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# Guide to Estimating Prestress Loss

Reported by Joint ACI-ASCE Committee 423



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## Guide to Estimating Prestress Loss

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**American Concrete Institute**  
**38800 Country Club Drive**  
**Farmington Hills, MI 48331**  
**Phone: +1.248.848.3700**  
**Fax: +1.248.848.3701**

[www.concrete.org](http://www.concrete.org)

# Guide to Estimating Prestress Losses

Reported by Joint ACI-ASCE Committee 423

Carin L. Roberts-Wollmann\*, Chair

Amy M. Reineke Trygestad, Secretary

Theresa M. Ahlborn\*  
Robert W. Barnes†  
Florian G. Barth  
Asit N. Baxi†  
Roger J. Becker  
Kenneth B. Bondy†  
Charles W. Dolan†  
James P. Donnelly  
Pierre Esselinck†

Martin J. Fradua  
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Shawn P. Gross  
Pawan R. Gupta  
William M. Hale  
H. R. Trey Hamilton III†  
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Mohammad Iqbal

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Larry B. Krauser†  
Jason J. Krohn‡  
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Sami H. Rizkalla  
James Rogers  
Bruce W. Russell  
David H. Sanders

Thomas C. Schaeffer  
Morris Schupack‡  
Richard W. Stone  
Miroslav F. Vejvoda†  
Jeffery S. Volz  
H. Carl Walker  
Zuming Xia†  
Paul Zia\*\*

\*Co-chairs of subcommittee responsible for preparation of report.

†Subcommittee members.

‡Deceased

## Consulting Members

Robert N. Bruce Jr.  
Ned H. Burns  
Chunsheng “Steve” Caid

Steven R. Close  
Henry J. Cronin Jr.  
Ward N. Marianos Jr.

Hani Melhem  
Antoine E. Naaman  
Thomas E. Nehil

Andrea J. Schokker

*This guide is intended for estimation of prestress losses in concrete structures. Methods presented include lump sum, simplified approaches addressing individual source of loss, and additional estimation methods. They address losses in pretensioned and post-tensioned members, including bonded, unbonded, and external tendons. Note that these estimation methods have not been evaluated for relative merits. A discussion of the variability of prestress losses caused by the variability in concrete properties is also presented. Several example problems are included.*

**Keywords:** creep; friction; post-tensioning; prestress loss; prestressed concrete; relaxation; shrinkage.

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**CHAPTER 1—INTRODUCTION****1.1—Introduction**

Estimating prestress loss at any given time during the life of a prestressed concrete member is a complex issue. In pretensioned and post-tensioned members, applying prestressing force causes shortening of the concrete member that, in turn, causes a loss of tendon stress. Over time, concrete creep,

concrete shrinkage, and steel relaxation result in additional reductions of tendon stress. In post-tensioned members, losses occur during the stressing operation due to friction between the tendon and sheathing or duct, which is caused by the intended and unintended tendon curvature. There are also losses due to seating of the wedges or nuts as the jacking force is transferred into the anchorage device. These and other sources of prestress loss are examined by the licensed design professional to get an estimate of the total prestress loss and resulting effective prestressing force.

Losses have inherent variability due to variations of material properties and environmental and curing conditions. Some losses may affect others. Time-dependent concrete properties are particularly difficult to estimate accurately, so losses due to creep and shrinkage are expected to be variable. Friction between the tendon and sheathing or duct, movement of wedges within the anchorage device, and modulus of elasticity of concrete are also variables. The variability within each component and the interdependence among the components make it understandable that studies comparing measured prestress losses to predictions have shown that accurate and consistent calculation of prestress loss is difficult to achieve.

The best effort to calculate prestress loss is only an estimate and, therefore, the licensed design professional should consider the consequences of actual losses being higher or lower than the estimated value. Estimation of prestress loss is an important factor for evaluating the serviceability of all types of prestressed members and the calculation of flexural strength of members with unbonded tendons. The estimation of prestress loss, however, is not a significant factor in determination of flexural strength of bonded prestressed members. When computing the shear strength of prestressed members with little or no transverse reinforcement, a conservative estimate of the effective prestressing force is warranted.

