reinforcement is brittle and the shear stress at failure is influenced by several design parameters, including member depth, longitudinal reinforcement ratio, and concrete strength (Rajagopalan and Ferguson 1968; Collins and Kuchma 1999; Reineck et al. 2003; Lubell et al. 2004). Unsafe predictions of shear capacity can result from the use of ACI 318 for lightly reinforced beams that are generally with ρ less than 1 percent. Laboratory tests (Hassan et al. 2008; Desalegne and Lubell 2010) demonstrate that these trends also occur for members containing ASTM A1035/A1035M (CS) bars as longitudinal reinforcement; therefore, all lightly reinforced beams should contain minimum shear reinforcement.

For members designed according to the simplified method for flexural design in 4.2 of this guide, the shear capacity can be determined from ACI 318. Such members should include shear reinforcement that satisfies the spacing and quantity requirements of ACI 318. ACI 318 limits the design yield strength f_{yt} for shear reinforcement to 60,000 psi (420 MPa), but the value can be increased to 80,000 psi (550 MPa) for welded deformed wire reinforcement. Similarly, ACI 318 limits the design yield strength f_{yt} for torsional reinforcement to 60,000 psi (420 MPa). The limitation of 60,000 psi (420 MPa) has been specified since ACI 318-71, in part to provide a control on diagonal crack width at service load levels.

ACI 318-95 raised the f_{yt} limit to 80,000 psi (550 MPa) for welded deformed wire reinforcement. Full-scale beam tests (Griezic et al. 1994) indicated that inclined shear crack widths at service load levels were less for beams with welded deformed wire reinforcement cages designed with a yield strength of 75,000 psi (520 MPa) than for beams reinforced with deformed Grade 60 (420) stirrups. For lack of comparative data, the limitation of 60,000 psi (420 MPa) remains for torsional reinforcement even when using welded deformed wire reinforcement.

Tests of full-scale beams by Munikrishna (2008) and Sumpter et al. (2009) compared the behavior of beams reinforced with ASTM A1035/A1035M (CS) Grade 100 (690) bars as stirrups with 135-degree hooks and designed with f_{yt} of 80,000 psi (550 MPa) with the behavior of similar beams reinforced with deformed Grade 60 (420) stirrups. The results indicated that the behavior of the two groups of beams was similar at failure. At service load levels, inclined shear crack widths were larger, as expected, for beams reinforced with ASTM A1035/A1035M (CS) Grade 100 (690) stirrups designed with a yield strength of 80,000 psi (550 MPa). In all cases, the crack widths were less than the commonly accepted limit of 0.016 in. (0.41 mm).

A design yield strength f_{yt} of 80,000 psi (550 MPa) for ASTM A1035/A1035M (CS) Grade 100 (690) stirrups as shear reinforcement is appropriate if appearance and serviceability due to shear cracking is not a critical design consideration. Otherwise, f_{yt} should be limited to 60,000 psi (420 MPa). For lack of research data, f_{yt} should also be limited to 60,000 psi (420 MPa) for shear reinforcement designed for torsion.

CHAPTER 5—COLUMNS

5.1—Introduction

Column design using ASTM A1035/A1035M (CS) bars should follow ACI 318, with modifications made to the specified minimum yield strength, f_y, for the longitudinal reinforcement and f_{yt} for the lateral reinforcement.

5.2—Specified minimum yield strength for longitudinal reinforcement

For members under axial load, the maximum compressive stress of the longitudinal reinforcement should be limited to 80,000 psi (550 MPa) when computing the design axial strength ϕP_n using ACI 318.

The steel strain ε_s corresponding to this stress is approximately 0.0028, nearly equal to the maximum usable strain (0.003) for concrete in compression assumed by ACI 318.

For members subjected to axial load combined with moment, f_y should be 80,000 psi (550 MPa) for the longitudinal reinforcement in compression for the reason given previously, and 100,000 psi (690 MPa) for the longitudinal reinforcement in tension as in the case of flexural design discussed in Chapter 4 of this guide.

5.3—Specified minimum yield strength for transverse reinforcement

Transverse reinforcement in a column serves three functions:

(1) Lateral support to the longitudinal reinforcement

(2) Confinement to the core concrete when the load supported by the column approaches its axial strength

(3) Shear reinforcement when the column is subjected to shear

The transverse reinforcement is most commonly rectangular or circular ties and spirals.

ACI 318 provides minimum size and maximum spacing of ties in compression members, irrespective of the bar strength. These requirements apply for ASTM A1035/A1035M (CS) bars.

When a column is required to resist shear, provide ties for the required shear strength as discussed in 4.11 of this guide.

For columns reinforced with ASTM A1035/A1035M (CS) spirals, determine the volumetric spiral reinforcement ratio ρ_s of ACI 318 with the specified minimum yield strength f_{yt} as 100,000 psi (690 MPa). Spirals should be continuous and lap splices should not be used.

5.4—Slenderness effect

When slenderness effect is considered, design the column by using the moment magnification procedure of ACI 318. Use the magnified moment to design the column according to the procedures outlined in Chapter 4 of this guide.

CHAPTER 6—SLAB SYSTEMS

6.1—One-way slabs

Slab depth is typically determined by deflection control considerations including ACI 318 depth limits table for one-way slabs modified for f_y = 100,000 psi (690 MPa), or by the depth required to provide adequate shear strength without shear reinforcement. Design for flexure and deflection in one-way slabs, with or without joists, is based on the same principles as for beams. Thus, the recommendations in Chapter 4 of this guide apply. To control flexural cracking, the distribution of flexural reinforcement should conform to ACI 318, as discussed in 4.9 of this guide. In addition, shrinkage and temperature reinforcement should be provided to meet the requirements of ACI 318.

An analytical study of one-way slab deflection at service load level (Tang and Lubell 2008) using Eq. (4.8) for the effective moment of inertia, I_e, indicated that minimum slab thickness for deflection control was less sensitive to reinforcement strength than suggested by ACI 318 and more heavily influenced by applied loading, span length, and the assumptions for cracking moment. Desalegne and Lubell (2012) showed similar trends for members with A1035/A1035M (CS) Grade 100 reinforcement.

6.2—Shear design of one-way slabs

Shear design of one-way slabs is the same as shear design of beams and should follow the provisions of ACI 318. Shear reinforcement, however, is rarely used in one-way slabs. As indicated in 4.11 of this guide, research has demonstrated that for members without shear reinforcement, the shear stress at failure decreases as the member depth increases and as the reinforcement ratio decreases.

Bentz et al. (2006) and Bentz and Collins (2006) developed a simplified shear capacity model that accounts for the influence of member depth, concrete strength, aggregate size, crack width, and longitudinal reinforcement strains on shear strength. Hoult et al. (2008) presented an enhanced version of the model to better predict shear strengths for members with larger reinforcement strains.

For one-way slabs containing ASTM A1035/A1035M (CS) longitudinal bars designed in accordance with Chapter 4 of this guide, a simplification to the Hoult et al. (2008) model for shear strength was developed by Desalegne and Lubell (2010). This simplification assumed a longitudinal reinforcement strain of 0.0042, which corresponds to a stress of 100,000 psi (690 MPa) according to Eq. (3.4b) of this guide. For lightly reinforced members (generally with ρ less than 1 percent) without significant axial load and not containing shear reinforcement, the shear strength may be taken as

$$V_c = \frac{73}{(39 + 2.1d)}\sqrt{f_c'}b_w d \quad \text{(in.-lb)}$$

$$V_c = \frac{154}{(1000 + 2.1d)}\sqrt{f_c'}b_w d \quad \text{(SI)} \tag{6.2}$$

Because Eq. (6.2) is based on the simplified flexural model from Mast et al. (2008) (4.2), it is not appropriate for members designed according to Appendix B of this guide. Desalegne and Lubell (2010) provide a generalized shear capacity model that can also be applied to members designed for flexure using Appendix B of this guide.

6.3—Two-way slabs

ACI 318 governs design of two-way slab systems. These provisions should be applicable when using ASTM A1035/A1035M (CS) bars with specified minimum yield strength of 100,000 psi (690 MPa) for tension and 80,000 psi (550 MPa) for compression. ACI 318 permits design of two-way slabs by the Direct Design Method (Section 8.10 of ACI 318-14) or the Equivalent Frame Method (Section 8.11 of ACI 318-14). Both methods are suitable for analysis of two-way slabs containing ASTM A1035/A1035M (CS) steel. As noted in 4.7 of this guide, however, the use of moment redistribution should not be used for members containing ASTM A1035/A1035M (CS) bars until further test data are available. Therefore, allowing up to 10 percent moment redistribution in accordance to ACI 318-14 in slabs with Grade 60 (420), steel should not be followed.

ACI 318 provides guidance on slab reinforcement details and illustrates the minimum extensions of reinforcement that are required in two-way slabs without beams. These provisions should also apply to slabs with ASTM A1035/A1035M (CS) steel if the designer directly checks for the impact on flexural capacity due to the longer bar development lengths compared with steel with f_y = 80,000 psi (550 MPa).

Only limited test data are available that examine the punching shear strength of two-way slab systems containing ASTM A1035/A1035M (CS) steel as flexural reinforcement without any shear reinforcement (Seliem et al. 2008). The punching shear strength of two specimens containing ASTM A1035/A1035M (CS) steel were similar to that of a third specimen containing Grade 60 (420) steel, and the capacities were in agreement with the existing ACI 318 punching shear analytical model. One-way shear should also be checked in accordance with 6.2 of this guide. Because of a lack of adequate research data, the specified minimum yield strength f_{yt} for shear reinforcement should be limited to 60,000 psi (420 MPa), where shear reinforcement is required to provide sufficient shear strength for a two-way slab system. Yang et al. (2010) reported that direct replacement of conventional steel bars with high-strength steel bars, having the same area, resulted in a 27 percent increase of the punching shear strength. This increase of punching shear resistance is because the higher-strength bars did not yield prior to punching failure.

When f_y = 100,000 psi (690 MPa) is used in the design of two-way slab systems, it is recommended that the minimum slab thickness without interior beams be determined according to ACI 318. When using ACI 318 depth limits table, direct extrapolation from f_y = 75,000 psi (520 MPa) to f_y = 100,000 psi (690 MPa) is recommended. When using ASTM A1035/A1035M (CS) bars as longitudinal reinforcement and members are lightly reinforced (generally with ρ less than 1 percent), direct calculations of deflection is recommended.

Due to the higher reinforcement stresses of ASTM A1035/A1035M (CS) steel at the service load level, it is essential to check for adequate control of cracking. Additional guidance on crack control in two-way structures is provided by ACI 224R.

CHAPTER 7—WALLS

7.1—Introduction

Design of walls using ASTM A1035/A1035M (CS) Grade 100 (690) bars should follow ACI 318, except that f_y for flexural reinforcement and for shear reinforcement should be limited as indicated in 7.2 through 7.4 of this guide.

7.2—Vertical reinforcement

Limit specified minimum yield strength f_y to 80,000 psi (550 MPa) for vertical reinforcement in compression, and to 100,000 psi (690 MPa) for vertical reinforcement in tension due to out-of-plane flexure or in-plane overturning forces.

7.3—Horizontal reinforcement

Limit the specified minimum yield strength f_y to 80,000 psi (550 MPa) for horizontal reinforcement in compression, and to 100,000 psi (690 MPa) for horizontal reinforcement in tension due to flexure.

7.4—Shear reinforcement

Limit the specified minimum yield strength f_y to 60,000 psi (420 MPa) where the vertical or horizontal reinforcement is required to resist shear. If appearance and serviceability due to shear cracking are not critical design considerations, f_y may be taken as 80,000 psi (550 MPa). Larger diagonal crack widths under service load are expected if f_y is taken as 80,000 psi (550 MPa). Refer to the ACI 318 Commentary on crack width and shear reinforcement.

7.5—Minimum reinforcement

Minimum reinforcement for the control of shrinkage cracking should meet the requirements of ACI 318.

CHAPTER 8—FOOTINGS AND PILE CAPS

8.1—Design

The design of footings and pile caps is governed by the provisions of ACI 318. The depth of footings is generally determined by its required shear strength.

For flexural design of shallow footings and pile caps, follow the design principles from Chapter 4 of this guide. For reinforcement in tension due to flexural forces, f_y may be taken up to 100,000 psi (690 MPa). For reinforcement in compression due to flexural forces, f_y should not be taken greater than 80,000 psi (550 MPa). For shear design of shallow footings, follow the principles in 4.11 and 6.2 of this guide.

For deep footings or regions of foundation structures meeting the criteria of ACI 318, the strut-and-tie modeling procedure in ACI 318 can be applied. Hassan et al. (2008) and Garay-Moran and Lubell (2008, 2016) reported tests of deep beams with ASTM A1035/A1035M (CS) bars as the primary tension ties, but without web reinforcement. In all cases, the failure mode was brittle and the ACI 318 provisions over-predicted the strength for beams with higher concrete strengths and larger shear span-depth ratios. Therefore, all members with ASTM A1035/A1035M (CS) ties should contain minimum web reinforcement in accordance with ACI 318 until additional test data become available.

Garay-Moran and Lubell (2008, 2016) reported tests of deep beams containing ASTM A1035/A1035M (CS) ties and Grade 60 (420) web reinforcement. Ductile failure modes were achieved, and ACI 318 strut-and-tie modeling provisions predicted strengths safely when the maximum tension tie stress was limited to 100,000 psi (690 MPa). No test data are available for the case of deep members with ASTM A1035/A1035M (CS) bars as the distributed web reinforcement. The strut-and-tie models should not be used in cases where the elastic flow of forces is not followed and also where considerable deformation capacity redistribution is needed based on the model geometry.

CHAPTER 9—MAT FOUNDATIONS

9.1—Design

ACI 318 governs the design of mat foundations. For design with ASTM A1035/A1035M (CS) bars, limit steel stress to 80,000 psi (550 MPa) for reinforcement in compression and 100,000 psi (690 MPa) for reinforcement in tension due to flexure.

Provide minimum reinforcement in each principal direction, with maximum spacing of 18 in. (460 mm), in accordance with ACI 318. Distribute reinforcement near the top or bottom of the section or allocate between the two section faces for specific conditions, such that the total area of continuous reinforcing steel satisfies ACI 318.

For shear design of mat foundations, follow the principles in 4.11, 6.2, and 6.3 of this guide.

CHAPTER 10—OTHER DESIGN CONSIDERATIONS

10.1—Seismic design limitations

ACI 318 provides design requirements for earthquake resistance but does not apply to structures assigned to Seismic Design Category (SDC) A. Structures assigned to SDC B through F should satisfy the applicable provisions of ACI 318. The recommendations presented in previous sections are applicable to structures assigned to SDC A, B, and C. For structures assigned to SDC D, E, or F, the application of this guide is limited to slab systems, foundations, and structural components not designated as part of the seismic-force-resisting system, but explicitly checked for the induced effects of the design displacements. The exception to this is the use of transverse reinforcement for concrete confinement with f_{yt} up to 100,000 psi (690 MPa) for special moment frames and special structural walls as permitted by ACI 318.

Numerous studies (Muguruma and Watanabe 1990; Muguruma et al. 1991; Sugano et al. 1990; Budek et al. 2002; Stephan et al. 2003) conducted on the use of high-strength reinforcement (f_{yt} up to and greater than 120,000 psi

[827 MPa]) show that there is no detriment to using high-strength transverse reinforcement as column confinement. The test program by Lepage et al. (2008) has demonstrated successful use of high-strength steel (HSS) in beams and columns subjected to reverse cyclic loadings (Rautenberg et al. 2010). With the exception of using f_{yt} up to 100,000 psi (690 MPa) for confinement, the following two recommendations are appropriate until comprehensive and systematic tests performed using ASTM A1035/A1035M (CS) bars justify its use in structural members of buildings assigned to SDC D and higher:

1. ASTM A1035/A1035M (CS) bars should not be used as longitudinal reinforcement in a structural member that is part of the seismic-force-resisting system of a building assigned to SDC D, E. or F. The use of ASTM A1035/A1035M (CS) bars as transverse reinforcement is permitted, provided f_{yt} is limited to 60,000 psi (420 MPa) for computing shear strength.

2. If Grade 60 (420) bar is used for column longitudinal reinforcement and ASTM A1035/A1035M (CS) bar is used for beam longitudinal reinforcement in an intermediate moment frame, it should be ensured that the beam-column joints exhibit strong column-weak beam behavior because the actual flexural strengths of beams using ASTM A1035/A1035M (CS) reinforcement exceed the nominal flexural strengths computed by the simplified method in Chapter 4 of this guide. In addition, the required shear strength of the beam should be determined based on the nominal moment M_n obtained from nonlinear analysis, as given in Appendix B of this guide.

10.2—Development and lap splice length

Design for tension development and splices of ASTM A1035/A1035M (CS) reinforcement should follow ACI 318. Some of the provisions require additional testing and evaluation for validation, as noted in the following.

There have been several investigations on bond and development length of ASTM A1035/A1035M bars of different sizes and with different concrete strengths. Earlier studies include 130 bond tests of beam-end specimens (Ahlborn and DenHartigh 2002), 15 pullout tests (El-Hacha and Rizkalla 2002), and six beam tests with unconfined spliced bars (Peterfreund 2003). Although these studies used pre-standardization high-strength bars, the bar deformations have not changed and the research results are still relevant. The results of these investigations indicated that the development length computed by Eq. (12-1) of ACI 318-99 and ACI 318-02 was sufficient for pre-standardized ASTM A1035/A1035M (CS) bars to develop the 0.2 percent offset yield strength of 120,000 psi (830 MPa).

However, a coordinated research program of bond tests of 69 large-scale specimens using ASTM A1035/A1035M bars (Seliem et al. 2009) indicated that ACI 318 should be used to predict only the strength of confined splices, whereas a similar equation proposed by ACI 408R provides a better strength estimate for both unconfined and confined splices using a strength reduction factor ϕ of 0.82.

Based on the aforementioned studies of ACI 318, it is found applicable for calculating the development and splice lengths of ASTM A1035/A1035M (CS) bars to develop the specified minimum yield strength of 100,000 psi (690 MPa) if the bars are confined. ACI 318 should not be applied to unconfined bars, as discussed in the following.

Alternatively, for both confined and unconfined spliced bars, the equation in ACI 408R with a revised strength reduction factor ϕ of 0.80, instead of 0.82, is recommended. The equations in ACI 408R for development length in the customary in.-lbf units are shown as Eq. (10.2a) through (10.2e).

$$\ell_d = \frac{\left(\frac{f_y}{f_c'^{1/4}} - \phi 2400\omega\right)\alpha\beta_c\lambda}{\phi 76.3\left(\frac{c_b\omega + K_{tr}}{d_b}\right)} d_b \quad \text{(in.-lb)}$$

$$\ell_d = \frac{\left(\frac{f_y}{f_c'^{1/4}} - \phi 57.4\omega\right)\alpha\beta_c\lambda}{\phi 1.83\left(\frac{c_b\omega + K_{tr}}{d_b}\right)} d_b \quad \text{(SI)} \qquad (10.2a)$$

where α = 1.3 for top cast bars, β_c = 1.0 for uncoated bars, λ = 1.0 for normalweight concrete, and

$$K_{tr} = (0.52 t_r t_d A_{tr}/sn)\sqrt{f_c'} \quad \text{(in.-lb)}$$

$$K_{tr} = (6.26 t_r t_d A_{tr}/sn)\sqrt{f_c'} \quad \text{(SI)} \qquad (10.2b)$$

$$t_r = 9.6R_r + 0.28 \leq 1.72 \qquad (10.2c)$$

$$t_d = 0.78d_b + 0.22 \quad \text{(in.-lb)}$$

$$t_d = 0.03d_b + 0.22 \quad \text{(SI)} \qquad (10.2d)$$

$$\left(\frac{c_b\omega + K_{tr}}{d_b}\right) \leq 4.0 \qquad (10.2e)$$

Although no test data are available, design for development and splices of ASTM A1035/A1035M (CS) Grade 100 (690) bars in compression may follow ACI 318, but the specified minimum yield strength f_y in compression is limited to 80,000 psi (550 MPa).

Tests by Harries et al. (2010) have shown that ASTM A1035/A1035M (CS) bars with 90- and 180-degree standard hooks can develop tensile bar stresses of at least 140,000 psi (965 MPa) and, in many cases, the strength of the bar. Instead of hooked bars, headed bars may be considered as an alternative. Section 10.3 of this guide discusses development of headed ASTM A1035/A1035M (CS) bars.

10.3—Mechanically spliced bars and headed bars

According to 10.2 of this guide, designs with f_y of 100,000 psi (690 MPa) for ASTM A1035/A1035M (CS) bars require relatively long development or splice lengths. Because these lengths may be uneconomical or impractical, the designer may consider using mechanical splices (mechanical connections) and mechanically headed bars.

ACI 318 provides minimum tensile strength requirements for mechanical splices and specifies these requirements as $1.25f_y$ for the Type 1 splice and $1.0f_u$ for the Type 2 splice. Accordingly, the requirements for Type 1 mechanical splices are intended to avoid a splice failure when the reinforcement is subject to expected stress levels in yielding regions. The additional requirement for a Type 2 mechanical splice is intended to result in a mechanical splice capable of sustaining inelastic strains through multiple cycles.

These requirements are functions of the specified strengths of the reinforcing bars being spliced (not actual bar strengths) and do not include any ductility requirements. These functions of the specified strengths were historically developed based on reinforcement with stress-strain behavior that is sharply yielding with a definite yield plateau. For gradually yielding reinforcement, such as ASTM A1035/A1035M (CS) reinforcement, strain-based requirements are more appropriate. Equivalent strain criteria for the typical Type 1 and Type 2 mechanical splices (ACI 318) can be developed for ASTM A1035/A1035M (CS) reinforcement by examination of the actual stress-strain behavior for sharply-yielding reinforcement, and establishing a strain that corresponds to the specified minimum strengths for the mechanical splice. On this basis, the Type 1 mechanical splice minimum strength corresponds to a strain in the spliced bars that is beyond the yield plateau and into the onset of strain hardening; the onset of strain hardening occurs at strains in the range of 1 to 2 percent for ASTM A615/A615M Grade 60 and ASTM A706/A706M Grade 60 reinforcement. The Type 2 mechanical splice minimum strength requirement corresponds to a strain in the spliced bars that is even larger and beyond the onset of strain hardening and into the strain hardening range, corresponding to strains in the range of 2.5 to 4 percent for ASTM A615/A615M Grade 60 and ASTM A706/A706M Grade 60 reinforcement. Consequently, to achieve stress-strain performance similar to those of the strength-based requirements for mechanical splices specified in ACI 318, new designated Type A and B mechanical splices are recommended by this guide with monotonic strain limits based on strain of the bars.

Based on the historical statistical analysis of ASTM A1035/A1035M (CS) bars' test data, it is recommended that bars be mechanically spliced to develop 2 percent static tensile strain for Type A connections and 3 percent static tensile strain for Type B connections. Testing of the mechanical splices for strain capacity should be performed under static tensile test loading, as any multiple inelastic cyclic excursions beyond the actual yield point of the bar (measured through 0.2% offset), such as defined in ICC-ES AC133, ICC-ES AC429, ASTM A1034/A1034M, or ISO 15835-2, will strain harden the reinforcing bar and reduce the strain capacity of the reinforcing steel. Inelastic cyclic testing is therefore not recommended for use in evaluation of strain capacity of mechanically spliced A1035/A1035 M (CS) Grade 100 (690) bars. Only test samples cut from original straight lengths of reinforcing steel, not from coil or post-fabricated steel, should be used for tensile strain evaluation.

ASTM A1035/A1035M (CS) bar can be developed in tension by using a mechanically attached head, as is the case with other ASTM types and grades of deformed reinforcing bars. Provisions for computing development length of headed deformed reinforcing bars were introduced in ACI 318, but they are currently limited to reinforcing bars with f_y not exceeding 60,000 psi (420 MPa) and, thus, precludes the use of these provisions with ASTM A1035/A1035M (CS) bars and other reinforcing bars with f_y in excess of 60,000 psi (420 MPa), such as ASTM A615/A615M Grade 75 (520) and Grade 80 (550) bars. However, headed ASTM A1035/A1035M (CS) bars may be used in accordance with ACI 318 "…provided that test results showing the adequacy of such attachment or device are approved by the building official."

ACI 439.3R summarizes available mechanical splices. Some manufacturers use certain methods to produce mechanical splices or headed bars that excessively heat the reinforcing bar, causing an unfavorable alteration to the microstructure of ASTM A1035/A1035M (CS) steel. The applicability of a mechanical splice or headed bar should be determined by consulting the manufacturer of the mechanical splices for the applicability with ASTM A1035/A1035M (CS) bar.

Mechanical splices and headed bars should be placed in accordance with the design drawings or as approved by the licensed design professional.

10.4—Bending and welding of bars

Experiments and field applications have shown that the ASTM A1035/A1035M (CS) reinforcing bars can be bent into standard hooks by following the requirements of ACI 318. Note, however, that because of the strain-hardening characteristics of the material, it is usually necessary to overbend the bar to achieve the desired bent angle. As it is with other reinforcing bars straightening of a bent bar embedded in concrete should be avoided, as it may cause the bar to break.

Although the chemistry (other than the chromium content) of ASTM A1035/A1035M (CS) steel is similar to that of ASTM A706/A706M steel, ASTM A1035/A1035M bars should not be welded. The heat of welding alters the microstructure of the steel. Welding of ASTM A1035 bars should be approached with caution because no specific provisions have been included in the ASTM A1035 to enhance weldability of the bars.

10.5—Use of ASTM A1035/A1035M (CS) bars with ASTM A615/A615M bars

ASTM A1035/A1035M (CS) bars may be used in direct contact with ASTM A615/A615M bars in a concrete structure because the difference in corrosion potential between these dissimilar metals is approximately 100 mV, which is well within the acceptable value of 250 mV (Hartt 2009).

Table 11—Specified minimum yield strengths for design of members using ASTM A1035/A1035M (CS) reinforcement

Type of member	Longitudinal reinforcement		Transverse reinforcement		
	Tension, psi (MPa)	Compression, psi (MPa)	Shear, psi (MPa)	Torsion, psi (MPa)	Confinement, psi (MPa)
Beams and one-way slabs	100,000 (690)	80,000 (550)	80,000 (550)	60,000 (420)	N/A
Columns	100,000 (690)	80,000 (550)	80,000 (550)	60,000 (420)	100,000 (690)
Tension ties	80,000 (550)	N/A	N/A	N/A	N/A
Compression struts	N/A	80,000 (550)	N/A	N/A	N/A
Two-way slabs	100,000 (690)	80,000 (550)	60,000 (420)	60,000 (420)	N/A
Walls	100,000 (690)	80,000 (550)	80,000 (550)	N/A	100,000 (690)
Footings and pile caps	100,000 (690)	80,000 (550)	80,000 (550)	60,000 (420)	N/A
Mat foundations	100,000 (690)	80,000 (550)	80,000 (550)	N/A	N/A

CHAPTER 11—SUMMARY

Presented in this document are guidelines for using ASTM A1035/A1035M (CS) Grade 100 (690) deformed reinforcing bars in reinforced concrete members, including beams, columns, slab systems, walls, footings, and mat foundations, for Seismic Design Category (SDC) A, B, or C. A structure assigned to SDC A, B, or C is required to be designed for all applicable gravity and environmental loads. In the case of SDC A structures, seismic forces are notional structural integrity forces. This document addresses all design required for SDC A, B, and C structures. Application of these guidelines for SDC D, E, or F is limited to slab systems, foundations, and structural components that are not designed as part of seismic-force-resisting system. Both advantages and disadvantages of using ASTM A1035/A1035M (CS) deformed bars are discussed in Chapter 1 of this guide. Design examples included in Appendixes A and B of this guide illustrate design procedures and proper applications of the recommended design criteria.

Tests indicated that ASTM A1035/A1035M (CS) Grade 100 (690) reinforcing bars exhibit a linear stress-strain relationship up to a proportional limit ranging from 60,000 to 80,000 psi (420 to 550 MPa) with a modulus of elasticity of 29,000,000 psi (200,000 MPa), but without a well-defined yield point. The yield strength determined by the 0.2 percent offset method exceeds 115,000 psi (790 MPa). The tensile strength exceeds 155,000 psi (1070 MPa) with corresponding strain ranging from 0.04 to 0.06. The elongation in 8 in. (200 mm) ranges from 0.08 to 0.13. An approximate lower-bound representation of the stress-strain curve of the Grade 100 (690) bar is given in Eq. (3.4a) through (3.4c) of this guide.

A simplified design method for flexural tension is developed based on an idealized "elastic-perfectly plastic" stress-strain curve with a yield plateau at f_y = 100 ksi (690 MPa). Because the yield plateau neglects the effect of strain hardening of the steel, the tension-controlled and compression-controlled strain limits and the strength reduction factor ϕ are adjusted to ensure satisfactory member behavior. The adjusted tension and compression strain limits are 0.009 and 0.004, respectively, and the adjusted strength reduction factor ϕ is given by Eq. (4.4) of this guide. For flexural and direct compression, f_y is taken as 80,000 psi (550 MPa) so that the compatible strain in concrete under compression will not exceed the maximum value of 0.003. Due to a lack of research data and experience, moment redistribution for flexural design using ASTM A1035/A1035M (CS) reinforcement is not recommended.

Flexural design using ASTM A1035/A1035M (CS) Grade 100 (690) bars with f_y = 100,000 psi (690 MPa) would result in higher steel stress at service load conditions, so the designer should exercise caution in applying the ACI 318 provisions on crack and deflection controls, especially for shallow members with reinforcement ratio less than 1 percent and where diagonal shear cracking is expected at the service load level.

