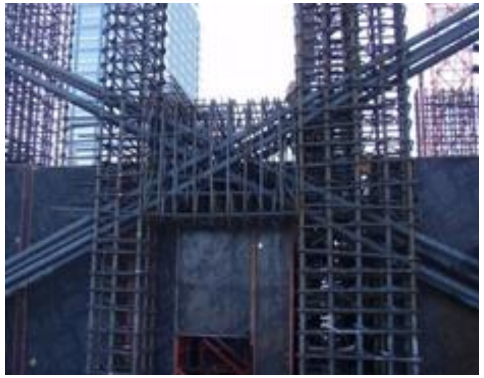
# Simplified Moment-Rotation Relationship for Plastic Hinge of Coupling Beams



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# Contents

# Coupling Beam Design Guidelines – ACI 318-19

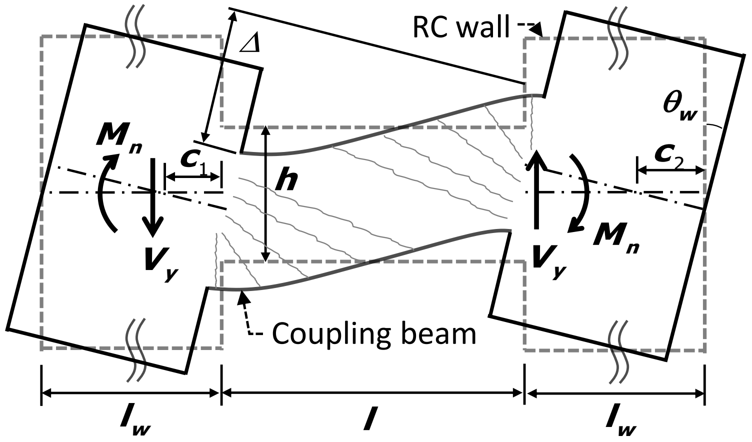
- **Length-to-height ($l/h$)**
  - $l/h \geq 4$ :  Special moment frames – Beam design
  - $l/h \leq 4$ : 18.10.7 Coupling beams
- **Shear strength of short coupling beams**
  - Essential application of diagonal bar: $l/h < 2$ and $V_u \geq 0.33\sqrt{f_c'}A_{cw}$
  - Shear strength(diagonal bars contribution):

$$V_n = 2A_{vd}f_y \sin\alpha < 0.83\sqrt{f_c'}A_{cw}$$

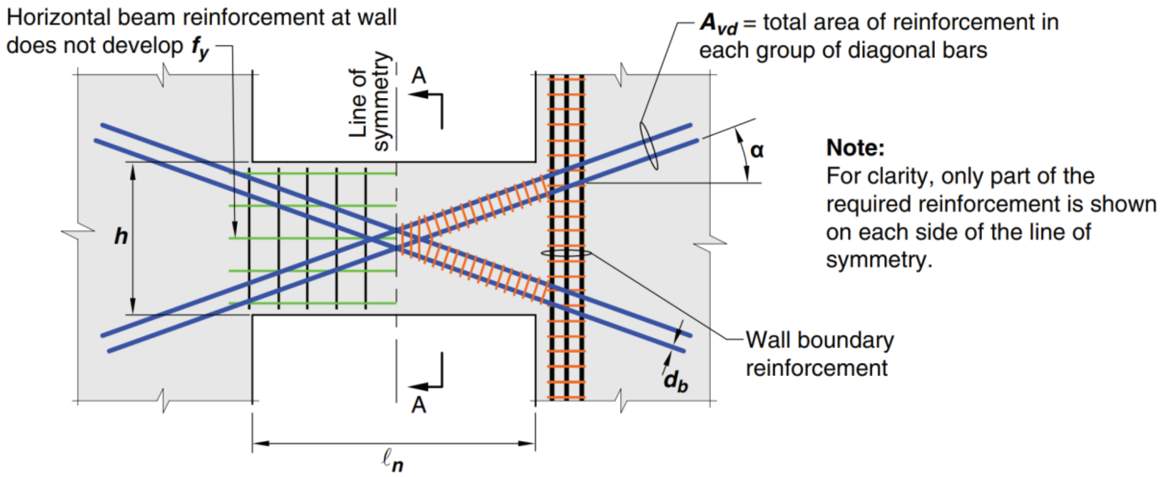
- **Transverse reinforcement**
  - $s$ shall not exceed the least of (a) ~ (d):

(a) $d/4$  (b) 150 mm

(c) For Grade 420, $6d_b$  (d) For Grade 550, $5d_b$



$$\theta = \gamma = \frac{\Delta}{l} = \frac{c_1\theta_w + (l_w - c_2)\theta_w}{l} \approx \frac{l_w}{l}\theta_w$$



# 1. Background

## Coupling Beam Design Guidelines – ASCE/SEI 41-17

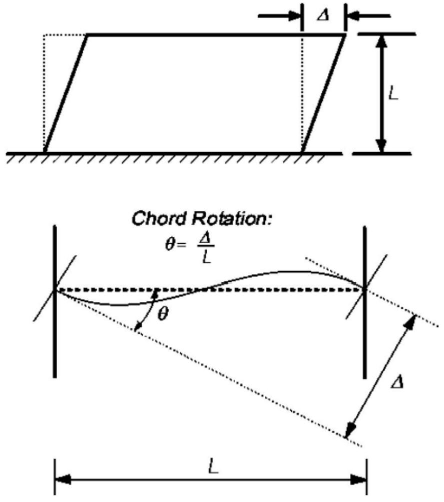
- ◆ **Modeling criteria**
  - Both bending and shear deformations shall be used
  - Diagonal reinforcement beam (ACI 318) : flexure only
- ◆ **Shear strength of short coupling beams**
  - Shear strength : $V_n = 2A_{vd} f_y \sin\alpha$ (ACI 318)
  - Modeling parameters

a) Controlled by Flexure

| Condition | | Modeling parameters | |
|---|---|---|---|
| Details | $\frac{V}{t_w l_w \sqrt{f'_{cE}}}$ | Plastic Hinge Rotation (rad.) | |
| | | a | b |
| Seismic detail section | $\leq 3$ | 0.025 | 0.050 |
| | $\geq 6$ | 0.020 | 0.040 |
| Non-seismic detail section | $\leq 3$ | 0.020 | 0.035 |
| | $\geq 6$ | 0.010 | 0.025 |
| Diagonal reinforcement | NA | 0.030 | 0.050 |

b) Controlled by Shear

| Condition | | Modeling parameters | |
|---|---|---|---|
| Details | $\frac{V}{t_w l_w \sqrt{f'_{cE}}}$ | Plastic Hinge Rotation (rad.) | |
| | | d | e |
| Seismic detail section | $\leq 3$ | 0.02 | 0.030 |
| | $\geq 6$ | 0.016 | 0.024 |
| Non-seismic detail section | $\leq 3$ | 0.012 | 0.025 |
| | $\geq 6$ | 0.008 | 0.014 |