**1.2—Scope**

ACI 318-11 requires that the design of prestressed concrete members allow for prestress loss; however, the required level of detail for calculating losses is unspecified. The friction loss provisions for post-tensioned construction that first appeared in ACI 318-63 were removed from ACI 318-11. Although ACI 318-11 Commentary indicates that the lump sum method is obsolete, the licensed design professional's requirement to choose a method to compute losses remains. This guide is intended to aid the designer in this choice by providing an overview of the various methods available.

Many participants in the design and construction process need information on prestress losses. The licensed design professional, precasters, and post-tensioners all need an understanding of, and method to estimate, aspects of losses. To which entity is responsible for calculation of each type of loss has to be clearly defined in the contract documents.

Total losses,  $\Delta f_{pT}$ , are losses due to friction and seating  $\Delta f_{pFS}$ , elastic shortening  $\Delta f_{pES}$ , creep of concrete  $\Delta f_{pCR}$ , shrinkage of concrete  $\Delta f_{pSH}$ , and relaxation of tendons  $\Delta f_{pRE}$ . This can be expressed as Eq. (1.2)

$$\Delta f_{pT} = \Delta f_{pFS} + \Delta f_{pES} + \Delta f_{pCR} + \Delta f_{pSH} + \Delta f_{pRE} \quad (1.2)$$

This guide presents background information and methods to calculate each type of loss.

Following the introduction and a list of notation and definitions, [Chapter 3](#) includes a historical account of the lump sum method, currently recommended values for preliminary design, and a summary of losses that have been measured in field and laboratory studies.

[Chapter 4](#) discusses the different types of initial losses and addresses the differences between pretensioned and post-tensioned members.

[Chapter 5](#) presents a simplified approach to estimate long-term losses due to creep, shrinkage, and relaxation for pretensioned and post-tensioned concrete members.

Detailed approaches to estimate long-term losses are presented in [Chapter 6](#), which also addresses changes in prestressing force caused by differential shrinkage and hydration of the concrete deck in composite members. The approaches can be used for pretensioned or post-tensioned members.

[Chapter 7](#) discusses the variability of prestress loss calculations caused by concrete material properties, including compressive strength at transfer, modulus of elasticity, and creep and shrinkage.

[Chapter 8](#) presents example problems and compares solutions from different methods.

### 1.3—Historical development

The concept of prestressing concrete dates back to the late 1800s ([Naaman 2012](#)). The performance of early prestressed concrete structures was adversely affected by time-dependent strains in the concrete—for example, creep and shrinkage, which were nearly as large as the initial steel strain due to prestressing. Before 1940, the initial steel strain induced by prestressing was limited by the low yield strength of steel. French engineer Eugene Freyssinet recognized the significance of prestress losses and the need for steels with high yield strength for prestressed applications. By 1945, higher strength steel became available, making it possible to produce the initial prestressing strain large enough so that the time-dependent strains developed in the concrete would not overcome the initial prestressing strain. As a result, the remaining prestressing force in the steel would be sufficiently large to be effective.

Prestress losses were first addressed by ACI 318 in 1963. Although the provisions catalogued the different causes of prestress loss, they only provided specific instruction on determining friction losses. These code provisions were based on an earlier committee publication that provided similar, slightly more detailed guidance on prestress loss ([ACI-ASCE Committee 323 1958](#)).

In the 1970s, the Precast/Prestressed Concrete Institute ([PCI Committee on Prestress Losses 1975](#)) and [Zia et al. \(1979\)](#) provided more detailed methods to estimate prestress losses. Since the 1970s, others have developed methods to estimate prestress losses ([Tadros et al. 2003](#); [Seguirant and Anderson 1985](#); [Youakim et al. 2007](#); [Garber et al. 2013](#)).

[Gilbert and Ranzi \(2011\)](#) and [Branson \(1977\)](#) provide general approaches to the calculation of a variety of time-dependent effects in concrete structures, including prestress losses. Computer programs have been developed to perform the tedious calculations required for stepwise analyses of prestress loss. However, due to the inherent uncertainties associated with material properties, construction practices, and in-service conditions, even the most refined calculations result in prestress loss predictions that differ from measured values.

**1.3.1 Currently available guidance on estimating prestress losses**—For pretensioned building products, the *PCI Design Handbook* ([PCI 2010](#)) presents a method to estimate prestress losses based on the method developed by Zia et al. (1979). This method is widely used for building structures and is referenced in the R18.6.1 commentary of [ACI 318-11](#), and presented in this guide in Chapter 5. For bridge beams, the “AASHTO LRFD Bridge Design Specification” ([AASHTO 2012](#)) presents two methods. One is an approximate method and the other a refined method based on several parameters to estimate prestress losses. The refined method could be applied to building products as well. These methods are presented in Chapters 5 and 6.