For shear design of beams, ASTM A1035/A1035M (CS) Grade 100 (690) bars can be used as shear reinforcement with f_{yt} = 80,000 psi (550 MPa), but at service load levels, slightly increased shear crack widths should be expected. In addition, the concrete contribution to shear strength, V_c, of lightly reinforced deep beams with longitudinal reinforcement ratio less than 1 percent may be less than predicted by ACI 318. It is advisable that such beams should contain minimum shear reinforcement as determined by ACI 318.

Development and splice length for ASTM A1035/A1035M (CS) reinforcement in tension may be determined by ACI 318, provided the splice is confined. Better predictions for both unconfined and confined splices, however, can be obtained by using Eq. (10.2a) through (10.2e) of this guide, which are recommended in ACI 408R. Splice lengths shall comply with ACI 318 if ACI 408R-modified equations provide shorter lengths.

Design of slab systems using ASTM A1035/A1035M (CS) Grade 100 (690) steel bars should follow ACI 318 provisions for shear in one-way slabs and for two-way slabs. Using f_y = 100,000 psi (690 MPa) for tension reinforcement in one-way slabs, the section may become lightly reinforced. If ρ is less than 1 percent, it is recommended that the reduced concrete contribution to shear strength, V_c, as given in Eq. (6.2) of this guide, be considered. For two-way slabs, the slab thickness should be determined according to

ACI 318 based on actual deflection calculations. When the member is lightly reinforced (generally with ρ less than 1 percent), it is recommended that direct deflection calculations be used (Yang et al. 2010). If shear reinforcement is required to provide sufficient shear strength for a two-way slab system, the specified minimum yield strength, f_{yt}, for shear reinforcement should be limited to 60,000 psi (420 MPa) because adequate research data are not available to justify a higher value.

Designs for columns, walls, footings and pile caps, and mat foundations using ASTM A1035/A1035M (CS) reinforcement are discussed in Chapters 5, 7, 8, and 9, respectively, of this guide. There are no exceptions from the requirements of ACI 318 for these structural members other than the adjustments of specified minimum yield strength f_y for tension and compression reinforcements, and f_{yt} for shear reinforcement. Some other design considerations, including seismic design limitations, spliced and headed bars, bending of bars, and use of ASTM A1035/A1035M (CS) bars together with ASTM A615/A615M bars, are discussed in Chapter 10 of this guide.

The recommended specified minimum yield strengths for design of various structural members using ASTM A1035/A1035M (CS) reinforcement are summarized for convenient reference in Table 11.

CHAPTER 12—REFERENCES

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American Concrete Institute (ACI)

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ACI 318-71—Building Code Requirements for Reinforced Concrete

ACI 318-95—Building Code Requirements for Structural Concrete and Commentary

ACI 318-99—Building Code Requirements for Structural Concrete and Commentary

ACI 318-02—Building Code Requirements for Structural Concrete and Commentary

ACI 318-05—Building Code Requirements for Structural Concrete and Commentary

ACI 318-08—Building Code Requirements for Structural Concrete and Commentary

ACI 318-11—Building Code Requirements for Structural Concrete and Commentary

ACI 318-14—Building Code Requirements for Structural Concrete and Commentary

ACI 408R-03—Bond and Development of Straight Reinforcing Bars in Tension

ACI 439.3R-07—Types of Mechanical Splices for Reinforcing Bars

ACI 439.4R-09(17)—Report on Steel Reinforcement—Material Properties and U.S. Availability

ACI ITG-6R-10—Design Guide for the Use of ASTM A1035/A1035M Grade 100 (690) Steel Bars for Structural Concrete

ASTM International

A615/A615M-18—Standard Specification for Deformed and Plain Carbon-Steel Bars for Concrete Reinforcement

A706/A706M-16—Standard Specification for Deformed and Plain Low-Alloy Steel Bars for Concrete Reinforcement

A1034/A1034M-10a(2015)—Standard Test Methods for Testing Mechanical Splices for Steel Reinforcing Bars

A1035/A1035M-16b—Standard Specification for Deformed and Plain, Low-Carbon, Chromium, Steel Bars for Concrete Reinforcement

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APPENDIX A—DESIGN EXAMPLES

A.1—Introduction

The following design examples illustrate the various design procedures recommended in this guide. The problems selected for these examples are from a Portland Cement Association (PCA) publication, *PCA Notes on ACI 318-05 Building Code Requirements for Structural Concrete* (PCA 2005). The term "CS" used in the examples refers to ASTM A1035/A1035M (CS) Grade 100 steel reinforcement.

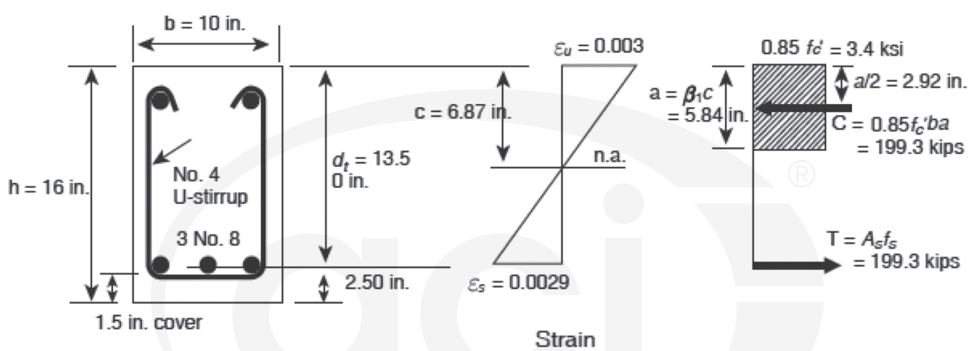
Examples are numbered as the chapter number of this guide. Examples are grouped to illustrate the various aspects of the recommended design procedures, not in ascending numerical order. Examples appear in customary in.-lb units except Examples 6.1(SI) and 4.6(SI), which are the same as Example 6.1 and Example 4.6, but in SI units for purposes of illustration.

Where appropriate, commentaries are added to indicate the differences in design when using ASTM A1035/A1035M (CS) bars as opposed to the ASTM A615/A615M Grade 40 and Grade 60 steel bars.

A.2—Design examples

Example 4.1—Moment strength in beams using equivalent rectangular stress distribution

Similar to Example 6.1 in the *PCA Notes*, this example represents direct bar-for-bar replacement of Grade 60 reinforcing bars by CS bars.



(a) For the beam section shown, calculate moment strength based on static equilibrium using the ACI equivalent rectangular stress distribution. Assume f_c' = 4000 psi and f_y = 100,000 psi. For simplicity, neglect hanger bars in the compression zone.

(b) Calculate the moment strength for the same concrete section reinforced with two No. 8 high-strength bars (f_y = 100,000 psi) instead of three No. 8 Grade 60 bars. For simplicity, neglect hanger bars in the compression zone.

This example represents replacement of the longitudinal tension force, A_sf_y, provided by the conventional reinforcement, which results in the use of fewer high-strength reinforcing bars.

Calculations and discussion

(a) Section reinforced with three No. 8, Grade 100 tension bars

1. Define rectangular concrete stress distribution.

$d = d_t = 16 - 2.5 = 13.50$ in.

$A_s = 3 \times 0.79 = 2.37$ in.^{2}

Assuming $\varepsilon_s < \varepsilon_y$ ($\varepsilon_y = 100/29{,}000 = 0.00345$)

$T = A_sE_s\varepsilon_s = 2.37 \times 29{,}000 \times \varepsilon_s = 68{,}730\varepsilon_s$

$C = 0.85f_c'ba = 0.85 \times 4 \times 10 \times a = 34a$

$$\varepsilon_s = \left(\frac{d_t - c}{c}\right)0.003 = \left(\frac{d_t - a/\beta_1}{a/\beta_1}\right)0.003$$

Satisfying equilibrium $T = C$

$$68,730\left(\frac{13.50-a/0.85}{a/0.85}\right)0.003=34a$$

$$40a^2 + 242.58a - 2783.57 = 0$$

a = 5.84 in.

2. Determine net tensile strain ε_s *and* ϕ.
$c = a/\beta_1 = 5.84/0.85 = 6.87$ in.

	c, in.	ϕ, curvature
Grade 60, PCA	4.92	0.00061
Grade 100, CS	6.87	0.00044

$$\varepsilon_s = \left(\frac{d_t - c}{c}\right)0.003 = \left(\frac{13.50-6.87}{6.87}\right)0.003 = 0.0029 < 0.004$$

(Refer to Section 4.4 of this guide.)

Therefore, the section is compression-controlled. This also confirms that $\varepsilon_s < \varepsilon_y = 0.00345$.

$\phi = 0.65$

(Refer to Section 4.4 of this guide.)

3. Determine nominal moment strength M_n *and design moment strength* ϕM_n.
$M_n = A_sE_s\varepsilon_s(d_t - (a/2)) = 68,730 \times 0.0029 \times (13.50 - 2.92) = 2108.8$ in.-kip = 175.7 ft-kip

$\phi M_n = 0.65(175.7) = 114.2$ ft-kip

	M_n, ft-kip	ϕM_n, ft-kip
PCA, Grade 60	135.2	121.9
CS, Grade 100	175.7	114.2

4. Minimum reinforcement.

$$A_{s,min} = \frac{3\sqrt{f_c'}}{f_y}b_w d \geq \frac{200 b_w d}{f_y}$$

Because f_c' = 4000 psi, $3\sqrt{4000} = 189.7 < 200$. Therefore, $200b_wd/f_y$ governs

$$\frac{200\ b_w d}{f_y} = \frac{200 \times 10 \times 13.50}{100,000} = 0.27\text{ in.}^2$$

A_s (provided) = 2.37 in.^{2} > $A_{s,min}$ = 0.27 in.^{2} **OK**

The nominal moment strength of the beam reinforced with CS is 30 percent higher than the nominal moment strength of 135.2 ft-kip for the same beam reinforced with Grade 60 steel (*PCA Notes*, Example 6.1). The change of failure mode from tension-controlled to compression-controlled, however, reduces the ϕ value, resulting in the beam design moment with CS bars 6 percent lower than the beam reinforced with Grade 60 steel bars.

(b) Section reinforced with two No. 8, Grade 100 tension bars.
1. Define rectangular concrete stress distribution.
$d = d_t = 16 - 2.5 = 13.50$ in.

$A_s = 2 \times 0.79 = 1.58$ in.^{2}

Assuming $\varepsilon_s > \varepsilon_y$

$T = A_s f_y = 1.58 \times 100 = 158$ kip

$$a = \frac{A_s f_y}{0.85 f_b' b} = \frac{158}{0.85 \times 4 \times 10} = 4.65 \text{ in.}$$

2. Determine net tensile strain ε_s and ϕ.
$c = a/\beta_1 = 4.65/0.85 = 5.47$ in.

	c, in.	**ϕ, Curvature**
PCA, 3 No. 8 Grade 60	4.92	0.00061
CS, 3 No. 8 Grade 100	5.47	0.00055

$$\varepsilon_s = \left(\frac{d_t - c}{c}\right)0.003 = \left(\frac{13.50 - 5.47}{5.47}\right)0.003 = 0.0044 > 0.004$$

Therefore, the section is in the transition zone. This also confirms that $\varepsilon_s > \varepsilon_y = 0.00345$

$\phi = 0.45 + 50\varepsilon_s = 0.45 + 50 \times 0.0044 = 0.67$

(Refer to Section 4.4 of this guide.)

3. Determine nominal moment strength M_n *and design moment strength* ϕM_n.

$$M_n = A_s f_y \left(d_t - \frac{a}{2}\right) = 158 \times (13.50 - 2.33) = 1764.9 \text{ in.-kip} = 147.1 \text{ ft-kip}$$

$\phi M_n = 0.67(147.1) = 98.5$ ft-kip

	M_n, ft-kip	**ϕM_n, ft-kip**
PCA, 3 No. 8 Grade 60	135.2	121.9
CS, 2 No.8 Grade 100	147.1	98.5

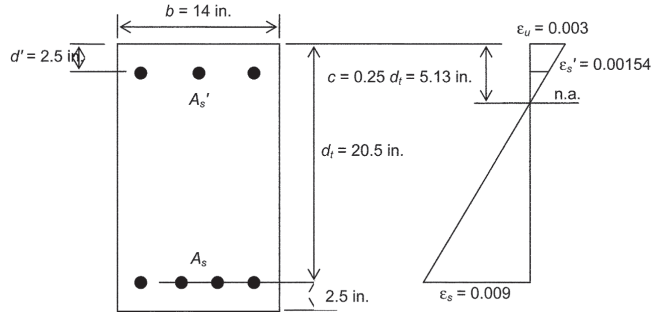
4. Minimum reinforcement.

A_s (provided) = 1.58 in.^{2} > $A_{s,min}$ = 0.27 in.^{2} **OK**

In this case, even though there is a reduction of one-third of the longitudinal steel area, the failure mode is in the transition region between compression control and tension control. The corresponding reduction of the ϕ value causes the beam design moment with two No. 8 CS bars to be 19 percent lower than for the beam reinforced with three No. 8 Grade 60 steel bars.

Example 4.2—Design of beam with compression reinforcement

Similar to Example 6.2 in *PCA Notes*, this example represents replacement of Grade 60 reinforcing bars by high-strength bars. This example demonstrates the benefit of using high-strength concrete with high-strength bars.



(a) A beam cross section is limited to the maximum size shown. Determine the required area of reinforcement for a factored moment M_u = 516 ft-kip; f_c' = 4000 psi; f_y = 100,000 psi (tension); and f_y = 80,000 psi (compression).

(b) Determine the required area of reinforcement for the same section using f_c' = 8000 psi.

(c) Determine the required area of reinforcement for the same section using f_c' = 8000 psi and f_y = 60,000 psi for comparison.

Calculations and discussion

(a) Grade 100 steel and f_c' = 4000 psi.

1. Determine the required nominal strength.

Design the section to be tension controlled, $\varepsilon_s = 0.009$ and $\phi = 0.9$.

(Refer to Section 4.4 of this guide.)

$M_n = M_u/\phi = 516/0.9 = 573$ ft-kip

2. Determine maximum moment without compression reinforcement.

$c = 0.25d = 0.25 \times 20.5 = 5.13$ in.

f_c' = 4000 psi	c, in.	ϕ, curvature
Grade 60, PCA	7.69	0.00040
Grade 100, CS	5.13	0.00058

$a = \beta_1 c = 0.85 \times 5.13 = 4.36$ in.

$C = T = 0.85 f_c' ab = 0.85 \times 4 \times 4.36 \times 14 = 208$ kip

$$M_{nt} = T\left(d_t - \frac{a}{2}\right) = 208\left(20.5 - \frac{4.36}{2}\right) = 3811 \text{ in.-kip} = 318 \text{ ft-kip}$$

3. Required area of tension steel to develop M_{nt}.

$A_{s,nt} = 208/100 = 2.08$ in.^{2}

4. Additional moment (573 – 318 = 255 ft-kip) should be developed in T-C *couple between tension steel and compression steel.*

$$\text{Additional tension steel required: } \Delta A_s = \frac{255(12)}{(20.5 - 2.5)(100)} = 1.70 \text{ in.}^2$$

Total tension steel required: $A_s = 2.08 + 1.70 = 3.78$ in.^{2}

The strain in the compression steel is 0.00154, as shown in the aforementioned strain diagram. The corresponding steel stress is 44.6 ksi.

Compression steel required: $A_s' = \dfrac{255(12)}{(20.5-2.5)(44.6)} = 3.81\text{ in.}^2$

PCA Notes solution: using Grade 60 steel, $A_s = 6.59$ in.^{2} and $A_s' = 1.43$ in.^{2}

Total area of reinforcement required for this design, including the tension steel of 3.78 in.^{2} and the compression steel of 3.81 in.^{2}, is 7.59 in.^{2} The original design in the *PCA Notes*, which uses Grade 60 steel, required a tension steel of 6.59 in.^{2} and a compression steel of 1.43 in.^{2} for a total 8.02 in.^{2} of reinforcing steel. The use of CS results in a reinforcing steel reduction of 5 percent. The total reduction is small because the compression steel increases to offset the smaller compressive force provided by the normal-strength concrete. High-strength steel (HSS) for flexural capacity is most advantageous when used with high-strength concrete, as illustrated in the following section.

(b) Grade 100 steel and f_c' = 8000 psi

1. Determine the required nominal strength.

$M_n = M_u/\phi = 516/0.9 = 573$ ft-kip

2. Determine maximum moment without compression reinforcement.

$c = 0.25d = 0.25 \times 20.5 = 5.13$ in.

f_c' = 8000 psi	c, in.	ϕ, curvature
Grade 60, PCA	5.98	0.00050
Grade 100, CS	5.13	0.00058

$a = \beta_1 c = 0.65 \times 5.13 = 3.33$ in.

$C = T = 0.85f_c'ab = 0.85 \times 8 \times 3.33 \times 14 = 317$ kip

$M_{nt} = 5971$ in.-kip = 498 ft-kip

3. Required area of tension steel to develop M_{nt}.

$A_{s,nt} = 317/100 = 3.17$ in.^{2}

4. Additional moment (573 – 498 = 75 ft-kip) should be developed in T-C *couple between tension steel and compression steel.*

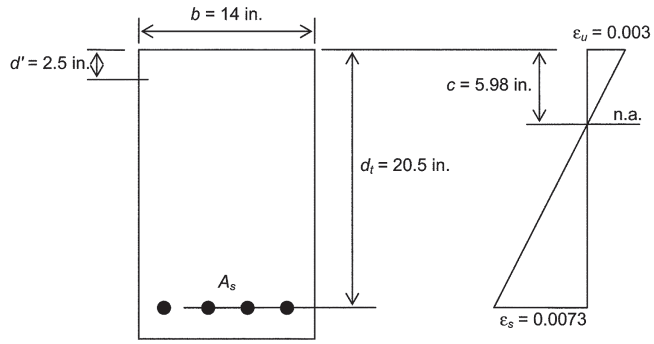
Additional tension steel required: $\Delta A_s = \dfrac{75(12)}{(20.5-2.5)(100)} = 0.05\text{ in.}^2$

Total tension steel required: $A_s = 3.17 + 0.50 = 3.67$ in.^{2}

Compression steel required: $A_s' = \dfrac{75(12)}{(20.5-2.5)(44.6)} = 1.12\text{ in.}^2$

By using Grade 100 steel with 8000 psi concrete, both the tension and the compression steels are reduced.

(c) f_c' = 8000 psi and f_y = 60,000 psi



1. Determine the required nominal strength.

$M_n = M_u/\phi = 516/0.9 = 573$ ft-kip = 6876 in.-kip

2. Define rectangular concrete stress distribution.

$C = T = 0.85f_c'ab$

$$M_n = T\left(d_t - \frac{a}{2}\right) = 0.85 f_c' ab\left(d_t - \frac{a}{2}\right)$$

$6876 = 0.85 \times 8 \times a \times 14 \times (20.5 - a/2)$

$47.6\,a^2 - 1951.6a + 6876 = 0$

$a = 3.89$ in.

3. Determine required area of tension steel.

$c = a/\beta_1 = 3.89 / 0.65 = 5.98$ in.

$$\varepsilon_s = \left(\frac{d_t - c}{c}\right)(0.003) = \left(\frac{20.5 - 5.98}{5.98}\right)0.003 = 0.0073 > 0.00207 = e_y$$

$A_s = T/f_y = 0.85f_c'ab/f_y = 0.85 \times 8 \times 3.89 \times 14/60 = 6.17$ in.^{2} No compression steel is required.

PCA Notes solution: using Grade 60 steel with 4000 psi concrete, $A_s = 6.59$ in.^{2} and $A_s' = 1.43$ in.^{2}

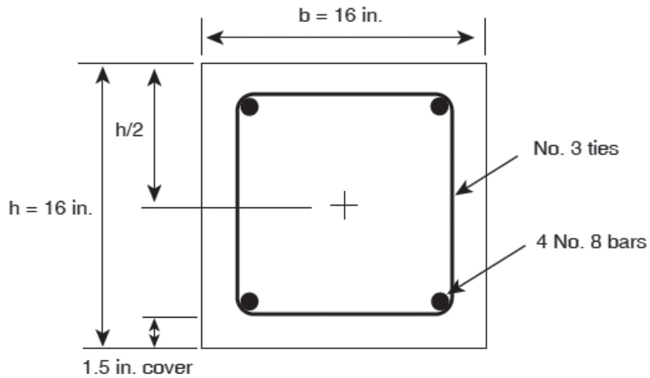
Using CS for flexural capacity is most beneficial when used with high-strength concrete, as illustrated in this example. Using CS requires a total area of reinforcement, including tension and compression steel, of 4.79 in.^{2} Grade 60 reinforcement requires a total of 6.17 in.^{2} of steel. The design using CS results in a 25 percent reduction of the required flexural reinforcement.

The design using Grade 60 steel does not require compression steel but using CS does. The tension-controlled strain limit for CS is 0.009 instead of 0.005 for Grade 60 steel. This high strain level results in a smaller compression zone depth c. Consequently, the high-strength compression reinforcement should provide additional compression force if $c > d'$.

	A_s, in.^{2}	A_s', in.^{2}	Total A_s, in.^{2}
Grade 60, PCA, $f_c' = 4000$ psi	6.59	1.43	8.02
Grade 100, CS, $f_c' = 4000$ psi	3.78	3.81	7.59
Grade 60, PCA, $f_c' = 8000$ psi	6.17	Not needed	6.17
Grade 100, CS, $f_c' = 8000$ psi	3.67	1.12	4.79

Example 5.1—Axial load-moment strength, P_n and M_n, for given strain conditions

Similar to Example 6.4 in *PCA Notes*, this example represents direct bar-for-bar replacement of Grade 60 reinforcing bars by high-strength bars.

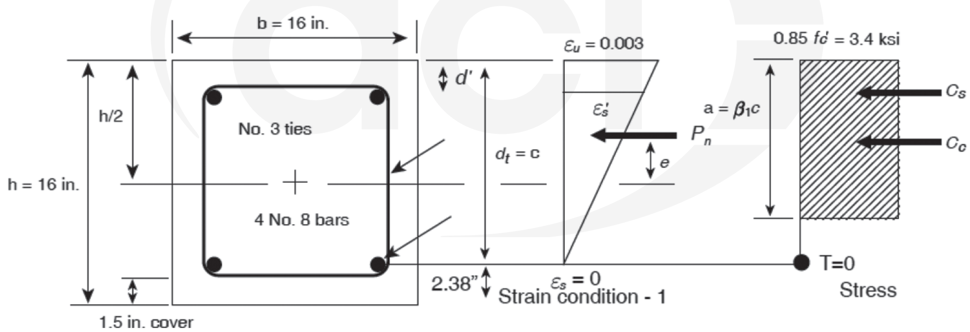


For the column section shown, calculate the load-moment strength, P_n and M_n, for four strain conditions.
(1) Bar stress near tension face of member equal to zero, $f_s = 0$
(2) Bar stress near tension face of member equal to $0.5f_y$ ($f_s = 0.5f_y$)
(3) At limit for compression-controlled section ($\varepsilon_t = 0.004$)
(4) At limit for tension-controlled section ($\varepsilon_t = 0.009$)

Use $f_c' = 4000$ psi, $f_y = 100{,}000$ psi in tension and $f_y = 80{,}000$ psi in compression.

Calculations and discussion

(a) Axial load-moment strength, P_n and M_n, for Strain Condition 1: $\varepsilon_s = 0$.



1. Define stress distribution and determine force values.

d' = Cover + No. 3 tie diameter + $d_b/2$ = 1.5 + 0.375 + 0.5 = 2.38 in.

$d_t = 16 - 2.38 = 13.62$ in.

Because $\varepsilon_s = 0$, $c = d_t = 13.62$ in.

$a = \beta_1 c = 0.85 \times 13.62 = 11.58$ in., where $\beta_1 = 0.85$ for $f_c' = 4000$ psi.

$C_c = 0.85 f_c' ba = 0.85 \times 4 \times 16 \times 11.58 = 630$ kip

Compression: $\varepsilon_y' = f_y'/E_s = 80/29{,}000 = 0.00276$

Tension: $\varepsilon_y = f_y/E_s = 100/29{,}000 = 0.00345$

From strain compatibility

$$\varepsilon_s' = \varepsilon_u\left(\frac{c-d'}{c}\right) = 0.003\left(\frac{13.62-2.38}{13.62}\right) = 0.00248 < \varepsilon_y' = 0.00276$$

Compression steel has not yielded

$C_s = A_s'E_s\varepsilon_s' = 1.58 \times 29{,}000 \times 0.00248 = 113.6$ kips

2. *Determine* P_n *and* M_n *from static equilibrium.*

$P_n = C_c + C_s = 630 + 113.6 = 743.6$ kip

$M_n = P_ne = C_c(ha/2 - a/2) + C_s(ha/2 - d') = 630(8 - 5.79) + 113.6(8 - 2.38) = 2030.7$ in.-kip $= 169.2$ ft-kip

$e = M_n/P_n = 2030.7/743.6 = 2.73$ in.

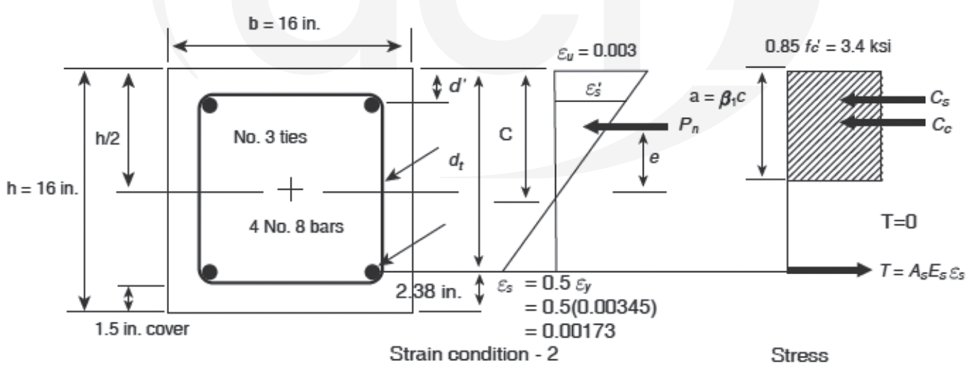
$\phi = 0.65$

(Refer to Section 4.4 of this guide.)

Therefore, for strain condition $\varepsilon_s = 0$; the design axial load strength, $\phi P_n = 0.65(743.6) = 483.3$ kip; and the design moment strength, $\phi M_n = 0.65(169.2) = 110.0$ ft-kip.

$\varepsilon_s = 0$	ϕP_n, kip	ϕM_n, ft-kip
Grade 60, PCA	471.1	104.3
Grade 100, CS	471.1	110.0

(b) Axial load-moment strength, P_n and M_n, for Strain Condition 2: $\varepsilon_s = 0.5\varepsilon_y$.



1. *Define stress distribution and determine force values.*

$d' = 2.38$ in. $d_t = 13.62$ in.

From strain compatibility

$$\left(\frac{c}{0.003}\right) = \left(\frac{d^t}{0.5\varepsilon_y} + 0.003\right)$$

$$c = \left(\frac{0.003d_t}{0.5\varepsilon_\gamma + 0.003}\right) = \frac{0.003 \times 13.62}{0.00173 + 0.003} = 8.64 \text{ in.}$$

Strain in compression reinforcement

$$\varepsilon_s' = \varepsilon_u\left(\frac{c-d'}{c}\right) = 0.003\left(\frac{8.64-2.38}{8.64}\right) = 0.00218 < \varepsilon_y' = 0.00276$$

Compression steel has not yielded

$a = \beta_1 c = 0.85 \times 8.64 = 7.34$ in.

$C_c = 0.85 f_c' ba = 0.85 \times 4 \times 16 \times 7.34 = 399.3$ kip

$C_s = A_s' E_s \varepsilon_s' = 1.58 \times 29{,}000 \times 0.00218 = 99.9$ kip

$T = A_s E_s \varepsilon_s = 1.58 \times 29{,}000 \times 0.00173 = 79.3$ kip

2. Determine P_n *and* M_n *from static equilibrium.*

$P_n = C_c + C_s - T = 399.3 + 99.9 - 79.3 = 419.9$ kip

$M_n = P_n e = C_c(ha/2 - a/2) + C_s(ha/2 - d') + T(d_t - ha/2)$
$= 399.3(8 - 3.67) + 99.9(8 - 2.38) + 79.3(13.62 - 8) = 2736.1$ in.-kip $= 228$ ft-kip

$e = M_n/P_n = 2736.1/419.9 = 6.52$ in.

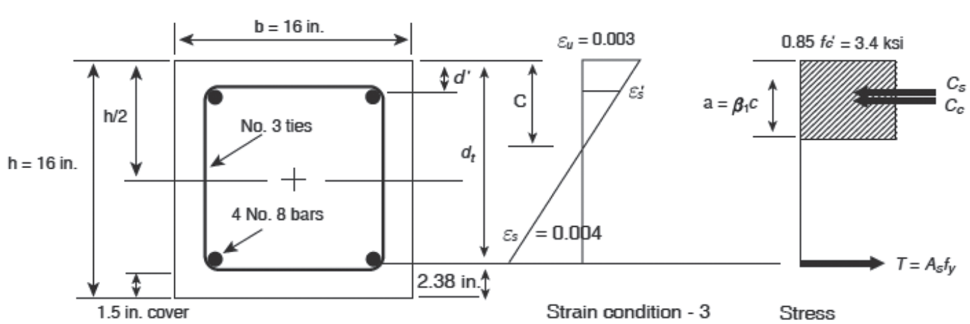
$\phi = 0.65$

(Refer to Section 4.4 of this guide.)