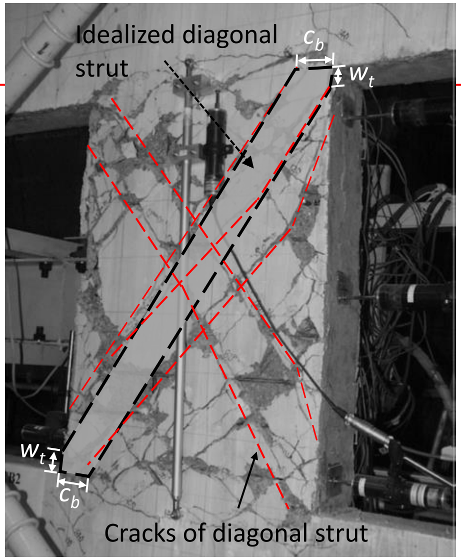
Chord rotation for coupling beams

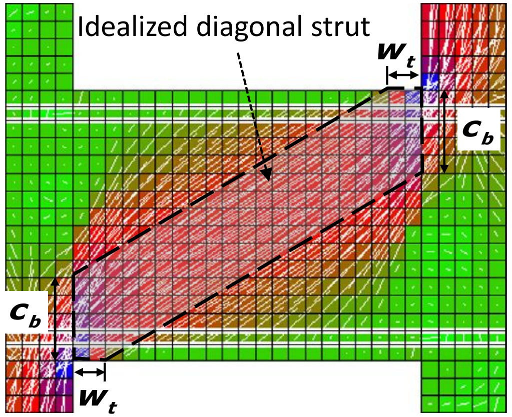
# 1. Background

## Existing Research

### ◆ Problem

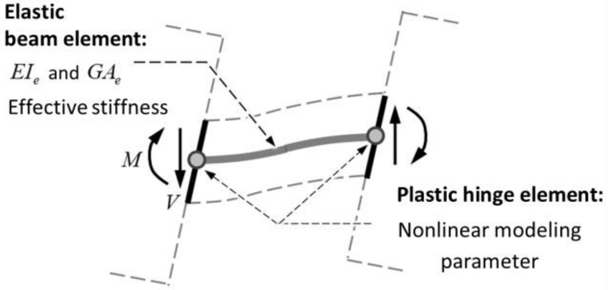
- ➢ Inelastic deformations increase → shear strength decrease
- ➢ Existing studies: shear strength degradation is not clearly defined
- ➢ Unified model addressing various parameter is required

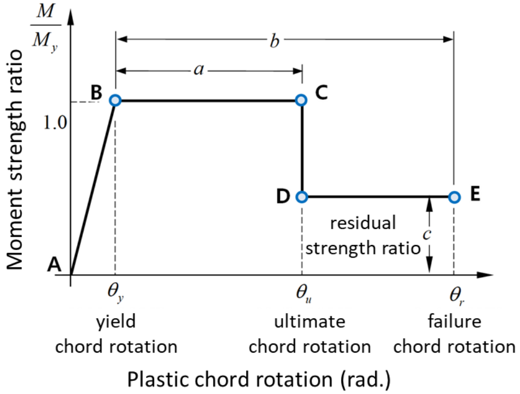




### ◆ Development goals

- ➢ Shear strength degradation of coupling beams
- ➢ Moment-rotation relationship for plastic hinge
- ➢ Nonlinear behavior model for coupling beams w/wo diagonal bars

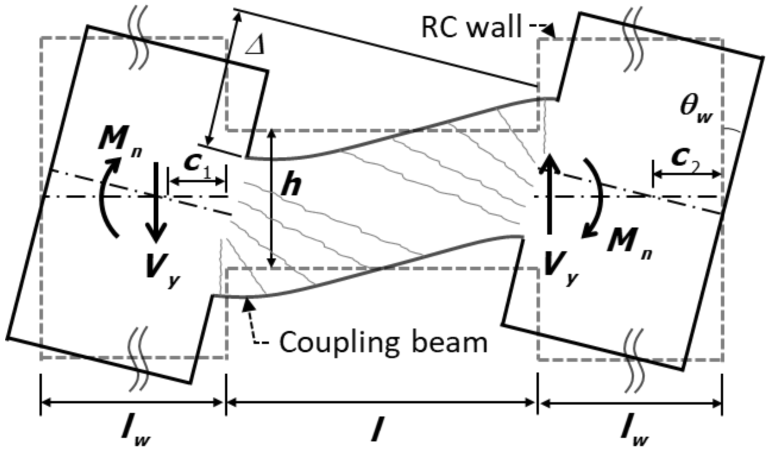




# 2. Coupling Beam Model

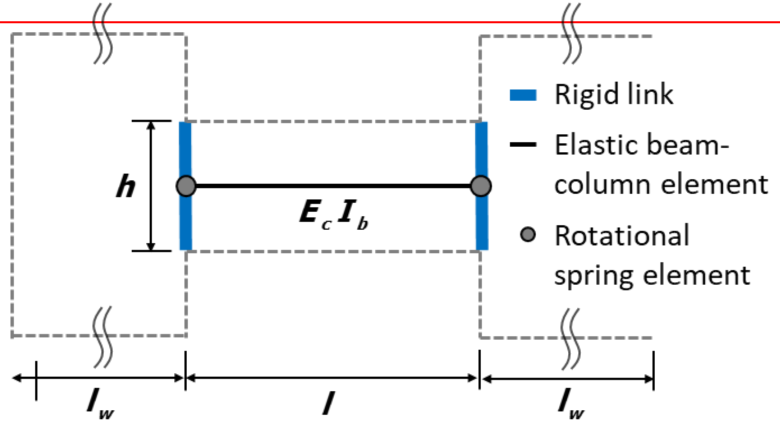
## Plastic Hinge Model

- ◆ **Elastic Beam + Plastic Hinge**

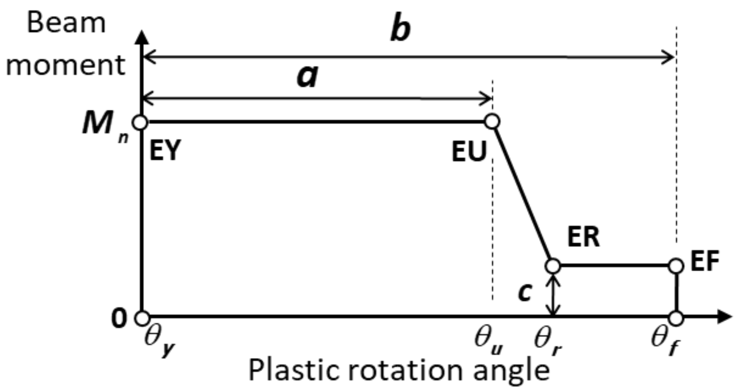


$$\theta = \gamma = \frac{\Delta}{l} = \frac{c_1\theta_w + (l_w - c_2)\theta_w}{l} \approx \frac{l_w}{l}\theta_w$$

**Short coupling beam**



(a) Plastic hinge model for coupling beams



(b) Envelope curve for rotational spring element

# 2. Coupling Beam Model

## Plastic Hinge Model

- **Elastic Beam + Plastic Hinge**
  - Effective stiffness*:

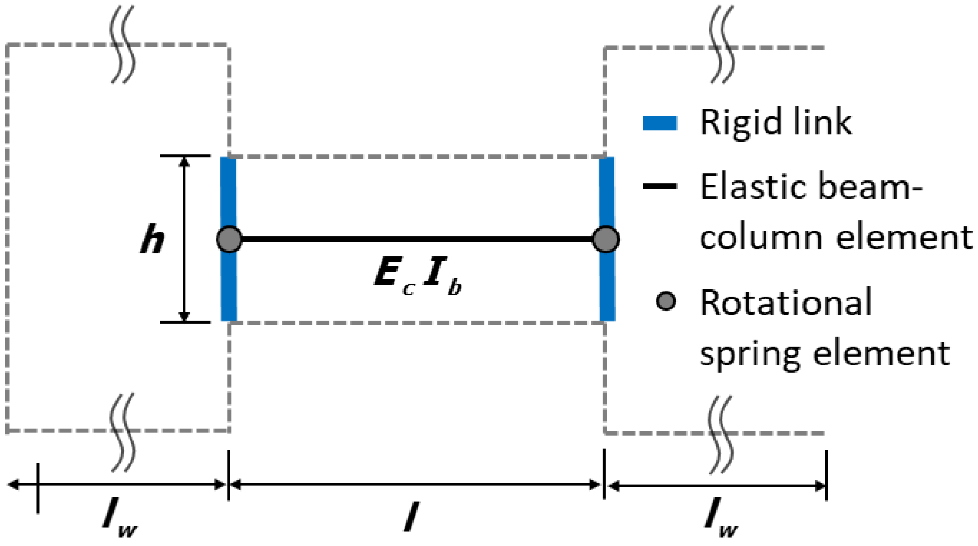
$$E_c I_b = \frac{0.3}{1+20\left(h/l\right)^3} E_c I_g$$