### 1.4—Guide organization and use

This guide presents a variety of approaches for estimating prestress losses in pretensioned and post-tensioned members. This section identifies relevant sections of interest in the guide, depending on member type (pretensioned or post-tensioned) and level of effort (lump sum, simplified, or detailed). The lump sum method is only recommended for preliminary designs. The simplified method is appropriate for most typical designs. Detailed methods are most often used for more complex structures, which may have staged construction and prestressing operations.

**1.4.1 Pretensioned members**—Losses for pretensioned members are classified as initial or long-term. One group of initial loss occurs during stressing and before transfer of prestress due to friction, seating losses, and temperature effects. It is the precaster’s responsibility to understand the magnitude of these losses and account for them to provide the specified strand stress before transfer. Information on these types of losses is found in:

- (a) Anchorage seating—[4.2.1](#)
- (b) Form and abutment deformations—[4.2.2](#)
- (c) Thermal effects—[4.2.4](#)
- (d) Steel relaxation—[4.2.5](#)

Another initial loss is elastic shortening of the member that occurs at the time of transfer. As the prestress force is transferred to concrete, the member shortens. The steel and concrete are fully bonded, so the steel shortens with the concrete. This shortening causes a loss in stress in the prestressing steel, known as the elastic shortening loss, which should be accounted for by the designer. Long-term losses occur due to concrete creep and shrinkage and prestressing steel relaxation. Other changes of tendon force can occur due to temperature effects and external loads placed on the member at the time of casting or in service.

**1.4.1.1 Pretensioned members/lump sum method**—The lump sum method presented in **Chapter 3** is often used for preliminary design. The values presented in sources referenced in Chapter 3 typically include all losses, both initial and long-term.

**1.4.1.2 Pretensioned members/simplified method**—The simplified method is a commonly employed approach to estimate prestress losses in typical pretensioned members. The designer needs to calculate four components of loss and add them together for the total prestress loss. Components and applicable sections are:

- (a) Elastic shortening—**4.3.2**
- (b) Creep—**5.2**
- (c) Shrinkage—**5.3**
- (d) Relaxation—**5.4**

**1.4.1.3 Pretensioned members/detailed method**—This guide provides information on more detailed methods of prestress loss estimation. Two alternate methods for a more detailed calculation of elastic shortening losses in pretensioned members are:

- (a) Transformed section method—**4.3.1**
- (b) Iterative gross section method with iteration—**4.3.3**

More detailed approaches to calculate long-term losses are presented in **Chapter 6**. These methods are used with a variety of creep and shrinkage models, as opposed to the simplified method, which uses a single model. Detailed methods also allow the designer to consider the influence of a cast-in-place composite deck if needed, whereas the simplified method only accounts for the weight of the deck, but not other factors such as differential shrinkage and internal stress redistributions between the beam and the deck, if acting compositely. The detailed methods are:

- (a) AASHTO LRFD refined method (AASHTO 2012)—**6.3.2**
- (b) General age-adjusted effective modulus method (Menn 1990)—**6.3.3**
- (c) Incremental time-step method (Nilson 1987)—**6.4**

Chapter 6 (**6.6**) also provides information on the approximation of changes due to thermal effects of deck casting.

**1.4.2 Post-tensioned members**—Several approaches can be used to approximate prestress losses in post-tensioned members. Initial losses encompass all prestress loss during the stressing operation, including friction due to wobble and curvature, seating losses, and elastic shortening losses. The estimation of long-term losses for bonded post-tensioned members is essentially the same as for pretensioned members. Calculation of long-term losses in unbonded post-tensioned members is different, because losses are related to the overall change in tendon length, rather than the change in strain at a specific section.

**1.4.2.1 Post-tensioned members/lump sum method**—The lump sum method, presented in **Chapter 3**, is typically used only for preliminary designs. Before adopting a value for use in preliminary design, the licensed design professional should determine if the presented value includes friction and seating losses.