Therefore, for strain condition $\varepsilon_s = 0.5\varepsilon_y = 0.00173$; design axial load strength, $\phi P_n = 0.65(419.9) = 272.9$ kip; and design moment strength $\phi M_n = 0.65\ (228) = 148.2$ ft-kip.

$\varepsilon_s = 0.5\varepsilon_y$	ϕP_n, kip	ϕM_n, ft-kip
Grade 60, PCA, $\varepsilon_s = 0.00104$	335.3	136.9
Grade 100, CS, $\varepsilon_s = 0.00173$	272.9	148.2

(c) Axial load-moment strength, P_n and M_n, for Strain Condition 3: $\varepsilon_s = 0.004$.



1. Define stress distribution and determine force values.

$d' = 2.38$ in.

$d_t = 13.62$ in.

From strain compatibility

$$\left(\frac{c}{0.003}\right) = \left(\frac{d_t}{0.004 + 0.003}\right)$$

$$c = \left(\frac{0.003d_t}{0.004 + 0.003}\right) = \frac{0.003 \times 13.62}{0.004 + 0.003} = 5.84 \text{ in.}$$

Strain in compression reinforcement

$$\varepsilon_s' = \varepsilon_u \left(\frac{c - d'}{c}\right) = 0.003\left(\frac{5.84 - 2.38}{5.84}\right) = 0.00178 < \varepsilon_y' = 0.00276$$

Compression steel has not yielded

$a = \beta_1 c = 0.85 \times 5.84 = 4.96$ in.

$C_c = 0.85 f_c' ba = 0.85 \times 4 \times 16 \times 4.96 = 269.8$ kip

$C_s = A_s' E_s' \varepsilon_s' = 1.58 \times 0.00178 \times 29{,}000 = 81.6$ kips

$T = A_s f_y = 1.58 \times 100 = 158.0$ kip

2. Determine P_n *and* M_n *from static equilibrium.*

$P_n = C_c + C_s - T = 269.8 + 81.6 - 158.0 = 193.4$ kip

$M_n = P_n e = C_c(ha/2 - a/2) + C_s(ha/2 - d') + T(d_t - ha/2)$
$= 269.8(8 - 2.48) + 81.6(8 - 2.38) + 158.0(13.62 - 8) = 2835.8$ in.-kip $= 236.3$ ft-kip

$e = M_n/P_n = 2835.8/193.4 = 14.66$ in.

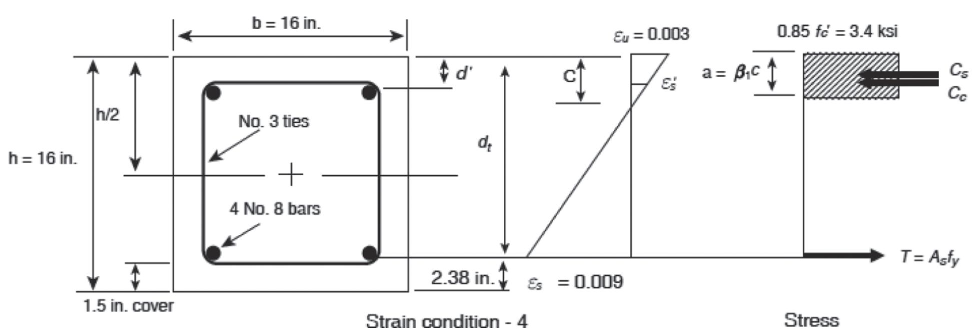
$\phi = 0.65$

(Refer to Section 4.4 of this guide.)

Therefore, for strain condition $\varepsilon_s = 0.004$; design axial load strength $\phi P_n = 0.65(193.4) = 125.7$ kip; and design moment strength, $\phi M_n = 0.65(236.3) = 153.6$ ft-kip.

ε_s	ϕP_n, kip	ϕM_n, ft-kip
Grade 60, PCA, $\varepsilon_s = 0.00207$	242.3	150.1
Grade 100, CS, $\varepsilon_s = 0.004$	125.7	153.6

(d) Axial load-moment strength, P_n and M_n, for Strain Condition 4: $\varepsilon_s = 0.009$.



1. Define stress distribution and determine force values.

$d' = 2.38$ in. $d_t = 13.62$ in.

From strain compatibility

$$\left(\frac{c}{0.003}\right)=\left(\frac{d_t}{0.009+0.003}\right)$$

$$c=\left(\frac{0.003d^t}{0.009+0.003}\right)=\frac{0.003\times 13.62}{0.009+0.003}= \ 3.41 \text{ in.}$$

Strain in compression reinforcement

$$\varepsilon_s{}'=\varepsilon_u\left(\frac{c-d'}{c}\right)=0.003\left(\frac{3.41-2.38}{3.41}\right)=0.00091<\varepsilon_y{}'=0.00276$$

Compression steel has not yielded

$a = \beta_1 c = 0.85 \times 3.41 = 2.90$ in.

$C_c = 0.85f_c'ba = 0.85 \times 4 \times 16 \times 2.90 = 157.8$ kip

$C_s = A_s'E_s\varepsilon_s' = 1.58 \times 29{,}000 \times 0.00091 = 41.7$ kip

$T = A_sf_y = 1.58 \times 100 = 158.0$ kips

2. Determine P_n *and* M_n *from static equilibrium.*

$P_n = C_c + C_s - T = 157.8 + 41.7 - 158.0 = 41.5$ kip

$M_n = P_ne = C_c(ha/2 - a/2) + C_s(ha/2 - d') + T(d_t - ha/2)$
$= 157.8(8 - 1.45) + 41.7(8 - 2.38) + 158.0(13.62 - 8) = 2155.9$ in.-kips $= 179.7$ ft-kip

$e = M_n/P_n = 2155.9/41.5 = 51.9$ in.

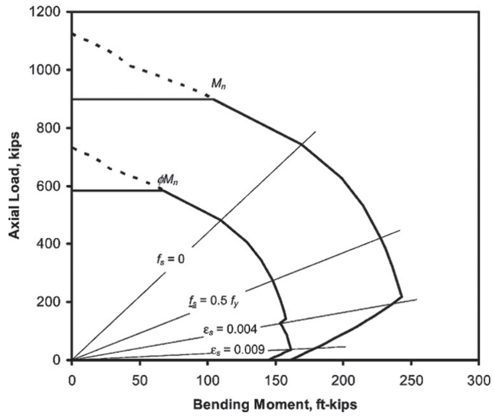
$\phi = 0.9$

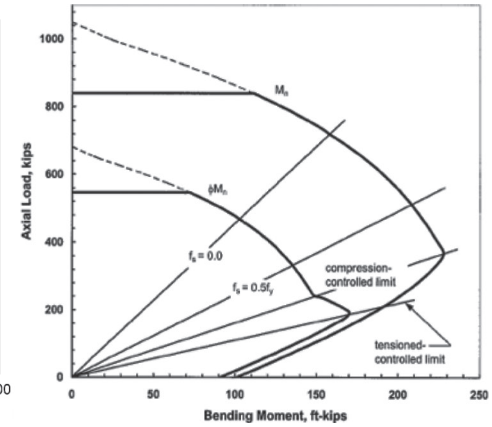
(Refer to Section 4.4 of this guide.)

Therefore, for strain condition $\varepsilon_s = 0.009$; the design axial load strength, $\phi P_n = 0.9\ (41.5) = 37.4$ kips; and the design moment strength, $\phi M_n = 0.9\ (179.7) = 161.7$ ft-kips.

ε_s	ϕP_n, kip	ϕM_n, ft-kip
Grade 60, PCA, $\varepsilon_s = 0.005$	193.4	174.2
Grade 100, CS, $\varepsilon_s = 0.009$	179.7	161.7

The following figure shows a complete interaction diagram for this column. Grade 100, CS versus PCA Figure 6-25 for Grade 60:





Example 6.4 in the *PCA Notes* shows the axial load-bending moment interaction diagram for the same section reinforced with Grade 60 steel. Comparing the behavior of the two columns indicates that, for a given axial load level, using CS reinforcement provides higher nominal bending moment strength than using Grade 60 steel. This difference is greater for tension-controlled sections. For the case of no axial load, the nominal bending moment strength of the column reinforced with CS is 60 percent greater than the column reinforced with Grade 60 steel. For larger axial load levels, the difference between the bending-moment strengths is less. For compression-controlled sections, the capacity of the concrete controls the capacity of the section. The type of reinforcement does not significantly affect the strength of the member.

A similar trend is observed for the axial compression design strength of the columns. For axial load levels between 150 and 250 kip, however, the design bending moment strength of the member reinforced with Grade 60 steel is higher than the column reinforced with CS. These loading conditions represent the transition region from tension-controlled failure to compression-controlled failure. The difference of the design bending moment strength is due to the lower strength reduction factor ϕ used for the high-strength column in this region.

Example 4.3—Design of rectangular beam with tension reinforcement only

Similar to Example 7.1 in the *PCA Notes*, this example highlights differences between beams designed with CS and Grade 60 reinforcement. Select a rectangular beam size and required reinforcement A_s to carry service load moments.

M_D = 56 ft-kip and M_L = 35 ft-kip. Select reinforcement to control flexural cracking.

Use f_c' = 4000 psi and f_y = 100,000 psi.

Calculations and discussion

(a) To illustrate a complete design procedure for rectangular sections with tension reinforcement only, compute a minimum beam depth using the maximum reinforcement permitted for tension-controlled flexural members. The design procedure follows the method in the *PCA Notes* and the recommendations in this guide.

1. Determine maximum tension-controlled reinforcement ratio for material strengths.

$$\rho_t = 0.002125\beta_1 f_c' = 0.002125 \times 0.85 \times 4 = 0.00723$$

(Refer to Table 4.3.4 of this guide.)

2. Compute bd² *required.*

Required moment strength

$$M_u = (1.2 \times 56) + (1.6 \times 35) = 123.2 \text{ ft-kip}$$

$$R_n = \frac{M_n}{bd^2} = \rho f_y\left(1 - \frac{0.5\rho f_y}{0.85 f_c'}\right) = 0.00723 \times 100{,}000 \times \left(1 - \frac{0.5 \times 0.00723}{0.85}\frac{100}{4}\right) = 646.1 \text{ psi}$$

$$bd^2 \text{ (required)} = \frac{M_u}{\phi R_n} = \frac{123.2 \times 12 \times 1000}{0.9 \times 646.1} = 2542.4 \text{ in.}^3$$

3. Size members so that bd^2 *provided* ≥ bd^2 *required.*

Set b = 10 in. (column width)

$$d = \sqrt{\frac{2542.4}{10}} = 15.9 \text{ in.}$$

Minimum beam depth ≈ 15.9 + 2.5 = 18.4 in.

For moment strength, a 10 x 19 in. beam size is adequate. (*PCA Notes*: a 10 x 16 in. beam size is adequate.) However, deflection is an essential consideration in designing beams by the Strength Design Method. Section 4.8 of this guide discusses deflection control, but it is not part of this example.

4. Using the 19 in. beam depth, compute a revised value of ρ.

$$d = 19 - 2.5 = 16.5 \text{ in.}$$

$$R_n = \frac{M_u}{\phi bd^2} = \frac{123.2 \times 12 \times 1000}{0.9 \times 10 \times 16.5^2} = 603.4 \text{ psi}$$

$$\rho = \frac{0.85 f_c'}{f_y}\left(1 - \sqrt{1 - \frac{2R_n}{0.85 f_c'}}\right) = \frac{0.85 \times 4}{100}\left(1 - \sqrt{\frac{2 \times 603.4}{0.85 \times 4000}}\right) = 0.00669$$

5. Compute A_s *required.*

$$A_s = (\text{revised } \rho)(bd \text{ provided}) = 0.00669 \times 10 \times 16.5 = 1.10 \text{ in.}^2$$

(b) Review the correctness of the computations by considering statics.

$T = A_s f_y = 1.10 \times 100 = 110$ kip

$$a = \frac{A_s f_y}{0.85 f_c' b} = \frac{110}{0.85 \times 4 \times 10} = 3.24 \text{ in.}$$

$c = a/\beta_1 = 3.24/0.85 = 3.81$ in.

$$\varepsilon_s = \left(\frac{d-c}{c}\right)0.003 = \left(\frac{16.5-3.81}{3.81}\right)0.003 = 0.01 > 0.009 \quad \textbf{OK}$$

Design moment strength

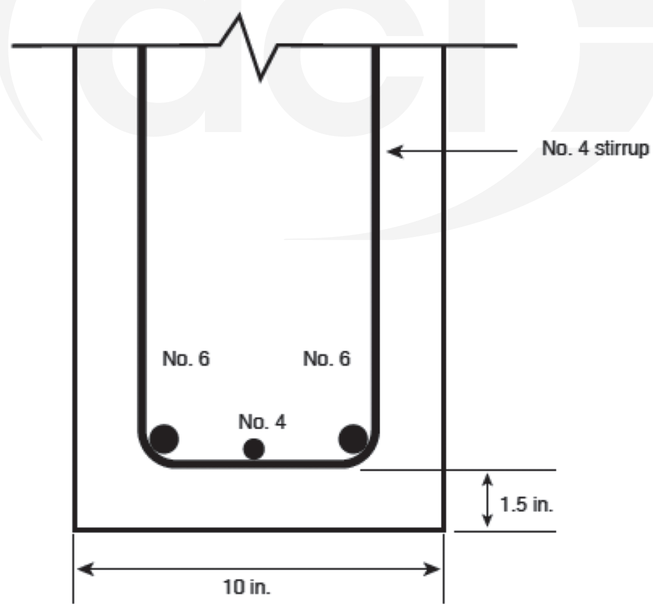
$$\phi M_n = \phi\left[A_s f_y\left(d - \frac{a}{2}\right)\right] = 0.9\left[110\left(16.5 - \frac{3.24}{2}\right)\right]$$

$= 1473.1$ in.-kip $= 122.8$ ft-kip $\approx$ required $M_u = 123.2$ ft-kip **OK**

(c) Select reinforcement to satisfy distribution of flexural reinforcement requirements of ACI 318.

A_s required = 1.10 in.^{2} (*PCA Notes*: A_s required = 2.40 in.^{2})

For illustrative purposes, select two No. 6 and one No. 4 bars (A_s = 1.08). For practical design and detailing, use one bar size for total A_s.



$c_c = 1.5 + 0.5 = 2.0$ in.

Maximum spacing allowed by ACI 318

$$s = 15\left(\frac{40{,}000}{f_s}\right) - 2.5c_c \le 12\left(\frac{40{,}000}{f_s}\right)$$

Use $f_s = 0.67 f_y = 67$ ksi

$$s = \frac{600}{67} - 2.5 \times 2.0 = 8.96 - 5 = 3.96 \text{ in. (governs)}$$

$$s = 12\left(\frac{40}{f_s}\right) = 12\left(\frac{40}{67}\right) = 7.16 \text{ in.}$$

$$\text{spacing provided} = \frac{1}{2}(10 - 2[1.5 + 0.5 + 3/8]) = 2.63 \text{ in.} < 3.96 \text{ in.} \quad \textbf{OK}$$

	Beam size, in.	**A_s, in.^{2}**
Grade 60, PCA	10 x 16	2.40
Grade 100, CS	10 x 19	1.10

The total area of reinforcement required for this design is 1.10 in.^{2} The original design in the *PCA Notes*, which uses Grade 60 steel, requires 2.40 in.^{2} of steel. The use of CS results in a 55 percent reduction of the required flexural reinforcement. The required beam depth, however, is 19 in.—a 20 percent increase compared with the original design. The increased beam depth provides sufficient depth for the compression zone to develop an internal moment within the beam.

Example 6.1—Design of one-way solid slab

Similar to Example 7.2 in the *PCA Notes*, this example highlights differences between slabs designed with CS and Grade 60 reinforcement. Determine required thickness and reinforcement for a one-way slab continuous over more than two equal spans. Clear span ℓ_n = 18 ft. Use f_c' = 4000 psi and f_y = 100,000 psi. Service loads: w_d = 75 psf (assume 6 in. slab), w_ℓ = 50 lb/ft^2.

Calculations and discussion

1. Compute required moment strengths using approximate moment analysis permitted by ACI 318 base design on end span.

Factored load $w_u = (1.2 \times 75) + (1.6 \times 50) = 170$ lb/ft^2

Positive moment at discontinuous end integral with support

$+M_u = w_u\ell_n^2/14 = 0.170 \times 18^2/14 = 3.93$ ft-kip/ft

Negative moment at exterior face of first interior support

$-M_u = w_u\ell_n^2/10 = 0.170 \times 18^2/10 = 5.51$ ft-kip/ft

2. Determine required slab thickness.

For deflection control, choose a reinforcement ratio ρ equal to $0.5\rho_t$, or half of the maximum permitted for tension-controlled sections.

$\rho_t = 0.002125\beta_1 f_c' = 0.002125 \times 0.85 \times 4 = 0.00723$

(Refer to Table 4.3.4 of this guide.)

Set $\rho = 0.5(0.00723) = 0.00362$

Design procedure follows the method outlined previously.

$$R_n = \frac{M_n}{bd^2} = \rho f_y\left(1 - \frac{0.5\rho}{0.85}\frac{f_y}{f_c'}\right) = 0.00362 \times 100,000 \times \left(1 - \frac{0.5 \times 0.00362}{0.85}\frac{100}{4}\right) = 342.7 \text{ psi}$$

$$\text{required } d = \sqrt{\frac{M_u}{\phi R_n b}} = \sqrt{\frac{5.51 \times 12 \times 1000}{0.9 \times 342.7 \times 12}} = 4.23 \text{ in.}$$

Assuming No. 3 bars, required h = 4.23 + 0.375/2 + 0.75 in. (cover) = 5.17 in.

These calculations indicate a slab thickness of 5.5 in. is adequate. Table 9.5(a) in ACI 318-11, however, indicates a minimum thickness of $(\ell/24) \times (0.4 + f_y/100,000) \geq 12.6$ in., unless deflections are computed. Also, note that Table 9.5(a) is applicable only to "members in one-way construction not supporting or attached to partitions or other construction likely to be damaged by large deflections." Otherwise, deflections should be computed.

For purposes of illustration, the required reinforcement is computed using h = 5.5 in., d = 4.56 in., and deflection is not considered. (*PCA Notes*: use h = 4.5 in. and d = 3.59 in.)

3. Compute required negative moment reinforcement.

$$\rho = \frac{0.85 f_c'}{f_y}\left(1 - \sqrt{1 - \frac{2R_n}{0.85 f_c'}}\right) = \frac{0.85 \times 4}{100}\left(1 - \sqrt{\frac{2 \times 294.4}{0.85 \times 4000}}\right) = 0.00308$$

Alternatively, by approximate proportion

revised ρ = (revised R_n/original R_n)(original ρ) = 294.4/342.7 × 0.00362 = 0.00311

$-A_s$ (required) = ρbd = 0.00308 × 12 × 4.56 = 0.17 in.^{2}/ft

Use No. 3 at 8 in. (A_s = 0.165 in.^{2}/ft ~ 0.17 in.^{2}/ft)

(*PCA Notes*: use No. 5 at 10 in. [A_s = 0.37 in.^{2}/ft].)

Check that the beam is tension controlled: $\rho = 0.00308 < \rho_t = 0.00723$ **OK**

4. Compute required positive moment reinforcement.

$$R_n = \frac{M_u}{\phi bd^2} = \frac{3.93 \times 12 \times 1000}{0.9 \times 12 \times 4.56^2} = 210.0 \text{ psi}$$

revised ρ = (revised R_n/original R_n)(original ρ) = 210.0/342.7 × 0.00362 = 0.00222

$+A_s$ (required) = ρbd = 0.00222 × 12 × 4.56 = 0.12 in.^{2}/ft

Use No. 3 at 12 in. (A_s = 0.11 in.^{2}/ft ~ 0.12 in.^{2}/ft)

Maximum spacing allowed by ACI 318

$$s = 15\left(\frac{40,000}{f_s}\right) - 2.5c_c \leq 12\left(\frac{40,000}{f_s}\right)$$

Use $f_s = 0.67f_y$ = 67 ksi

$$s = \frac{600}{f_s} - 2.5c_c = \frac{600}{67} - 2.5 \times 0.75 = 7.08 \text{ in. (governs)}$$

$$s = 12\left(\frac{40}{f_s}\right) = 12\left(\frac{40}{67}\right) = 7.16 \text{ in.}$$

Use No. 3 at 7 in. (A_s = 0.189 in.^{2}/ft) to satisfy spacing requirements (ρ = 0.00345). (*PCA Notes*: use No. 4 at 9 in. [A_s = 0.27 in.^{2}/ft.])

Check that the beam is tension controlled: $\rho = 0.00345 < \rho_t$ **OK**

	Total area of reinforcement	**Slab thickness, in.**
Grade 60, PCA	0.64 in.^{2}	4.5
Grade 100, CS	0.38 in.^{2}	5.5

The total area of reinforcement provided for this design, including positive and negative moment region reinforcement, is 0.38 in.^{2}/ft. The original design in *PCA Notes*, which uses Grade 60 steel, requires a total of 0.64 in.^{2}/ft of steel. The slab thickness in this design is 5.5 in. as compared with 4.5 in. in the original design. Both designs fail to meet the minimum slab thickness for deflection control.

For a complete design, check serviceability requirements such as cracking and deflection, especially for lightly reinforced flexural members with CS reinforcement. Verify adequate shear capacity of the member.

Example 6.1 (SI)—Design of one-way solid slab (SI units)

This example is identical to Example 6.1, but presented in SI units for purposes of illustration. Determine required thickness and reinforcement for a one-way slab continuous over more than two equal spans. Clear span ℓ_n = 5.49 m. Use f_c' = 27.6 MPa and f_y = 690 MPa. Service loads: w_d = 3.59 kN/m^2 (assume 150 mm slab), w_ℓ = 2.39 kN/m^2.

Calculations and discussion

1. Compute required moment strengths using approximate moment analysis permitted by ACI 318M base design on end span.

Factored load $w_u = (1.2 \times 3.59) + (1.6 \times 2.39) = 8.13$ kN/m^2

Positive moment at discontinuous end integral with support

$+M_u = w_u \ell_n^2/14 = 8.13 \times (5.49)^2/14 = 17.5$ kN-m/m

Negative moment at exterior face of first interior support

$-M_u = w_u \ell_n^2/10 = 8.13 \times (5.49)^2/10 = 24.5$ kN-m/m

2. Determine required slab thickness.

For deflection control, choose a reinforcement ratio ρ equal to approximately $0.5\rho_t$, or half of the maximum permitted for tension-controlled sections.

$\rho_t = 0.0003079\beta_1 f_c' = 0.0003079 \times 0.85 \times 27.6 = 0.00722$

(Refer to Table 4.3.4 of this guide.)

Set $\rho = 0.5(0.00722) = 0.00361$

Design procedure follows the method outlined previously.

$$R_n = \frac{M_n}{bd^2} = \rho f_y\left(1 - \frac{0.5\rho f_y}{0.85 f_c'}\right) = 0.00361 \times 690 \times \left(1 - \frac{0.5 \times 0.00361}{0.85}\frac{690}{27.6}\right) = 2.36 \text{ MPa}$$

Assuming 10M bars, required $h = 107 + 11.3/2 + 19$ mm (cover) $= 132$ mm

These calculations indicate a slab thickness of 140 mm is adequate. Table 9.5(a) of ACI 318-11, however, indicates a minimum thickness of $(\ell/24) \times (0.4 + f_y/690) \geq 320$ mm, unless deflections are computed. Also, note that Table 9.5(a) is applicable only to "members in one-way construction not supporting or attached to partitions or other construction likely to be damaged by large deflections." Otherwise, compute deflections.

For purposes of illustration, the required reinforcement will be computed using $h = 140$ mm, $d = 116$ mm, and deflection is not considered. (The design in the *PCA Notes* uses $h = 114$ mm and $d = 91$ mm.)

3. Compute required negative moment reinforcement.

$$R_n = \frac{M_u}{\phi bd^2} = \sqrt{\frac{24.5 \times 1000 \times 1000}{0.9 \times 1000 \times (116)^2}} = 2.02 \text{ MPa}$$

$$\rho = \frac{0.85 f_c'}{f_y}\left(1 - \sqrt{1 - \frac{2R_n}{0.85 f_c'}}\right) = \frac{0.85 \times 27.6}{690}\left(1 - \sqrt{1 - \frac{2 \times 2.02}{0.85 \times 27.6}}\right) = 0.00307$$

Alternatively, by approximate proportion

revised ρ = (revised R_n/original R_n)(original ρ) = $(2.02/2.36) \times 0.00361 = 0.00309$

$-A_s$ (required) $= \rho bd = 0.00307 \times 1000 \times 116 = 356$ mm^2/m

Use 10M bars at 250 mm ($A_s = 400$ mm^2/m > 356 mm^2/m).

Check that the beam is tension controlled: $\rho = 0.00345 < \rho_t = 0.00722$ **OK**

4. Compute required positive moment reinforcement.

$$R_n = \frac{M_u}{\phi bd^2} = \frac{17.5\times1000\times1000}{0.9\times1000\times(116)^2} = 1.44 \text{ MPa}$$

revised ρ = (revised R_n/original R_n)(original ρ) = (1.44/2.36) × 0.00361 = 0.0022

+A_s (required) = ρbd = 0.0022 × 1000 × 116 = 255 mm^2/m

Use 10M bars at 350 mm (A_s = 286 mm^2/m > 255 mm^2/m)

Maximum spacing allowed by ACI 318M

$$s = 380\left(\frac{280}{f_s}\right) = 2.5c_c \leq 300\left(\frac{280}{f_s}\right)$$

Use f_s = (2/3) f_y = 460 MPa

$$s = 380\left(\frac{280}{f_s}\right) = 2.5c_c = \frac{106,400}{460} - 2.5\times19 = 184 \text{ mm}$$

$$s = 300\left(\frac{280}{f_s}\right) = 300\left(\frac{280}{460}\right) = 183 \text{ mm (governs)}$$

Use 10M bars at 180 mm (A_s = 556 mm^2/m) to satisfy spacing requirements (ρ = 0.00479).

Check that the beam is tension controlled: ρ = 0.00479 < ρ_t = 0.00722 **OK**

	Total area of reinforcement	Slab thickness, mm
Grade 60 (420), PCA	1312 mm^2/m	114
Grade 100 (690), CS	956 mm^2/m	140

The total area of reinforcement provided for this design, including positive and negative moment region reinforcement, is 956 mm^2/m. The original design in *PCA Notes*, which uses Grade 420 steel, requires a total of 1312 mm^2/m of steel. The slab thickness in this design is 140 mm as compared with 114 mm in the original design. Both designs fail to meet the minimum slab thickness for deflection control.

For a complete design, check serviceability requirements such as cracking and deflection, especially for lightly reinforced flexural members with CS reinforcement. Verify adequate shear capacity of the member.

Example 6.1A—Design of one-way solid slab (alternate solution)

This example presents an alternative to Example 6.1 of this guide by considering the deflection criterion neglected in Example 6.1. To meet the minimum thickness requirement of Table 9.5(a) in ACI 318-11, the slab should be at least 12.6 in. thickness (refer to Example 6.1). A slab of 13 in. thick is selected.

Determine required thickness and reinforcement for a one-way slab continuous over more than two equal spans. Clear span ℓ_n = 18 ft. Use f_c' = 4000 psi and f_y = 100,000 psi. Service loads: w_d = 162.5 lb/ft^2 (use 13 in. slab), w_l = 50 lb/ft^2.

Calculations and discussion

1. Compute required moment strengths using approximate moment analysis permitted by ACI 318 base design on end span.

Factored load w_u = (1.2 × 162.5) + (1.6 × 50) = 275 lb/ft^2

Positive moment at discontinuous end integral with support

$+M_u = w_u \ell_n^2/14 = 0.275 \times 18^2/14 = 6.36$ ft-kips/ft

Negative moment at exterior face of first interior support

$-M_u = w_u \ell_n^2/10 = 0.275 \times 18^2/10 = 8.91$ ft-kips/ft

2. Compute required negative moment reinforcement.

For 13 in. thick slab, use d = 11 in.

$$R_n = \frac{M_u}{\phi b d^2} = \sqrt{\frac{8.91 \times 12 \times 1000}{0.9 \times 12 \times (11)^2}} = 81.8 \text{ psi}$$

$$\rho = \frac{0.85 f_c'}{f_y}\left(1 - \sqrt{1 - \frac{2R_n}{0.85 f_c'}}\right) = \frac{0.85 \times 4}{100}\left(1 - \sqrt{1 - \frac{2 \times 81.8}{0.85 \times 4000}}\right) = 0.00083 << \rho_t = 0.00723 \quad \textbf{OK}$$

So the slab is tension-controlled.

(Refer to Table 4.3.4 of this guide.)

$-A_s$ (required) = $\rho bd = 0.00083 \times 12 \times 11 = 0.11$ in.^{2}/ft

Check the minimum reinforcement requirement by ACI 318

$$-A_{s,min} = \frac{3\sqrt{f_c'}}{f_y} bd = \frac{3\sqrt{4000}}{100,000}(12)(11) = 0.25 \text{ in.}^2/\text{ft}$$

$$\text{or } -A_{s,min} = \frac{200bd}{f_y} = \frac{200(12)(11)}{100,000} = 0.264 \text{ in.}^2/\text{ft (governs)}$$

Use No. 3 at 5 in. (A_s = 0.264 in.^{2}/ft)

3. Compute required positive moment reinforcement.

By inspection, the positive moment reinforcement should be No. 3 at 5 in.

4. Check maximum reinforcement spacing allowed by ACI 318.

$$s = 15\left(\frac{40,000}{f_s}\right) = 2.5c_c = 12\left(\frac{40,000}{f_s}\right)$$

$c_c = 13 - 11 - 0.375/2 = 1.81$ in.