$E_c$ = Elastic modulus of concrete(= $4700\sqrt{f_c'}$)

$I_g$ = second-order moment of inertia of the gross cross section in the coupling beam

$h$ = coupling beam height

$l$ = coupling beam length



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* Eom, T.S. et al. "Nonlinear Modeling Parameters of Reinforced Concrete Coupling Beams,"

# 2. Coupling Beam Model

## Plastic Hinge Model

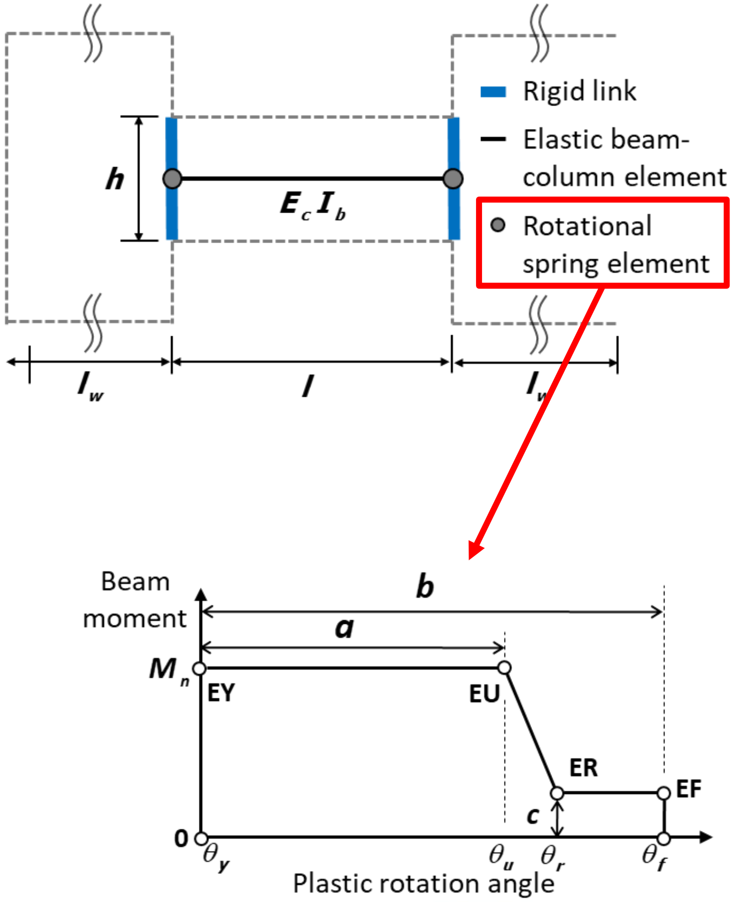
- **Elastic Beam +** **Plastic Hinge**
  - In moment-rotation relationship

    ***a*** = inelastic deformation prior to a sudden strength degradation

    ***b*** = ultimate deformation at failure

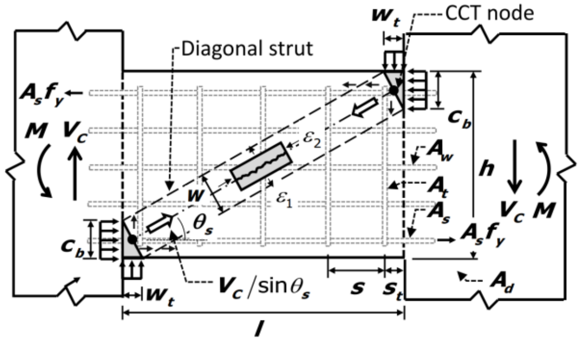
    ***c*** = residual strength
  - Moment-rotation relationship of rotational spring element:

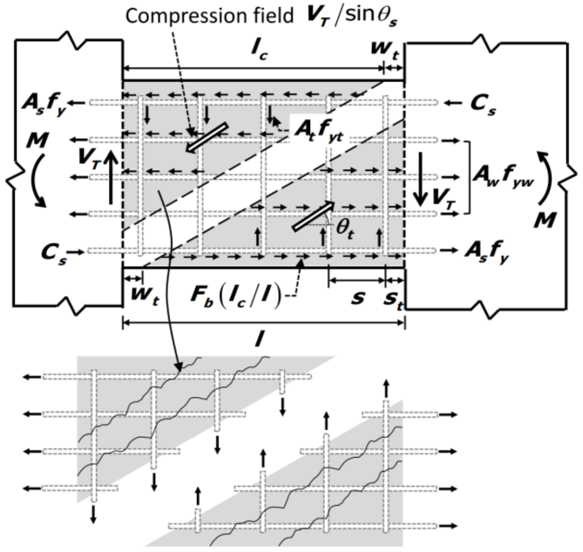
    Defined by shear strength degradation after flexural yielding

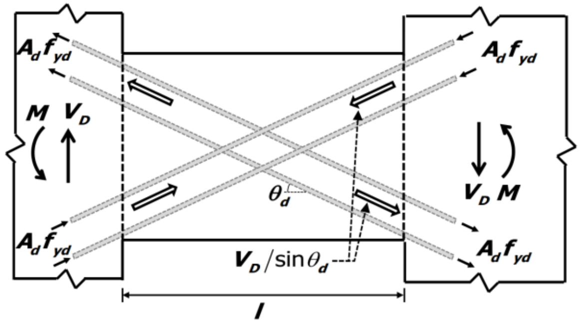


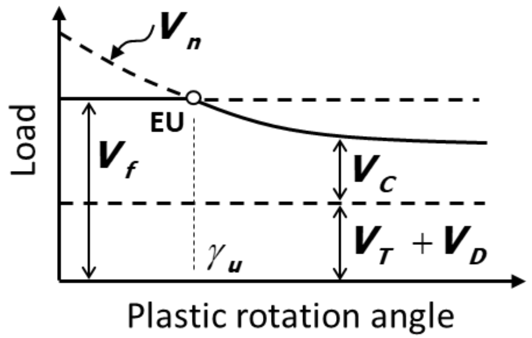
# Shear Resistance Mechanisms of Short Coupling Beam

◆ Strut mechanism($V_C$) + Truss mechanism($V_T$) + Diagonal bar resistance($V_D$)









**Load – plastic rotation angle relationship**

- ➢ Truss mechanism($V_T$) & Diagonal bar resistance($V_D$):

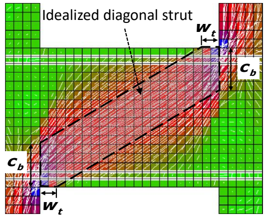
  Constant shear contribution regardless of deformation

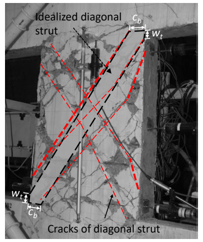
- ➢ Strut mechanism($V_C$) :