**1.4.2.2 Post-tensioned members/simplified method**—The simplified method can be used to estimate prestress losses in

typical post-tensioned members. The designer needs to calculate five components of loss and add them together for the total prestress loss. Components and applicable sections are:

- (a) Friction and seating loss—**4.4**
- (b) Elastic shortening loss—**4.5**
- (c) Creep loss—**5.2.1** (bonded)
- (d) Creep loss—**5.2.2** (unbonded)
- (e) Shrinkage loss—**5.3**
- (f) Relaxation loss—**5.4**

Note that elastic shortening losses only occur in post-tensioned members with multiple tendons when the tendons are stressed sequentially. Tendons stressed first will incur losses as the concrete shortens due to the stressing of subsequent tendons.

**1.4.2.3 Post-tensioned members/detailed methods**—As with pretensioned members, long-term prestress loss in post-tensioned members are estimated using more detailed methods presented in Chapter 6, with a detailed description in 1.4.1.3. Initial losses are calculated per 4.3.3 and 4.4.

## CHAPTER 2—NOTATION AND DEFINITIONS

### 2.1—Notation

$a$	= constant
$ab$	= eccentricity from the centroid of the beam (gross section) to the centroid of the deck (also centroid of the deck reinforcing steel), in. (mm)
$A_c$	= area of concrete, in. <sup>2</sup> (mm <sup>2</sup> )
$A_{comp}$	= transformed area of the composite section, in. <sup>2</sup> (mm <sup>2</sup> )
$A_d$	= area of composite concrete deck, in. <sup>2</sup> (mm <sup>2</sup> )
$A_g$	= area of gross concrete section at the cross section considered, in. <sup>2</sup> (mm <sup>2</sup> )
$A_{ps}$	= area of prestressing steel, in. <sup>2</sup> (mm <sup>2</sup> )
$A_{sd}$	= area of deck steel, in. <sup>2</sup> (mm <sup>2</sup> )
$A_{tr}$	= transformed cross-sectional area, in. <sup>2</sup> (mm <sup>2</sup> )
$b(y)$	= width of cross-section at depth $y$ relative to centroid of section, in. (mm)
$C$	= factor in calculation of prestress loss due to relaxation according to the <i>PCI Design Handbook</i> (PCI 2010) method
$C_c$	= creep coefficient
$d$	= friction loss over length $L$ , psi (MPa)
$e$	= base of Napierian logarithms
$e_b$	= basic elongation, in. (mm)
$e_c$	= eccentricity of centroid of tendons with respect to the centroid of the gross cross section at the center of the beam, in. (mm)
$e_d$	= distance between centroid of deck and centroid of composite beam, in. (mm)
$e_e$	= eccentricity of centroid of tendons with respect to the centroid of the gross cross section at the ends of the beam, in. (mm)
$e_p$	= eccentricity of centroid of tendons with respect to the centroid of the gross concrete at the cross section considered, in. (mm)
$e_{pc}$	= eccentricity of centroid of tendons with respect to the centroid of the composite cross section, in. (mm)