Use $f_s = 0.67 f_y = 67$ ksi

$$s = \frac{600}{f_s} = 2.5c_c = \frac{600}{67} - 2.5 \times 1.81 = 4.43 \text{ in. (governs)}$$

$$s = 12\left(\frac{40}{f_s}\right) = 12\left(\frac{40}{67}\right) = 7.16 \text{ in.}$$

Use No. 3 at 4.25 in. for positive and negative reinforcements to satisfy the spacing requirements.

5. Check shear strength at the face of the first interior support.

$V_u = 1.15 w_u \ell_n/2 = 1.15 \times 0.275 \times 18/2 = 2.85$ kip/ft wide of slab

$$V_u = 2\sqrt{f_c'}bd = 2\sqrt{4000}(12)(11)/1000 = 16.7 \text{ kip/ft wide of slab} > V_u/\phi$$

If Eq. (6.2) of this guide is used

$$V_u = \frac{73}{38+2.1d}\sqrt{f_c'}bd = \frac{73}{38+2.1(11)}\sqrt{4000}(12)(11)/1000 = 9.98 \text{ kip/ft wide of slab} >> V_u/\phi$$

No shear reinforcement required.

6. Calculate deflections.

Slab thickness has been selected to meet the minimum requirement in Table 9.5(a) of ACI 318-11, but the slab deflections are calculated for purposes of illustration. The deflection of the end span is considered.

Compute modulus of rupture, modulus of elasticity, and modular ratio.

$$f_r = 7.5\sqrt{f_c'} = 7.5\sqrt{4000} = 474 \text{ psi}$$

$$E_c = w_c^{1.5} 33\sqrt{f_c'} = 150^{1.5} 33\sqrt{4000} = 3.83\times10^6 \text{ psi}$$

$N = E_s/E_c = 29 \times 10^6/3.83 \times 10^6 = 7.6$

Compute moment of inertia of gross section and cracking moment.

$I_g = bh^3/12 = 12 \times (13)^3/12 = 2197$ in.^{4}

$M_{cr} = f_r I_g/(h/2) = 474 \times 2197/6.5 = 160{,}212$ in.-lb $= 13.35$ ft-kip

Compute dead load and live load moments.

Negative moments at interior face of first support

$-M_D = 162.5(18)^2/10 = 5265$ ft-lb $= 5.265$ ft-kip

$-M_L = 50(18)^2/10 = 1620$ ft-lb $= 1.62$ ft-kip

Positive moments at discontinuous end integral with support

$M_D = 162.5(18)^2/14 = 3761$ ft-lb $= 3.761$ ft-kip

$M_L = 50(18)^2/14 = 1157$ ft-lb $= 1.157$ ft-kip

Because $M_D + M_L$ in both cases is much less than M_{cr}, use

I_e (equivalent moment of inertia) $= I_g = 2197$ in.^{4}

Based on the commentary of ACI 318, compute the midspan deflection of the end span as

$\Delta = K(5/48)M_a \ell_n^2/E_c I_e$

where $K = 1.2 – 0.2(M_o/M_a)$; $M_o = w\ell^2/8$ (w = load on the span, ℓ = span length); and M_a = positive moment in the span. Therefore,

Under dead load

$M_o = (162.5)(18)^2/8 = 6581$ ft-lb, $M_a = 3761$ ft-lb

$K = 1.2 – 0.2(6581/3761) = 0.85$

Δ_{Di} = immediate deflection due to dead load

$= 0.85(5/48)(3761)(18)^2(1728)/(3.83 \times 10^6)(2197) = 0.022$ in.

Under live load

Δ_{Li} = immediate deflection due to live load by direct proportion

$= 0.022(1157)/(3761) = 0.007$ in. $<< \ell_n/360 = 0.6$ in.

So the immediate deflection under dead and live loads = 0.022 + 0.007 = 0.029 in. $<< \ell_n / 360$.

Consider the effect of long-term deflection based on ACI 318.

$$\lambda_\Delta = \frac{\xi}{1 + 50\rho'}$$

$\xi = 2.0$ for 5 years or more, $\rho' = 0.264/12 \times 11 = 0.002$

$\lambda_\Delta = 2/(1 + 50 \times 0.002) = 1.82$

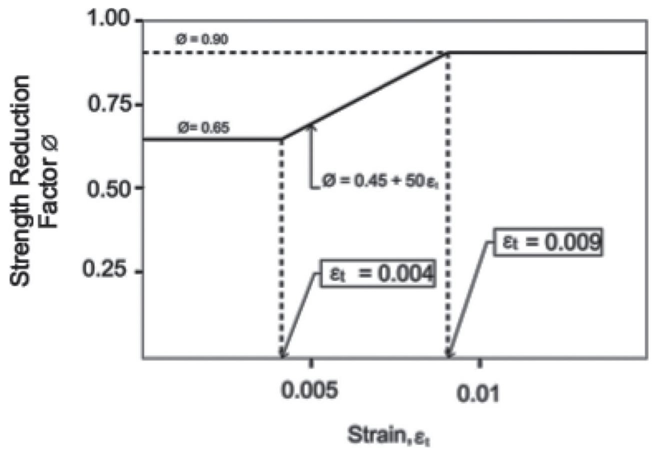
Long-term deflection of the slab under dead load = 0.022 × 1.82 = 0.04 in.

Long-term deflection plus live load deflection = 0.04 + 0.007 = 0.047 in. $<< \ell_n/480 = 0.45$ in.

This example demonstrates that the ACI 318 minimum depth table results were conservative. Based on the deflection control criteria of ACI 318, the slab thickness in this example requires little reinforcement to meet the minimum requirement. Further, the slab remains uncracked under the service load ($D + L$), and the immediate and long-term deflections of the slab are well within the permissible limits. The nominal shear strength of the slab exceeds the factored shear load, so no shear reinforcement is required.

Example 4.4—Design of rectangular beam with compression reinforcement

Similar to Example 7.3 in *PCA Notes*, this example illustrates the benefits of using CS reinforcing bars for design of heavily-reinforced flexural members. A beam cross section is limited to the size shown. Determine the required area of reinforcement for service load moments.



M_D = 430 ft-kip and M_L = 175 ft-kip
Check crack control requirements of Section 10.6.4 of ACI 318-08.
Use f_c' = 4000 psi, f_y = 100,000 psi (tension), and f_y' = 80,000 psi (compression)

Calculations and discussion

(a) Determine required reinforcement.

1. Determine if compression reinforcement is needed.

$M_u = 1.2M_D + 1.6M_L = (1.2 \times 430) + (1.6 \times 175) = 796$ ft-kip

$M_n = M_u/\phi = 796/0.9 = 884$ ft-kip

$$R_n = \frac{M_u}{bd^2} = \sqrt{\frac{884 \times 12 \times 1000}{12 \times 30^2}} = 982 \text{ psi}$$

Determine the maximum value of R_n without compression reinforcement at the tension-controlled strain limit.

$$\rho_t = 0.2125\beta_1 \frac{f_c'}{f_y} = 0.2125(0.85)\frac{4}{100} = 0.00723$$

(Refer to Table 4.3.4 of this guide.)

$$\int \omega_t = \rho_t \frac{f_y}{f_c'} = 0.00723\frac{100}{4} = 0.1808$$

$$\frac{M_{nt}}{f_c' bd^2} = 0.1614 \text{ from Table 7-1 of PCA Notes}$$

$$R_{nt} = \frac{M_{nt}}{bd^2} = 646 \text{ psi} < 982 \text{ psi}$$

Therefore, compression reinforcement is required.

2. Find the nominal strength moment resisted by the concrete section, without compression reinforcement.

$M_{nt} = R_{nt}bd^2 = 646 \times 12 \times 30^2/(12 \times 1000) = 581$ ft-kip resisted by the concrete

Required moment strength resisted by the compression reinforcement

$M_n' = 884 - 581 = 303$ ft-kip

3. Determine the compression steel stress f_s'.

Check yielding of the compression steel. Because the section was designed at the tension-controlled strain limit $\varepsilon_t = 0.009$, $c = 0.25d$

$c = 0.25d = 7.5$ in.

$$\varepsilon_s' = \left(\frac{c-d'}{c}\right)(0.003) = \left(\frac{7.5-2.5}{7.5}\right)(0.003) = 0.002 < \varepsilon_y' = \frac{f_y'}{E_s} = \frac{80}{29,000} = 0.00276$$

Compression reinforcement is elastic at the nominal strength.

$f_s' = E_s\varepsilon_s' = 29,000 \times 0.002 = 58$ ksi

4. Determine the total required reinforcement.

$$A_s' = \frac{M_n'}{f_s'(d-d')} = \frac{303\times12}{58(30-2.5)} = 2.28 \text{ in.}^2$$

$$A_s = f_s'/f_y A_s' + \rho_t bd = 58/100 \times 2.28 + 0.00723 \times 12 \times 30 = 3.93 \text{ in.}^2$$

	A_s, in.^{2}	A_s', in.^{2}
Grade 60, PCA	7.29	0.79
Grade 100, CS	3.93	2.28

5. Check moment capacity.

When the compression reinforcement is elastic.

$$a = \frac{A_s f_y - A_s' f_s'}{0.85 f_c' b} = \frac{3.93\times100 - 2.28\times58}{0.85\times4\times12} = 6.36 \text{ in.}$$

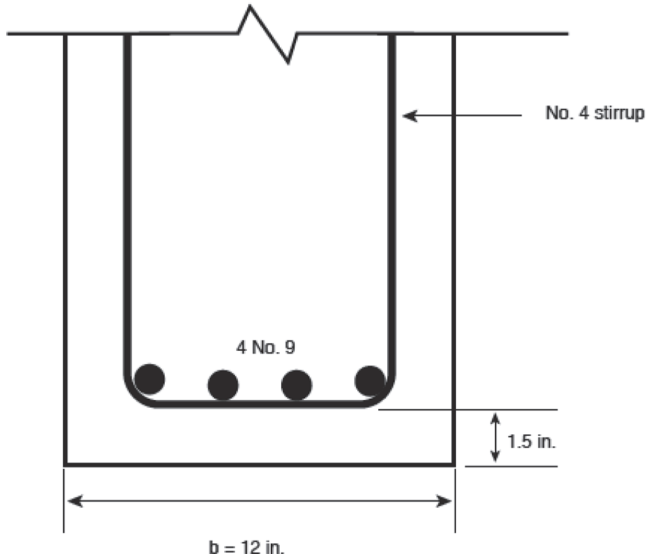
$$\begin{aligned}\phi M_n &= \phi(\rho b d f_y[d - a/2] + A_s' f_s[d - d'])\\ &= 0.9[0.00723 \times 12 \times 30 \times 100 \times (30 - 3.18) + 2.30 \times 58 \times (30 - 2.5)]\\ &= 9584 \text{ in.-kip} = 799 \text{ ft-kip} > M_u = 796 \text{ ft-kip} \quad \textbf{OK}\end{aligned}$$

(b) Select reinforcement to satisfy ACI 318 for control of flexural cracking.

Select three No. 8 bars ($A_s' = 2.37$ in.^{2} > 2.28 in.^{2}) for compression reinforcement.

Select four No. 9 bars ($A_s = 4.00$ in.^{2} > 3.93 in.^{2}) for tension reinforcement.

	Tension reinforcement	Compression reinforcement
Grade 60, PCA	Two No. 6 (0.88 in.^{2})	Six No. 10 (7.62 in.^{2})
Grade 100, CS	Three No. 8 (2.37 in.^{2})	Four No. 9 (4.0 in.^{2})



Maximum spacing allowed by ACI 318

$$s = \frac{600}{f_s} - 2.5c_c \leq 12\left(\frac{40}{f_s}\right)$$

c_c = 1.5 + 0.5 = 2.0 in.

Use $f_s = 0.67f_y$ = 67 ksi

$$s = \frac{600}{f_s} - 2.5c_c = \frac{600}{67} - 2.5 \times 2.0 = 3.96 \text{ in. (governs)}$$

$$s = 12\left(\frac{40}{f_s}\right) = 12\left(\frac{40}{67}\right) = 7.16 \text{ in.}$$

Spacing provided = (1/3)(12 – 2[1.5 + 0.5 + 9/16]) = 2.29 in. < 3.96 in. **OK**

(c) Stirrups or ties are required throughout the distance where compression reinforcement is required for strength.

maximum spacing = 16 × longitudinal bar diameter = 16 × 1.125 = 18 in.
= 48 × tie bar diameter = 48 × 0.5 = 24 in. = least dimension of member = 12 in. (governs)

Use s_{max} = 12 in. for No. 4 stirrups

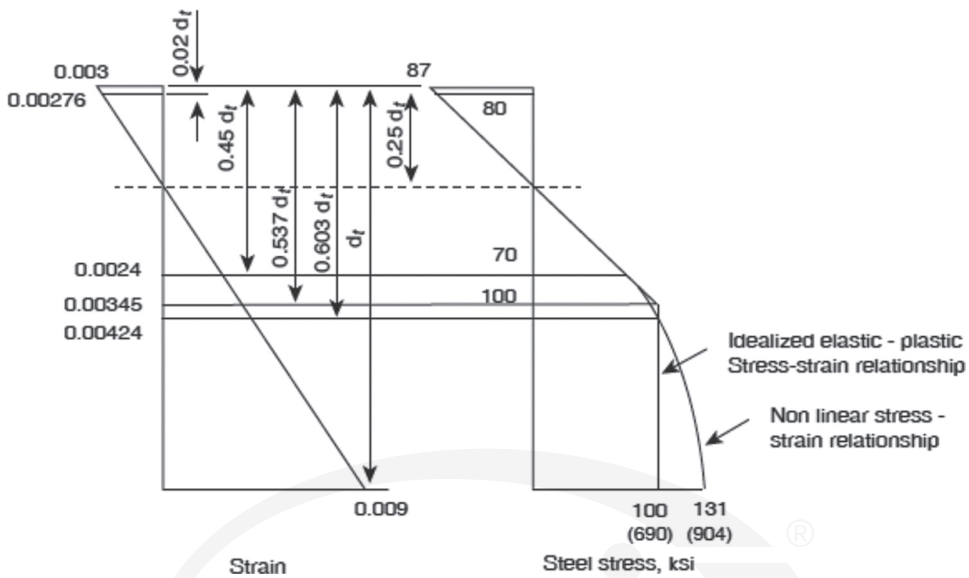
The total area of reinforcement used for this design, including tension and compression reinforcement is 6.37 in.^{2} The original design in the *PCA Notes*, which uses Grade 60 steel, uses a total of 8.5 in.^{2} of reinforcing steel. The use of CS results in a 25 percent reduction of the required flexural reinforcement. The selected high-strength tension reinforcement can be placed as one layer at the beam bottom compared with two layers of conventional steel required by the original design. The design using CS requires less labor to place the reinforcement and tie the reinforcing cages.

Example 4.5—Design of flanged section with tension reinforcement only

Similar to Example 7.4 in *PCA Notes*, this example illustrates the design procedure for flanged beams reinforced with CS. Select reinforcement for the flanged section shown to carry service dead and live load moments of

M_D = 72 ft-kip and M_L = 88 ft-kip

Use f_c' = 4000 psi and f_y = 100,000 psi



Calculations and discussion

1. Determine the required flexural strength.

$M_u = 1.2M_D + 1.6M_L = (1.2 \times 72) + (1.6 \times 88) = 227$ ft-kip

2. Determine depth of equivalent stress block, a, *for a rectangular section. Assume* $\phi = 0.9$.

$M_u = \phi C(d - a/2) = \phi 0.85 f_c' ab(d - a/2)$

$227 \times 12 = 0.9 \times 0.85 \times 4 \times a \times 30 \times (19 - a/2)$

$45.9a^2 - 1744.2a + 2724 = 0$

$a = 1.63$ in. < 2.5

with $a < h_f$, determine A_s for a rectangular section (refer to Example 4.6 for the case when $a > h_f$)

Check ϕ

$c = a/\beta_1 = 1.63/0.85 = 1.92$ in.

$$\varepsilon_s = \left(\frac{d-c}{c}\right)(0.003) = \left(\frac{19-1.92}{1.92}\right)(0.003) = 0.0267 > 0.009$$

Section is tension-controlled, and $\phi = 0.9$

3. Compute A_s *required.*

$$A_s = \frac{0.85 f_c' ab}{f_y} = \frac{0.85 \times 4 \times 1.63 \times 30}{100} = 1.66 \text{ in.}^2$$

Use three No. 7 bars (A_s = 1.8 in.^{2} > 1.66 in.^{2})

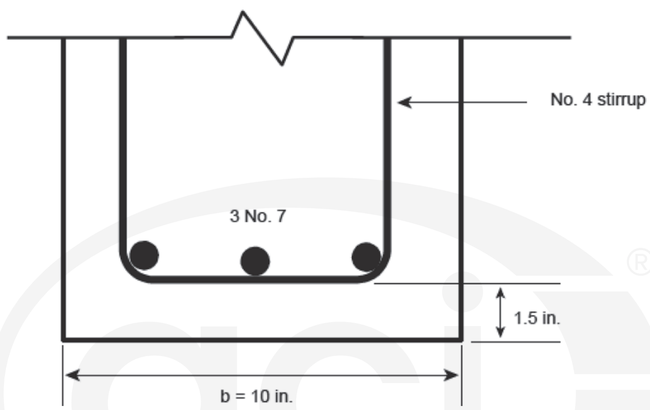
	Tension reinforcement	A_s, in.^{2}
Grade 60, PCA	Three No. 9	3.0 in.^{2}
Grade 100, CS	Three No. 7	1.8 in.^{2}

4. Check minimum required reinforcement.

For f_c' = 4000 psi, the governing requirement is

$$A_{s,min} = \frac{200}{f_y} b_w d = \frac{200}{100,000} \times 10 \times 19 = 0.38 \text{ in.}^2 < A_s \text{ provided} = 1.8 \text{ in.}^2 \quad \textbf{OK}$$

5. Check distribution of reinforcement.



Maximum spacing allowed by ACI 318

$$s = \frac{600}{f_s} - 2.5c_c \le 12\left(\frac{40}{f_s}\right)$$

c_c = 1.5 + 0.5 = 2.0 in.

Use $f_s = 0.67 f_y$ = 67 ksi

$$s = \frac{60}{f_s} - 2.5c_c = \frac{600}{67} - 2.5 \times 2.0 = 3.96 \text{ in. (governs)}$$

$$s = 12\left(\frac{40}{f_s}\right) = 12\left(\frac{40}{67}\right) = 7.16 \text{ in.}$$

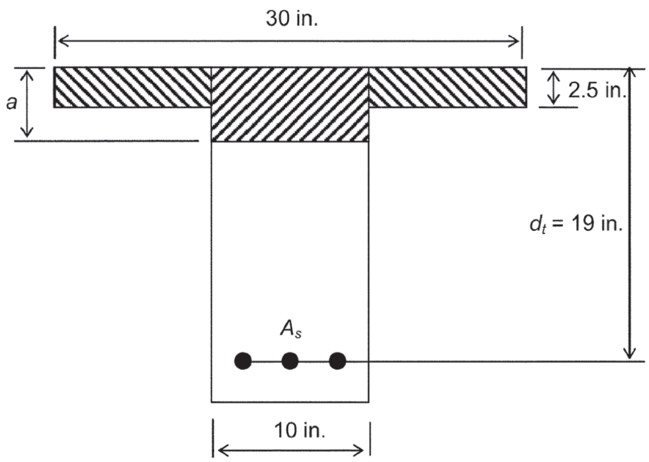
Spacing provided = (1/2)(10 – 2([1.5 + 0.5 + 7/16]) = 2.56 in. < 3.96 in. **OK**

The total area of flexural reinforcement provided by this design is 1.8 in.^{2} The original design in *PCA Notes*, which uses Grade 60 steel, uses 3.0 in.^{2} of reinforcing steel. This represents a 40 percent reduction of the required flexural reinforcement. Flanged sections provide a wide compression block compared with rectangular sections. The wide flange provides adequate compression force to balance the tension force provided by the CS without the need for additional compression reinforcement.

Example 4.6—Design of flanged section with tension reinforcement only

Similar to Example 7.5 in *PCA Notes*, this example illustrates the design procedure for flanged beams reinforced with CS with a neutral axis that lies in the beam web. Select reinforcement for the flanged section shown to carry a factored moment of M_u = 400 ft-kip.

Use f_c' = 4000 psi and f_y = 100,000 psi.



Calculations and discussion

(a) Determine required reinforcement.

1. Determine the depth of equivalent stress block, a, *as for a rectangular section.*

Assume tension-controlled section, $\phi = 0.9$

$M_u = \phi 0.85 f_c' a b_w (d - a/2)$

$400 \times 12 = 0.9 \times 0.85 \times 4 \times a \times 30 \times (19 - a/2)$

$45.9a^2 - 1744.2a + 4800 = 0$

a = 2.98 in. > 2.5 in.

Because the value of a as a rectangular section exceeds the flange thickness, the equivalent stress block extends into the web, and the design should be based on T-section behavior. (Refer to Example 4.5 when a is less than the flange depth.)

2. Compute required reinforcement A_{sf} *and nominal moment strength* M_{nf} *corresponding to the overhanging beam flange in compression.*

Compressive strength of flange

$C_f = 0.85 f_c'(b - b_w)h_f = 0.85 \times 4 \times (30 - 10) \times 2.5 = 170$ kip

Required A_{sf} to equilibrate C_f

$A_{sf} = C_f/f_y = 170/100 = 1.70$ in.^{2}

Nominal moment strength of flange

$$M_{nf} = A_{sf} f_y \left(d - \frac{h_f}{2}\right) = 1.70 \times 100 \times (19 - 1.25) = 3018 \text{ in.-kip} = 252 \text{ ft-kip}$$

3. Required nominal moment strength carried by beam web.

$M_{nw} = M_u/\phi - M_{nf} = 400/0.9 - 252 = 193$ ft-kip

4. Compute reinforcement A_{sw} *required to develop moment strength to be carried by the web.*

$M_{nw} = 0.85f_c'a_w b_w(d - a_w/2)$

$193 \times 12 = 0.85 \times 4 \times a_w \times 10 \times (19 - a_w/2)$

$17a_w^2 - 646a_w + 2316 = 0$

$a_w = 4.01$ in. $> h_f = 2.5$ in.

5. Check to see if section is tension-controlled.

$c_w = a_w/\beta_1 = 4.01/0.85 = 4.72$ in.

$$\varepsilon_s = \left(\frac{d - c_w}{c_w}\right)(0.003) = \left(\frac{19 - 4.72}{4.72}\right)(0.003) = 0.0091 > 0.009$$

Therefore, the section is tension-controlled and $\phi = 0.9$

(Refer to Section 4.4 of this guide.)

$$A_{sw} = \frac{0.85 f_c' a_w b_w}{f_y} = \frac{0.85 \times 4 \times 4.01 \times 10}{100} = 1.36 \text{ in.}^2$$

6. Total reinforcement required to carry factored moment M_u *= 400 ft-kip.*

$A_s = A_{sf} + A_{sw} = 1.70 + 1.36 = 3.06$ in.^{2}

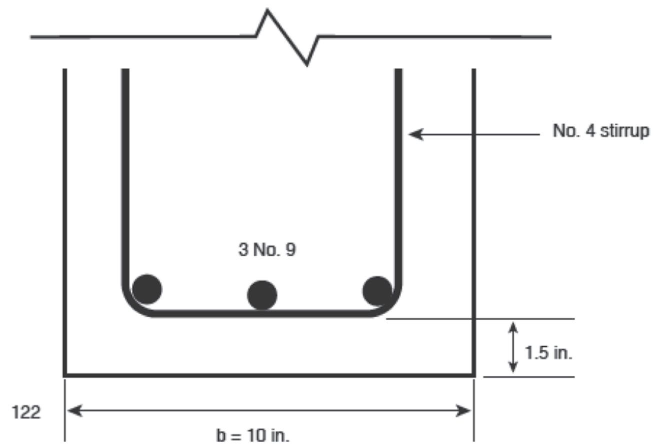
	A_{sf}	A_{sw}
Grade 60, PCA (5.10 in.^{2})	2.83 in.^{2}	2.27 in.^{2}
Grade 100, CS (3.06 in.^{2})	1.70 in.^{2}	1.36 in.^{2}

7. Check moment capacity.

$$\phi M_n = \phi\left(A_{sf} f_y\left[d - \frac{h_f}{2}\right] + A_{sw} f_y\left[d - \frac{a_w}{2}\right]\right) = 0.9\left(1.70 \times 100\left[19 - \frac{2.5}{2}\right] + 1.36 \times 100\left[19 - \frac{4.01}{2}\right]\right)$$

$= 4796$ in.-kip $= 400$ ft-kip **OK**

(b) Select reinforcement to satisfy crack control criteria.



Use three No. 9 bars (A_s = 3.0 in.^{2}, 2 percent less than required, assumed sufficient)

Maximum spacing allowed by ACI 318

$c_c = 1.5 + 0.5 = 2.0$ in.

Use $f_s = 0.67$, $f_y = 67$ ksi

$$s = \frac{600}{f_s} - 2.5c_c = \frac{600}{67} - 2.5\times 2.0 = 3.96 \text{ in. (governs)}$$

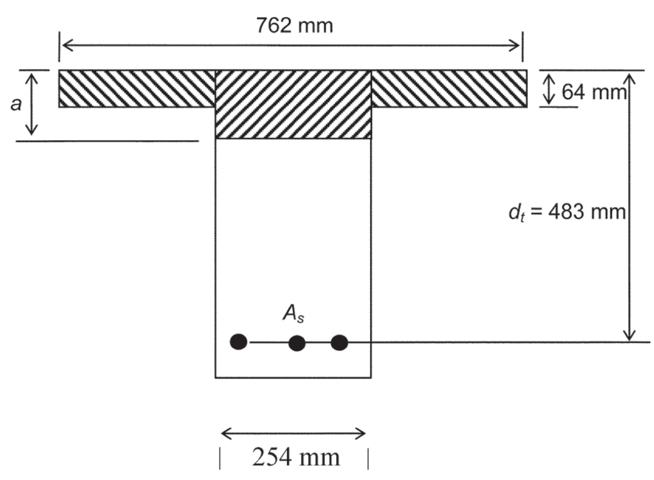
$$s = 12\left(\frac{40}{f_s}\right) = 12\left(\frac{40}{67}\right) = 7.16 \text{ in.}$$

Spacing provided = (1/2)(10 – 2[1.5 + 0.5 + 9/16]) = 2.44 in. < 3.96 in. **OK**

The total area of flexural reinforcement provided in this design is 3.0 in.^{2} The original design in *PCA Notes*, which uses Grade 60 steel, uses 5.0 in.^{2} of reinforcing steel. This represents a 40 percent reduction of the required flexural reinforcement. For members exhibiting T-beam behavior at nominal strength, reduce the required flexural reinforcement by using CS with a suitable design approach.

Example 4.6 (SI)—Design of flanged section with tension reinforcement only (SI units)

This example is identical to Example 4.6 but presented in SI units. Select reinforcement for the flanged section shown, to carry a factored moment of M_u = 542.4 kN-m. Use f_c' = 27.6 MPa, and f_y = 690 MPa.



Calculations and discussion

(a) Determine required reinforcement.

1. Determine the depth of equivalent stress block, a, *as for a rectangular section.*

Assume tension-controlled section, $\phi = 0.9$

$M_u = \phi 0.85 f_c' a b_w (d - a/2)$

$542.4 \times 1000 = 0.9 \times 0.85 \times 27.6 \times a \times 0.762 \times (483 - a/2)$

$8.04a^2 - 7770a + 542{,}400 = 0$

$a = 75.8 \text{ mm} > 64 \text{ mm}$

Because the value of a as a rectangular section exceeds the flange thickness, the equivalent stress block extends into the web, and the design should be based on T-section behavior. (Refer to Example 4.5 when a is less than the flange depth.)