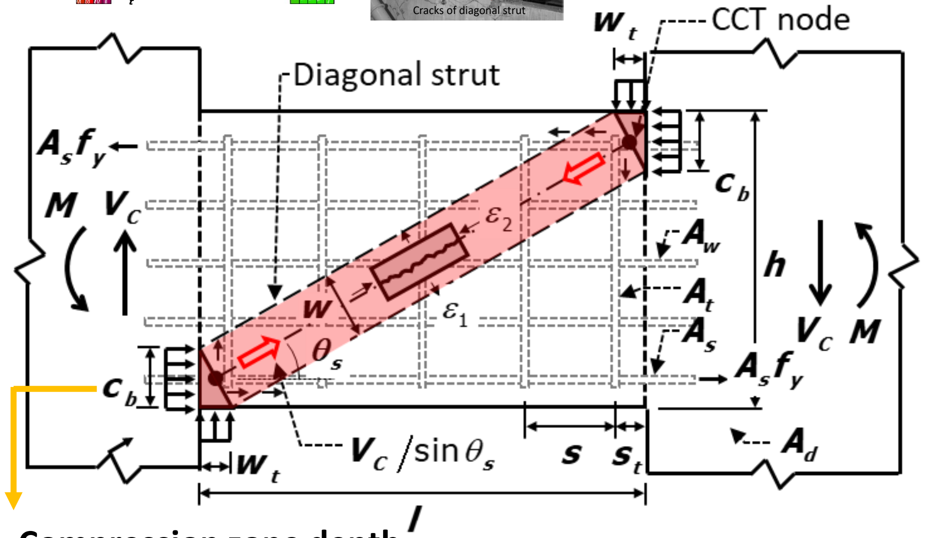
  As deformation increases, the shear contribution decreases due to the diagonal cracks

# Shear Resistance Mechanisms of Short Coupling Beam

◆ Diagonal strut resistance $V_C$







- **Compression zone depth**

$$c_b = \frac{A_s f_y + A_d f_{yd}\cos\theta_d}{0.85 f_c' b}$$

($A_d f_{yd}\cos\theta_d$: contribution of diagonal tension bars to $c_b$)

- **Maximum width of CCT node**

$$w_t = \frac{A_t f_{yt}}{0.8(0.85 f_c')b} \le 2s_t$$

- **Width & angle of diagonal strut**

$$w = c_b\cos\theta_s + w_t\sin\theta_s \qquad \theta_s = \operatorname{atan}\left(\frac{h - c_b}{l - w_t}\right)$$

$\theta_s$: determined from the geometric property and crack angle based on existing test result

- **Effective concrete compressive strength (MCFT)**

$$f_{ce} \approx \frac{f_c'}{0.8 + 170\left[\frac{\gamma}{2}\tan\theta_s + \varepsilon_{yt}\right]} \le f_c'$$

γ: inelastic shear distortion of the coupling beam

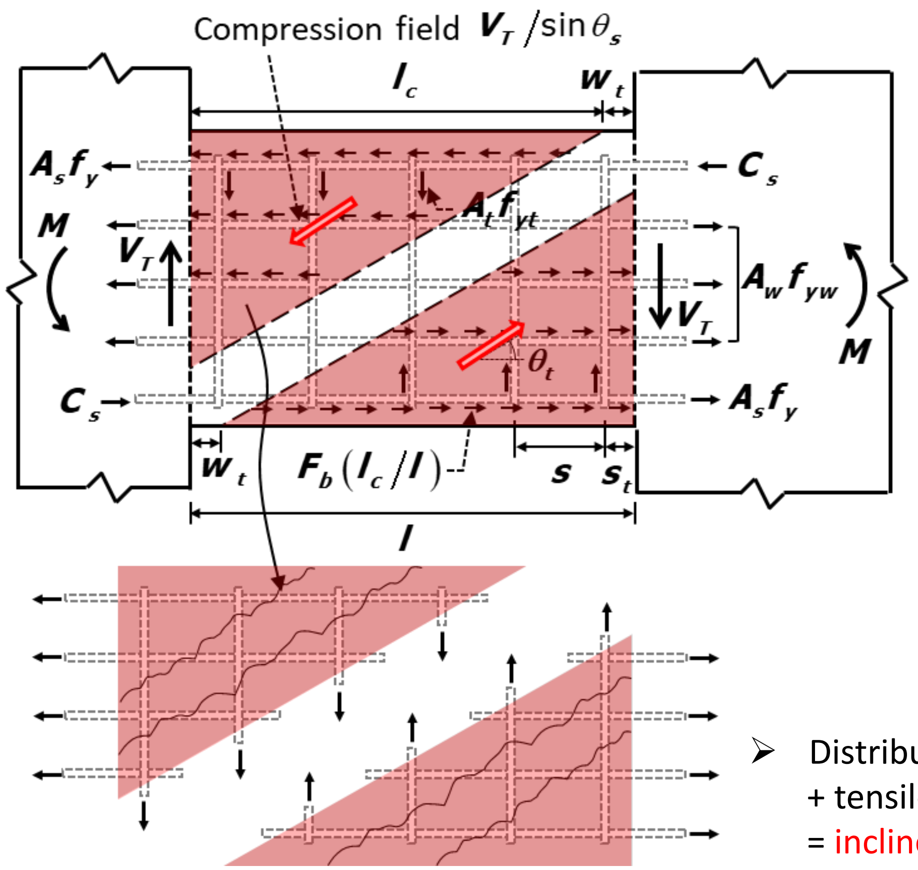
(As the shear deformations increases, $f_{ce}$ decreases)

- **Shear resistance of diagonal strut**

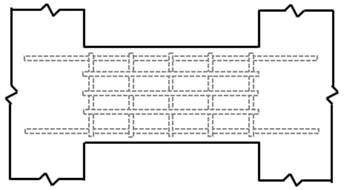
$$V_C = f_{ce}(b_w)\sin\theta_s$$

# Shear Resistance Mechanisms of Short Coupling Beam

- ◆ Truss mechanism resistance $V_T$



- **Shear strength by longitudinal bars**



$$V_{T1} = \left(A_s f_y + \alpha_c A_w f_{yw}\right)\tan\theta_t$$

$\alpha_c$: factor related to cut-off bars

(=0.6 for cut-off distributed longitudinal web bars)

- **Crack angle in compression stress field**

$$\theta_t = \max\left[\theta_s, 26.5°\right]$$

$\theta_s$: diagonal strut angle

26.5: $\cot\theta_t = 2.0$, according to bearing pressure(or compressive stress) distribution in the compression field