$e_{tr}$	= eccentricity of centroid of tendons with respect to the centroid of the $f_{bpt}$ transformed concrete at the cross section considered, in. (mm)	$I_c$	= moment of inertia of the composite cross section, in. <sup>4</sup> (mm <sup>4</sup> )
$E(t)$	= modulus of elasticity at any time $t$ , psi (MPa)	$I_d$	= moment of inertia of the deck, in. <sup>4</sup> (mm <sup>4</sup> )
$E_c$	= modulus of elasticity of concrete, psi (MPa)	$I_g$	= moment of inertia of gross concrete section about centroidal axis, neglecting reinforcement, in. <sup>4</sup> (mm <sup>4</sup> )
$E_c'$	= effective modulus of elasticity of concrete, psi (MPa)	$I_{tr}$	= moment of inertia of transformed concrete section about centroidal axis, including reinforcement, in. <sup>4</sup> (mm <sup>4</sup> )
$E_c''$	= age-adjusted effective modulus of elasticity of concrete, psi (MPa)	$J$	= factor in calculation of prestress loss due to relaxation according to the <i>PCI Design Handbook</i> (PCI 2010) method
$E_{cd}$	= modulus of elasticity of the composite deck, psi (MPa)	$k$	= wobble friction coefficient per unit length of tendon, per ft (per m)
$E_{ci}$	= modulus of elasticity of concrete at time of application of prestress, psi (MPa)	$k_f$	= factor for the effect of concrete strength
$E_c(t_i)$	= modulus of elasticity of concrete at time $t_i$ , psi (MPa)	$k_{hc}$	= humidity factor for creep
$E_p$	= modulus of elasticity of the prestressing steel, psi (MPa)	$k_s$	= factor for the effect of volume-to-surface ratio
$f_{anchor}$	= strand stress at the anchorage device after seating, psi (MPa)	$k_{td}$	= time development factor
$f_c$	= concrete compressive stress, psi (MPa)	$K_{cir}$	= modification factor in <i>PCI Design Handbook</i> (PCI 2010) method in calculation of concrete stress due to prestressing force immediately after the prestress has been applied to the concrete
$f_c'$	= specified compressive strength of concrete, psi (MPa)	$K_{cr}$	= coefficient in the <i>PCI Design Handbook</i> (PCI 2010) method to account for loss due to creep
$f_{cd}$	= concrete stress at center of gravity of prestressing force due to all superimposed permanent loads that are applied to the member after it has been prestressed, psi (MPa)	$K_{df}$	= transformed section coefficient
$f_{ci}'$	= specified compressive strength of concrete at transfer of prestress, psi (MPa)	$K_{es}$	= factor in calculation of elastic shortening losses in <i>Zia et al. (1979)</i> method
$f_{ci}$	= concrete compressive stress immediately after transfer at fiber under investigation, psi (MPa)	$K_{id}$	= section modification factor from <i>AASHTO (2012)</i> prestress loss method
$f_{cir}$	= net compressive concrete stress at center of gravity of prestressing force immediately after the prestress has been applied to the concrete, psi (MPa)	$K_{re}$	= factor in calculation of prestress loss due to relaxation in <i>PCI Design Handbook</i> (PCI 2010) method
$f_{cpa}$	= average compressive concrete stress at the center of gravity of the tendons immediately after the prestress has been applied to the concrete, psi (MPa)	$K_{sh}$	= factor in calculation of prestress losses due to shrinkage in <i>PCI Design Handbook</i> (PCI 2010) method
$f_{cps}$	= concrete stress at center of gravity of prestressing force due to all prestress and applied loads, psi (MPa)	$L$	= strand length from anchorage to anchorage, ft or in. (m or mm)
$f_{dead}$	= tendon stress at nonstressing end, psi (MPa)	$L_{beam}$	= length of beam, ft or in. (m or mm)
$f_{jack}$	= jacking stress, psi (MPa)	$L_{free}$	= length of strand outside of beam, ft or in. (m or mm)
$f_L$	= stress in prestressing steel at a distance $L$ from jacking end, psi (MPa)	$M$	= bending moment experienced by cross section immediately after transfer (usually due to self-weight), in.-lb (N-mm)
$f_{L/2}$	= stress in prestressing steel at a distance $L/2$ from the jacking end, psi (MPa)	$M_b^o$	= initial creep-producing moment in the girder, in.-lb (N-mm)
$f_{max}$	= maximum stress in the prestressing steel along the tendon length, psi (MPa)	$M_{deck}$	= moment in beam due to the weight of the deck, in.-lb (N-mm)
$f_{pbt}$	= stress in prestressing steel immediately before transfer, psi (MPa)	$M_g$	= bending moment due to dead weight of prestressed member and any other permanent loads in place at the time of prestressing, in.-lb (N-mm)
$f_{pi}$	= prestressing steel stress immediately following transfer, psi (MPa)	$M_{sd}$	= moment due to all superimposed permanent loads applied after prestressing, in.-lb (N-mm)
$f_{po}$	= prestressing steel stress after jacking and seating, psi (MPa)	$n_p$	= modular ratio; modulus of prestressing steel divided by modulus of concrete
$f_{ps}(t)$	= stress in prestressing steel at time $t$ , psi (MPa)	$N$	= number of sequentially stressed tendons
$f_{pt}$	= stress in prestressing steel immediately after transfer, psi (MPa)	$N_b^o$	= initial creep-producing force in the girder, lb (N)
$f_{pu}$	= specified tensile strength of prestressing steel, psi (MPa)	$P$	= applied tension force, lb (N)
$f_{py}$	= specified yield strength of prestressing steel, psi (MPa)	$P_{avg}$	= average force in the tendon, lb (N)
$f_x$	= stress in prestressing steel at a distance $x$ from the jacking end, psi (MPa)	$P_i$	= initial prestress force after anchorage seating loss, lb (N)