2. Compute required reinforcement A_{sf} *and nominal moment strength* M_{nf} *corresponding to the overhanging beam flange in compression.*

Compressive strength of flange

$C_f = 0.85 f_c'(b - b_w)h_f = 0.85 \times 27.6 \times (762 - 254) \times 64 = 762{,}730 \text{ N}$

Required A_{sf} to equilibrate C_f

$A_{sf} = C_f/f_y = 762{,}730/690 = 1105 \text{ mm}^2$

Nominal moment strength of flange

$$M_{nf} = A_{sf} f_y \left(d_t - \frac{h_f}{2}\right) = 1105 \times 690 \times (483 - 32) = 343{,}865{,}000 \text{ N-mm} = 343.9 \text{ kN-m}$$

3. Required nominal moment strength carried by beam web.

$M_{nw} = M_u/\phi - M_{nf} = 542.4/0.9 - 343.9 = 258.8 \text{ kN-m}$

4. Compute reinforcement A_{sw} *required to develop moment strength to be carried by the web.*

$M_{nw} = 0.85 f_c' a_w b_w (d - a_w/2)$

$258.8 \times 1000 = 0.85 \times 27.6 \times a_w \times 0.254 \times (483 - a_w/2)$

$2.98a_w^2 - 966a_w + 86{,}846 = 0$

$a_w = 100 \text{ mm} > h_f = 64 \text{ mm}$

5. Check to see if section is tension-controlled.

$c_w = a_w/\beta_1 = 100/0.85 = 118 \text{ mm}$

$$\varepsilon_s = \left(\frac{d - c_w}{c_w}\right)(0.003) = \left(\frac{483 - 118}{118}\right)(0.003) = 0.0093 > 0.009$$

Therefore, the section is tension-controlled and $\phi = 0.9$

(Refer to Section 4.4 of this guide.)

$$A_{sw} = \frac{0.85 f_c' a_w b_w}{f_y} = \frac{0.85 \times 27.6 \times 100 \times 254}{690} = 864 \text{ mm}^2$$

6. Total reinforcement required to carry factored moment M_u *= 542.4 kN-m.*

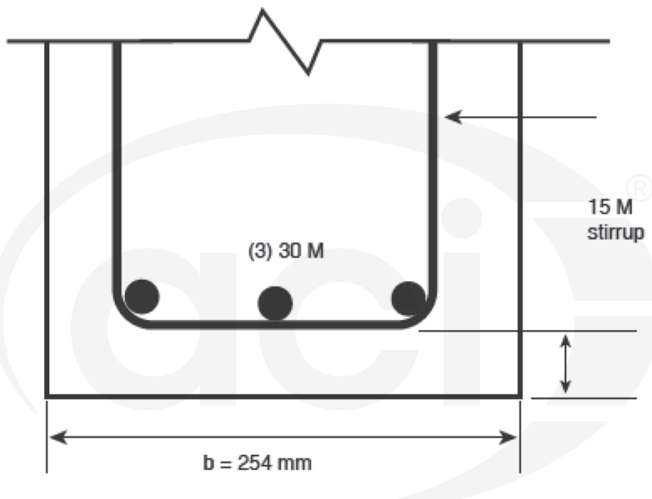
$A_s = A_{sf} + A_{sw} = 1105 + 864 = 1969 \text{ mm}^2$

7. Check moment capacity.

$$\phi M_n = \phi\left(A_{sf} f_y\left[d - \frac{h_f}{2}\right] + A_{sw} f_y\left[d - \frac{a_w}{2}\right]\right) = 0.9\left(1105 \times 690\left[483 - \frac{64}{2}\right] + 864 \times 690\left[483 - \frac{100}{2}\right]\right)$$

$$= 541{,}802{,}000 \text{ N·mm} = 542 \text{ kN·m} \quad \textbf{OK}$$

(b) Select reinforcement to satisfy crack control criteria.



Use three 30M bars ($A_s = 2100 \text{ mm}^2 > 1969 \text{ mm}^2$ required).

Maximum spacing allowed by ACI 318M

$$s = 380\left(\frac{280}{f_s}\right) - 2.5c_c \leq 300\left(\frac{280}{f_s}\right)$$

$c_c = 38 + 16 = 54$ mm

Use $f_s = 0.67 f_y = 462$ MPa

$$s = \frac{106{,}400}{f_s} - 2.5c_c = \frac{106{,}400}{462} - 2.5 \times 54 = 95 \text{ mm (governs)}$$

$$s = 300\left(\frac{280}{f_s}\right) = 300\left(\frac{280}{462}\right) = 182 \text{ mm}$$

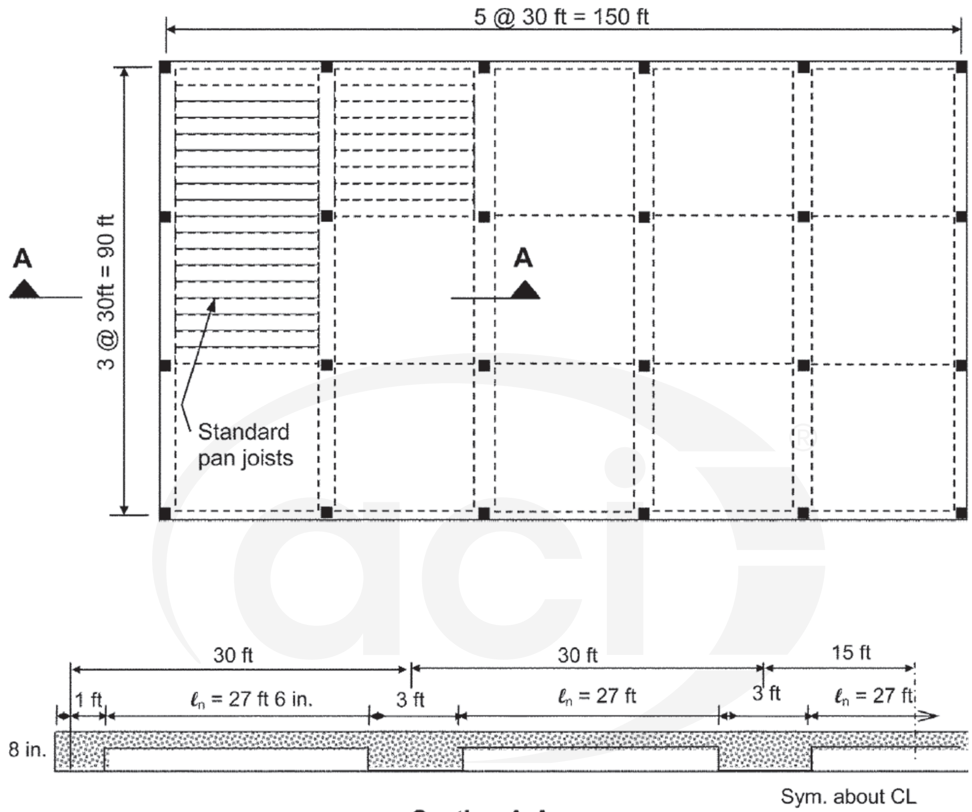
Spacing provided = (1/2)(254 – 2[38 + 16 + 29.9/2]) = 58 mm < 95 mm **OK**

The total area of flexural reinforcement provided in this design is 1969 mm^2. The original design in the *PCA Notes*, which uses Grade 420 steel, uses 3226 mm^2 of reinforcing steel. This represents a 39 percent reduction of the required flexural reinforcement. For members exhibiting T-beam behavior at nominal strength, reduce the required flexural reinforcement by using CS with a suitable design approach.

Example 6.2—Design of one-way joist

Similar to Example 7.6 in *PCA Notes*, this example illustrates the design procedure for continuous one-way joist systems reinforced with CS. Determine the required depth and reinforcement for the one-way joist system shown in the following. The joists are 6 in. wide and are spaced 36 in. on center. The slab is 3.5 in. thick. Use f_c' = 4000 psi and f_y = 100,000 psi.

Service DL = 130 lb/ft^2 (assumed total for joists and beams plus superimposed dead loads)
Service LL = 60 lb/ft^2
Width of spandrel beam = 20 in., width of interior beams = 36 in.
Columns: interior = 18 x 18 in., exterior = 16 x 16 in.
Story height (typ.) = 13 ft



Calculations and discussion

(a) Compute the factored moments at the faces of the supports and determine the depth of the joists.

w_u = [(1.2 × 0.13) + (1.6 × 0.06)] × 3 = 0.756 ft-kip

Using the approximate coefficients, the following table summarizes the factored moments along the span.

Location	M_u, ft-kip
End span	
Exterior negative	$w_u\ell_n^2/24 = 0.756 \times 27.5^2/24 = 23.8$
Positive	$w_u\ell_n^2/14 = 0.756 \times 27.5^2/14 = 40.8$
Interior negative	$w_u\ell_n^2/10 = 0.756 \times 27.25^2/10 = 56.1$
Interior span	
Positive	$w_u\ell_n^2/16 = 0.756 \times 27^2/16 = 34.4$
Negative	$w_u\ell_n^2/11 = 0.756 \times 27^2/11 = 50.1$

For reasonable deflection control, choose a reinforcement ratio ρ equal to approximately one-half ρ_t.

$\rho_t = 0.002125\beta_1 f_c' = 0.002125 \times 0.85 \times 4 = 0.00723$

(Refer to Table 4.3.4 of this guide.)

Set $\rho = 0.5 \times 0.00723 = 0.00362$

Determine the required depth of the joist based on $M_u = 56.1$ ft-kip

$$\omega = \frac{\rho f_y}{f_c'} = \frac{0.00362 \times 100}{4} = 0.0905$$

From Table 7-1 in the *PCA Notes*, $M_u/\phi f_c' bd^2 = 0.0855$

$$d = \sqrt{\frac{M_u}{\phi f_c' b_w (0.0855)}} = \sqrt{\frac{56.1 \times 12}{0.9 \times 4 \times 6 \times 0.0855}} = 19.1 \text{ in.}$$

Allowing 1.25 in. for concrete cover and half bar diameter, then $h \approx 19.1 + 1.25 = 20.4$ in.

To satisfy the requirements for joist construction in ACI 318, $h_{max} = 3.5 \times b_w = 3.5 \times 6 = 21$ in.

These calculations indicate a 21 in. joist depth is adequate. ACI 318, however, indicates a minimum thickness of $(\ell/18.5) \times (0.4 + f_y/100{,}000) = 27$ in., unless deflections are computed. This is applicable only to "members in one-way construction not supporting or attached to partitions or other construction likely to be damaged by large deflections." Otherwise, deflections must be computed.

For purposes of illustration, compute the required reinforcement for a 21 in. deep joist without considering deflection further. Assume $d = 21$ in. – 1.25 in. = 19.75 in.

(b) Compute required reinforcement.

1. End span, exterior negative.

$$\frac{M_u}{\phi f_c' bd^2} = \frac{23.8 \times 12}{0.9 \times 4 \times 6 \times 19.75^2} = 0.0339$$

From Table 7-1 in *PCA Notes,* $\omega \approx 0.0346$

$$A_s = \frac{\omega bd\ f_c'}{f_y} = \frac{0.0346 \times 6 \times 19.75 \times 4}{100} = 0.16 \text{ in.}^2$$

For $f_c' = 4000$ psi, use

$$A_{s,min} = \frac{200 b_w d}{f_y} = \frac{200 \times 6 \times 19.75}{100{,}000} = 0.24 \text{ in.}^2 > 0.16 \text{ in.}^2$$

Use $A_s = 0.24$ in.^{2}

Distribute bars uniformly in top of slab

$A_s = 0.24/3 = 0.08$ in.^{2}/ft

Maximum spacing allowed by ACI 318

$$s = \frac{600}{f_s} - 2.5c_c \leq 12\left(\frac{40}{f_s}\right)$$

Assuming No. 3 bars

$c_c = h - d - 3/16 = 21 - 19.75 - 3/16 = 1.1$ in.

Use $f_s = 0.67f_y = 67$ ksi

$$s = \frac{600}{f_s} - 2.5c_c = \frac{600}{67} - 2.5 \times 1.1 = 6.21 \text{ in. (governs)}$$

$$s = 12\left(\frac{40}{f_s}\right) = 12\left(\frac{40}{67}\right) = 7.16 \text{ in.}$$

Use No. 3 at 6 in. ($A_s = 0.22$ in.^{2}/ft). $A_s = 6 \times 0.11$ in.^{2} = 0.66 in.^{2} in 36 in. for each joist.

(*PCA Notes*: use No. 3 at 10 in.^{2})

Check if the joist is tension-controlled

$$\rho = \frac{A_s}{b_w d} = \frac{0.66}{6 \times 19.75} = 0.0056 < \rho_t = 0.00723 \quad \textbf{OK}$$

2. End span, positive.

$$\frac{M_u}{\phi f_c' bd^2} = \frac{40.8 \times 12}{0.9 \times 4 \times 36 \times 19.75^2} = 0.0097$$

From Table 7-1 in *PCA Notes*, $\omega \approx 0.01$

$$A_s = \frac{\omega b d f_c'}{f_y} = \frac{0.01 \times 36 \times 19.75 \times 4}{100} = 0.28 \text{ in.}^2$$

Check rectangular section behavior

$$a = \frac{A_s f_y}{0.85 f_c' b} = \frac{0.28 \times 100}{0.85 \times 4 \times 36} = 0.22 < 3.5 \text{ in.} \quad \textbf{OK}$$

Use one No. 5 bar ($A_s = 0.31$ in.^{2}) (*PCA Notes*: use two No. 5 bars)

3. End span, interior negative.

$$\frac{M_u}{f_c' bd^2} = \frac{56.1 \times 12}{0.9 \times 4 \times 6 \times 19.75^2} = 0.0799$$

From Table 7-1 in *PCA Notes*, $\omega \approx 0.084$

$$A_s = \frac{\omega b d\ f_c'}{f_y} = \frac{0.084 \times 6 \times 19.75 \times 4}{100} = 0.40 \text{ in.}^2$$

Distribute bars uniformly in top of slab

$A_s = 0.40/3 = 0.13$ in.^{2}/ft

Use No. 3 at 6 in. for crack control considerations as previously described.

(*PCA Notes*: use No. 5 at 12 in.^{2})

4. Obtain reinforcement for the other sections in similar fashion. The following table summarizes the results. Note that at all sections, the requirements in ACI 318 for crack control are satisfied.

Location	M_u, ft-kip	A_s, in.^{2} CS 100 ksi	A_s, in.^{2} PCA 60 ksi	Details CS 100 ksi	Details PCA 60 ksi
End span exterior negative	23.8	0.24	0.37	No.3 @ 6 in.	No.3 @ 10 in.
End span positive	40.8	0.28	0.53	1 No.5	2 No.5
End span interior negative	56.1	0.40	0.73	No.3 @ 6 in.	No.5 @ 12 in.
Interior span positive	34.4	0.23	0.42	2 No.3	2 No.5
Interior span negative	50.1	0.35	0.65	No.3 @ 6 in.	No.5 @ 6 in.

5. The slab reinforcement normal to the joists is located at mid-depth of the slab to resist positive and negative moments.

Use $M_u = \frac{w_u \ell^2}{12} = \frac{0.185 \times 2.5^2}{12} = 0.096$ ft-kip , where w_u = 1.2(44 + 30) + 1.6(60) = 185 lb/ft^2 = 0.185 kip/ft^2.

$$\frac{M_u}{\phi f_c' bd^2} = \frac{0.096 \times 12}{0.9 \times 4 \times 12 \times 1.75^2} = 0.0087$$

From Table 7-1 in *PCA Notes,* $\omega \approx 0.0087$.

$$A_s = \frac{\omega bd\ f_c'}{f_y} = \frac{0.087 \times 12 \times 1.75 \times 4}{100} = 0.01 \text{ in.}^2$$

ACI 318 governs minimum slab reinforcement.

$$A_{s,min} = 0.0018 \times 12 \times 3.5 \times \frac{60,000}{100,000} = 0.05 \text{ in.}^2/\text{ft}$$

$s_{max} = 5h = 5 \times 3.5 = 17.5$ in. but not more than 18 in.

Use No. 3 at 16 in. (A_s = 0.08 in.^{2}/ft)

(*PCA Notes*: Use No. 3 at 16 in.)

(c) Shear at supports should be checked, but it is beyond the scope of this example.

Based on flexural strength requirements, using CS results in a reduction of the required reinforcement between 30 to 65 percent within the joist system as compared with using Grade 60 steel (PCA 2005). One notable exception is at the exterior end of the end span. This corresponds to the location of lowest negative moment. Due to the low magnitude of the applied moment at this location, cracking requirements for the design govern the required steel. Due to the higher level of stress in the reinforcement at the service load level, the maximum allowable spacing for high-strength reinforcement is lower than the spacing used for conventional steel. Consequently, at this location, the amount of CS provided is 67 percent greater than that of the design using Grade 60 steel. The required joist depth for the design using CS is 10 percent greater than using Grade 60 steel. This is due to the higher tension-controlled strain limit of the high-strength reinforcement. The deflection of the joist should be calculated to determine if it is within acceptable limits.

Example 6.3—Flexural design of support beam for one-way joist

Similar to Example 7.7 in *PCA Notes*, this example illustrates the design procedure for continuous beams reinforced with CS. Determine the required depth and reinforcement for the support beams along the interior column line in Example 6.2. The width of the beams is 36 in. Use $f_c' = 4000$ psi and $f_y = 100{,}000$ psi.

Service DL = 130 lb/ft^2 (assumed total for joists and beams plus superimposed dead loads)
Service LL = 60 lb/ft^2
Width of spandrel beam = 20 in.
Width of interior beams = 36 in.
Columns: interior = 18 x 18 in.
exterior = 16 x 16 in.
Story height (typ.) = 13 ft

Calculations and discussion

(a) Compute the factored moments at the faces of the supports and determine the beam depth.

$w_u = [(1.2 \times 0.13) + (1.6 \times 0.06)] \times 30 = 7.56$ kip/ft

Using the approximate coefficients, the following table summarizes the factored moments along the span.

Location	M_u, ft-kip
End span	
Exterior negative	$w_u\ell_n^2/16 = 7.56 \times 28.58^2/16 = 385.9$
Positive	$w_u\ell_n^2/14 = 7.56 \times 28.58^2/14 = 441.1$
Interior negative	$w_u\ell_n^2/10 = 7.56 \times 28.54^2/10 = 615.8$
Interior span	
Positive	$w_u\ell_n^2/16 = 7.56 \times 28.5^2/16 = 383.8$

For overall economy, choose a beam depth equal to the joist depth used in Example 6.2.

Check the 21-in. depth for $M_u = 615.8$ ft-kip.

$\omega_t = 0.2125\beta_1 = 0.2125 \times 0.85 = 0.1806$

$\phi R_{nt} = \omega_t f_c'(1 - 0.59\omega_t) = 0.1806 \times 4 \times (1 - 0.59 \times 0.1806) \times 1000 = 645$ psi

$M_{ut} = \phi R_{nt} bd^2 = 645 \times 36 \times 18.5^2/(12 \times 1000) = 662$ ft-kip $> M_{u,max} = 615.8$ ft-kip **OK**

Section will be tension-controlled without compression reinforcement.

These calculations indicate a beam depth of 21 in. is adequate. ACI 318, however, indicates a minimum thickness of $\ell/18.5 \times (0.4 + f_y/100{,}000) = 27$ in., unless deflections are computed. This is applicable only to "members in one-way construction not supporting or attached to partitions or other construction likely to be damaged by large deflections." Otherwise, deflections must be computed.

For purposes of illustration, compute the required reinforcement for a 21 in. deep beam without considering deflection further.

(b) Compute required reinforcement.

1. End span, exterior negative.

$$\frac{M_u}{\phi f_c' bd^2} = \frac{385.9 \times 12}{0.9 \times 4 \times 36 \times 18.5^2} = 0.1044$$

From Table 7-1 in *PCA Notes*, $\omega \approx 0.112$.

$$A_s = \frac{\omega bd\ f_c'}{f_y} = \frac{0.112 \times 36 \times 18.5 \times 4}{100} = 2.98\text{ in.}^2$$

Check for minimum required reinforcement according to ACI 318.

For $f_c' = 4000$ psi, use

$$A_{s,min} = \frac{200 b_w d}{f_y} = \frac{200 \times 36 \times 18.5}{100,000} = 1.33\text{ in.}^2 < A_s \quad \textbf{OK}$$

Use 10 No. 5 bars ($A_s = 3.10$ in.^{2}).

(*PCA Notes*: use seven No. 8 bars [$A_s = 5.53$ in.^{2}].)

Check distribution of flexural reinforcement requirements of ACI 318

Maximum spacing allowed by ACI 318.

$$s = \frac{600}{f_s} - 2.5c_c \leq 12\left(\frac{40}{f_s}\right)$$

$c_c = 1.5 + 0.5 = 2.0$ in.

Use $f_s = 0.67f_y = 67$ ksi

$$s = \frac{600}{f_s} - 2.5c_c = \frac{600}{67} - 2.5 \times 2.0 = 3.96\text{ in. (governs)}$$

$$s = 12\left(\frac{40}{f_s}\right) = 12\left(\frac{40}{67}\right) = 7.16\text{ in.}$$

$$\text{spacing provided} = \frac{1}{9}\left(36 - 2\left[1.5 + 0.5 + \frac{5}{16}\right]\right) = 3.49\text{ in.} < 3.96\text{ in.} \quad \textbf{OK}$$

2. End span, positive.

$$\frac{M_u}{\phi f_c' bd^2} = \frac{441.1 \times 12}{0.9 \times 4 \times 36 \times 18.5^2} = 0.1193$$

From Table 7-1 in *PCA Notes*, $\omega \approx 0.129$.

$$A_s = \frac{\omega bd\ f_c'}{f_y} = \frac{0.129 \times 36 \times 18.5 \times 4}{100} = 3.44\text{ in.}^2$$

Use 11 No. 5 bars ($A_s = 3.41$ in.^{2}) (one percent less than required, assumed sufficient).

(*PCA Notes*: use 11 No. 7 bars [$A_s = 6.60$ in.^{2}].)

Note that this reinforcement satisfies the cracking requirements in ACI 318 and fits within the beam width. It also can be used conservatively at the midspan section of the interior span.

3. End span, interior negative.

$$\frac{M_u}{\phi f_c' bd^2} = \frac{615.8 \times 12}{0.9 \times 4 \times 36 \times 18.5^2} = 0.1666$$

From Table 7-1 in *PCA Notes*, $\omega \approx 0.187$.

$$A_s = \frac{\omega bd\ f_c'}{f_y} = \frac{0.187 \times 36 \times 18.5 \times 4}{100} = 4.98 \text{ in.}^2$$

Use nine No. 7 bars (A_s = 5.40 in.^{2}). (*PCA Notes*: use 10 No. 9 bars [A_s = 10.0 in.^{2}].)

$$\text{spacing provided } = \frac{1}{8}\left(36 - 2\left[1.5 + 0.5 + \frac{7}{2 \times 8}\right]\right) = 3.89 \text{ in.} < 3.96 \text{ in.} \quad \textbf{OK}$$

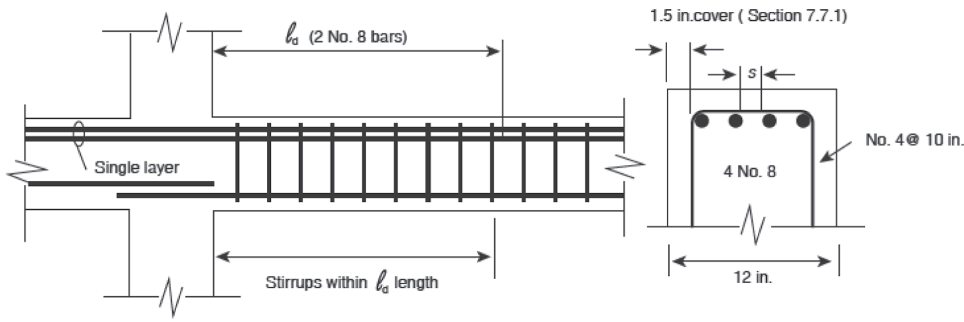
	PCA Grade 60 A_s, in.^{2}	PCA Grade 60 details	CS Grade 100 A_s, in.^{2}	CS Grade 100 details
End span, exterior negative	3.10	10 No.5	5.53	7 No.8
End span, positive	3.41	11 No.5	6.60	11 No.7
End span, interior negative	5.40	9 No.7	10.00	10 No.9

High-strength steel (HSS) reinforcement was 3.10 in.^{2} to 5.40 in.^{2} at locations throughout the beam. The original design in *PCA Notes* provided 5.53 in.^{2} to 10.0 in.^{2} of Grade 60 reinforcement. This is a reduction of 45 to 50 percent at the corresponding locations in the beam. The beam depth was 10 percent greater using CS. Calculate beam deflection to determine if it is within acceptable limits.

Example 10.1—Development of bars in tension

Similar to Example 4.3 in *PCA Notes*, this example illustrates the calculation of the development for CS reinforcing bars using ACI 408R equations.

Calculate required development length for the inner two No. 8 bars in the beam shown in the following. Make the two No. 8 outer bars are continuous along the full length of beam. Use f_c' = 4000 psi (normalweight concrete) and f_y = 100,000 psi, and uncoated bars. Stirrups provide the minimum code requirements for beam shear reinforcement.



Calculations and discussion

Determine the development length of No. 8 bar with f_y = 100,000 psi.

Nominal bar diameter of No. 8 bar = 1.0 in.

$$\ell_d = \frac{\left(\frac{f_y}{(f_c')^{1/4}} - \phi 2400\omega\right)\alpha\beta_c\lambda}{\phi 76.3\left(\frac{c\omega + K_{tr}}{d_b}\right)} d_b$$

(Refer to Section 10.2 of this guide.)

α = 1.3 for top cast bars; β_c = 1.0 for uncoated bars.

λ = 1.0 for normalweight concrete.

Clear spacing = [12 – 2(cover) – two (No. 4 stirrups) – four No. 8 bars)]/three spaces
= [12 – 2(1.5) – 2(0.5) – 4(1.0)]/3 = 1.33 in.

c_{si} = 1/2 clear spacing = 0.67 in.

$c_{so} = c_b$ = cover + No. 4 stirrup = 1.5 + 0.5 = 2.0 in.

$c_s = \min(c_{so}, c_{si} + 0.25) = \min(2.0, 0.92) = 0.92$ in.

$c_{min} = \min(c_s, c_b) = \min(0.92, 2.0) = 0.92$ in.

$c_{max} = \max(c_s, c_b) = \max(0.92, 2.0) = 2.0$ in.

$c = c_{min} + 0.5d_b = 0.92 + 0.5(1.0) = 1.42$ in.

$$\omega = 0.1\left(\frac{c_{max}}{c_{min}}\right) + 0.9 \leq 1.25 = 0.1\left(\frac{2.0}{0.92}\right) + 0.9 = 1.12 < 1.25$$

$$K_{tr} = \frac{0.52 t_r t_d A_{tr}}{sn}\sqrt{f_c'}$$

$t_r = 9.6R_r + 0.28 \leq 1.72 = 9.6(0.07) + 0.28 = 0.95 \leq 1.72$

$t_d = 0.78d_b + 0.22 = 0.78(1.0) + 0.22 = 1.0$ in.

$s = 10$ in. spacing of stirrups

$n =$ two bars being developed

A_{tr} (two No. 4) = 2(0.2) = 0.4 in.^{2}

$$K_{tr} = \frac{(0.52)(0.94)(1.0)(0.4)}{(10)(2)}\sqrt{4000} = 0.62 \text{ in.}$$

$$\frac{c\omega + K_{tr}}{d_b} = \frac{(1.42)(1.12) + 0.62}{1.0} = 2.21 < 4$$

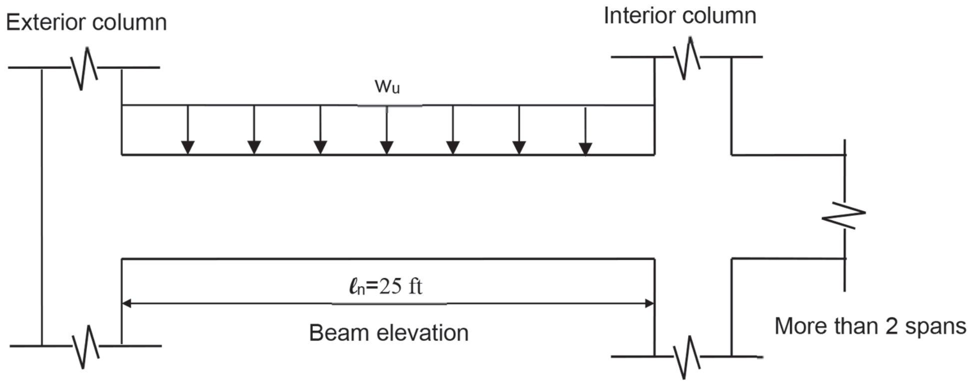
$$\ell_d = \frac{\left(\dfrac{100{,}000}{(4000)^{1/4}} - (0.8)(2400)(1.12)\right)(1.3)(1.0)(1.0)}{(0.8)(76.3)(2.21)}(1.0) = 100.5 \text{ in.}$$

The development length ℓ_d for two conventional No. 8 Grade 60 bars (*PCA Notes*, Example 4.3) was calculated using two different methods allowed in ACI 318 as 61.7 in. and 47.0 in. The calculated development length for the high-strength reinforcing bars represents an increase of 63 and 114 percent. This indicates that with a yield strength of 100,000 psi, CS bars require increased development length.

Example 10.2—Development of flexural reinforcement

Similar to Example 4.4 in *PCA Notes*, this example illustrates the calculation of the required bar lengths for continuous beams reinforced with CS. This example also highlights design differences between continuous beams reinforced with CS as compared with Grade 60 reinforcement.

Determine the lengths of the top and bottom bars for the exterior span of the continuous beam shown in the following. The concrete is normal weight and the bars are Grade 100. The total uniformly distributed factored gravity load on the beam is w_u = 6.0 kip/ft (including weight of the beam). Use f_c' = 4000 psi and f_y = 100,000 psi; b = 16 in.; and h = 22 in. Concrete cover = 1.5 in.