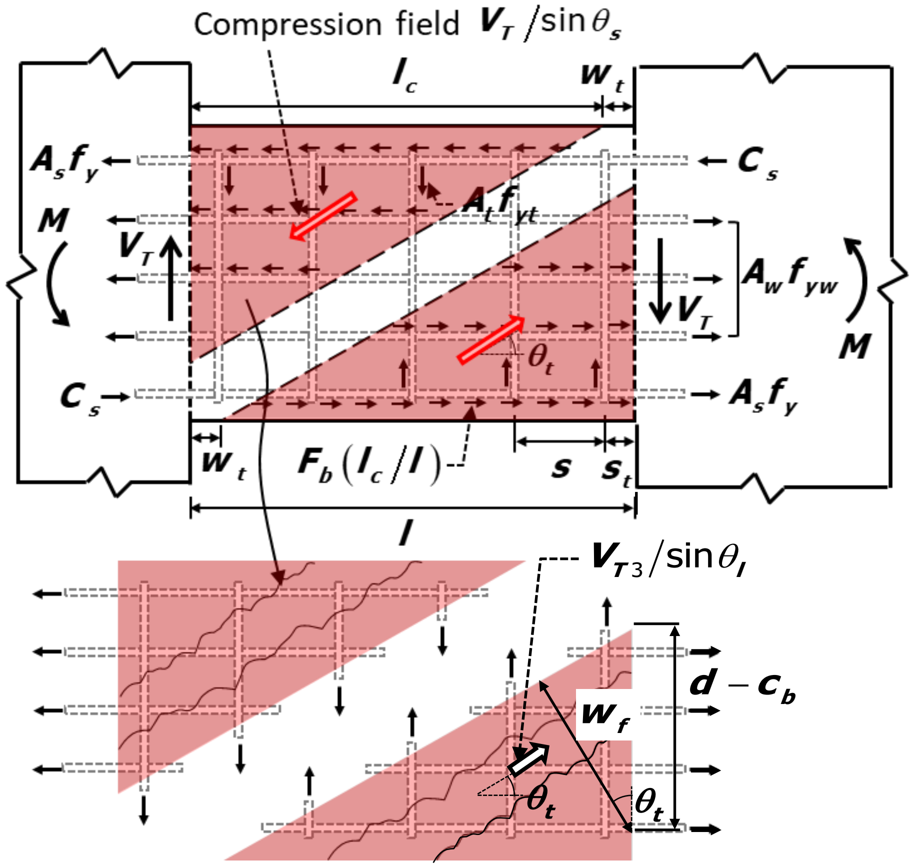
- **Shear strength by transverse bars**

$$V_{T2} = A_t f_{yt} n_t = A_t f_{yt}\frac{d}{s\cdot\tan\theta_t}$$

- Distributed bond stress of longitudinal bars
+ tensile force of transverse reinforcement
= inclined compression stress field

# Shear Resistance Mechanisms of Short Coupling Beam

◆ Truss mechanism resistance $V_T$



- **Local failure of concrete**

$$V_{T3} = 0.35 f_{ce0}\left(b w_f\right)\sin\theta_t$$

Strength reduction factor of 0.35
due to Bi-directional tension of concrete
+ Diagonal cracks of opposite strut

$$f_{ce0} \approx \frac{f_c'}{0.8+170\varepsilon_{yt}} \geq 0.85\times 0.75 f_c'$$

Strut strength of ACI 318-19

- **Local diagonal strut width**

$$w_f = \left(d - c_b\right)\cos\theta_t$$

*d*: Beam effective depth

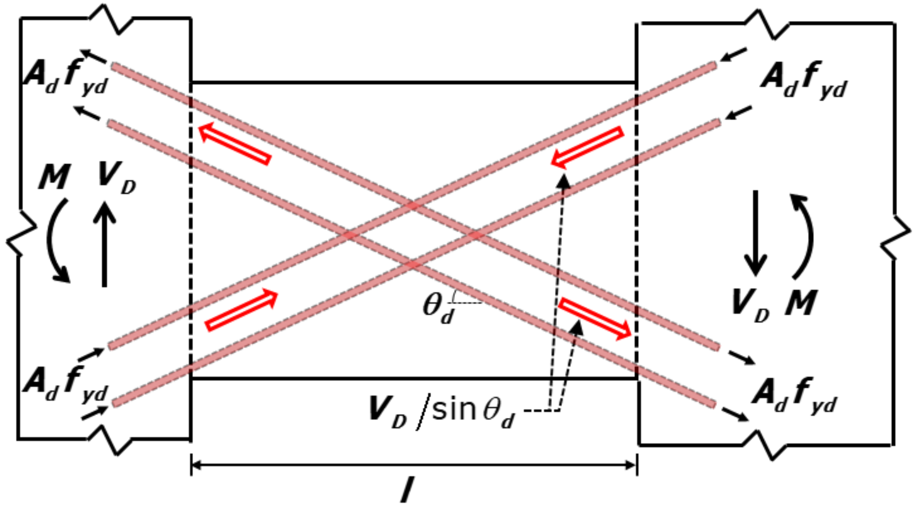
- **Shear strength of truss mechanism**

$$V_T = \min[V_{T1}, V_{T2}, V_{T3}]$$

# 2. Coupling Beam Model

## Shear Resistance Mechanisms of Short Coupling Beam

◆ Diagonal bar resistance $V_D$



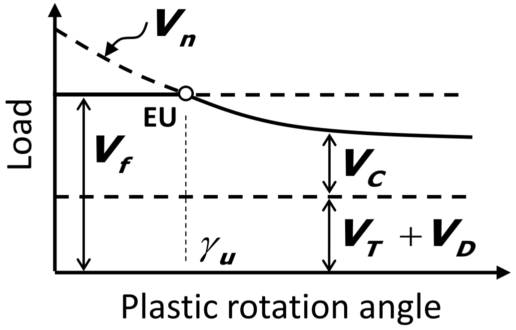
- Cyclic loading
  - → Large tensile plastic deformation of diagonal bars
  - → Residual tensile strain
  - → Early development of the compressive stress
  - → Contribution to tension and compression
  - → Shear resistance of the diagonal bars
- **Shear resistance of diagonal bars**

$$V_D = 2A_d f_{yd} \sin \theta_d$$

- **Diagonal bars resist the shear force directly**
  - → When high-strength rebar (or cable) is applied, concrete damage may occur before diagonal bar yielding: upper limit of $f_{yd}$ should be considered

# Deformation Capacity of Coupling Beam

◆ Shear capacity($V_n$) of a coupling beam



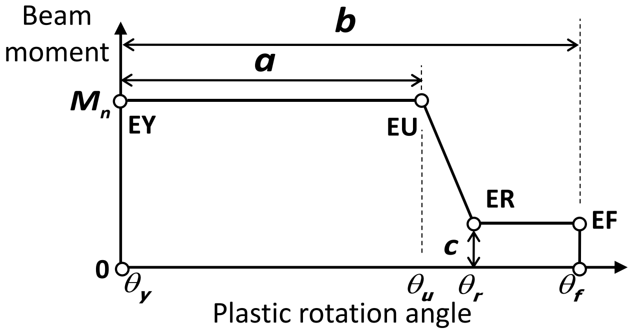
- **Shear capacity**

$$V_n = V_C + V_T + V_D$$

- **Shear distortion increase**

  → $V_C$ decreases & $V_T$ and $V_D$ are maintained

◆ Moment-rotation relationship of rotational spring element



- Rotational spring elements at the interface between beam and wall

- **EY** $(0, M_n)$: Yielding point

  **EU** $(\theta_u, M_n)$: Ultimate point

  **ER** $(\theta_r, 0.2M_n)$: Residual point

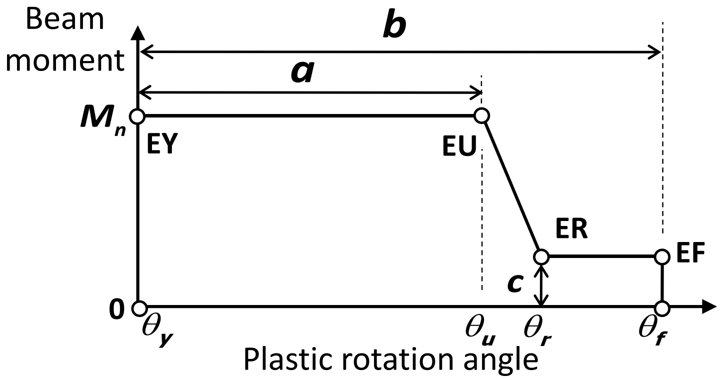
  **EF** $(\theta_f, 0.2M_n)$: Failure point

aci CONCRETE CONVENTION

# 2. Coupling Beam Model

## Moment-Rotation Relationship of Plastic Hinge Model

◆ Proposed model



- **EY** $(0, M_n)$ **– Yielding Point**

$$M_n = \left(A_s f_y + A_d f_{yd} \cos\theta_d\right)\left(d - \frac{c_b}{2}\right)$$