$P_j$	= maximum prestress force during jacking operation, lb (N)	$\Delta f_{pCR1}$	= prestress loss due to creep of girder concrete between transfer and deck placement, psi (MPa); (AASHTO 2012)
$P_o$	= prestress force before release, lb (N)	$\Delta f_{pEG}$	= increase in prestress (elastic gain) due to addition of superimposed permanent loads, psi (MPa)
$P_{shd}$	= force to fully restrain the shrinkage of the composite deck, lb (N)	$\Delta f_{pES}$	= prestress loss due to elastic shortening, psi (MPa)
$RH$	= average ambient relative humidity in percent	$\Delta f_{pFS}$	= prestress loss due to friction and seating, psi (MPa)
$s$	= slope of stress in prestress versus distance line, psi/ft (MPa/m)	$\Delta f_{pLT}$	= long-term prestress loss, psi (MPa)
$S_b$	= section modulus with respect to the bottom fiber of the beam, in. <sup>3</sup> (mm <sup>3</sup> )	$\Delta f_{pR1}$	= prestress loss due to relaxation of prestressing strands between time of transfer and deck placement, psi (MPa)
$t$	= time under consideration from time of release, days	$\Delta f_{pR2}$	= prestress loss due to relaxation of prestressing strands in composite section between time of deck placement and final time, psi (MPa)
$t_c$	= age of concrete, days	$\Delta f_{pRE}$	= prestress loss due to relaxation, psi (MPa)
$t_d$	= time since end of cure to time of deck placement, days	$\Delta f_{pSD}$	= prestress loss due to shrinkage of girder concrete between time of deck placement and final time, psi (MPa)
$t_f$	= final time under consideration from time of release, days	$\Delta f_{pSH}$	= prestress loss due to shrinkage, psi (MPa)
$t_i$	= 1-day steam cured	$\Delta f_{pSR}$	= prestress loss due to shrinkage of girder concrete between transfer and deck placement, psi (MPa)
$t_o$	= time of initial loading from time of release, days	$\Delta f_{pSS}$	= prestress gain due to shrinkage of deck in composite section, psi (MPa)
$T_0$	= prestressing force at stressing end, lb (N)	$\Delta f_{pT}$	= total prestress loss, psi (MPa)
$T_x$	= prestressing force at point $x$ , lb (N)	$\Delta L_{jack}$	= elongation within the jack, in. (mm)
$T(y)$	= temperature of cross section at distance $y$ from centroid, °F (°C)	$\Delta M_b$	= change in moment in the beam, in.-lb (N-mm)
$V/S$	= ratio of volume to surface area of concrete element, in. <sup>3</sup> /in. <sup>2</sup> (mm <sup>3</sup> /mm <sup>2</sup> )	$\Delta M_d$	= change in moment in the deck, in.-lb (N-mm)
$w_c$	= unit weight of normalweight concrete or equilibrium density of lightweight concrete, lb/ft <sup>3</sup> (kg/m <sup>3</sup> )	$\Delta N_b$	= change in force in the beam, lb (N)
$w_t$	= weight of topping, lb/ft <sup>2</sup> (N/m <sup>2</sup> )	$\Delta N_d$	= change in force in the deck, lb (N)
$x$	= length of tendon from stressing end to point $x$ , ft or in. (m or mm)	$\Delta N_{ps}$	= change in prestress force, lb (N)
$x_s$	= length influenced by anchor set, ft or in. (m or mm)	$\Delta N_{relax}$	= change in prestress force due to relaxation (no associated strain), lb (N)
$y$	= distance from centroid of cross section to location under consideration, in. (mm)	$\Delta N_{sd}$	= change in the force in the deck reinforcement, lb (N)
$y_{bott}$	= the distance from the centroid of the composite section to the bottom of the section, in. (mm)	$\Delta P$	= change in force, lb (N)
$y_{tr}$	= distance from centroid of transformed section to concrete fiber under investigation, in. (mm)	$\Delta P_c$	= change in force in concrete, lb (N)
$\alpha$	= total angular change from jacking end to point $x$ , radians	$\Delta P_{ps}$	= change in force in prestressing steel, lb (N)
$\alpha_c$	= coefficient of thermal expansion of concrete, /°F (°C)	$\Delta s$	= anchor set, in. (mm)
$\alpha_{ps}$	= coefficient of thermal expansion of prestress, /°F (°C)	$\Delta T_1$	= temperature rise, °F (°C)
$\beta$	= constant such that $\beta = (28 - \alpha)/28$	$\Delta T_2$	= temperature change, °F (°C)
$\gamma_h$	= correction factor for ambient relative humidity	$\Delta \epsilon_b$	= change in strain in the beam at the gross section centroid
$\gamma_{st}$	= correction factor for specified concrete compressive strength at transfer	$\Delta \epsilon_c$	= change in strain in concrete at the center of gravity of the prestressing force
$\Delta$	= tendon elongation, in. (mm)	$\Delta \epsilon_{cr}$	= change in strain in concrete due to creep
$\Delta f_{cdf}$	= change in concrete stress at the center of gravity of the prestressing force due to the differential shrinkage force, psi (MPa)	$\Delta \epsilon_d$	= change in strain at the centroid of the deck
$\Delta f_{cgp}$	= change in concrete stress at the center of gravity of the prestressing force due to application of superimposed load, psi (MPa)	$\Delta \epsilon_p$	= change in strain in prestressing steel
$\Delta f_{pA}$	= change in stress due to anchor set, psi (MPa)	$\Delta \epsilon_{pA}$	= change in strain in prestressed reinforcement due to anchor set
$\Delta f_{pCD}$	= prestress loss due to creep of girder concrete between time of deck placement and final time, psi (MPa)	$\Delta \epsilon_{free}$	= change in strain in free length of prestressing steel
$\Delta f_{pCR}$	= prestress loss due to creep, psi (MPa)	$\Delta \epsilon_{shb}$	= change in strain in the beam due to shrinkage
		$\Delta \epsilon_{shb}(t_d)$	= shrinkage strain in the beam concrete at the time the deck is placed
		$\Delta \epsilon_{shb}(t_f)$	= shrinkage strain in the beam concrete at the final time considered
		$\Delta \epsilon_{sd}$	= change in strain in the deck steel
		$\Delta \kappa$	= change in curvature, /in. (/mm)
		$\Delta \sigma(t_i)$	= change in stress at time $t_i$ , psi (MPa)