Calculations and discussion

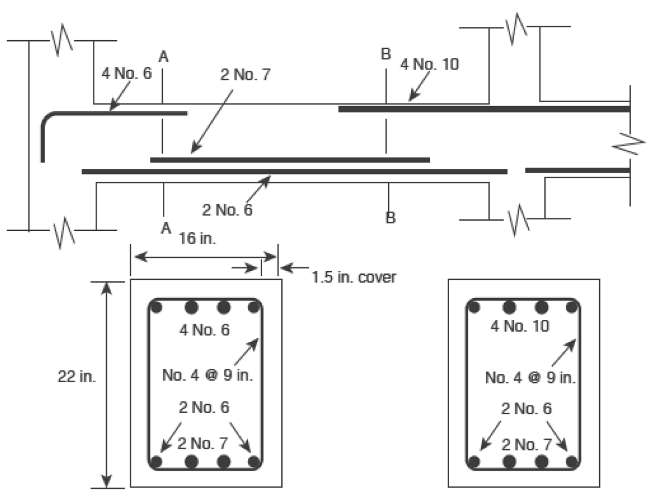
(a) Preliminary design for moment and shear reinforcement

1. Use approximate analysis for moment and shear.

Location	Factored moments and shears
Interior face of exterior support	$-M_u = w_u\ell_n^2/16 = 6(25^2)/16 = -234.4$ ft-kip
End span positive	$+M_u = w_u\ell_n^2/14 = 6(25^2)/14 = 267.9$ ft-kip
Exterior face of first interior support	$-M_u = w_u\ell_n^2/10 = 6(25^2)/10 = -375.0$ ft-kip
Exterior face of first interior support	$V_u = 1.15w_u\ell_n/2 = 1.15(6)(25)/2 = 86.3$ kip

2. Determine required flexural reinforcement using procedures of Chapter 4 of this guide. With 1.5 in. cover, No. 4 bar stirrups, and No. 6 or 7 flexural bars, d ≈ *19.6 in.*

M_u	A_s required	Bars	A_s provided
–234.4 ft-kip	1.74 in.^{2}	Four No. 6	1.76 in.^{2}
267.9 ft-kip	2.02 in.^{2}	Two No. 6 and Two No. 7	2.08 in.^{2}
–375 ft-kip	4.64 in.^{2}	Four No. 10	5.08 in.^{2}



Using the reinforcement outlined in the previous table, the beam exhibits a compression-controlled failure at the exterior face of the first interior support and therefore $\phi = 0.65$ is used. Due to the higher tension-controlled strain limit for the high-strength reinforcing bars, the section cannot be designed as tension-controlled without including additional compression reinforcement. The bars on the bottom of the beam do not have adequate length to develop the full yield strength in compression at the exterior face of the first interior support. Make the bars continuous across the support to provide the necessary anchorage. This reduces the required top reinforcement at this section by approximately 18 percent ($A_s = 3.35$ in.^{2}). Check the bar spacing of the proposed reinforcing to satisfy crack control limits. For illustration purposes, continue with the reinforcement as outlined.

3. Determine required shear reinforcement using V_c *per ACI 318.*

V_u at d, distance from face of support

$V_u = 86.3 - 6(19.6/12) = 76.5$ kip

$$\phi V_c = \phi(2\sqrt{f_c'}b_w d) = 0.75(2)(\sqrt{4000})(16)(19.6)/1000 = 29.8 \text{ kip}$$

Use No. 4 stirrups at 9 in. spacing < $s_{max} = d/2 = 9.8$ in.

$$\phi V_s = \frac{\phi A_v f_y d}{s} = \frac{0.75(0.4)(80)(19.6)}{9} = 52.3 \text{ kip}$$

$\phi V_n = \phi V_c + \phi V_s = 29.8 + 52.3 = 82.1$ kip > 76.6 kip **OK**

Distance from support where stirrups not required

$V_u < 1/2\phi V_c = 29.8 / 2 = 14.9$ kip

$V_u = 86.3 - 6x = 14.9$ kip

$x = 11.9$ ft ≈ 1/2 span

Use No. 4 U stirrups at 9 in. (entire span).

(*PCA Notes*: use No. 4 U stirrups at 7 in. [entire span].)

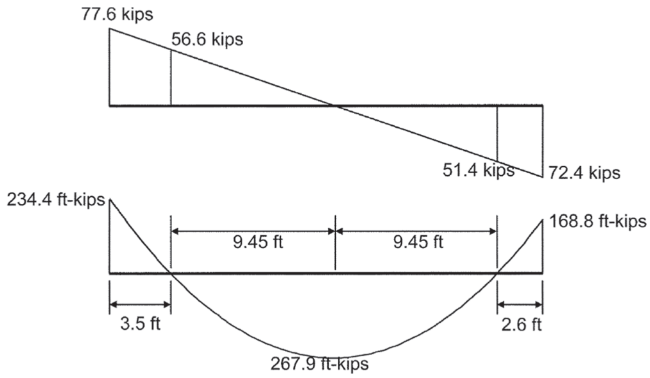
(b) Bar lengths for bottom reinforcement

1. Required number of bars to be extended into supports.

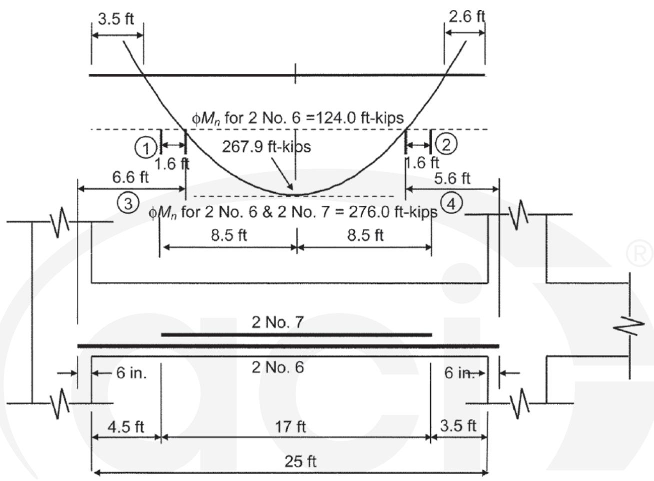
Extend 1/4 of ($+A_s$) at least 6 in. into the supports. With a longitudinal bar required at each corner of the stirrups of ACI 318, at least two bars should be extended full length. Extend the two No. 6 bars full span length (plus 6 in. into the supports) and cut off the two No. 7 bars within the span.

2. Determine cut-off locations for the two No. 7 bars and check other development requirements.

Shear and moment diagrams for loading condition causing maximum factored positive moment are shown in the following figure.



The positive moment portion of the M_u diagram is shown in the following at a larger scale, including the design moment strengths ϕM_n for the total A_s (two No. 6 and two No. 7) and for two No. 6 bars separately. For two No. 6 and two No. 7, ϕM_n = 276.0 ft-kip. For two No. 6, ϕM_n = 124 ft-kip.



The two No. 6 bars extend full span length plus 6 in. into the supports. Cut the two No. 7 bars at 4.5 ft and 3.5 ft from the exterior and interior supports. Determine these cutoff locations as follows.

Dimensions (1) and (2) should be the larger of d or $12d_b$

d = 19.6 in.^{2} = 1.6 ft (governs)

$12d_b$ = 12(0.88) = 10.5 in.

Within the development length ℓ_d, only two No. 6 bars are developed (two No. 7 bars are already developed in the length of 8.5 ft).

Development for No. 6 corner bars (refer to *PCA Notes* Example 4.3 for a detailed calculation of development length for CS reinforcing bars) is as follows.

ℓ_d = 31 in. = 2.6 ft

Dimension (3): 6.6 ft > 2.6 ft **OK**

Dimension (4): 5.6 ft > 2.6 ft **OK**

Check required development length ℓ_d for two No. 7 bars. Two No. 6 bars are already developed in length 2.6 ft from bar end.

ℓ_d = 44 in. = 3.7 ft < 8.5 ft **OK**

For No. 6 bars, check development requirements at point of inflection (PI):

$$\ell_d \leq \frac{M_n}{V_u} + \ell_a$$

For two No. 6 bars, M_n = 124.0/0.9 = 137.8 ft-kip

At the left PI, V_u = 77.6 – 6(3.5) = 56.6 kip

ℓ_a = larger of $12d_b$ = 9 in. or d = 19.6 in. (governs)

$$\ell_d \leq \frac{137.8 \times 12}{56.6} + 19.6 = 48.8 \text{ in.}$$

For No. 6 bars, ℓ_d = 31 in. < 48.8 in. **OK**

At the right PI, V_u = 51.4 kip; by inspection, the development requirements for the No. 6 bars are satisfactory.

With both tentative cutoff points located in a zone of flexural tension, one of the three conditions of ACI 318 should be satisfied. The applicability of this provision to high-strength reinforcing bars has not been demonstrated experimentally. The code provisions are checked herein for illustrative purposes.

At left cutoff point (4.5 ft from support).

V_u = 77.6 – (6)(4.5) = 50.6 kip

ϕV_n = 82.1 kip (No. 4 U-stirrups at 9 in.)

2/3(82.1) = 54.7 kip > 50.6 kip **OK**

For illustrative purposes, determine if the bar cutoff point of Section 12.10.5.3 of ACI 318-11 is also satisfied:

M_u = 54.1 ft-kip at 4.5 ft from support

A_s = 0.38 in.^{2}

For two No. 6 bars, A_s provided = 0.88 in.^{2}

0.88 in.^{2} > 2(0.38) = 0.76 in.^{2} **OK**

3/4(82.1) = 61.6 kip > 50.6 kip **OK**

Therefore, ACI 318 is also satisfied at cutoff location.

At right cutoff point (3.5 ft from support)

V_u = 72.4 – (3.5)(6) = 51.4 kip

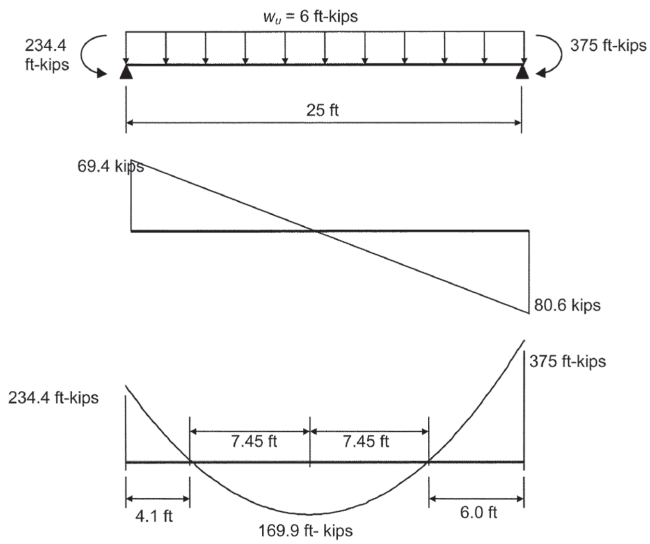
2/3(ϕV_n) = 54.7 kip > 51.4 kip **OK**

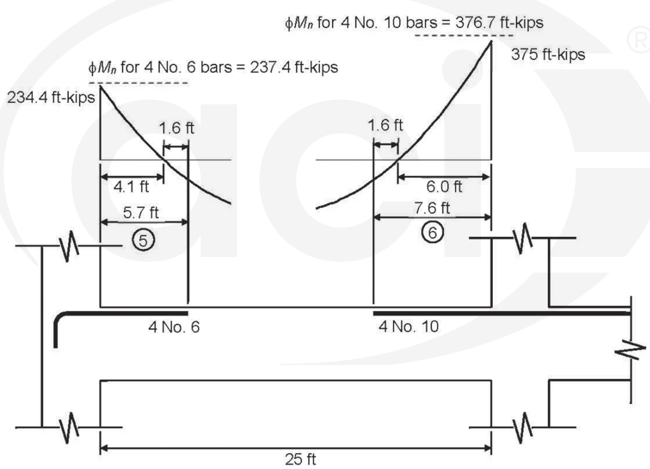
Summary: The tentative cutoff locations for the bottom reinforcement meet all proposed development requirements. Place the two No. 7 bars by 17 ft unsymmetrically within the span. Ensure proper placement of the No. 7 bars by specifying an 18 ft length for symmetrical bar placement within the span, 3.5 ft from each support. The recommended bar arrangement is shown at the end of this example.

(c) Bar lengths for top reinforcement

Shear and moment diagrams for loading condition causing maximum factored negative moments are shown in the following.

The negative moment portions of the M_u diagram are shown in the following at a larger scale, including the design moment strengths ϕM_n for the total negative A_s at each support (four No. 6 at exterior support and four No. 10 at interior support). For four No. 6, ϕM_n = 237.4 ft-kip (ϕ = 0.9). For four No. 10, ϕM_n = 376.7 ft-kip (ϕ = 0.65).





(d) Development requirements for four No. 6 bars at exterior support

1. Required number of bars to be extended.

Extend 1/3 of ($-A_s$) provided at supports beyond the inflection point a distance equal to the greater of d, $12d_b$, or $\ell_n/16$.

d = 19.6 = 1.6 ft (governs)

$12d_b$ = 12(0.75) = 9.0 in.

$\ell_n/16$ = 25 × 12/16 = 18.8 in.

Because the inflection point is 4.1 ft from the support, the total length of the No. 6 bars will be short even with the required 1.6 ft extension beyond the inflection point. Check the required development length ℓ_d for a cutoff location at 5.75 ft (5 ft 9 in.) from the support face.

Dimension (5) in the aforementioned figure should be at least equal to ℓ_d.

For No. 6 bars, ℓ_d = 31 in.

With four No. 6 bars developed at same location (support face)

Including top bar effect, $\ell_d = 1.3(31) = 40.3$ in.

For No. 6 top bars, $\ell_d = 40.3$ in. = 3.35 ft < 5.75 ft **OK**

2. Anchorage into exterior column.

The No. 6 bars can be anchored into the column with a standard end hook following the provisions of ACI 318. Recent tests have shown that ASTM A1035/A1035M bars with standard hooks can develop tensile bar stresses of at least 140,000 psi (965 MPa) and, in many cases, the strength of the bar. (Refer to Section 10.2 of this guide.)

$$\ell_{dh} = \left(0.02\psi_e \frac{f_y}{\lambda}\sqrt{f_c'}\right)d_b$$

$\psi_e = 1.0$ for uncoated bars

$\lambda = 1.0$ for normalweight concrete

$$\ell_{dh} = (0.02)(1.0)\left(\frac{100,000}{1.0}\right)\sqrt{4000}(0.75) = 23.7 \text{ in.}$$

The required ℓ_{dh} for the hook is reduced if excess reinforcement is considered. Because A_s (provided) ≈ A_s (required), however, this effect is not considered in this example.

Overall depth of column required is 23.7 + 2 = 25.7 in.

(e) Development requirements for four No. 10 bars at interior column.
1. Required extension for one-third of (–A_s)

d = 19.6 in. = 1.6 ft (governs)

$12d_b = 12(1.27) = 15.2$ in.

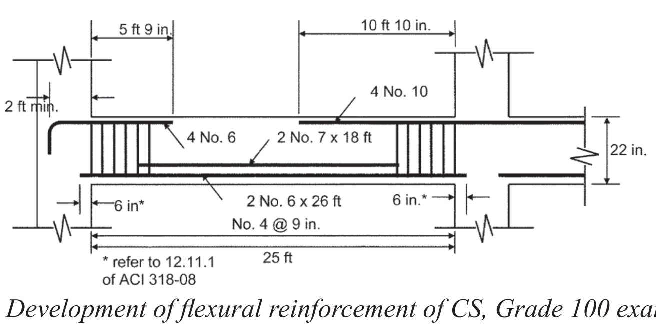
$\ell_n/16 = 18.8$ in.

For No. 10 bars, $\ell_d = 129.6$ in. = 10.8 ft

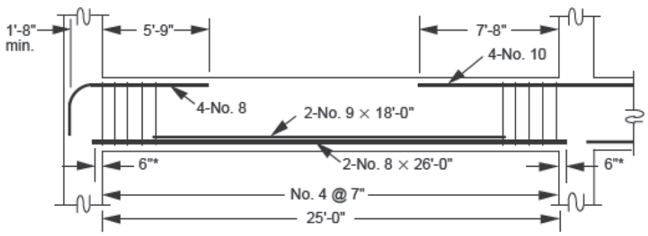
Dimension (6) in the aforementioned figure = 7.6 ft < ℓ_d = 10.8 ft **NG**

Extend the No. 10 bars to provide the required ℓ_d beyond the face of the column.

(f) Summary: selected bar lengths for the top and bottom reinforcement is shown in the following.



Development of flexural reinforcement of CS, Grade 100 example



Development of flexural reinforcement of PCA Grade 60 example

(g) Supplementary requirements

If the beam was part of a primary lateral load-resisting system, the two No. 6 bottom bars extending into the supports would have to be anchored to develop the bar yield strength at the face of supports. At the exterior column, anchorage should be provided by using end hooks or headed bars.

At the interior column, the two No. 6 bars could be extended a distance ℓ_d beyond the support face into the adjacent span, mechanically coupled or lap spliced with extended bars from the adjacent span. A Class A lap splice or a mechanical splice may be considered to satisfy the intent of ACI 318.

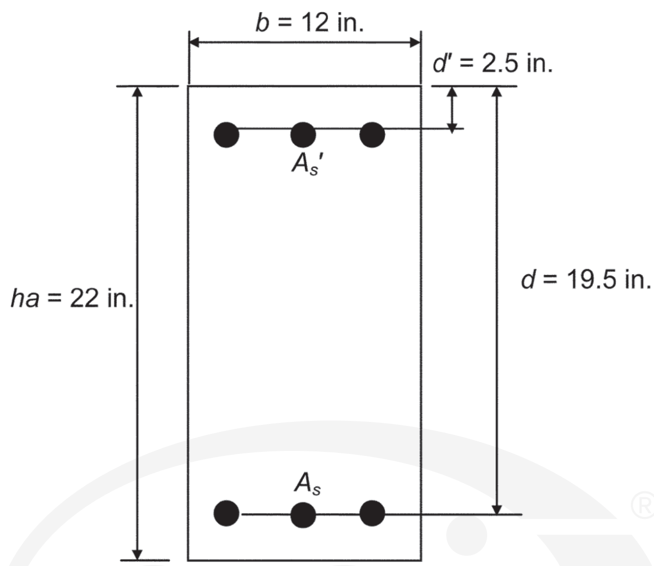
The design using CS requires an equivalent number of smaller-diameter bars as compared with a similar design using conventional steel reinforcement (PCA 2005). The development length required for the smaller-diameter high-strength reinforcing bars is similar to that required for the larger-diameter conventional steel bars.

In this example, using high-strength reinforcing bars results in an overall savings of the required amount of flexural steel of approximately 20 percent as compared with the design using Grade 60 steel bars. Wider spacing of the CS stirrups provides a 20 percent reduction of the shear reinforcement.

Example 4.7—Deflections of simple-span rectangular beam

(Similar to Example 10.1 in *PCA Notes*) This example illustrates the calculation of short- and long-term deflections for a rectangular beam reinforced with CS bars. The section was designed to have similar flexural strength and the same failure mode as the beam in the original example, which uses Grade 40 reinforcing bars. The current design represents a 60 percent reduction of the tension steel provided and a 45 percent reduction of the compression steel as compared with the original design using Grade 40 reinforcement.

Required: Analysis of short-term deflections, and long-term deflections at 3 months and 5 years (ultimate value). (Refer to Section 4.8 of this guide.)



f_c' = 3000 psi (normalweight concrete)
f_y = 100,000 psi
f_y' = 80,000 psi
A_s = two No. 5 and one No. 3 = 0.73 in.^{2}
E_s = 29,000,000 psi
$\rho = A_s/bd$ = 0.0031
A_s' = one No. 3 = 0.33 in.^{2}
(A_s' not required for strength)
$\rho' = A_s'/bd$ = 0.0014
Superimposed dead load = 120 lb/ft (not including beam weight)
Live load = 300 lb/ft (50 percent sustained)
Span = 25 ft

Calculations and discussion

(a) Minimum beam thickness for members not supporting or attached to partitions or other construction likely to be damaged by large deflections

According to Table 9.5(a) of ACI 318-11, footnotes for CS

h_{min} = span/16 multiplied by 1.4 factor for 100,000 psi steel

$$= \frac{25 \times 12}{16} \times 1.4 = 26 \text{ in.} > 22 \text{ in.}$$

Therefore, calculate the beam deflection to confirm that the beam depth is satisfactory.

(b) Moments

$w_d = 0.12 + (12)(22)(0.150)/144 = 0.395$ kip/ft

$$M_d = \frac{w_d \ell^2}{8} = \frac{(0.395)(25)^2}{8} = 30 \text{ ft-kip}$$

$$M_\ell = \frac{w_\ell \ell^2}{8} = \frac{(0.300)(25)^2}{8} = 23.4 \text{ ft-kip}$$

$M_{d+\ell}$ = 54.3 ft-kip

$M_{sus} = M_d + 0.5M_l$ = 30.9 + (0.5)(23.4) = 42.6 ft-kip

(c) Modulus of rupture, modulus of elasticity, modular ratio

$$f_r = 7.5\sqrt{f_c'} = 7.5\sqrt{3000} = 411 \text{ psi}$$

$$E_c = \omega_c^{1.5} 33\sqrt{f_c'} = (150)^{1.5} 33\sqrt{3000} = 3.32 \times 10^6 \text{ psi}$$

$$n_s = \frac{E_s}{E_c} = \frac{29 \times 10^6}{3.32 \times 10^6} = 8.7$$

(d) Gross and cracked section moments of inertia, using Table 10-2 from *PCA Notes*

$$I_g = \frac{bh^3}{12} = \frac{(12)(22)^3}{12} = 10{,}650 \text{ in.}^4$$

$$B = \frac{b}{nA_s} = \frac{12}{(8.7)(0.73)} = 1.89 \text{ in.}$$

$$r = \frac{(n-1)A_s'}{(nA_s)} = \frac{(7.7)(0.33)}{(8.7)(0.73)} = 0.400$$

$$\begin{aligned} kd &= \left[\sqrt{2dB(1 + rd'/d) + (1+r)^2} - (1+r)\right] / B \\ &= \left[\sqrt{2(19.5)(1.89)\left(1 + (0.400)(2.5)/(19.5)\right) + (1+0.400)^2} - (1+0.400)\right] / 1.89 = 3.98 \end{aligned}$$

$$\begin{aligned} I_{cr} &= \frac{bk^3d^3}{3} + nA_s(d-kd)^2 + (n-1)A_s'(kd-d')^2 \\ &= \frac{(12)(3.98)^3}{3} + (8.7)(0.73)(19.5-3.98)^2 + (7.2)(0.33)(3.98-2.5)^2 \\ &= 1788 \text{ in.}^4 \end{aligned}$$

I_g/I_{cr} = 5.96

(e) Effective moment of inertia, I_{eff}, per ACI 318, and as per Eq. (4.8) of this guide.

$$M_{cr} = \frac{f_r I_g}{y_t} = \frac{(411)(10{,}650)}{(11)} / 12{,}000 = 33.2 \text{ ft-kip}$$

1. Under dead load only.

$$\frac{M_{cr}}{M_d} = \frac{33.2}{30.9} > 1$$

Hence, $(I_e)_d = I_g = 10{,}650\text{ in.}^4$

2. Under sustained load.

$$\left(\frac{M_{cr}}{M_{sus}}\right)^3 = \left(\frac{33.2}{42.6}\right)^3 = 0.473$$

$$\begin{aligned}(I_e)_{sus} &= (M_{cr}/M_a)^3 I_g + \left(1 - [M_{cr}/M_a]^3\right) I_{cr} \le I_g \\ &= (0.473)(10{,}650) + (1 - 0.473)(1788) \\ &= 5980\text{ in.}^4\end{aligned}$$

$(I_e)_{sus} = 3612\text{ in.}^4$ (Eq. (4.8))

3. Under dead + live load.

$$\left(\frac{M_{cr}}{M_{d+\ell}}\right)^3 = \left(\frac{33.2}{54.3}\right)^3 = 0.229$$

$$\begin{aligned}(I_e)_{d+\ell} &= (0.229)(10{,}650) + (1 - 0.229)(1788) \\ &= 3817\text{ in.}^4\end{aligned}$$

$(I_e)_{d+\ell} = 2595\text{ in.}^4$ (Eq. (4.8))

(f) Initial or short-time deflections, using Eq. (3) of *PCA Notes*

$$(\Delta_i)_d = \frac{K(5/48)M_d\ell^2}{E_c(I_e)_d} = \frac{(1)(5/48)(30.9)(25)^2(12)^3}{(3320)(10{,}650)} = 0.098\text{ in.}$$

$K = 1$ for simple spans (refer to Table 8-3 of *PCA Notes*)

$$(\Delta_i)_{sus} = \frac{K(5/48)M_{sus}\ell^2}{E_c(I_e)_d} = \frac{(1)(5/48)(42.6)(25)^2(12)^3}{(3320)(5980)} = 0.241\text{ in.} \quad \text{(0.495 in. per this guide)}$$

$$(\Delta_i)_{d+1} = \frac{K(5/48)M_{d+1}\ell^2}{E_c(I_e)_d} = \frac{(1)(5/48)(54.3)(25)^2(12)^3}{(3320)(3817)} = 0.482\text{ in.} \quad \text{(0.689 in. per this guide)}$$

$(\Delta_i)_\ell = (\Delta_i)_{d+\ell} - (\Delta_i)_d = 0.482 - 0.098 = 0.384$ in. (0.591 in. per this guide)

Allowable deflections (Table 9.5(b) of ACI 318-08).

Flat roofs not supporting and not attached to nonstructural elements likely to be damaged by large deflections

$(\Delta_i)_\ell \le \text{span}/180 = 300/180 = 1.67\text{ in.} > 0.384$ in. **OK** (> 0.591 in. **OK** per this guide)

Floors not supporting and not attached to nonstructural elements likely to be damaged by large deflections

$(\Delta_i)_\ell \le \text{span}/360 = 300/360 = 0.83\text{ in.} > 0.384$ in. **OK** (> 0.591 in. **OK** per this guide)

(g) Additional long-term deflections at 3 months and 5 years (ultimate value).

Combined creep and shrinkage deflections of ACI 318 and Eq. (4) of *PCA Notes*: $\Delta_{cp} + \Delta_{sh} = \lambda(\Delta_i)_{sus}$

Duration	ξ	$\lambda = \dfrac{\xi}{1+50\rho'}$	$(\Delta_i)_{sus}$, in.	$(\Delta_i)_l$, in.	$\Delta_{cp} + \Delta_{sh} = \lambda(\Delta_i)_{sus}$, in.	$\Delta_{cp} + \Delta_{sh} + (\Delta_i)_\ell$, in.
5 years	2.0	1.87	0.241	0.384	0.451	0.84
3 months	1.0	0.93	0.241	0.384	0.224	0.61

Separate creep and shrinkage deflections, using Eq. (5) and (6) of *PCA Notes*

For $\rho = 0.0031$; $\rho' = 0.0014$

For $\rho = 100$, $\rho = 0.31$ and $\rho' = 100$, $\rho' = 0.14$, read $A_{sh} = 0.287$ (Fig. 10.3 of *PCA Notes*) and $K_{sh} = 0.125$ for simple spans (Table 10-5 of *PCA Notes*).

Duration	C_t	$\lambda_{cp} = \dfrac{0.85C_t}{1+50\rho'}$	$\Delta_{cp} = \lambda_{cp}(\Delta_i)_{sus}$, in.	ε_{sh}, in./in.	$\phi_{sh} = \dfrac{A_{sh}\mu_{sh}}{h}$, 1/in.	$\Delta_{sh} = K_{sh}\phi_{sh}l^2$, in.	$\Delta_{cp} + \Delta_{sh} + (\Delta_i)_\ell$, in.
5 years	1.6 (ultimate)	1.27	0.31	400×10^{-6}	$\dfrac{0.287 \times 400 \times 10^{-6}}{22}$ $= 5.22 \times 10^{-6}$	$\dfrac{1}{8} \times 5.22 \times 10^{-6} \times (25 \times 12)^2$ $= 0.059$	0.31 + 0.059 + 0.384 = 0.753
3 months	$0.56 \times 1.6 = 0.9$	0.71	0.17	$0.6 \times 400 \times 10^{-6} = 240 \times 10^{-6}$	3.13×10^{-6}	= 0.035	0.17 + 0.035 + 0.384 = 0.59

Allowable deflection of ACI 318.

Roof or floor construction supporting or attached to nonstructural elements are likely to be damaged by large deflections (very stringent limitation).

$\Delta_{cp} + \Delta_{sh} + (\Delta_i)_l \leq$ Span/480 = 300/480 = 0.63 in. **NG** by both methods

Roof or floor construction supporting or attached to nonstructural elements not likely to be damaged by large deflections

$\Delta_{cp} + \Delta_{sh} + (\Delta_i)_l \leq$ Span/240 = 300/240 = 1.25 in. **OK** by both methods and this guide.

For beams reinforced with high-strength reinforcing steel, the calculated short-term and long-term deflections are 25 to 40 percent higher than the corresponding deflections of an equivalent beam reinforced with conventional steel reinforcement. This is due to the reduced area of CS required to achieve equal moment capacity of the section. The reduction of the required steel area results in a corresponding reduction of the section's effective moment of inertia, which increases the calculated beam deflections. Deflection limitations, rather than strength requirements, may govern the design of concrete beams reinforced with CS bars. Deflection calculations are based on ACI 318 using an effective moment of inertia and a long-term multiplier and are used for illustrative purposes. The applicability of these provisions to high-strength reinforcing bars has not been demonstrated experimentally.

Example 4.8—Design for shear: members subject to shear and flexure only

Similar to Example 12.1 in *PCA Notes*, this example illustrates the calculation of the required high-strength shear reinforcement to resist a given level of applied load. This example considers the allowable increase of the shear reinforcement yield strength.

Determine required size and spacing for vertical U-stirrups for a 30 ft span, simply-supported beam. (Refer to Section 4.11 of this guide.)

b_w = 13 in.
d = 20 in.
f_c' = 3000 psi
f_{yt} = 80,000 psi
w_u = 4.5 kip/ft

Calculations and discussion

Assume the live load is present on the full span so that the design shear at the centerline of the span is zero. (Obtain a design shear greater than zero at midspan by using partial live loading of the span.) Using design procedure for shear reinforcement outlined in this part.

1. Determine factored shear forces.

At support: $V_u = 4.5(15) = 67.5$ kip

At distance d from support: $V_u = 67.5 - 4.5(20/12) = 60$ kip

2. Determine shear strength provided by concrete.

$$\phi V_c = \phi 2\sqrt{f_c'} b_w d$$

$\phi = 0.75$

$$\phi V_c = (0.75)2\sqrt{3000}(13)(20) = 21.4 \text{ kip}$$

$V_u = 60 \text{ kip} > \phi V_c = 21.4$ kip

Therefore, shear reinforcement is required.