- **EU** $(\theta_u, M_n)$ **– Ultimate Point**
  - Intersection point between the shear capacity and shear demand

$$V_f = \frac{2M_n}{l} = \frac{2\left(A_s f_y + A_d f_{yd} \cos\theta_d\right)\left(d - \frac{c_b}{2}\right)}{l}$$

$$\theta_u = \gamma_u = \frac{1}{85\tan\theta_s}\left[\frac{f_c'(bw)\sin\theta_s}{V_f - V_T - V_D} - 0.8 - 170\varepsilon_{yt}\right]$$

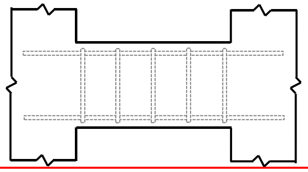
- Compression zone failure
  - For conventional reinforcement $\theta_u$ = 0.03, for distributed reinforcement $\theta_u$ = 0.035, for diagonal reinforcement $\theta_u$ = 0.045

- **ER** $(\theta_r, 0.2M_n)$**, EF** $(\theta_f, 0.2M_n)$ **– Residual & Failure Points**
  - A linear strength degradation between **EU** and **ER**, on the basis of the strength degradation of existing test

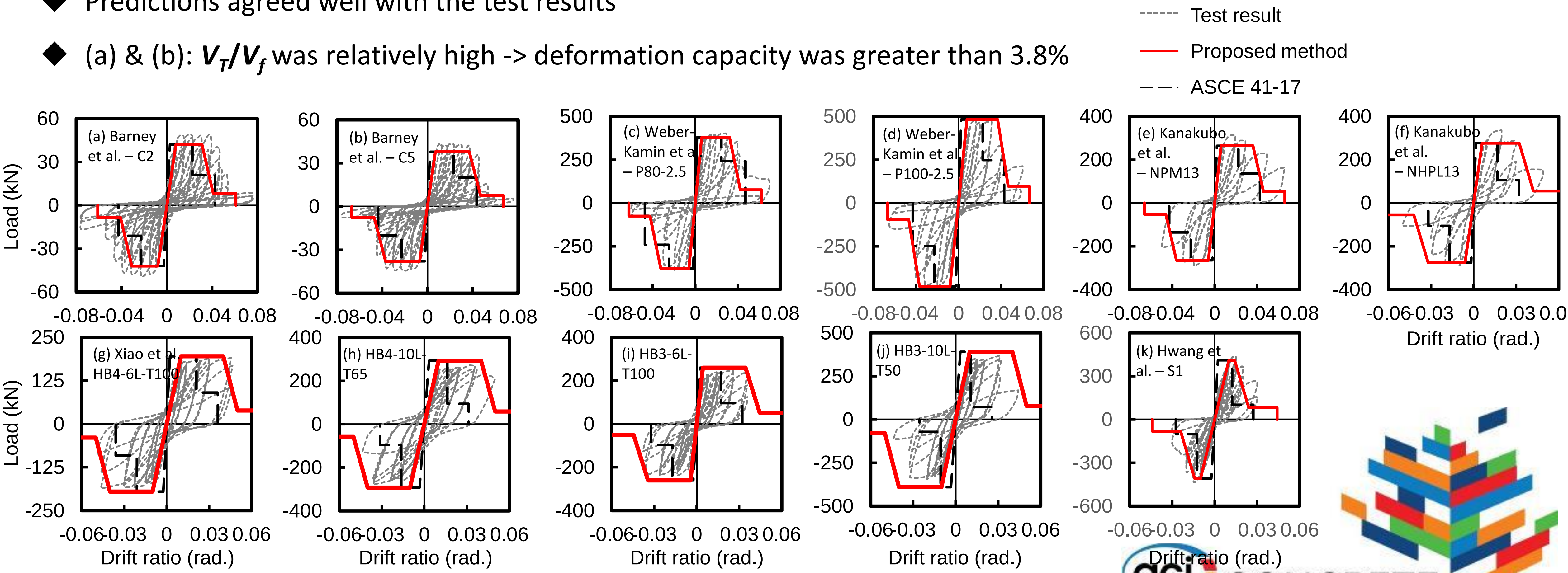
$$\theta_r = \theta_u + 0.01 \text{ rad.} \qquad \theta_f = \theta_u + 0.03 \text{ rad.}$$

# 3. Comparison with Test Result

## Conventional Reinforcement

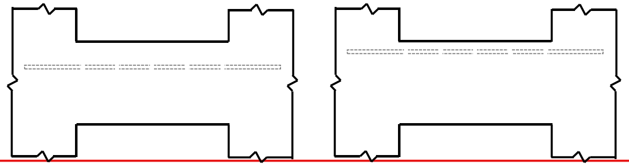


- Predictions agreed well with the test results
- (a) & (b): $V_T/V_f$ was relatively high -> deformation capacity was greater than 3.8%

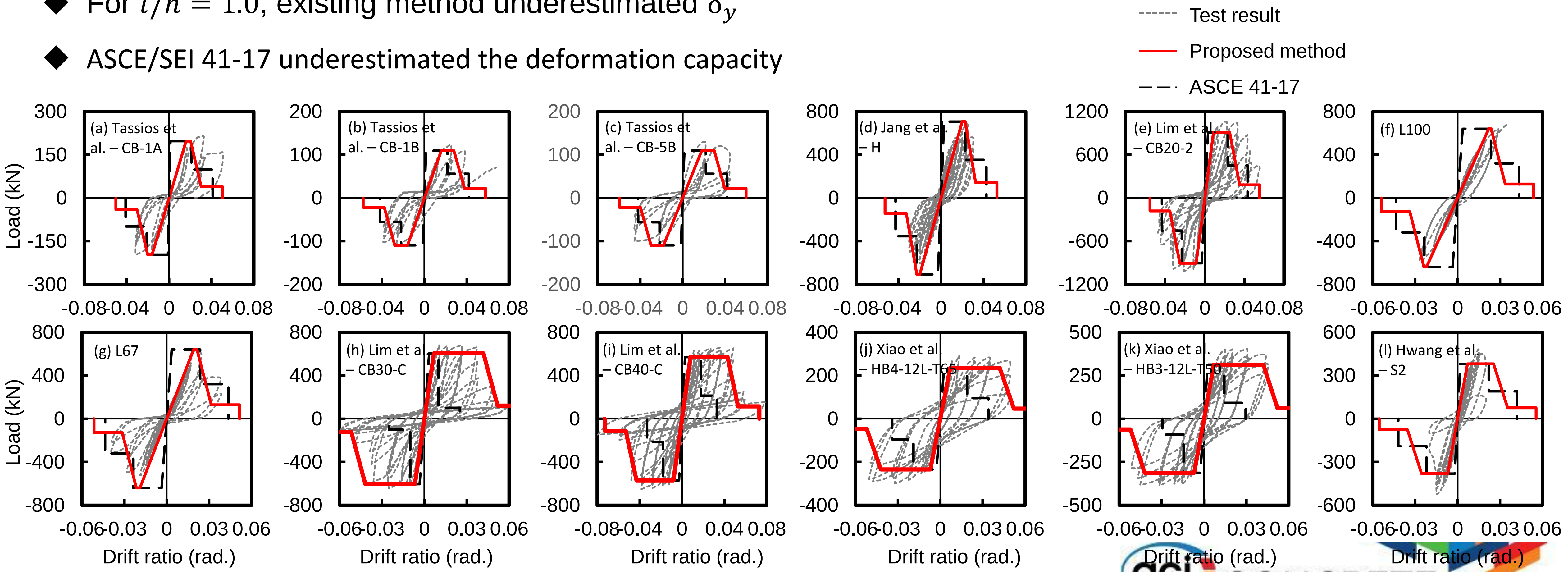


# 3. Comparison with Test Result

## Distributed Reinforcement

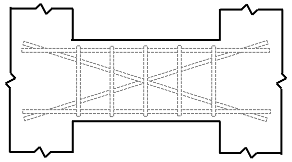


- For $l/h = 1.0$, existing method underestimated $\delta_y$
- ASCE/SEI 41-17 underestimated the deformation capacity