- $\varepsilon(t)$  = concrete strain at time  $t$   
 $\varepsilon(t_o)$  = initial concrete strain  
 $\varepsilon_{shb}(t)$  = shrinkage strain in the prestressed beam at time  $t$  following the end of cure  
 $\varepsilon_{shb}(t_d)$  = shrinkage strain of the girder at the time the deck is placed  
 $\varepsilon_{shb}(t_f)$  = total shrinkage strain of the beam at the time of analysis  
 $\varepsilon_{shd}(t)$  = shrinkage strain in the composite deck at time  $t$  following the end of cure  
 $\varepsilon_{shd}(t_f)$  = shrinkage of the deck concrete at final time under consideration  
 $\mu$  = curvature friction coefficient  
 $\sigma(t_o)$  = stress at time  $t_o$ , psi (MPa)  
 $\phi(t, t_i)$  = creep coefficient at time  $t$  for loads applied at time  $t_i$   
 $\phi(t, t_o)$  = creep coefficient at time  $t$  for loads applied at time  $t_o$   
 $\phi(t_f, t_d)$  = creep coefficient at time  $t_f$  for loads applied at time  $t_d$   
 $\phi(t_f, t_o)$  = creep coefficient at time  $t_f$  for loads applied at time  $t_o$   
 $\phi_d(t_f, t_d)$  = creep coefficient of the deck concrete at time  $t_f$  for loads applied at time  $t_d$   
 $\chi$  = aging coefficient

## 2.2—Definitions

ACI provides a comprehensive list of definitions through an online resource, “ACI Concrete Terminology,” <https://www.concrete.org/store/productdetail.aspx?ItemID=CT16>. Definitions provided herein complement this source.

**anchorage**—in post-tensioning, a device used to anchor the tendon to the concrete member; in pretensioning, a device used to maintain the elongation of a tendon during the time interval between stressing and release.

**modulus of elasticity**—the ratio of normal stress to corresponding strain for tensile or compressive stress below the proportional limit of the material.