3. Compute $V_u - \phi V_c$ *at critical section.*

$$V_u - \phi V_c = 60 - 21.4 = 38.6 \text{ kip} < \phi 8\sqrt{f_c'} b_w d = 85.4 \text{ kip}$$ **OK**

4. Determine distance x_c *from support beyond which minimum shear reinforcement is required* ($V_u = \phi V_c$).

$$x_c = \frac{V_u \text{ at support} - \phi V_c}{w_u} = \frac{67.5 - 21.4}{4.5} = 10.2 \text{ ft}$$

Determine distance x_m from support beyond which concrete can carry total shear force ($V_u = \phi V_c/2$).

$$x_m = \frac{V_u \text{ at support} - (\phi V_c/2)}{w_u} = \frac{67.5 - (21.4/2)}{4.5} = 12.6 \text{ ft}$$

5. Determine required spacing of vertical U-stirrups.

At critical section, $V_u = 60 \text{ kip} > \phi V_c = 21.4$ kip

$$s \text{ (required)} = \frac{\phi A_v f_{yt} d}{V_u - \phi V_c}$$

Assuming No. 4 U-stirrups (A_v = 0.40 in.^{2})

$$s \text{ (required)} = \frac{0.75(0.4)(80)(20)}{38.6} = 12.4 \text{ in.}$$

Check maximum permissible spacing of stirrups.

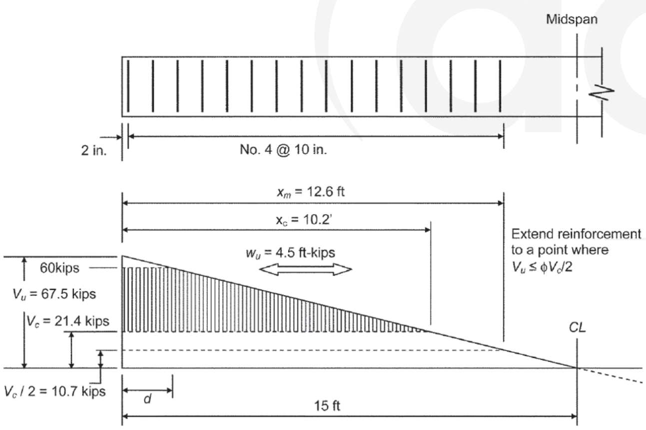
$$s \text{ (max)} \leq d/2 = 20/2 = 10 \text{ in. (governs)}$$
$$\leq 24 \text{ in. because } V_u - \phi V_c = 38.6 < \phi 4\sqrt{f_c'} b_w d = 42.7 \text{ kip}$$

Maximum stirrup spacing based on minimum shear reinforcement.

$$s \text{ (max)} \leq \frac{A_v f_{yt}}{0.75\sqrt{f_c'} b_w} = \frac{0.4(80,000)}{0.75\sqrt{3000}(13)} = 60 \text{ in.}$$
$$\leq \frac{A_v f_{yt}}{50 b_w} = \frac{0.4(80,000)}{50(13)} = 49$$

Because s (required) = 12 in. > s (max) = 10 in., provide stirrups at 10 in. spacing.

Stirrup spacing using No. 4 U-stirrups

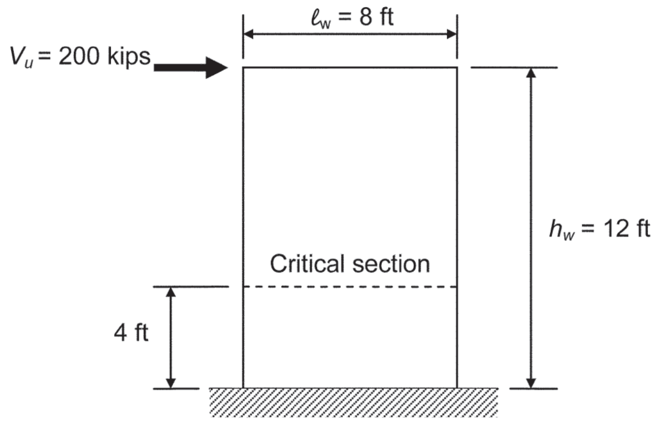


The design with CS resulted in a 25 percent reduction of the shear reinforcement as compared with the design with Grade 40 steel (*PCA Notes*). Due to the increased yield strength of the CS shear reinforcement, minimum spacing or area requirements, rather than strength requirements, may govern the spacing of the shear reinforcement. The design with high-strength shear reinforcement results in constant spacing along the region requiring shear reinforcement. This minimizes construction errors, which can occur from different spacing requirements along the beam length.

Example 7.1—Shear design of wall

Similar to Example 21.4 in *PCA Notes*, this example illustrates the design of a reinforced concrete wall using CS reinforcing bars to carry lateral loads.

Determine the shear and flexural reinforcement for the wall shown.



h (wall thickness) = 8 in.
$f_c' = 3000$ psi
$f_y = 100{,}000$ psi
$f_{yt} = 80{,}000$ psi
$f_y' = 80{,}000$ psi

Calculations and discussion

1. Check maximum shear strength permitted.

$$\phi V_n = \phi 10\sqrt{f_c'}hd$$

where $d = 0.8\ell_w = 0.8 \times 8 \times 12 = 76.8$ in.

$$\phi V_n = 0.75 \times 10\sqrt{3000} \times 8 \times 76.8/1000 = 252.4 \text{ kip} > 200 \text{ kip} \quad \textbf{OK}$$

2. Calculate shear strength provided by concrete V_c.

Critical section for shear

$\ell_w/2 = 8/2 = 4$ ft (governs)

or

$h_w/2 = 12/2 = 6$ ft

$$V_c = 3.3\sqrt{f_c'}hd + \frac{N_u d}{4\ell_w}$$
$$= 3.3\sqrt{3000}(8)(76.8)/1000 + 0 = 111 \text{ kips}$$

or

$$V_c = \left[0.6\sqrt{f_c'} + \frac{\ell_w \left(1.25\sqrt{f_c'} + \dfrac{0.2N_u}{\ell_w h} \right)}{\dfrac{M_u}{V_u} - \dfrac{\ell_w}{2}} \right] hd$$

$$= \left[0.6\sqrt{3000} + \frac{96(1.25\sqrt{3000} + 0)}{96 - 48} \right] \left(\frac{8 \times 76.8}{1000} \right) = 104 \text{ kips (governs)}$$

where $M_u = (12 - 4)V_u = 8V_u$ ft-kip $= 96V_u$ in.-kip

3. Determine required horizontal shear reinforcement.

$V_u = 200$ kip $> \phi V_c/2 = 0.75(104)/2 = 39.0$ kip

Provide shear reinforcement in accordance with ACI 318.

$$\begin{aligned} V_u &\le \phi V_n \\ &\le \phi(V_c + V_s) \\ &\le \phi V_c + \phi A_v f_y d/s_2 \end{aligned}$$

$$\frac{A_v}{s_2} = \frac{(V_u - \phi V_c)}{\phi f_y d} = \frac{[200 - (0.75 \times 104)]}{0.75 \times 80 \times 76.8} = 0.0265$$

For two No. 3: $s_2 = \dfrac{2 \times 0.11}{0.0265} = 8.3$ in.

For two No. 4: $s_2 = \dfrac{2 \times 0.20}{0.0265} = 15.1$ in.

For two No. 5: $s_2 = \dfrac{2 \times 0.31}{0.0265} = 23.4$ in.

Use two No. 4 at 15 in.

$$\rho_t = \frac{A_v}{A_g} = \frac{2 \times 0.2}{8 \times 15} = 0.0033 > 0.0025$$

OK

$$\text{maximum spacing} = \begin{cases} \ell_w/5 = 96/5 = 19.2 \text{ in.} \\ 3h = 3 \times 8 = 24 \text{ in.} \\ 18 \text{ in. (governs)} \end{cases}$$

Use two No. 4 at 15 in.

4. Determine vertical shear reinforcement.

$$\rho_\ell = 0.0025 + 0.5\left(2.5 - \frac{h_w}{\ell_w}\right)(\rho_t - 0.0025) \ge 0.0025$$

$$= 0.0025 + 0.5\left(2.5 - \frac{12}{8}\right)(0.0033 - 0.0025) \ge 0.0025$$

$$= 0.0029$$

$$\text{maximum spacing} = \begin{cases} \ell_w/3 = 96/3 = 32 \text{ in.} \\ 3h = 3 \times 8 = 24 \text{ in.} \\ 18 \text{ in. (governs)} \end{cases}$$

Use two No. 4 at 15 in. ($\rho_t = 0.0033$).

5. Design for flexure.

$M_u = V_u h_w = 200 \times 12 = 2400$ ft-kip

Assume tension-controlled section ($\phi = 0.9$)

with $d = 0.8\ell_w = 0.8 \times 96 = 76.8$ in.

(Note: An exact value of d is determined by a strain compatibility analysis in the following.)

$$R_n = \frac{M_u}{\phi bd^2} = \frac{2400 \times 12{,}000}{0.9 \times 8 \times 76.8^2} = 678 \text{ psi}$$

$$\rho = \frac{0.85 f_c'}{f_y}\left(1 - \sqrt{1 - \frac{2R_n}{0.85 f_c'}}\right)$$
$$= \frac{0.85 \times 3}{100}\left(1 - \sqrt{1 - \frac{2 \times 678}{0.85 \times 3000}}\right) = 0.00805$$

$A_s = \rho bd = 0.00805 \times 8 \times 76.8 = 4.95$ in.^{2}

Use nine No. 6 ($A_s = 3.96$ in.^{2}) at each end of wall, which provides less area of steel than that determined based on $d = 0.8\ell_w$.

Check moment strength of the wall with nine No. 6 bars using a strain compatibility analysis (refer to the following figure for reinforcement layout) based on the idealized elastic-plastic stress-strain curve of steel of the simplified method discussed in Chapter 4 of this guide.

From strain compatibility analysis (including No. 4 vertical bars)

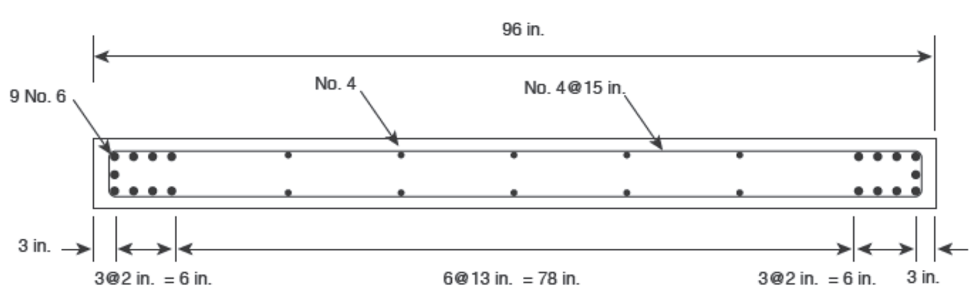
$c = 20$ in.

$d \approx 90$ in. (to centroid of No. 6 bars)

$\varepsilon_t = 0.011$ (at extreme tension steel)

$M_n = 3215$ ft-kip

$\phi M_n = 0.9 \times 3215 = 2894$ ft-kip > 2400 ft-kip **OK**



Use nine No. 6 bars each side ($A_s = 3.96$ in.^{2})

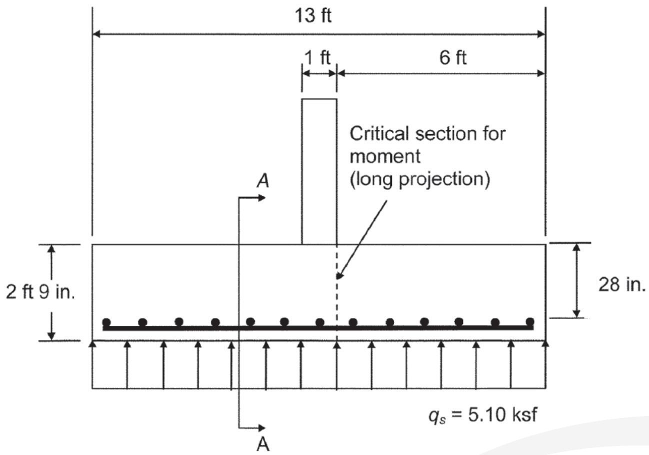
Including the main flexural reinforcement and the vertical shear reinforcement, this design provides 9.92 in.^{2} of vertical CS reinforcement. The design using conventional steel reinforcement requires 16.22 in.^{2} of vertical reinforcement. The use of CS reinforcement results in a 40 percent reduction of the amount of vertical steel. The use of CS results in a 33 percent reduction of the required horizontal shear reinforcement.

No research data are available to validate the use of the equations for V_c in Step 2 of the solution when using CS reinforcement.

Example 8.1—Design for flexural reinforcement of footing

Similar to Example 22.3 in *PCA Notes*, this example illustrates the design of the main flexural reinforcement of a typical footing using CS bars. Determine the required reinforcement for the footing shown per linear ft.

$f_c' = 3000$ psi
$f_y = 100{,}000$ psi
$P_u = 860$ kip
$q_s = 5.10$ kip/ft^2
Footing width b = 13 ft, column dimensions = 12 x 30 in.



Calculations and discussion

1. Critical section for moment is at column face.

$M_u = 5.10 \times 13 \times 6^2/2 = 1193$ ft-kip

2. Compute required A_s *assuming tension-controlled section* ($\phi = 0.9$).

$$\text{required } R_n = \frac{M_u}{\phi b d^2} = \frac{1193 \times 12 \times 1000}{0.9 \times 156 \times 28^2} = 130 \text{ psi}$$

$$\rho = \frac{0.85 f_c'}{f_y}\left(1 - \sqrt{1 - \frac{2R_n}{0.85 f_c'}}\right) = \frac{0.85 \times 3}{100}\left(1 - \sqrt{1 - \frac{2 \times 130}{0.85 \times 3000}}\right) = 0.0013$$

Based on gross area, $\rho = (d/h) \times 0.0013 = (28/33) \times 0.0013 = 0.0011$

Check minimum A_s required for footings of uniform thickness; for Grade 100 reinforcement

$$\rho_{min} = \frac{0.0018 \times 60{,}000}{f_y} = \frac{0.0018 \times 60{,}000}{100{,}000} = 0.0011 < 0.0014 \text{ (required)}$$

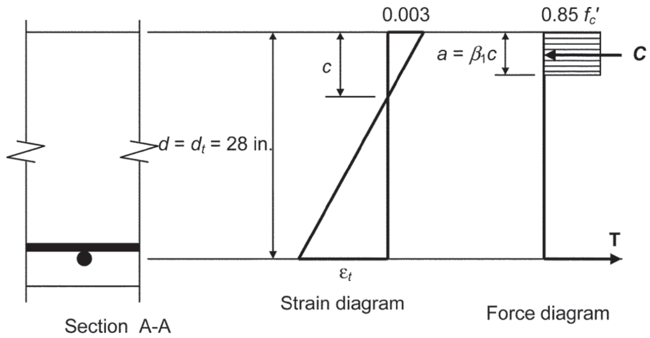
$\rho_{min} = 0.0014$ (controls)

Therefore, required $A_s = \rho_{min} bh = 0.0014 \times 156 \times 33 = 7.21$ in.^{2}

Use 12 No. 7 bars (A_s = 7.20 in.^{2}) each way.

A lesser amount of reinforcement is required in the perpendicular direction due to lesser M_u, but for ease of placement, use the same uniformly distributed reinforcement each way. For perpendicular direction, d_t = 27.1 in.

3. Check net tensile strain (ε_t).



$$a = \frac{A_s f_y}{0.85 f_c' b} = \frac{7.20 \times 100}{0.85 \times 3 \times 156} = 1.81 \text{ in.}$$

$$c = \frac{a}{\beta_1} = \frac{1.81}{0.85} = 2.13 \text{ in.}$$

$$\varepsilon_t = \left(\frac{0.003}{c}\right) d_t - 0.003 = \frac{0.003}{2.13} \times 28 - 0.003 = 0.036 > 0.009$$

Therefore, section is tension-controlled and initial assumption is valid. **OK**

Use 12 No. 7 bars each way.

4. Check development of reinforcement.

Critical section for development is the same as that for moment (at column face)

$$\ell_d = \frac{\left(\frac{f_y}{(f_c')^{1/4}} - \phi 2400\omega\right)\alpha\beta_c\lambda}{\phi 76.3\left(\frac{c\omega + K_{tr}}{d_b}\right)} d_b$$

α = 1.0 for bottom cast bars

β_c = 1.0 for uncoated bars

λ = 1.0 for normalweight concrete

Clear spacing = [156 – 2(cover) – 12 (No. 7 bars)]/11 spaces

= [156 – 2(3.0) – 12(7/8)]/11 = 12.7 in.

c_{si} = 1/2 clear spacing = 6.35 in.

$c_{so} = c_b$ = cover = 3.0 in.

$c_s = \min(c_{so}, c_{si} + 0.25) = \min(3.0, 6.6) = 3.0$ in.

$c_{min} = \min(c_s, c_b) = \min(3.0, 3.0) = 3.0$ in.

$c_{max} = \max(c_s, c_b) = \max(3.0, 3.0) = 3.0$ in.

$c = c_{min} + 0.5d_b = 3.0 + 0.5(7/8) = 3.44$ in.

$$\omega = 0.1\left(\frac{c_{max}}{c_{min}}\right) + 0.9 \leq 1.25$$

$$= 0.1\left(\frac{3.0}{3.0}\right) + 0.9 = 1.0 < 1.25$$

K_{tr} = 0 (no transverse reinforcement)

$$\frac{c\omega + K_{tr}}{d_b} = \frac{(3.44)(1.0)+0}{7/8} = 3.93 < 4$$

$$\ell_d = \frac{\left(\frac{100,000}{3000^{1/4}} - 0.82(2400)(1.0)\right)(1.0)(1.0)(1.0)}{(0.82)(76.3)(3.93)}(7/8) = 41.1 \text{ in.} > 12 \text{ in.}$$ **OK**

Because ℓ_d = 41.4 in. is less than the available embedment length in the short direction

$$\left(\frac{156}{2} - \frac{30}{2} - 3 = 60 \text{ in.}\right)$$

the No. 7 bars can be fully developed.

Use 12 No. 7 each way.

The current design using CS reinforcing bars represents a 30 percent reduction of the total area of flexural reinforcing steel provided in the footing compared with using Grade 60 steel (PCA 2005). Minimum reinforcement requirements for shrinkage and temperature govern the use of high-strength bars. Designers need to check for shear and punching shear. Strength requirements govern design using conventional steel.

APPENDIX B—FLEXURAL ANALYSIS USING NONLINEAR STRESS-STRAIN CURVE OF ASTM A1035/A1035M (CS) GRADE 100 (690) REINFORCEMENT

B.1—Introduction

The flexural strength of members reinforced with ASTM A1035/A1035M (CS) Grade 100 steel may be determined by using the nonlinear stress-strain curve of the reinforcement described in Chapter 3 of this guide.

A general solution is adopted for calculating the flexural strength of beam sections by a method of successive approximations for the neutral axis depth. This solution may be implemented by using an electronic spreadsheet, "439.6R-18 Flexural Analysis Spreadsheet.xlsx," which can be accessed online through the ACI Bookstore. Figure B.1 shows a flowchart for the solution by successive approximation for the general case of a doubly-reinforced beam. All variables with a prime correspond to the top reinforcement.

When using the nonlinear stress-strain curve of reinforcement in flexural design, the tensile stress in the reinforcement under service load may exceed the proportional limit. The impact of this higher tensile stress on member deflection and crack control should be considered.

Furthermore, when the longitudinal reinforcement has been designed according to the method in this appendix, the possible nonlinear response and higher strains of the longitudinal reinforcement may have significant impact on shear capacity and development length. Due to a lack of experimental data, it is recommended that members designed by this method should contain at least the minimum shear reinforcement specified by ACI 318. Desalegne and Lubell (2010, 2015) present shear design models for members with and without shear reinforcement where the longitudinal reinforcement is designed according to Appendix B. To fully develop the higher tensile stress f_s in the longitudinal reinforcement, the shear reinforcement may also be used as confinement reinforcement when determining the required development length of the longitudinal bars.

B.2—Design assumptions

The strength design of members for flexure and axial loads is based on ACI 318. However, when using ASTM A1035/A1035M (CS) Grade 100 steel bars, use Eq. (3.4a) to (3.4c) of this guide instead of ACI 318. In addition, the stress in compression reinforcement should not be taken greater than 80 ksi.

The design moment strength determined by using the nonlinear stress-strain curve of ASTM A1035/A1035M (CS) Grade 100 steel uses a strength reduction factor ϕ from 0.65 to 0.90, similar to the range given in ACI 318 and in Section 4.3 of this guide. ACI 318 defines the compression-controlled strain limit using the balanced strain conditions, which refers to the strain in the reinforcement at first yield. Similarly, consider the 0.2 percent offset yield strength of 100 ksi as the strain in the reinforcement at first yield. This strain occurs at 0.00424 according to Eq. (3.4b) and defines the compression-controlled strain limit. The tension-controlled strain limit is 0.0066 as discussed in Section 4.3 of this guide.

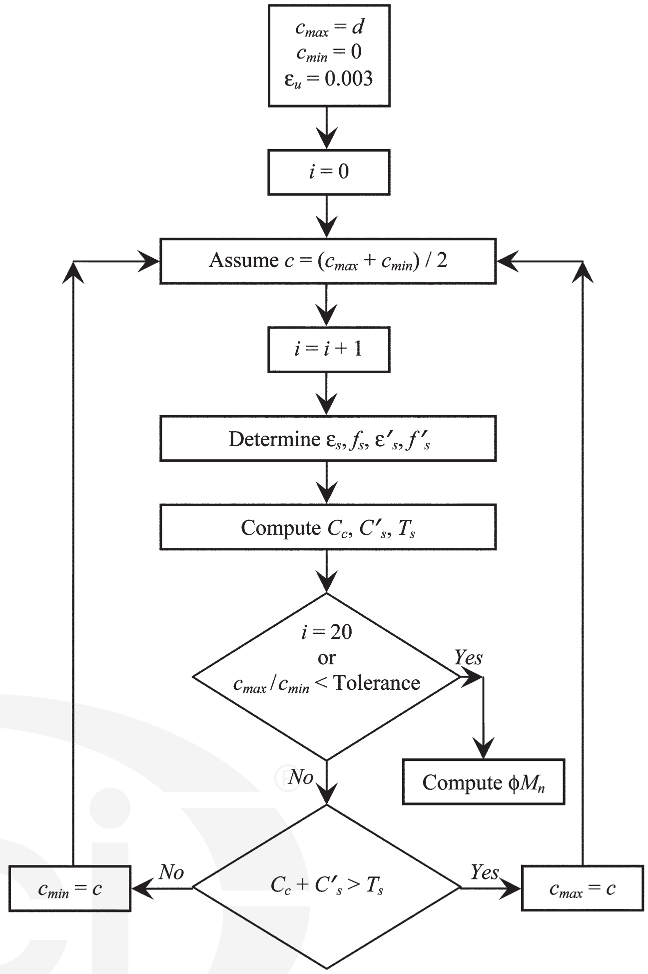


Fig. B.1—Flowchart for successive approximation solution for neutral axis, c.

In this appendix, a simple and conservative expression of ϕ is adopted after assigning $\phi = 0.65$ corresponding to a tensile strain of 0.0042 (f_s =100 ksi) and assigning $\phi = 0.90$ corresponding to a tensile strain of 0.0067 (f_s =120 ksi).

$$0.65 \leq (\phi = 0.23 + 100\varepsilon_t) \leq 0.9 \quad \text{(B.2)}$$

where the value of ε_t is positive for tension.

B.3—Spreadsheet implementation

An electronic spreadsheet is programmed to compute the flexural strength of a doubly-reinforced T-beam section as shown in Fig. B.3, where a singly-reinforced rectangular section corresponds to setting h_f and A_s' to zero. The spreadsheet, "439.6R-18 Flexural Analysis Spreadsheet.xlsx," can be accessed online through the ACI Bookstore. The spreadsheet is organized by examples with flexural strength calculations as shown in Figs. B.4a to B.4j, where all cell values are easily reproduced using hand calculations following the equations given in the following.

The spreadsheet contains two blocks of cells. Use the top block, Cells A1 to J19, to highlight the main input variables. These variables, except for f_c', correspond to the general T-beam cross section of Fig. B.3. Use the bottom block,

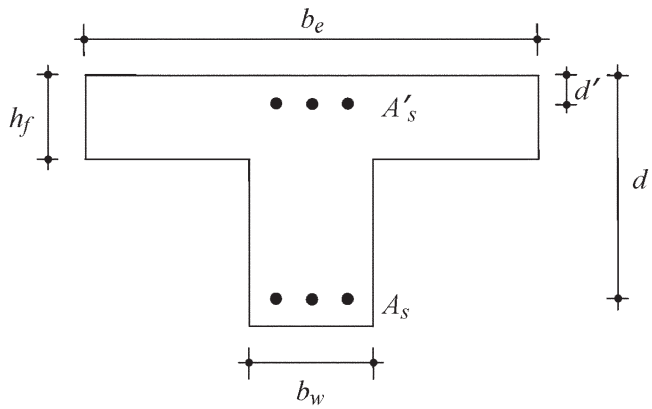


Fig. B.3—Doubly-reinforced T-beam section.

Cells A20 to J50, to perform the calculations that support the value of ϕM_n in Cell B47.

Trial values of c in Cells B25 to B44 correspond to the midpoint between c_{max} and c_{min} of Cells I25 and J25, to I44 and J44. The number of iterations is fixed to 20 even though solutions of reasonable accuracy are obtained with less iterations. After each iteration, update the values of c_{max} and c_{min} depending on the value for the summation of internal forces associated with the assumed c, as recorded by Cells G25 to G44 using Eq. (B.3a)

$$\sum \text{forces} = C_c + C_s' + T_s \tag{B.3a}$$

where

$$C_c = 0.85f_c' \times b_e \times \beta_1 \times c \text{ (for } \beta_1 \times c \le h_f) \tag{B.3b(a)}$$

$$C_c = 0.85f_c'[(b_e - b_w)h_f + b_w \times \beta_1 \times c] \text{ (for } \beta_1 \times c > h_f) \tag{B.3b(b)}$$

$$C_s' = A_s' \times f_s' \tag{B.3c}$$

$$T_s = A_s \times f_s \tag{B.3d}$$

The variables are illustrated in Fig. B.3 and explained in Fig. B.4a to B.4j. Apply Eq. (3.4a), (3.4b) and (3.4c), as stated in Section B.2, to obtain the stress in the top reinforcement, f_s', and the stress in the bottom reinforcement, f_s. The strains ε_s' and ε_s to determine the stresses f_s' and f_s are based on Eq. (B.3e) and (B.3f)

$$\varepsilon_s' = \frac{\varepsilon_u}{c}(c - d') \tag{B.3e}$$

$$\varepsilon_s = \frac{\varepsilon_u}{c}(c - d) \tag{B.3f}$$

where ε_u is the usable compressive strain of concrete, defined as 0.003 in accordance with ACI 318. Compressive strains are taken as positive, therefore in Eq. (B.3a) compression forces are taken as positive. If the summation in Eq. (B.3a) results positive, reduce the value of c (and its value in the current iteration defines the new high bound c_{max}) in Cells I25 to I44. If the summation in Eq. (B.3a) results negative, increase the value of c (and its value in the current iteration defines the new low bound c_{min}) in Cells J25 to J44.

Use Eq. (B.2) and (B.3g(a)) to obtain the design flexural strength ϕM_n computed in Cell B47.

For $\beta_1 \times c \le h_f$

$$M_n = 0.85 f_c' b_e \beta_1 c\left(d - \frac{\beta_1 c}{2}\right) + A_s' f_s'(d - d') \tag{B.3g(a)}$$

For $\beta_1 \times c > h_f$

$$M_n = 0.85 f_c'(b_e - b_w)h_f\left(d - \frac{h_f}{2}\right) + 0.85 f_c' b_w \beta_1 c\left(d - \frac{\beta_1 c}{2}\right) + A_s' f_s'(d - d') \tag{B.3g(b)}$$

B.4—Design examples

The spreadsheet described in Section B.3 is applied to several of the design examples in Appendix A. The spreadsheet, "439.6R-18 Flexural Analysis Spreadsheet.xlsx," can be accessed online through the ACI Bookstore. Figures B.4a to B.4j includes solutions to examples involving flexural strength calculations. Brief comments are made after each example. The results are summarized in Table B.4. It can be seen that by using the nonlinear analysis presented in this appendix, a saving of over 20 percent of the amount of reinforcement is realized in each case.

Example 4.1a

	A	B	C	D	E	F	G	H	I	J
1										
2		Analysis of doubly-reinforced beam sections:								
3										
4										
5	*units*	*s*		(*s* for standard or *m* for metric)						
6										
7	b_e =	0.0	in	width of effective compression flange (use 0 in rectangular sections)						
8	b_w =	10.0	in	web width						
9	h_f =	0.0	in	flange thickness (use 0 in rectangular sections)						
10	d =	13.50	in	effective depth, tension reinforcement						
11	d' =	0.00	in	effective depth, compression reinforcement						
12										
13	f'_c =	4.0	ksi	concrete strength						
14	$f_{y,A1035}$ =	100	ksi	yield strength of reinforcement (0.2% offset)						
15	E_s =	29000	ksi	steel modulus						
16										
17	A'_s =	0.00	in^2	area of compression reinforcement						
18	A_s =	2.37	in^2	area of tension reinforcement						
19										
20										
21	β_1 =	0.85			ACI 318-08 Section 10.2.7.3					
22	ε_u =	0.003			ACI 318-08 Section 10.2.3					
23										
24	*Iteration*	*c*	ε_s	f_s	ε'_s	f'_s	Σ *forces*		c_{max}	c_{min}
25	1	6.75	-0.0030	-82.2	0.0030	80.0	0.2		13.50	0.00
44	20	6.75	-0.0030	-82.3	0.0030	80.0	0.0		6.75	6.75
45										
46	ϕ =	0.65								
47	ϕM_n =	112.3	k-ft							
48										
49										
50										

Fig. B.4a—Example 4.1a.