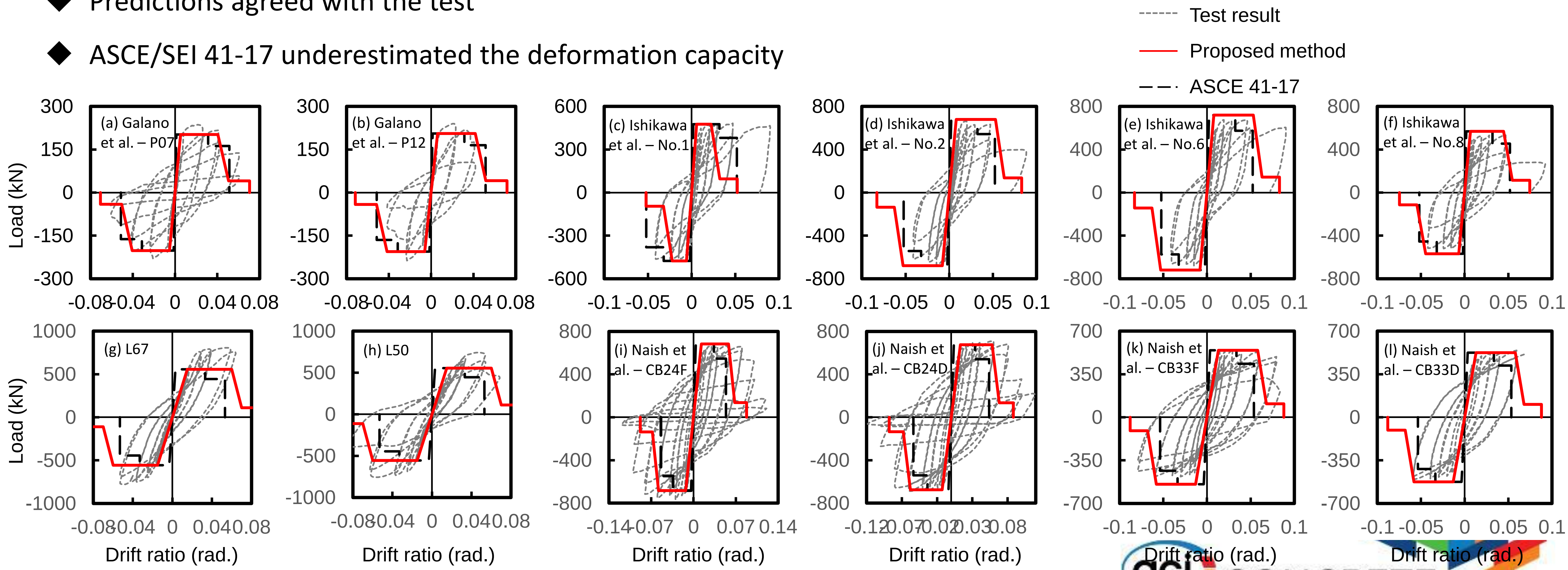


# 3. Comparison with Test Result

## Diagonal Reinforcement



- Predictions agreed with the test
- ASCE/SEI 41-17 underestimated the deformation capacity

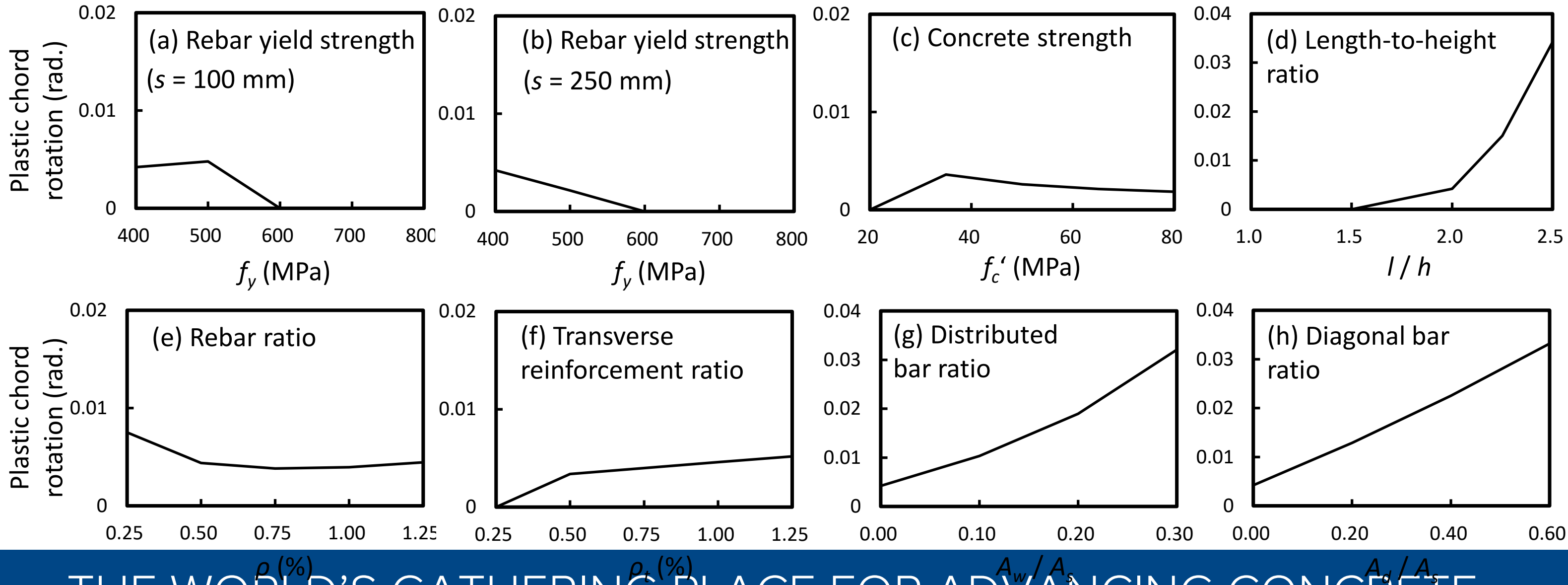


# 4. Parameter Effect Analysis

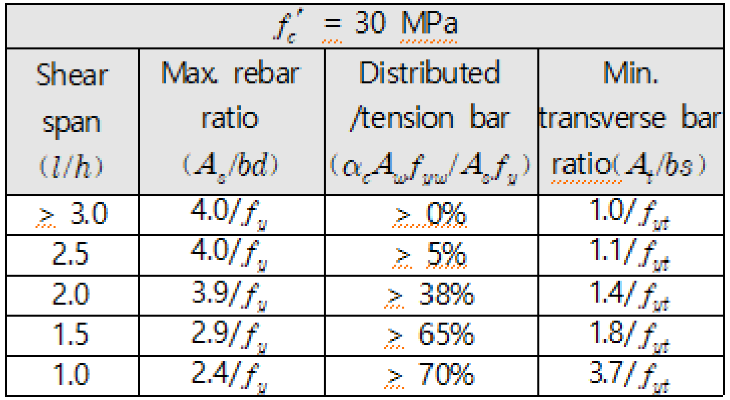
## Effects of design parameters

- Transverse reinforcement
  - Shear strength increases as transvers reinforcement increases
- Distributed bars
  - Shear strength increases as distributed bar ratio increases
  - Distributed bars do not increase shear demand