**post-tensioning**—method of prestressing reinforced concrete in which tendons are tensioned after concrete has attained a specified minimum strength or specified minimum age.

## CHAPTER 3—LUMP-SUM METHOD

### 3.1—Scope

In some cases, use of a simple lump-sum value for prestress loss may be adequate instead of detailed calculations. This chapter summarizes lump-sum values from past codes and literature, and discusses their use in design. In addition, data from the measurement of prestress losses are presented and summarized.

### 3.2—Historical code requirements

The Federal Highway Administration (FHWA), known as the **Bureau of Public Roads (1954)** at the time, published design criteria for prestressed concrete bridges in which losses due to creep, relaxation, shrinkage, and elastic deformation were specified as

$$\begin{aligned} \text{pretensioned concrete} &= 6000 + 16f_{cps} + 0.04f_{pi} \text{ (psi)} \\ \text{pretensioned concrete} &= 41.4 + 16f_{cps} + 0.04f_{pi} \text{ (MPa)} \end{aligned} \quad (3.2a)$$

$$\begin{aligned} \text{post-tensioned concrete} &= 3000 + 11f_{cps} + 0.04f_{pi} \text{ (psi)} \\ \text{post-tensioned concrete} &= 20.7 + 11f_{cps} + 0.04f_{pi} \text{ (MPa)} \end{aligned} \quad (3.2b)$$

where  $f_{cps}$  is the concrete stress at the center of gravity of the prestressing force due to all prestress and applied loads, psi (MPa); and  $f_{pi}$  is prestressing steel stress immediately following transfer, psi (MPa).

In formulating the criteria, it was assumed that the efficiency of the anchorage was 100 percent with no seating loss and applicable to normalweight concrete only. The amount of losses was to be determined by tests for lightweight concrete. In Eq. (3.2a) and (3.2b), the first term represents loss due to shrinkage, the second represents combined losses due to elastic shortening and creep, and the third represents loss due to steel relaxation. For post-tensioned concrete, elastic shortening was ignored. In the early 1950s, only lower strengths of concrete and steel were available for prestressed concrete applications. For typical designs, the value of  $f_{cps}$  was approximately 1300 psi (8.96 MPa) and the value of  $f_{pi}$  was a maximum of 200,000 psi (1380 MPa). Using these typical values for  $f_{cps}$  and  $f_{pi}$ , the criteria stated (Eq. (3.2a) and Eq. (3.2b)) for losses equates to 34,800 psi (240 MPa) for pretensioned concrete and 25,300 psi (174 MPa) for post-tensioned concrete.

Requirements for evaluating prestress loss first appeared in **ACI 318-63**, in new Chapter 29 on prestressed concrete. ACI 318-63 commentary recommended the following lump-sum losses, excluding friction and seating losses, based on the **ACI-ASCE Committee 323 (1958)** report: pretensioning = 35,000 psi (241 MPa); and post-tensioning = 25,000 psi (172 MPa).

In published discussions, **Abeles (1958)** argued that these recommended lump-sum values should be eliminated or increased to allow designers to assess the losses for specific conditions. If the lump-sum values were to remain, he suggested the following ranges of values: pretensioning = 30,000 to 40,000 psi (207 to 276 MPa); and post-tensioning = 20,000 to 30,000 psi (138 to 207 MPa).

Similarly, the Precast/Prestressed Concrete Institute (PCI) published building code requirements for prestressed concrete (**Structural Engineers Association of Northern California 1959**) specifying the following lump-sum values for estimating prestress losses, excluding friction prestressing steel loss: pretensioning = 25,000 to 35,000 psi (172 to 241 MPa); and post-tensioning = 15,000 to 25,000 psi (103 to 172 MPa).

In **ACI 318-83**, specific lump sum recommendations in the commentary were replaced by the following statement: “Lump-sum values of prestress losses for both pretensioned and post-tensioned members that were indicated before the 1983 Commentary are considered obsolete.” The commentary also provides several references that can be used to compute prestress loss (ACI-ASCE Committee 323 1958; **ACI 435R**; **PCI Committee on Prestress Losses 1975**; **Zia et al. 1979**). Although **ACI 318-11** does not explicitly prohibit use of lump-sum losses, the commentary is clear in warning that previously cited lump-sum losses are considered obsolete.