The compression-controlled section has the tension reinforcement at a strain of approximately 0.0030 for an actual steel stress lower than the stress based on a linear stress-strain relationship, as assumed in Appendix A. The computed ϕM_n following Appendix B is 112.3 ft-kip compared with 114.2 ft-kip following Appendix A, a difference of less than 2 percent.

Example 4.1b

	A	B	C	D	E	F	G	H	I	J
1										
2		Analysis of doubly-reinforced beam sections:								
3										
4										
5	*units*	*s*		(*s* for standard or *m* for metric)						
6										
7	b_e =	0.0	in	width of effective compression flange (use 0 in rectangular sections)						
8	b_w =	10.0	in	web width						
9	h_f =	0.0	in	flange thickness (use 0 in rectangular sections)						
10	d =	13.50	in	effective depth, tension reinforcement						
11	d' =	0.00	in	effective depth, compression reinforcement						
12										
13	f'_c =	4.0	ksi	concrete strength						
14	$f_{y,A1035}$ =	100	ksi	yield strength of reinforcement (0.2% offset)						
15	E_s =	29000	ksi	steel modulus						
16										
17	A'_s =	0.00	in^2	area of compression reinforcement						
18	A_s =	1.58	in^2	area of tension reinforcement						
19										
20										
21	β_1 =	0.85			ACI 318-08 Section 10.2.7.3					
22	ε_u =	0.003			ACI 318-08 Section 10.2.3					
23										
24	*Iteration*	c	ε_s	f_s	ε'_s	f'_s	Σ *forces*		c_{max}	c_{min}
25	1	6.75	-0.0030	-82.2	0.0030	80.0	65.1		13.50	0.00
44	20	5.52	-0.0043	-101.0	0.0030	80.0	0.0		5.52	5.52
45										
46	ϕ =	0.66								
47	ϕM_n =	98.4 k-ft								
48										
49										
50										

Fig. B.4b—Example 4.1b.

A steel tensile stress of 101.0 ksi at a strain of 0.00433 balances the concrete compression forces. In this case, Appendix B gives a flexural strength of ϕM_n = 98.4 ft-kip, nearly identical to that obtained in Appendix A, where ϕM_n = 98.5 ft-kip. The solution in Appendix B uses a slightly lower ϕ factor.

Example 4.2a

	A	B	C	D	E	F	G	H	I	J
1										
2		Analysis of doubly-reinforced beam sections:								
3										
4										
5	*units*	*s*		(*s* for standard or *m* for metric)						
6										
7	b_e =	0.0	in	width of effective compression flange (use 0 in rectangular sections)						
8	b_w =	14.0	in	web width						
9	h_f =	0.0	in	flange thickness (use 0 in rectangular sections)						
10	d =	20.50	in	effective depth, tension reinforcement						
11	d' =	2.50	in	effective depth, compression reinforcement						
12										
13	f'_c =	4.0	ksi	concrete strength						
14	$f_{y,A1035}$ =	100	ksi	yield strength of reinforcement (0.2% offset)						
15	E_s =	29000	ksi	steel modulus						
16										
17	A'_s =	2.44	in^2	area of compression reinforcement						
18	A_s =	3.21	in^2	area of tension reinforcement						
19										
20										
21	β_1 =	0.85			ACI 318-08 Section 10.2.7.3					
22	ε_u =	0.003			ACI 318-08 Section 10.2.3					
23										
24	*Iteration*	c	ε_s	f_s	ε'_s	f'_s	Σ *forces*		c_{max}	c_{min}
25	1	10.25	-0.0030	-82.2	0.0023	65.8	311.4		20.50	0.00
44	20	6.34	-0.0067	-120.0	0.0018	52.7	0.0		6.34	6.34
45										
46	ϕ =	0.90								
47	ϕM_n =	516.0	k-ft							
48										
49										
50										

Fig. B.4c—Example 4.2a.

The doubly-reinforced rectangular section is designed so that the tension reinforcement is at a strain of 0.0067 with a corresponding steel stress of 120 ksi instead of the 100 ksi assumed in Appendix A. The solution in Appendix B uses compression reinforcement at a strain of 0.00182 for a stress of 52.7 ksi, whereas Appendix A uses a strain of 0.00154 for a stress of 44.6 ksi. The total area of steel, $A_s' + A_s$, of 5.65 in.^{2} obtained in Appendix B represents a 26 percent reduction compared with the 7.59 in.^{2} obtained in Appendix A.

Example 4.2b

	A	B	C	D	E	F	G	H	I	J
1										
2		Analysis of doubly-reinforced beam sections:								
3										
4										
5	*units*	*s*		(*s* for standard or *m* for metric)						
6										
7	b_e =	0.0	in	width of effective compression flange (use 0 in rectangular sections)						
8	b_w =	14.0	in	web width						
9	h_f =	0.0	in	flange thickness (use 0 in rectangular sections)						
10	d =	20.50	in	effective depth, tension reinforcement						
11	d' =	2.50	in	effective depth, compression reinforcement						
12										
13	f'_c =	8.0	ksi	concrete strength						
14	$f_{y,A1035}$ =	100	ksi	yield strength of reinforcement (0.2% offset)						
15	E_s =	29000	ksi	steel modulus						
16										
17	A'_s =	0.00	in^2	area of compression reinforcement						
18	A_s =	3.01	in^2	area of tension reinforcement						
19										
20										
21	β_1 =	0.65			ACI 318-08 Section 10.2.7.3					
22	ε_u =	0.003			ACI 318-08 Section 10.2.3					
23										
24	*Iteration*	c	ε_s	f_s	ε'_s	f'_s	Σ *forces*		c_{max}	c_{min}
25	1	10.25	-0.0030	-82.2	0.0023	65.8	386.7		20.50	0.00
44	20	5.99	-0.0073	-123.1	0.0017	50.7	0.0		5.99	5.99
45										
46	ϕ =	0.90								
47	ϕM_n =	515.6	k-ft							
48										
49										
50										

Fig. B.4d—Example 4.2b.

This example uses 8 ksi concrete on the same cross section of Example 4.2a. Appendix A requires compression reinforcement for a total amount of steel, $A_s' + A_s$, of 4.79 in^2. The solution in Appendix B, where A_s = 3.01 in.^{2} without need of compression reinforcement, offers a steel reduction of 37 percent.

Example 4.3

	A	B	C	D	E	F	G	H	I	J
1										
2		Analysis of doubly-reinforced beam sections:								
3										
4										
5	*units*	*s*		(*s* for standard or *m* for metric)						
6										
7	b_e =	0.0	in	width of effective compression flange (use 0 in rectangular sections)						
8	b_w =	10.0	in	web width						
9	h_f =	0.0	in	flange thickness (use 0 in rectangular sections)						
10	d =	16.50	in	effective depth, tension reinforcement						
11	d' =	0.00	in	effective depth, compression reinforcement						
12										
13	f'_c =	4.0	ksi	concrete strength						
14	$f_{y,A1035}$ =	100	ksi	yield strength of reinforcement (0.2% offset)						
15	E_s =	29000	ksi	steel modulus						
16										
17	A'_s =	0.00	in^2	area of compression reinforcement						
18	A_s =	0.82	in^2	area of tension reinforcement						
19										
20										
21	β_1 =	0.85			ACI 318-08 Section 10.2.7.3					
22	ε_u =	0.003			ACI 318-08 Section 10.2.3					
23										
24	*Iteration*	*c*	ε_s	f_s	ε'_s	f'_s	Σ *forces*		c_{max}	c_{min}
25	1	8.25	-0.0030	-82.2	0.0030	80.0	171.0		16.50	0.00
44	20	3.80	-0.0100	-133.9	0.0030	80.0	0.0		3.80	3.80
45										
46	ϕ =	0.90								
47	ϕM_n =	122.6	k-ft							
48										
49										
50										

Fig. B.4e—Example 4.3.

For a target flexural strength of ϕM_n = 123 ft-kip, Appendix A requires A_s = 1.10 in.^{2} Using the same beam dimensions, Appendix B requires A_s = 0.82 in.^{2}, a 25 percent reduction.

Example 4.4

	A	B	C	D	E	F	G	H	I	J
1										
2		Analysis of doubly-reinforced beam sections:								
3										
4										
5	*units*	*s*		(*s* for standard or *m* for metric)						
6										
7	b_e =	0.0	in	width of effective compression flange (use 0 in rectangular sections)						
8	b_w =	12.0	in	web width						
9	h_f =	0.0	in	flange thickness (use 0 in rectangular sections)						
10	d =	30.00	in	effective depth, tension reinforcement						
11	d' =	2.50	in	effective depth, compression reinforcement						
12										
13	f'_c =	4.0	ksi	concrete strength						
14	$f_{y,A1035}$ =	100	ksi	yield strength of reinforcement (0.2% offset)						
15	E_s =	29000	ksi	steel modulus						
16										
17	A'_s =	1.28	in^2	area of compression reinforcement						
18	A_s =	3.36	in^2	area of tension reinforcement						
19										
20										
21	β_1 =	0.85			ACI 318-08 Section 10.2.7.3					
22	ε_u =	0.003			ACI 318-08 Section 10.2.3					
23										
24	*Iteration*	c	ε_s	f_s	ε'_s	f'_s	Σ *forces*		c_{max}	c_{min}
25	1	15.00	-0.0030	-82.2	0.0025	72.3	336.4		30.00	0.00
44	20	9.28	-0.0067	-120.0	0.0022	63.6	0.0		9.28	9.28
45										
46	ϕ =	0.90								
47	ϕM_n =	796.6	k-ft							
48										
49										
50										

Fig. B.4f—Example 4.4.

For the doubly-reinforced rectangular section designed for ϕM_n = 796 ft-kip while attaining a tension-controlled strain limit, Appendix A requires a total steel area, $A_s' + A_s$, of 6.21 in.^{2} with a tensile steel strain of 0.009 and a compression steel strain of 0.002. The solution in Appendix B requires a total steel area, $A_s' + A_s$, of 4.64 in.^{2} with a tensile steel strain of 0.0067 and a compression steel strain of 0.00219. This represents a 25 percent reduction of steel with respect to Appendix A.

Example 4.5

	A	B	C	D	E	F	G	H	I	J
1										
2		Analysis of doubly-reinforced beam sections:								
3										
4										
5	*units*	*s*		(*s* for standard or *m* for metric)						
6										
7	b_e =	30.0	in	width of effective compression flange (use 0 in rectangular sections)						
8	b_w =	10.0	in	web width						
9	h_f =	2.5	in	flange thickness (use 0 in rectangular sections)						
10	d =	19.00	in	effective depth, tension reinforcement						
11	d' =	0.00	in	effective depth, compression reinforcement						
12										
13	f'_c =	4.0	ksi	concrete strength						
14	$f_{y,A1035}$ =	100	ksi	yield strength of reinforcement (0.2% offset)						
15	E_s =	29000	ksi	steel modulus						
16										
17	A'_s =	0.00	in^2	area of compression reinforcement						
18	A_s =	1.11	in^2	area of tension reinforcement						
19										
20										
21	β_1 =	0.85			ACI 318-08 Section 10.2.7.3					
22	ε_u =	0.003			ACI 318-08 Section 10.2.3					
23										
24	*Iteration*	c	ε_s	f_s	ε'_s	f'_s	Σ *forces*		c_{max}	c_{min}
25	1	9.50	-0.0030	-82.2	0.0030	80.0	353.3		19.00	0.00
44	20	1.92	-0.0267	-150.0	0.0030	80.0	0.0		1.92	1.92
45										
46	ϕ =	0.90								
47	ϕM_n =	227.1	k-ft							
48										
49										
50										

Fig. B.4g—Example 4.5.

A singly-reinforced T-beam section is designed for ϕM_n = 227 ft-kip. The beam requires 1.66 in.^{2} according to Appendix A. The steel area of 1.11 in.^{2} obtained in Appendix B, with a tensile steel strain of 0.0267 and a steel stress of 150 ksi, represents a 33 percent reduction in steel area. The strain exceeds the 0.015 threshold indicated in Section 4.2 of this design guide. Cracking and deflections may be critical and should be investigated.

Example 4.6

	A	B	C	D	E	F	G	H	I	J
1										
2		Analysis of doubly-reinforced beam sections:								
3										
4										
5	*units*	*s*		(*s* for standard or *m* for metric)						
6										
7	b_e =	30.0	in	width of effective compression flange (use 0 in rectangular sections)						
8	b_w =	10.0	in	web width						
9	h_f =	2.5	in	flange thickness (use 0 in rectangular sections)						
10	d =	19.00	in	effective depth, tension reinforcement						
11	d' =	0.00	in	effective depth, compression reinforcement						
12										
13	f'_c =	4.0	ksi	concrete strength						
14	$f_{y,A1035}$ =	100	ksi	yield strength of reinforcement (0.2% offset)						
15	E_s =	29000	ksi	steel modulus						
16										
17	A'_s =	0.00	in^2	area of compression reinforcement						
18	A_s =	2.34	in^2	area of tension reinforcement						
19										
20										
21	β_1 =	0.85			ACI 318-08 Section 10.2.7.3					
22	ε_u =	0.003			ACI 318-08 Section 10.2.3					
23										
24	*Iteration*	c	ε_s	f_s	ε'_s	f'_s	Σ *forces*		c_{max}	c_{min}
25	1	9.50	-0.0030	-82.2	0.0030	80.0	252.1		19.00	0.00
44	20	4.71	-0.0091	-130.9	0.0030	80.0	0.0		4.71	4.71
45										
46	ϕ =	0.90								
47	ϕM_n =	400.0	k-ft							
48										
49										
50										

Fig. B.4h—Example 4.6.

The steel area required for ϕM_n = 400 ft-kip is 2.34 in.^{2} according to Appendix B, whereas Appendix A requires 3.06 in.^{2}, a reduction of 24 percent.

Example 4.6(SI)

	A	B	C	D	E	F	G	H	I	J
1										
2		Analysis of doubly-reinforced beam sections:								
3										
4										
5	*units*	*m*		(s for standard or m for metric)						
6										
7	b_e =	762.0	mm	width of effective compression flange (use 0 in rectangular sections)						
8	b_w =	254.0	mm	web width						
9	h_f =	63.5	mm	flange thickness (use 0 in rectangular sections)						
10	d =	482.6	mm	effective depth, tension reinforcement						
11	d' =	0.0	mm	effective depth, compression reinforcement						
12										
13	f'_c =	27.6	MPa	concrete strength						
14	$f_{y,A1035}$ =	690	MPa	yield strength of reinforcement (0.2% offset)						
15	E_s =	200000	MPa	steel modulus						
16										
17	A'_s =	0.00	mm^2	area of compression reinforcement						
18	A_s =	1510	mm^2	area of tension reinforcement						
19										
20										
21	β_1 =	0.85			ACI 318-08 Section 10.2.7.3					
22	ε_u =	0.003			ACI 318-08 Section 10.2.3					
23										
24	*Iteration*	c	ε_s	f_s	ε'_s	f'_s	Σ *forces*		c_{max}	c_{min}
25	1	241.30	-0.0030	-567.2	0.0030	551.7	1121.7		482.60	0.00
44	20	119.75	-0.0091	-902.6	0.0030	551.7	0.0		119.75	119.75
45										
46	ϕ =	0.90								
47	ϕM_n =	542.5	kN-m							
48										
49										
50										

Fig. B.4i—Example 4.6(SI).

The steel area required for ϕM_n = 542 kN-m is 1510 mm^2 according to Appendix B, whereas Appendix A requires 1970 mm^2, a reduction of 24 percent.

Example 6.3

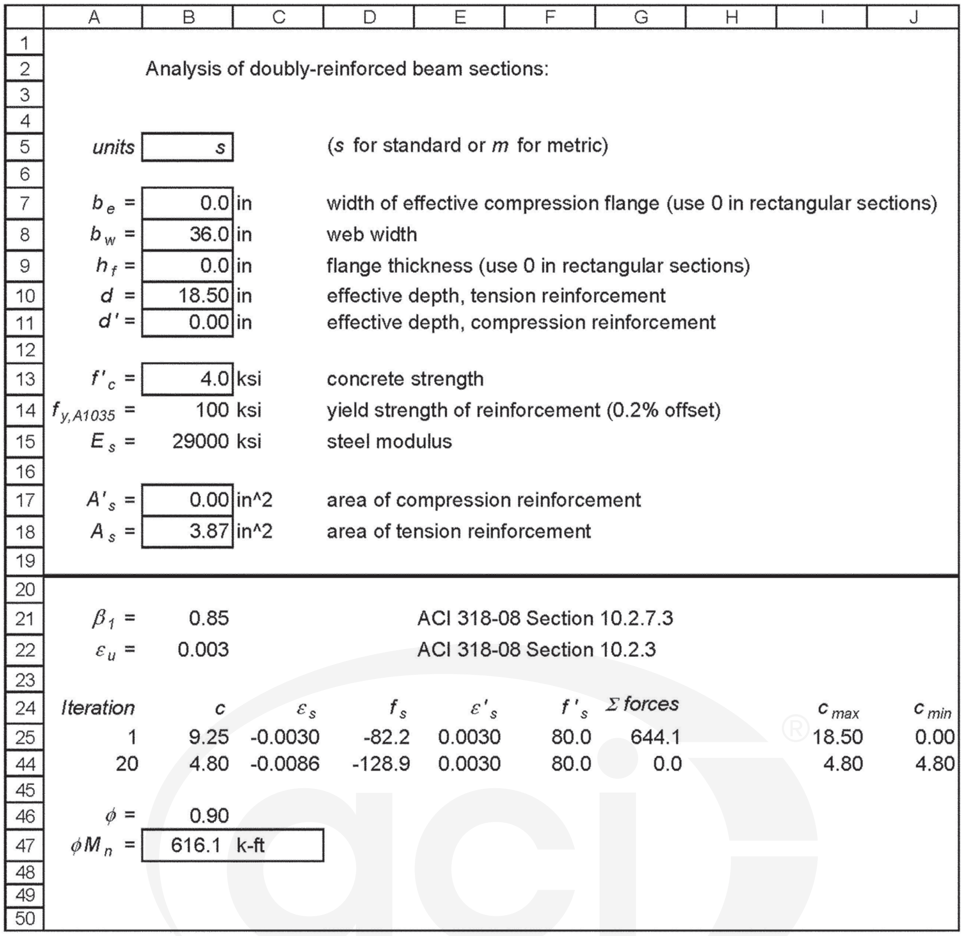


Fig. B.4j—Example 6.3.

The negative moment on the one-way joist needs a flexural strength of ϕM_n = 616 ft-kip. Appendix A gives a solution with A_s = 4.98 in.^{2}, whereas Appendix B requires A_s = 3.87 in.^{2} for 22 percent reduction in steel area.

Table B.4—Summary of required flexural reinforcement in selected examples

Example	Appendix A, in.^{2}	Appendix B, in.^{2}	Ratio B/A
4.2a	7.59	5.65	0.74
4.2b	4.79	3.01	0.63
4.3	1.10	0.82	0.75
4.4	6.21	4.64	0.75
4.5	1.66	1.11	0.67
4.6	3.06	2.34	0.76
6.3	4.98	3.87	0.78

APPENDIX C—FLEXURAL BEHAVIOR OF BEAMS REINFORCED WITH ASTM A1035/A1035M BARS

The stress-strain curve of ASTM A1035/A1035M (CS) Grade 100 (690) bar differs from that of ASTM A615/A615M Grade 60 (420) bar in that the former is without a well-defined yield plateau. The Grade 100 (690) bar has a specified minimum yield strength of 100,000 psi (690 MPa) as determined by the 0.2% offset method, whereas the Grade 60 (420) bar has a specified minimum yield strength of 60,000 psi (410 MPa), usually determined by observation of a distinct yield point or knee in the stress-strain curve. These differences affect the flexural behavior of beams significantly.

Figure C.1 shows the load-deflection curves obtained by Yotakhong (2003) from flexural tests of three 12 x 18 in. (300 x 450 mm) beams over a simple span of 15 ft (4.6 m) with two-point loading producing a constant moment region at midspan. The number of bars and the grade of steel of the tension reinforcement are shown in the figure. Each beam contained two No. 4 Grade 60 (420) bars as compression reinforcement. The beams also contained No. 4 Grade 60 (420) stirrups at 8 in. (200 mm) on center within the shear spans and at 9 in. (225 mm) on center in the constant moment region.

Before the initial flexural cracking occurred at an applied load of approximately 8 kip (36 kN), the behavior of all three beams was virtually identical. After the initial cracking, the load-deflection response of all beams was nearly linear. Beams A and C showed similar stiffness after cracking because both contained three No. 6 bottom bars. Beam B, however, deflected more than Beams A and C at a similar load because it contained only two No. 6 bottom bars and, therefore, the stiffness was less, the neutral axis depth was smaller, and the bar stress was 50 percent higher.

At the load level of 33 kip (147 kN), the reinforcement in Beam A reached its yield strength and the beam deflection continued to increase with only a slight increase in loading. Failure of Beam A occurred when the concrete reached the maximum measured strain of 0.004 in the extreme compression fiber. At this same load level of 33 kip (147 kN), however, Beam B had deflected more than Beam A because Beam B contained only two No. 6 bottom bars and the bar was stressed to 96,000 psi (662 MPa) compared with 64,000 psi (441 MPa) for Beam A. Being reinforced with Grade 100 (690) bars that lack a well-defined yield plateau, both Beams B and C continued to carry more load with increasing deflections and decreasing stiffness (as the reinforcement stress progressed into the inelastic range) until failure occurred when the concrete reached the measured maximum strain of 0.003 in the extreme compression fiber. All three beams failed in flexure with crushing of the concrete near midspan.

Yotakhong (2003) compared the test results with the theoretical predictions based on the actual stress-strain relationships of the reinforcements. The comparisons are close, as shown in Table C.1.

It is worth noting that both Beams B and C showed considerable deflection before failure, even though the deflection at failure was less than that of Beam A. Furthermore, both beams carried proportionally more load than Beam A, which was reinforced with Grade 60 (420) bars.

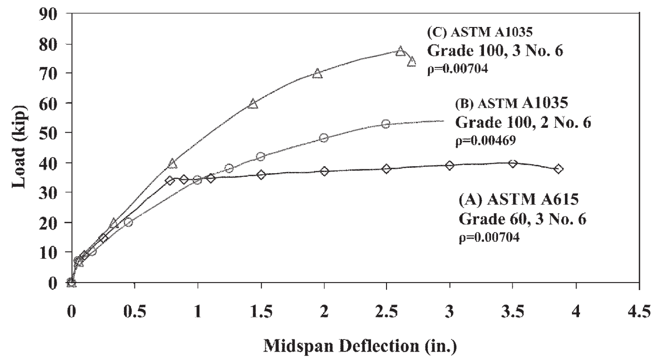


Fig. C.1—Load-deflection curves of beams reinforced with ASTM A1035/A1035M (CS) and ASTM A615/A615M reinforcing bars (Yotakhong 2003). (Note: 1 kip = 4.45 kN; 1 in. = 25.4 mm.).

The flexural behavior of the beams described previously also indicates the difference in the internal moment-resisting mechanism between Beam A reinforced with Grade 60 (420) bars and Beams B and C reinforced with Grade 100 (690) bars. Initially, under increasing load, the internal forces T (tension in the reinforcement) and C (compression in the concrete) of all three beams gradually increase and the lever arm between T and C also slowly increases as the neutral axis moves slowly toward the extreme compression fiber, gradually reducing the area of the compression zone. At the higher load near failure, the internal forces T and C of Beam A must remain constant because T has reached its yield value, so any increase in moment is accompanied with an increase in the lever arm between T and C, causing the neutral axis to move quickly closer to the extreme compression fiber, reducing the area of compression block, and producing larger curvature and deflection. On the other hand, for Beams B and C under increasing load toward failure, the internal forces T and C continue to increase (because the reinforcement lacks a yield plateau) without the need of a pronounced increase in the lever arm between T and C, thus producing smaller curvatures and deflections for a given applied load.

Figure C.2 shows the relationship for a singly reinforced rectangular beam between the nominal flexural strength M_n and reinforcement ratio ρ for three different reinforcement yield strengths f_y: 60,000 psi (420 MPa) for Grade 60 (420) bar, 80,000 psi (550 MPa) for the upper limit permitted by ACI 318, and 100,000 psi (690 MPa) for the simplified design method (Mast et al. 2008) recommended in this guide. Also shown in the figure is the theoretical prediction obtained by using a nonlinear stress-strain relationship for ASTM A1035/A1035M (CS) Grade 100 (690) reinforcement closely similar to Eq. (3.4a), (3.4b), and (3.4c) recommended in this guide.

It is important to note that each curve consists of two segments connected at the point corresponding to the balanced reinforcement ratio ρ_b. Tension-controlled failure of a beam occurs when its reinforcement ratio is less than ρ_b, whereas compression-controlled failure of a beam occurs when its reinforcement ratio is greater than ρ_b. As the reinforcement yield strength f_y increases, the balanced reinforcement ratio is greatly reduced. For beams with tension-

Table C.1—Comparisons between measured and predicted ultimate load and the corresponding deflection of beams tested by Yotakhong (2003)

Beam ID	Ultimate load			Deflection at ultimate load		
	Measured, kip (kN) (a)	Calculated, kip (kN) (b)	(a)/(b)	Measured, in. (mm) (c)	Calculated, in. (mm) (d)	(c)/(d)
Beam A	40.7 (181)	41 (182)	0.99	3.87 (98)	3.1 (79)	1.25
Beam B	54.7 (243)	58 (258)	0.94	3.1 (79)	3.1 (79)	1.0
Beam C	77.9 (347)	80 (356)	0.97	2.7 (69)	2.5 (64)	1.08

Table C.2—Details of flexural tests by Malhas (2002) and Ansley (2002)

Beam ID	Bottom bars	Top bars	Stirrups	f_c', psi (MPa)	Measured M_n, ft-kip (m-kN) (a)	Calculated M_n, ft-kip (m-kN) (b)	(a)/(b)
Malhas (2002) tests							
A1	Two No. 8	Two No. 4	No. 4 at 6 in.	6200 (43)	227 (309)	194 (264)	1.17
A2	Four No. 4	Two No. 4	No. 4 at 6 in.	6200 (43)	145 (197)	102 (139)	1.42
A3	Two No. 4	Two No. 4	No. 4 at 6 in.	6200 (43)	73 (99)	54 (73)	1.35
Ansley (2002) test							
1	Two No. 6	None	None	6683 (46)	202.5 (275)	112.6 (153)	1.80

controlled failure, the nominal flexural strength is governed by the reinforcement yield strength; therefore, as the yield strength increases, the flexural strength increases substantially. On the other hand, for beams with compression-controlled failure, the nominal flexural strength is governed by the concrete strength; therefore, the three curves merge into a single curve for large values of ρ.

Also shown in Fig. C.2 are flexural test results obtained by Malhas (2002) of the University of North Florida and by Ansley (2002) of the Florida Department of Transportation, both using 12 x 18 in. (300 x 450 mm) beams reinforced with ASTM A1035/A1035M (CS) bars. The details of these test beams are shown in Table C.2. The calculated nominal flexural strength M_n given in the table is based on the simplified method recommended in this guide.

It can be seen from Table C.2 that the test results exceeded the predictions using the simplified method by a substantial margin. Furthermore, Fig. C.2 shows that the test results compared closely with the theoretical predictions from nonlinear analysis. These comparisons suggest that it is justifiable to use the ϕ factor specified by ACI 318 for the simplified design method. Additionally, because the failure is dominated by yielding of tension reinforcement, reliability of the calculated flexural strength is directly related to reliability of the steel properties of ASTM A1035/A1035M (CS) bars, which are comparable to reliability of the steel properties of ASTM A615/A615M bars.

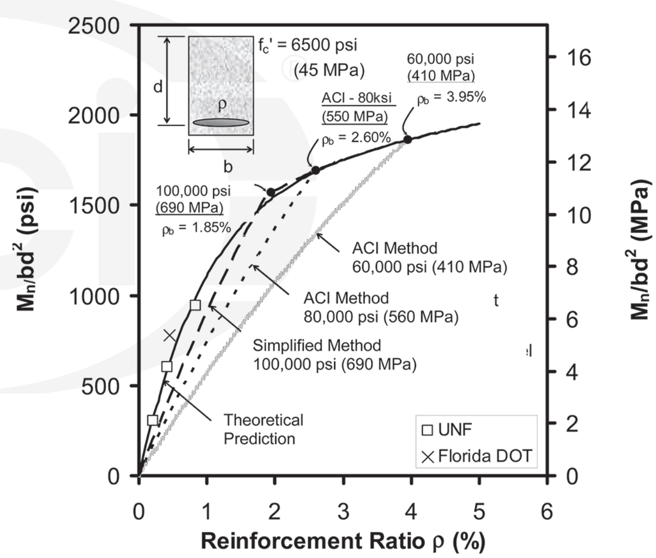


Fig. C.2—Comparisons of nominal flexural strength versus reinforcement ratio for different reinforcement yield strength (Mast et al. 2008).

Designers using ASTM A1035/A1035M (CS) should check for cracking and deflection of members being designed.

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