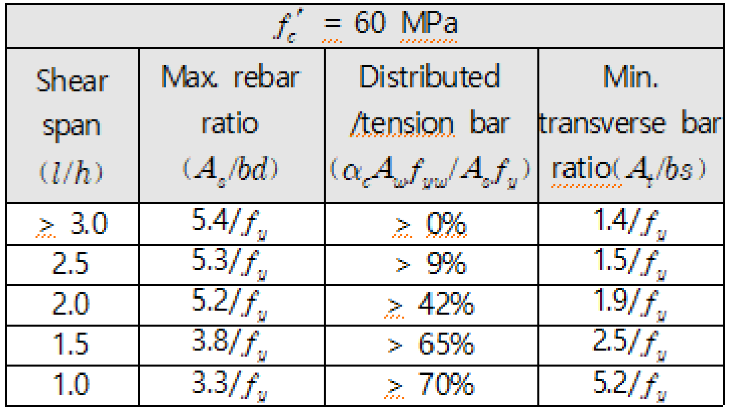
→ increasing the transverse reinforcement and distributed bars is effective



Design recommendation for the plastic hinge rotation of *a* = 0.03 rad.



| $f_c'$ = 30 MPa | | | |
|---|---|---|---|
| Shear span $(l/h)$ | Max. rebar ratio $(A_s/bd)$ | Distributed /tension bar $(\alpha_c A_w f_{yw} / A_s f_y)$ | Min. transverse bar ratio$(A_t/bs)$ |
| ≥ 3.0 | $4.0/f_y$ | ≥ 0% | $1.0/f_{yt}$ |
| 2.5 | $4.0/f_y$ | ≥ 5% | $1.1/f_{yt}$ |
| 2.0 | $3.9/f_y$ | ≥ 38% | $1.4/f_{yt}$ |
| 1.5 | $2.9/f_y$ | ≥ 65% | $1.8/f_{yt}$ |
| 1.0 | $2.4/f_y$ | ≥ 70% | $3.7/f_{yt}$ |



| $f_c'$ = 60 MPa | | | |
|---|---|---|---|
| Shear span $(l/h)$ | Max. rebar ratio $(A_s/bd)$ | Distributed /tension bar $(\alpha_c A_w f_{yw} / A_s f_y)$ | Min. transverse bar ratio$(A_t/bs)$ |
| ≥ 3.0 | $5.4/f_y$ | ≥ 0% | $1.4/f_y$ |
| 2.5 | $5.3/f_y$ | > 9% | $1.5/f_y$ |
| 2.0 | $5.2/f_y$ | ≥ 42% | $1.9/f_y$ |
| 1.5 | $3.8/f_y$ | > 65% | $2.5/f_y$ |
| 1.0 | $3.3/f_y$ | ≥ 70% | $5.2/f_y$ |

# 5. Simplified model

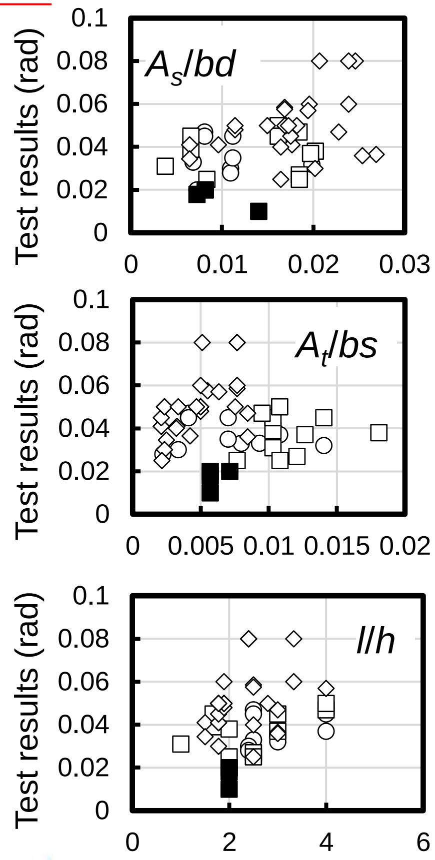
## Simplification

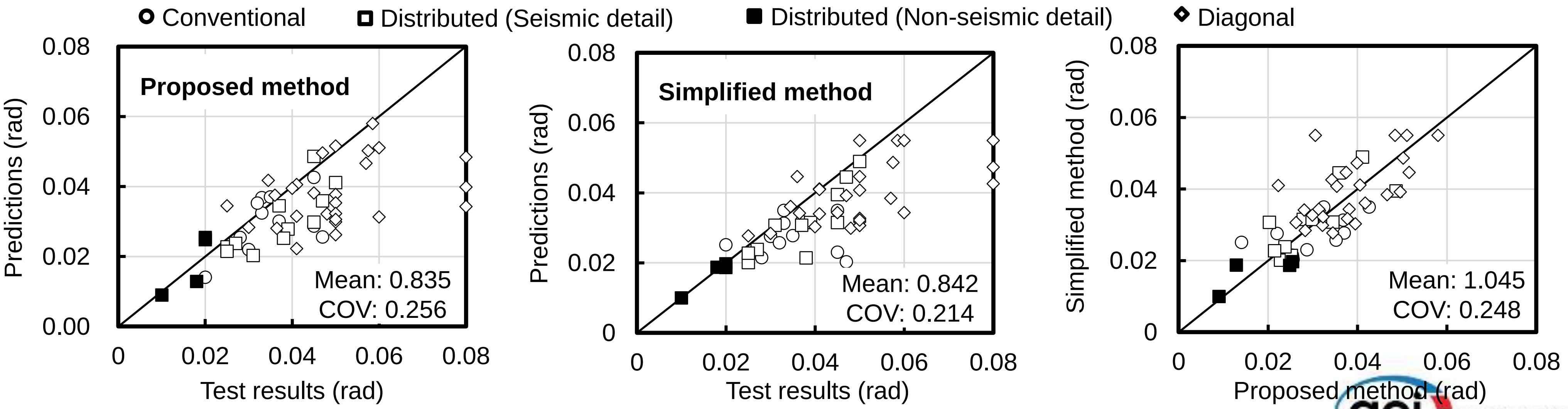
- Shear strength model in ACI 318-19
  - $V_n = V_c + V_s + V_d$
  - $V_c = \sqrt{f_c'}/6\,(bd)$
  - $V_s = A_t f_{yt} d/s \leqq 4V_c$
  - $V_d = 2A_d f_{yd} \sin\theta_d$

Mean value = 0.84

→ Close to 1 sigma range of reliability (overestimation ratio = 16%)

| Rebar details | | Plastic rotation *a* (rad) |
|---|---|---|
| Conventional rebars<br>$V_u \leqq V_n$ | Seismic detail | $0.02(V_n/V_u) - \theta_y \leqq 0.035 - \theta_y$ |
| | Non-seismic detail | $0.016(V_n/V_u) - \theta_y \leqq 0.025 - \theta_y$ |
| Distributed rebars<br>$V_u \leqq V_n$ | Seismic detail | $0.025(V_n/V_u) - \theta_y \leqq 0.05 - \theta_y$ |
| | Non-seismic detail | $0.016(V_n/V_u) - \theta_y \leqq 0.035 - \theta_y$ |
| Diagonal rebars | All cases | $0.03(V_n/V_u) - \theta_y \leqq 0.055 - \theta_y$ |
| Conventional & Distributed<br>$V_u > V_n$ | All cases | $0.01 - \theta_y$ |





# 6. Summary and conclusions

1. For the nonlinear numerical analysis of short coupling beams, a plastic hinge model was developed
2. Shear strength of coupling beam was defined as $V_n = V_C + V_T + V_D$
3. A rotational spring element was used to describe inelastic deformation
4. To describe the shear strength degradation, moment-chord rotation relationship of the rotational spring element was developed
5. Distributed bars and transverse reinforcement can increase shear strength without increasing shear demand
6. Simplified method based on shear strength of ACI 318-19 was proposed

# Thank you for your attention!

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